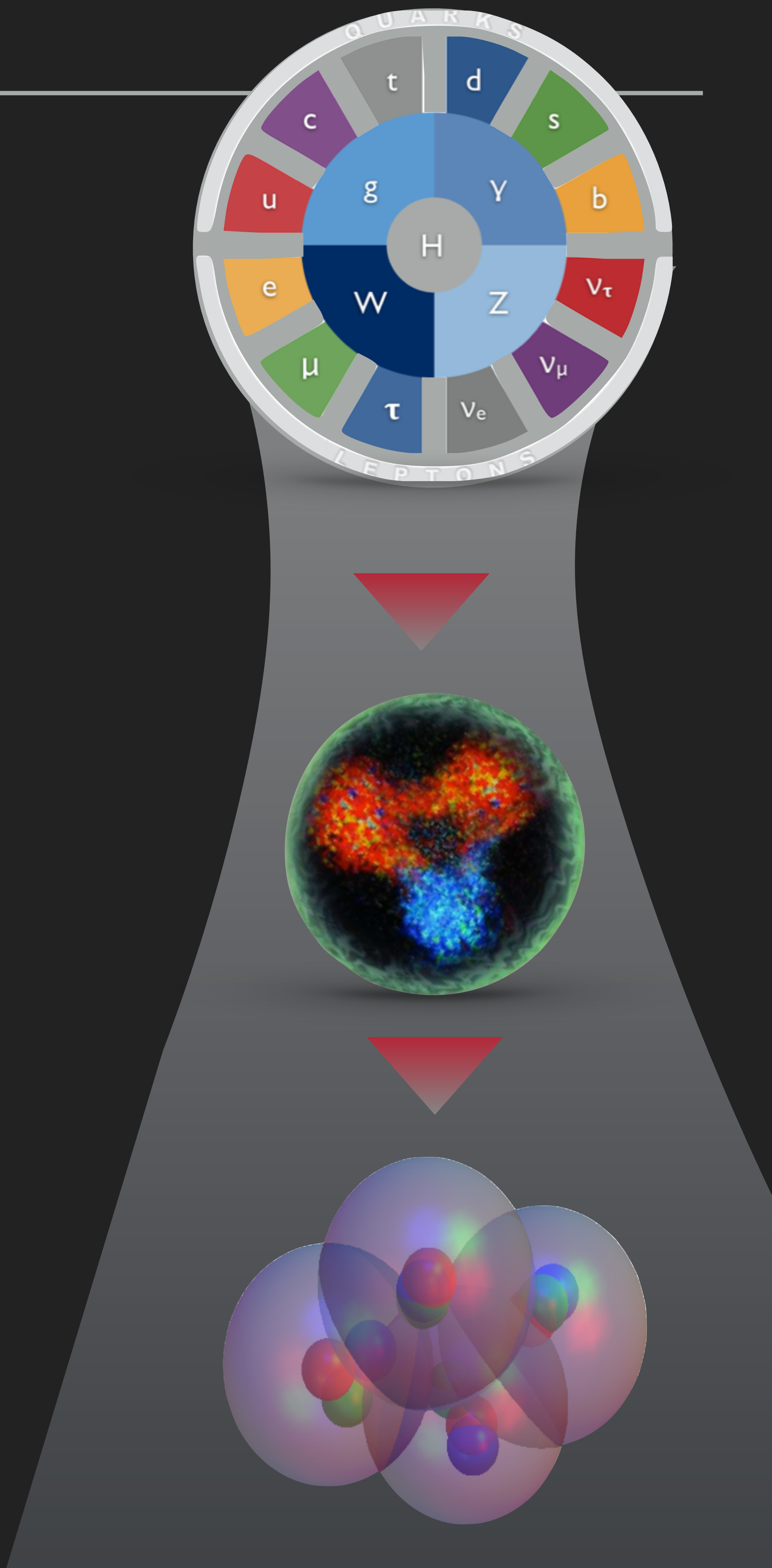
WILL DETMOLD MIT

QUARK STRUCTURE OF NUCLEI

Novel Quark and Gluon effects in nuclei, Trento, April 17th 2018

EMERGENCE OF NUCLEI

- QCD+EW encodes nuclear physics
- Computational challenge to see QCD produce nuclear physics
  - Study emergence of layered complexity of nucleons and nuclei
  - Input for intensity frontier experiments seeking BSM physics
- Lattice QCD calculations can potentially make this connection



NPLQCD: UNPHYSICAL NUCLEI

- Case study LQCD with unphysical quark masses (m_π~800 MeV, 450 MeV)

1. Spectrum and scattering of light nuclei (A<5) [PRD 87 (2013), 034506]
2. Nuclear structure: magnetic moments, polarisabilities (A<5) [PRL **113**, 252001 (2014), PRL **116**, 112301 (2016)]
3. Nuclear reactions: np→dγ [PRL **115**, 132001 (2015)]
4. Gamow-Teller transitions: pp→deν, $g_A(^3H)$ [PRL **119** 062002 (2017)]
5. Double β decay: pp→nn [PRL **119**, 062003 (2017)]
6. Gluon structure (A<4) [PRD **96** 094512 (2017)]
7. Scalar/tensor currents (A<4) [1712.03221➡PRL]

+ Arjun Gambhir (WM→LLNL)

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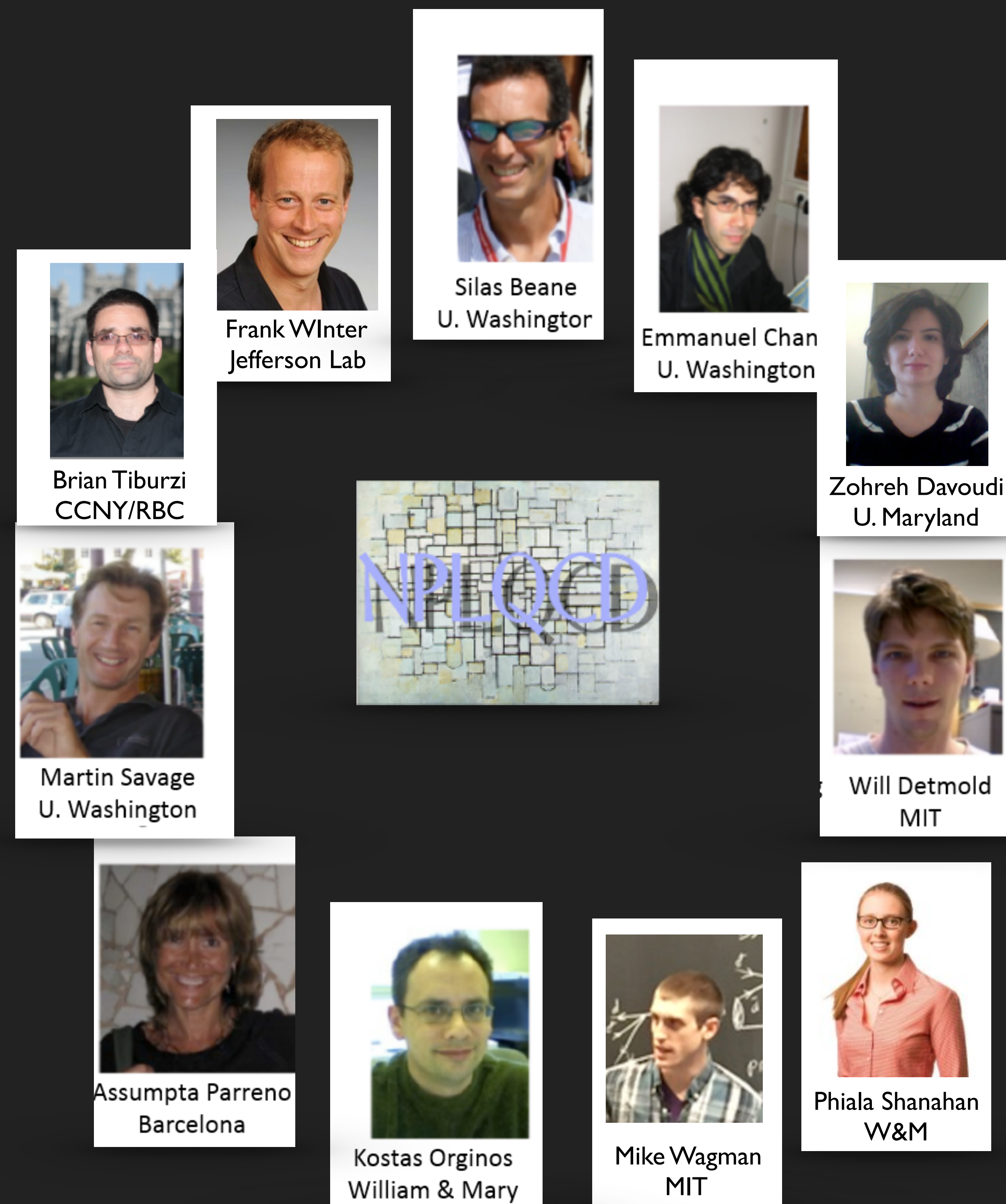

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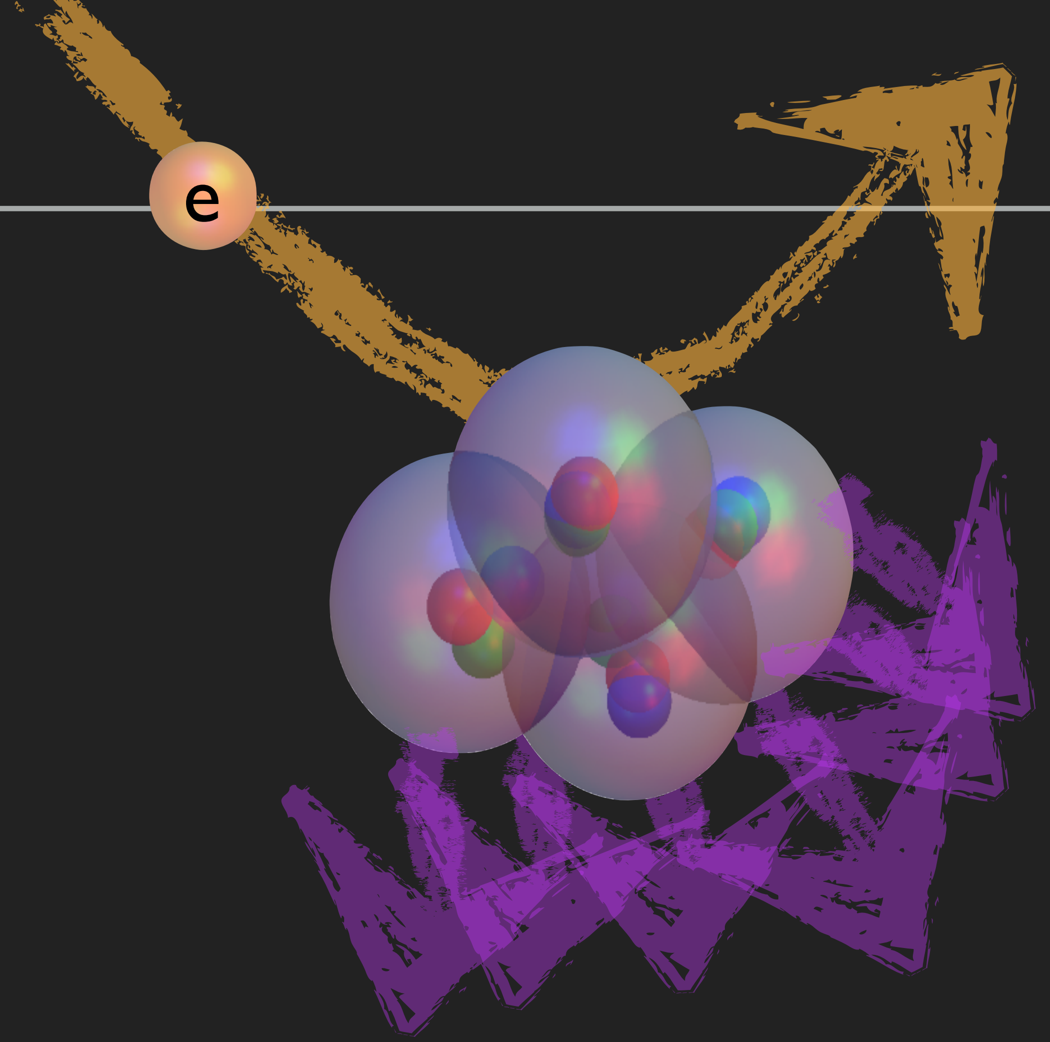
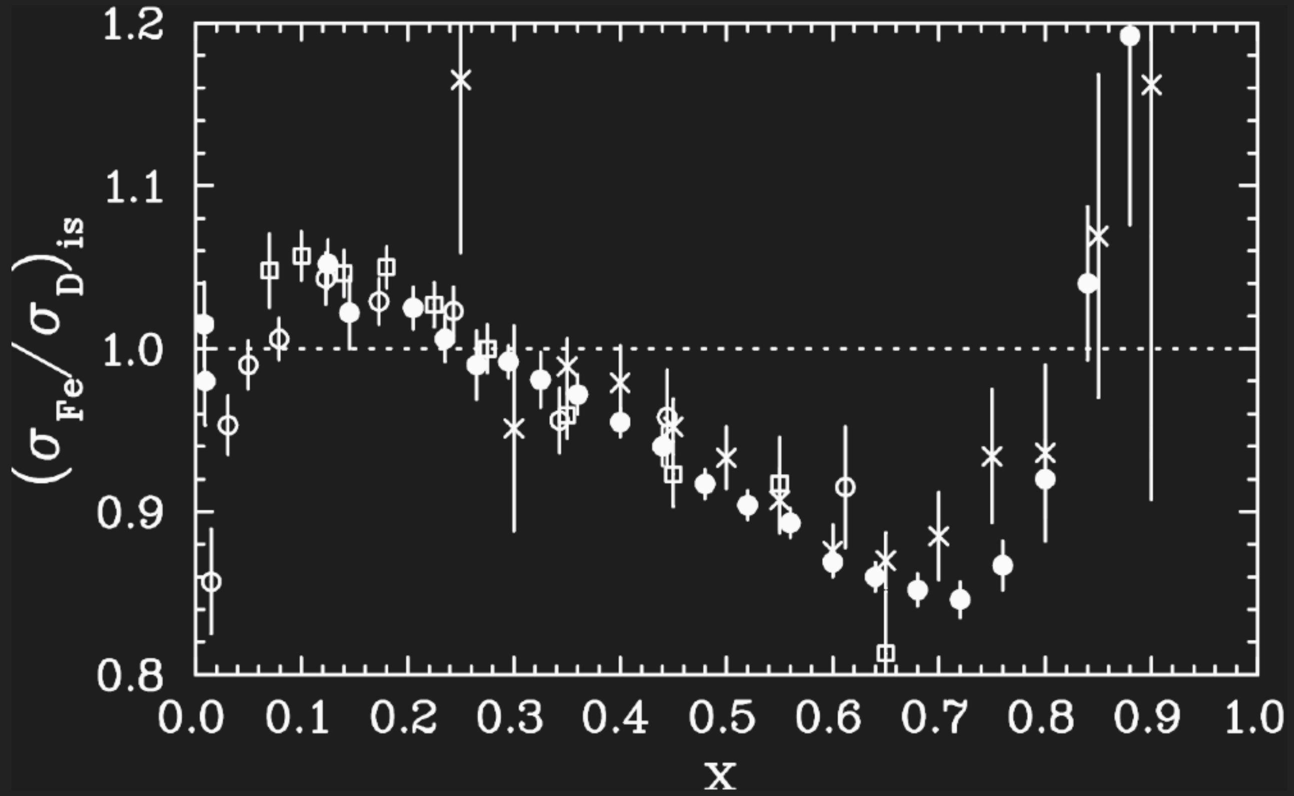


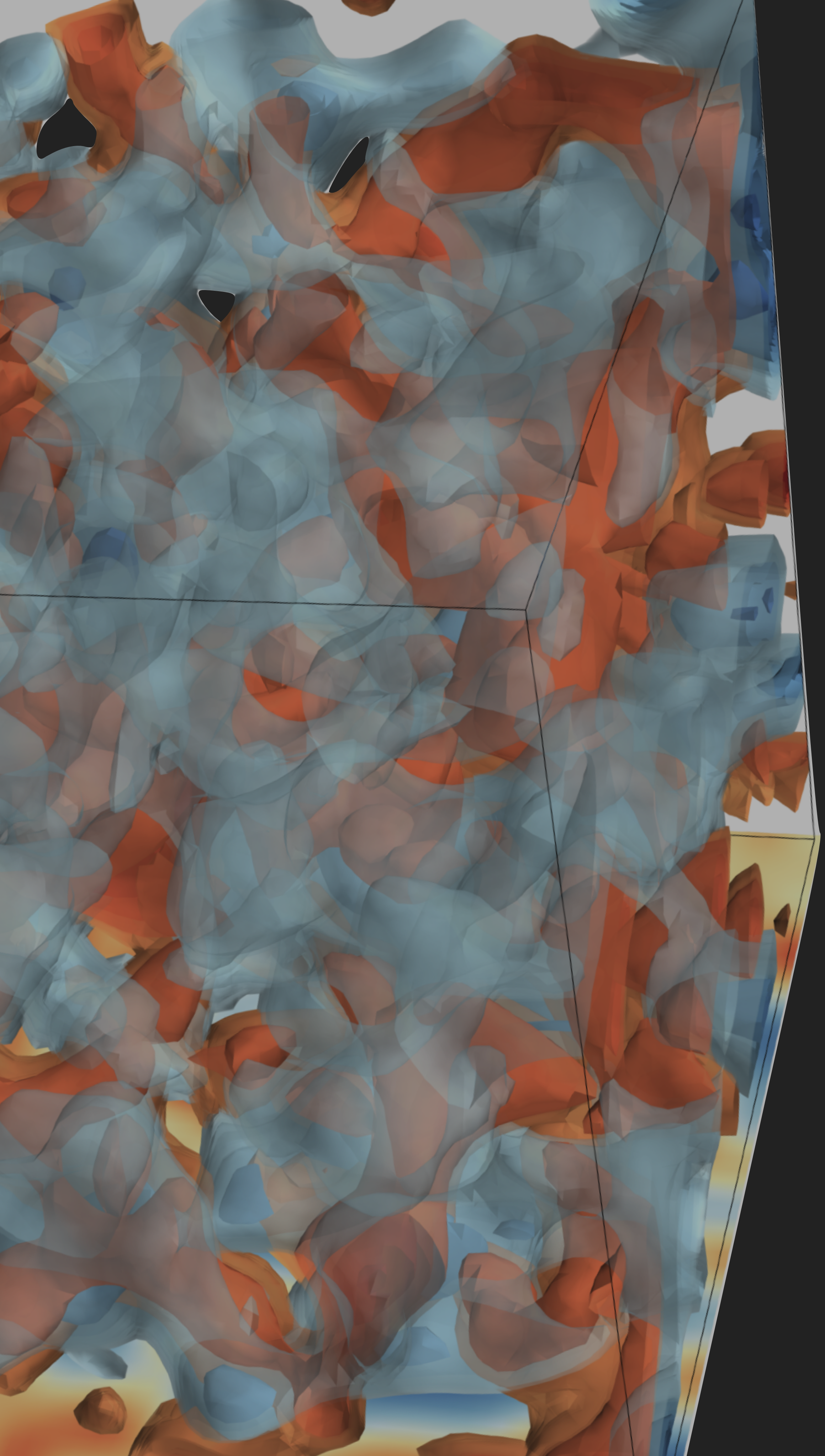
PARTONIC STRUCTURE

- Focus at this workshop is partonic structure
- Currently extending light nuclear calculations to moments of quark PDFs

$$\langle x^n \rangle_{q;A} \sim \langle A, Z | \bar{\psi} \gamma_{\{\mu_0} D_{\mu_1} \ldots D_{\mu_n\}} | A, Z \rangle$$

- Future possibility: calculations of nuclear PDFs using quasi-PDFs accessible in Euclidean space
- … but not today…



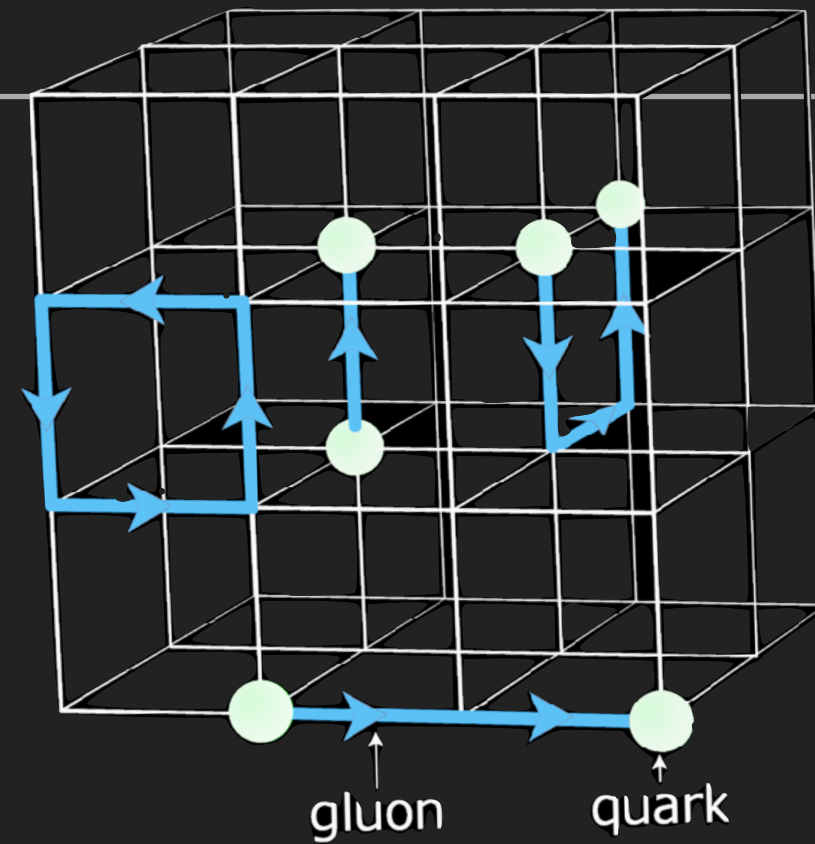


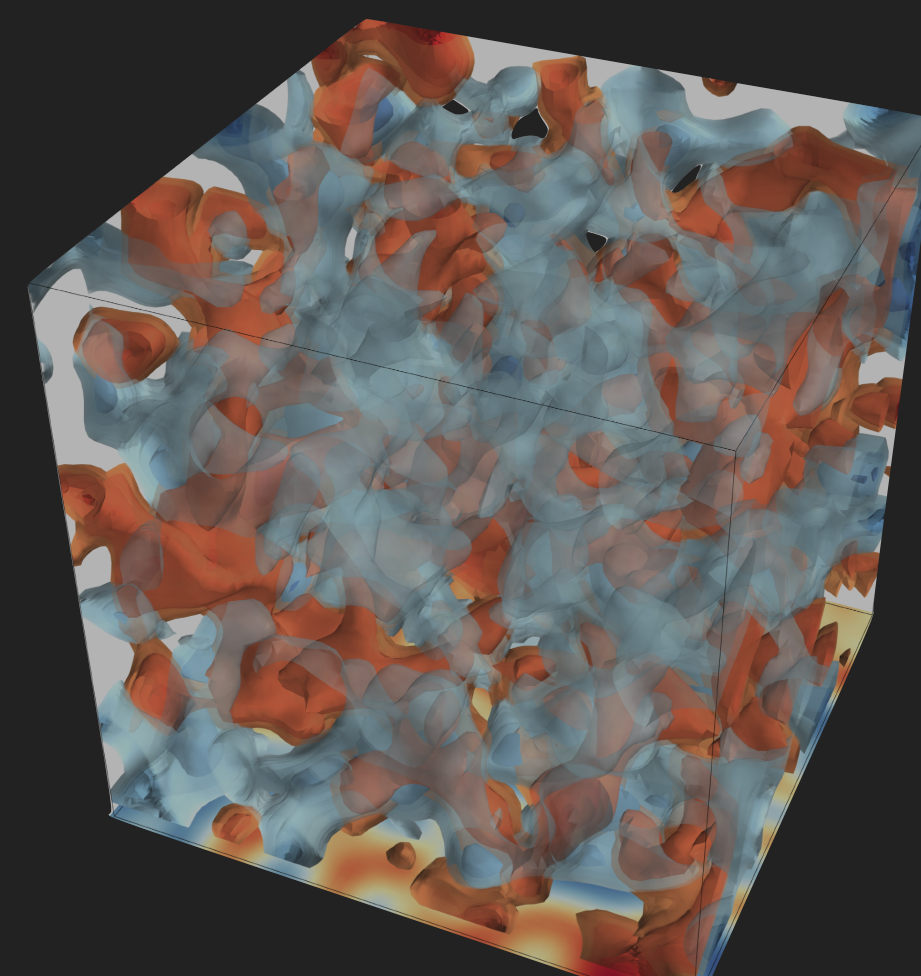
NUCLEAR PHYSICS FROM

LQCD

HIGH FIDELITY LATTICE QCD

- LQCD: strong coupling definition of QCD and method to handle quarks & gluons
- Numerical LQCD entering exciting era
- Modern calculations of simple quantities control all systematics
  - Physical quark masses,
    infinite volume and continuum limits
  - Multiple independent groups
  - Include QED in numerical calculations
- *QCD is the theory of <u>strong</u> strong interactions*





