A covariant calculation of deuteron GPDs

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Relativistic structure

Non-relativistically...

- Point particles are delta functions in configuration space.
  Point particle described by plane wave in momentum space.
  (These are related by Fourier transform.)
- Charge distribution is a function $\rho(\mathbf{r})$ of $\mathbf{r}$.
  Momentum space description given by form factor $F(Q^2)$.
  (These are related by Fourier transform.)

Relativistic structure

Relativistically...

- Point particles are **not** delta functions in configuration space.
  Point particle described by plane wave in momentum space.
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- Charge distribution is **not** a function $\rho(\mathbf{r})$ of $\mathbf{r}$.
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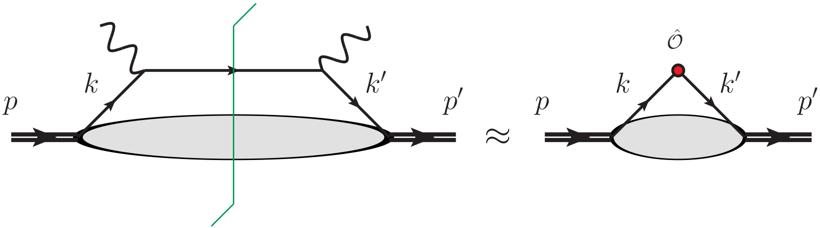
  (These are **not** related by Fourier transform.)

(*cf.* any comment by Jerry.)

- Relativity mucks up configuration space descriptions of many-body systems.
- We opt to take relativity seriously.
- Any description of 3D partonic structure should be a **Lorentz-invariant** function of **Lorentz-invariant** variables.
- **Generalized parton distributions** (GPDs) are exactly the kind of structure we need.

Generalized parton distributions

Look at **leading twist** GPDs. The **partonic correlator** is defined diagrammatically (leading twist approximation):



The operator $\hat{\mathcal{O}}$ depends on helicity-independent, axial, *etc.*:

$$\hat{\mathcal{O}}_V = \not{n}\delta(n \cdot [xP - K]) \qquad \hat{\mathcal{O}}_A = \not{n}\gamma_5\delta(n \cdot [xP - K])$$

The GPDs are Lorentz-invariant functions that fall out of these matrix elements.

GPDs of the nucleon

An example: leading-twist GPDs of the nucleon.

$$\langle p', \lambda' | \not{n} \delta(n \cdot [xP - K]) | p, \lambda \rangle = \bar{u}(p', \lambda') \left[\not{n} H_N(x, \xi, t) + \frac{i\sigma^{n\Delta}}{2m_N} E_N(x, \xi, t) \right] u(p, \lambda)$$

- Similar to form factor decomposition.
- H_N and E_N are the nucleon GPDs.
- Defined wrt a lightlike four-vector n.
- Depend on four Lorentz scalars, the last being a factorization scale μ_F (dependence not notated here).
- Lorentz-covariant 3D structure!

$$P = \frac{1}{2}(p + p') \qquad x = \frac{K \cdot n}{P \cdot n}$$

$$K = \frac{1}{2}(k + k') \qquad \xi = -\frac{\Delta \cdot n}{2P \cdot n}$$

$$\Delta = P' - P \qquad t = \Delta^2$$

Caveat: twist decomposition depends on choice of n.
cf. [V. Braun *et al.*, PRD89 (2014), 074022]

Choice of n should be informed by process.

Reductions of nucleon GPDs

Nucleon GPDs correspond to familiar quantities through limits or sum rules.

- Sum rules for Dirac and Pauli form factors:

$$\int_{-1}^{1} H_N^q(x,\xi,t)dx = F_{1N}^q(t) \qquad \int_{-1}^{1} E_N^q(x,\xi,t)dx = F_{2N}^q(t)$$

  Note that ξ dependence vanishes in integral. (Special case of **polynomiality**)

- Forward limit for PDFs:

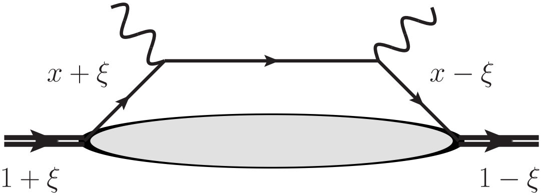
$$H_N^q(x,0,0) = q(x)$$

  We use $x \in [-1,1]$, with the meaning:

$$q(-x) = -\bar{q}(x)$$

The variables in the GPDs

Let's take a quick pictorial look at the GPD variables.



$$P = \frac{1}{2}(p + p') \qquad x = \frac{K \cdot n}{P \cdot n}$$

$$K = \frac{1}{2}(k + k') \qquad \xi = -\frac{\Delta \cdot n}{2P \cdot n}$$

$$\Delta = P' - P \qquad t = \Delta^2$$

- Light cone fractions defined wrt $(P \cdot n)$.
- x is average momentum fraction of parton at both vertices.
- $x > 0$ quarks/gluons; $x < 0$ antiquarks/gluons.
- The **skewness** (2ξ) is the fraction lost by the target.

