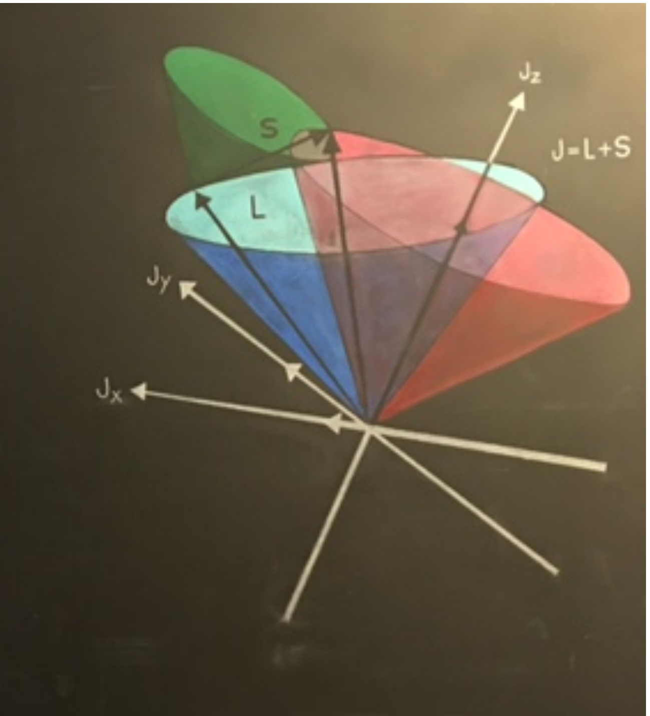
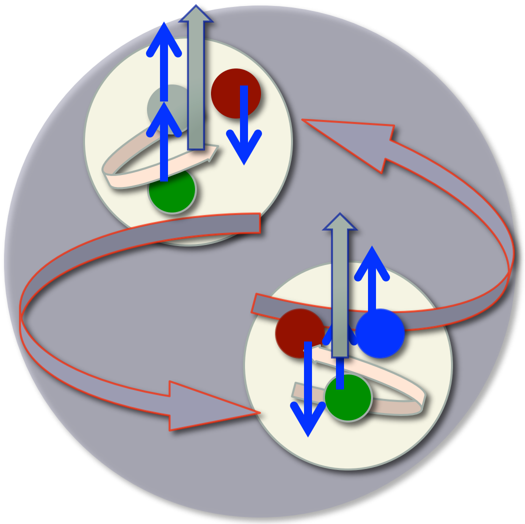
THE QCD ENERGY MOMENTUM TENSOR IN SPIN 0, 1/2 AND 1 NUCLEI

ECT* TRENTO WORKSHOP, APRIL 18, 2018



Simonetta Liuti
University of Virginia



Based on

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Parton transverse momentum and orbital angular momentum distributions

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Lorentz Invariance and QCD Equation of Motion Relations for Generalized Parton Distributions and the Dynamical Origin of Proton Orbital Angular Momentum

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We derive new Lorentz Invariance and Equation of Motion Relations between twist-three Generalized Parton Distributions (GPDs) and moments in the parton transverse momentum, k_T, of twist-two Generalized Transverse Momentum-Dependent Distributions (GTMDs), as a function of the parton longitudinal momentum fraction x. Although GTMDs in principle define the observables for partonic orbital motion, experiments that can unambiguously detect them appear remote at present. The relations presented here provide a solution to this impasse in that, e.g., the orbital angular momentum density is connected to directly measurable twist-three GPDs. Out of 16 possible Equation of Motion relations that can be written in the T-even sector, we focus on three helicity configurations that can be detected analyzing specific spin asymmetries: two correspond to longitudinal proton polarization and are associated with quark orbital angular momentum and spin-orbit correlations; the third, obtained for transverse proton polarization, is a generalization of the relation obeyed by the g_2 structure function. We also exhibit an additional relation connecting the off-forward extension of the Sivers function to an off-forward Qiu-Sterman term.

arXiv:1709.05770

"We shall therefore think of the proton as a box of partons sharing the momentum and practically free…..assume ***finite energy*** *of interaction among parts so as time goes on they change their momenta, are created annihilated etc. in* ***finite times****. But moving at large momentum* ***P*** *these times are dilated by the relativistic transformation so as* ***P*** *rises things change more and more slowly until ultimately they appear as not interacting at all" …*

Feynman (Photon-Hadron Interactions)

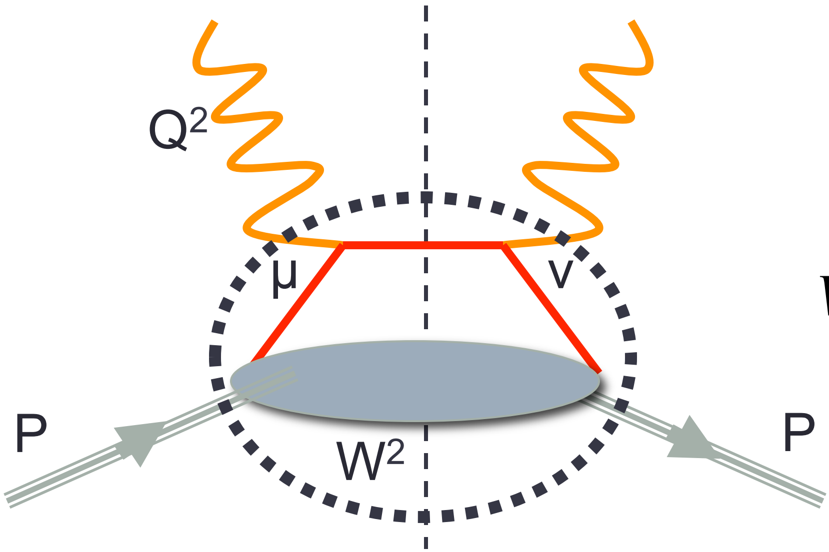
Outline

1. DIS and OPE
2. DVCS and OPE $\mapsto$ Moments of GPDs
3. EMT
4. EMT in QCD
5. Physical Interpretation
6. Parametrization for spin 0 ½ 1
7. Sum rules $\mapsto$ connection with moments (issue of gauge invariance)
8. Wandzura Wilczek relations

1. DIS AND OPE

From DIS Structure Functions to Parton Distributions

$\mathcal{I}$m part of **forward** Compton Amplitude



$Q^2 \gg M^2$ ➔ "deep"
$W^2 \gg M^2$ ➔ "inelastic"

$$W^{\mu\nu} = \frac{1}{4\pi}\int d^4\xi\, e^{iq\xi}\left\langle P\right|\left[J^\mu(\xi),J^\nu(0)\right]\left|P\right\rangle$$

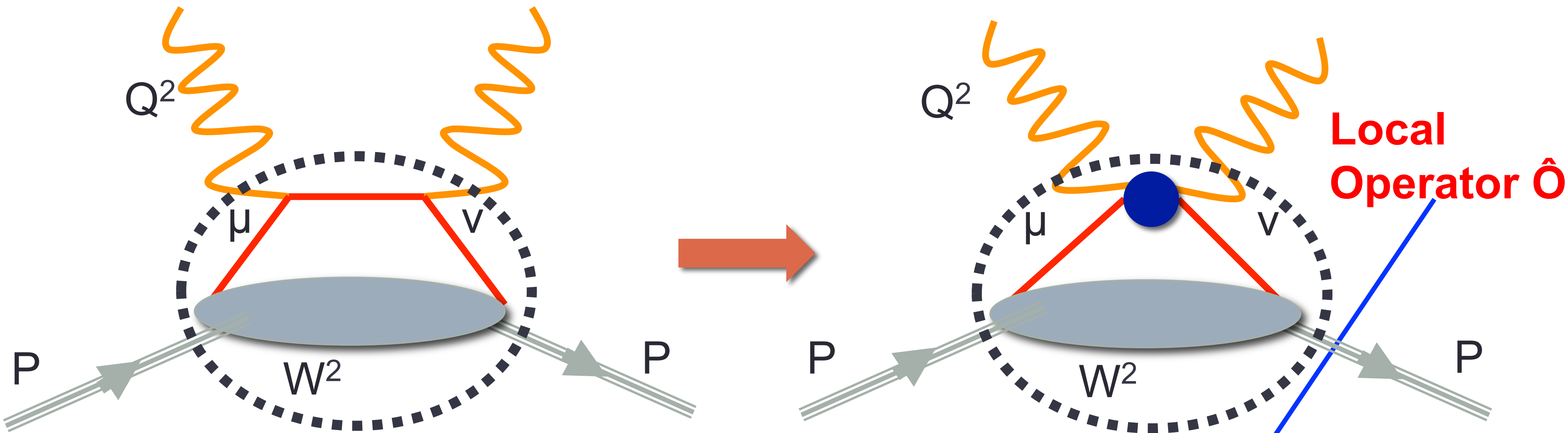
$$W^{\mu\nu} = \frac{1}{4\pi}\sum_X (2\pi)^4 \left\langle P\right|J^\mu(0)\left|X\right\rangle\left\langle X\right|J^\nu(0)\left|P\right\rangle\, \delta^4(P+q-P_X)$$

