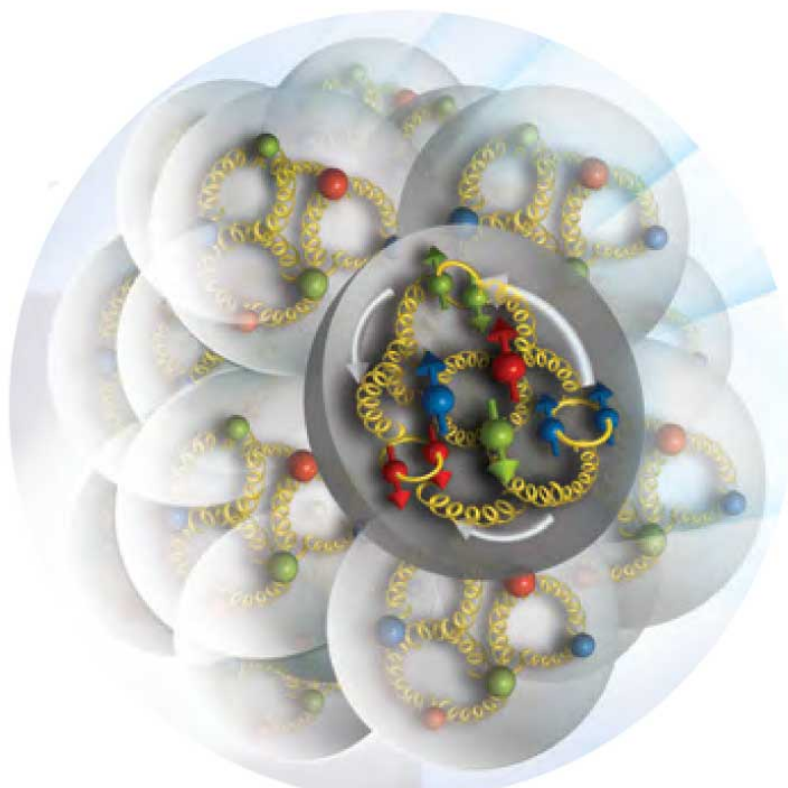
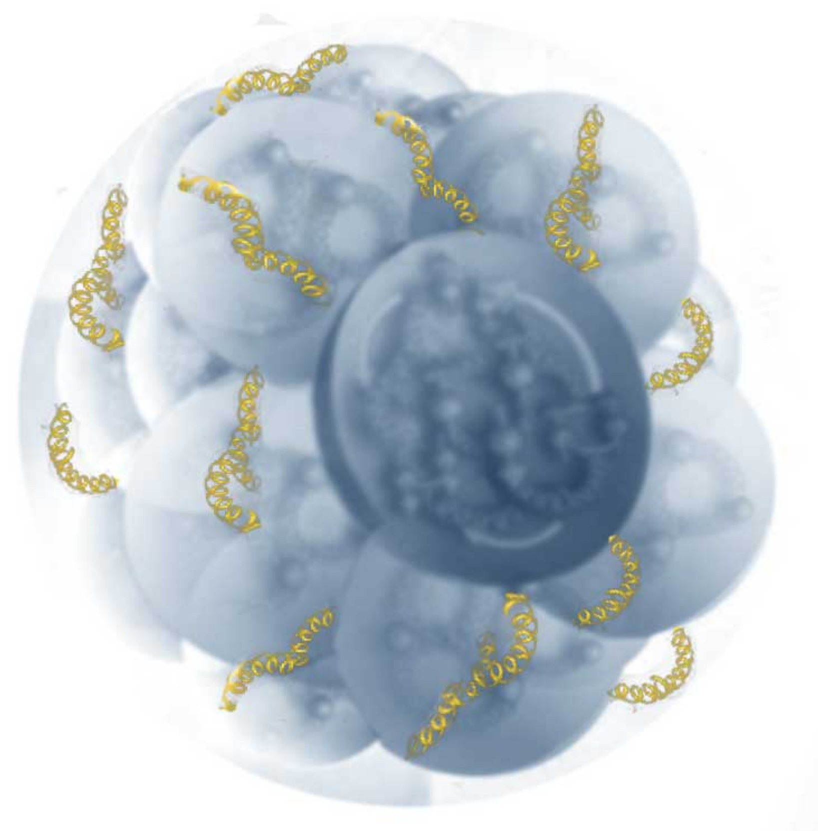
Gluon structure from lattice QCD





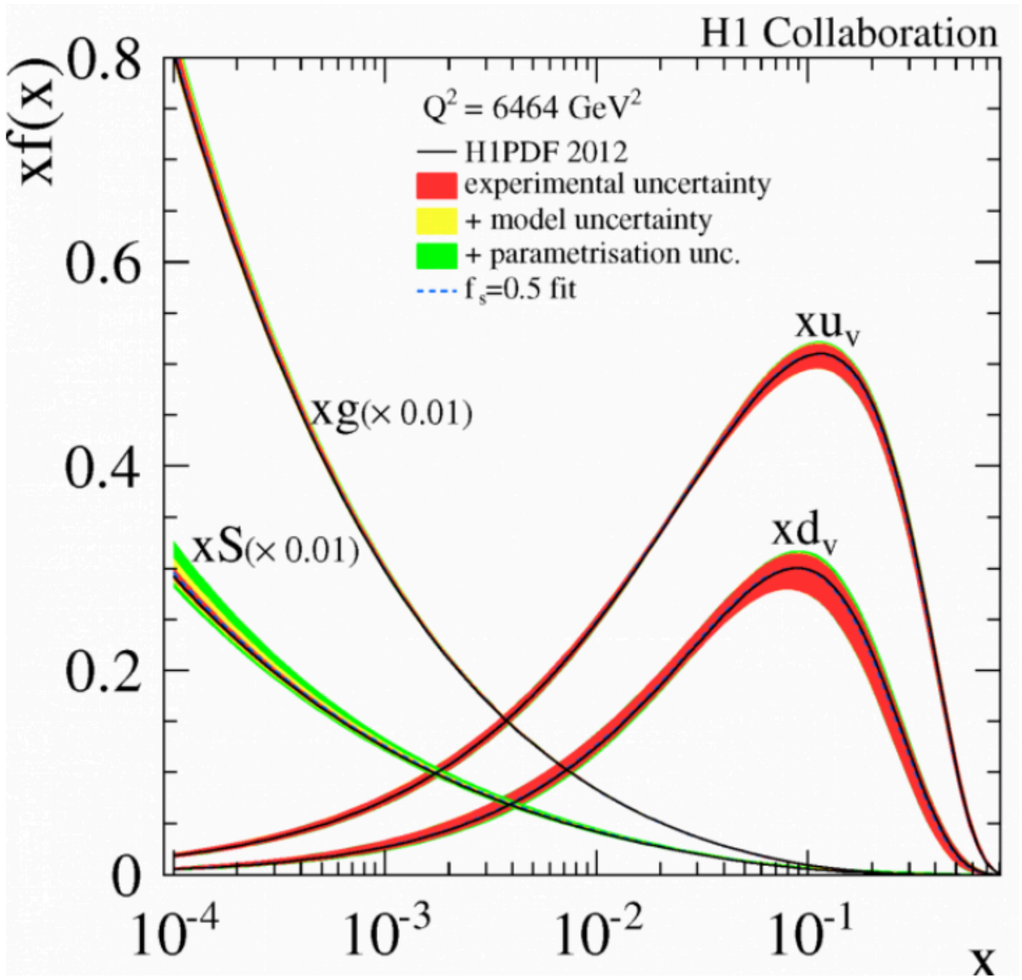
Phiala Shanahan
William and Mary
Thomas Jefferson National Accelerator Facility

Gluon structure

Gluons offer a new window on nuclear structure

- Past 60+ years: detailed view of quark structure of nucleons
- Gluon structure also important
  - Unpolarised gluon PDF dominant at small longitudinal momentum fraction
- Other aspects of gluon structure relatively unexplored

Parton distributions in the proton



Longitudinal momentum fraction carried by parton

Gluon structure

First-principles QCD calculations

→ QCD benchmarks and predictions ahead of experiment

Cover image from EIC whitepaper arXiv::1212.1701

Gluon Structure from LQCD

How much do gluons contribute to the proton's

- Momentum
- Mass
- Spin

1

What is the 3D gluon distribution of a proton

- PDFs
- TMDs
- GPDs
- 'Gluon radius'

2

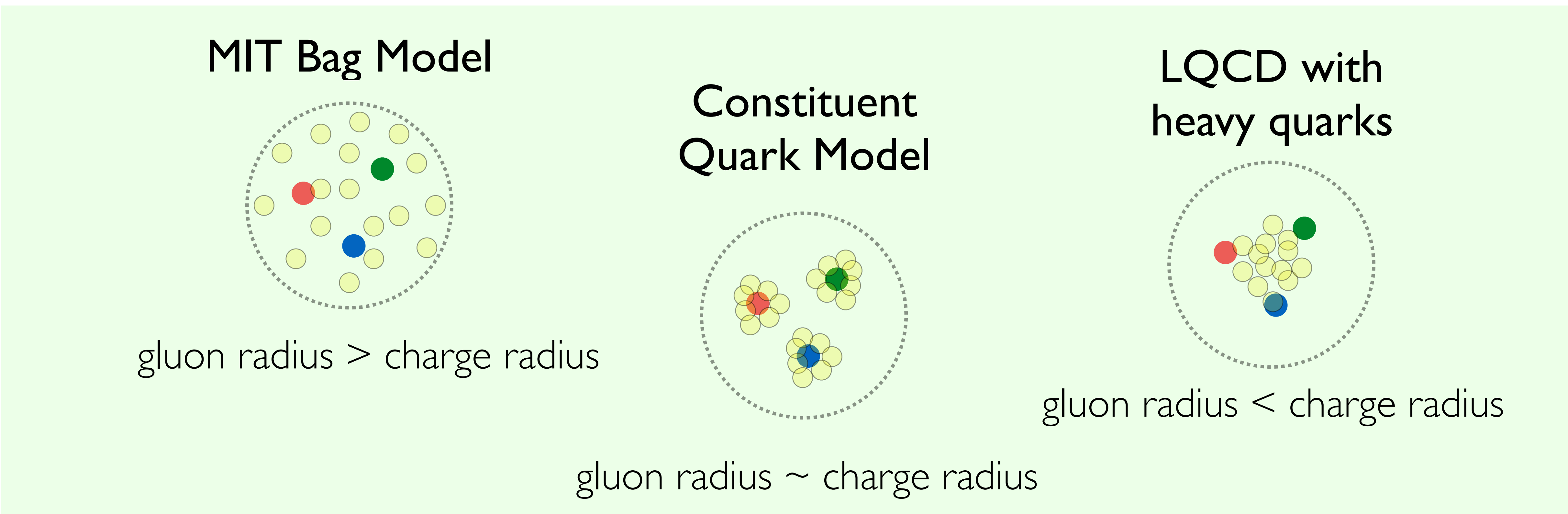
How is the gluon structure of a proton modified in a nucleus

- Gluon 'EMC' effect
- Exotic glue

Gluon radii

1

How does the gluon radius of a proton compare to the quark/charge radius?



Charge radius:
slope of electric form factor with respect to momentum transfer

Gluon radius:
slope of gluon form factor with respect to momentum transfer

Gluon structure of nuclei

2A

How does the gluon structure of a nucleon change in a nucleus?

Ratio of structure function F_2 per nucleon for iron and deuterium

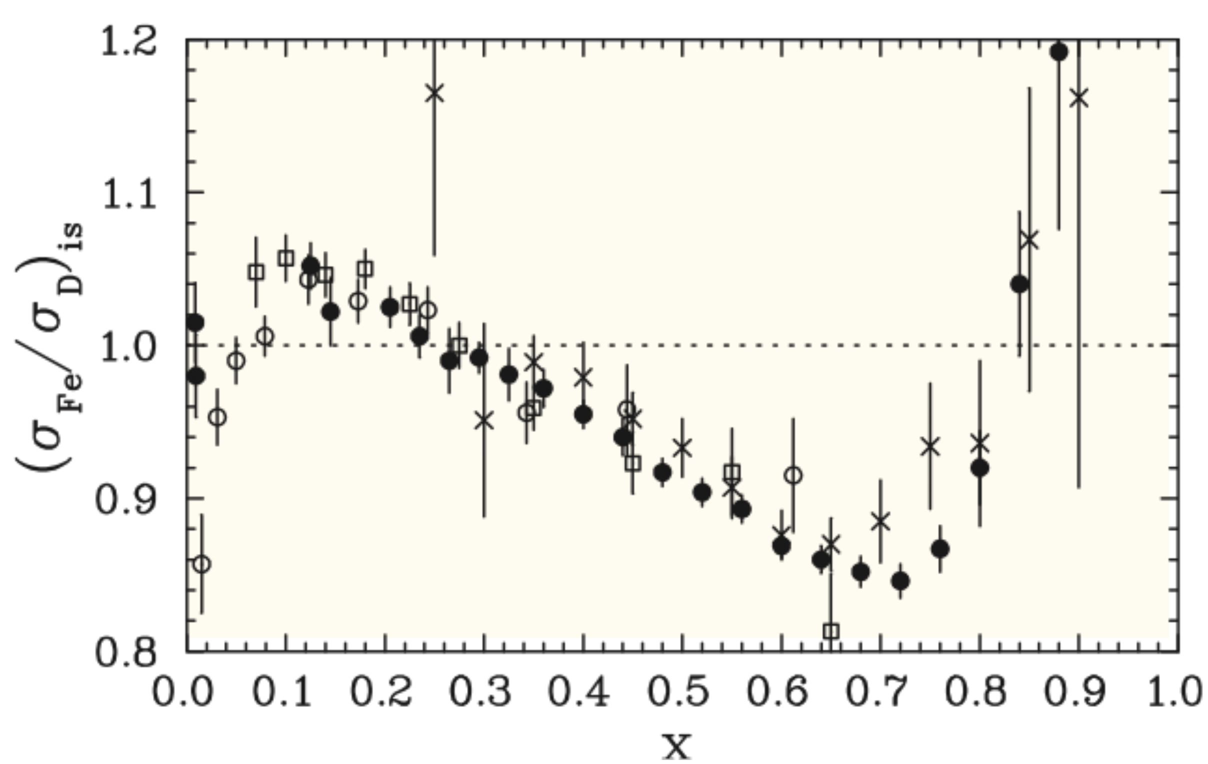
$$F_2(x, Q^2) = \sum_{q=u,d,s\dots} x\, e_q^2 \left[q(x, Q^2) + \overline{q}(x, Q^2)\right]$$

Number density of partons of flavour q

European Muon Collaboration (1983): "EMC effect"

Modification of per-nucleon cross section of nucleons bound in nuclei

Gluon analogue?



Longitudinal momentum fraction

Gluon structure of nuclei

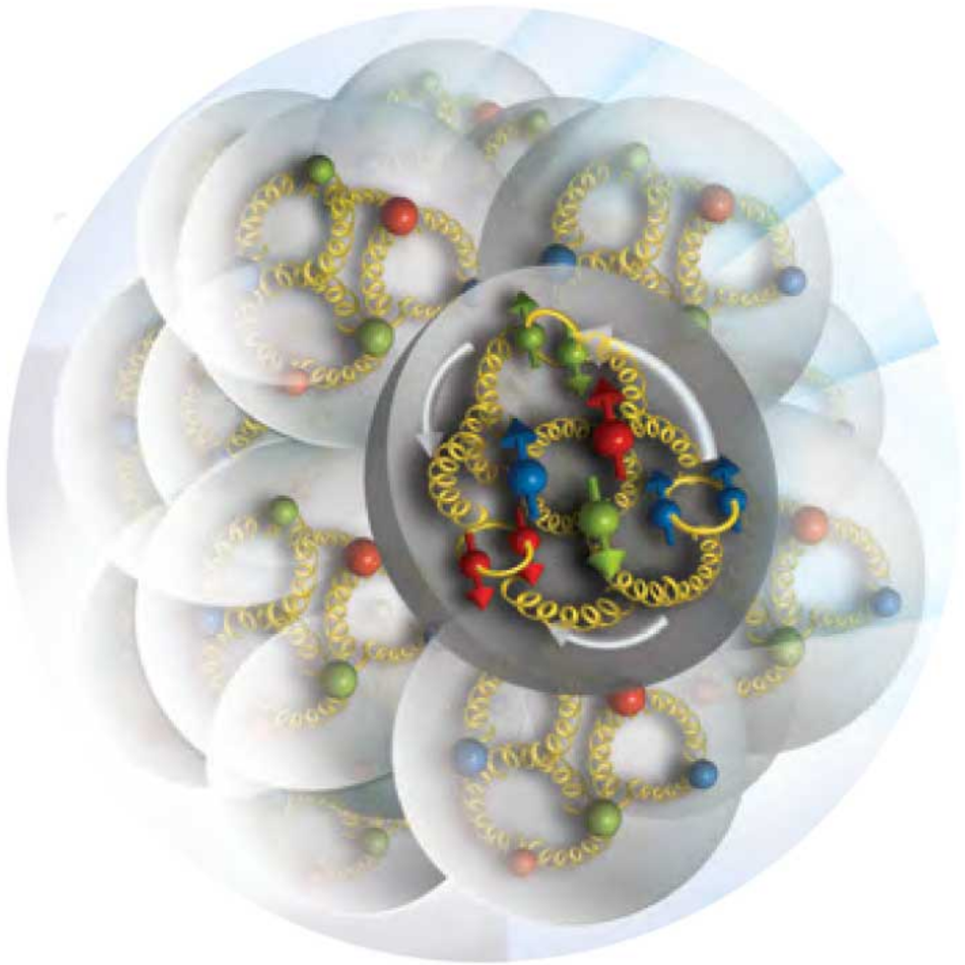
2B **Exotic Glue**

Contributions to nuclear structure from gluons not associated with individual nucleons in nucleus