SPECTROSCOPY

- Correlation decays exponentially with distance

$$C(t) = \sum_n Z_n \exp(-E_n t)$$

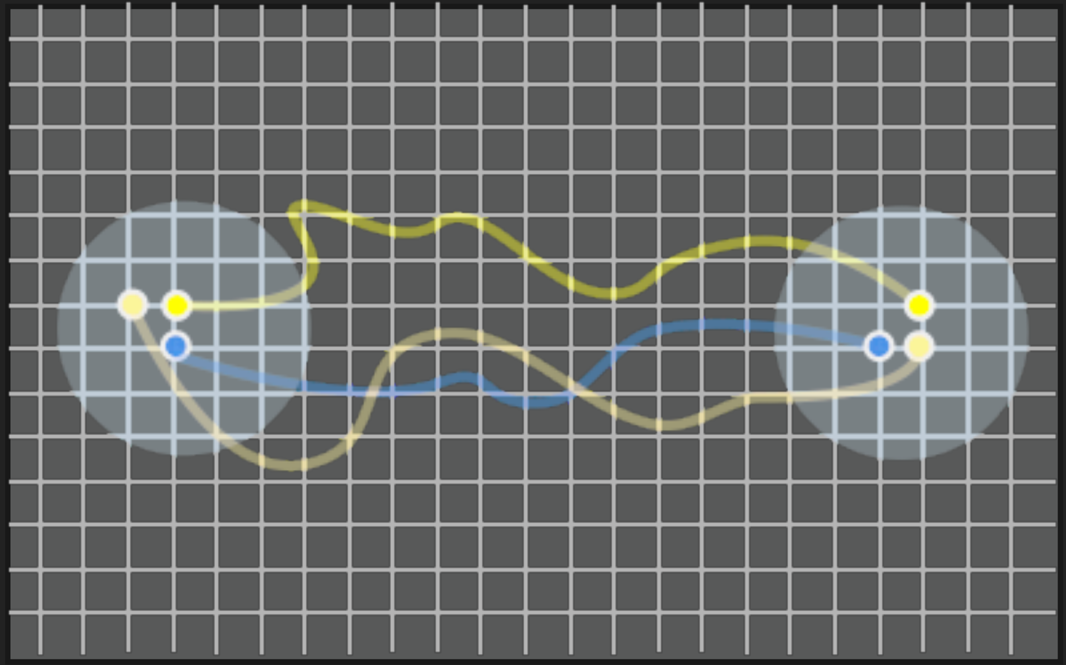
all eigenstates with q#'s of proton

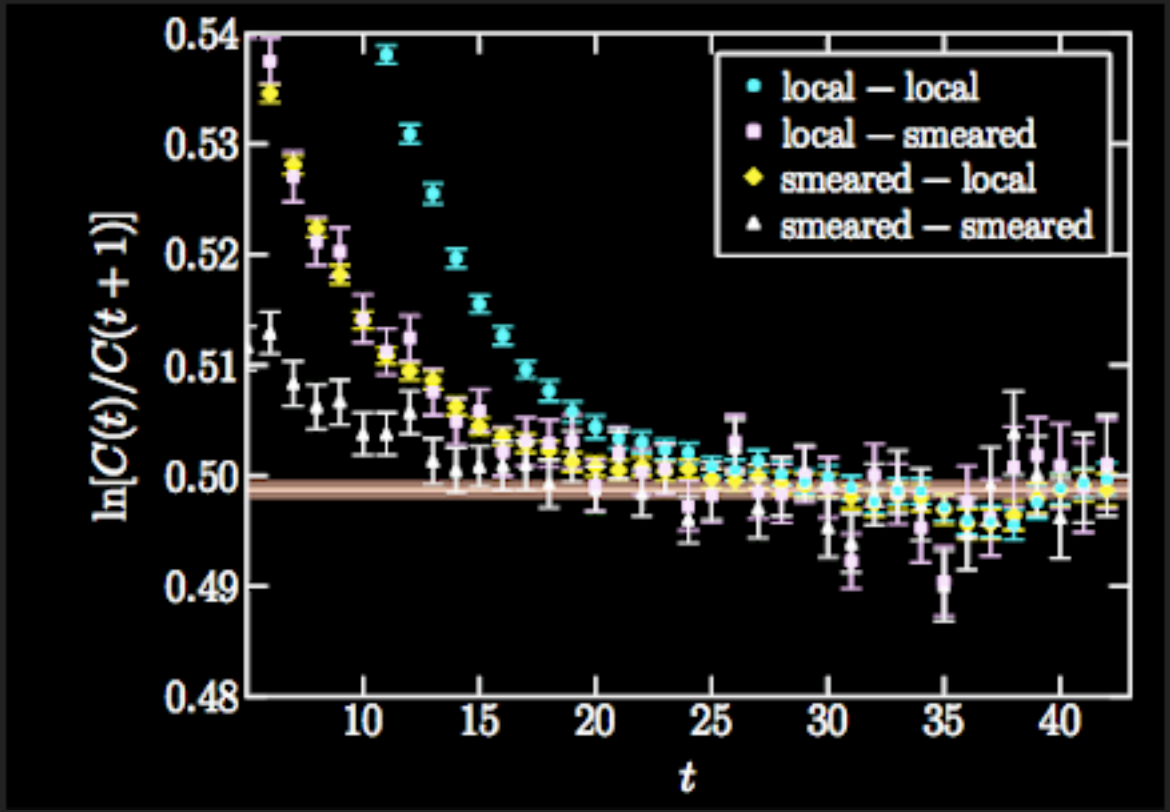
at late times

$$\to Z_0 \exp(-E_0 t)$$

- Ground state mass revealed through "effective mass plot"

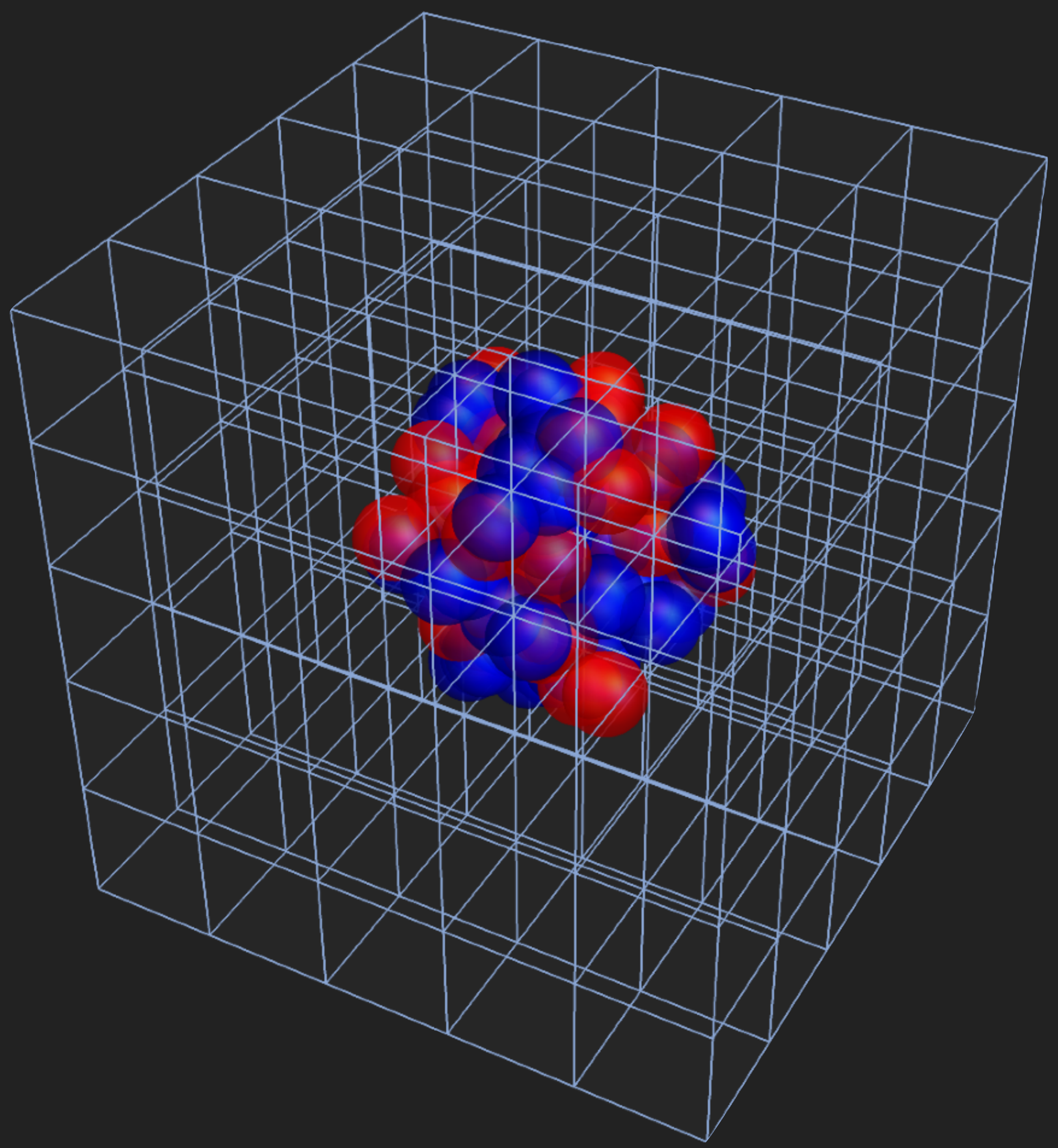
$$M(t) = \ln\left[\frac{C(t)}{C(t+1)}\right] \overset{t\to\infty}{\longrightarrow} E_0$$





QCD FOR NUCLEAR PHYSICS

- Nuclear physics is Standard Model physics
  - Can compute the mass of lead nucleus … in principle



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  - QCD in non-perturbative domain
  - Physics at multiple scales

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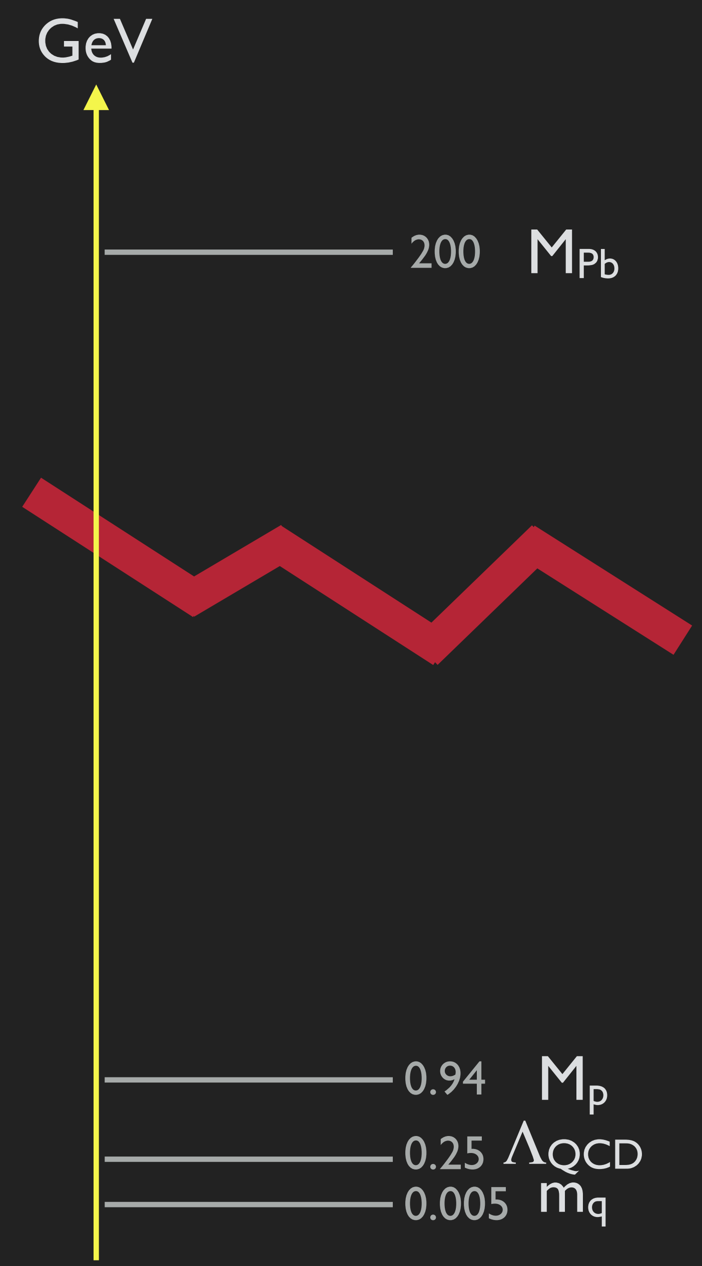

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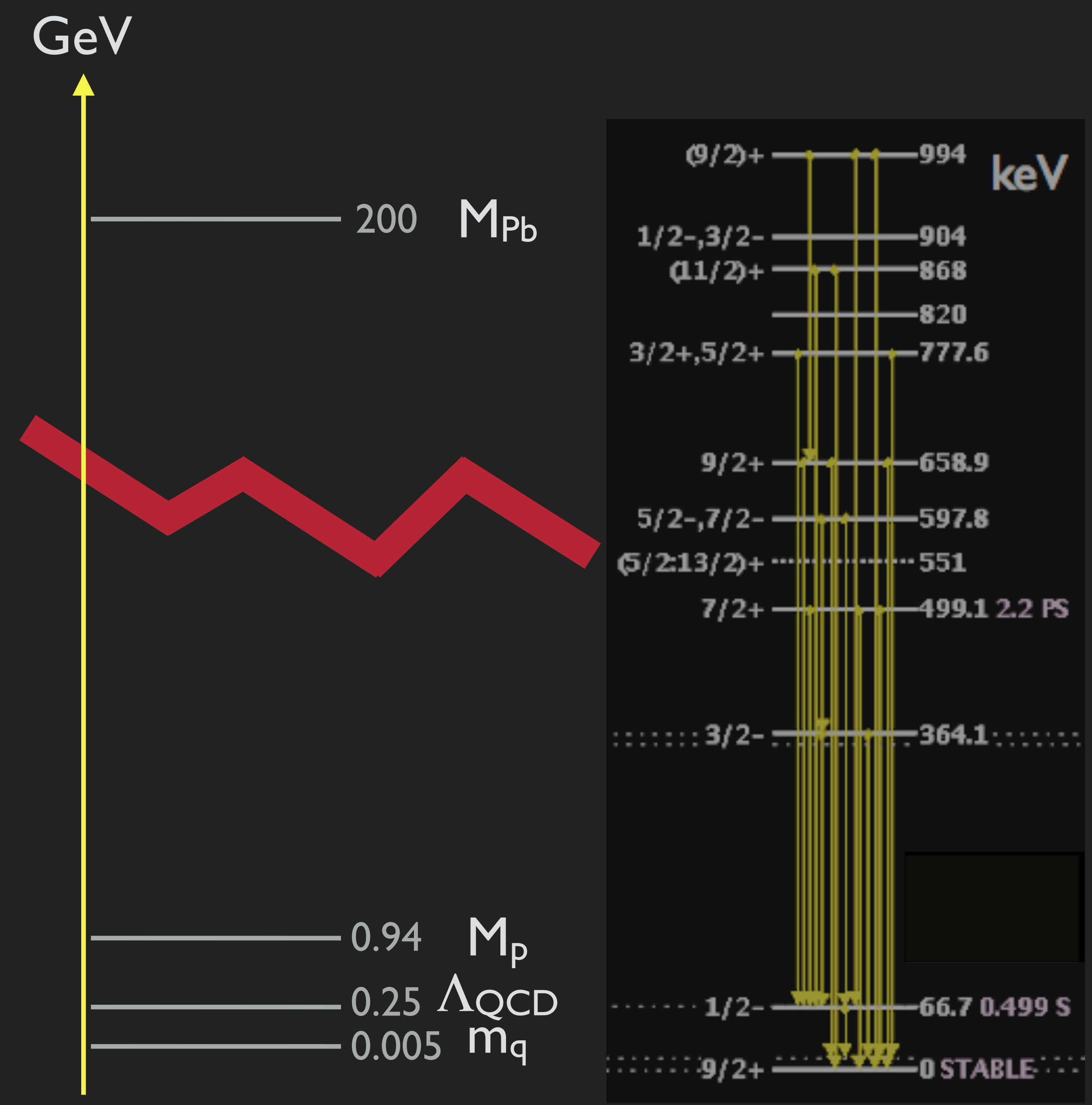

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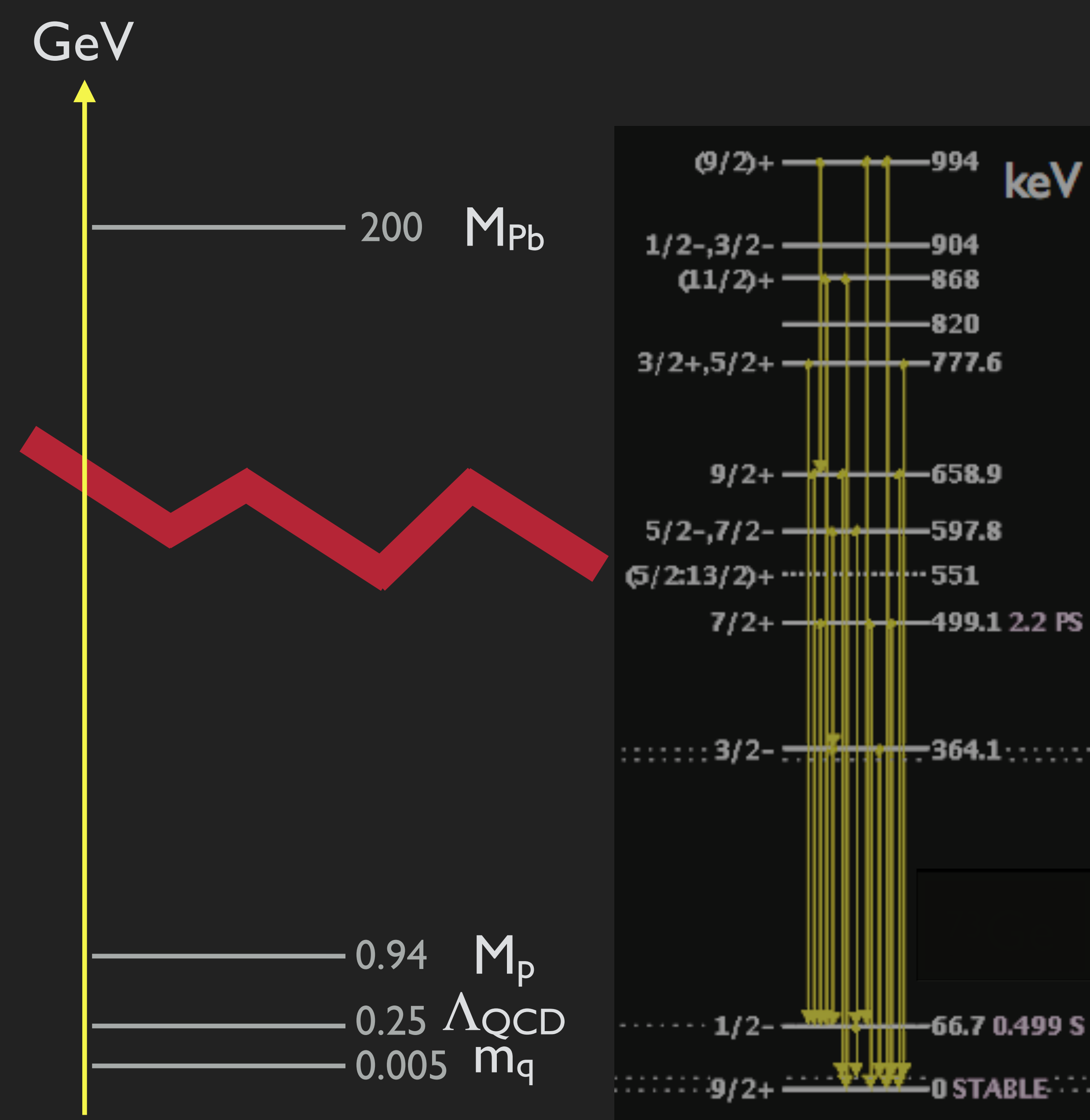
- Nuclear physics is Standard Model physics
  - Can compute the mass of lead nucleus … in principle
- In practice: a hard problem
  - QCD in non-perturbative domain
  - Physics at multiple scales
- At least two exponentially difficult computational challenges
  - Noise: statistical uncertainty grows exponentially with A
  - Contraction complexity grows factorially



STATISTICAL SAMPLING

[Parisi 84, Lepage '89]