$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)F_1\left(x,Q^2\right) + \left(P^\mu + \frac{(Pq)}{q^2}q^\mu\right)\left(P^\nu + \frac{(Pq)}{q^2}q^\nu\right)\frac{F_2\left(x,Q^2\right)}{(Pq)}$$

$$F_2\left(x,Q^2\right) = xq(x,Q^2) + x\bar{q}(x,Q^2)$$

$$F_2(x) = \sum_q x f_1^q(x)$$

Theoretical Framework: OPE and geometric twist



… at twist 2

$$W_{\mu\nu} = i\,\Im m\, T_{\mu\nu} = i \int d^4\xi e^{iq\xi} \langle P \mid \bar{\psi}(\xi)\gamma_\mu \frac{1}{Q^2 + i2D\cdot q}\psi(0) \mid P\rangle q_\nu + \ldots$$

$$\frac{i}{Q^2}\sum_n \left(\frac{-2i}{Q^2}\right)^n q^{\mu_1}\ldots q^{\mu_n} \langle P \mid \bar{\psi}(0)\gamma_\mu D_{\mu_1}\ldots D_{\mu_n}\psi(0) \mid P\rangle$$

short distance behavior $\xi^2 \to 0$

quark/gluon large distance structure

$$\frac{i}{Q^2}\sum_n\left(\frac{-2i}{Q^2}\right)^n q^{\mu_1}...q^{\mu_n}\langle P \mid \bar{\psi}(0)\gamma_\mu D_{\mu_1}...D_{\mu_n}\psi(0) \mid P\rangle$$

$$\int_0^1 dx x^j F_2(x,Q^2)$$

Mellin Moments

The matrix element of a bi-local operator contains operators of different twist

Example

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)\gamma_\mu \psi(\lambda n)|PS\rangle = 2\left\{f_1(x)p_\mu + f_4(x)n_\mu\right\}.$$

tw 2 tw 4

Each one of the PDFs corresponds to its own tower of operators/ OPE expansion

In a nutshell:

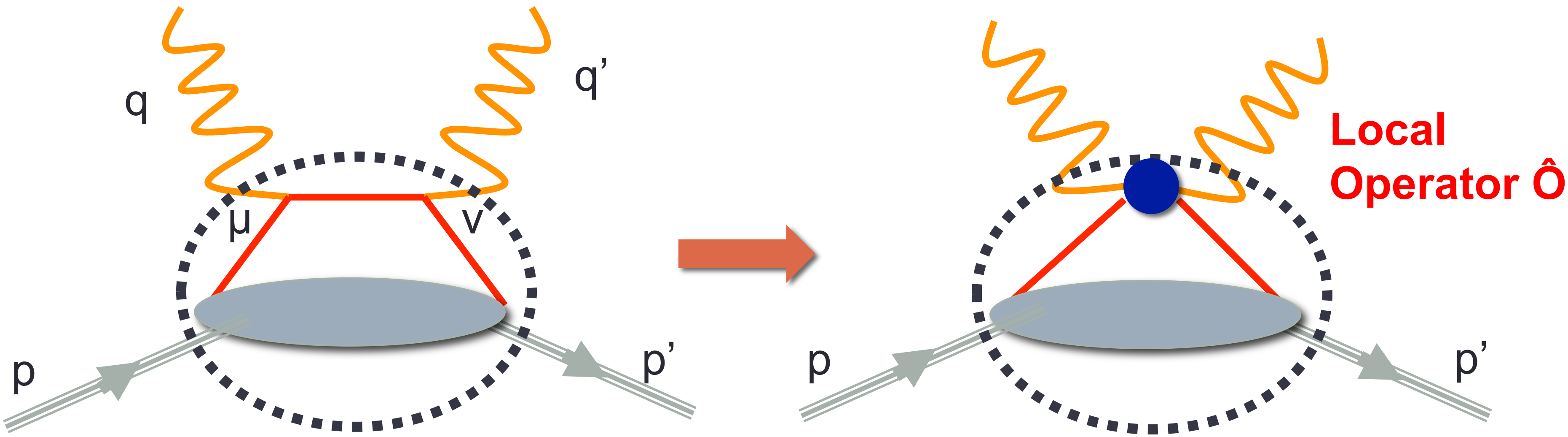
1. OPE in QCD allows us to order the matrix element by dominant LC singularities while separating out the long and short distance components

2. At a given value of the hard scale, Q, the quark-quark operators of twist two are the leading terms

3. Higher twist structure functions associated with operators of higher dimension involve covariant derivatives i.e. gluon… and they are suppressed by powers of 1/Q

4. Lorentz invariant concept, defined through space-time properties/geometry, independently from quark and gluon dynamics

2. DVCS AND OPE



$Q^2 >> M^2$ ➔ "deep"
$W^2 >> M^2$ ➔ equivalent to inelastic but not directly accessible

$$\left\{ \begin{array}{l} P = \dfrac{p + p'}{2} \\ \Delta = p' - p = q - q' \end{array} \right.$$

OPE (X. Ji, 1998)

$$n_{\mu_1}\ldots n_{\mu_n}\langle P'|O_q^{\mu_1\ldots\mu_n}|P\rangle = \overline{U}(P')\,\not{n}U(P)H_{qn}(\xi,t) + \overline{U}(P')\frac{\sigma^{\mu\alpha}n_\mu i\Delta_\alpha}{2M}U(P)E_{qn}(\xi,t)$$

helicity conserving — helicity flip

$$H_{qn}(\xi,t) = \sum_{i=0}^{[\frac{n-1}{2}]} A_{qn,2i}(t)(-2\xi)^{2i} + \mathrm{Mod}(n+1,2)C_{qn}(t)(-2\xi)^n$$

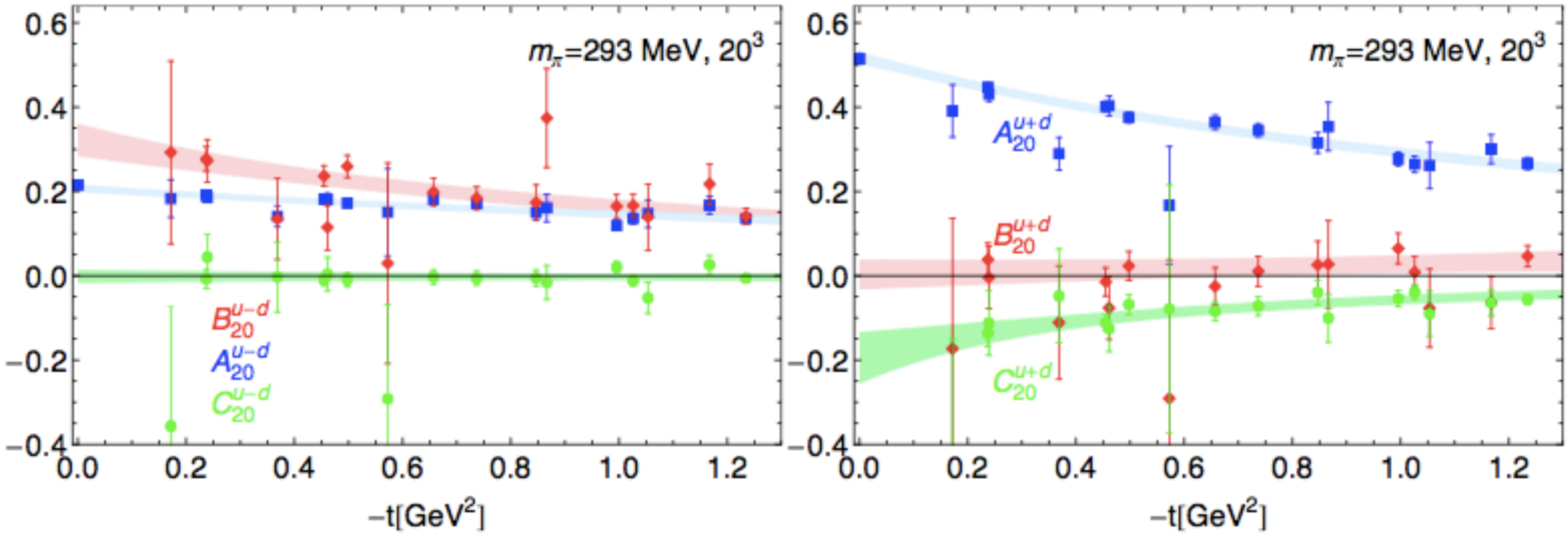
$$E_{qn}(\xi,t) = \sum_{i=0}^{[\frac{n-1}{2}]} B_{qn,2i}(t)(-2\xi)^{2i} - \mathrm{Mod}(n+1,2)C_{qn}(t)(-2\xi)^n.$$

Mellin Moments

$$\int_{-1}^{1} dx\, x^{n-2} H_q(x,\xi,t) = H_{qn}$$

$$\int_{-1}^{1} dx\, x^{n-2} E_q(x,\xi,t) = E_{qn}$$

Ph. Haegler, JoP: **295** (2011) 012009



Gluons: see Phiala Shanahan's talk

3. ENERGY MOMENTUM TENSOR

Scalar Field

Lagrangian density

$$\mathcal{L}(x) = \frac{1}{2}\left(\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x_\mu} - \lambda^2\phi^2\right) = \frac{1}{2}\left(\partial_\mu\phi\,\partial^\mu\phi - \lambda^2\phi^2\right)$$