Exotic glue operator:

nucleon $\langle p|\mathcal{O}|p\rangle = 0$

nucleus $\langle N, Z|\mathcal{O}|N, Z\rangle \neq 0$



Jaffe and Manohar, "Nuclear Gluonometry" Phys. Lett. B223 (1989) 218

Gluon structure of nuclei

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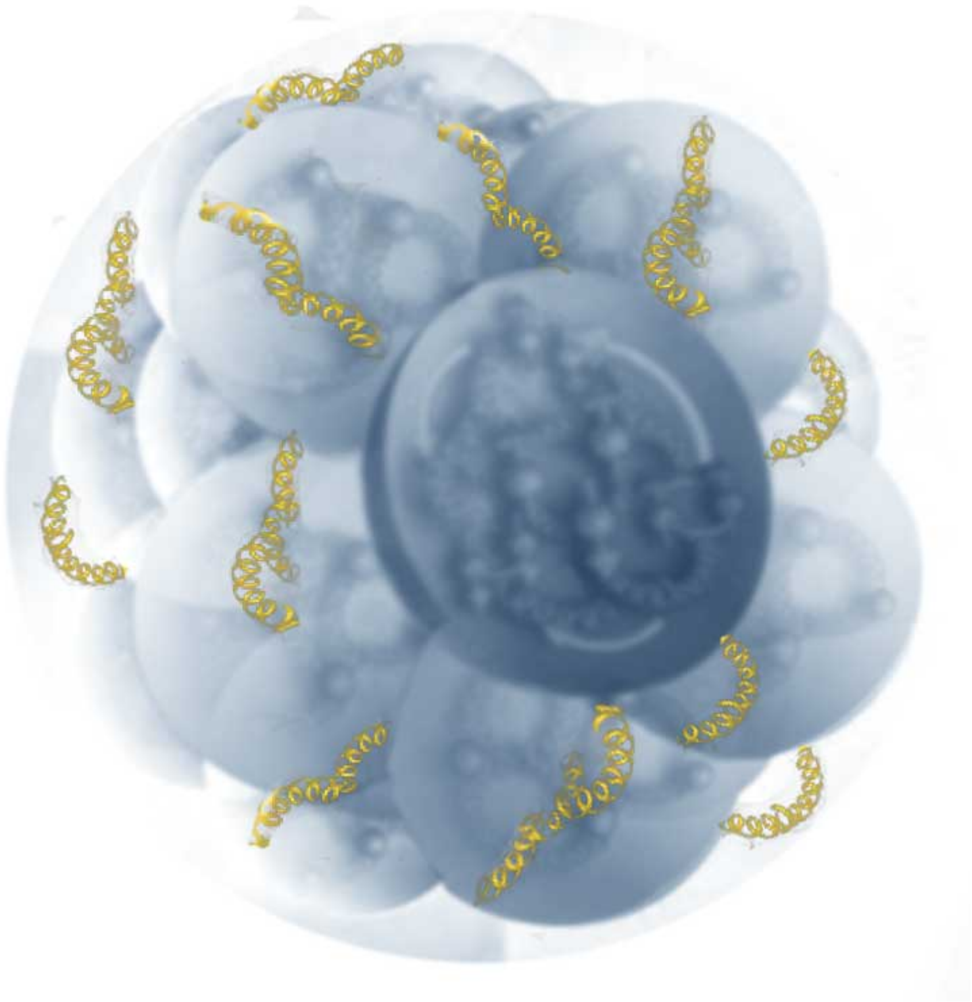

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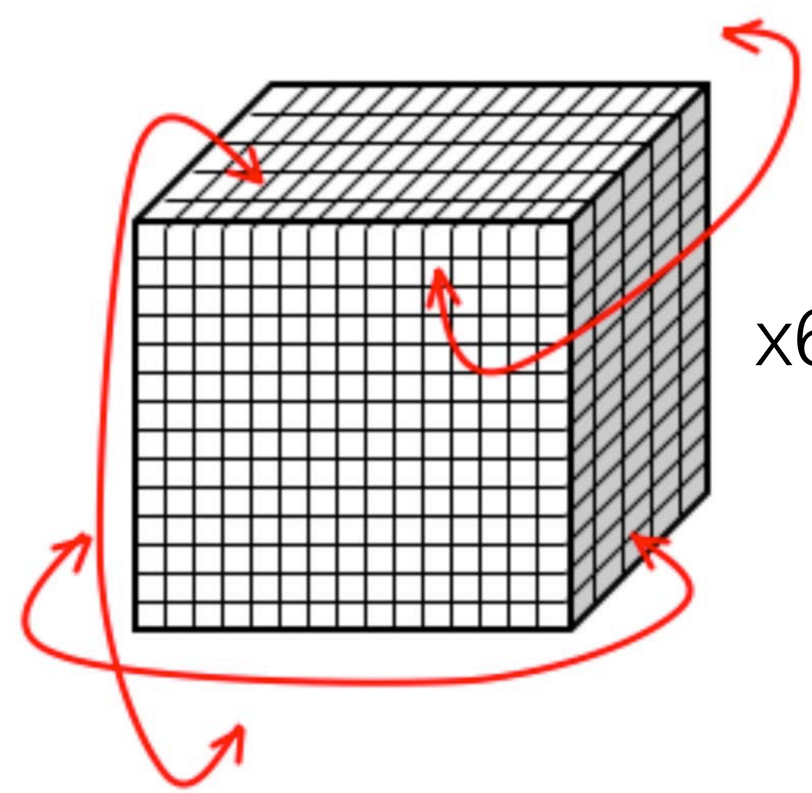
Lattice QCD

- Numerical first-principles approach to non-perturbative QCD
- Euclidean space-time $t \to i\tau$
  - Finite lattice spacing a
  - Volume $L^3 \times T \approx 32^3 \times 64$
  - Boundary conditions
- Some calculations use larger-than-physical quark masses (cheaper)



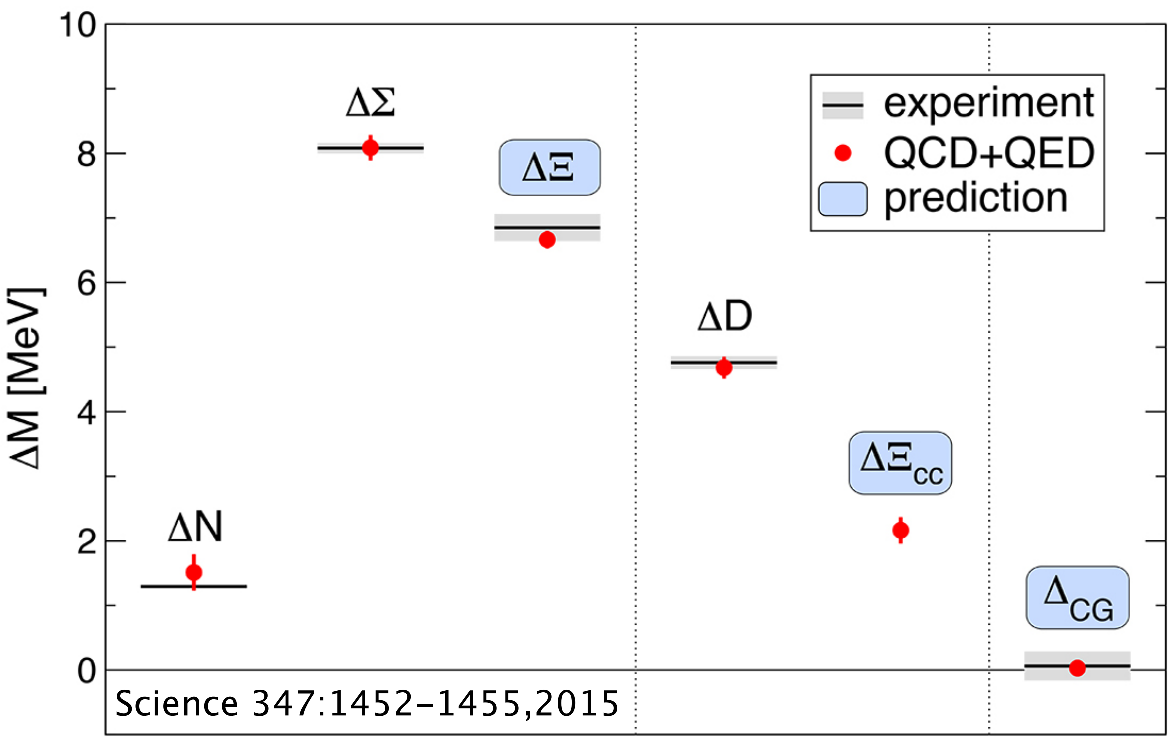
Approximate the QCD path integral by **Monte Carlo**

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\overline{\psi} \mathcal{D}\psi \mathcal{O}[A, \overline{\psi}\psi] e^{-S[A,\overline{\psi}\psi]} \rightarrow \langle \mathcal{O} \rangle \simeq \frac{1}{N_{\mathrm{conf}}} \sum_{i}^{N_{\mathrm{conf}}} \mathcal{O}([U^i])$$

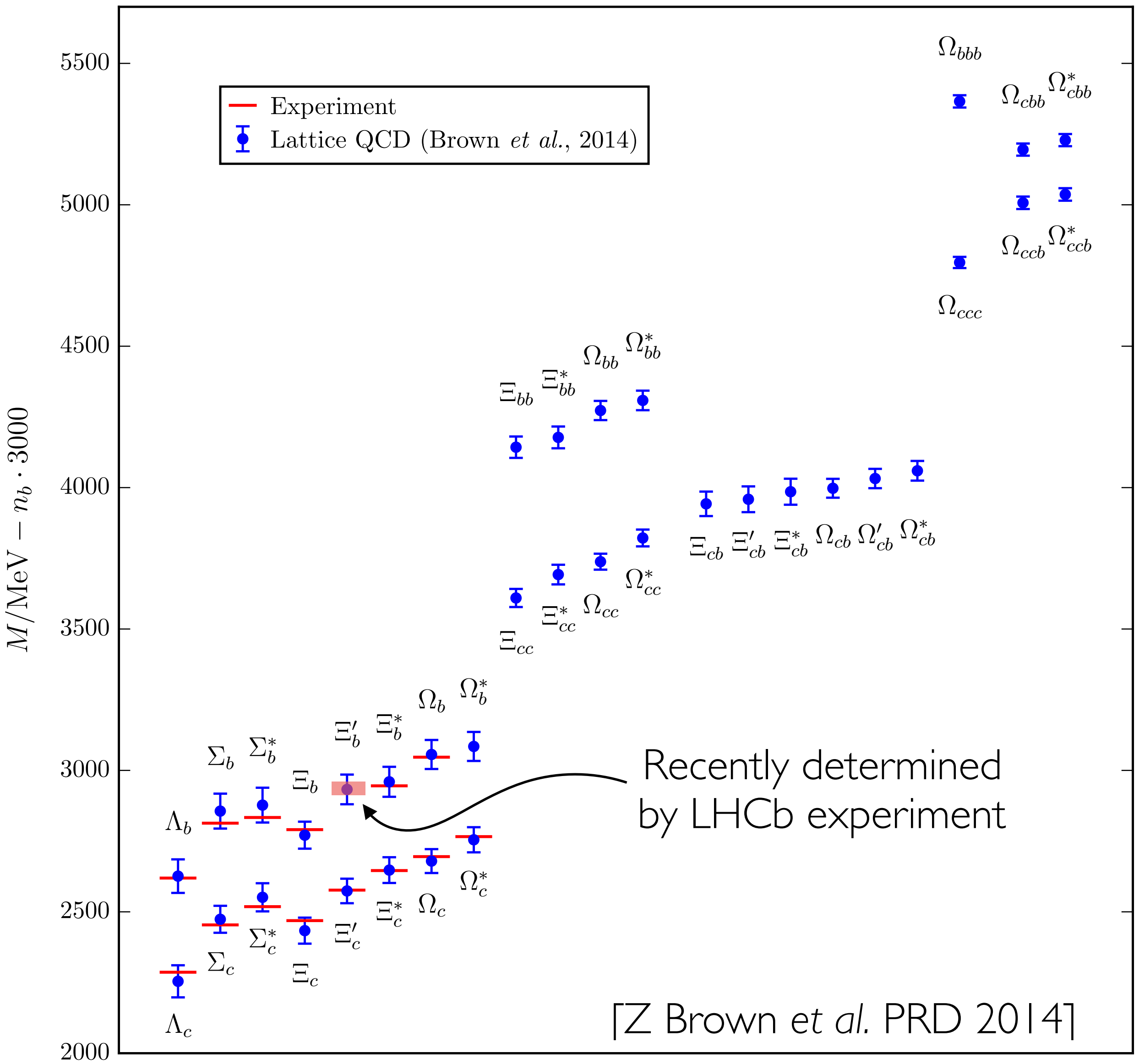
with field configurations U^i distributed according to $e^{-S[U]}$

Lattice QCD works

- Ground state hadron spectrum reproduced
- p-n mass splitting reproduced
- ...



- Predictions for new states with controlled uncertainties



Doing lattice QCD

- Correlation decays exponentially with distance in time:

$$C_2(t) = \sum_{n} Z_n \exp(-E_n t)$$

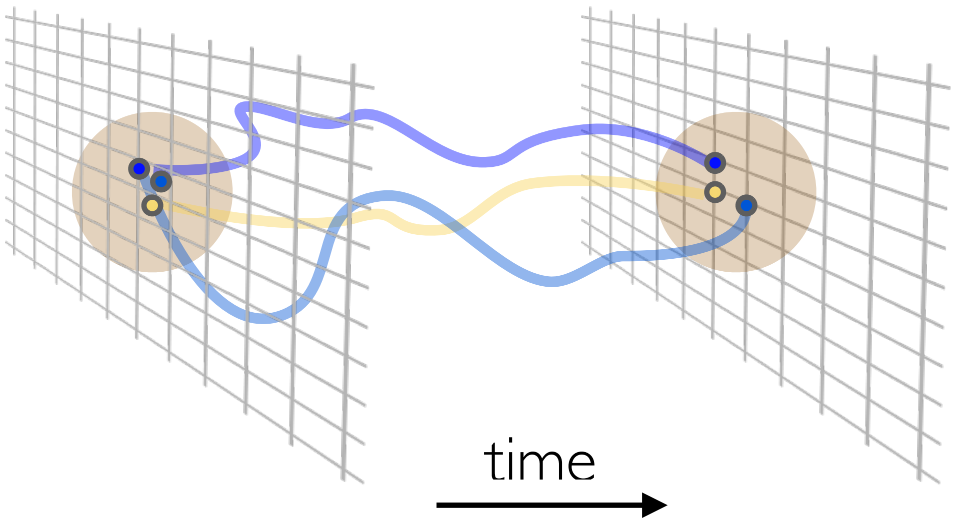
all eigenstates with q#'s of proton

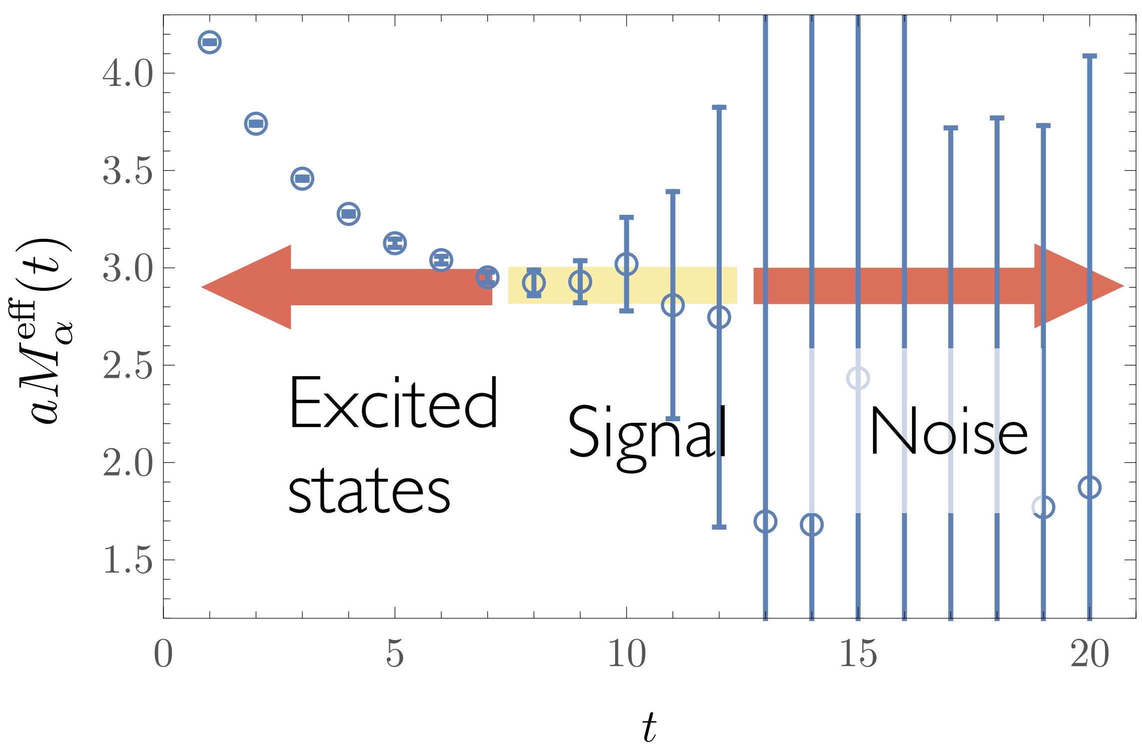
At late times:

$$\rightarrow Z_0 \exp(-E_0 t)$$

- Ground state mass revealed through "effective mass plot"