STATISTICAL SAMPLING

- Importance sampling of QCD functional integrals
  - ➤ correlators determined stochastically
- Proton

$$\text{signal} \sim \langle C \rangle \sim \exp[-M_p t]$$

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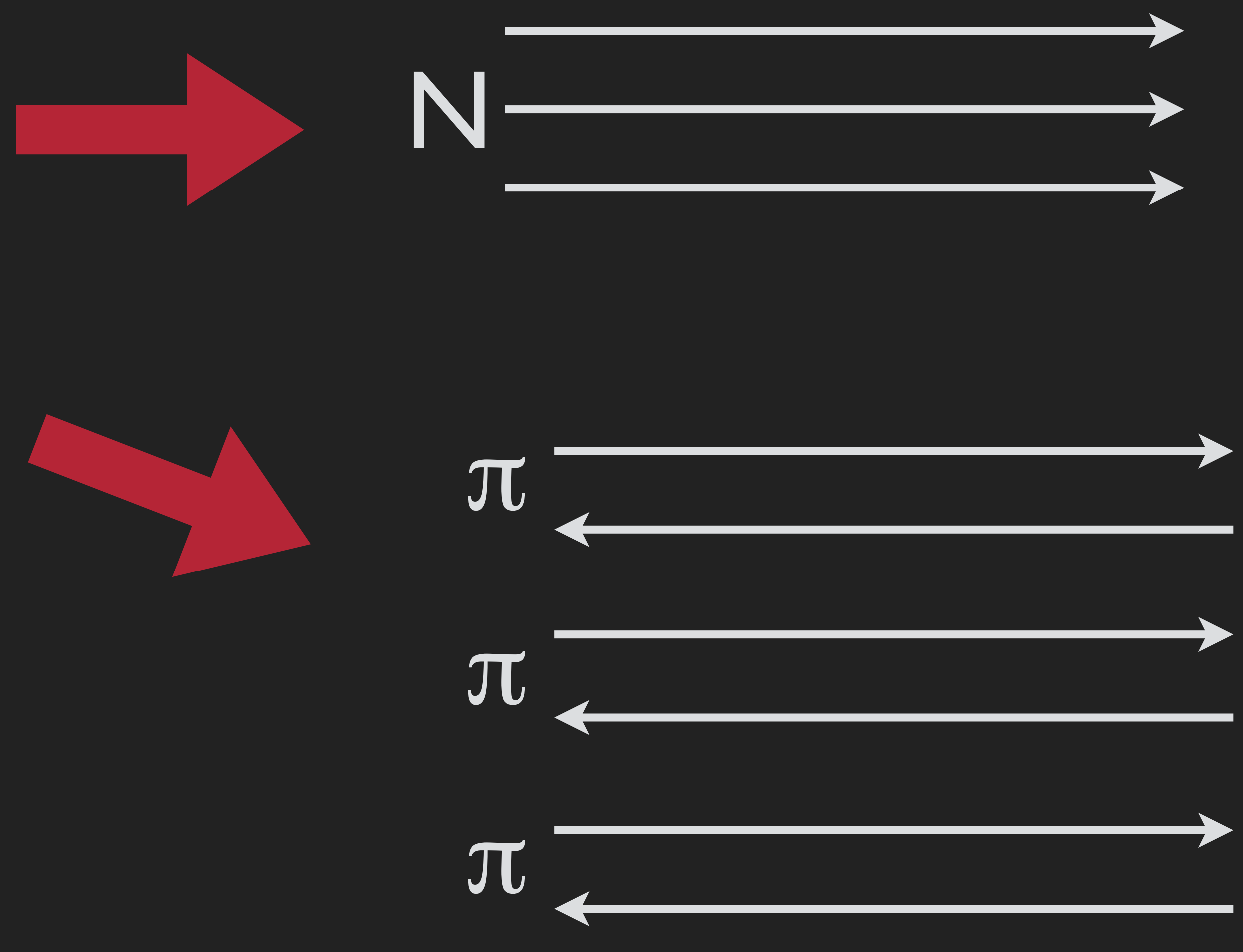

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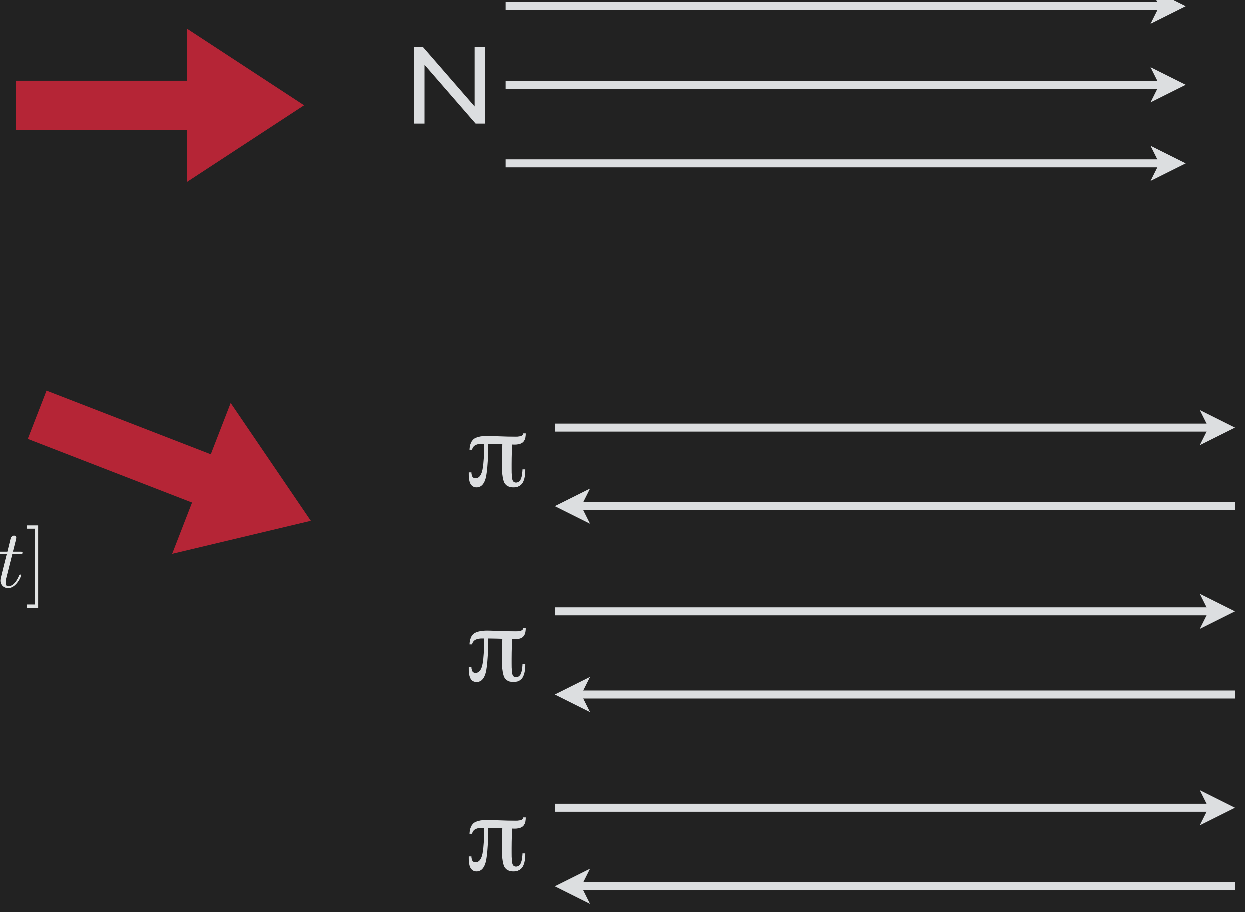
$$\sigma^2(C) = \langle CC^\dagger \rangle - |\langle C \rangle|^2$$

$$\text{noise} \sim \sqrt{\langle CC^\dagger \rangle} \sim \exp[-3/2 m_\pi t]$$

$$\frac{\text{signal}}{\text{noise}} \sim \exp[-(M_p - 3/2 m_\pi)t]$$



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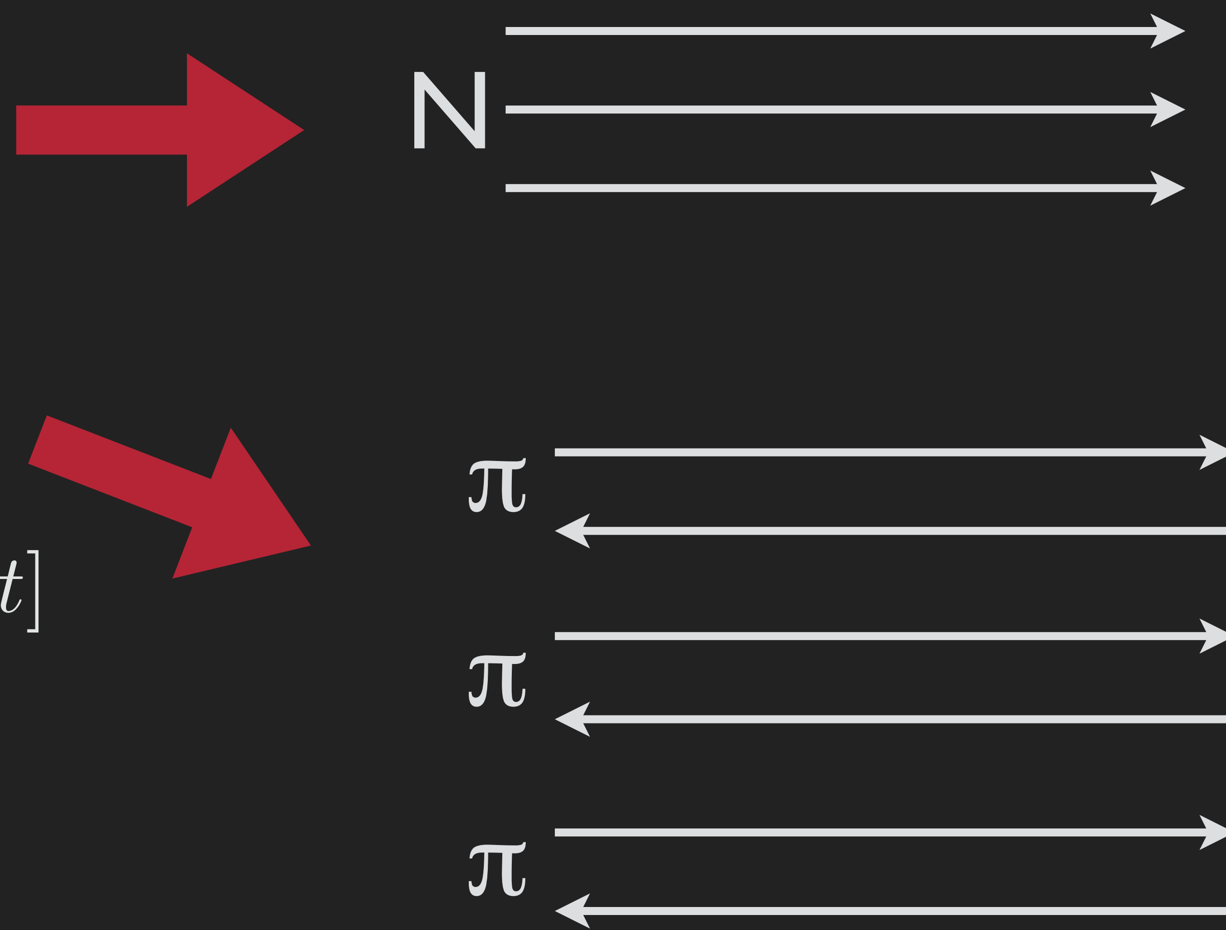

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$$\text{noise} \sim \sqrt{\langle CC^\dagger \rangle} \sim \exp[-3/2 m_\pi t]$$

$$\frac{\text{signal}}{\text{noise}} \sim \exp[-(M_p - 3/2 m_\pi)t]$$

- For nucleus A:

$$\frac{\text{signal}}{\text{noise}} \sim \exp[-A(M_p - 3/2 m_\pi)t]$$

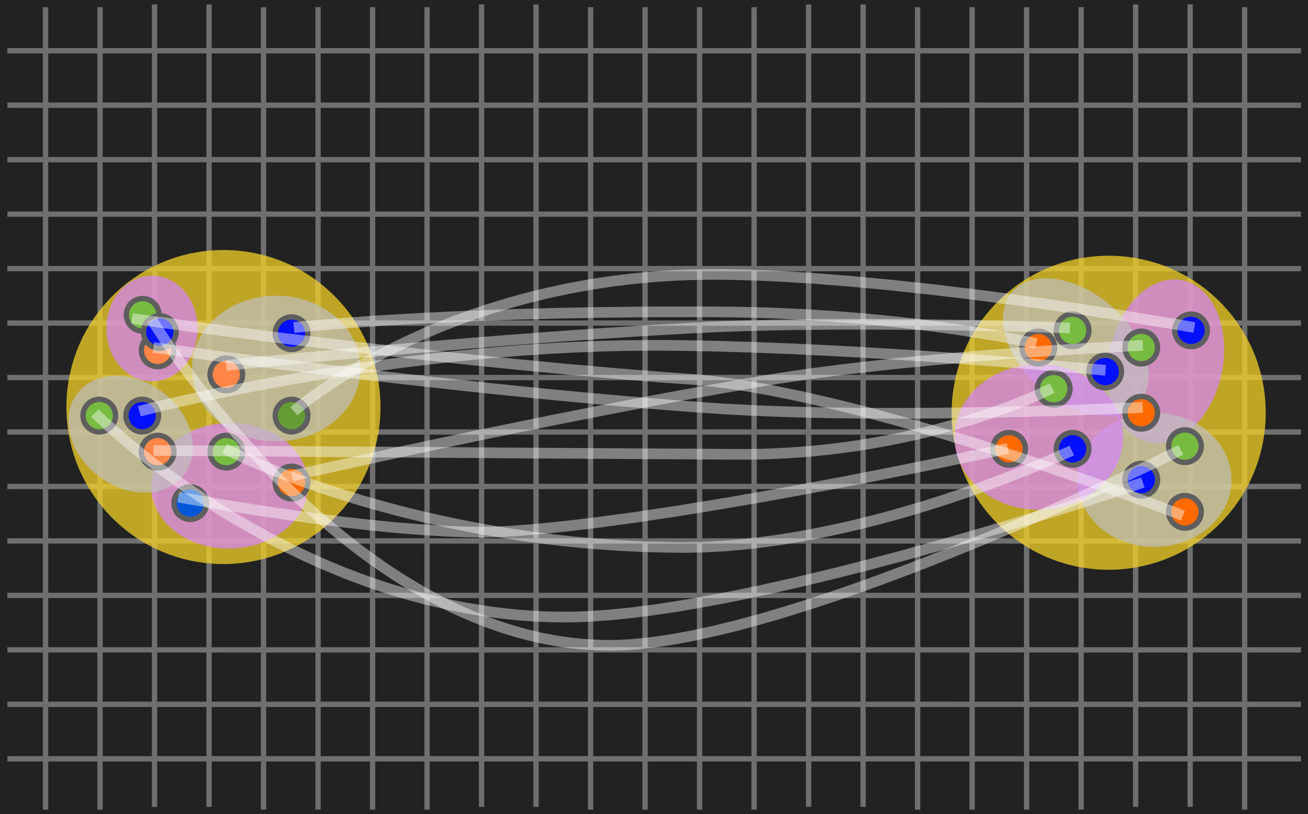
[Parisi 84, Lepage '89]

STATISTICAL SAMPLING

- Valid asymptotically but interpolator choice can suppress overlap onto noise
  - Golden window of time slices from which to extract physics
- Use variational operator construction to optimise overlap onto low eigenstates at earlier times [Michael,Lüscher&Wolff]
- Optimisation problem involving variance correlation function $<CC^\dagger>$ to maximise signal-noise ratio [WD & Mike Endres, PRD 2014]
- New method of phase reweighing/unwrapping [Wagman, Savage 2016,7]

CONTRACTIONS

- Quarks need to be tied together in all possible ways
  - $N_{contractions} = N_u!N_d!N_s!$ (eg~10^{1500} for ^{208}Pb)



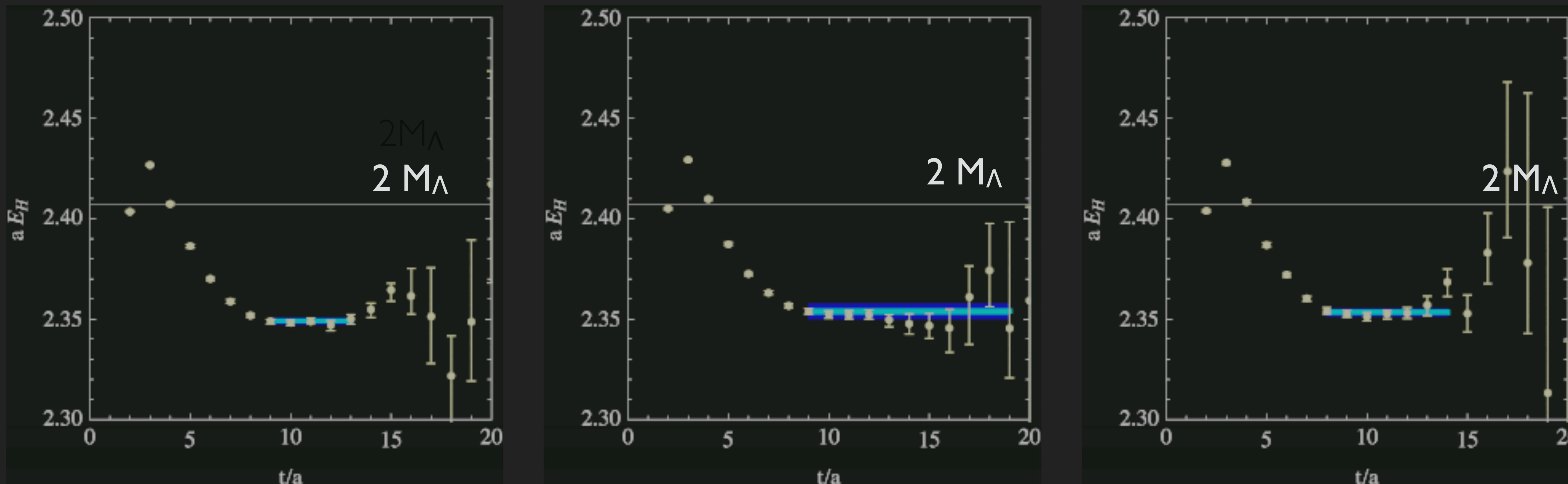
- Managed using algorithmic trickery [WD & Savage, WD & Orginos; Doi & Endres, Günther et al]
  - Study up to N=72 pion systems, A=5 (and 28) nuclei

CASE STUDY OF

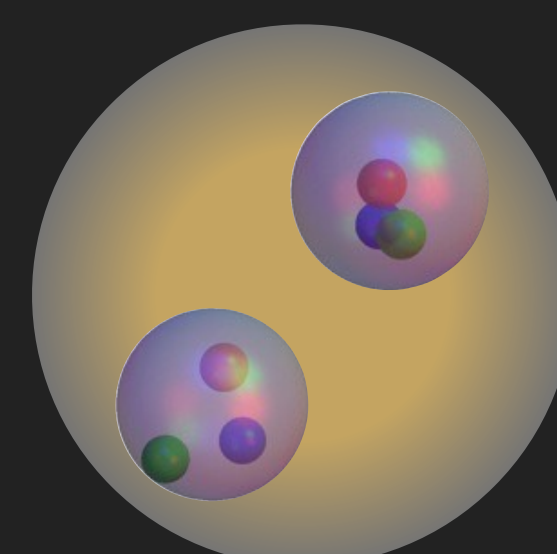
NUCLEI IN LQCD

EX: H DIBARYON (ΛΛ)

$24^3 \times 48$ | $32^3 \times 48$ | $48^3 \times 64$



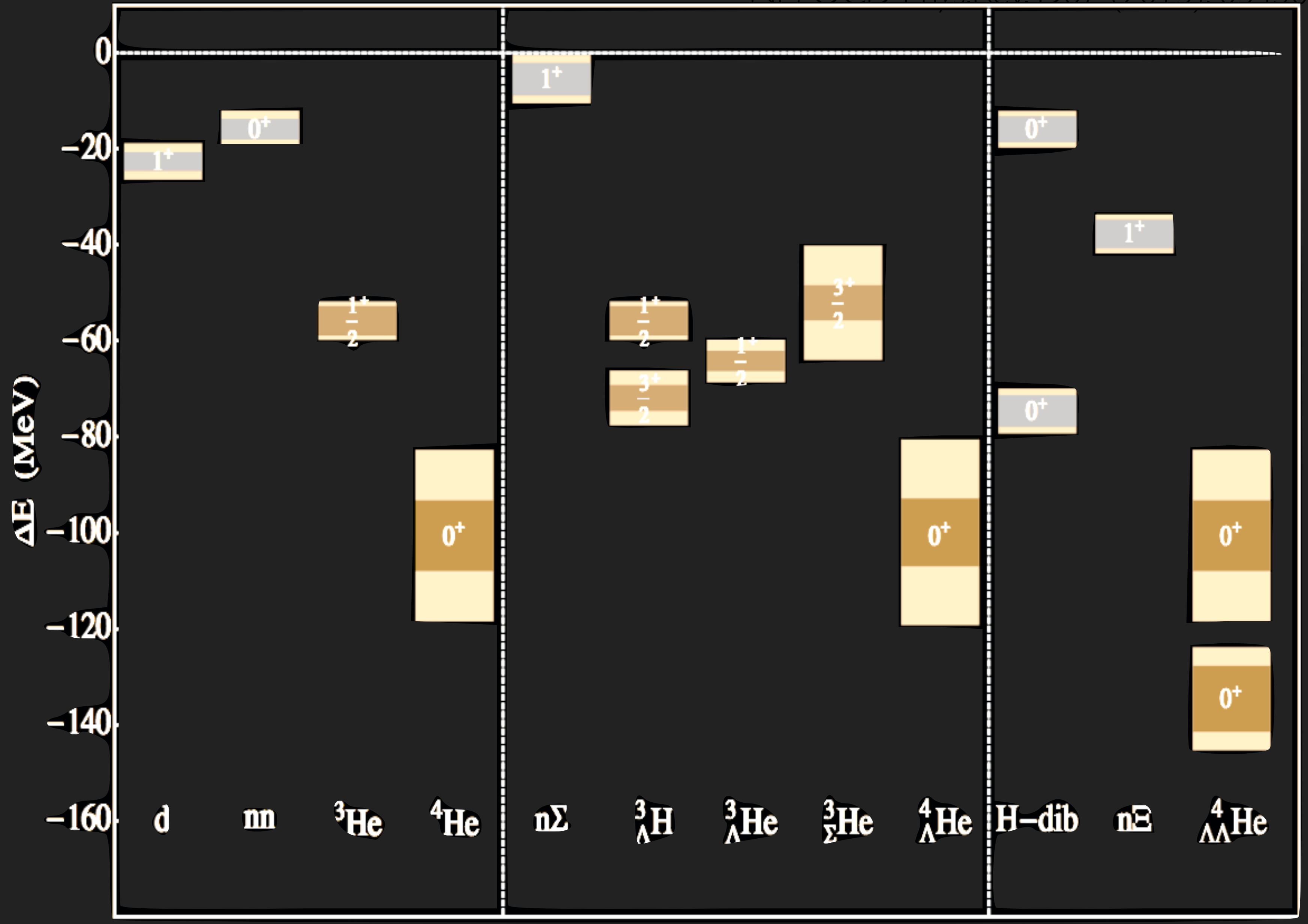
- Effective mass plots of energies
- Multiple volumes needed to disentangle bound state from attractive scattering state