Differing conventions

Personally, I get confused because there are **many differing notational conventions**. I'll explicitly define the conventions I'm using here:

$$P = \frac{1}{2}(p + p') \qquad K = \frac{1}{2}(k + k')$$

$$x = \frac{K \cdot n}{P \cdot n} \in [-1, 1] \qquad x_A = Ax \in [-A, A]$$

$$\xi = -\frac{\Delta \cdot n}{2P \cdot n} \in [-1, 1] \qquad \xi_A = A\xi \in [-A, A]$$

Note that if I write x rather than x_A, it is **not scaled by** A.

Yes, there's a good reason to use x (rather than x_A) even for nuclear systems: higher Mellin moments in x_A pick up extra powers of A that make things messy and confusing!

I'll use x_A at the end, but it will be explicitly notated.

Covariance and Polynomiality

Polynomiality rules for the nucleon

Nucleon GPDs are known to obey **polynomiality sum rules** [X. Ji, J.Phys. G24 (1998) 1181]:

$$\int_{-1}^{1} x^s H_N(x,\xi,t)dx = \sum_{\substack{l=0 \\ 2|l}}^{s} A_{s+1,l}(t)(2\xi)^l + \mathrm{mod}(s,2)C_N(t)(2\xi)^{s+1}$$

$$\int_{-1}^{1} x^s E_N(x,\xi,t)dx = \sum_{\substack{l=0 \\ 2|l}}^{s} B_{s+1,l}(t)(2\xi)^l - \mathrm{mod}(s,2)C_N(t)(2\xi)^{s+1}$$

- A, B, and C are called **generalized form factors**.
- These rules are a **result of Lorentz covariance**.
- They are **violated for models that break covariance** (*e.g.*, models with Fock space truncations or which use non-relativistic nuclear wave functions).

Spin-1 systems will have polynomiality rules too (due to Lorentz symmetry).

Polynomiality sum rules for the deuteron

I have derived the following sum rules for spin-1 systems (with $x \in [-1, 1]$ convention):

$$\int_{-1}^{1} x^s H_1(x,\xi,t)dx = \sum_{\substack{l=0 \\ 2|l}}^{s} \mathcal{A}_{s+1,l}(t)(2\xi)^l + \mathrm{mod}(s,2)\mathcal{F}_{s+1}(t)(2\xi)^{s+1}$$

$$\int_{-1}^{1} x^s H_2(x,\xi,t)dx = \sum_{\substack{l=0 \\ 2|l}}^{s} \mathcal{B}_{s+1,l}(t)(2\xi)^l$$

$$\int_{-1}^{1} x^s H_3(x,\xi,t)dx = \sum_{\substack{l=0 \\ 2|l}}^{s} \mathcal{C}_{s+1,l}(t)(2\xi)^l + \mathrm{mod}(s,2)\mathcal{G}_{s+1}(t)(2\xi)^{s+1}$$

$$\int_{-1}^{1} x^s H_4(x,\xi,t)dx = \sum_{\substack{l=1 \\ 2\nmid l}}^{s} \mathcal{D}_{s+1,l}(t)(2\xi)^l$$

$$\int_{-1}^{1} x^s H_5(x,\xi,t)dx = \sum_{\substack{l=0 \\ 2|l}}^{s-1} \mathcal{E}_{s+1,l+1}(t)(2\xi)^l.$$

Only H_1 and H_3 (related to **electric charge distribution**, but not magnetic) have the $(2\xi)^{s+1}$ term.

Special cases of generalized form factors

The first Mellin moments ($s = 0$) give **electromagnetic form factors**:

$$\int_{-1}^{1} H_1(x,\xi,t)dx = F_1(t) \qquad \int_{-1}^{1} H_2(x,\xi,t)dx = F_2(t)$$

$$\int_{-1}^{1} H_3(x,\xi,t)dx = F_3(t) \qquad \int_{-1}^{1} H_4(x,\xi,t)dx = \int_{-1}^{1} H_5(x,\xi,t)dx = 0.$$

Forward limits relate the GPDs to **parton distribution functions**:

$$H_1(x,0,0) = f(x) \qquad H_5(x,0,0) = b_1(x)$$

(No forward limits defined for the other GPDs.)
Combining these entails valence version of Kumano-Close sum rule:

$$\int_{-1}^{1} b_1(x)dx = 0$$

Violation of the usual Kumano-Close sum rule is possible, and would indicate tensor polarization in the sea.

Special cases of generalized form factors

The second Mellin moments ($s = 1$) give **gravitational form factors**:

$$\int_{-1}^{1} xH_1(x,\xi,t)dx = \mathcal{G}_1(t) + (2\xi)^2\mathcal{G}_3(t) \qquad \int_{-1}^{1} xH_2(x,\xi,t)dx = \mathcal{G}_5(t)$$

$$\int_{-1}^{1} xH_3(x,\xi,t)dx = \mathcal{G}_2(t) + (2\xi)^2\mathcal{G}_4(t) \qquad \int_{-1}^{1} xH_4(x,\xi,t)dx = (2\xi)\mathcal{G}_6(t)$$

$$\int_{-1}^{1} xH_5(x,\xi,t)dx = \mathcal{G}_7(t) \qquad \sum_{\text{partons}} \mathcal{G}_7(t) = -\frac{t}{2M_D^2}\sum_{\text{partons}} \mathcal{G}_6(t)$$

(Notation from Simonetta's papers, modulo small differences like factors of 2.)
The last relation is a consequence of energy-momentum conservation.
It also entails:

$$\sum_{\text{partons}} \int_{-1}^{1} xb_1(x)dx = 0$$

Violation of this sum rule over quarks alone would indicate momentum-sharing with gluons.