$$M(t) = \ln\left[\frac{C_2(t)}{C_2(t+1)}\right] \xrightarrow{t\to\infty} E_0$$



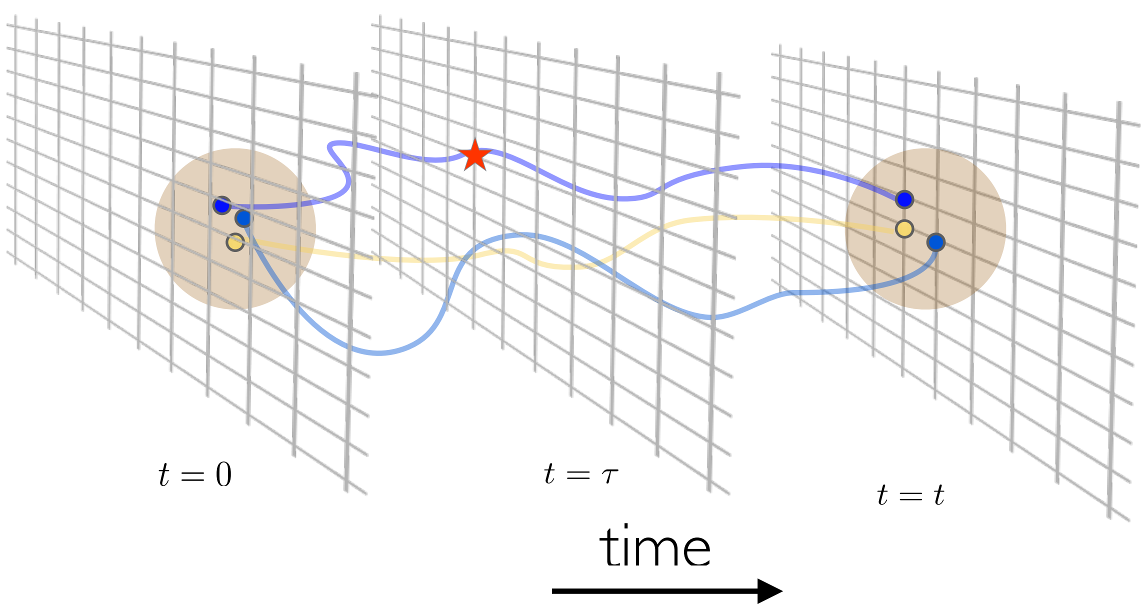


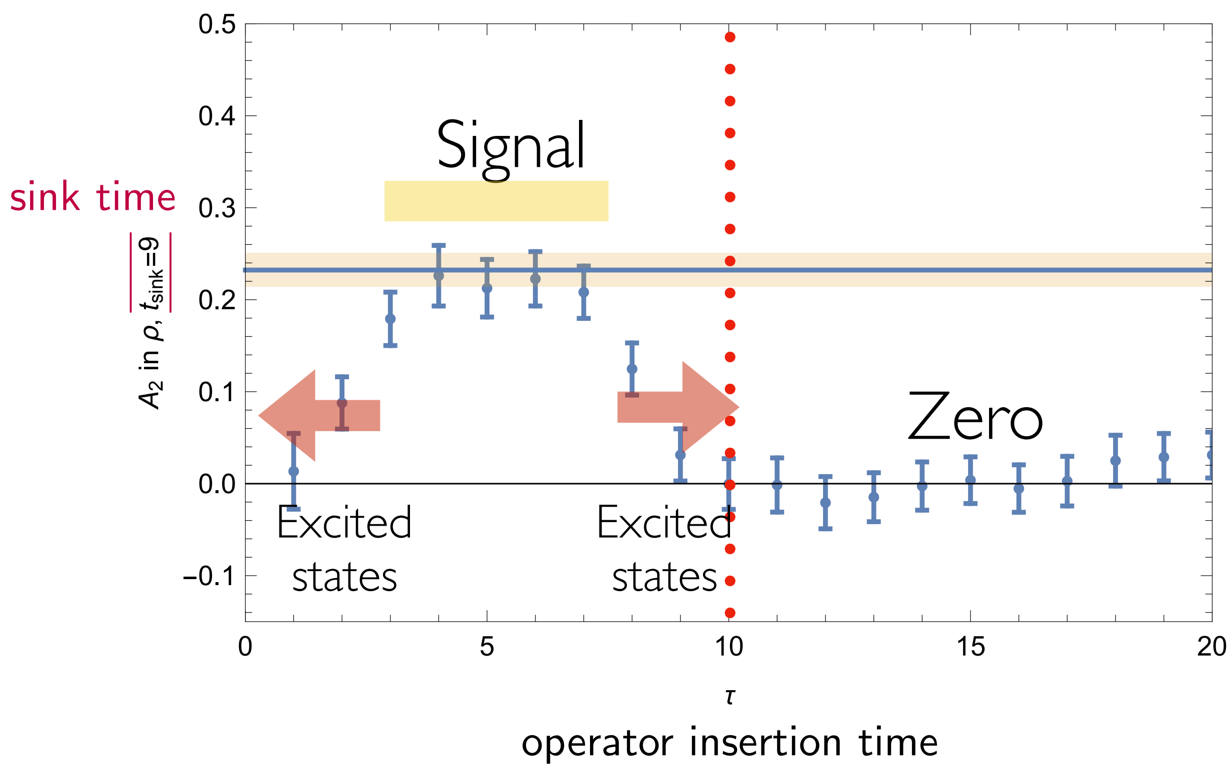
LQCD matrix elements

Calculate matrix elements

- Create three quarks (correct quantum numbers) at a source and annihilate the three quarks at sink far from source
- Insert operator at intermediate timeslice
- Remove time-dependence by dividing out with two-point correlators:

$$\frac{C_3(t,\tau,\vec{p},\vec{q})}{C_2(\tau,\vec{p})} \xrightarrow{0\ll\tau\ll t} \langle N(p')|\mathcal{O}(q)|N(p)\rangle$$



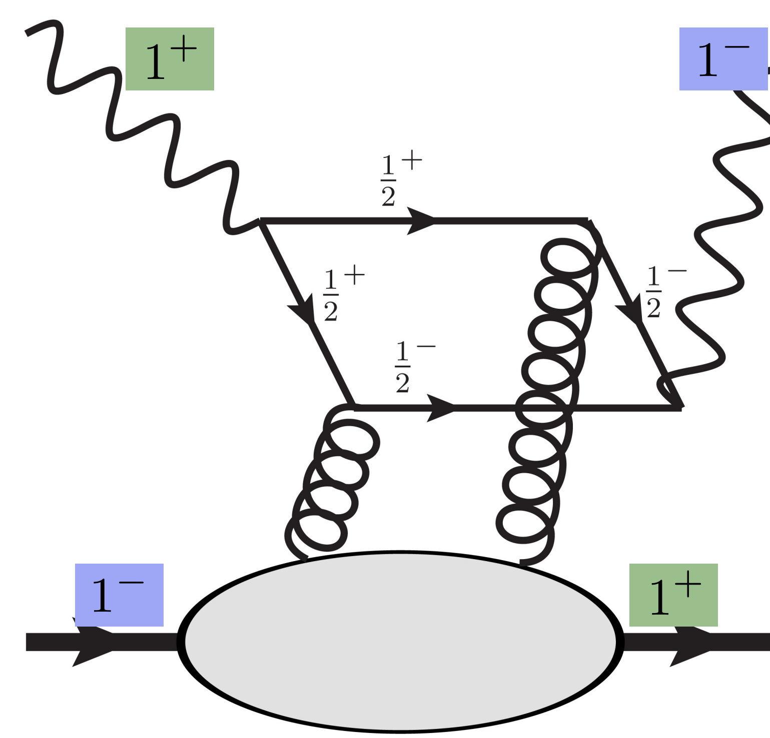


Gluon transversity

Leading twist gluon parton distribution Δ(x,Q²)
[Jaffe & Manohar 1989]

- **Unambiguously gluonic:** no analogous quark PDF at twist-2
- **Double helicity flip**: non-vanishing in forward limit for targets with spin ≥ 1
- **Experimentally measurable** in unpolarised electron DIS on polarised target
  - Nitrogen target: JLab LoI 2015
  - Polarised nuclei at EIC

$$\Delta(x,Q^2) = A_{+-,-+}$$



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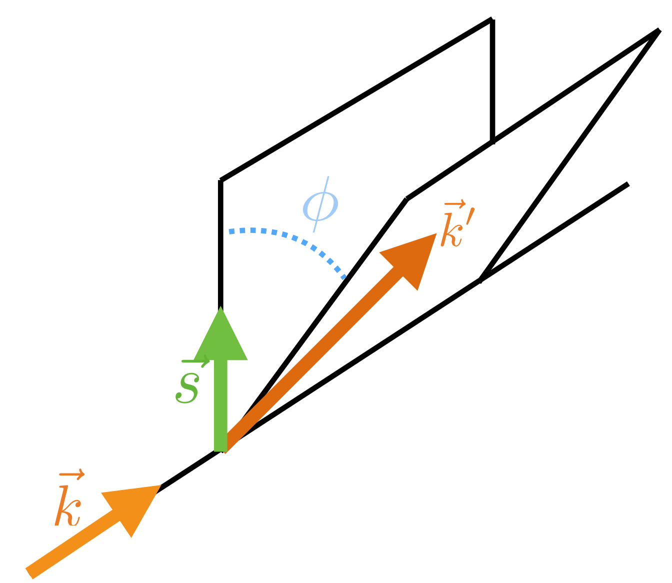


Measure azimuthal variation

$$\lim_{Q^2\to\infty}\frac{d\sigma}{dx\,dy\,d\phi}=\frac{e^4ME}{4\pi^2Q^4}\Big[xy^2F_1(x,Q^2)+(1-y)F_2(x,Q^2) - \frac{x(1-y)}{2}\Delta(x,Q^2)\cos 2\phi\Big]$$

Gluon transversity

Double helicity flip distribution $\Delta(x,Q^2)$

- **Hadrons:** Gluonic Transversity (parton model interpretation)

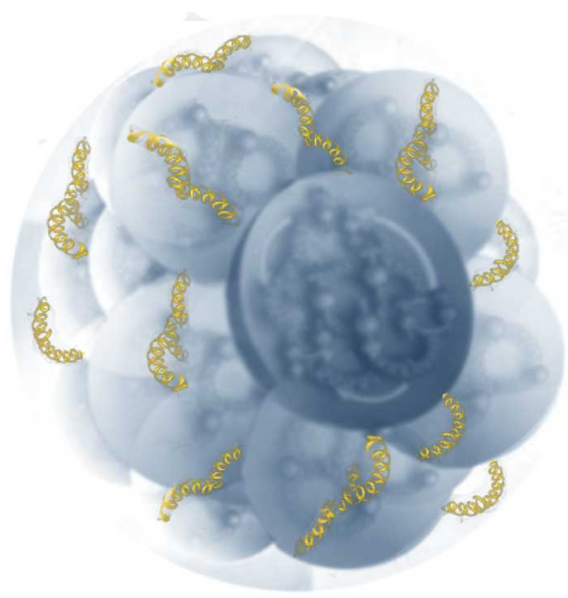
$$\Delta(x,Q^2) = -\frac{\alpha_s(Q^2)}{2\pi}\mathrm{Tr}\mathcal{Q}^2 x^2 \int_x^1 \frac{dy}{y^3}\left[g_{\hat{x}}(y,Q^2) - g_{\hat{y}}(y,Q^2)\right]$$

$g_{\hat{x},\hat{y}}(y,Q^2)$: probability of finding a gluon with momentum fraction y linearly polarised in $\hat{x},\hat{y}$ direction in a target polarised in $\hat{x}$ direction

- **Nuclei:** Exotic Glue

gluons not associated with individual nucleons in nucleus

$$\langle p|\mathcal{O}|p\rangle = 0$$
$$\langle N,Z|\mathcal{O}|N,Z\rangle \neq 0$$



Gluon transversity

Calculating lightcone distributions is challenging in Euclidean space

Moments of Δ(x,Q²) are calculable in lattice QCD

Moments of distribution

Reduced Matrix Element

$$\boxed{\int_0^1 dx x^{n-1} \Delta(x, Q^2)} = \frac{\alpha_s(Q^2)}{3\pi} \frac{\boxed{A_n(Q^2)}}{n+2}, \qquad n = 2, 4, 6 \ldots,$$

Determined by matrix elements of local gluonic operators

Gluon field strength tensor

Polarisation vector (spin-1)

Symmetrise in $\mu_1, \ldots, \mu_n$, trace subtract in all free indices

$$\langle p\underline{E'} | \underline{S} \left[\underline{G_{\mu\mu_1}} \overleftrightarrow{D}_{\mu_3} \ldots \overleftrightarrow{D}_{\mu_n} \underline{G_{\nu\mu_2}} \right] | p\underline{E} \rangle$$

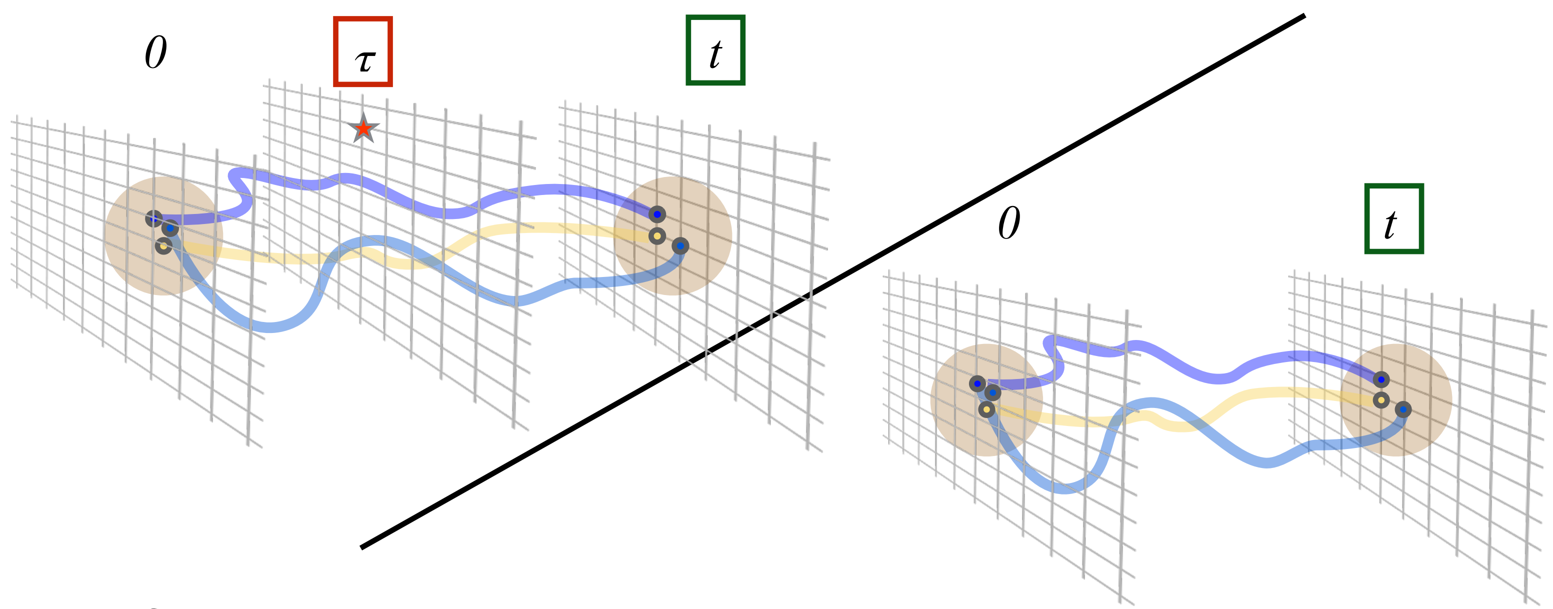
$$= (-2i)^{n-2} \underline{S} \left[(p_\mu E'^*_{\mu_1} - p_{\mu_1} E'^*_\mu)(p_\nu E_{\mu_2} - p_{\mu_2} E_\nu) + (\mu \leftrightarrow \nu) \right] p_{\mu_3} \ldots p_{\mu_n} \boxed{A_n(Q^2)} \ldots,$$

Reduced Matrix Element

Gluon transversity

- Discrete lattice: rotational symmetry ➡ hypercubic symmetry
- Take linear combinations of operators that transform irreducibly under hypercubic group