LIGHT NUCLEI AND HYPERNUCLEI

- Light hypernuclear binding energies @ m_π=800 MeV

NPLQCD Phys.Rev. D87 (2013), 034506



Magnetic moments and polarisabilities of nuclei

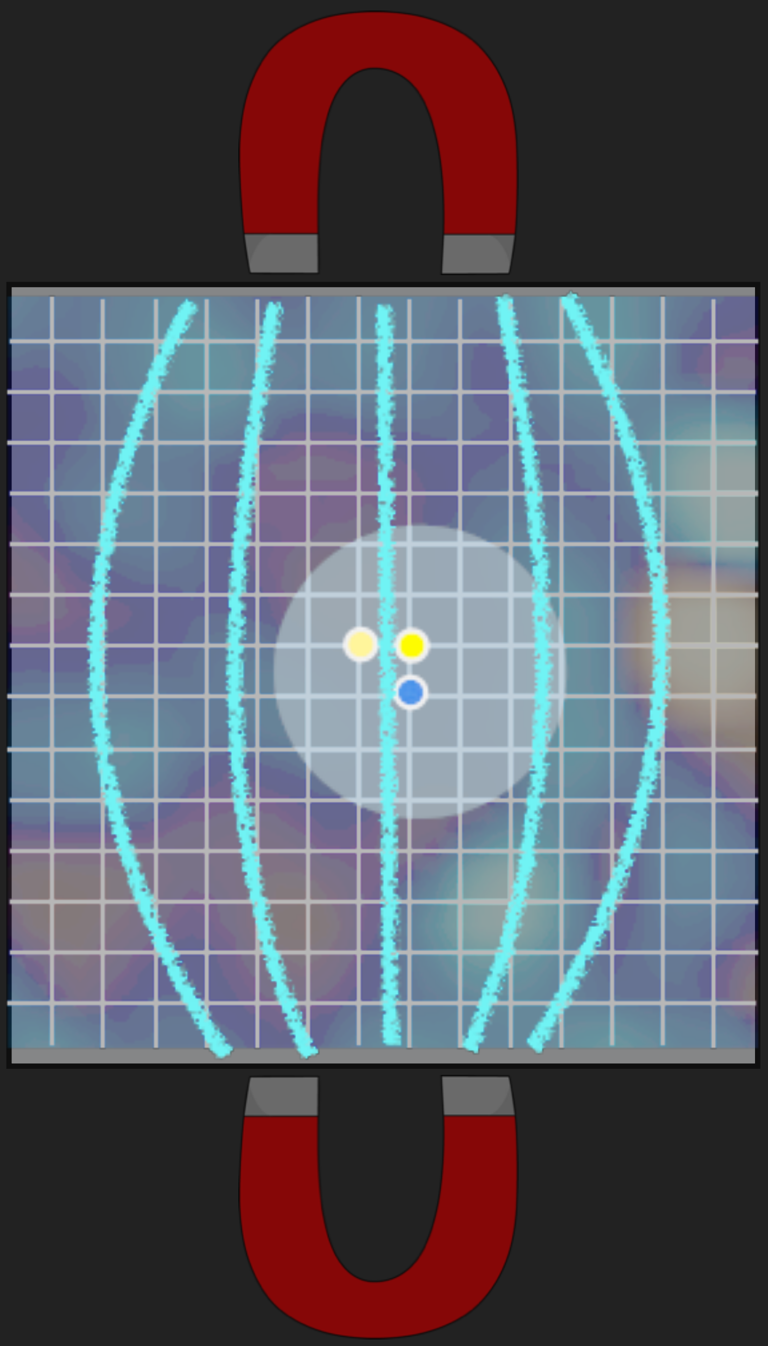
[NPLQCD PRL **113**, 252001 (2014)]

MAGNETIC MOMENTS

- Hadron/nuclear energies are modified by presence of fixed external fields
- Eg: fixed B field

$$E_{h;j_z}(\mathbf{B}) = \sqrt{M_h^2 + (2n+1)|Q_h eB|} - \boldsymbol{\mu}_h \cdot \mathbf{B} - 2\pi\beta_h^{(M0)}|\mathbf{B}|^2 + \ldots$$

- QCD calculations with multiple fields enable extraction of coefficients of response
    - Magnetic moments, polarisabilities, …



[NPLQCD PRL **113**, 252001 (2014)]

MAGNETIC MOMENTS OF NUCLEI

- Magnetic field in *z*-direction (quantised n)*

$$U_\mu^{\rm QCD} \longrightarrow U_\mu^{\rm QCD} \cdot U_\mu^{(Q)} \qquad \text{(gluon links)}$$

$$U_\mu^{(Q)}(x) \;=\; e^{i\frac{6\pi Q_q \tilde{n}}{L^2} x_1 \delta_{\mu,2}} \times e^{-i\frac{6\pi Q_q \tilde{n}}{L} x_2 \delta_{\mu,1}\delta_{x_1,L-1}}$$

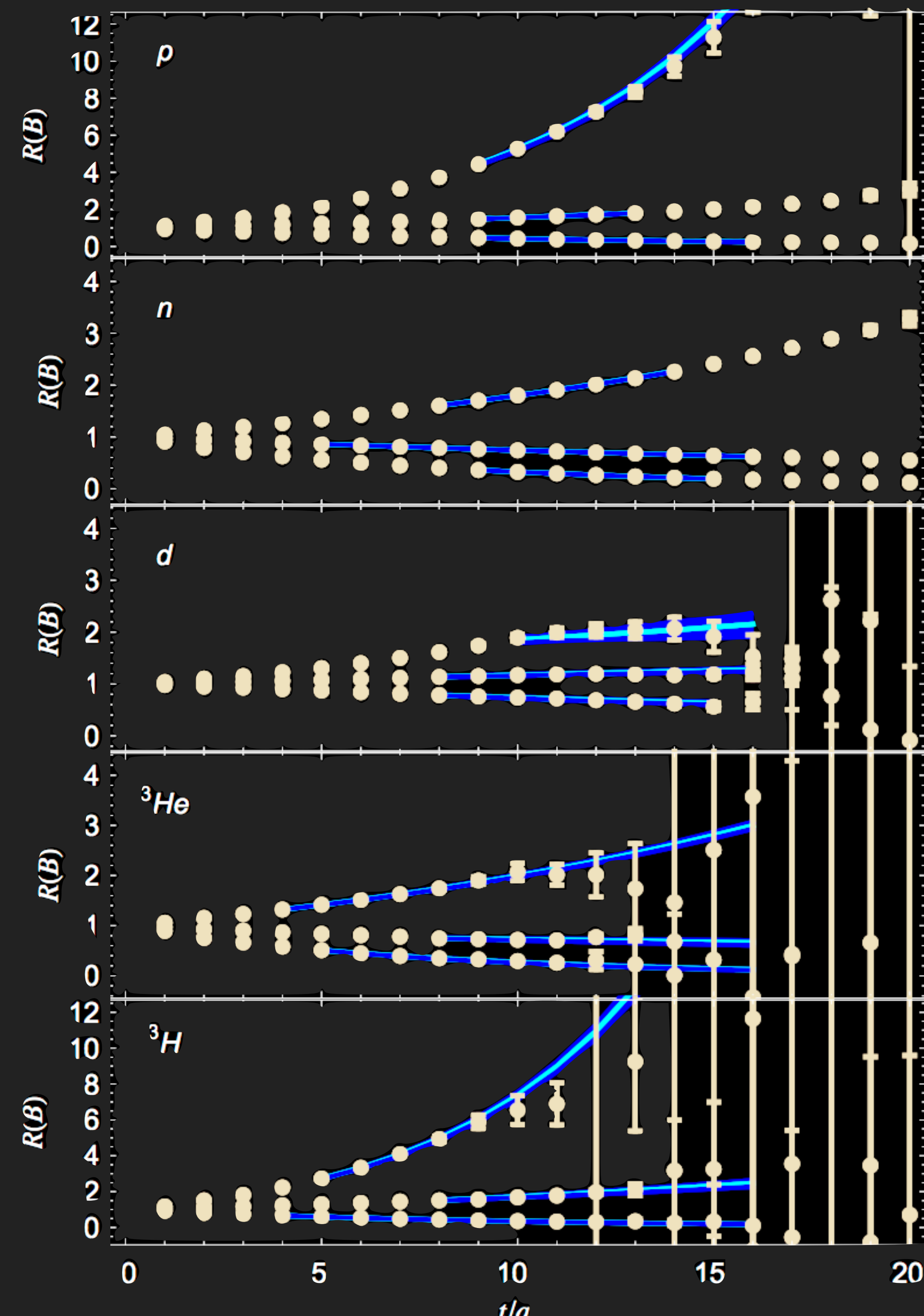
- Magnetic moments from spin splittings

$$\delta E^{(B)} \equiv E^{(B)}_{+j} - E^{(B)}_{-j} = -2\mu|\mathbf{B}| + \gamma|\mathbf{B}|^3 + \ldots$$

- Extract splittings from ratios of two-point correlation functions

$$R(B) = \frac{C_j^{(B)}(t)\; C_{-j}^{(0)}(t)}{C_{-j}^{(B)}(t)\; C_j^{(0)}(t)} \;\stackrel{t\to\infty}{\longrightarrow}\; Z e^{-\delta E^{(B)} t}$$

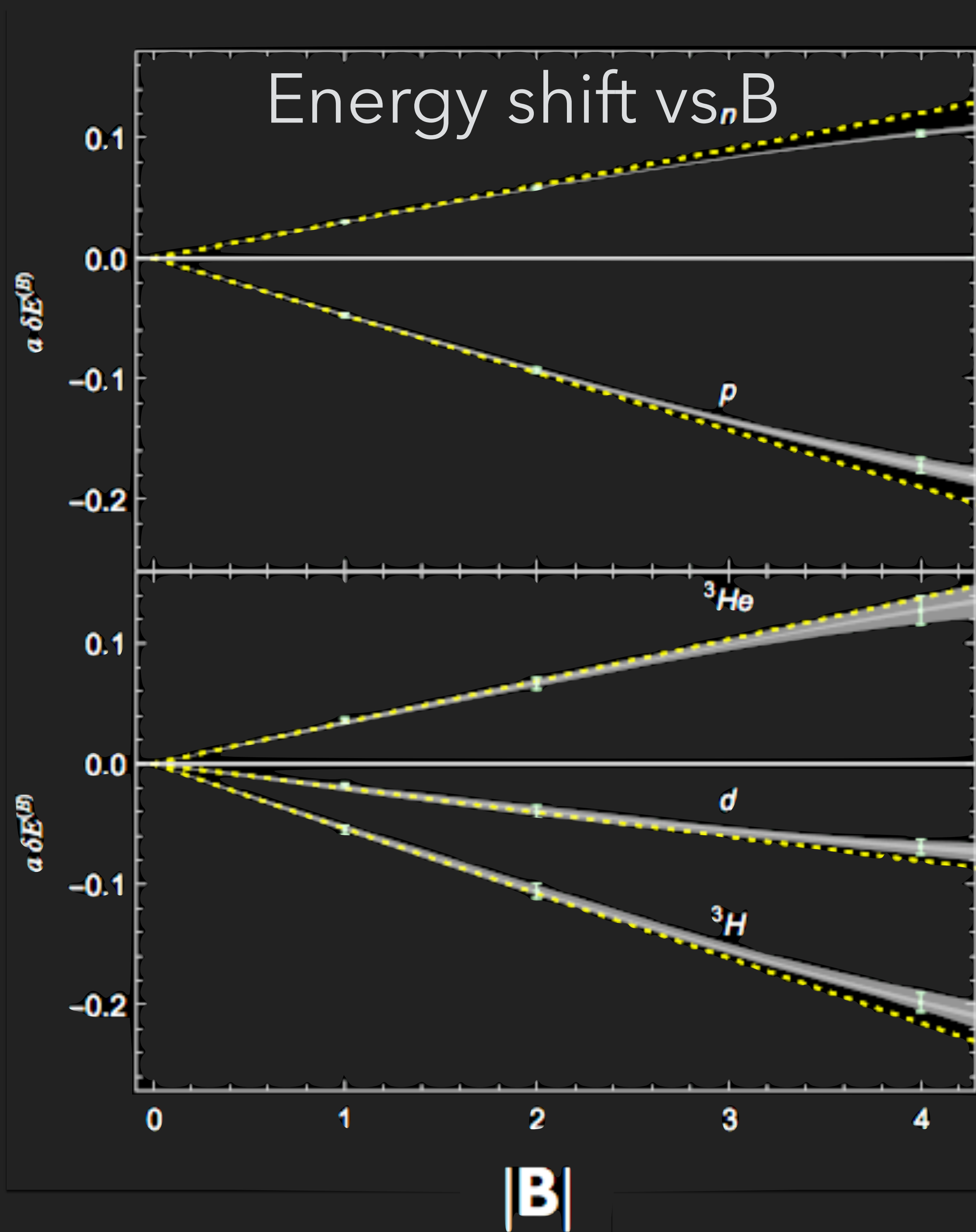
- Careful to be in single exponential region of each correlator

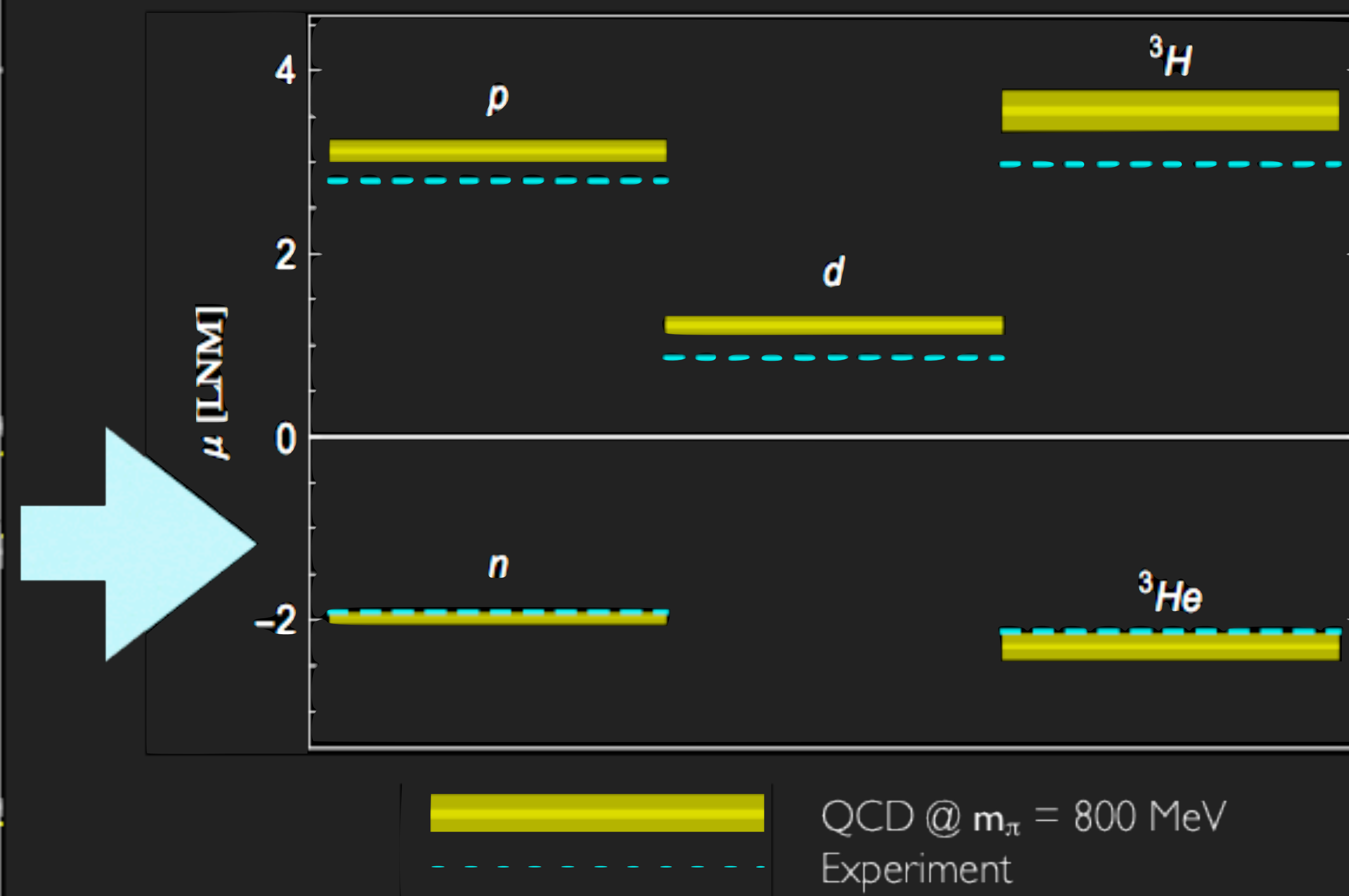


* post applied U(1) field exact since tr[Q]=0

[NPLQCD PRL **113**, 252001 (2014)]

MAGNETIC MOMENTS OF NUCLEI





	n	p	d	^{3}He	^{3}H
μ	-1.98(1)(2)	3.21(3)(6)	1.22(4)(9)	-2.29(3)(12)	3.56(5)(18)

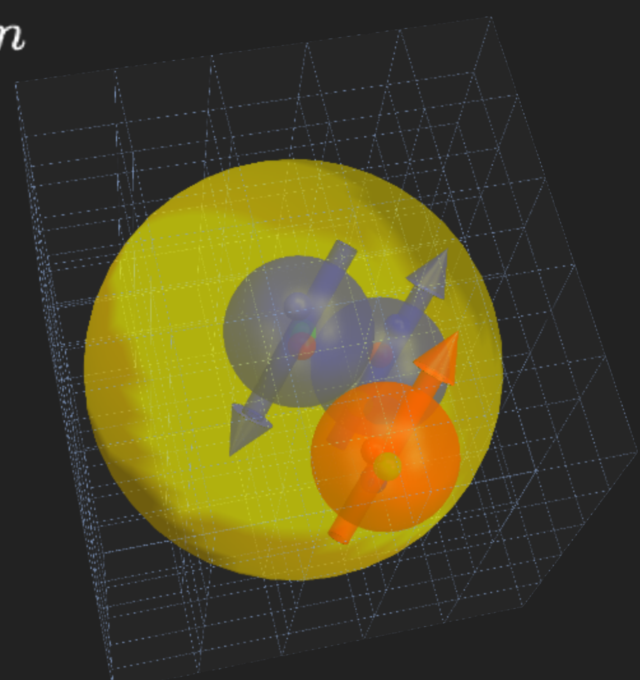
In units of appropriate nuclear magnetons (heavy M_N)

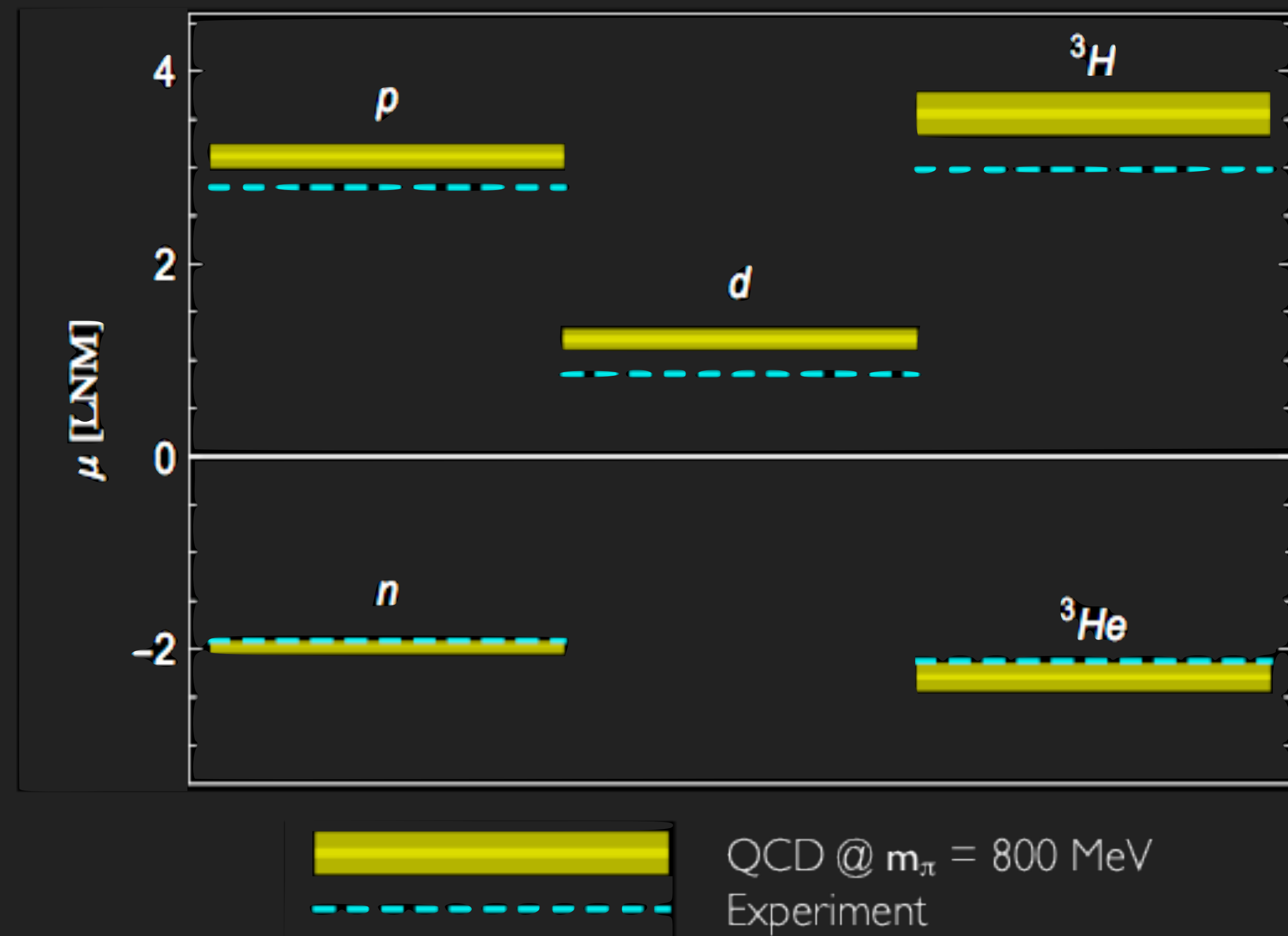
[NPLQCD PRL **113**, 252001 (2014)]

MAGNETIC MOMENTS OF NUCLEI

- Numerical values are surprisingly interesting
- Shell model expectations

$$\mu_d = \mu_p + \mu_n$$
$$\mu_{^3\mathrm{H}} = \mu_p$$
$$\mu_{^3\mathrm{He}} = \mu_n$$





- Lattice results appear to suggest heavy quark nuclei are shell-model like!