GFFs and the matter distribution

One can construct a **monopole gravitational form factor** for a spin-one system:

$$\mathcal{G}_N(t) = \left(1 + \frac{2}{3}\tau\right)\mathcal{G}_1(t) - \frac{2}{3}\tau\mathcal{G}_5(t) + \frac{2}{3}\tau(1+\tau)\mathcal{G}_2(t)$$

where $\tau = -t/(4M_D^2)$.
(Analogous to the Coulomb electric form factor. N is for Newton.)
Related to a **gravitational radius**:

$$\langle r_G^2 \rangle = 6\frac{d}{dt}\left[\mathcal{G}_N(t)\right]$$

This should look very familiar to the charge radius! But:

$$r_G \neq r_E,$$

The model I'll present soon gives:

$$r_E = 2.09 \text{ fm} \qquad\qquad r_G = 1.95 \text{ fm}$$

GFFs and much more

There's even more information hidden in GFFs.

- That's either **generalized** or **gravitational** form factor.

Taneja *et al.* (Phys.Rev. **D86** (2012) 036008) tell us that

$$J(t) = \frac{1}{2}\mathcal{G}_5(t)$$

We can learn how quarks and gluons share and distribute their angular momentum!

Also, the **purely spacelike** components of the stress-energy tensor give **shears and pressures**:

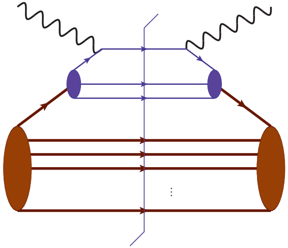
- $\mathcal{G}_3(t)$, $\mathcal{G}_4(t)$, and $\mathcal{G}_6(t)$ all contribute to these.
- In particular, both $\mathcal{G}_3(t)$ and $\mathcal{G}_6(t)$ contribute to Polyakov's and Schweitzer's coveted static quantity D.
- This is an ongoing topic of research with Simonetta, Whit, and Ian.

Convolution Formalism

GPD convolution

- A real nucleus has modifications (**EMC effect**—the whole point of this workshop!).
- But the "EMC effect" exists *as a discrepancy with a baseline prediction*.
- So what do nuclear GPDs look like if we assume unmodified nucleons?

$$\langle p', \lambda' | \mathcal{O}_q | p, \lambda \rangle = \sum_{\text{nucleons}}$$



Unmodified convolution formalism

To have an EMC effect for the GPDs, we need an **unmodified convolution formalism**.
This is ostensibly straightforward:

- Get a model for the nucleon GPDs H_N and E_N.
- Compute the matrix element

$$\langle p', \lambda' | \left[\not{n} H_N + \frac{i\sigma^{n\Delta}}{2m_N} E_N \right] | p, \lambda \rangle$$

  assuming pointlike, on-shell nucleons.
  (The factors H_N and E_N fold in the non-pointlike structure.)
- An **ambiguity** arises: identities like Gordon decomposition that are true for on-shell nucleons **will lead to different results for kinematically off-shell nucleons**.
- This turns out to matter for the nucleon D-terms.

The D-term and Gordon decomposition

In models such as [Goeke *et al.*, Prog. Part. Nucl. Phys. 47 (2001)], the nucleon GPD is broken into a **double distribution** and a **D-term**:

$$H_N(x,\xi,t) = H_{DD}(x,\xi,t) + D\left(\frac{x}{\xi},t\right) \qquad E_N(x,\xi,t) = E_{DD}(x,\xi,t) - D\left(\frac{x}{\xi},t\right)$$

- The D-term here contributes to the $(2\xi)^{s+1}$ GFF in the polynomiality sum rules.
- The same D-term enters both H_N and E_N with opposite sign.
- **This is due to Lorentz invariance.** [X. Ji, J.Phys. G24 (1998) 1181]

Using **Gordon decomposition**, we can write:

$$\bar{u}(\mathbf{p}',\sigma')\left[\not{n}H_N + \frac{i\sigma^{n\Delta}}{2m_N}E_N\right]u(\mathbf{p},\sigma) = \bar{u}(\mathbf{p}',\sigma')\left[\not{n}H_{DD} + \frac{i\sigma^{n\Delta}}{2m_N}E_{DD} + \frac{p\cdot n}{m_N}D_N\right]u(\mathbf{p},\sigma)$$

for *on-shell* spinors.
We must decide between the LHS and RHS for the “unmodified” deuteron GPD.
(I’ve chosen the RHS since it emphasizes there is *one nucleon D-term*.)