$R_{jk}(t, \tau, \vec{p}) =$



$\propto$ Matrix element

operator insertion time

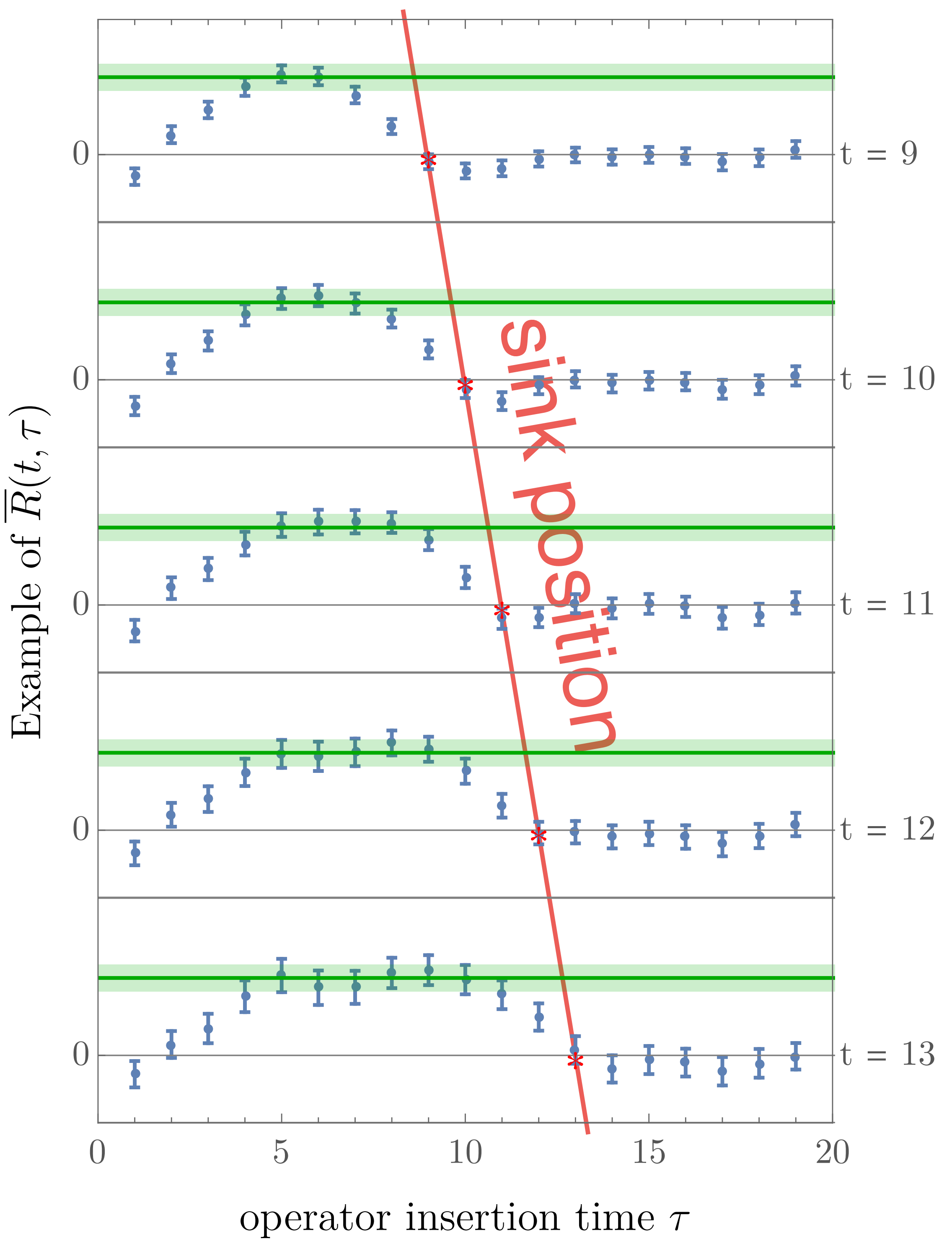
$0 \ll \boxed{\tau} \ll \boxed{t}$

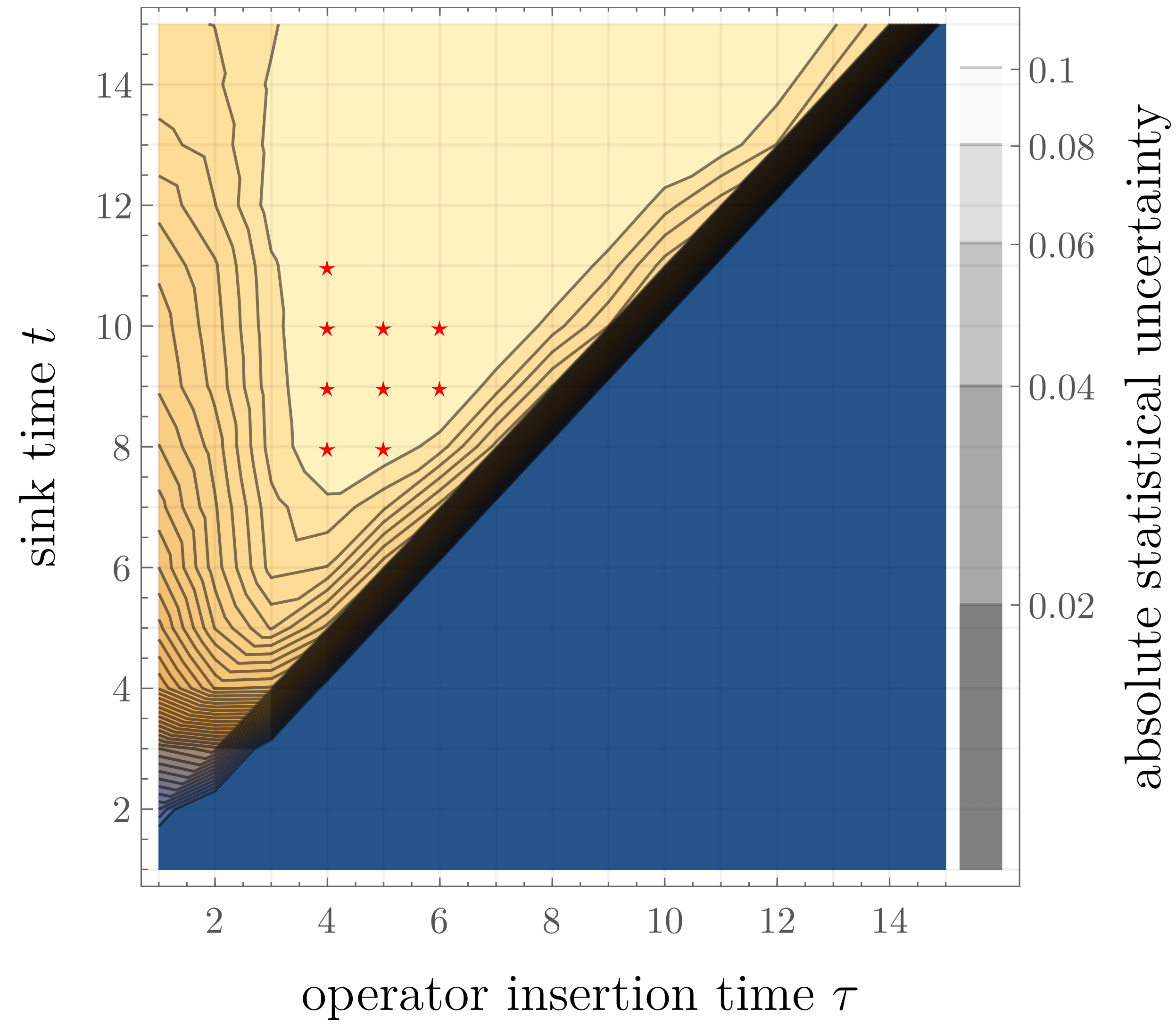
sink time

Calculate ratio for

- All source-sink polarisation combinations (j,k)
- Boost momenta up to (1,1,1)
- All operators in each hypercubic irrep.

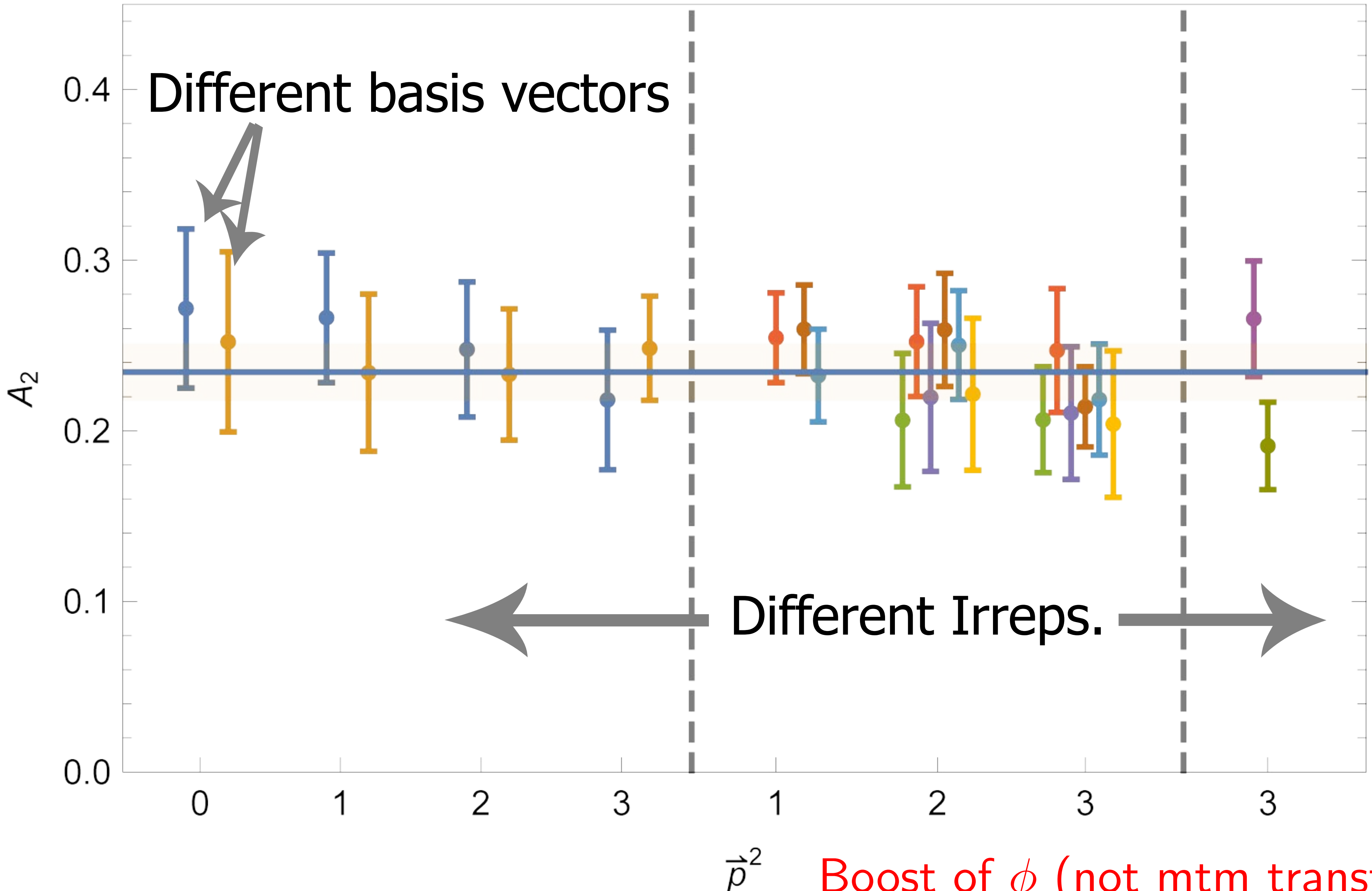
Gluon transversity





LQCD calculation

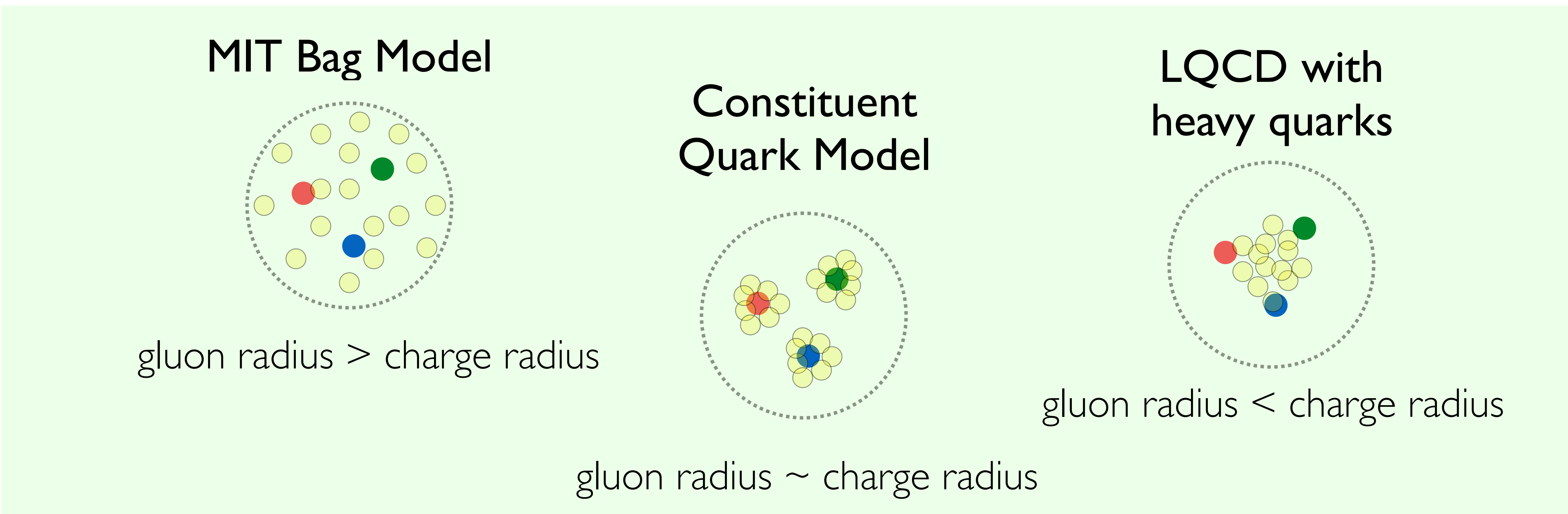
W. Detmold, PES, PRD 94 (2016), 014507



Boost of ϕ (not mtm transfer)

Gluon radii

1 How does the gluon radius of a proton compare to the quark/charge radius?



Charge radius:
slope of electric form factor with respect to momentum transfer

Gluon radius:
slope of gluon form factor with respect to momentum transfer

Gluon radii

Off-forward matrix elements (momentum transfer through operator)

- Moments of $\Delta(x,Q^2)$ related to many generalised form factors

polarisation vectors Average momentum

$$
\begin{aligned}
&\left\langle p'E' \middle| S\left[G_{\mu\mu_1} i\overleftrightarrow{D}_{\mu_3}\dots i\overleftrightarrow{D}_{\mu_n} G_{\nu\mu_2}\right] \middle| pE \right\rangle \\
&= \sum_{\substack{m\ \text{even} \\ m=2}}^{n} \Big\{ A^{(n)}_{1,m-2}(\Delta^2)\, S\left[(P_\mu E_{\mu_1} - E_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2})\Delta_{\mu_3}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right]
\end{aligned}
$$

Momentum transfer

$$
\begin{aligned}
&+ A^{(n)}_{2,m-2}(\Delta^2)\, S\left[(\Delta_\mu E_{\mu_1} - E_\mu \Delta_{\mu_1})(\Delta_\nu E'^*_{\mu_2} - E'^*_\nu \Delta_{\mu_2})\Delta_{\mu_3}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right] \\
&+ A^{(n)}_{3,m-2}(\Delta^2)\, S\Big[\big((\Delta_\mu E_{\mu_1} - E_\mu \Delta_{\mu_1})(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2}) - (\Delta_\mu E'^*_{\mu_1} - E'^*_\mu \Delta_{\mu_1})(P_\nu E_{\mu_2} - E_\nu P_{\mu_2})\big) \\
&\qquad\qquad \times \Delta_{\mu_3}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\Big] \\
&+ A^{(n)}_{4,m-2}(\Delta^2)\, S\left[(E_\mu E'^*_{\mu_1} - E_{\mu_1} E'^*_\mu)(P_\nu \Delta_{\mu_2} - P_{\mu_2}\Delta_\nu)\Delta_{\mu_3}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right] \\
&+ \frac{A^{(n)}_{5,m-2}(\Delta^2)}{M^2} S\Big[\big((E\cdot P)(P_\mu \Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(\Delta_\nu E'^*_{\mu_2} - E'^*_\nu \Delta_{\mu_2}) \\
&\qquad\qquad + (E'^*\cdot P)(P_\mu \Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(\Delta_\nu E_{\mu_2} - E_\nu \Delta_{\mu_2})\big)\,\Delta_{\mu_3}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\Big] \\
&+ \frac{A^{(n)}_{6,m-2}(\Delta^2)}{M^2} S\Big[\big((E\cdot P)(P_\mu \Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2}) \\
&\qquad\qquad - (E'^*\cdot P)\,(P_\mu \Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu E_{\mu_2} - E_\nu P_{\mu_2})\big)\,\Delta_{\mu_3}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\Big] \\
&+ \frac{A^{(n)}_{7,m-2}(\Delta^2)}{M^2} (E'^*\cdot E) S\left[(P_\mu \Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu \Delta_{\mu_2} - \Delta_\nu P_{\mu_2})\Delta_{\mu_3}\dots\Delta_{\mu_{m-1}} P_{\mu_m}\dots P_{\mu_n}\right] \\
&+ \frac{A^{(n)}_{8,m-2}(\Delta^2)}{M^4} (E\cdot P)(E'^*\cdot P) S\left[(P_\mu \Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu \Delta_{\mu_2} - \Delta_\nu P_{\mu_2})\Delta_{\mu_3}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right] \Big\}
\end{aligned}
$$

Gluon radii

Off-forward matrix elements (momentum transfer through operator)

- Moments of $\Delta(x,Q^2)$ related to many generalised form factors

$$\left\langle p'E' \left| S\left[G_{\mu\mu_1} i\overleftrightarrow{D}_{\mu_3}\ldots i\overleftrightarrow{D}_{\mu_n} G_{\nu\mu_2}\right]\right| pE\right\rangle$$

polarisation vectors | Average momentum

$$= \sum_{\substack{m \text{ even} \\ m=2}}^{n} \Big\{ \boxed{A^{(n)}_{1,m-2}(\Delta^2)} S\left[(P_\mu E_{\mu_1} - E_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2})\Delta_{\mu_3}\ldots\Delta_{\mu_m}P_{\mu_{m+1}}\ldots P_{\mu_n}\right]$$

Momentum transfer

$$+ \boxed{A^{(n)}_{2,m-2}(\Delta^2)} S\left[(\Delta_\mu E_{\mu_1} - E_\mu \Delta \right.$$

$$+ \boxed{A^{(n)}_{3,m-2}(\Delta^2)} S\left[((\Delta_\mu E_{\mu_1} - E \quad \ldots \quad E_\nu P_{\mu_2}))\right.$$

$$\times \Delta_{\mu_3} \ldots \Delta_{\mu_m}$$

$$+ \boxed{A^{(n)}_{4,m-2}(\Delta^2)} S\left[(\Delta_\mu E'^*_{\mu_1}\right.$$

$$+ \boxed{\frac{A^{(n)}_{5,m-2}(\Delta^2)}{M^2}} S\left[((E\cdot P)(P_\mu\Delta_\mu\right.$$

$$+ (E'^* \cdot P)(P \quad \ldots \quad P_{\mu_n}\Big]$$

$$+ \boxed{\frac{A^{(n)}_{6,m-2}(\Delta^2)}{M^2}} S\left[((E\cdot P)(P_\mu\Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2})\right.$$

$$\left. - (E'^* \cdot P)\,(P_\mu\Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu E_{\mu_2} - E_\nu P_{\mu_2}))\Delta_{\mu_3}\ldots\Delta_{\mu_m}P_{\mu_{m+1}}\ldots P_{\mu_n}\right]$$

$$+ \boxed{\frac{A^{(n)}_{7,m-2}(\Delta^2)}{M^2}} (E'^* \cdot E) S\left[(P_\mu\Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu\Delta_{\mu_2} - \Delta_\nu P_{\mu_2})\Delta_{\mu_3}\ldots\Delta_{\mu_{m-1}}P_{\mu_m}\ldots P_{\mu_n}\right]$$

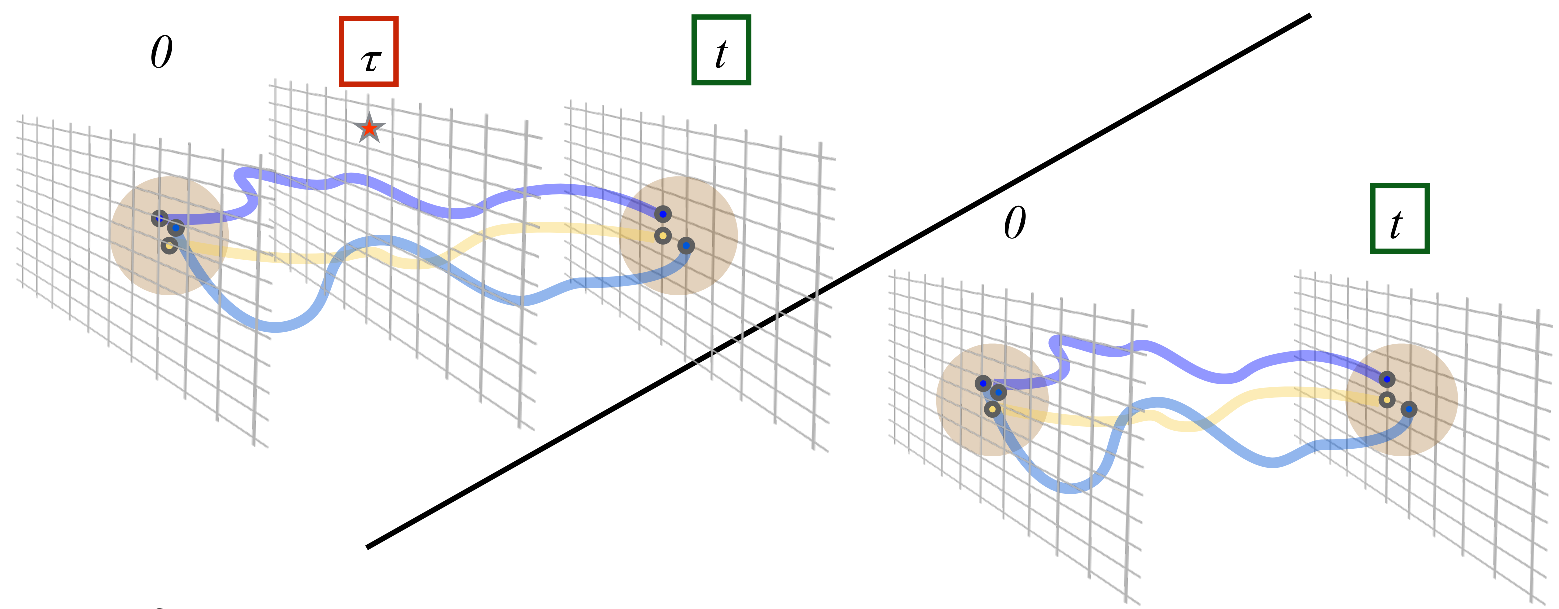
$$+ \boxed{\frac{A^{(n)}_{8,m-2}(\Delta^2)}{M^4}} (E\cdot P)(E'^* \cdot P) S\left[(P_\mu\Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu\Delta_{\mu_2} - \Delta_\nu P_{\mu_2})\Delta_{\mu_3}\ldots\Delta_{\mu_m}P_{\mu_{m+1}}\ldots P_{\mu_n}\right]\Big\}$$

Many gluonic radii:
Defined by slope of each form factor at $Q^2=\Delta^2=0$ (or linear combinations)

Gluon radii

- Discrete lattice: rotational symmetry ➡ hypercubic symmetry
- Take linear combinations of operators that transform irreducibly under hypercubic group

$$R_{jk}(t,\tau,\vec{p}) =$$



$\propto$ Matrix element

operator insertion time

$$0 \ll \boxed{\tau} \ll \boxed{t}$$

sink time

Calculate ratio for

- All source-sink polarisation combinations (j,k)
- Boost momenta up to (1,1,1)
- All operators in each hypercubic irrep.