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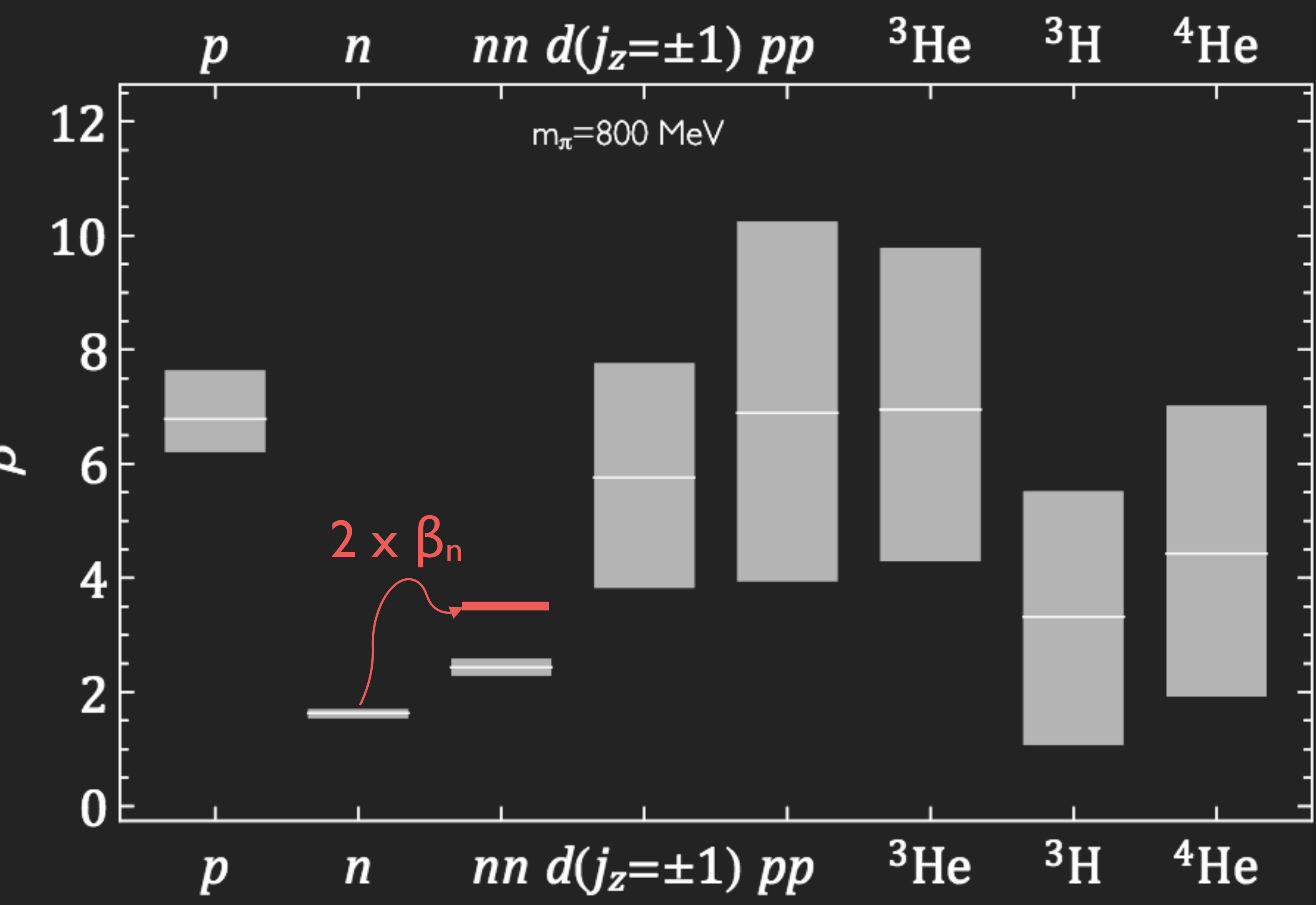
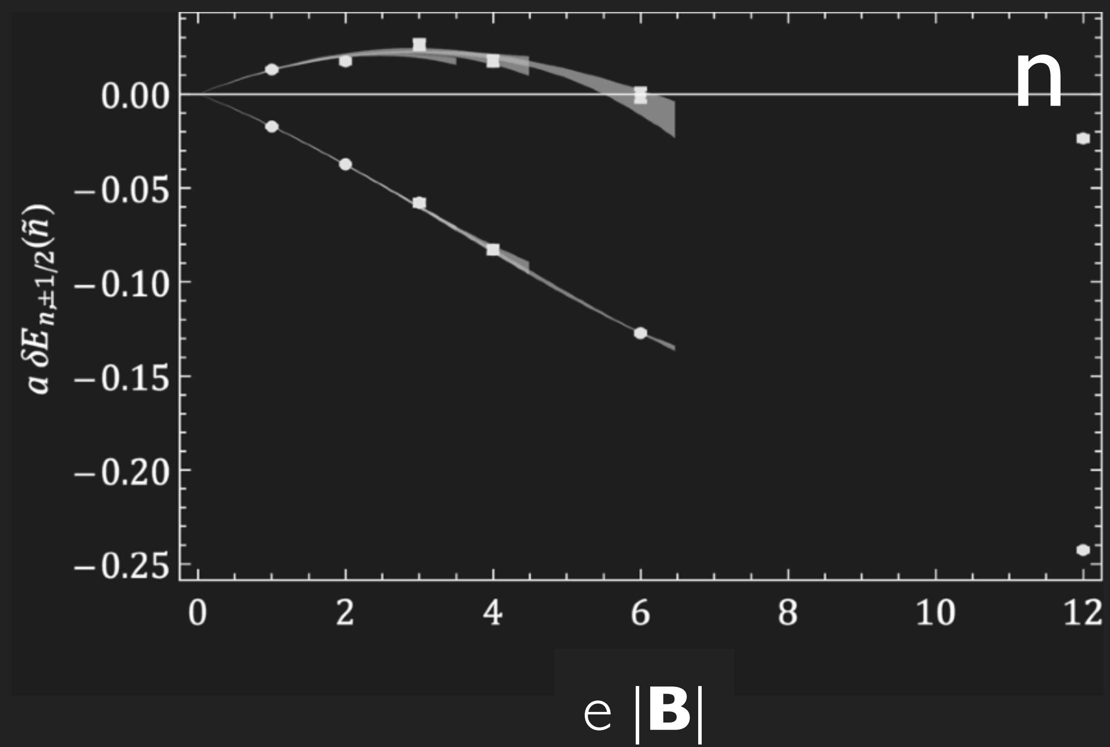
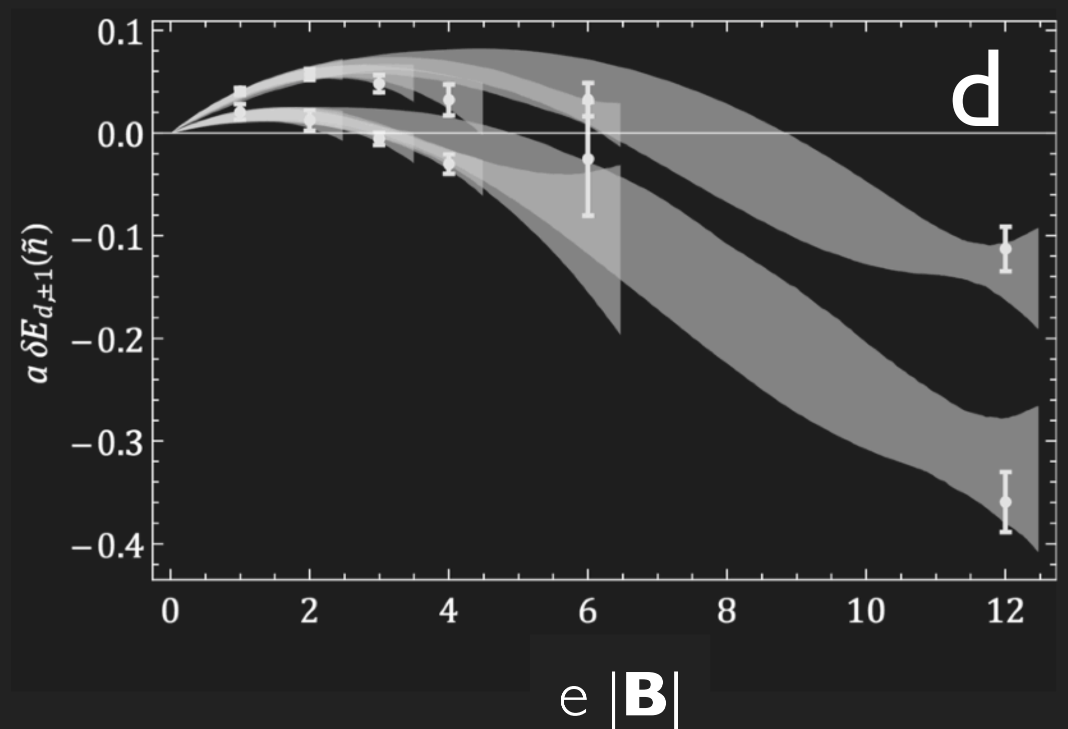
In units of appropriate nuclear magnetons (heavy M_N)

MAGNETIC POLARISABILITIES

- Care required with Landau levels
- Polarisabilities (dimensionless units)





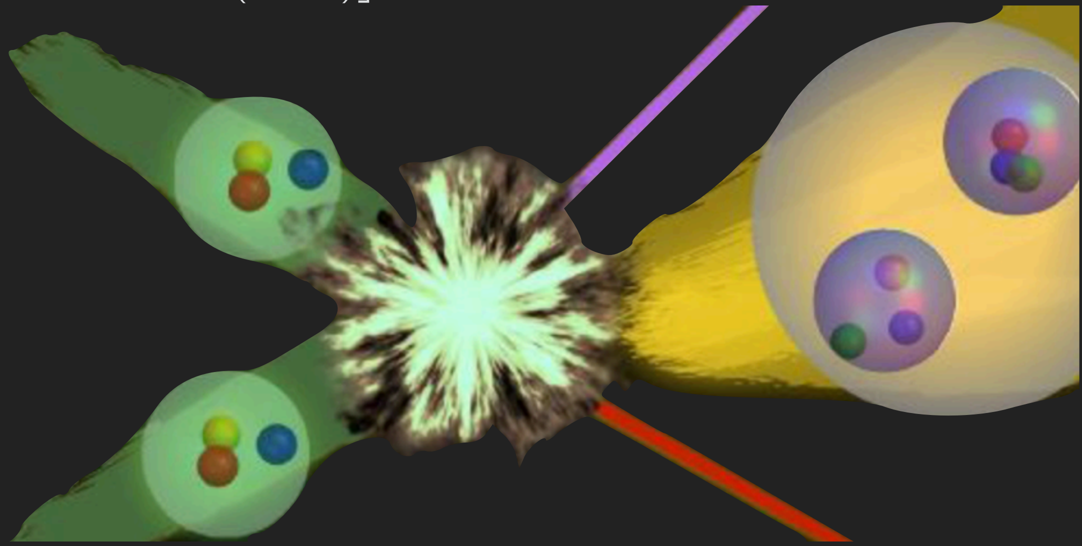


[NPLQCD Phys.Rev. D92 (2015), 114502]

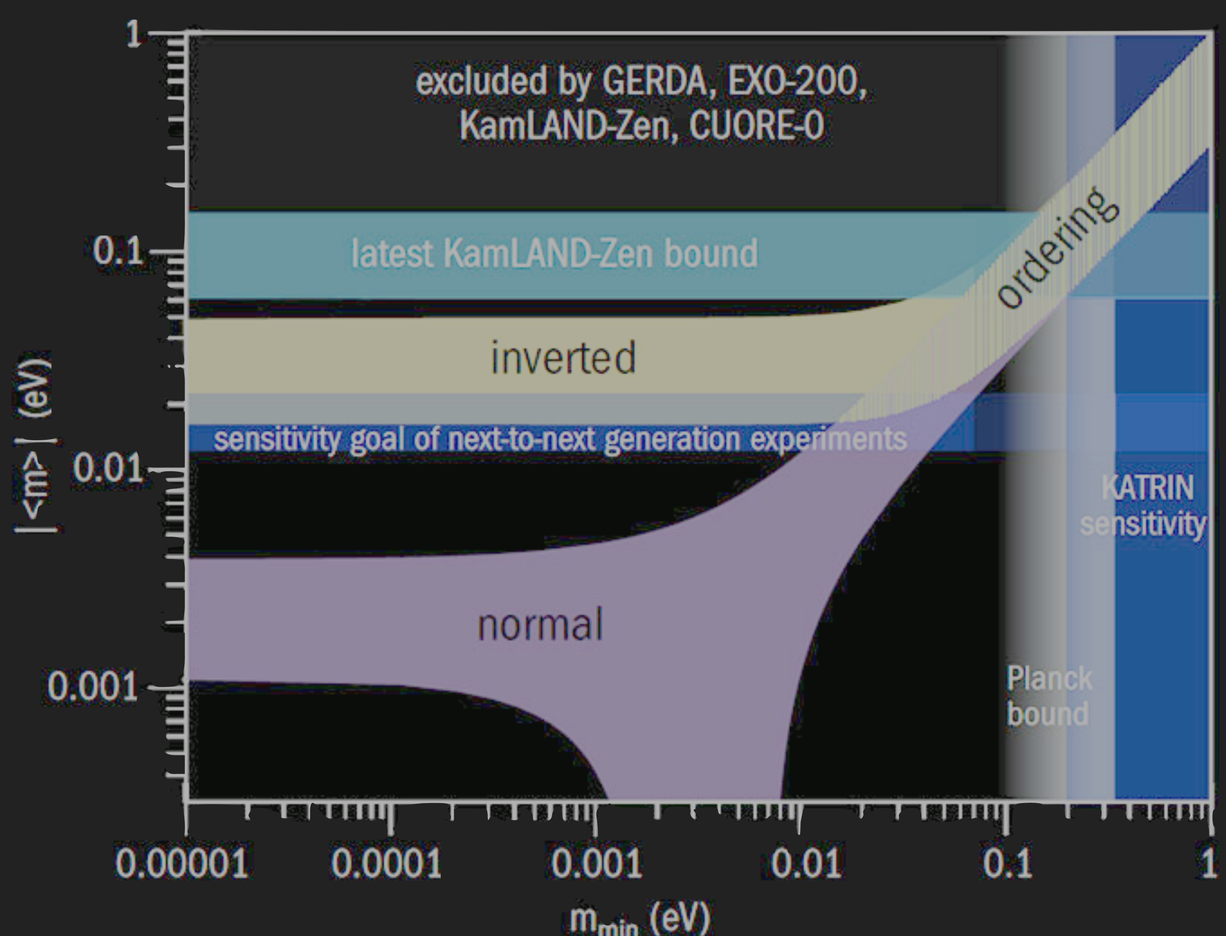
Axial matrix elements

[NPLQCD PRL **119**, 062002 (2017), PRL **119**, 062003 (2017)]

ELECTROWEAK PROCESSES



- Electroweak processes in light nuclei: first LQCD calculations
- Tritium decay $\langle {}^3\mathrm{He}|\overline{\mathbf{q}}\gamma_k\gamma_5\tau^-\mathbf{q}|{}^3\mathrm{H}\rangle$
- Proton-proton fusion [PRL 119, 062002 (2017)]
  - Instigating process in solar fusion but hard to measure
  - Calculations reaching level of precision of phenomenology.
- Double-β decay: nn→pp [PRL 119, 062003 (2017)]
  - Improve nuclear matrix element uncertainties



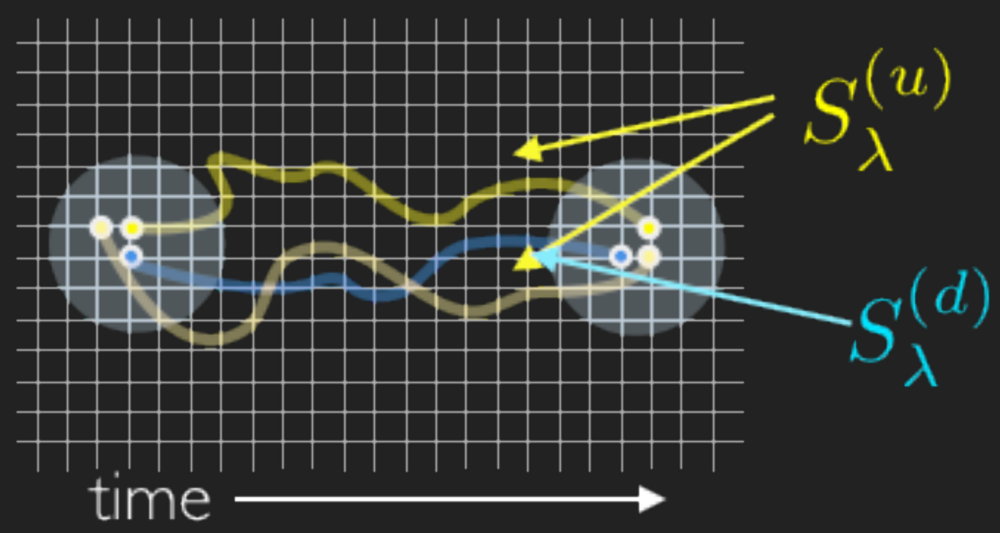
AXIAL BACKGROUND FIELD

- Fixed axial background field
- Construct correlation functions from quark propagators modified in axial field

compound propagator — constant

$$\boxed{S_\lambda^{(q)}(x,y)} = S^{(q)}(x,y) + \boxed{\lambda_q} \int dz\, S^{(q)}(x,z)\,\gamma_3\gamma_5\,S^{(q)}(z,y)$$

$$C_{\lambda_u;\lambda_d}(t) =$$



- Linear response gives axial matrix element

[NPLQCD 1610.04545]

AXIAL BACKGROUND FIELD

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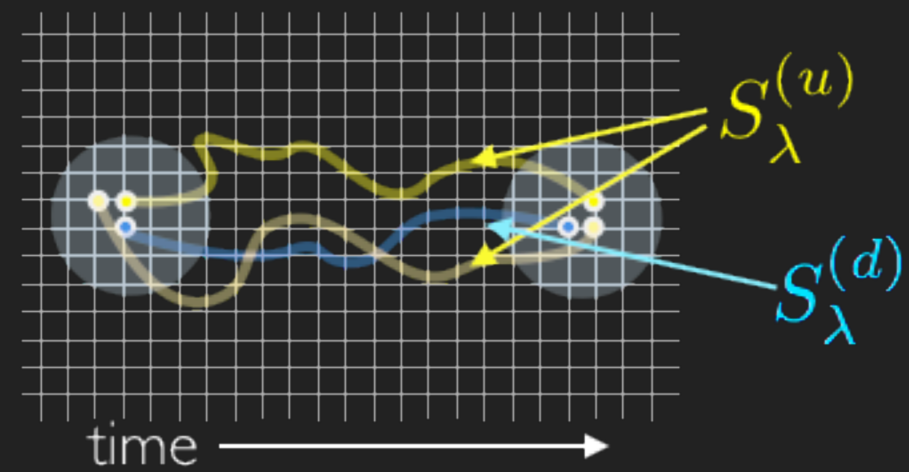

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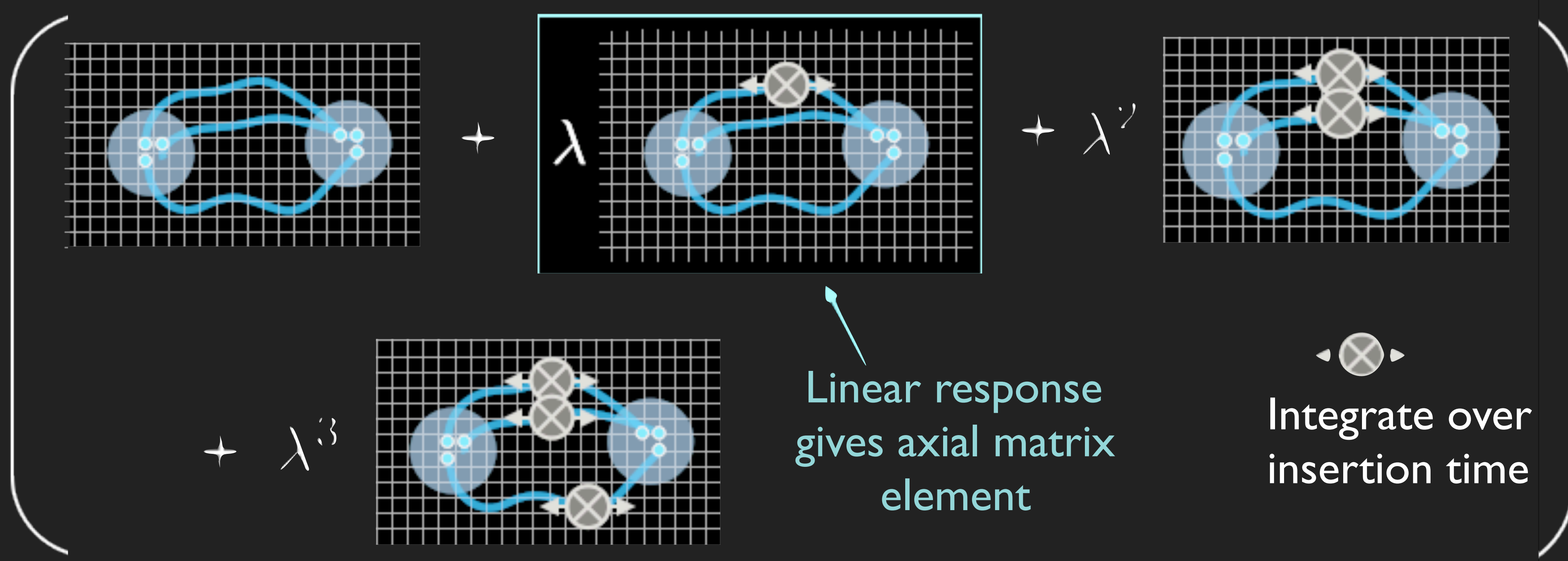
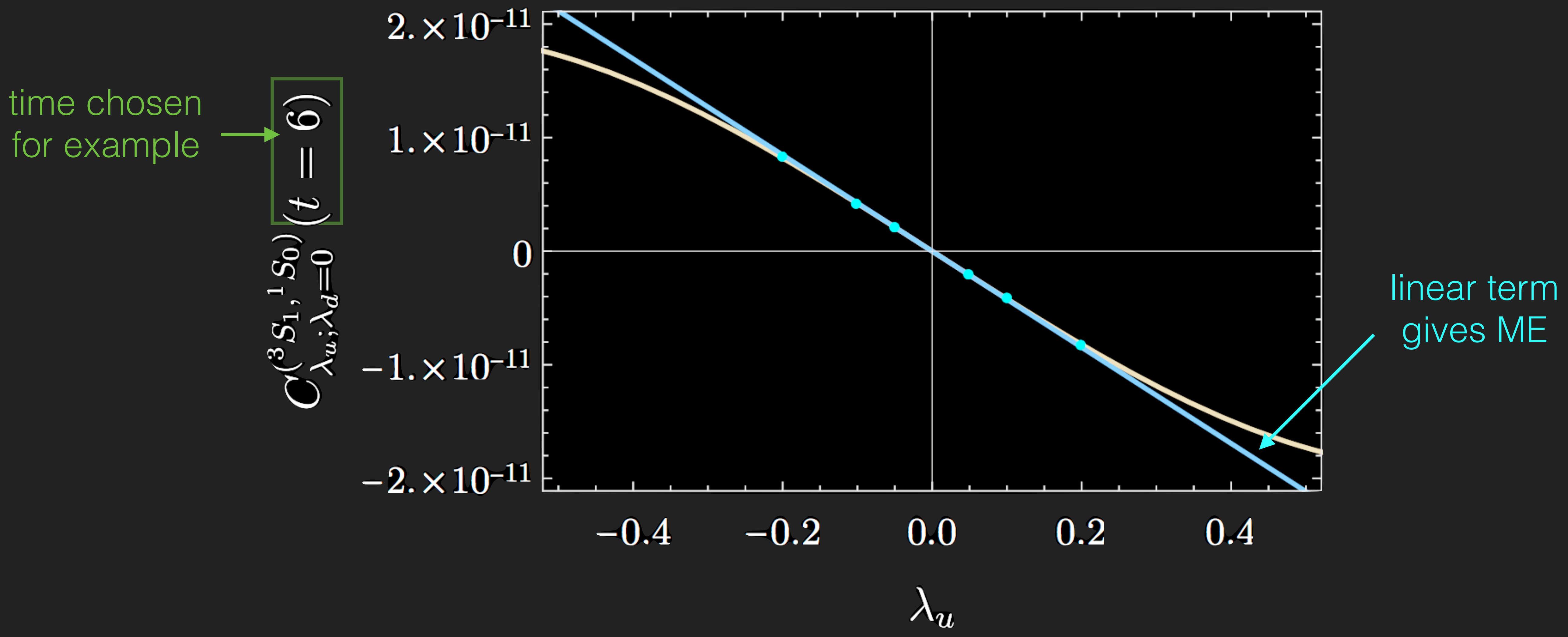