Energy, momentum and angular momentum conservation follow from the invariance of $\mathcal{L}$ under **translations** and **rotations**

$$\frac{\partial}{\partial x^\mu}\left[\frac{\partial\mathcal{L}}{\partial_\mu(\partial_\alpha\phi)}\frac{\partial\phi}{\partial x_\nu} - \mathcal{L}g^{\mu\nu}\right] = 0$$

translation inv. continuity equation

define → $T^{\mu\nu}$

$$\frac{\partial}{\partial x^\mu}\left[x^\nu T^{\mu\rho} - x^\rho T_{\mu\nu}\right] = 0$$

rotation inv. continuity equation

Four conserved quantities from translation invariance

$$P^{\mu} = \int d^3\mathbf{x}\, T^{o\mu}$$

Six conserved quantities from rotational invariance

$$M^{\mu\nu} = \int d^3\mathbf{x}\, M^{o\mu\nu} = \int d^3\mathbf{x}\, \left[x^{\mu} T^{o\nu} - x^{\nu} T^{o\mu}\right]$$

QED/QCD Energy Momentum Tensor $T^{\mu\nu}$

Energy density

Momentum density

$\frac{E^2+B^2}{2}$	S_x	S_y	S_z
S_x	σ_{xx}	σ_{xy}	σ_{xz}
S_y	σ_{yx}	σ_{yy}	σ_{yz}
S_z	σ_{zx}	σ_{zy}	σ_{zz}

Shear stress

Pressure

$$\vec{S} = \vec{E} \times \vec{B}$$

Several (still) open issues in QCD…

$$\mathcal{L}_{QCD} = \overline{\psi}\left(i\gamma_\mu D^\mu - m\right)\psi - \frac{1}{4}F_{a,\mu\nu}F_a^{\mu\nu}$$

Conditions of gauge invariance and conservation: symmetric, traceless

The canonical QCD EMT is not symmetric and traceless: Belinfante and Coleman, Jackiw “fix” using superpotentials (terms with divergences)

The energy momentum tensor matrix element in spherically symmetric systems

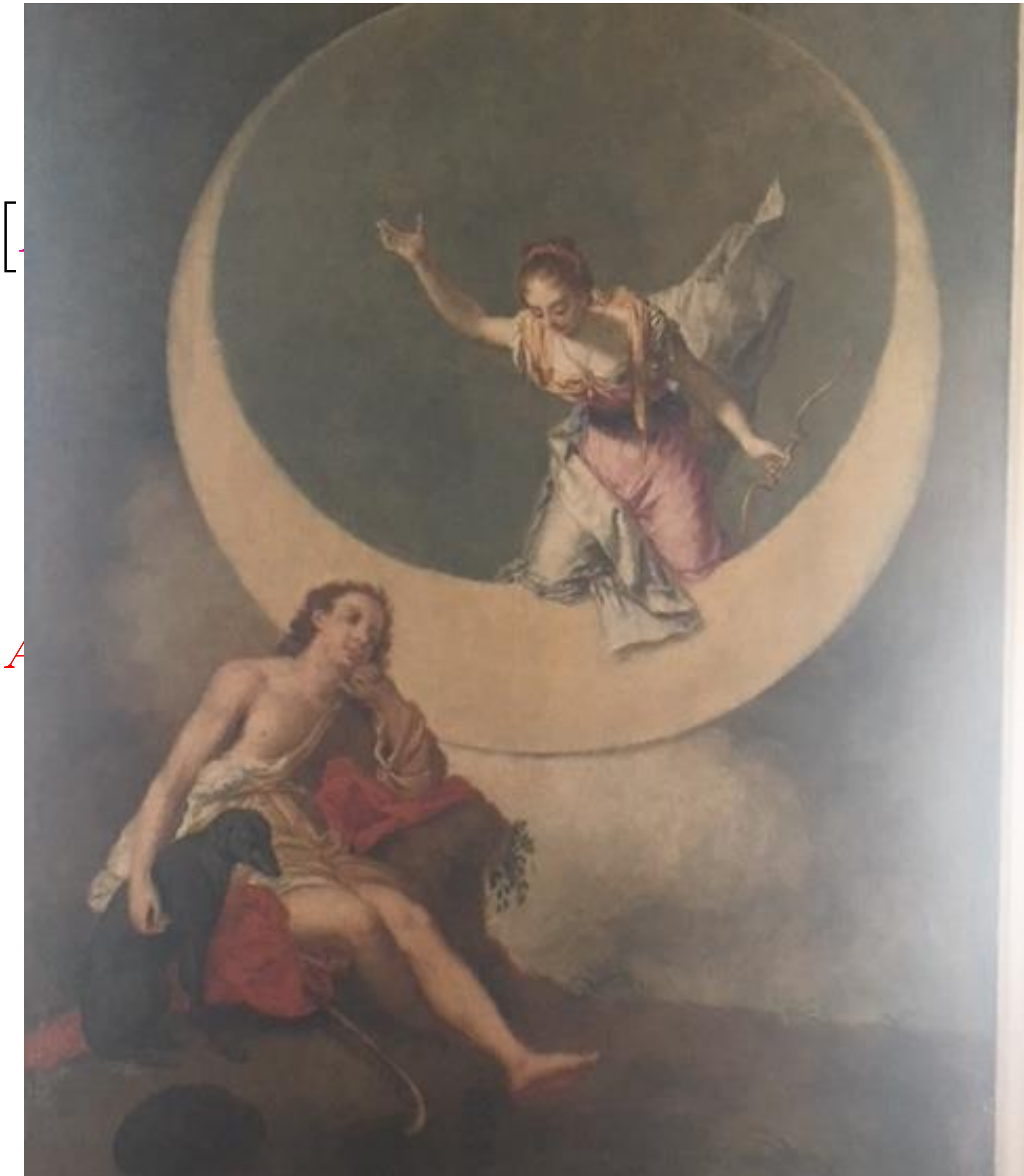
S=0

$$\langle p' \mid T^{\mu\nu} \mid p \rangle = 2 \Big[\quad \ldots \quad g^{\mu\nu}$$

S=1/2

$$\langle p', \Lambda \mid T^{\mu\nu} \mid p, \Lambda \rangle = A \quad \ldots \quad \Lambda') i \frac{\sigma^{\mu(\nu}\Delta^{\nu)}}{2M} U(p,\Lambda)$$

$$\ldots \quad) g^{\mu\nu} \bar{U}(p',\Lambda') U(p,\Lambda)$$



Energy Momentum Tensor in a spin 1 system

(Taneja, Kathuria, SL, Goldstein, PRD86(2012))

$$\langle p'|T^{\mu\nu}|p\rangle = -\frac{1}{2}P^\mu P^\nu(\epsilon'^*\epsilon)\mathcal{G}_1(t)$$

$$- \frac{1}{4}P^\mu P^\nu\frac{(\epsilon P)(\epsilon'^* P)}{M^2}\mathcal{G}_2(t) - \frac{1}{2}[\Delta^\mu\Delta^\nu \ldots$$ [illegible]

$$\times\ \mathcal{G}_3(t) - \frac{1}{4}[\Delta^\mu\Delta^\nu \ldots$$ [illegible] $$\mathcal{G}_4(t)$$

$$+\ \frac{1}{\ldots}$$ [illegible] $$P^\nu + \mu\leftrightarrow\nu]\,\mathcal{G}_5(t)$$

$$+$$ [illegible] $$(\epsilon P) - \epsilon^\mu(\epsilon'^* P))\,\Delta^\nu + \mu\leftrightarrow\nu$$

$$+\ 2g_{\mu\nu}(\epsilon P)(\epsilon'^* P) - (\epsilon'^{*\mu}\epsilon^\nu + \epsilon'^{*\nu}\epsilon^\mu)\Delta^2\Big]\,\mathcal{G}_6(t)$$

$$+\ \frac{1}{2}[\epsilon^{*\prime\mu}\epsilon^\nu + \epsilon'^{*\nu}\epsilon^\mu]\,\mathcal{G}_7(t) + g^{\mu\nu}(\epsilon'^*\epsilon)M^2\mathcal{G}_8(t) \qquad (5)$$

Angular momentum sum rule for spin one hadronic systems

Swadhin K. Taneja,[1,*] Kunal Kathuria,[2,†] Simonetta Liuti,[2,‡] and Gary R. Goldstein[3,§]

General rule to count form factors: t-channel J^{PC} q. numbers

Match $\langle P\bar{P}|$ to RHS → $\langle P\bar{P}|\bar{\psi}(0)\Gamma iD^{\{\mu_1}iD^{\mu_2}\cdots iD^{\mu_n\}}\psi(0)|0\rangle$

Haegler, PLB(2004)
Z.Chen&Ji, PRD(2005)

Nucleon

	$L=0$	1	2	3	4	...
$S=0$	J^{PC} 0^{-+}	1^{+-}	2^{-+}	3^{+-}	4^{-+}	
$S=1$	1^{--}	0^{++}	1^{--}	2^{++}	3^{--}	
		1^{++}	2^{--}	3^{++}	4^{--}	
		2^{++}	3^{--}	4^{++}	5^{--}	

TABLE I: J^{PC} of the $N\bar{N}$ states.

3

Deuteron

	$L=0$	1	2	3	4	...
$S=0$	J^{PC} 0^{++}	1^{--}	2^{++}	3^{--}	4^{++}	
$S=1$	1^{+-}	0^{-+}	1^{+-}	2^{-+}	3^{+-}	
		1^{-+}	2^{+-}	3^{-+}	4^{+-}	
		2^{-+}	3^{+-}	4^{-+}	5^{+-}	
$S=2$	2^{++}	1^{--}	0^{++}	1^{--}	2^{++}	
		2^{--}	1^{++}	2^{--}	3^{++}	
		3^{--}	2^{++}	3^{--}	4^{++}	
			3^{++}	4^{--}	5^{++}	
			4^{++}	5^{--}	6^{++}	

TABLE II: J^{PC} of the $d\bar{d}$ states.