Gluon Generalised FFs

- Complicated over and under-determined systems of equations (different choices of polarisation and boost at same momentum transfer)
- Some GFFs suppressed by orders of magnitude
- Some GFFs related by symmetries at some momenta

Simplest example:
Transversity GFFs
One basis (2 vectors)
Mtm 1 (lattice units)

$$
\begin{pmatrix}
0.604 & 0.0424 & 0 & 0 & 0 & 0 & 0.0588 & 0 \\
0.592 & -2.45\times10^{-3} & 0.0785 & -0.0785 & 6.58\times10^{-3} & -0.0992 & -0.103 & -4.15\times10^{-3} \\
0.485 & 0.0429 & 0 & 0 & 0 & 0 & 0.0379 & 0 \\
0.481 & 0.0431 & -3.02\times10^{-5} & 3.02\times10^{-5} & -2.53\times10^{-6} & -4.03\times10^{-7} & 0.0374 & -1.69\times10^{-8} \\
0.475 & -3.29\times10^{-3} & 0.0791 & -0.0791 & 6.59\times10^{-3} & -0.0791 & -0.0824 & -3.29\times10^{-3} \\
0.353 & -7.97\times10^{-4} & 0.0385 & -0.0385 & 3.28\times10^{-3} & -0.0598 & -0.0631 & -2.54\times10^{-3} \\
0.347 & -0.0382 & 0 & 0 & 0 & 0 & 0.0962 & 0 \\
0.258 & 0.0806 & 0 & 0 & 0 & 0 & -0.0374 & 0 \\
0.258 & 0.0808 & 0 & 0 & 0 & 0 & -0.0379 & 0 \\
0.253 & 0.101 & -8.60\times10^{-4} & 8.60\times10^{-4} & -7.20\times10^{-5} & 6.32\times10^{-7} & -0.0588 & 2.65\times10^{-8} \\
0.239 & -1.66\times10^{-3} & 0.0401 & -0.0401 & 3.29\times10^{-3} & -0.0393 & -0.0402 & -1.61\times10^{-3} \\
0.238 & -1.65\times10^{-3} & 0.0396 & -0.0396 & 3.29\times10^{-3} & -0.0396 & -0.0412 & -1.65\times10^{-3} \\
0.228 & -0.0581 & 8.30\times10^{-4} & -8.30\times10^{-4} & 6.94\times10^{-5} & -1.04\times10^{-6} & 0.0962 & -4.33\times10^{-8} \\
0.228 & -0.0379 & 0 & 0 & 0 & 0 & 0.0758 & 0 \\
0.0590 & -0.0109 & 0.139 & -0.139 & 0.0112 & -4.97\times10^{-3} & -3.94\times10^{-4} & -8.24\times10^{-6} \\
0.0578 & -2.56\times10^{-4} & 9.42\times10^{-3} & -9.42\times10^{-3} & 3.89\times10^{-4} & -4.65\times10^{-3} & 2.51\times10^{-4} & 5.25\times10^{-6} \\
0.0338 & 1.59\times10^{-3} & -0.128 & 0.128 & -0.0107 & 3.18\times10^{-4} & 0.0154 & 1.33\times10^{-5} \\
0.0183 & 6.36\times10^{-3} & -1.29\times10^{-4} & 1.29\times10^{-4} & 3.84\times10^{-4} & 4.84\times10^{-3} & 5.99\times10^{-3} & 5.18\times10^{-6} \\
0.0155 & -4.78\times10^{-3} & -0.128 & 0.128 & -0.0111 & -4.52\times10^{-3} & 9.41\times10^{-3} & 8.14\times10^{-6} \\
1.19\times10^{-3} & -0.0106 & 0.129 & -0.129 & 0.0108 & -3.22\times10^{-4} & -6.45\times10^{-4} & -1.35\times10^{-5} \\
0.549 & 2.44\times10^{-3} & 0 & 0 & 0 & 0 & 0.0895 & 0 \\
0.546 & -1.88\times10^{-3} & 0.0676 & -0.0676 & 5.69\times10^{-3} & -0.0918 & -0.0960 & -3.86\times10^{-3} \\
0.498 & 0.0710 & 0 & 0 & 0 & 0 & 0.0123 & 0 \\
0.480 & -2.37\times10^{-3} & 0.0685 & -0.0685 & 5.70\times10^{-3} & -0.0799 & -0.0828 & -3.33\times10^{-3} \\
0.429 & 0.0714 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.424 & 0.0834 & -5.14\times10^{-4} & 5.14\times10^{-4} & -4.30\times10^{-5} & 1.33\times10^{-7} & -0.0123 & 5.55\times10^{-9} \\
0.412 & 2.85\times10^{-3} & 0 & 0 & 0 & 0 & 0.0657 & 0 \\
0.412 & -2.85\times10^{-3} & 0.0685 & -0.0685 & 5.70\times10^{-3} & -0.0685 & -0.0714 & -2.85\times10^{-3} \\
0.409 & -8.65\times10^{-3} & 4.61\times10^{-4} & -4.61\times10^{-4} & 3.86\times10^{-5} & -8.30\times10^{-7} & 0.0771 & -3.47\times10^{-8} \\
0.0674 & -6.43\times10^{-3} & 0.0856 & -0.0856 & 6.70\times10^{-3} & -5.55\times10^{-3} & -8.26\times10^{-5} & -1.73\times10^{-6} \\
0.0656 & 4.96\times10^{-4} & -9.21\times10^{-4} & 9.21\times10^{-4} & -6.37\times10^{-6} & -0.0119 & -0.0132 & -5.32\times10^{-4} \\
0.0514 & -0.0685 & 0 & 0 & 0 & 0 & 0.0771 & 0 \\
0.0347 & -0.0124 & 0.155 & -0.155 & 0.0127 & -3.05\times10^{-3} & -6.00\times10^{-4} & -1.26\times10^{-5} \\
0.0327 & 5.99\times10^{-3} & -0.0692 & 0.0692 & -6.03\times10^{-3} & -2.50\times10^{-3} & 5.17\times10^{-4} & 1.08\times10^{-5} \\
0.0301 & 4.59\times10^{-3} & -0.0738 & 0.0738 & -5.95\times10^{-3} & 2.98\times10^{-3} & 0.0123 & 1.07\times10^{-5} \\
0.0285 & -1.84\times10^{-3} & -0.147 & 0.147 & -0.0126 & -2.43\times10^{-3} & 0.0143 & 1.24\times10^{-5} \\
0.0171 & 0.0685 & 0 & 0 & 0 & 0 & -0.0657 & 0 \\
0.0146 & 0.0920 & -9.75\times10^{-4} & 9.75\times10^{-4} & -8.17\times10^{-5} & 9.63\times10^{-7} & -0.0895 & 4.03\times10^{-8} \\
1.59\times10^{-3} & 6.43\times10^{-3} & 0.0736 & -0.0736 & 6.61\times10^{-3} & 5.40\times10^{-3} & -1.97\times10^{-3} & -1.71\times10^{-6}
\end{pmatrix}
\begin{pmatrix}
A^{(2)}_{1,0}(1) \\ A^{(2)}_{2,0}(1) \\ A^{(2)}_{3,0}(1) \\ A^{(2)}_{4,0}(1) \\ A^{(2)}_{5,0}(1) \\ A^{(2)}_{6,0}(1) \\ A^{(2)}_{7,0}(1) \\ A^{(2)}_{8,0}(1)
\end{pmatrix}
=
\begin{pmatrix}
0.179(36) \\ 0.150(38) \\ 0.152(30) \\ 0.154(37) \\ 0.129(32) \\ 0.056(31) \\ 0.067(41) \\ 0.056(35) \\ 0.069(21) \\ 0.093(36) \\ 0.028(32) \\ 0.041(27) \\ 0.012(33) \\ 0.029(30) \\ 0.024(11) \\ -0.005(21) \\ -0.0056(96) \\ -0.002(11) \\ 0.009(16) \\ 0.0162(91) \\ 0.086(26) \\ 0.131(31) \\ 0.155(33) \\ 0.086(33) \\ 0.098(16) \\ 0.094(17) \\ 0.088(27) \\ 0.114(25) \\ 0.075(27) \\ 0.034(25) \\ -0.006(22) \\ -0.001(31) \\ 0.022(11) \\ 0.014(16) \\ 0.0010(16) \\ 0.0008(85) \\ 0.018(23) \\ 0.001(29) \\ 0.005(18)
\end{pmatrix}
$$

Gluon Generalised FFs

- Complicated over and under-determined systems of equations (different choices of polarisation and boost at same momentum transfer)
- Some GFFs suppressed by orders of magnitude
- Some GFFs related by symmetries at some momenta

Simplest example:
Transversity GFFs
One basis (2 vectors)
Mtm 1 (lattice units)

$$
\begin{pmatrix}
0.604 & 0.0424 & 0 & 0 & 0 & 0 & 0.0588 & 0 \\
0.592 & -2.45\times10^{-3} & 0.0785 & -0.0785 & 6.58\times10^{-3} & -0.0992 & -0.103 & -4.15\times10^{-3} \\
0.485 & 0.0429 & 0 & 0 & 0 & 0 & 0.0379 & 0 \\
0.481 & 0.0431 & -3.02\times10^{-5} & 3.02\times10^{-5} & -2.53\times10^{-6} & -4.03\times10^{-7} & 0.0374 & -1.69\times10^{-8} \\
0.475 & -3.29\times10^{-3} & 0.0791 & -0.0791 & 6.59\times10^{-3} & -0.0791 & -0.0824 & -3.29\times10^{-3} \\
0.353 & -7.97\times10^{-4} & 0.0385 & -0.0385 & 3.28\times10^{-3} & -0.0598 & -0.0631 & -2.54\times10^{-3} \\
0.347 & -0.0382 & 0 & 0 & 0 & 0 & 0.0962 & 0 \\
0.258 & 0.0806 & 0 & 0 & 0 & 0 & -0.0374 & 0 \\
0.258 & 0.0808 & 0 & 0 & 0 & 0 & -0.0379 & 0 \\
0.253 & 0.101 & -8.60\times10^{-4} & 8.60\times10^{-4} & -7.20\times10^{-5} & 6.32\times10^{-7} & -0.0588 & 2.65\times10^{-8} \\
0.239 & -1.66\times10^{-3} & 0.0401 & -0.0401 & 3.29\times10^{-3} & -0.0393 & -0.0402 & -1.61\times10^{-3} \\
0.238 & -1.65\times10^{-3} & 0.0396 & -0.0396 & 3.29\times10^{-3} & -0.0396 & -0.0412 & -1.65\times10^{-3} \\
0.228 & -0.0581 & 8.30\times10^{-4} & -8.30\times10^{-4} & 6.94\times10^{-5} & -1.04\times10^{-6} & 0.0962 & -4.33\times10^{-8} \\
0.228 & -0.0379 & 0 & 0 & 0 & 0 & 0.0758 & 0 \\
0.0590 & -0.0109 & 0.139 & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} \\
0.0578 & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} \\
0.0338 & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} \\
0.0183 & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} \\
0.0155 & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} \\
1.19\times10^{-3} & -0.0106 & 0.129 & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} \\
0.549 & 2.44\times10^{-3} & 0 & 0 & 0 & 0 & 0.0895 & 0 \\
0.546 & -1.88\times10^{-3} & 0.0676 & -0.0676 & 5.69\times10^{-3} & -0.0918 & -0.0960 & -3.86\times10^{-3} \\
0.498 & 0.0710 & 0 & 0 & 0 & 0 & 0.0123 & 0 \\
0.480 & -2.37\times10^{-3} & 0.0685 & -0.0685 & 5.70\times10^{-3} & -0.0799 & -0.0828 & -3.33\times10^{-3} \\
0.429 & 0.0714 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.424 & 0.0834 & -5.14\times10^{-4} & 5.14\times10^{-4} & -4.30\times10^{-5} & 1.33\times10^{-7} & -0.0123 & 5.55\times10^{-9} \\
0.412 & 2.85\times10^{-3} & 0 & 0 & 0 & 0 & 0.0657 & 0 \\
0.412 & -2.85\times10^{-3} & 0.0685 & -0.0685 & 5.70\times10^{-3} & -0.0685 & -0.0714 & -2.85\times10^{-3} \\
0.409 & -8.65\times10^{-3} & 4.61\times10^{-4} & -4.61\times10^{-4} & 3.86\times10^{-5} & -8.30\times10^{-7} & 0.0771 & -3.47\times10^{-8} \\
0.0674 & -6.43\times10^{-3} & 0.0856 & -0.0856 & 6.70\times10^{-3} & -5.55\times10^{-3} & -8.26\times10^{-5} & -1.73\times10^{-6} \\
0.0656 & 4.96\times10^{-4} & -9.21\times10^{-4} & 9.21\times10^{-4} & -6.37\times10^{-6} & -0.0119 & -0.0132 & -5.32\times10^{-4} \\
0.0514 & -0.0685 & 0 & 0 & 0 & 0 & 0.0771 & 0 \\
0.0347 & -0.0124 & 0.155 & -0.155 & 0.0127 & -3.05\times10^{-3} & -6.00\times10^{-4} & -1.26\times10^{-5} \\
0.0327 & 5.99\times10^{-3} & -0.0692 & 0.0692 & -6.03\times10^{-3} & -2.50\times10^{-3} & 5.17\times10^{-4} & 1.08\times10^{-5} \\
0.0301 & 4.59\times10^{-3} & -0.0738 & 0.0738 & -5.95\times10^{-3} & 2.98\times10^{-3} & 0.0123 & 1.07\times10^{-5} \\
0.0285 & -1.84\times10^{-3} & -0.147 & 0.147 & -0.0126 & -2.43\times10^{-3} & 0.0143 & 1.24\times10^{-5} \\
0.0171 & 0.0685 & 0 & 0 & 0 & 0 & -0.0657 & 0 \\
0.0146 & 0.0920 & -9.75\times10^{-4} & 9.75\times10^{-4} & -8.17\times10^{-5} & 9.63\times10^{-7} & -0.0895 & 4.03\times10^{-8} \\
1.59\times10^{-3} & 6.43\times10^{-3} & 0.0736 & -0.0736 & 6.61\times10^{-3} & 5.40\times10^{-3} & -1.97\times10^{-3} & -1.71\times10^{-6}
\end{pmatrix}
\begin{pmatrix}
\text{[illegible]} \\
A^{(2)}_{5,0}(1) \\
A^{(2)}_{6,0}(1) \\
A^{(2)}_{7,0}(1) \\
A^{(2)}_{8,0}(1)
\end{pmatrix}
\begin{pmatrix}
0.179(36) \\ 0.150(38) \\ 0.152(30) \\ 0.154(37) \\ 0.129(32) \\ 0.056(31) \\ 0.067(41) \\ 0.056(35) \\ 0.069(21) \\ 0.093(36) \\ 0.028(32) \\ 0.041(27) \\ 0.012(33) \\ \text{[illegible]} \\ 0.086(26) \\ 0.131(31) \\ 0.155(33) \\ 0.086(33) \\ 0.098(16) \\ 0.094(17) \\ 0.088(27) \\ 0.114(25) \\ 0.075(27) \\ 0.034(25) \\ -0.006(22) \\ -0.001(31) \\ 0.022(11) \\ 0.014(16) \\ 0.0010(16) \\ 0.0008(85) \\ 0.018(23) \\ 0.001(29) \\ 0.005(18)
\end{pmatrix}
$$

Target a subset of "dominant GFFs"