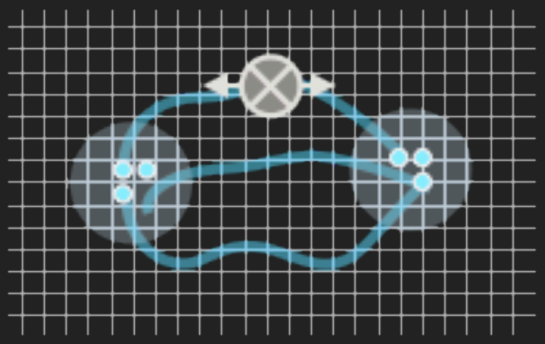
TRITIUM BETA DECAY

- Example: correlator formed with background field coupling to u quark



AXIAL BACKGROUND FIELD

$$C_{\lambda_u;\lambda_d}(t)\Big|_{\mathcal{O}(\lambda)} =$$



Implicit sum over current insertion times

- Example: determination of the proton axial charge

$$C_{\lambda_u;\lambda_d}(t)\Big|_{\mathcal{O}(\lambda)} = \sum_{\tau=0}^{t} \langle 0|\chi^\dagger(t) J(\tau)\chi(0)|0\rangle$$
$$= \dots$$
$$= Z_0 e^{-M_p t}\left[C + t\,\langle p|A_3^{(u)}(0)|p\rangle + \mathcal{O}(e^{-\delta t})\right]$$

Uninteresting constant

Excited states

Matrix element

- Time difference isolates matrix element part

$$\left(C_{\lambda_u;\lambda_d}(t+1) - C_{\lambda_u;\lambda_d}(t)\right)\Big|_{\mathcal{O}(\lambda)} = Z_0 e^{-M_p t}\langle p|A_3^{(u)}(0)|p\rangle + \mathcal{O}(e^{-\delta t})$$

PROTON AXIAL CHARGE

- Extract matrix element through linear response of correlators to the background field

- Form ratios to cancel leading time-dependence

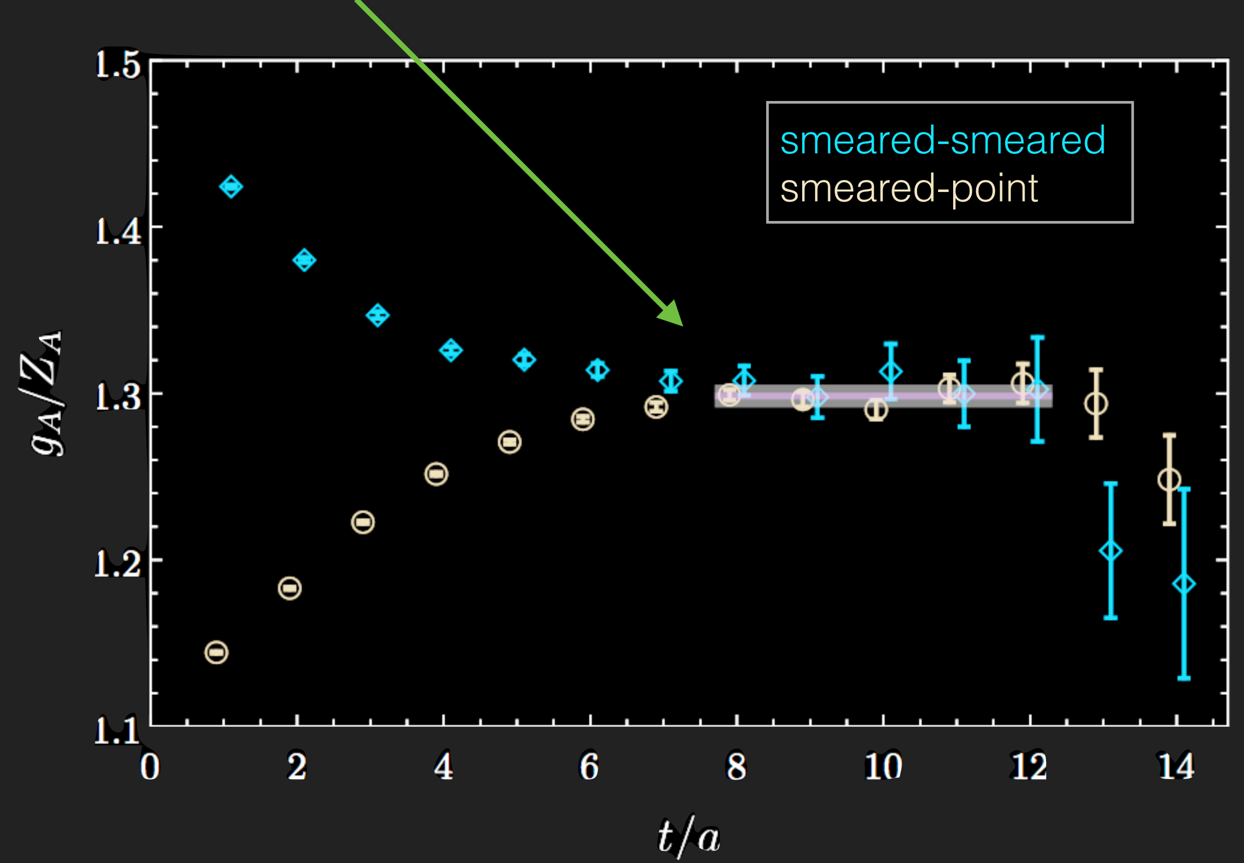
$$R_p(t) = \frac{\left(C^{(p)}_{\lambda_u;\lambda_d=0}(t) - C^{(p)}_{\lambda_u=0;\lambda_d}(t)\right)\Big|_{\mathcal{O}(\lambda)}}{C^{(p)}_{\lambda_u=0;\lambda_d=0}(t)}$$

At late times:

$$R_p(t+1) - R_p(t) \xrightarrow{t\to\infty} \frac{g_A}{Z_A}$$

- Matrix element revealed through "effective matrix elt. plot"

constant fit to plateau region



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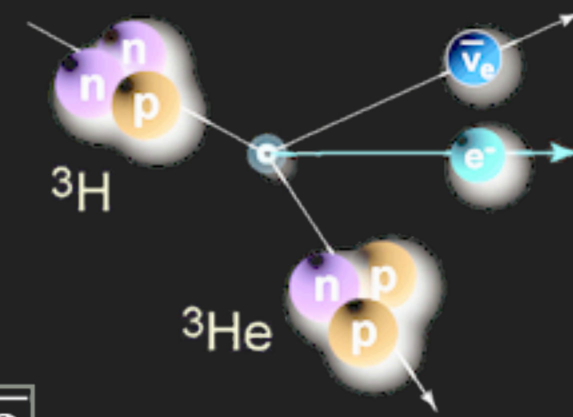


- Tritium decay half life

$$\frac{(1+\delta_R)f_V}{K/G_V^2}\, t_{1/2} = \frac{1}{\langle \mathbf{F}\rangle^2 + f_A/f_V\, g_A^2 \langle \mathbf{GT}\rangle^2}$$

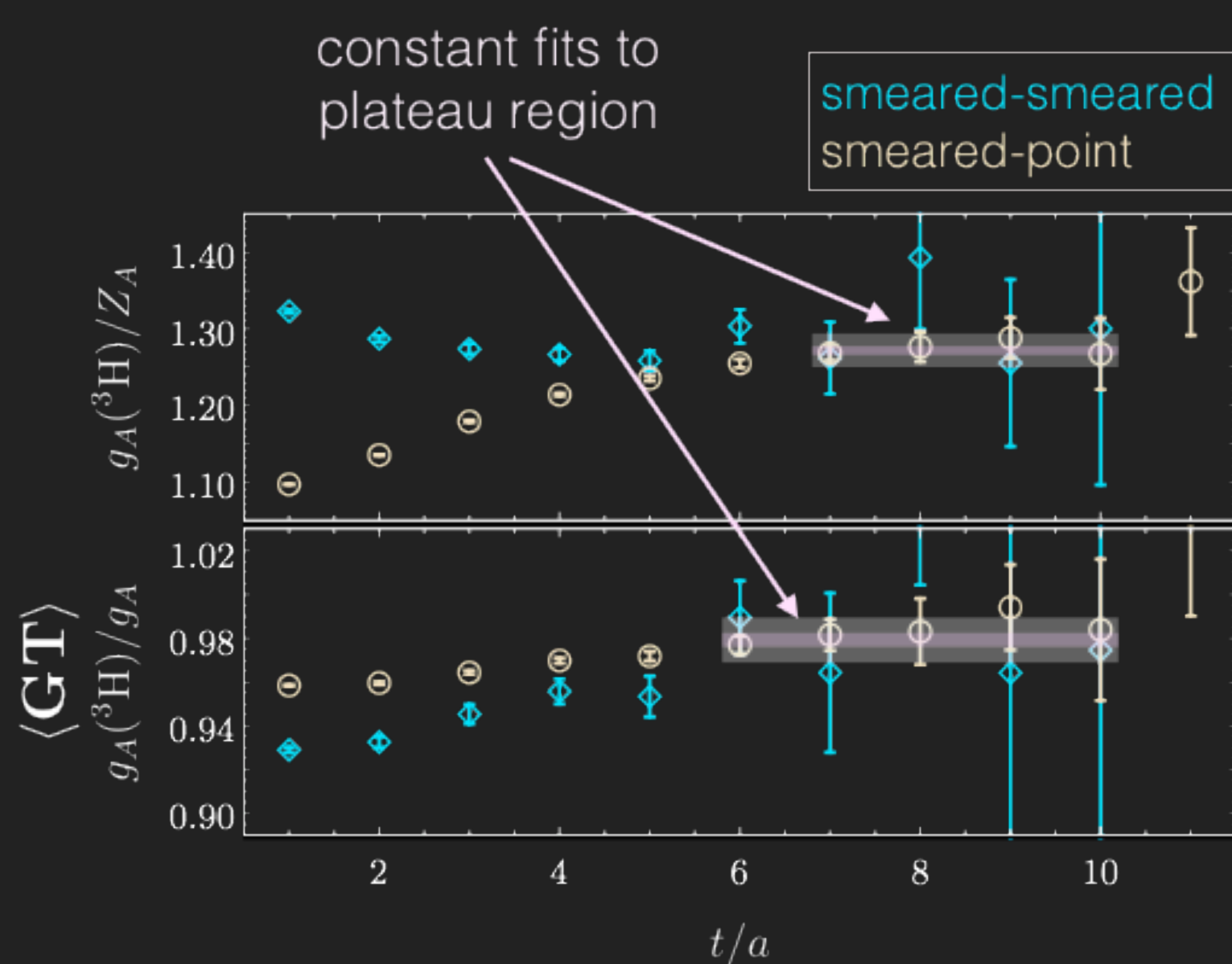
known from theory or expt. · half-life · vector ME · axial ME

- Biggest uncertainty in

$$g_A\langle \mathbf{GT}\rangle = \langle {}^3\mathrm{He}|\overline{\mathbf{q}}\gamma_{\mathbf{k}}\gamma_{\mathbf{5}}\tau^-\mathbf{q}|{}^3\mathrm{H}\rangle$$

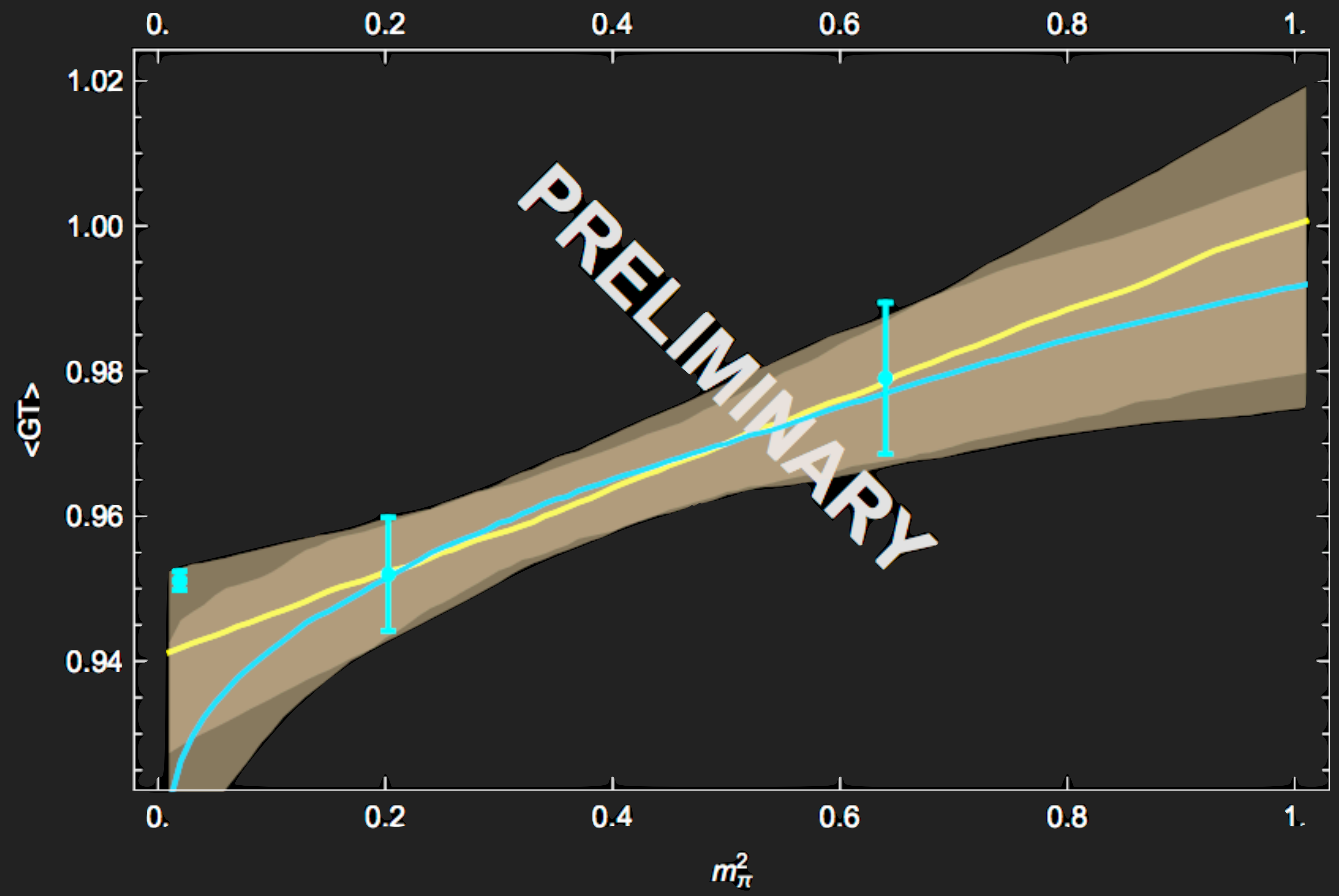
- Form ratios of correlators to cancel leading time-dependence:

$$\frac{\overline{R}_{^3\mathrm{H}}(t)}{\overline{R}_p(t)} \xrightarrow{t\to\infty} \frac{g_A(^3\mathrm{H})}{g_A} = \langle \mathbf{GT}\rangle$$



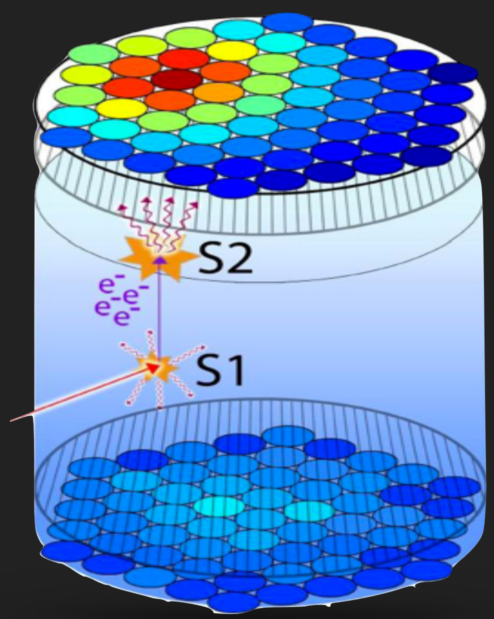
TRITIUM BETA DECAY

- Quark mass dependence (m_π~800,450 MeV)



Nuclear matrix elements for dark matter

NUCLEAR SIGMA TERMS



- One possible DM interaction is through scalar exchange

$$\mathcal{L} = \frac{G_F}{2} \sum_q a_S^{(q)} (\overline{\chi}\,\chi)(\overline{q}\,q)$$

- Direct detection depends on nuclear matrix element

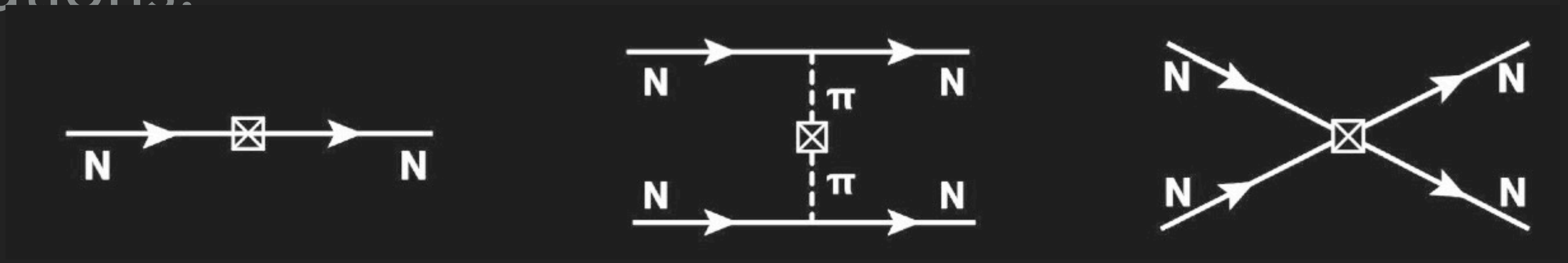
$$\sigma_{Z,N} \; = \; \overline{m} \langle Z, N(\mathrm{gs}) |\; \overline{u}u + \overline{d}d \;| Z, N(\mathrm{gs}) \rangle \; = \; \overline{m} \frac{d}{d\overline{m}} E_{Z,N}^{(\mathrm{gs})}$$

  - Accessible via Feynman-Hellman theorem

- At hadronic/nuclear level

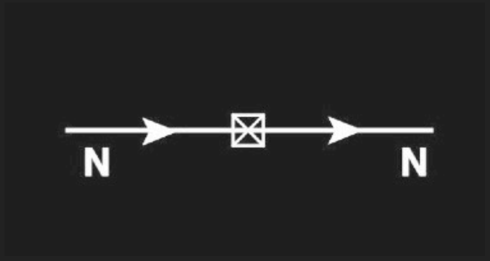
$$\mathcal{L} \to G_F \, \overline{\chi}\chi \, \Big(\frac{1}{4} \langle 0|\overline{q}q|0\rangle \; \mathrm{Tr}\left[a_S \Sigma^\dagger + a_S^\dagger \Sigma\right] \; + \; \frac{1}{4} \langle N|\overline{q}q|N\rangle N^\dagger N \mathrm{Tr}\left[a_S \Sigma^\dagger + a_S^\dagger \Sigma\right]$$
$$- \; \frac{1}{4} \langle N|\overline{q}\tau^3 q|N\rangle \left(N^\dagger N \mathrm{Tr}\left[a_S \Sigma^\dagger + a_S^\dagger \Sigma\right] \; - \; 4 N^\dagger a_{S,\xi} N \right) \; + \; ... \Big)$$

  - Contributions:



NUCLEON SIGMA TERM

- Single nucleon contribution



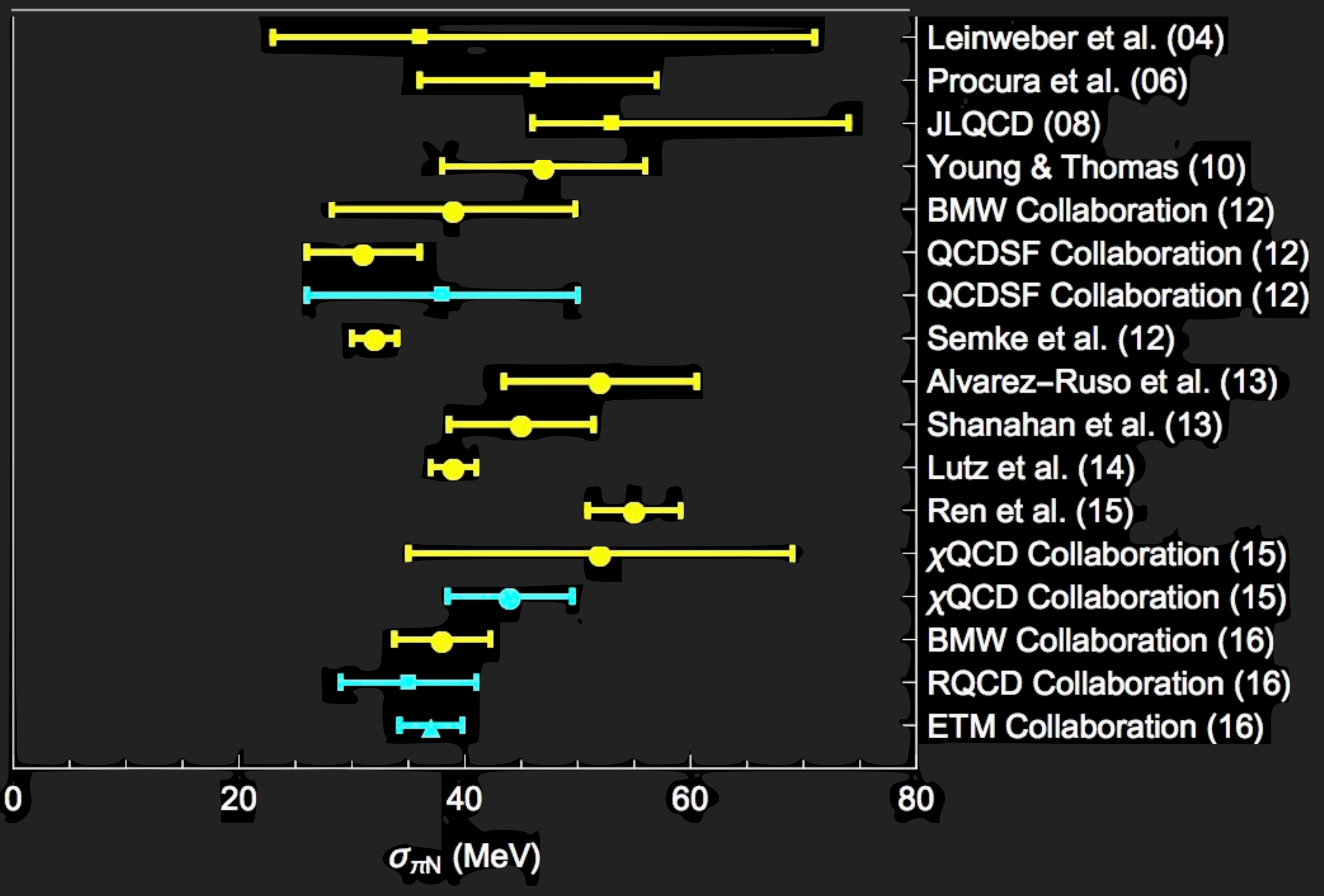
calculated by many lattice groups

- Results stabilised

- NB: interesting ~3σ tension with recent πN dispersive analysis

[Hoferichter et al, PRL. **115** (2015) 092301]