7

Both S and L states considered

➢ Spin and 3D structure of Deuteron

Angular Momentum Sum Rule in deuteron

$$\frac{1}{2}\int_{-1}^{1} dx x\, H_2^q(x,0,0) = J_q$$

Taneja et al., Phys. Rev. D 86, 036008 (2012)

Angular Momentum Sum Rule in proton (Ji)

$$\frac{1}{2}\int_{-1}^{1} dx x\, [H_q(x,0,0) + E_q(x,0,0)] = J_q$$

In terms of gravitomagnetic form factors

Proton

Momentum

$$\mathrm{A}^q + \mathrm{A}^g = 1$$

OAM

$$\frac{1}{2}\left(\mathrm{A}^{q,g} + B^{q,g}\right) = J_z^{q,g}$$

Deuteron

Momentum

$$\langle p' | \int d^3x \, T_{q,g}^{0i} \, | p \rangle = p^i \langle p' | p \rangle = \mathcal{G}_1^{q,g} p^i \int d^3x \, 2p^0$$

$$\Rightarrow \mathcal{G}_1^q + \mathcal{G}_1^q = 1$$

OAM

$$\langle p' | \int d^3x \left(x_1 T_{q,g}^{02} - x_2 T_{q,g}^{01} \right) | p \rangle = \mathcal{G}_5^{q,g} \int d^3x \, p^0$$

$$\Rightarrow \frac{1}{2} \mathcal{G}_5^{q,g} = J_z^{q,g}$$

Physical Interpretation

Inclusive Observables

$$H_1(x,0,0) = \frac{1}{3}\left(q^1(x) + q^{-1}(x) + q^0(x)\right) = f_1(x)$$

$$H_5(x,0,0) = \left(q^0(x) - \frac{q^1(x) + q^{-1}(x)}{2}\right) = b_1(x)$$

Form factors

$$\int H_1(x,\xi,t)\,dx = G_1(t)$$

$$\int H_2(x,\xi,t)\,dx = G_2(t)$$

$$\int H_3(x,\xi,t)\,dx = G_3(t)$$

$$\int H_4(x,\xi,t)\,dx = 0$$

$$\int H_5(x,\xi,t)\,dx = 0$$

2^{nd} moments

Nucleon

$$\int dx\, x H(x,\xi,t) = A(t) + \xi^2 C(t)$$

← D-term

$$\int dx\, x E(x,\xi,t) = B(t) - \xi^2 C(t)$$

Deuteron

$$\int dx\, x \left[H_1(x,\xi,t) - \frac{1}{3}H_5(x,\xi,t)\right] = \mathcal{G}_1(t) + \xi^2\mathcal{G}_3(t) \quad (7)$$ Charge/Momentum

$$\int dx\, x H_2(x,\xi,t) = \mathcal{G}_5(t) \quad (8)$$ Angular Momentum

$$\int dx\, x H_3(x,\xi,t) = \mathcal{G}_2(t) + \xi^2\mathcal{G}_4(t) \quad (9)$$ Quadrupole

$$\int dx\, x H_4(x,\xi,t) = \xi\mathcal{G}_6(t) \quad (10)$$ T-odd

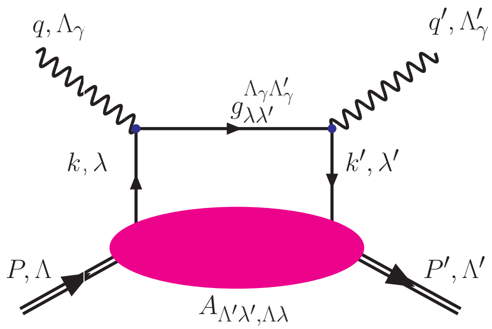
$$\int dx\, x H_5(x,\xi,t) = \mathcal{G}_7(t) \quad (11)$$ Connected to b_1 SR

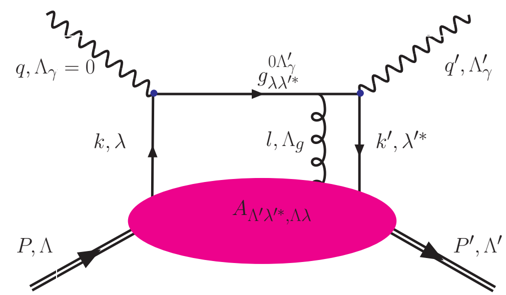
Two independent D-terms!

Interpretation of D-term(s) in terms of shear forces: work in progress with Adam Freese, Ian Cloet and Whitney Armstrong

IS OAM OF TWIST TWO OR THREE AND WHY?

dynamical twist ≠ geometric twist

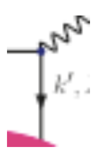




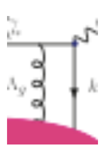
- The diagrammatic approach is based on light front quantization.
- **Power suppressed terms appear through a different construct than OPE!**
- Quark fields are decomposed into "good" and "bad" components:

$$\psi = \begin{pmatrix} \phi_\pm \\ \chi_\pm \end{pmatrix}$$

good



bad



The good components are the independent fields in the Equations of Motion

The bad components are functions of the good components and transv.gluon field

$$\chi = \mathcal{F}[\phi, A_\perp]$$

...and as it goes... the bad components are always more interesting than the good ones....

Origin of Wandzura Wilczek relations

- The difference between the Lorentz invariant structure of OPE and light front quantization produces a mismatch in the geometric and dynamic twist terms of higher twist
- The Wandzura-Wilczek (WW) relations are a rendition of this mismatch.

In polarized electron scattering

$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(x) + \bar{g}_2(x)$$

dynamical tw 3 (measured)

twist 2

geometric tw 3

… and why are they important

$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(x) + \bar{g}_2(x)$$

dynamical tw 3 (measured)

twist 2

geometric tw 3

1. A fundamental construct to cleanly extract the quark-gluon-quark interaction aka genuine twist three term

2. A window on the transverse structure of hadrons

The Wandzura Wilczek relations are derived in two steps

1. Lorentz Invariance Relations and gauge link structure

$$\int d^2k_T\,\frac{k_T^2}{M^2}\,g_{1T}(x,k_T^2) = -\int_x^1 g_2(y)dy + \widehat{g}_T$$

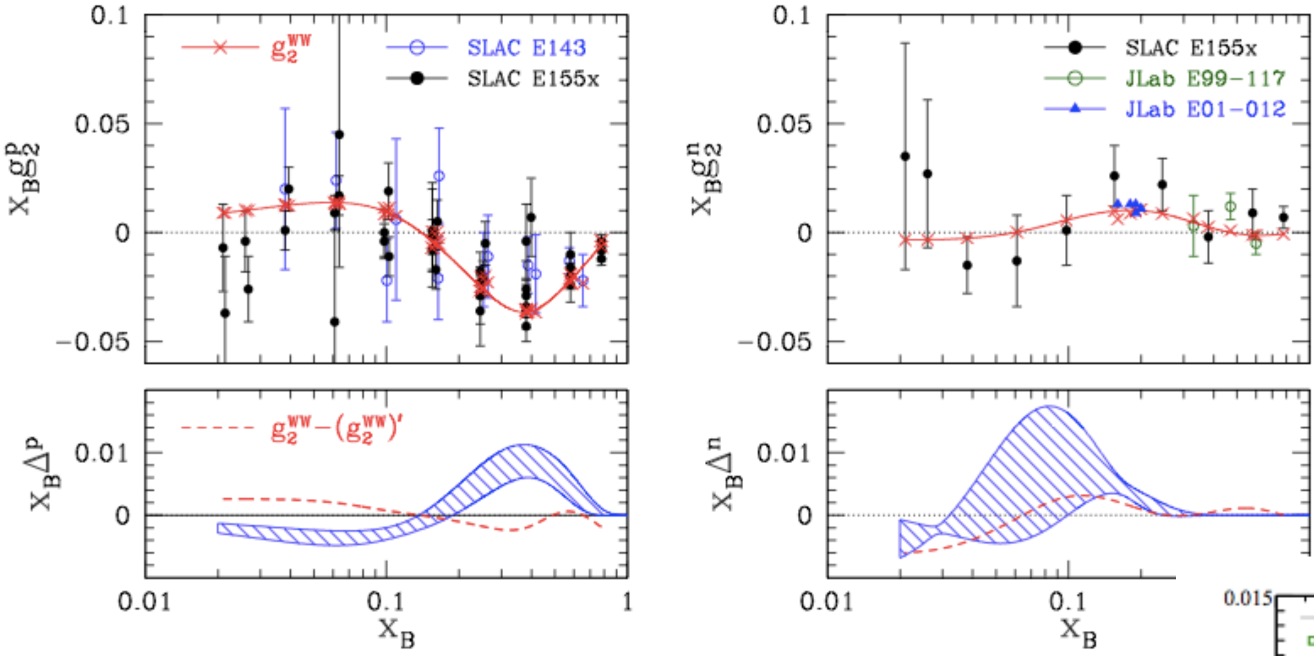
twist 3 str.func.