The master convolution formula

Evaluating the matrix element

$$\langle p', \lambda' | \left[\not{n} H_{DD} + \frac{i\sigma^{n\Delta}}{2m_N} E_{DD} + \frac{P_N \cdot n}{m_N} D_N \right] | p, \lambda \rangle$$

gives a **master convolution formula**:

$$H_i(x,\xi,t) = \int_{-1}^{1} \frac{dy}{y} \left[h_i(y,\xi,t) H_{DD}\left(\frac{x}{y}, \frac{\xi}{y}, t\right) + e_i(y,\xi,t) E_{DD}\left(\frac{x}{y}, \frac{\xi}{y}, t\right) + y d_i \left(\frac{y}{\xi}, t\right) D_N \left(\frac{x}{\xi}, t\right) \right]$$

(*cf.* also work by Sergio, Simonetta.)

- h_i, e_i, and d_i describe how the nucleons are distributed in the nucleus, using GPD language. Call them **generalized nucleon distributions** (GNDs).
- By construction, H_{DD}, E_{DD}, and D_N already obey polynomiality.
- We can prove that when the GNDs obey polynomiality sum rules, so do the deuteron GPDs.
- **The only ingredient needed to ensure the GNDs observe polynomiality is a Lorentz-covariant model of nuclear structure.**

Discrete convolution formulas

The master convolution formula entails a collection of **discrete convolution formulas** for the generalized form factors.

- Define "generalized body form factors" from the GNDs, *e.g.*,

$$\int_{-1}^{1} y^s h_1(y,\xi,t)dy = \sum_{\substack{l=0 \\ 2|l}}^{s} \mathcal{A}^H_{s+1,l}(t)(2\xi)^l + \text{mod}(s,2)\mathcal{F}^H_{s+1}(t)(2\xi)^{s+1}$$

  with similar relations for all i, for e_i, and for d_i.

- The full GFFs can be found through, *e.g.*:

$$\mathcal{A}_{s+1,l}(t) = \sum_{\substack{r=0 \\ 2|r}}^{l} \left[\mathcal{A}^H_{s-r+1,l-r}(t)A^N_{s+1,r}(t) + \mathcal{A}^E_{s-r+1,l-r}(t)B^N_{s+1,r}(t)\right]$$

- Similar relations for $\mathcal{B}$ through $\mathcal{E}$, except $\mathcal{D}$ involves odd r.
- $\mathcal{F}$ and $\mathcal{G}$ are slightly more complicated (they have an extra term).

Discrete convolution formulas

Convolution formulas for $\mathcal{F}$ and $\mathcal{G}$:

$$\mathcal{F}_{s+1}(t) = \sum_{\substack{r=0 \\ 2|r}}^{s} \left[\mathcal{F}^H_{s-r+1}(t) A^N_{s+1,r}(t) + \mathcal{F}^E_{s-r+1}(t) B^N_{s+1,r}(t) \right] + \mathcal{A}^D_{1,0} C^N_{s+1}(t)$$

$$\mathcal{G}_{s+1}(t) = \sum_{\substack{r=0 \\ 2|r}}^{s} \left[\mathcal{G}^H_{s-r+1}(t) A^N_{s+1,r}(t) + \mathcal{G}^E_{s-r+1}(t) B^N_{s+1,r}(t) \right] + \mathcal{C}^D_{1,0} C^N_{s+1}(t)$$

A reminder that these originate from Mellin moments of H_1 and H_3:

$$\int_{-1}^{1} x^s H_1(x,\xi,t) dx = \sum_{\substack{l=0 \\ 2|l}}^{s} \mathcal{A}_{s+1,l}(t)(2\xi)^l + \mathrm{mod}(s,2)\mathcal{F}_{s+1}(t)(2\xi)^{s+1}$$

$$\int_{-1}^{1} x^s H_3(x,\xi,t) dx = \sum_{\substack{l=0 \\ 2|l}}^{s} \mathcal{C}_{s+1,l}(t)(2\xi)^l + \mathrm{mod}(s,2)\mathcal{G}_{s+1}(t)(2\xi)^{s+1}$$

Utility of discrete convolution

There is a point to these discrete convolutions.
Say we want **deuteron electric form factors**:

$$F_{iD}(t) = F_{iV}(t)[F_{1p}(t) + F_{1n}(t)] + F_{iT}(t)[F_{2p}(t) + F_{2n}(t)]$$

- F_{iV} and F_{iT} ("body form factors") are matrix elements of $\not{n}$ and $i\sigma^{n\Delta}/(2m_N)$, respectively.
- This is an easy calculation, bypassing a full GPD computation followed by an additional integration over x (which would be expensive in core-hours!).

Say we want **deuteron gravitational form factors**:

$$\mathcal{G}_i(t) = \mathcal{G}_{iV}(t)[A_p(t) + A_n(t)] + \mathcal{G}_{iT}(t)[B_p(t) + B_n(t)] + \mathcal{G}_{(i-2)D}(t)[C_p(t) + C_n(t)]$$

(the pink terms contribute only for $i = 3, 4$).

- $\mathcal{G}_{iV}$, $\mathcal{G}_{iT}$, and $\mathcal{G}_{(i-2)D}$ are matrix elements of $i(n \cdot \overleftrightarrow{\partial})\not{n}$, $i(n \cdot \overleftrightarrow{\partial})\sigma^{n\Delta}/(2m_N)$, and $1/m_N$, respectively.
- This again bypasses an expensive computation.

Covariant Contact Model

Motivation for a contact model

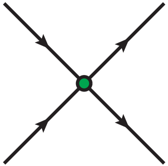
For computing the GPDs themselves, covariance is of the utmost importance.