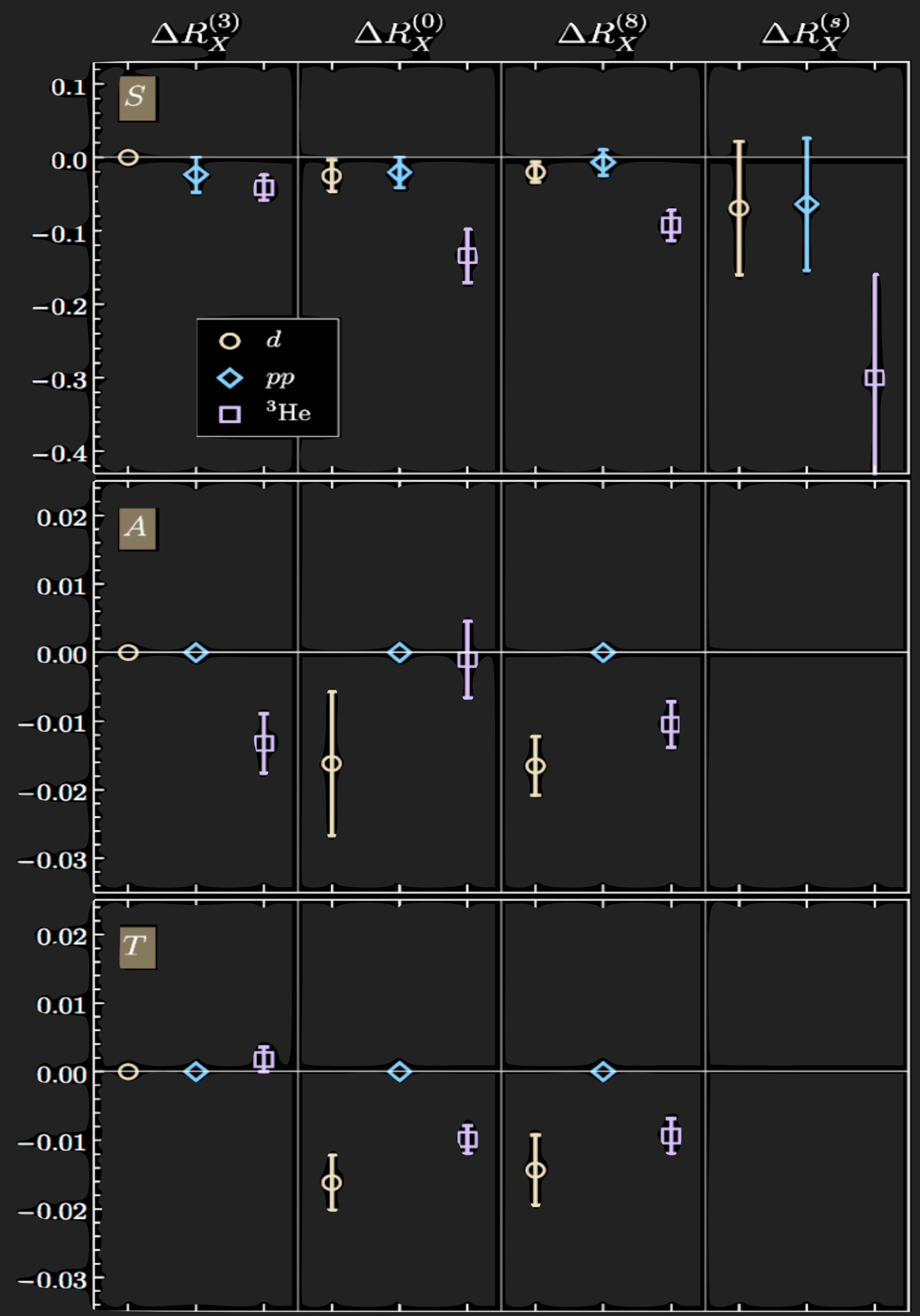
[summary by P Shanahan 2016]



[NPLQCD, PRL **120**, 152002 (2018)]

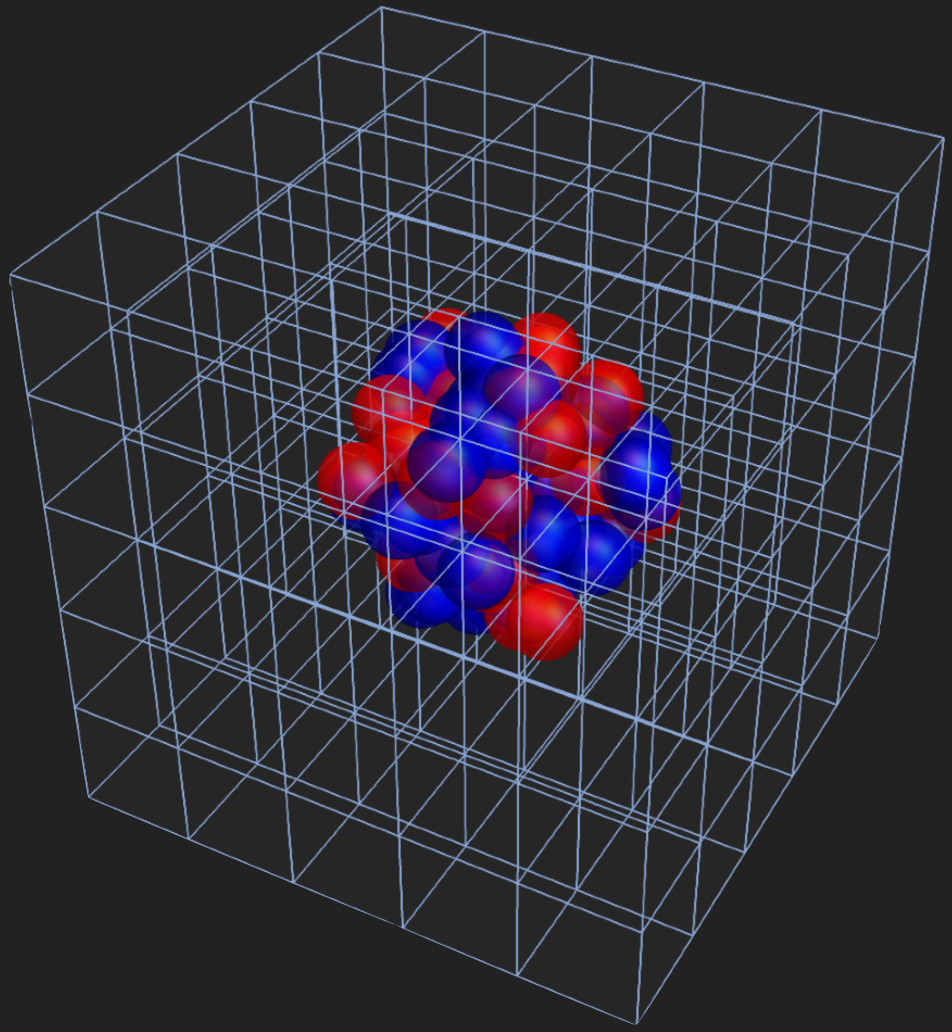
NUCLEAR EFFECTS

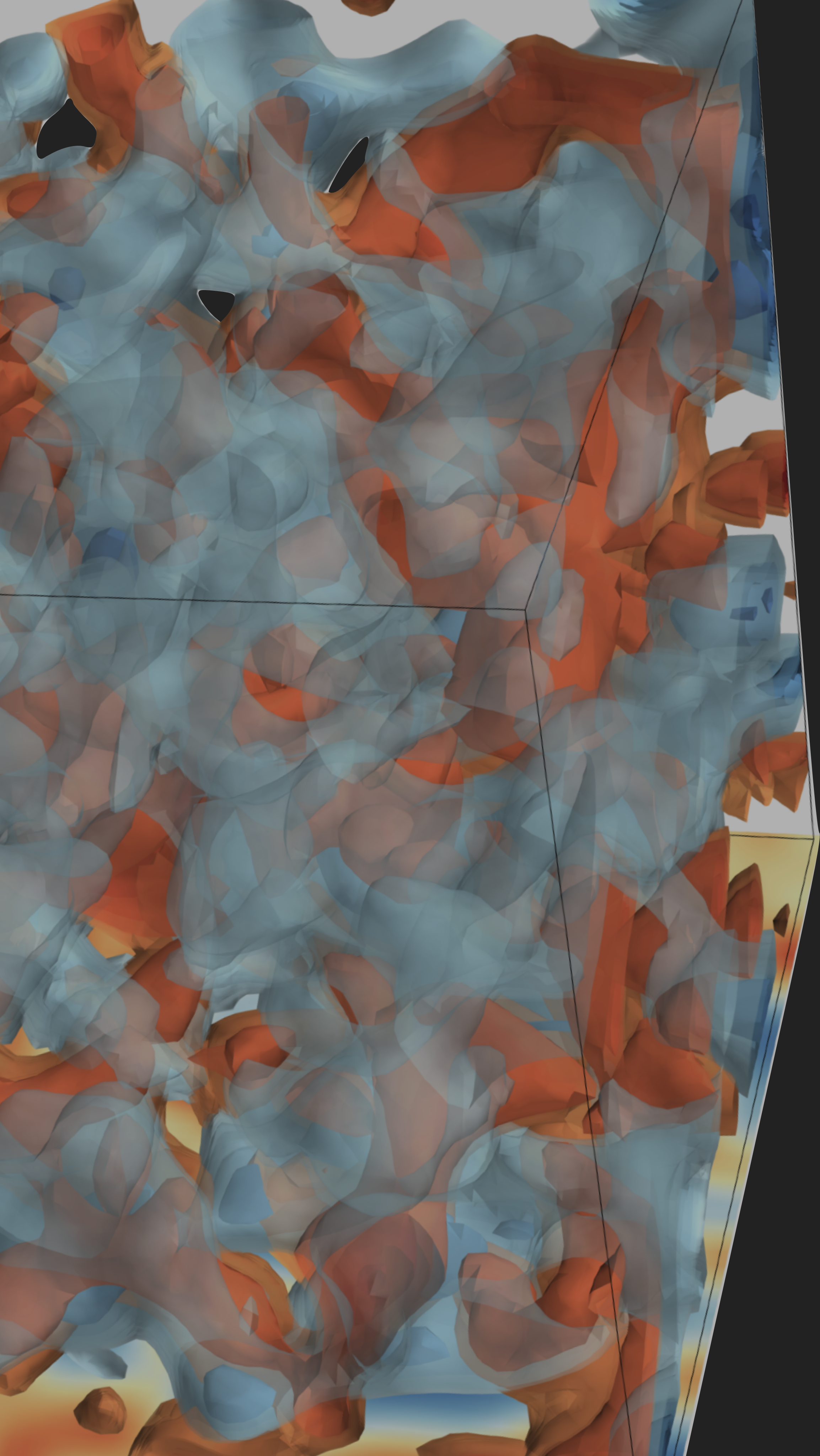
- Background fields for scalar (and also axial and tensor) quark bilinear
- Calculate forward limit MEs for A=2,3
- Scalar MEs has large (~10%) deviation from sum of nucleon MEs for A=3
  - Consequences for larger nuclei?
  - Repeating calculations at lighter quark masses
- Future: second order response gives scalar polarisability



OUTLOOK

- Nuclei are under study directly from QCD
  - Spectroscopy of light nuclei and exotic nuclei
  - Structure: magnetic moments, axial couplings…
  - Interactions: $np \to d\gamma$, $pp \to de^+\nu$, $nn \to pp$, DM
- Prospect of a quantitative connection to QCD makes this an exciting time for nuclear physics
  - Critical role in current and upcoming intensity frontier experimental program
  - Learn many interesting things about the nature of hadrons and nuclei along the way

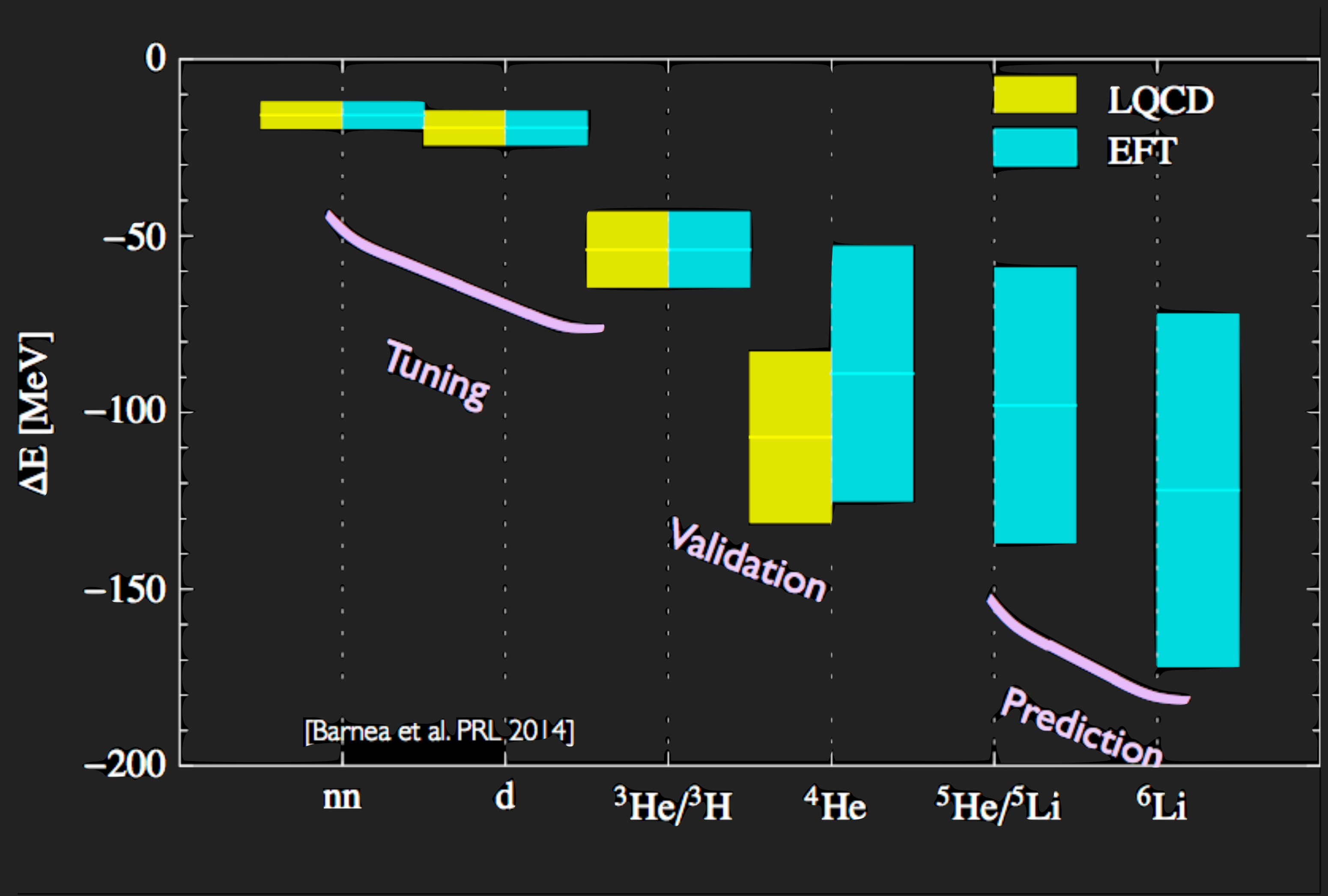




FIN

HEAVY QUARK UNIVERSE

- Combine LQCD and pionless EFT
- EFT matching to LQCD determines NN, NNN interactions: allows predictions for larger nuclei

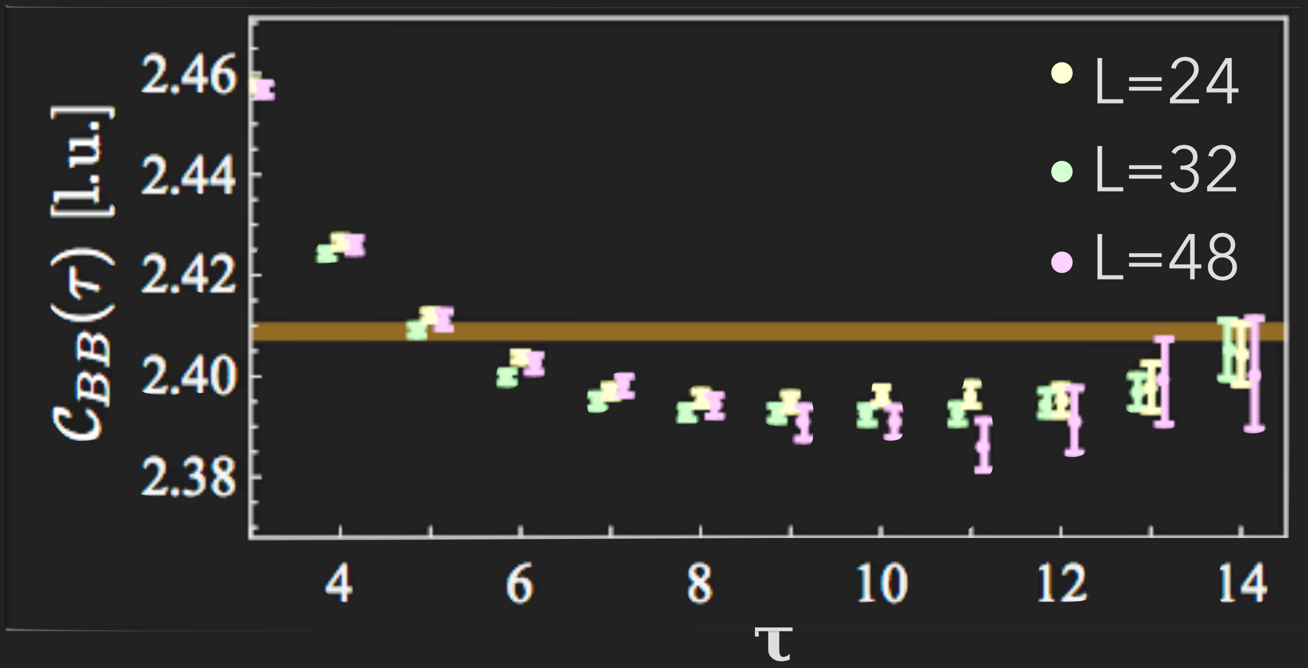


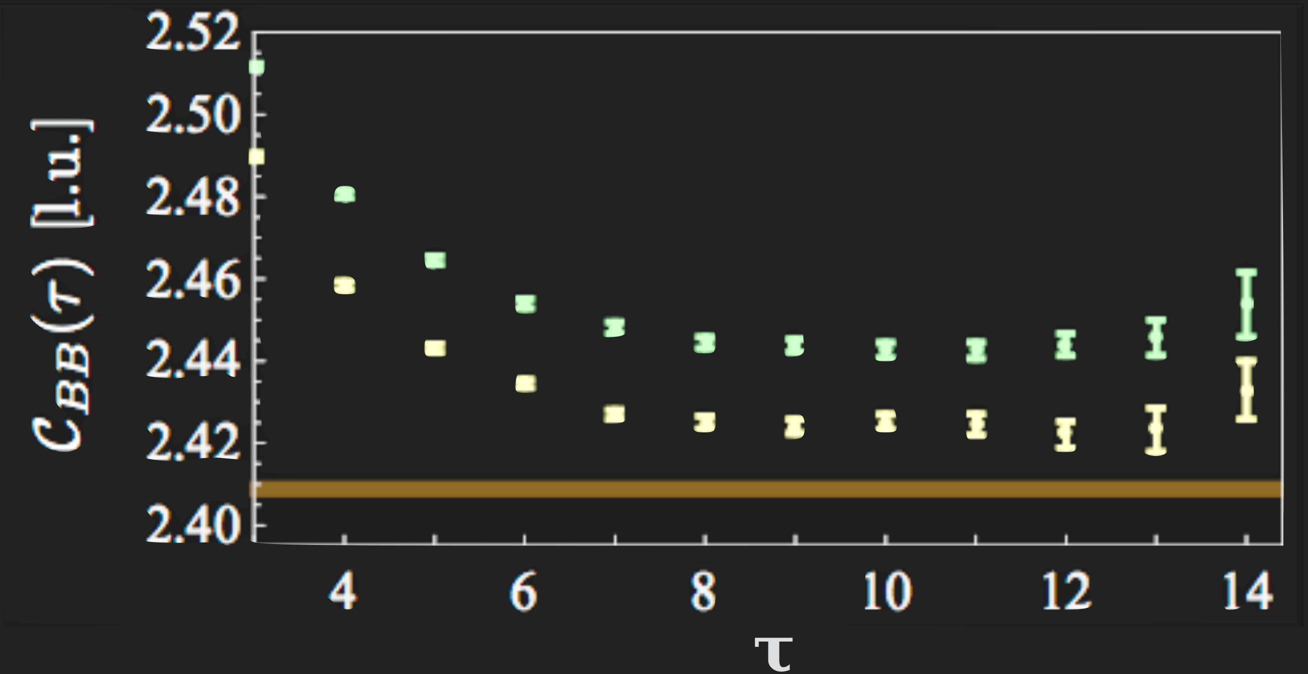
In a world
@ m_π = 800 MeV

- Other many-body methods significantly extend reach [Barnea et al. PRL 2014; see also Kirscher et al. 1506.09048, Contessi et al. 1701.06516]

NN BOUND STATES

- Potential for fake plateaus? [Iritani et al.]
    - Scattering states combine with relative signs to give negative-shifted flat behaviour
- Very unlikely
- Study at 3 volumes with same source structure
- Negative shifted states
    - Correlators fully consistent at L=24, 32, 48
- Excited state
    - Scales as $1/L^3$ consistent with scattering state

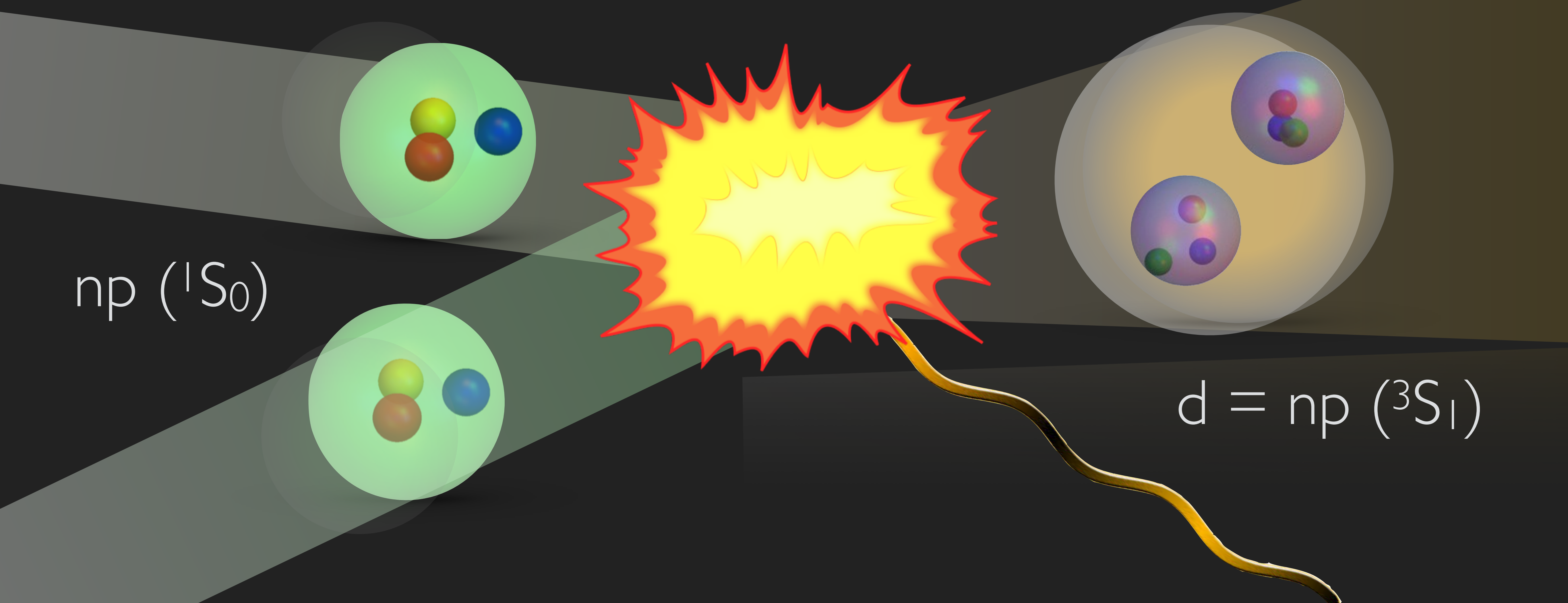




[NPLQCD PRL **115**, 132001(2015)]

THERMAL NEUTRON CAPTURE CROSS-SECTION

- Thermal neutron capture cross-section: $np \to d\gamma$
  - Critical process in Big Bang Nucleosynthesis
  - Historically important: 2-body contributions ~10%
  - First QCD nuclear reaction!