tw 3 from gauge link

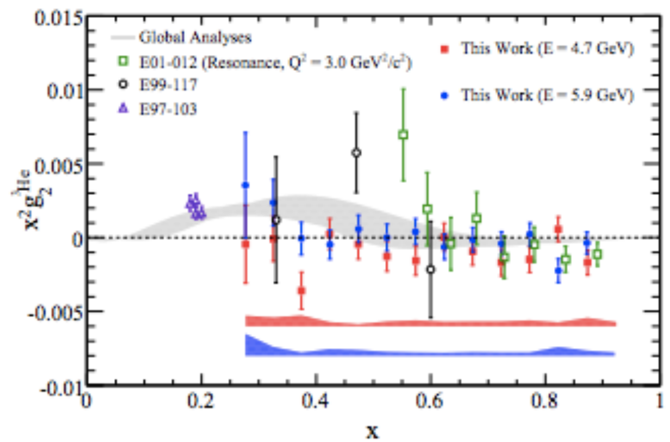
2. QCD Equations of Motion

$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(x) + \bar{g}_2(x)$$

sum of tw 3 from gauge link plus genuine qgq



Accardi, Bacchetta, Melnitchouk, Schlegel JHEP (2009)

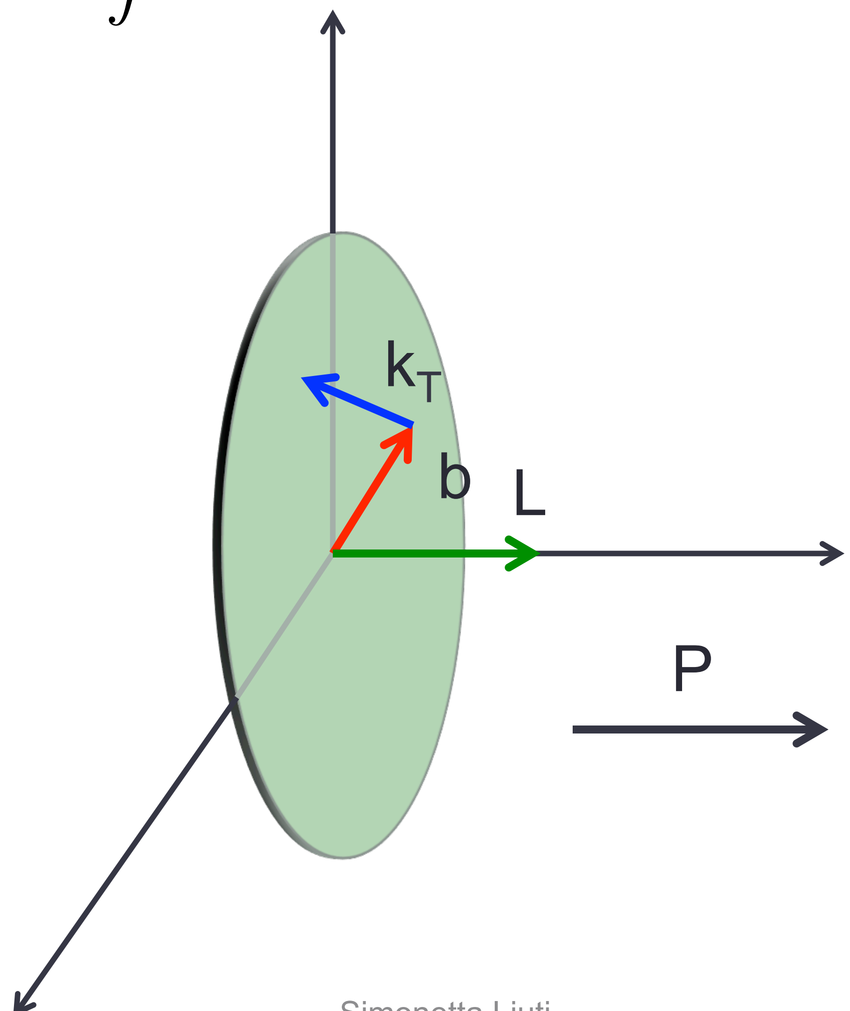


D. Flay et al, PRC 2016

The effect of the two twist-three terms combined might be small, each individual contribution can be large, however, it cannot be disentangled.

Partonic OAM: Wigner Distributions

$$L_q^{\mathcal{U}} = \int dx \int d^2\mathbf{k}_T \int d^2\mathbf{b}\,(\mathbf{b}\times\mathbf{k}_T)_z\,\mathcal{W}^{\mathcal{U}}(x,\mathbf{k}_T,\mathbf{b})$$

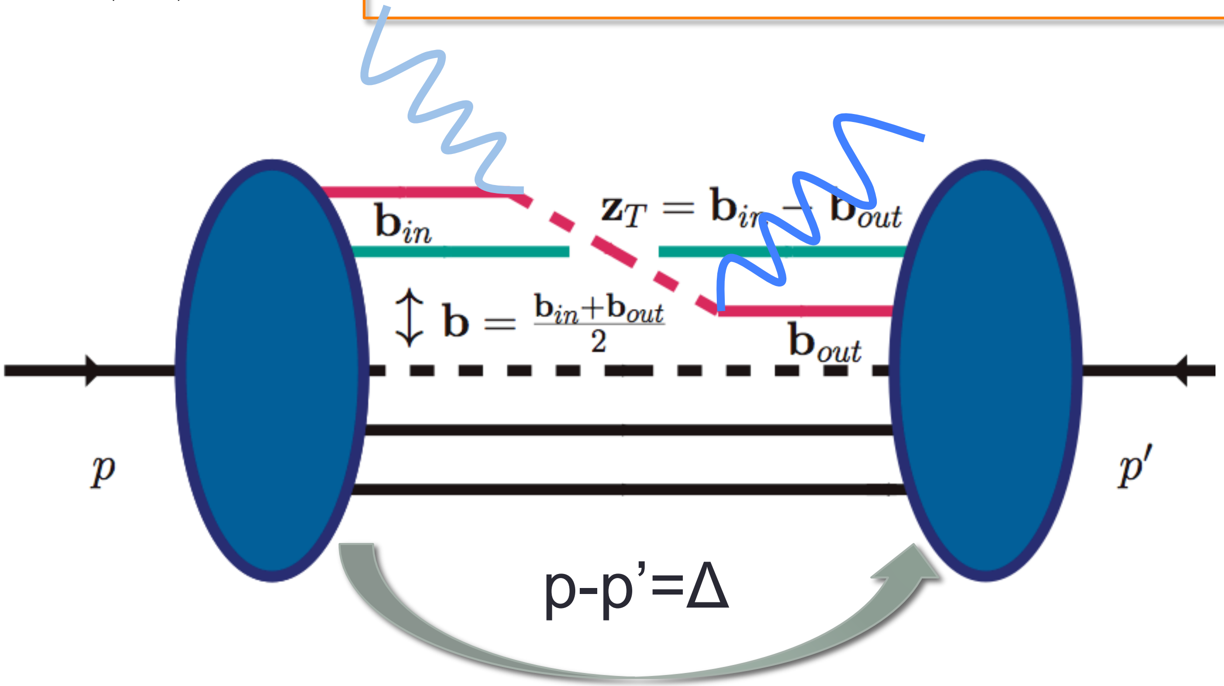


Hatta
Lorce, Pasquini,
Xiong, Yuan
Mukherjee

Wigner Distribution and qq correlation function

$$\mathcal{W}^{\mathcal{U}} = \frac{1}{2}\int \frac{d^2\mathbf{\Delta}_T}{(2\pi)^2} e^{i\Delta_T \cdot b} \int dz^- d^2\mathbf{z}_T\, e^{ikz} \langle P-\Delta, \Lambda' \mid \bar{q}(0)\gamma^+ \mathcal{U}(0,z) q(z) \mid P, \Lambda \rangle \mid_{z^+=0}$$

GTMD



- Δ_T Fourier conjugate: **b** = transverse position of the quark inside the proton
- k_T Fourier conjugate: $\mathbf{z}_T$= transverse distance traveled by the struck quark between the initial and final scattering

Possible Observable for L_q

$$\frac{1}{M}\int d^2k_T\, k_T^2\, F_{14}(x,0,k_T^2,0,0) = \langle b_T \times k_T \rangle_3(x)$$

$L_q(x)$

k_T moment of a GTMD
(Lorce and Pasquini)

$\xi=0$
$k_T \bullet \Delta_T=0$
$\Delta_T^2=0$

CAN IT BE MEASURED?

Is there any observable that we can identify OAM with?