- Can be difficult to maintain covariance while solving a bound state equation.
- Covariantly solving a four-Fermi contact interaction is tractable.
- Success of the Nambu-Jona-Lasinio (NJL) model suggests this approach has promise.
- The skeptic may ask: what about the deuteron's D-wave?
  What about the deuteron's huge quadrupole moment?
- The magic of relativity will produce these things, even in a contact interaction model.

Lagrangian

Construct most general possible NN Lagrangian that:

- Has four-fermi contact interactions.
- Has no derivatives in interaction terms.
- Obeys $\mathrm{SU}(2)_V \times \mathrm{SU}(2)_A$ isospin symmetry.
- Satisfies Pauli exclusion principle (enforced by ψ being Grassmann-number-valued!).

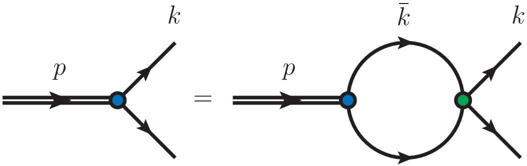


$$\begin{aligned}\mathcal{L}_{NN} &= \bar{\psi}(i\partial\!\!\!/ - m)\psi \\ &- G_S\left[\left(\bar{\psi}\tau_j C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\tau_j\psi\right) - \left(\bar{\psi}\tau_j\gamma^5 C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\gamma^5\tau_j\psi\right)\right] \\ &- G_V\left[\left(\bar{\psi}\tau_j\gamma^5\gamma^\mu C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\gamma^5\gamma_\mu\tau_j\psi\right) + \left(\bar{\psi}\gamma^\mu C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\gamma_\mu\psi\right)\right] \\ &- \frac{1}{2}G_T\left[\left(\bar{\psi}i\sigma^{\mu\nu}C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2 i\sigma_{\mu\nu}\psi\right)\right]\end{aligned}$$

Neglect charge-symmetry violation (assume $m_p = m_n \equiv m_N$).
Interactions decouple into separate isoscalar and isovector sectors.

Bethe-Salpeter vertex

Bethe-Salpeter equation in the covariant contact model:



Solution is the Bethe-Salpeter vertex:

$$\Gamma_D(p,\lambda) = \left[\alpha_V \not{\varepsilon}(p,\lambda) + i\alpha_T \frac{\sigma^{\varepsilon p}}{2M_D}\right] C\tau_2$$

We can solve for α_V and α_T in terms of couplings G_V and G_T ... and a UV regulator Λ (from proper time regularization).

Solution and static observables

Solution has parameters: G_V, G_T, and Λ.
These must be chosen somehow.
Fit to static observables:

- Deuteron binding energy
- Deuteron electromagnetic moments
- ${}^3S_1$-${}^3D_1$ scattering parameters.

	Contact model	Empirical
ϵ_D (MeV)	2.18	2.22
r_E (fm)	2.09	2.14
μ_D	0.879	0.857
$\mathcal{Q}_D$ (fm^2)	0.285	0.286
${}^3a_1$ (fm)	5.26	5.42
${}^3r_1$ (fm)	1.78	1.76
Λ (MeV)	139	—
G_V (GeV^{-2})	-683	—
G_T (GeV^{-2})	-715	—

- $\Lambda = 139$ MeV is a result of a fit—is not chosen by us.
- Suggests the model "knows" it breaks down when pion exchange becomes relevant.
- Note we have a non-zero, almost correct quadrupole moment.
- We do actually have a D-wave!

Origin of the D-wave

Whence the D-wave?
Bethe-Salpeter wave function takes the form

$$\psi_D(p,k,\lambda) = S(k)\Gamma_D(p,\lambda)S^T(p-k)$$

The numerator of the top-right 2×2 corner (where both nucleons have **positive energy**):

$$\psi_D^{(++)}(p,k,\lambda) \propto m_N(M_D + m_N)(\alpha_V + \alpha_T)(\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}) + 2(\alpha_V - \alpha_T)(\mathbf{k}\cdot\boldsymbol{\varepsilon})(\mathbf{k}\cdot\boldsymbol{\sigma})$$

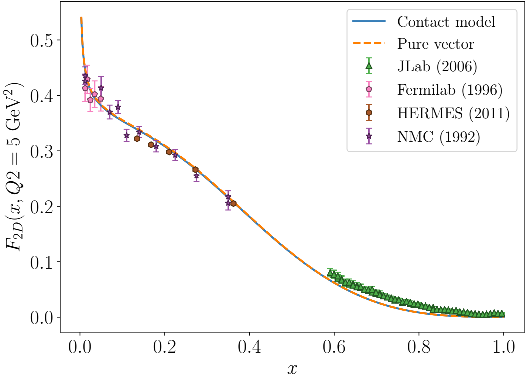
D-wave comes from second part of structure.
Ensures that even non-relativistic reductions, with:

