# The top-quark at the LHC and beyond

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LFC21: Strong Interactions from QCD to New Strong Dynamics at LHC and Future Colliders, September 8, 2021

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# Outline

- Introduction
- Top quark production and decay
  - a new NNLO calculation
- Top mass
  - NNLO cross sections in the $\overline{\mathrm{MS}}$ scheme
- Summary

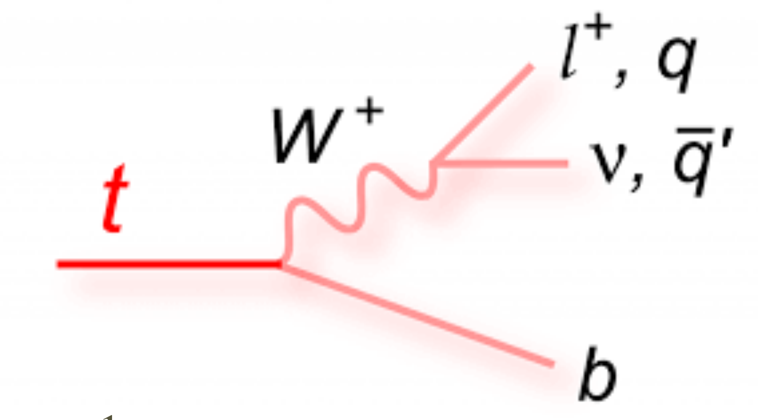
# Introduction

The top quark has a very special place in the Standard Model

- It is the heaviest elementary particle $m_t \sim 173\,\text{GeV}$ ➡ $y_t \sim 1$
- It couples strongly to the Higgs boson $m_t = y_t v/\sqrt{2} \sim 173\,\text{GeV}$
- It decays through EW interaction before hadronizing

- $\tau_{\text{had}} \sim 1/\Lambda_{QCD} \sim 10^{-23}\,s$

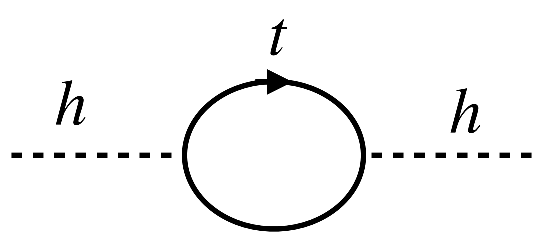
- $\tau_t = 1/\Gamma_t \sim \left(G_F m_t^3 |V_{tb}|^2\right)^{-1} \sim 5 \cdot 10^{-25}\,s$




( for the bottom quark instead $\tau_b \sim \left(G_F^2 m_b^5 |V_{bc}|^2\right)^{-1} \sim 10^{-12}\,s$ )