A New Relation

A. Rajan, A. Courtoy, M. Engelhardt, S.L., PRD (2016), arXiv:1601.06117
A. Rajan, M. Engelhardt, S.L., submitted to PRD arXiv:1709.05770

$$\frac{1}{M}\int d^2k_T\, k_T^2\, F_{14}(x,0,k_T^2,0,0) = -\int_x^1 dy\, \left[\widetilde{E}_{2T} + H + E\right]$$

Through Generalized Lorentz Invariance Relation (LIR) F_{14} is connected to twist 3 GPDs

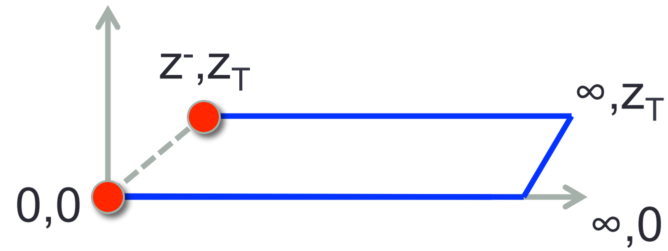
*Different notation! $G_2 \rightarrow \tilde{E}_{2T} + H + E$

Polyakov et al. Meissner, Metz and Schlegel, JHEP(2009)

Generalized LIR

$$\frac{1}{M}\int d^2k_T\, k_T^2\, F_{14}(x,0,k_T^2,0,0) = -\int_x^1 dy\,\left[\widetilde{E}_{2T} + H + E\right]$$

Generalized LIR for a staple link



$$\frac{d}{dx}\int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14} = \tilde{E}_{2T} + H + E + \mathcal{A}$$

LIR violating term

$$\mathcal{A}_{F_{14}} = v^- \frac{(2P^+)^2}{M^2}\int d^2k_T \int dk^- \left[\frac{k_T\cdot\Delta_T}{\Delta_T^2}(A_{11}^F + xA_{12}^F) + A_{14}^F + \frac{k_T^2\Delta_T^2 - (k_T\cdot\Delta_T)^2}{\Delta_T^2}\left(\frac{\partial A_8^F}{\partial(k\cdot v)} + x\frac{\partial A_9^F}{\partial(k\cdot v)}\right)\right]$$

Using the QCD EoM allows us to eliminate F_{14}

$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right]$$

genuine twist three term

Ji Sum Rule

L = J - S + 0

*Staple link

$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right] - \int_x^1 \frac{dy}{y}\mathcal{A}_{F_{14}}$$

Integrating over x we re-obtain the OPE based relation

Polyakov et al.(2000), Hatta(2012)

$$\int_0^1 dx\, x G_2 = -\frac{1}{2}\int_0^1 dx\, x(H+E) + \frac{1}{2}\int_0^1 dx\, \tilde{H}$$

$$J_q = L_q + \frac{1}{2}\Delta\Sigma_q$$

A generalized Wandzura Wilczek relation obtained using OPE for twist 2 and twist 3 operators for the off-forward matrix elements

Twist 3 GPDs are sensitive to the combination of genuine tw 3 + LIR violating terms

$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right] - \int_x^1 \frac{dy}{y}\mathcal{A}_{F_{14}}$$

$$\tilde{E}_{2T} = \tilde{E}_{2T}^{WW} + \tilde{E}_{2T}^{(3)} + \tilde{E}_{2T}^{LIR}$$

$$\tilde{E}_{2T}^{WW} = -\int_x^1 \frac{dy}{y}(H+E) - \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right]$$

$$-\left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right] - \int_x^1 \frac{dy}{y}\mathcal{A}_{F_{14}}$$

Jaffe Manohar vs. Ji OAM

Difference between JM and Ji independently worked out by Y. Hatta, M. Burkardt 2012

$$\mathcal{M}^i_{\Lambda\Lambda'} = \frac{1}{4}\int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{ixP^+ z^- - ik_T\cdot z_T}$$

$$\langle p', \Lambda' \mid \overline{\psi}(-z/2) \left[(\overrightarrow{\partial} - ig\not{A})\mathcal{U}\Gamma\Big|_{-z/2} + \Gamma\mathcal{U}(\overleftarrow{\partial} + ig\not{A})\Big|_{z/2} \right] \psi(z/2) \mid p, \Lambda \rangle_{z^+=0}$$

In our approach

$$-x\tilde{E}_{2T} = \tilde{H} - \int d^2 k_T \frac{k_T^2}{M^2} F_{14}^{\text{staple}} + \mathcal{M}_{F_{14}}^{\text{staple}} \quad \text{JM OAM}$$

$$-x\tilde{E}_{2T} = \tilde{H} - \int d^2 k_T \frac{k_T^2}{M^2} F_{14}^{\text{straight}} + \mathcal{M}_{F_{14}}^{\text{straight}} \quad \text{Ji OAM}$$

LIR violating term is the difference between JM and Ji

$$L^{JM}(x) - L^{Ji}(x) = \mathcal{M}_{F_{14}} - \mathcal{M}_{F_{14}}|_{v=0} = -\int_x^1 dy \, \mathcal{A}_{F_{14}}(y).$$

Generalized Qiu Sterman term

$$\int d^2k_T \, \frac{k_T^2}{M^2} \, F_{14}^{JM} - \int d^2k_T \, \frac{k_T^2}{M^2} \, F_{14}^{Ji} = T_F(x, x, \Delta)$$

$$-\int dx \;\; F_{14}^{(1)}\Big|_{\text{diff}}\Big|_{\Delta_T=0} = -\frac{\partial}{\partial \Delta_i} i\epsilon^{ij} g v^- \frac{1}{2P^+} \int_0^1 ds \, \langle p', +|\bar{\psi}(0)\gamma^+ U(0, sv) F^{+j}(sv) U(sv, 0)\psi(0)|p, +\rangle\Big|_{\Delta_T=0}$$

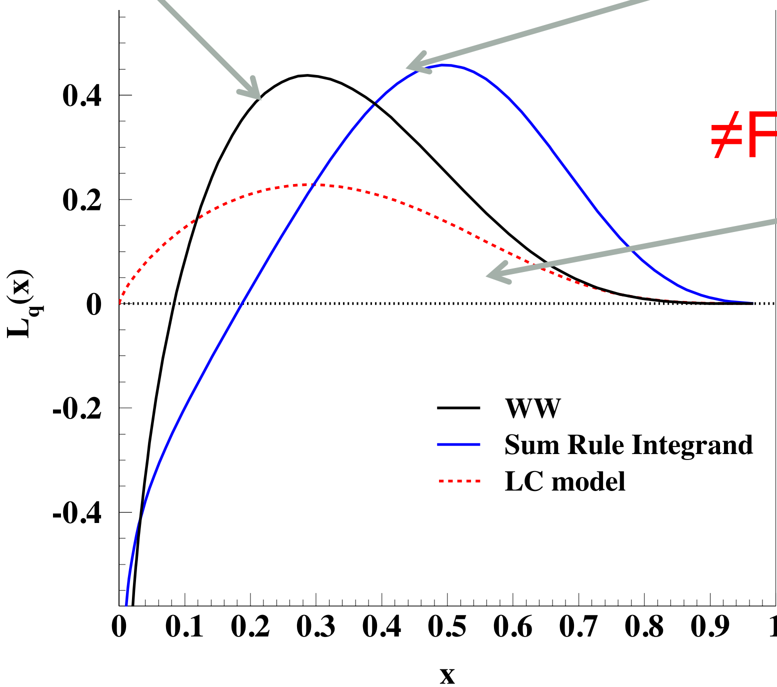
Clarifying a few open issues in the literature

- The genuine twist 3 term for a staple link contains an extra term that gives JM OAM
- The genuine twist 3 term for a straight link does not have this term
- An experimental measurement of twist 3 GPDs is sensitive to OAM but it cannot disentangle the difference between JM and Ji decompositions
- Similarly, the LIR term does not impact the extraction of the genuine twist 3 contribution to polarized DIS (g_2).

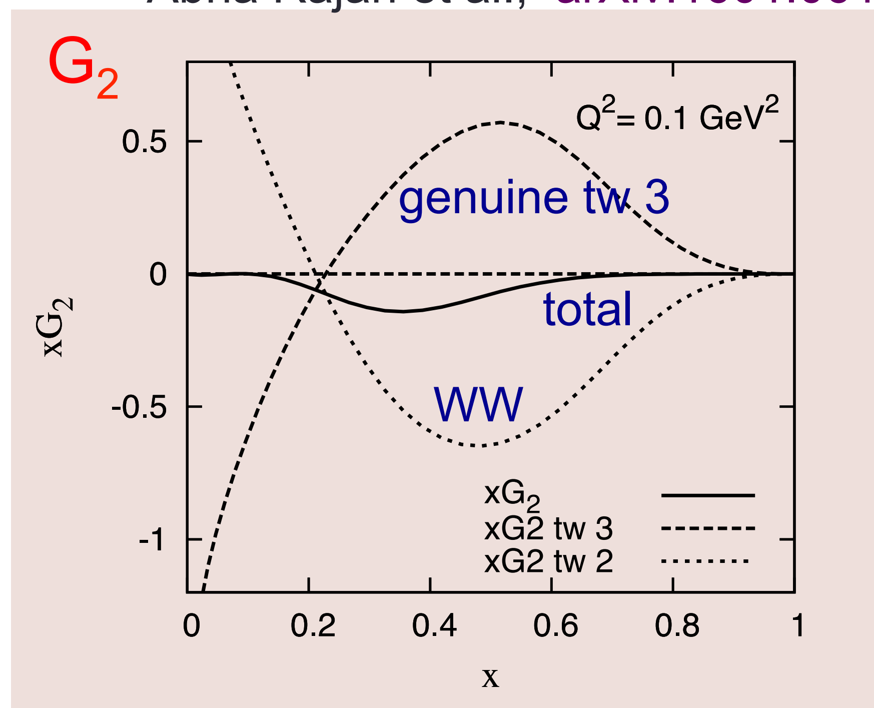
$$L_q(x,0,0) = x\int_x^1 \frac{dy}{y}(H_q(y,0,0)+E_q(y,0,0)) - x\int_x^1 \frac{dy}{y^2}\widetilde{H}_q(y,0,0),$$

Low x!