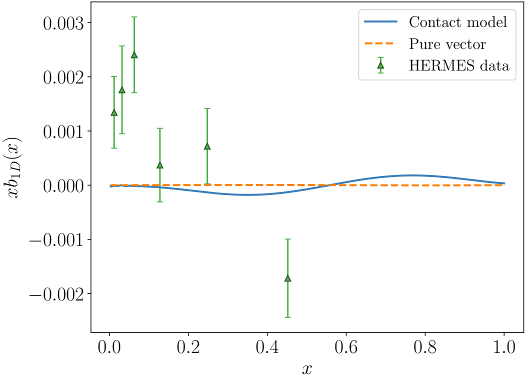
$$\psi_{\mathrm{NR}}(p,k,\lambda) \propto \bar{u}(k,s_1)\Gamma_D(p,\lambda)\bar{u}^T(p-k,s_2)$$

have D-wave—that is, $(\mathbf{k}\cdot\boldsymbol{\varepsilon})(\mathbf{k}\cdot\boldsymbol{\sigma})$ terms—in them.
Answer to **whence**: the **lower components** of u! **This is a relativistic effect.**

DIS structure functions

How well can this model describe DIS structure functions?
(Use CJ15 for nucleon PDFs.)

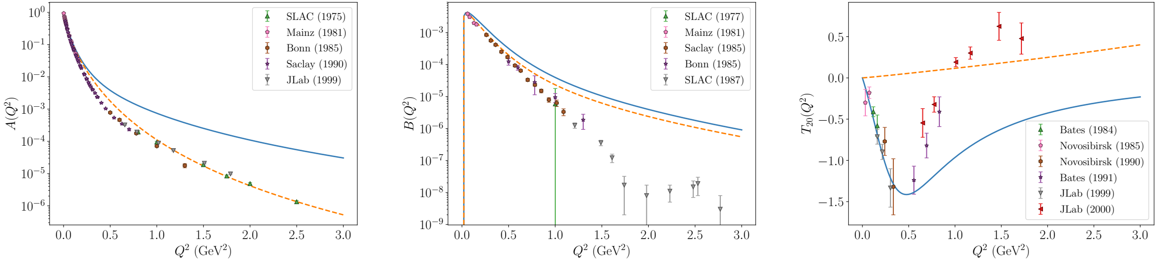




- Not bad for $F_2(x, Q^2)$ (underestimate at high x due to lack of short range correlations).
- Doesn't describe HERMES data for $b_1(x, Q^2)$, but that's expected.

Electromagnetic form factors

What about electromagnetic form factors?
(Use Kelly-Riordan nucleon form factors ... Blue is full model, orange is G_V only.)



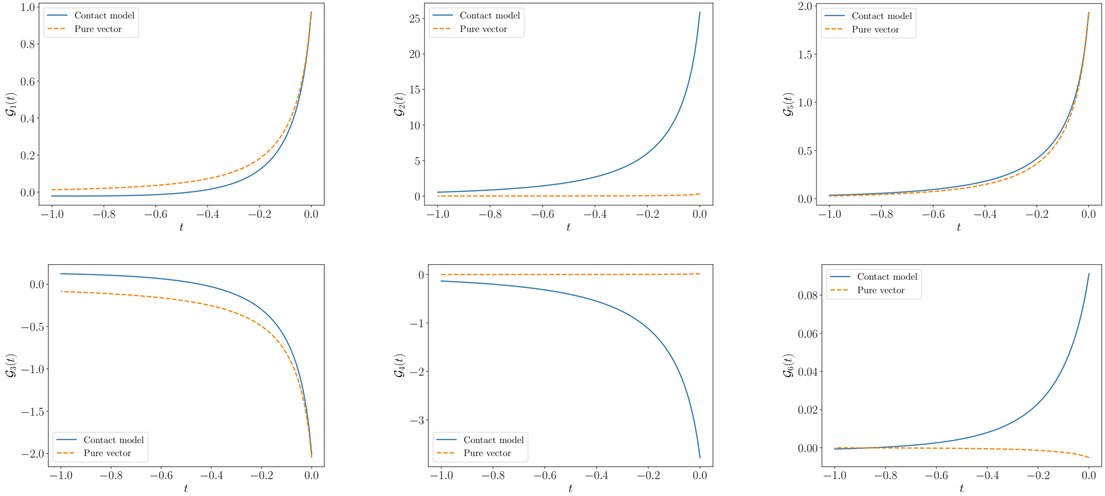
- Absolute size is too big at moderate-to-large Q^2.
- Agreement is OK for $Q^2 \lesssim 0.5$ GeV^2.
- Suggests our GPDs will be applicable to only low $-t$.

Gravitational form factors

Let's see what the **gravitational** form factors look like.

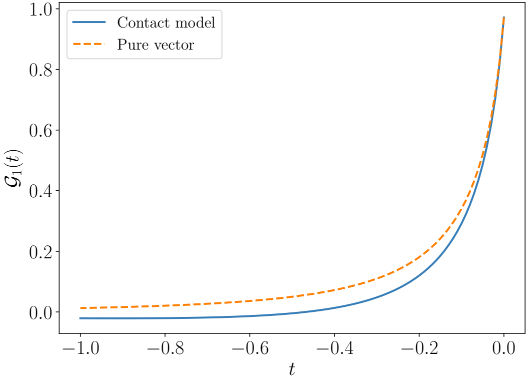
(Summed over all partons ... nucleon GPD model from [Goeke *et al.*, Prog. Part. Nucl. Phys 47 (2001)].)