Gluon transversity GFFs

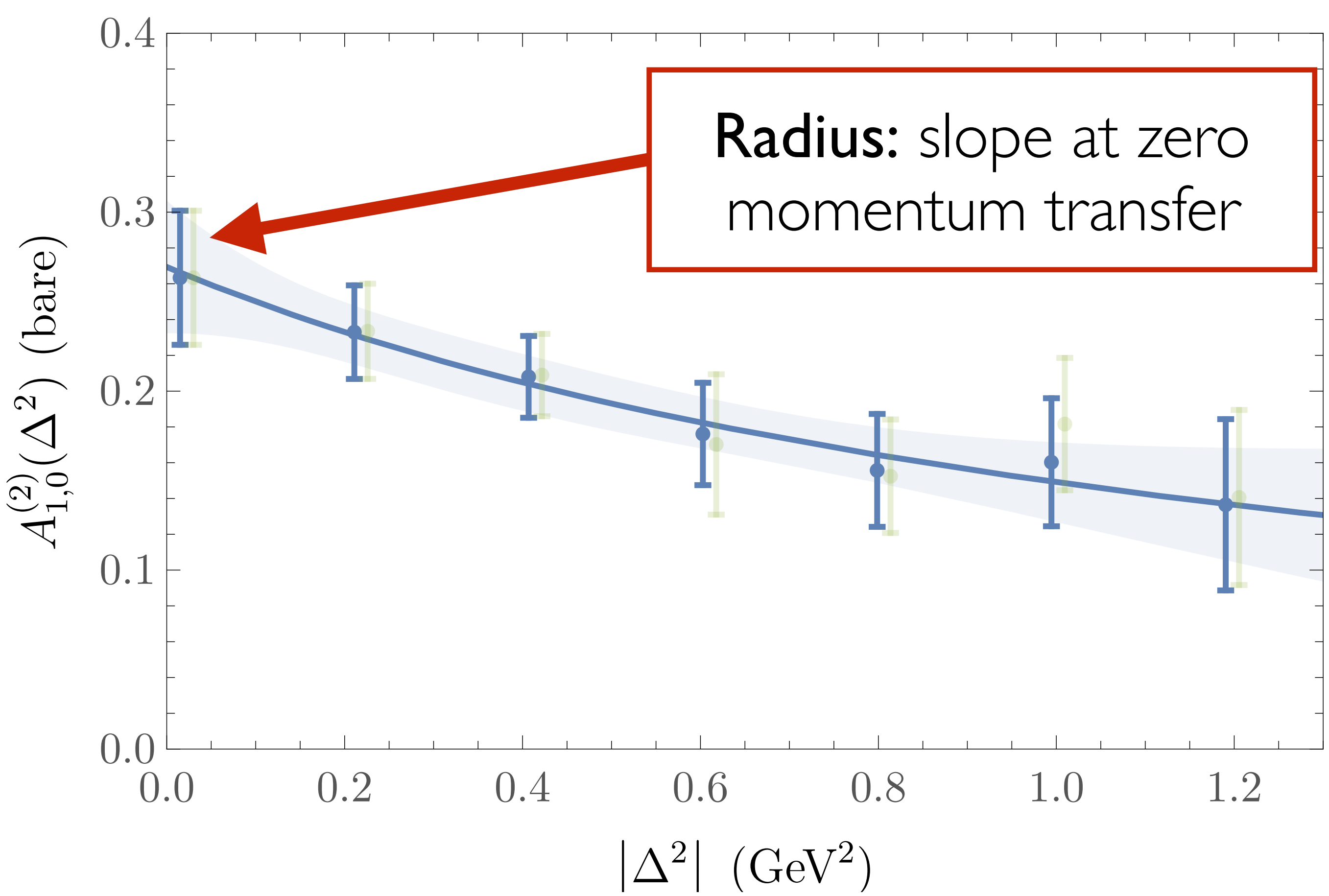
Detmold, Shanahan, PRD 94 (2016), 014507, Detmold, Pefkou, Shanahan, PRD 95 (2017), 114515

Calculate $\langle p'E'|\mathcal{O}|pE\rangle$ for all accessible $p, E, \mathcal{O}$ → Complicated analysis → Extract GFFs

ϕ($\bar{s}s$) meson (simplest spin-1 system):

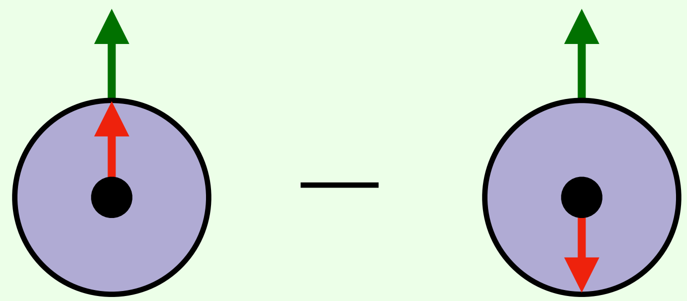
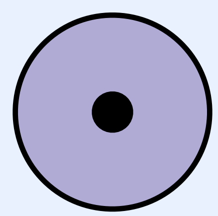
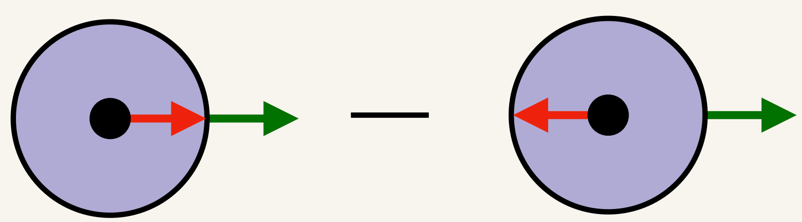
One transversity generalised form factor can be resolved at all momenta

Dipole-like fall-off vs $|\Delta^2|$



Squared momentum transfer

Gluon distributions

	Parton model	Local gluon operators
Transversity		$S\left[G_{\mu\mu_1}\overleftrightarrow{D}_{\mu_3}\ldots\overleftrightarrow{D}_{\mu_n}G_{\nu\mu_2}\right]$
Momentum (spin-independent)		$S\left[G_{\mu\alpha}\overleftrightarrow{D}_{\mu_1}\ldots\overleftrightarrow{D}_{\mu_n}G_{\nu}{}^{\alpha}\right]$
Helicity (spin-dependent)		$S\left[\tilde{G}_{\mu\alpha}\overleftrightarrow{D}_{\mu_1}\ldots\overleftrightarrow{D}_{\mu_n}G_{\nu}{}^{\alpha}\right]$

Spin-indep. gluon GFFs

Matrix elements of the spin-independent gluon structure function

- "Gravitational form factors" for n=0
- Gluon momentum fraction in forward limit

- Similarly complicated

$$\left\langle p'E' \left| S\left[G_{\mu\alpha} i\overleftrightarrow{D}_{\mu_1} \dots i\overleftrightarrow{D}_{\mu_n} G_\nu{}^\alpha\right] \right| pE \right\rangle$$

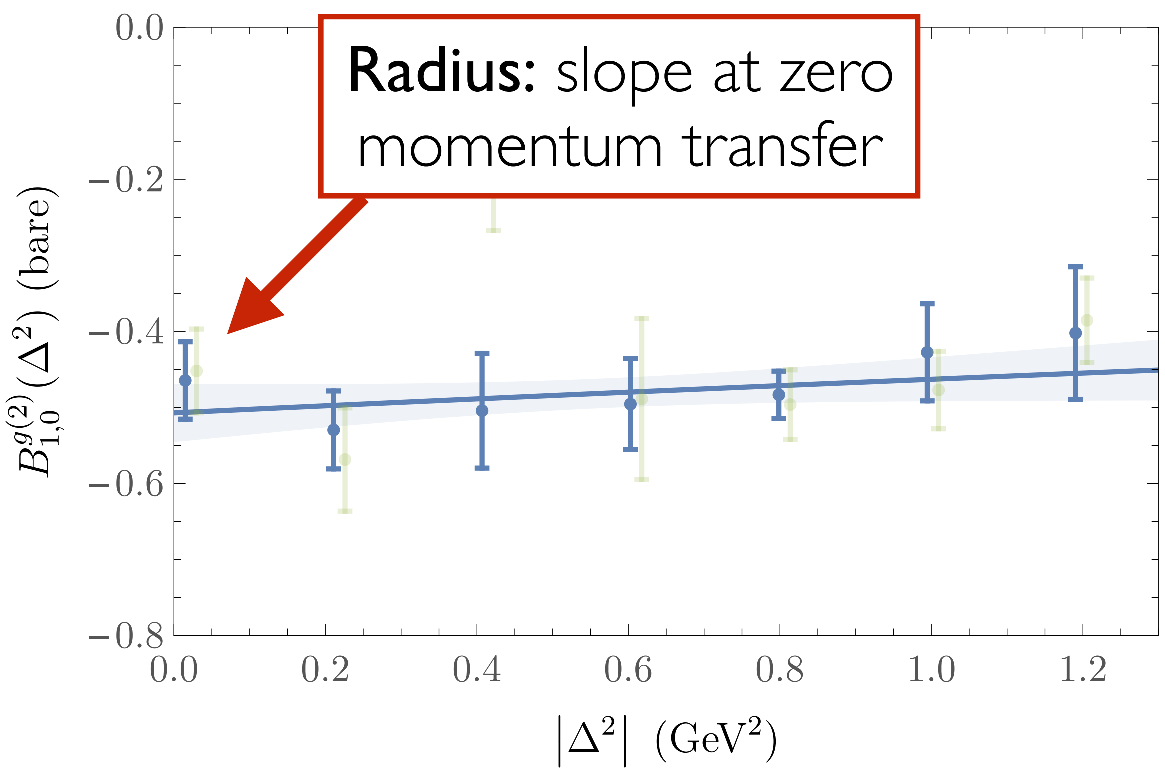
$$\begin{aligned}
= \sum_{\substack{m \text{ even} \\ m=0}}^{n} \Bigg\{ & \boxed{B_{1,m}^{(n+2)}(\Delta^2)} M^2 S\left[E_\mu E'^*_\nu \Delta_{\mu_1} \dots \Delta_{\mu_m} P_{\mu_{m+1}} \dots P_{\mu_n}\right] \\
& + \boxed{B_{2,m}^{(n+2)}(\Delta^2)} S\left[(E \cdot E'^*) P_\mu P_\nu \Delta_{\mu_1} \dots \Delta_{\mu_m} P_{\mu_{m+1}} \dots P_{\mu_n}\right] \\
& + \boxed{B_{3,m}^{(n+2)}(\Delta^2)} S\left[(E \cdot E'^*) \Delta_\mu \Delta_\nu \Delta_{\mu_1} \dots \Delta_{\mu_m} P_{\mu_{m+1}} \dots P_{\mu_n}\right] \\
& + \boxed{B_{4,m}^{(n+2)}(\Delta^2)} S\left[\left((E'^* \cdot P) E_\mu P_\nu + (E \cdot P) E'^*_\mu P_\nu\right) \Delta_{\mu_1} \dots \Delta_{\mu_m} P_{\mu_{m+1}} \dots P_{\mu_n}\right] \\
& + \boxed{B_{5,m}^{(n+2)}(\Delta^2)} S\left[\left((E'^* \cdot P) E_\mu \Delta_\nu - (E \cdot P) E'^*_\mu \Delta_\nu\right) \Delta_{\mu_1} \dots \Delta_{\mu_m} P_{\mu_{m+1}} \dots P_{\mu_n}\right] \\
& + \frac{\boxed{B_{6,m}^{(n+2)}(\Delta^2)}}{M^2} S\left[(E \cdot P)(E'^* \cdot P) P_\mu P_\nu \Delta_{\mu_1} \dots \Delta_{\mu_m} P_{\mu_{m+1}} \dots P_{\mu_n}\right] \\
& + \frac{\boxed{B_{7,m}^{(n+2)}(\Delta^2)}}{M^2} S\left[(E \cdot P)(E'^* \cdot P) \Delta_\mu \Delta_\nu \Delta_{\mu_1} \dots \Delta_{\mu_m} P_{\mu_{m+1}} \dots P_{\mu_n}\right] \Bigg\}.
\end{aligned}$$

Spin-indep. gluon GFFs

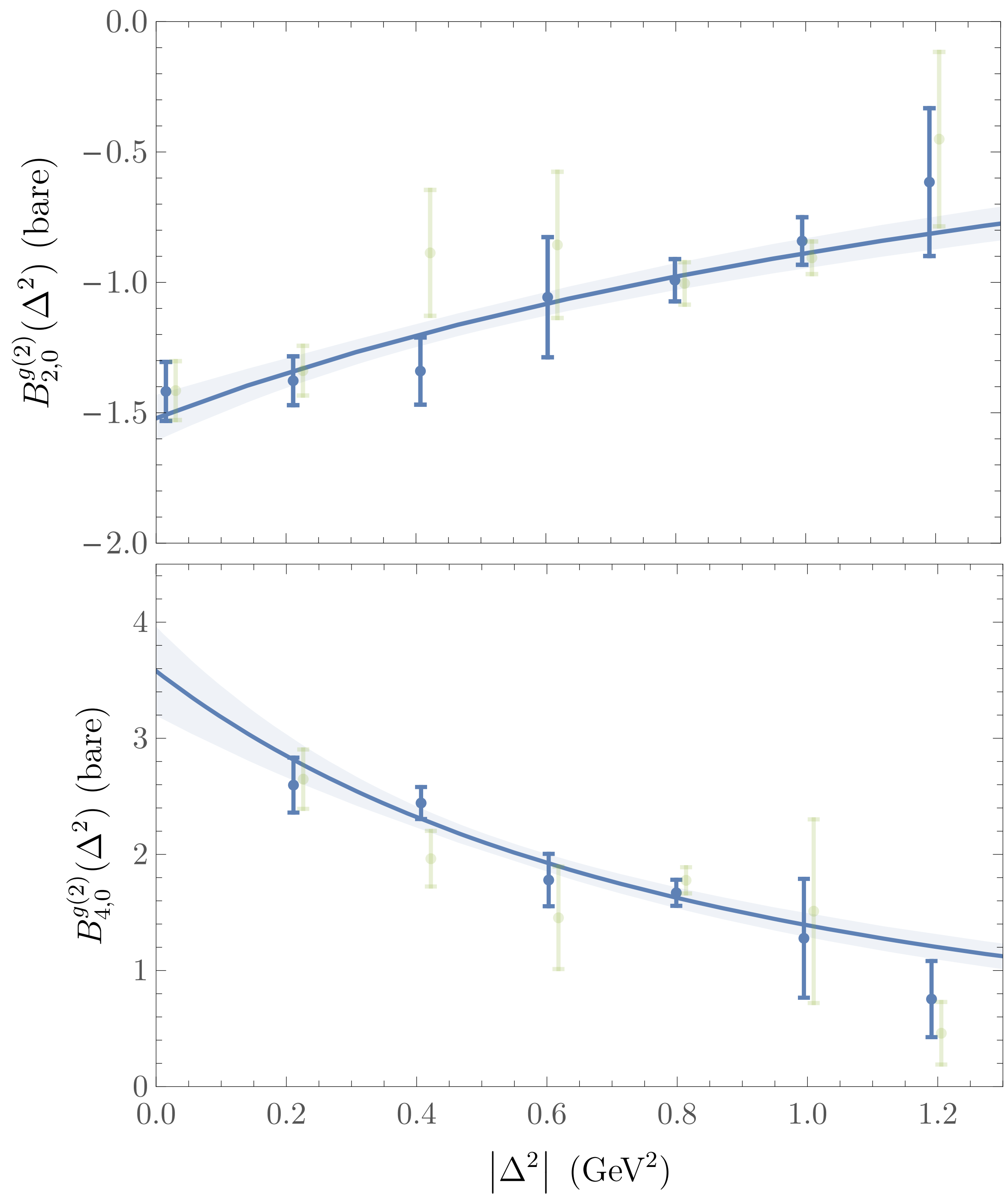
Detmold, Pefkou, Shanahan, PRD 95 (2017), 114515

"Gravitational form factors"