≠F_{14}!



Abha Rajan et al., arXiv:1601.06117



INTERPETATION OF X-MOMENTS

W. Armstrong, M. Engelhardt, SL, A. Rajan

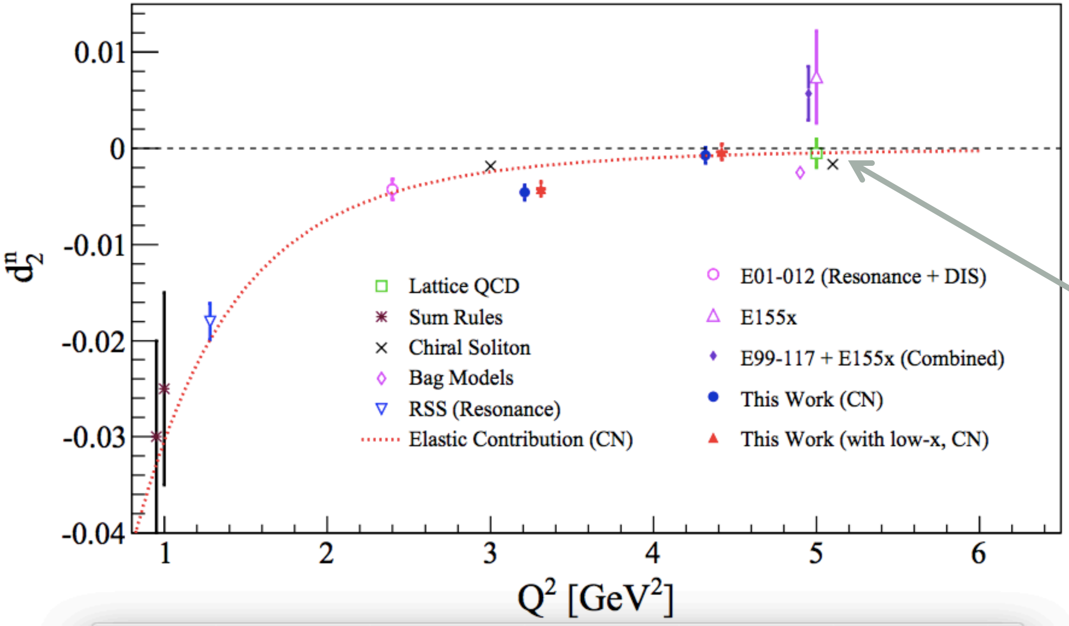
x-Moments

Generalized Burkhardt Cottingham

M$_1$ $$\int dx \tilde{E}_{2T} = -\int dx(H+E) \quad \Rightarrow \int dx \left(\tilde{E}_{2T} + H + E\right) = 0$$

M$_2$ OAM Sum Rule $$\int dx x \tilde{E}_{2T} = -\frac{1}{2}\int dx x(H+E) - \frac{1}{2}\int dx \tilde{H}$$

M$_3$ $$\int dx x^2 \tilde{E}_{2T} = -\frac{1}{3}\int dx x^2(H+E) - \frac{2}{3}\int dx x \tilde{H} - \frac{2}{3}\int dx x\, \mathcal{M}_{F_{14}}$$



Genuine twist three d$_2$

Lattice calculation

Interpretation

$M_2(v^-)$

Force acting on quark

$$\int dx \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = \quad i\epsilon^{ij} g v^- \frac{1}{2P^+} \int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0)\gamma^+ U(0, sv) F^{+j}(sv) U(sv, 0)\psi(0)|p, \Lambda\rangle$$

$$\int dx \int d^2k_T\, \mathcal{M}^{i,A}_{\Lambda'\Lambda} = \quad -g v^- \frac{1}{2P^+} \int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0)\gamma^+\gamma^5 U(0, sv) F^{+i}(sv) U(sv, 0)\psi(0)|p, \Lambda\rangle$$

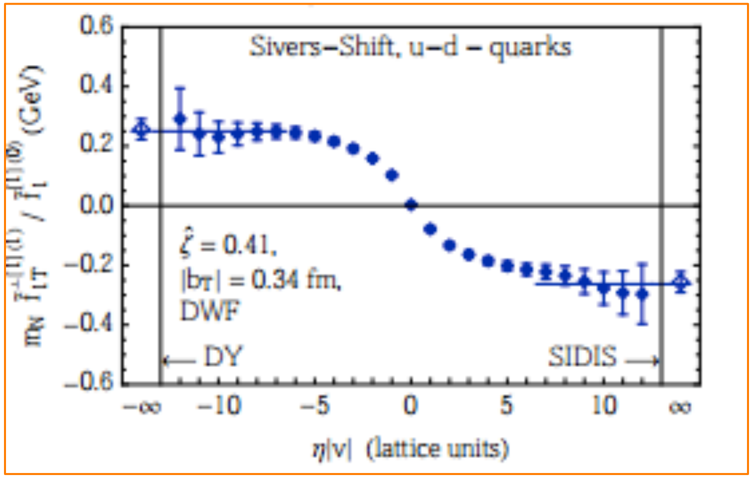
Non zero only for staple link

$M_3(v^-=0)$

$$\int dx\, x \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = \frac{ig}{4(P^+)^2} \langle p', \Lambda' | \bar{\psi}(0)\gamma^+\gamma^5 F^{+i}(0)\psi(0)|p, \Lambda\rangle$$

$$\int dx\, x \int d^2k_T\, \mathcal{M}^{i,A}_{\Lambda'\Lambda} = \frac{g}{4(P^+)^2} \epsilon^{ij} \langle p', \Lambda' | \bar{\psi}(0)\gamma^+ F^{+j}(0)\psi(0)|p, \Lambda\rangle$$

Relations between derivatives



$$\frac{d}{dv^-}\mathcal{M}^{i,S(n=2)}_{\Lambda\Lambda'}\Big|_{v^-=0} = i(2P^+)\mathcal{M}^{i,A(n=3)}_{\Lambda\Lambda'}$$

$$\frac{d}{dv^-}\mathcal{M}^{i,A(n=2)}_{\Lambda\Lambda'}\Big|_{v^-=0} = -i(2P^+)\mathcal{M}^{i,S(n=3)}_{\Lambda}$$

Proton transverse spin configuration

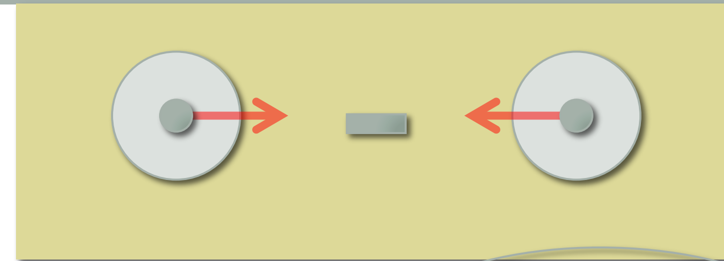
Genuine twist three d_2

$$\frac{d}{dv^-}\int dx F^{(1)}_{12}\Big|_{v^-=0} = \frac{d}{dv^-}\int dx \mathcal{M}_{F_{12}}\Big|_{v^-=0} = i(2P^+)\int dx\, x\frac{\Delta_i}{\Delta_T^2}\left(\mathcal{M}^{i,A}_{++} - \mathcal{M}^{i,A}_{--}\right) = \mathcal{M}^{n=3}_{G_{12}}$$

$$\frac{d}{dv^-}\int dx G^{(1)}_{12}\Big|_{v^-=0} = \frac{d}{dv^-}\int dx \mathcal{M}_{G_{12}}\Big|_{v^-=0} = i(2P^+)\int dx\, x\frac{\Delta_i}{\Delta_T^2}\left(\mathcal{M}^{i,S}_{++} + \mathcal{M}^{i,S}_{--}\right) = \mathcal{M}^{n=3}_{F_{12}}$$

Slope of Sivers function in staple length quantitative results being worked on now…

Other integrated relations: SPIN ORBIT!



$$\int dx x \left(E'_{2T} + 2\tilde{H}'_{2T}\right) = -\frac{1}{2}\int dx x \tilde{H} - \frac{1}{2}\int dx H + \frac{m}{2M}\int dx (E_T + 2\tilde{H}_T)$$

$$(L_z S_z)_q = \int dx x \left(E'_{2T} + 2\tilde{H}'_{2T} + \tilde{H}\right), \quad \kappa_T = \int dx \, (E_T + 2\tilde{H}_T), \quad e_q = \int dx \, H$$