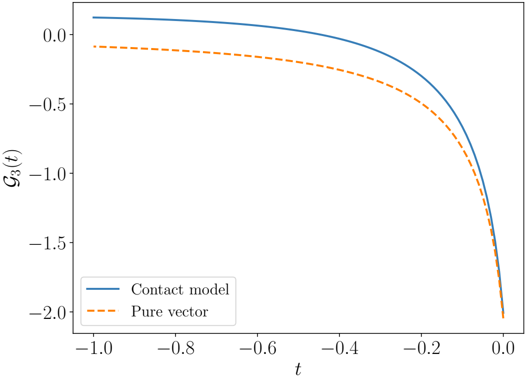
No data due to lack of graviton exchange dominated scattering experiments...



Gravitational form factors

Features of the monopole-dominated GFFs.

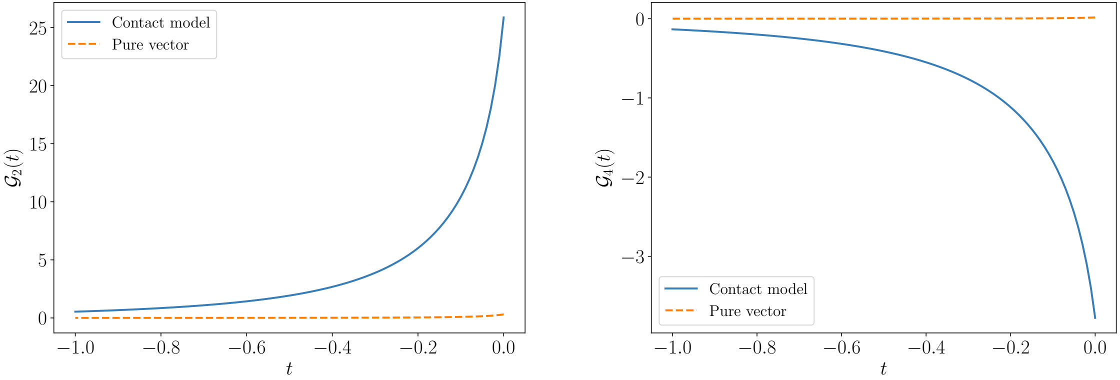




- $\mathcal{G}_1(t)$ is the analogoue of electromagnetic $F_1(t)$.
- $\mathcal{G}_1(0) = 1$ is a mass sum rule—it's a statement of **energy conservation**.
- $\mathcal{G}_3(t)$ dominates the Polyakov-Schweitzer D. It's related to internal balancing of forces.

Gravitational form factors

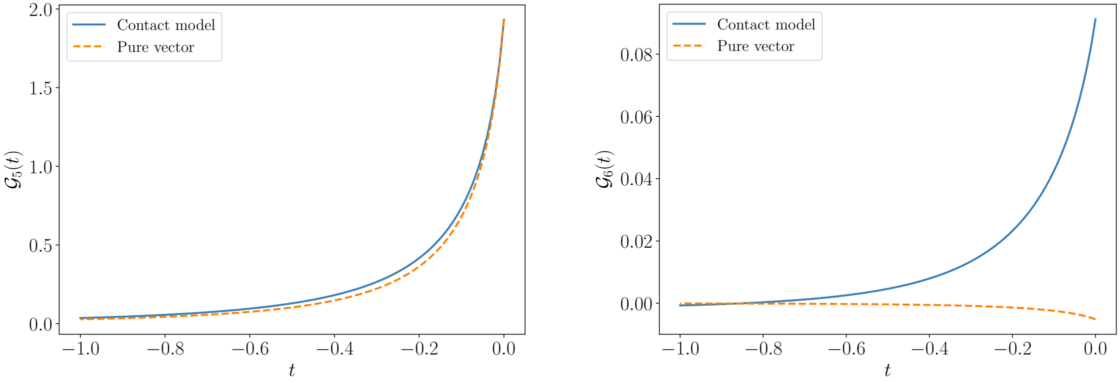
Features of the quadrupole-dominated GFFs.



- $\mathcal{G}_2(t)$ is the analogoue of electromagnetic $F_3(t)$.
- Deuteron has a *large gravitational quadrupole moment*. Large tidal forces for a small nucleus.
- $\mathcal{G}_4(t)$ is a $(2\xi)^{s+1}$ term—it seems like a "second deuteron D-term"—mandated by the deuteron's quadrupole moment.