[NPLQCD PRL **115**, 132001(2015)]

$$Z_d = 1/\sqrt{1-\gamma_0 r_3}$$

NP→Dγ IN PIONLESS EFT

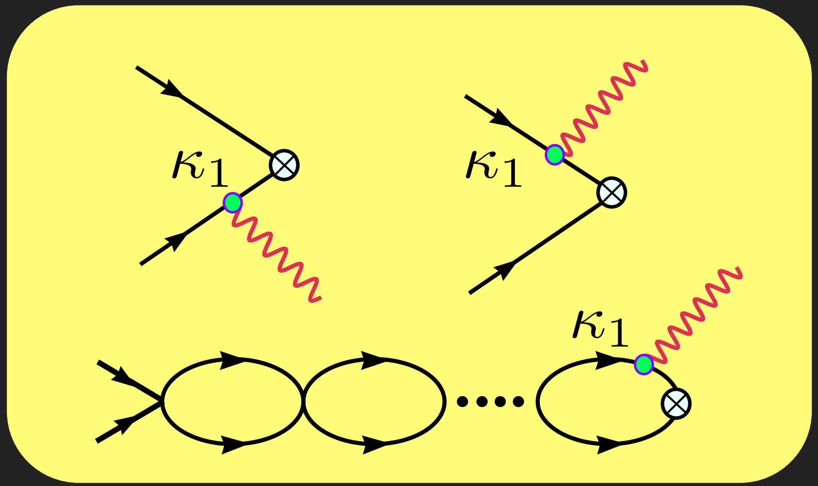
- Cross-section at threshold calculated in pionless EFT

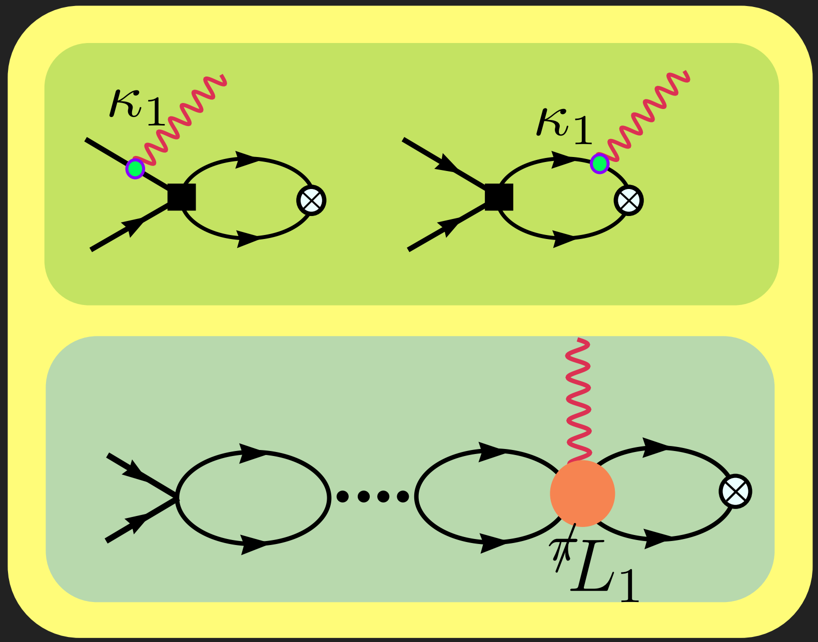
$$\sigma(np \to d\gamma) = \frac{e^2(\gamma_0^2 + |\mathbf{p}|^2)^3}{M^4\gamma_0^3|\mathbf{p}|}|\tilde{X}_{M1}|^2 + \ldots$$

- EFT expansion at LO given by mag. moments NLO contributions from short-distance two nucleon operators

$$\tilde{X}_{M1} = \frac{Z_d}{-\frac{1}{a_1} + \frac{1}{2}r_1|\mathbf{p}|^2 - i|\mathbf{p}|} \times \left[\frac{\kappa_1\gamma_0^2}{\gamma_0^2 + |\mathbf{p}|^2}\left(\gamma_0 - \frac{1}{a_1} + \frac{1}{2}r_1|\mathbf{p}|^2\right) + \frac{\gamma_0^2}{2}l_1\right]$$

- Phenomenological description with 1% accuracy for E< 1MeV
  - Short distance (MEC) contributes ~10%





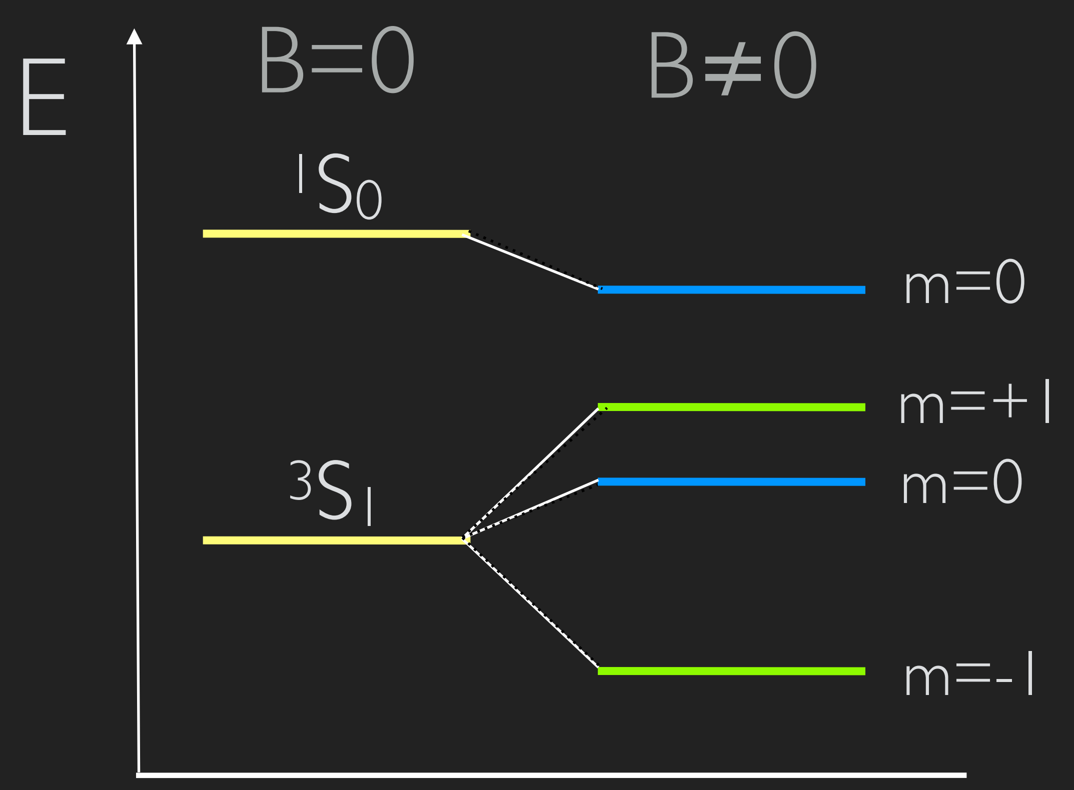
MECs: Riska, Phys.Lett. B38 (1972) 193
Hokert et al, Nucl.Phys. A217 (1973) 14
EFT: Chen et al.,Nucl.Phys. A653 (1999) 386
Chen et al, Phys.Lett. B464 (1999) 1
Rupak Nucl.Phys. A678 (2000) 405

BACKGROUND FIELDS

- Consider QCD in the presence of a constant background magnetic field
    - Implement by adding term to the action (careful with boundaries)
- Shifts spin-1/2 particle masses

$$M_{\uparrow\downarrow} = M_0 \pm \mu|\mathbf{B}| + 4\pi\beta|\mathbf{B}|^2 + \ldots$$

    - Changing strength of background field allows μ, β to be extracted
- Two nucleon states
    - Levels split and mix
    - Similar for electro-weak fields and twist-two fields



ENERGY LEVELS IN BF

- Background field modifies eigenvalue equation for m=±1 states

$$p\cot\delta(p) - \frac{1}{\pi L} S\left(\frac{L^2}{4\pi^2}\left[p^2 \pm e|\mathbf{B}|\kappa_0\right]\right) \mp \frac{e|\mathbf{B}|}{2}\left(L_2 - r_3\kappa_0\right) = 0$$

- Asymptotic expansion of lowest scattering level

$$E_0^{m=\pm 1} = \mp\frac{e|\mathbf{B}|\kappa_0}{M} + \frac{4\pi A_3}{ML^3}\left[1 - c_1\frac{A_3}{L} + c_2\left(\frac{A_3}{L}\right)^2 + \ldots\right]$$

where $\frac{1}{A_3} = \frac{1}{a_3} \pm \frac{e|\mathbf{B}|L_2}{2}$

- Mixes $^1S_0$ and $^3S_1$ m=0 states (coupled channels – but perturbative)

$$\left[p\cot\delta_1(p) - \frac{S_+ + S_-}{\pi L}\right]\left[p\cot\delta_3(p) - \frac{S_+ + S_-}{\pi L}\right] = \left[\frac{e|\mathbf{B}|L_1}{2} + \frac{S_+ - S_-}{2\pi L}\right]^2$$

where $S_\pm = S\left(\frac{L^2}{4\pi^2}\left[p^2 \pm e|\mathbf{B}|\kappa_1\right] + \ldots\right)$

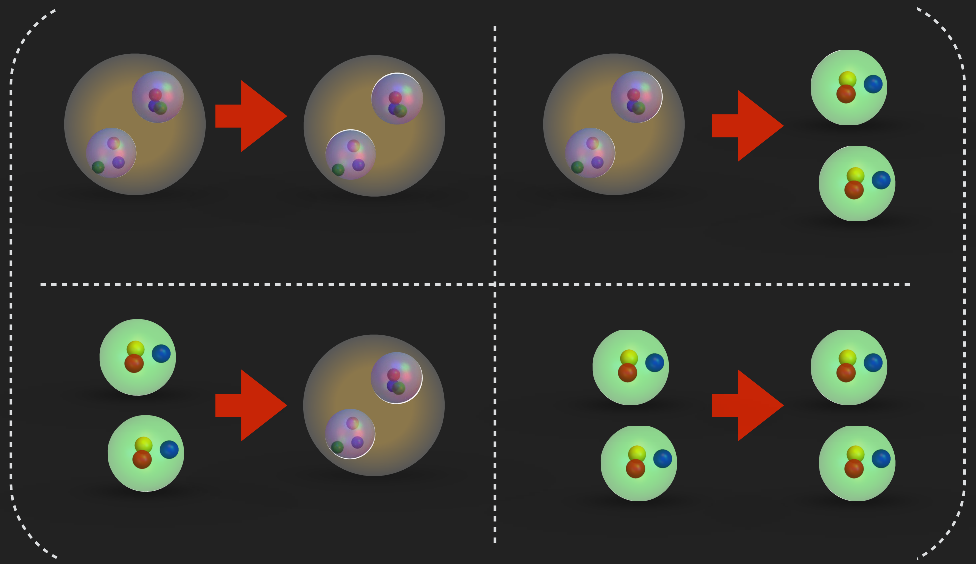
[WD & MJ Savage Nucl Phys A 743, 170]

[NPLQCD PRL **115**, 132003 I (2015)]

NP→Dγ

- Presence of magnetic field mixes $I_z=J_z=0$ $^3S_1$ and $^1S_0$ np systems

H =



- Wigner SU(4) super-multiplet (spin-flavour) symmetry relates $^3S_1$ and $^1S_0$ states (diagonal elements approximately equal)
  - Shift of eigenvalues determined by transition amplitude

$$\Delta E_{^3S_1,^1S_0} = \mp \left(\kappa_1 + \overline{L}_1\right) \frac{eB}{M} + \ldots$$

- More generally eigenvalues depend on transition amplitude

[WD, & M Savage 2004, H Meyer 2012]

[NPLQCD PRL **115**, 132001(2015)]

NP→Dγ

Lattice correlator with $^3S_1$ source and $^1S_0$ sink

- Iz=Jz=0 correlation matrix

$$\mathbf{C}(t;\mathbf{B}) = \begin{pmatrix} C_{^3S_1,^3S_1}(t;\mathbf{B}) & C_{^3S_1,^1S_0}(t;\mathbf{B}) \\ C_{^1S_0,^3S_1}(t;\mathbf{B}) & C_{^1S_0,^1S_0}(t;\mathbf{B}) \end{pmatrix}$$

- Generalised eigenvalue problem

$$[\mathbf{C}(t_0;\mathbf{B})]^{-1/2}\mathbf{C}(t;\mathbf{B})[\mathbf{C}(t_0;\mathbf{B})]^{-1/2}v = \lambda(t;\mathbf{B})v$$

- Ratio of correlator ratios to extract 2-body

$$R_{^3S_1,^1S_0}(t;\mathbf{B}) = \frac{\lambda_+(t;\mathbf{B})}{\lambda_-(t;\mathbf{B})} \overset{t\to\infty}{\longrightarrow} \hat{Z}\exp\left[2\,\Delta E_{^3S_1,^1S_0} t\right]$$

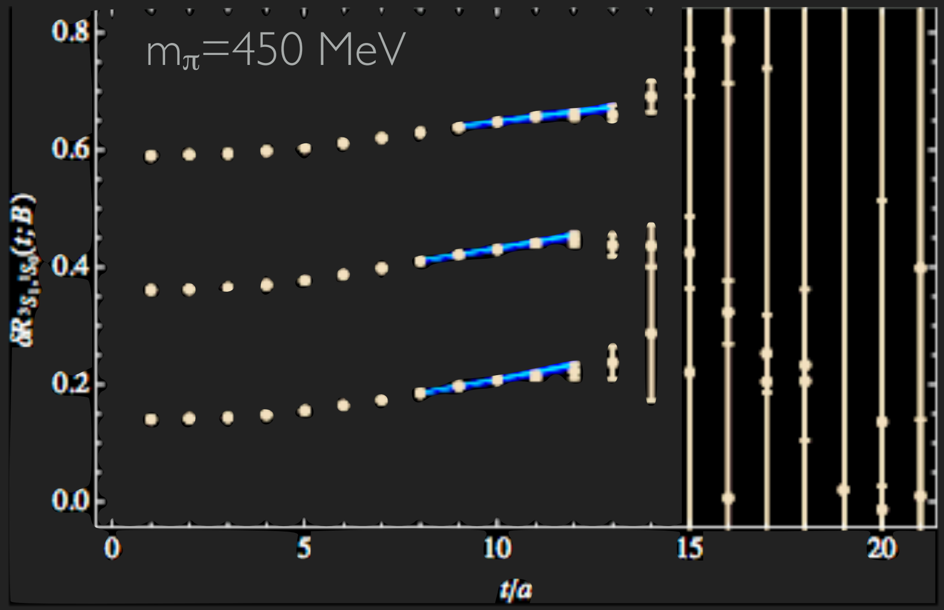
$$\delta R_{^3S_1,^1S_0}(t;\mathbf{B}) = \frac{R_{^3S_1,^1S_0}(t;\mathbf{B})}{\Delta R_p(t;\mathbf{B})/\Delta R_n(t;\mathbf{B})} \to A\, e^{-\delta E_{^3S_1,^1S_0}(\mathbf{B})t}$$

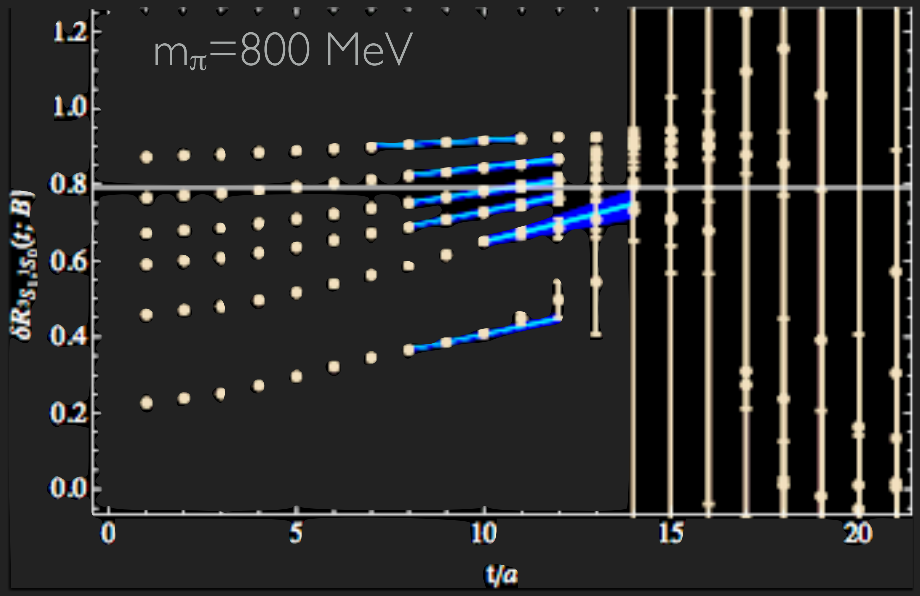
$$\begin{aligned}\delta E_{^3S_1,^1S_0} &\equiv \Delta E_{^3S_1,^1S_0} - [E_{p,\uparrow} - E_{p,\downarrow}] + [E_{n,\uparrow} - E_{n,\downarrow}] \\ &\to 2\overline{L}_1|e\mathbf{B}|/M + \mathcal{O}(\mathbf{B}^2)\,,\end{aligned}$$

[NPLQCD PRL **115**, 1320031(2015)]

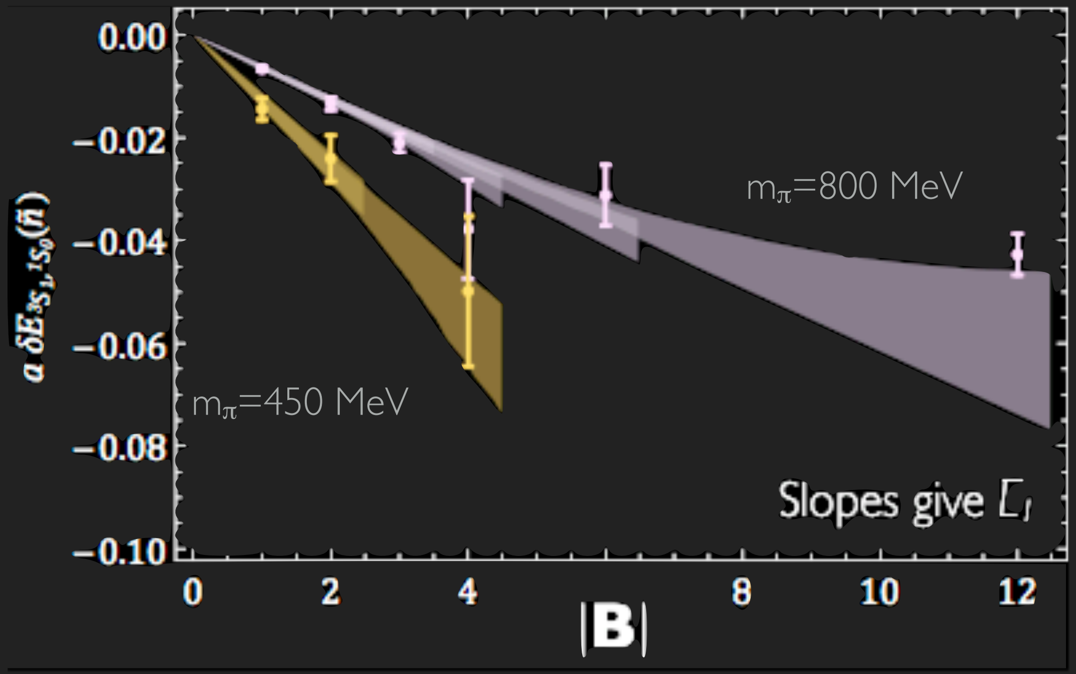
NP→Dγ

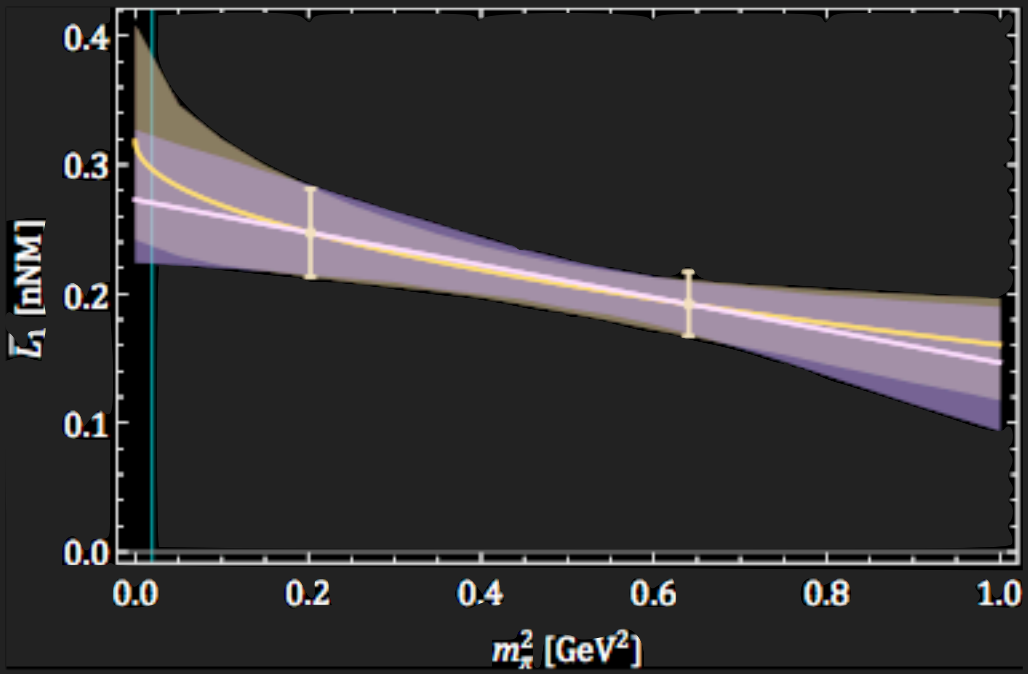
- Correlator ratios for different field strengths





- Field strength & mass dependence





[NPLQCD PRL **115**, 132001(2015)]

NP→Dγ

- Extracted short-distance contribution at physical mass

$$\overline{L}_1^{\mathrm{lqcd}} = 0.285(\,^{+63}_{-60}\,)\ \mathrm{nNM} \qquad l_1^{\mathrm{lqcd}} = -4.48(\,^{+16}_{-15}\,)\ \mathrm{fm}$$

- Combine with phenomenological nucleon magnetic moment, scattering parameters at incident neutron velocity v=2,200 m/s

$$\sigma^{\mathrm{lqcd}}(np \to d\gamma) = 307.8(1 + 0.273\ \overline{L}_1^{\mathrm{lqcd}})\ \mathrm{mb}$$

$$\sigma^{\mathrm{lqcd}}(np \to d\gamma) = 332.4(\,^{+5.4}_{-4.7}\,)\ \mathrm{mb}$$

c.f. phenomenological value

$$\sigma^{\mathrm{expt}}(np \to d\gamma) = 334.2(0.5)\ \mathrm{mb}$$

- NB: at m_π=800 MeV, use LQCD for all inputs (ab initio)

$$\sigma^{800\ \mathrm{MeV}}(np \to d\gamma) \sim 10\ \mathrm{mb}$$