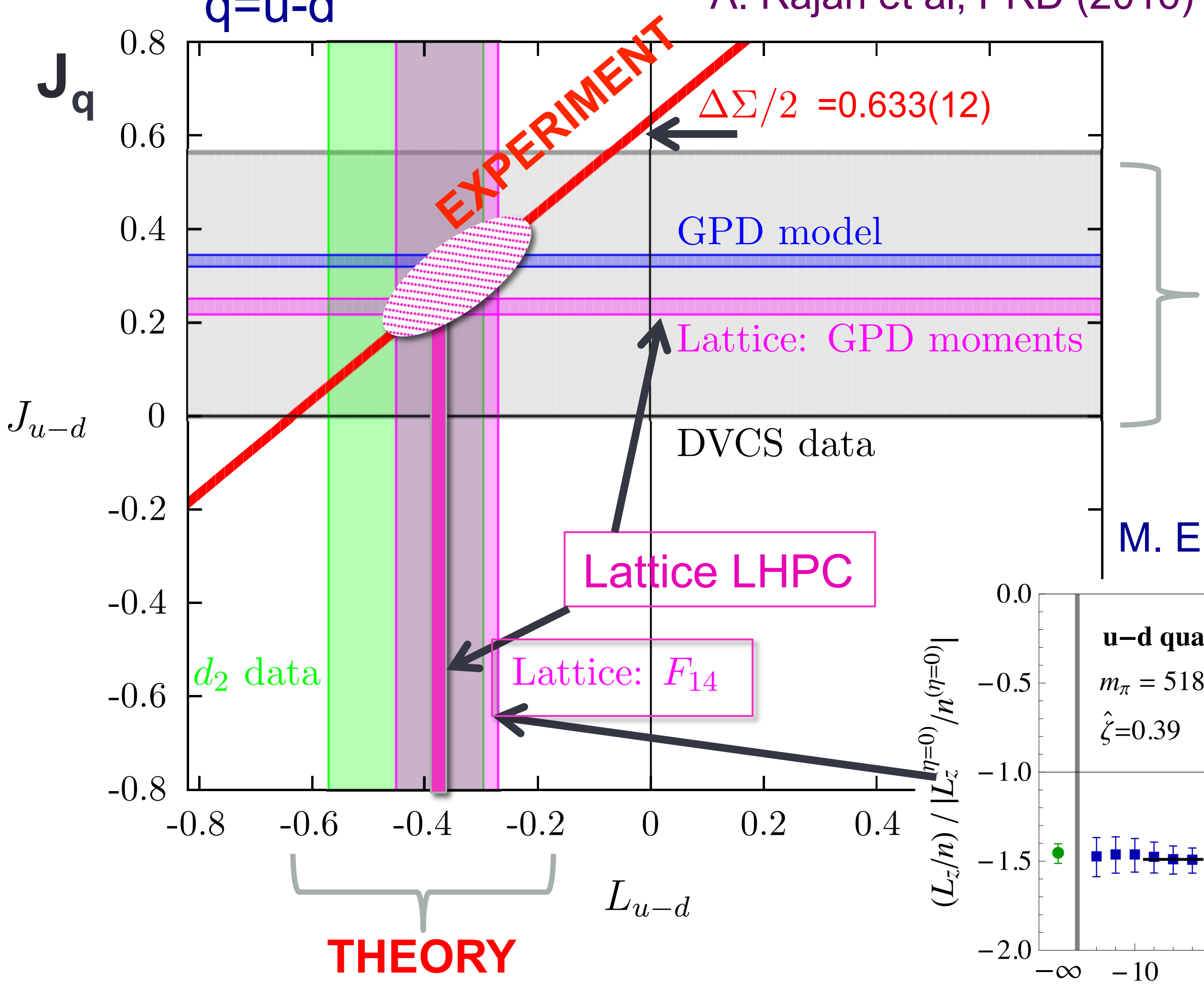
$$\boxed{\frac{1}{2}\int dx x \tilde{H} = (L_z S_z)_q + \frac{1}{2} e_q - \frac{m_q}{2M} \kappa_T^q}$$

➢ Integral relation without connecting to spin-orbit Polyakov et al. (2000)

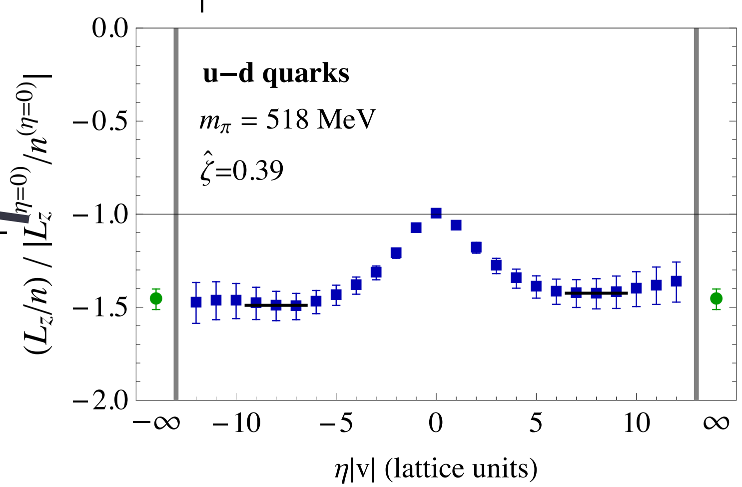
Chiral symmetry breaking test!

Quark sector : $J_q = L_q + \frac{1}{2}\Delta\Sigma_q$

A. Rajan et al, PRD (2016) arXiv:1601.06117



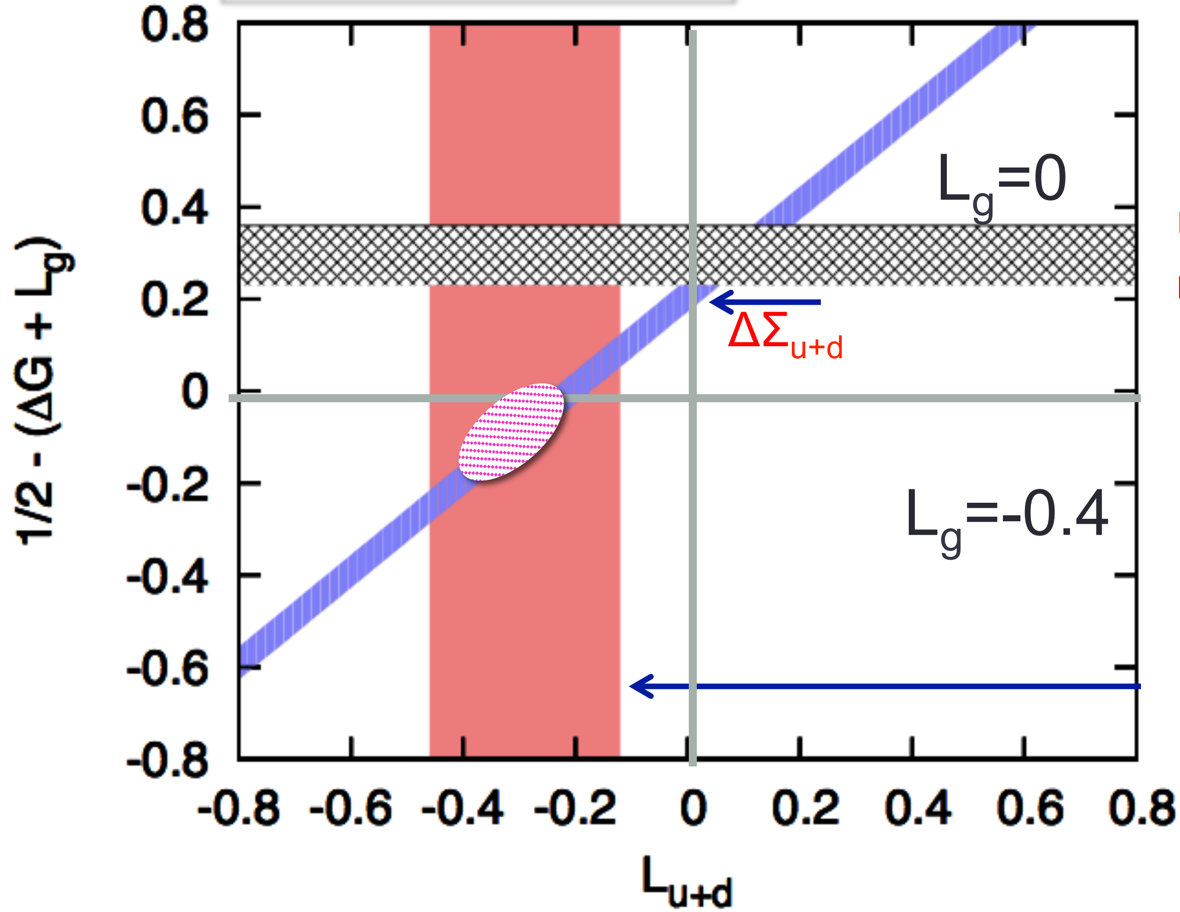
M. Engelhardt (PRD, 2017)



EIC → Adding gluons: Jaffe Manohar Sum Rule

$$\frac{1}{2}\Delta\Sigma_q + L_q + \Delta G + L_g = \frac{1}{2}$$

$$\frac{1}{2} - (\Delta G + L_g^{JM}) = L_q^{JM} + \frac{1}{2}\Delta\Sigma_q$$



Using the "estimated" measured value of ΔG

M. Engelhardt, preliminary

Lattice QCD evaluation of GTMD F_{14}+ gauge link

Conclusions and Outlook

The connection established through the new relations between (G)TMDs and Twist 3 GPDs, not only allows us to evaluate the angular momentum sum rule, it also opens many interesting avenues:

- It allows us to study in detail the role of quark-gluon correlations, in a framework where the role of k_T and off-shellness, k^2, is manifest.
- OAM was obtained so far by subtraction (also in lattice). We can now both calculate OAM on the lattice (GTMD) and validate this through measurements (twist 3 GPD)
- It provides an ideal setting to test renormalization issues, evolution etc…
- QCD studies at the amplitude level shed light on chiral symmetry breaking
- TWIST THREE OBJECTS ARE CRUCIAL TO STUDY QCD AT THE AMPLITUDE LEVEL