Similarly complicated decomposition
Three GFFs can be resolved
for all momenta



Squared momentum transfer

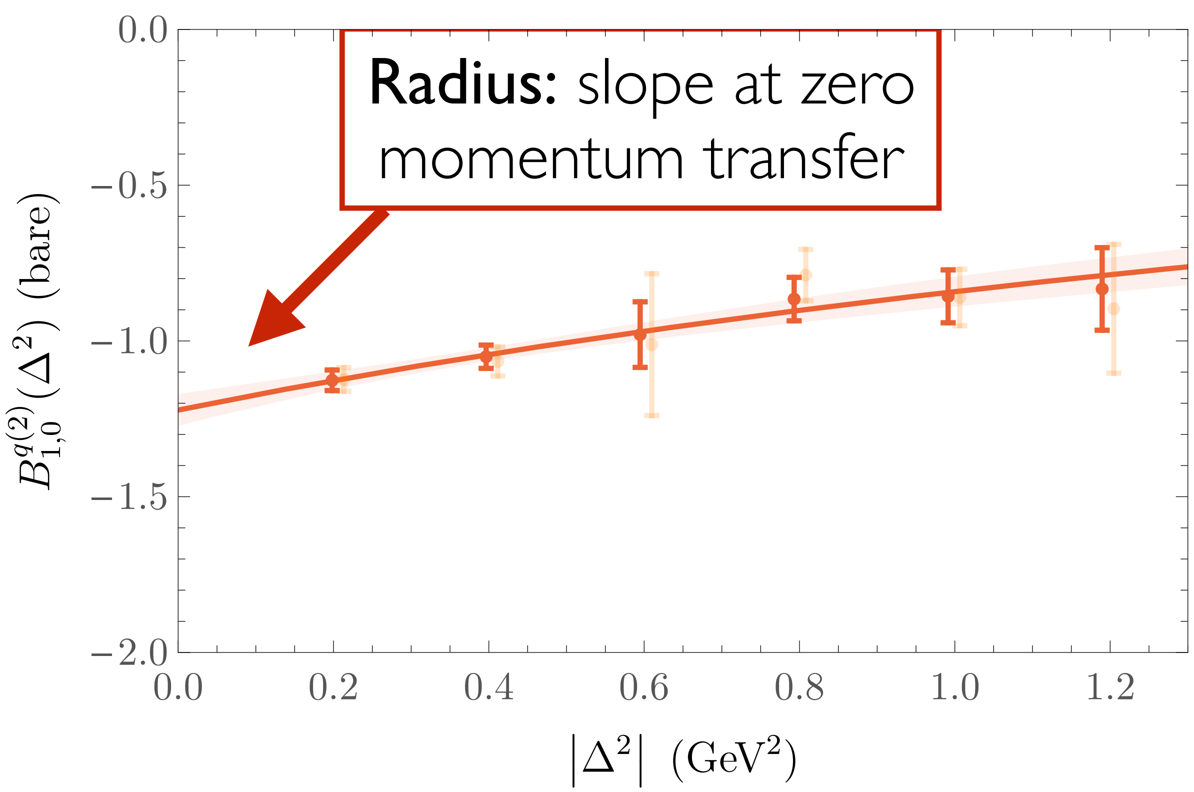


Squared momentum transfer

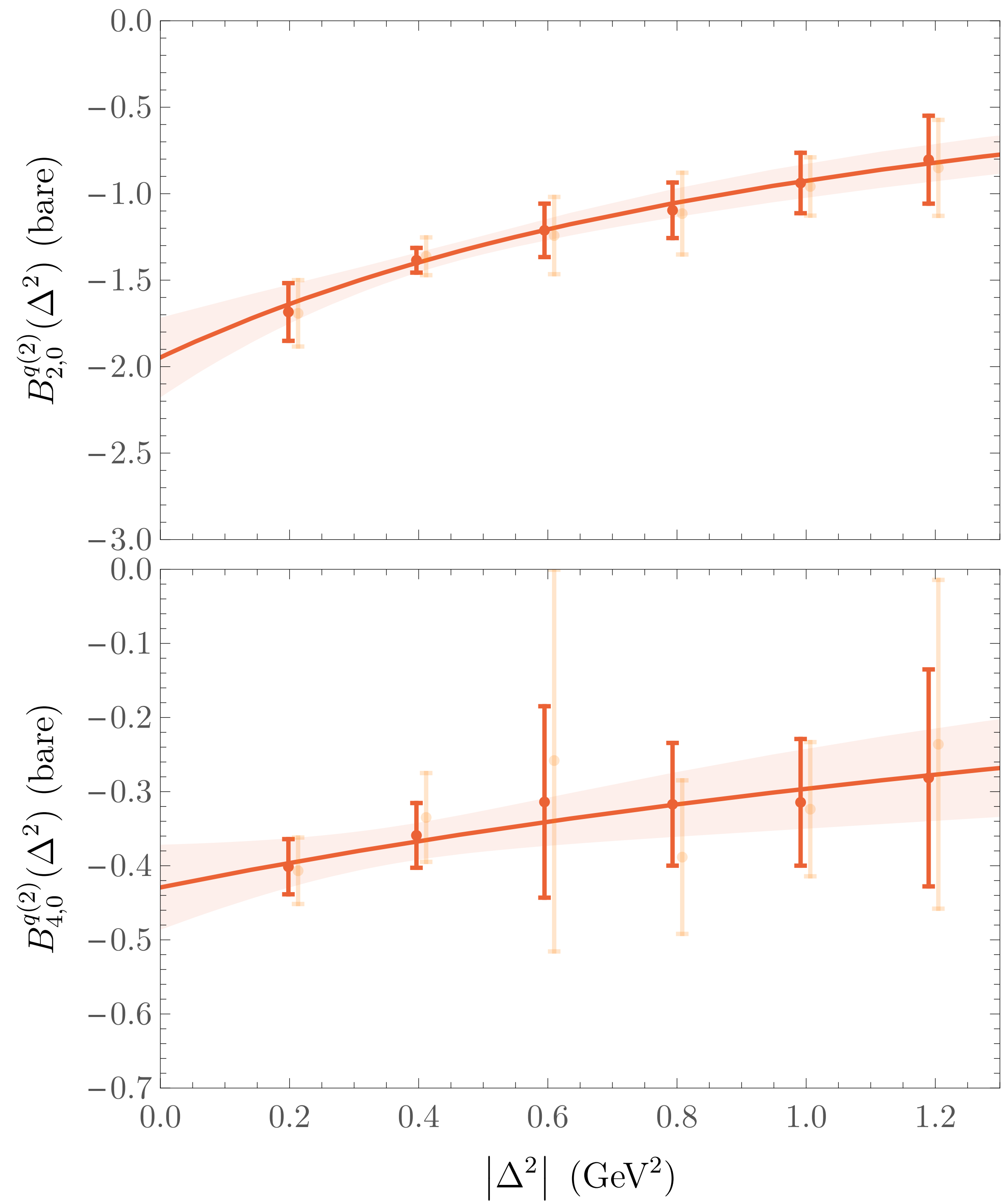
Spin-indep. quark GFFs

Detmold, Pefkou, Shanahan, PRD 95 (2017), 114515

GFF decomposition has precisely the same structure as in the spin-independent gluon case



Squared momentum transfer

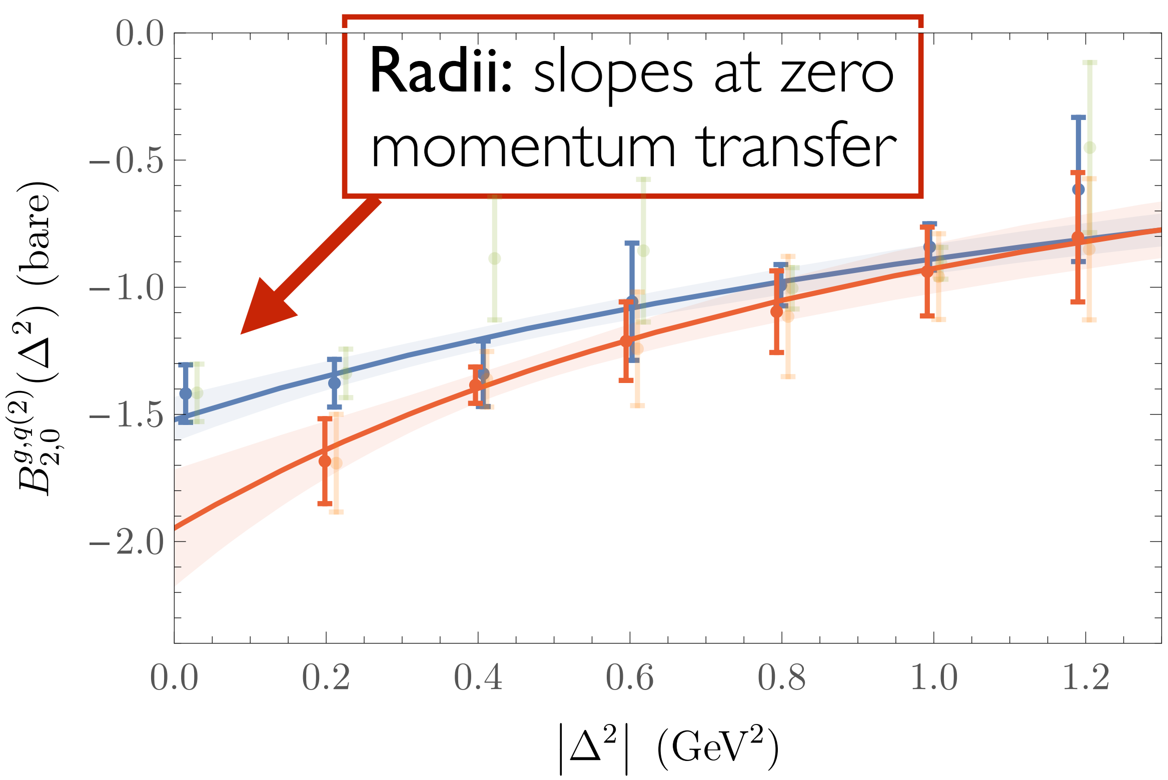


Squared momentum transfer

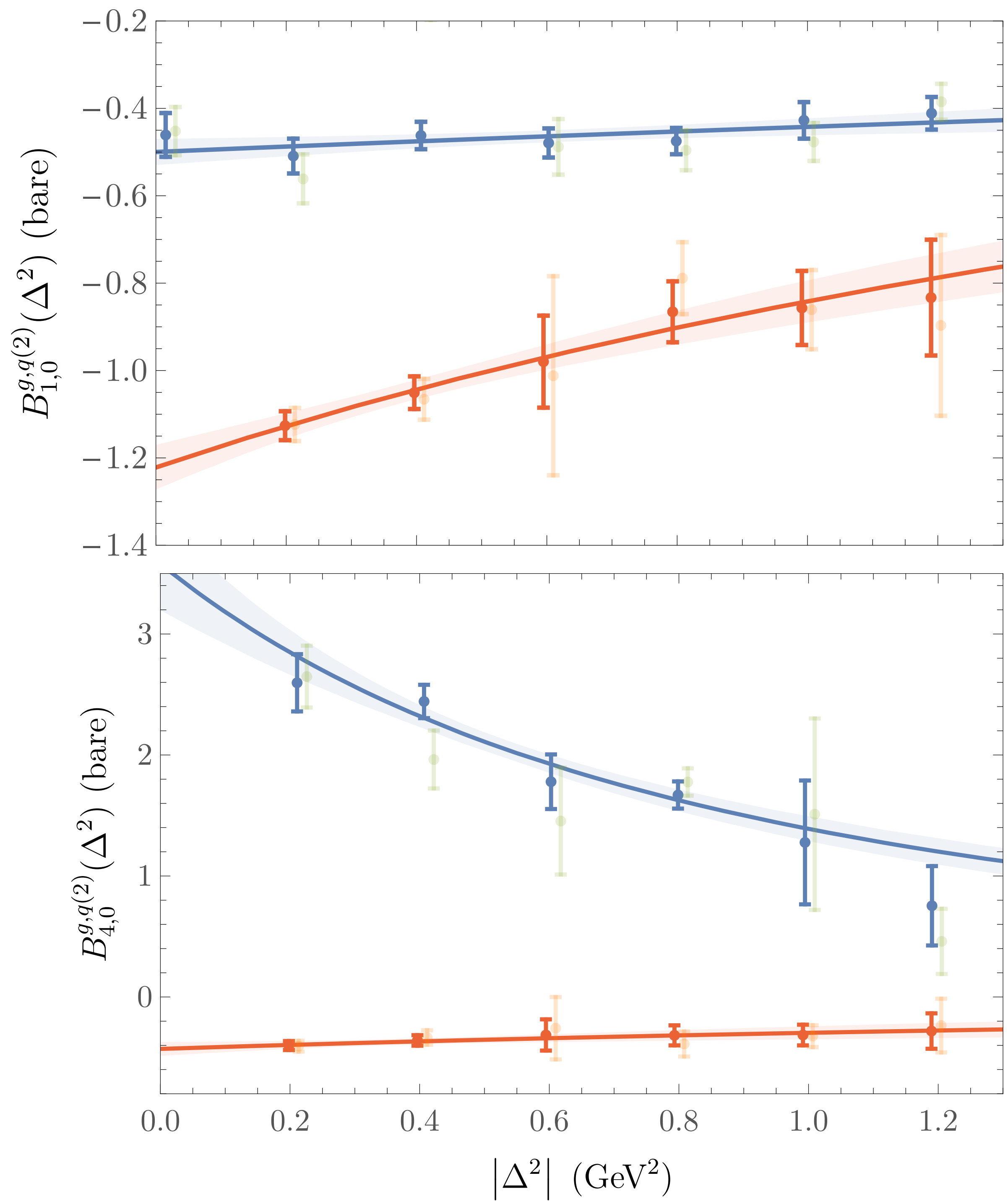
Spin-indep. quark GFFs

Detmold, Pefkou, Shanahan, PRD 95 (2017), 114515

Gluon vs quark radius depends strongly on which aspect of structure is being probed



Squared momentum transfer



Squared momentum transfer

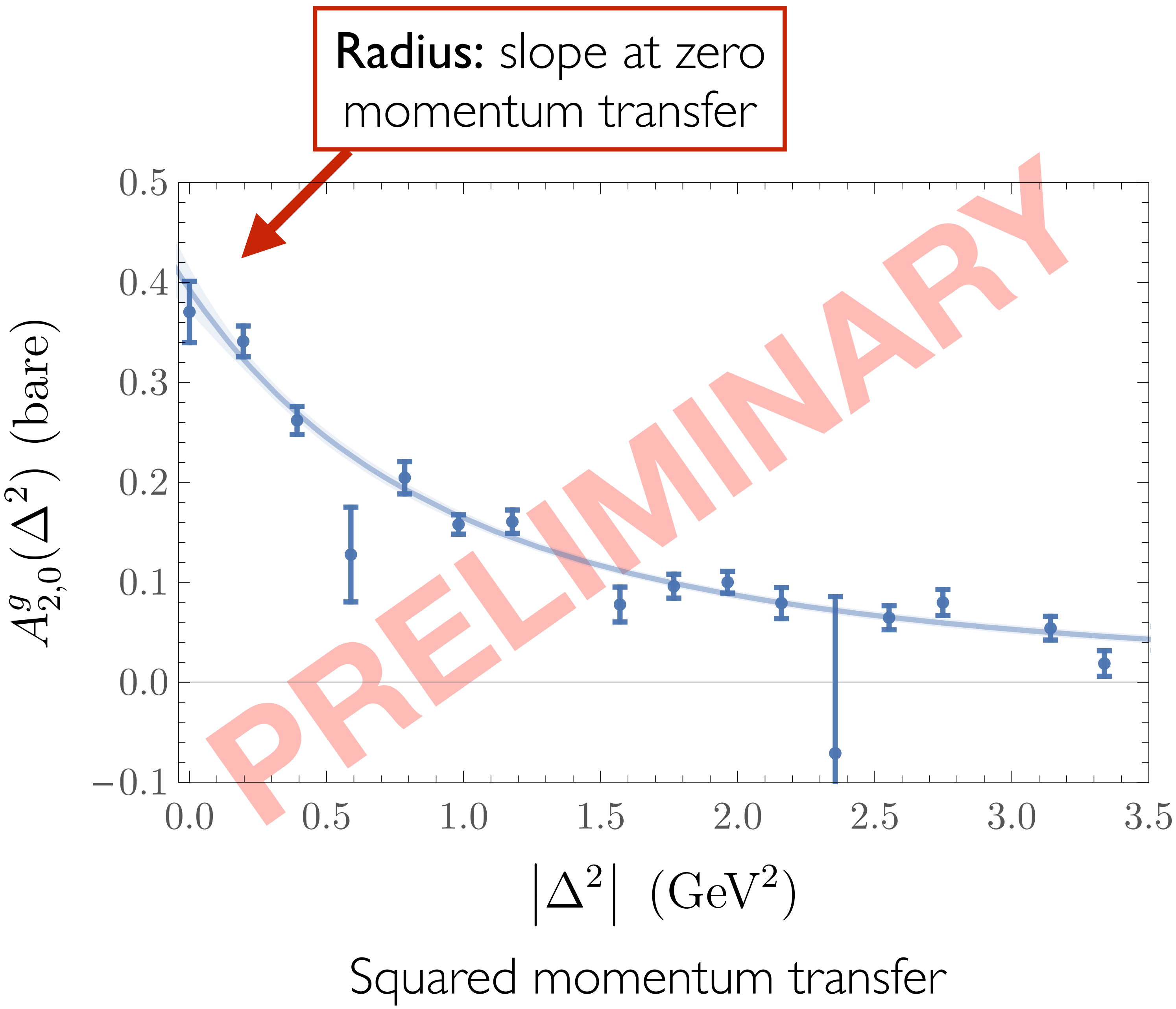
Nucleon spin-indep. gluon GFFs

Nucleon, m_π ~450 MeV

Three spin-independent generalised form factors, one can be resolved from zero at present statistics

Dipole-like fall-off vs $|\Delta^2|$

Renormalisation + comparison with quark GFFs in progress



Gluon structure of nuclei

2A

How does the gluon structure of a nucleon change in a nucleus?

Ratio of structure function F_2 per nucleon for iron and deuterium

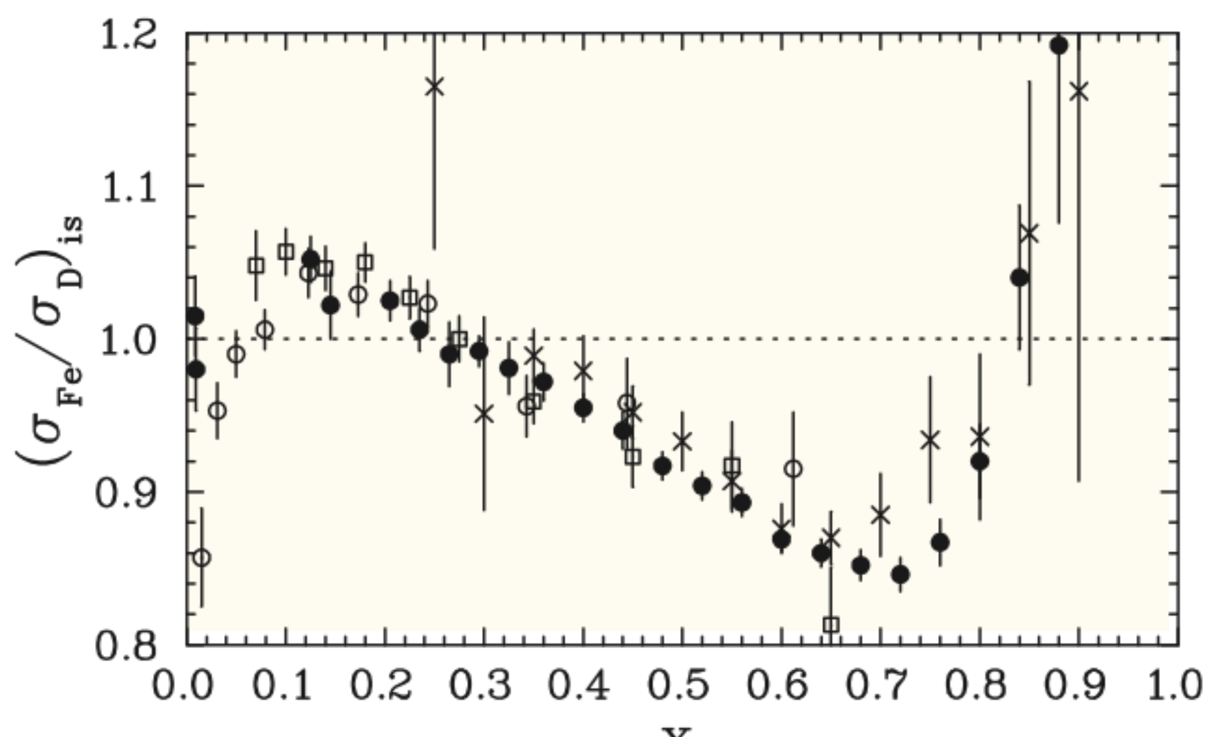
$$F_2(x, Q^2) = \sum_{q=u,d,s\ldots} x\, e_q^2\, [q(x,Q^2) + \overline{q}(x,Q^2)]$$

Number density of partons of flavour q

European Muon Collaboration (1983): "EMC effect"

Modification of per-nucleon cross section of nucleons bound in nuclei

Gluon analogue?