Gravitational form factors

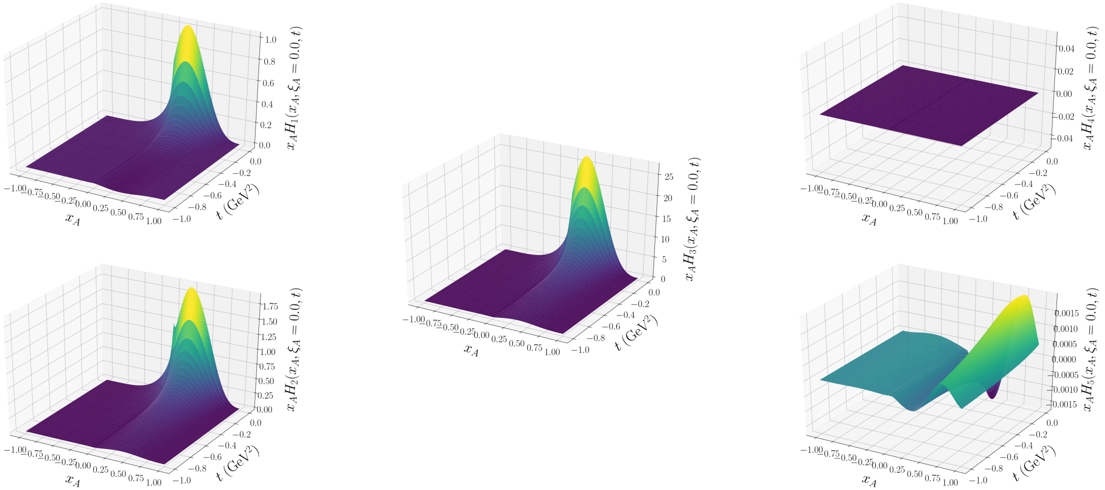
Features of the other GFFs.



- $\mathcal{G}_5(t)$ is angular momentum distribution. (See Simonetta's talk.)
- $\mathcal{G}_6(t)$ is a tensor polarization. (It's small, like $b_1(x)$.)

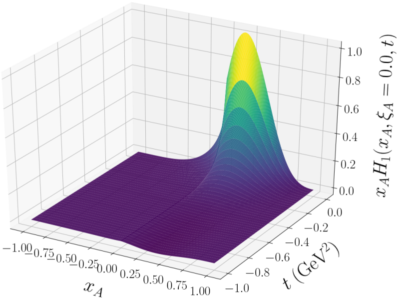
Generalized parton distributions

So, what do the coveted deuteron GPDs actually look like?
Look at $\xi = 0$; has clearer correspondences with well-known PDFs and form factors.

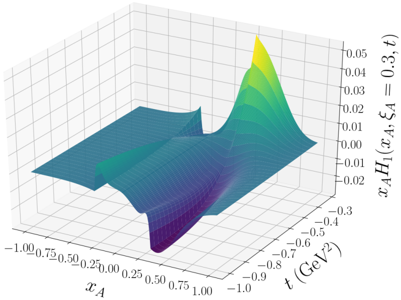


Generalized parton distributions

Let's take a closer look... at $H_1(x_A, \xi_A, t)$.



finite skewness $\longrightarrow$



- $H_1(x_A, \xi_A, t)$ is monopole-dominated.
- Forward limit ($\xi_A = 0$, $t = 0$) is old-fashioned PDF.
- Region $-\xi_A < x_A < \xi_A$ ("ERBL region") is dominated by D-term.

Generalized parton distributions

Closer look at $H_2(x_A, \xi_A, t)$.

- $H_2(x_A, \xi_A, t)$ is spin-dominated.
- Ridges at $x_A = \pm\xi_A$, despite lack of D-term.

Generalized parton distributions

Closer look at $H_3(x_A, \xi_A, t)$.

- $H_3(x_A, \xi_A, t)$ is quadrupole-dominated.
- The absolute magnitude is large because of the large quadrupole moment.
- ERBL region is dominated by D-term.

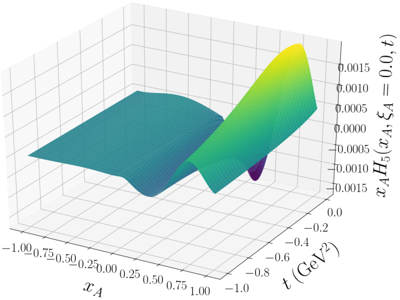
Generalized parton distributions

Closer look at $H_4(x_A, \xi_A, t)$.

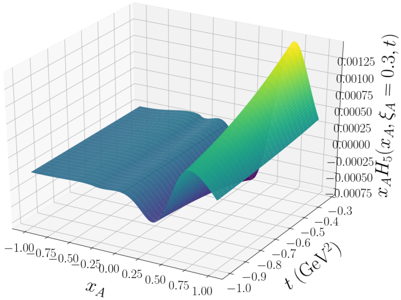
- $H_4(x_A, \xi_A, t)$ is dominated by tensor polarization.
- This GPD has neither a forward limit, nor a relation to EM form factors.
- It's zero at $\xi_A = 0$ because it is odd in ξ_A.

Generalized parton distributions

Closer look at $H_5(x_A, \xi_A, t)$.



finite skewness →



- $H_5(x_A, \xi_A, t)$ is tensor polarization dominated.
- Forward limit ($\xi_A = 0$, $t = 0$) is partonic $b_1(x)$.

Conclusions and outlook

In conclusion:

- We have calculated deuteron GPDs in a manifestly covariant contact model.
- Our GPDs obey polynomiality sum rules, and allow an unambiguous extraction of generalized form factors.
- We have computed gravitational form factors within this model, too.

Future work to be done:

- The model will be extended to other light nuclei (triton and helium).
- The NJL model can be used to compute covariant *nucleon* GPDs.
- A deeper understanding of the stress-energy tensor and gravitational form factors for spin-1 systems is needed.