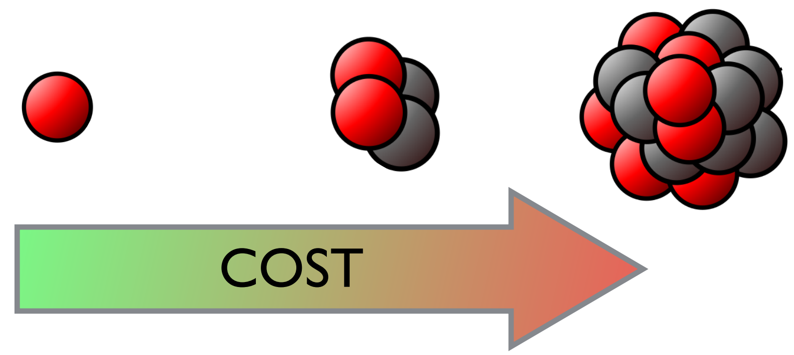


Longitudinal momentum fraction

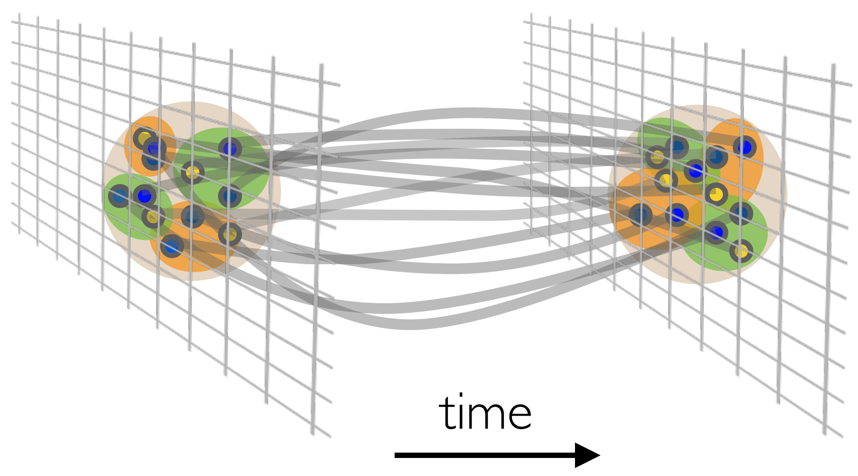
Nuclear physics from LQCD

Nuclei on the lattice: HARD

- **Noise:**
  Statistical uncertainty grows exponentially with number of nucleons



- **Complexity:**
  Number of contractions grows factorially



Calculations possible for A<5

Nuclear glue, m_π ~450 MeV

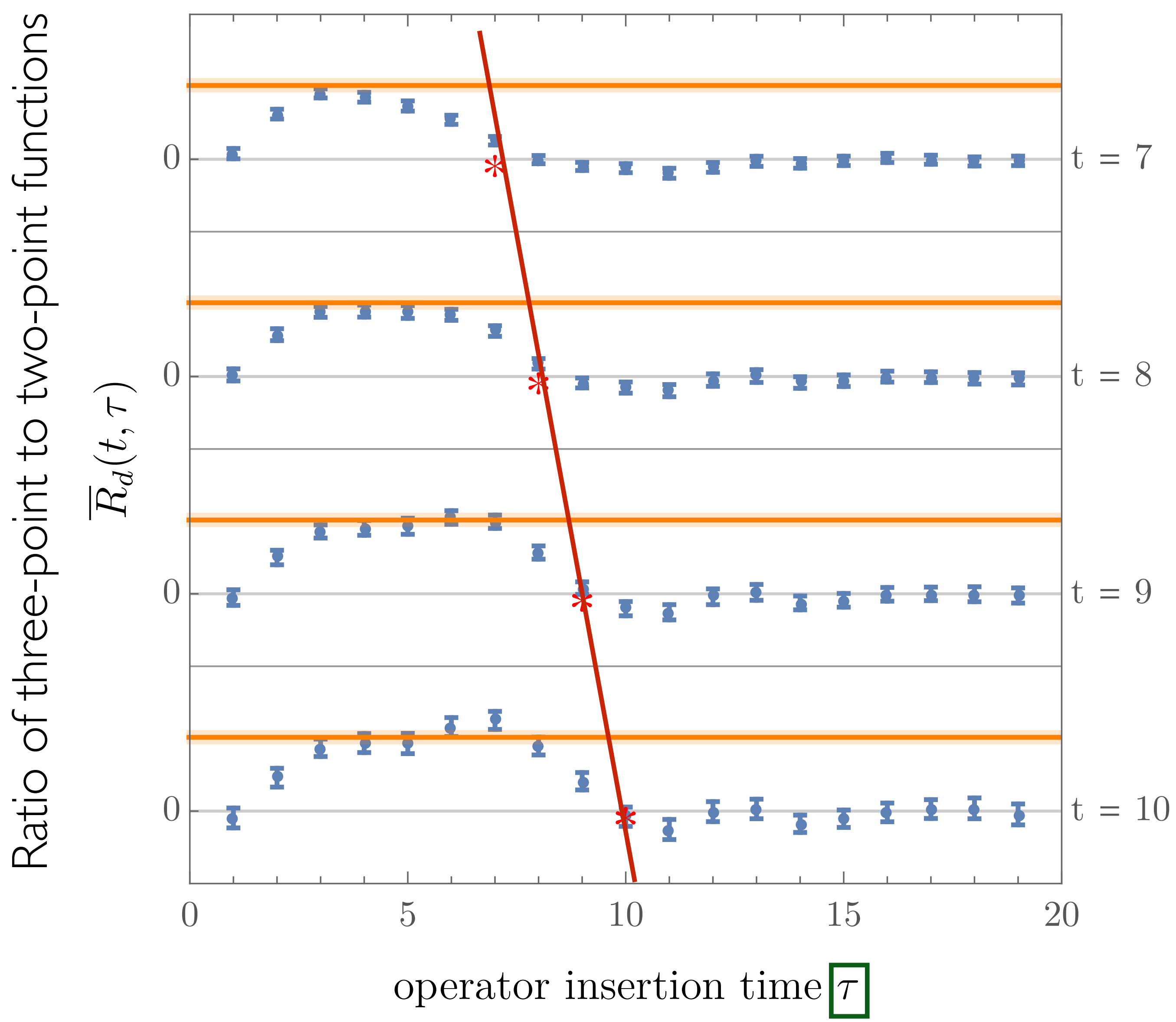
Look for **nuclear (EMC-type) effects** in the first moments of the spin-independent gluon structure function

Doubly challenging

- Nuclear matrix element
- Gluon observable (suffer from poor signal-to-noise)

Deuteron gluon momentum fraction

Ratio ∝ matrix element for $0 \ll \boxed{\tau} \ll \boxed{t}$



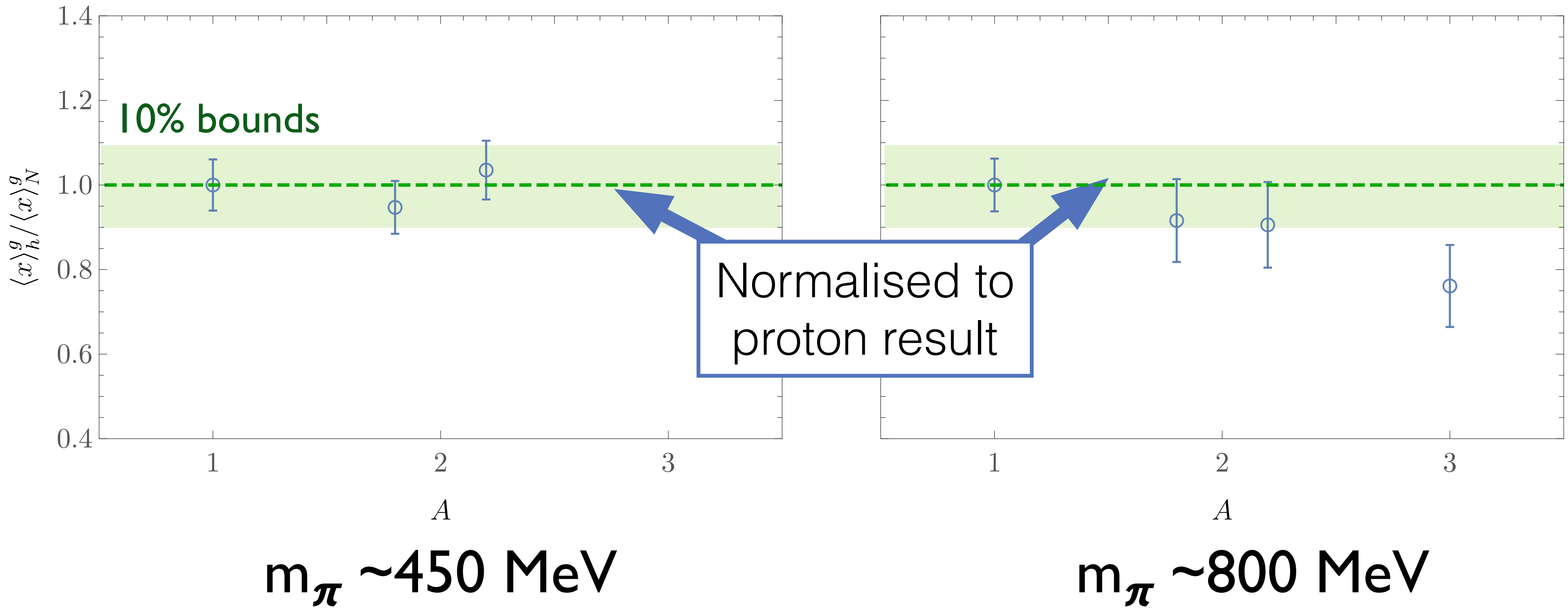
NPLQCD Collaboration PRD96 094512 (2017)

Gluon momentum fraction

NPLQCD Collaboration PRD96 094512 (2017)

- Matrix elements of the **spin-independent gluon operator** in nucleon and light nuclei
- Present statistics: can't distinguish from no-EMC effect scenario
- Small additional uncertainty from mixing with quark operators

Ratio of gluon momentum fraction in nucleus to nucleon



Gluon structure of nuclei

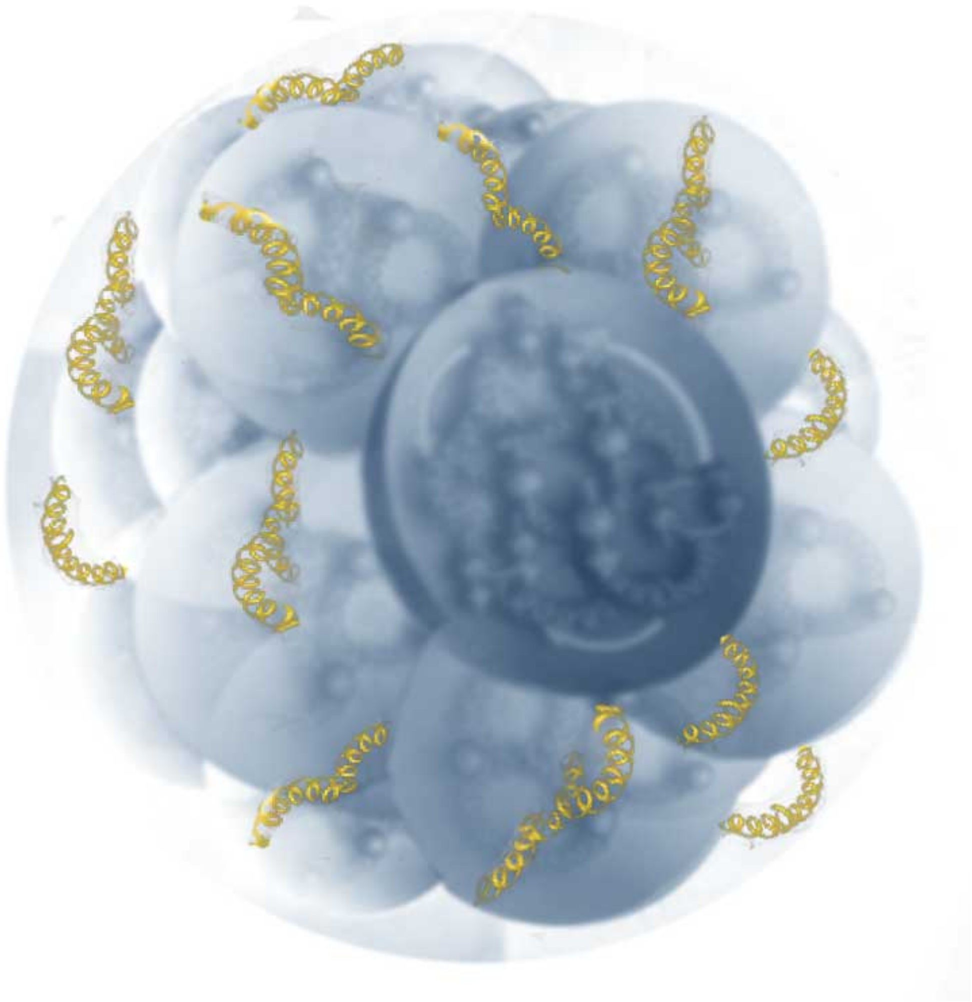
2B Exotic Glue

Contributions to nuclear structure from gluons not associated with individual nucleons in nucleus

Exotic glue operator:

nucleon $\langle p|\mathcal{O}|p\rangle = 0$

nucleus $\langle N,Z|\mathcal{O}|N,Z\rangle \neq 0$



Jaffe and Manohar, "Nuclear Gluonometry" Phys. Lett. B223 (1989) 218

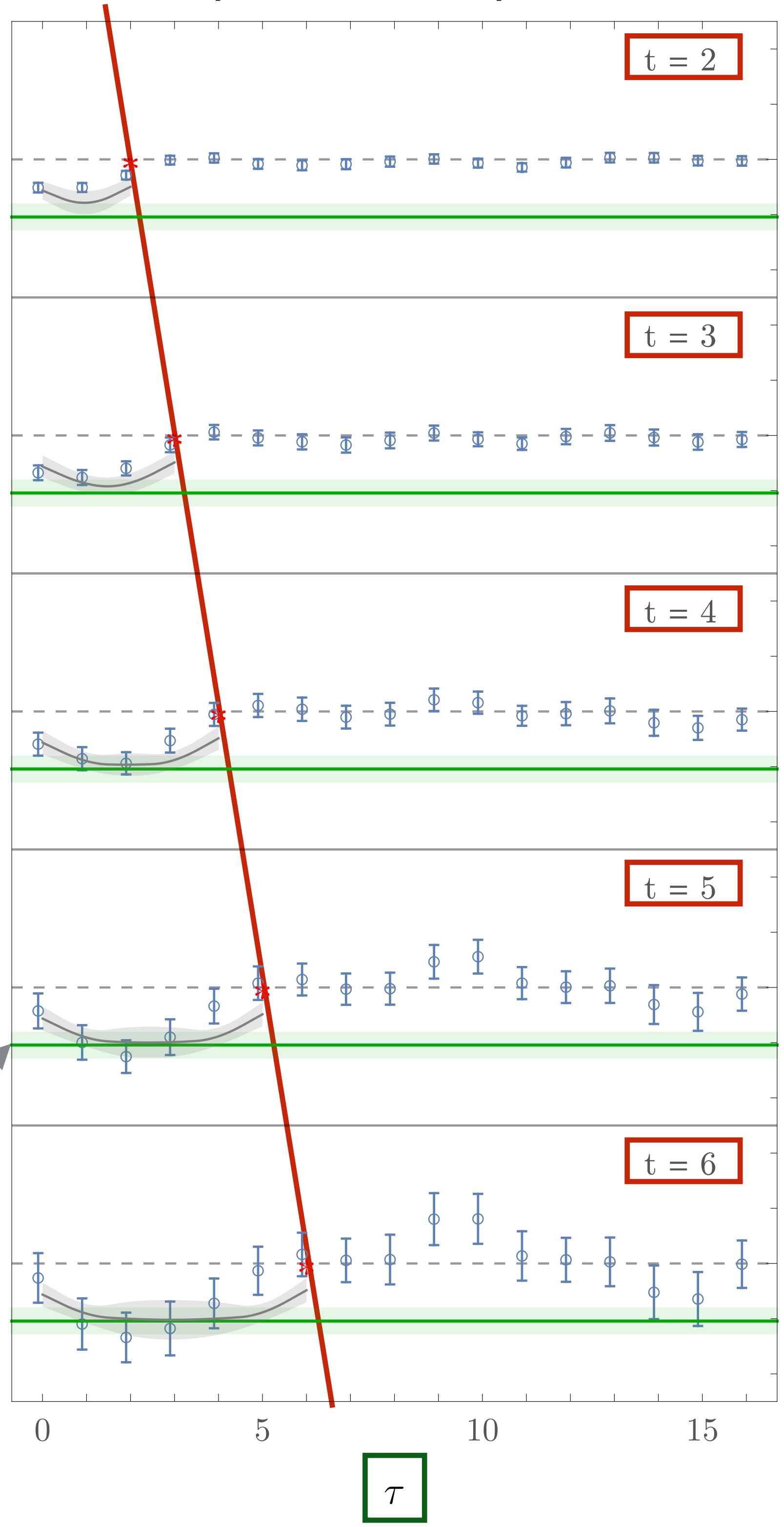
Non-nucleonic glue in deuteron

NPLQCD Collaboration PRD96 094512 (2017)

First moment of gluon transversity distribution in the deuteron,
m_{π} ~800 MeV

- **First evidence for non-nucleonic gluon contributions to nuclear structure**
- **Hypothesis of no signal ruled out to better than one part in 10^7**
- Magnitude relative to momentum fraction as expected from large-N_c

Ratio of 3pt and 2pt functions



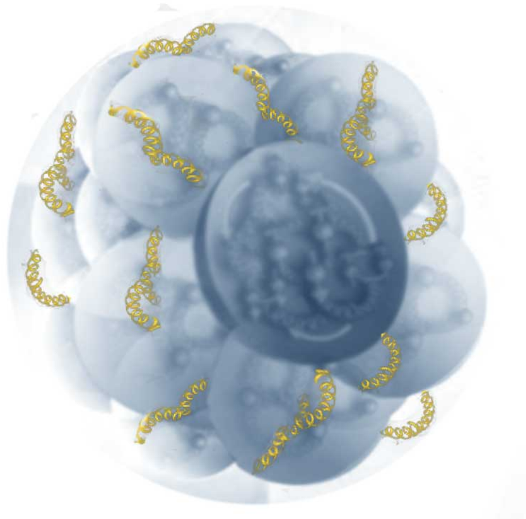
Ratio ∝ matrix element for $0 \ll \boxed{\tau} \ll \boxed{t}$

Gluon structure from LQCD

- Electron-Ion collider will dramatically alter our knowledge of the gluonic structure of hadrons and nuclei
  - Work towards a complete 3D picture of parton structure (moments, x-dependence of PDFs, GPDs, TMDs)
  - $\Delta(x,Q^2)$ has an interesting role

    Purely gluonic

    Non-nucleonic: directly probe nuclear effects
  - Compare quark and gluon distributions in hadrons and nuclei



- Lattice QCD calculations in hadrons and light nuclei will complement and extend understanding of fundamental structure of nature