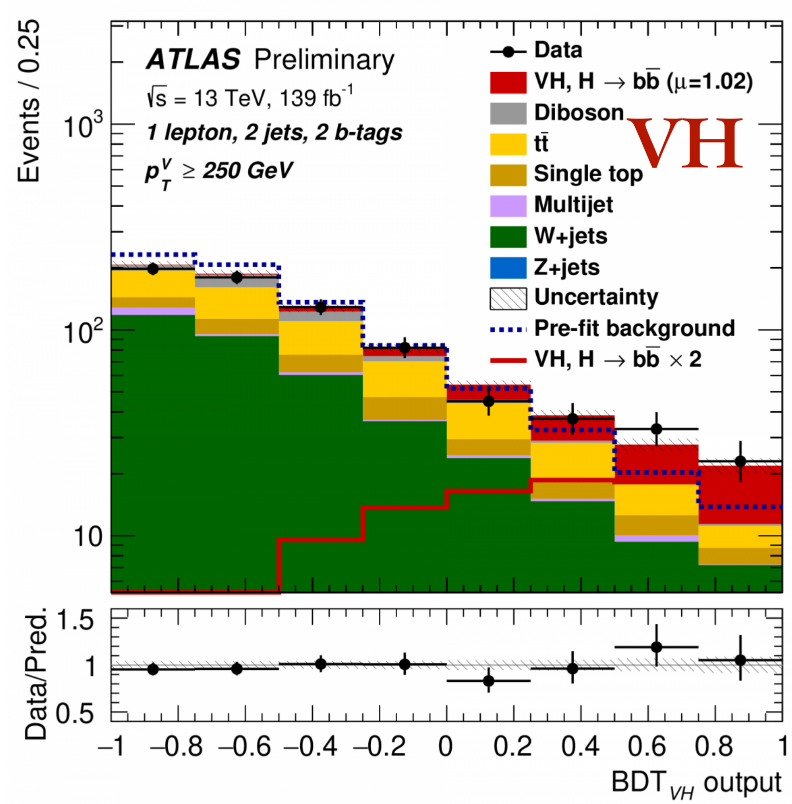
# Introduction

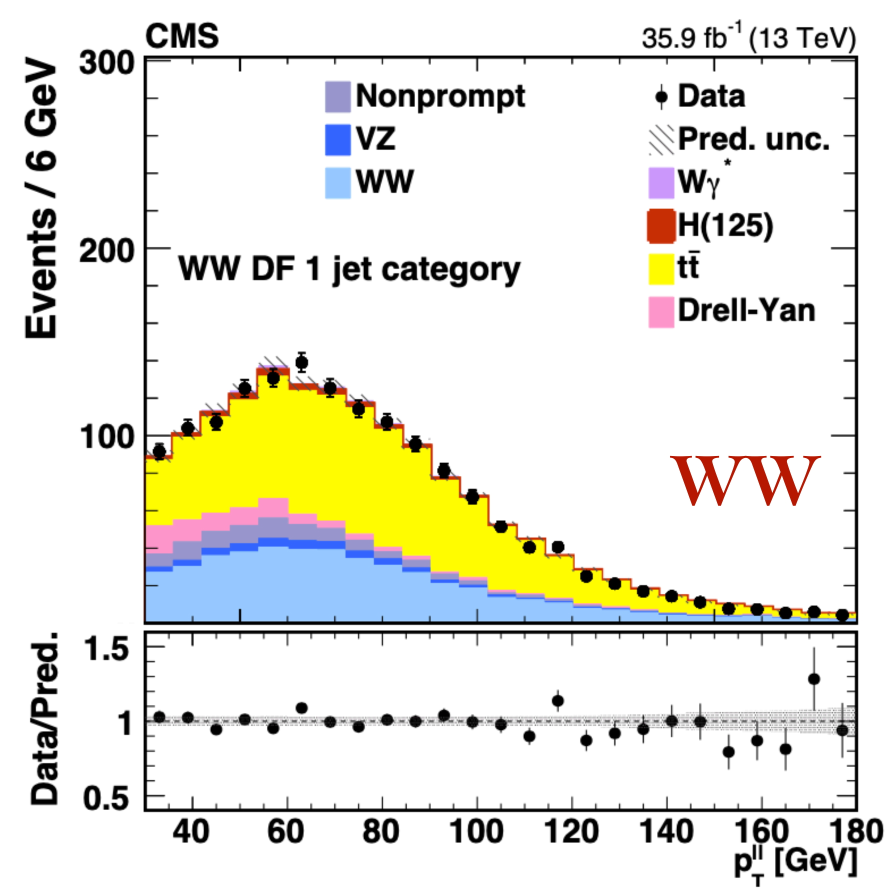
- Possible window on new physics




$$\sim \frac{y_t^2}{8\pi^2}\Lambda_{UV}^2$$

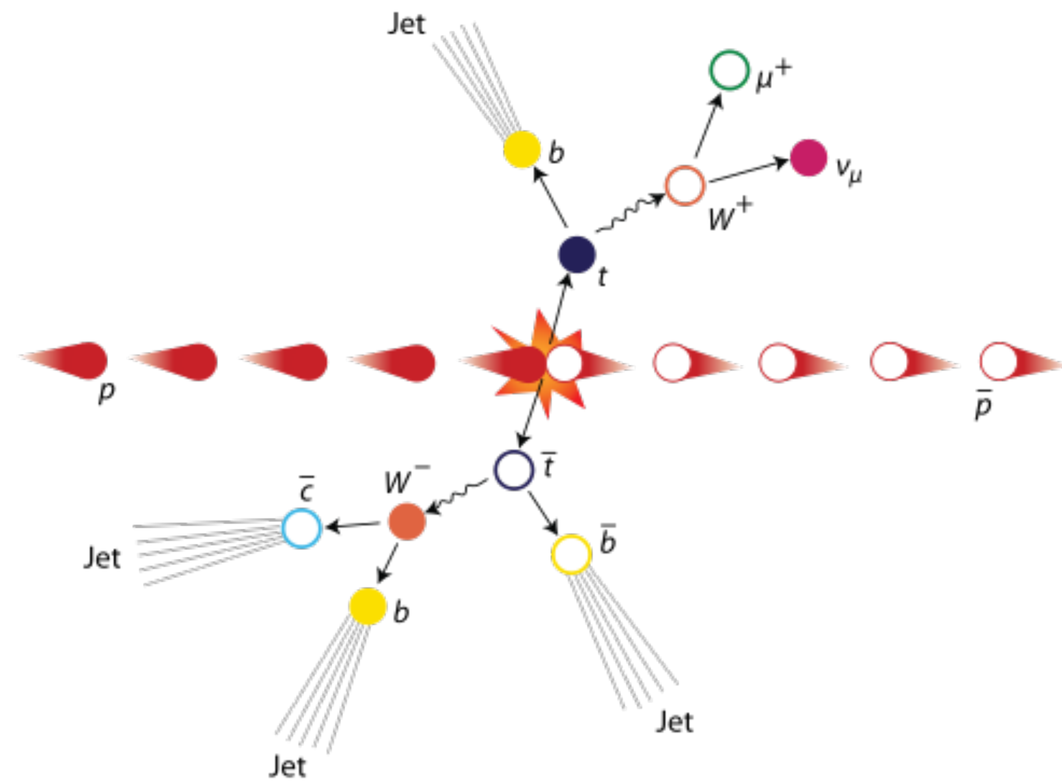
- Events with top quarks provide an ubiquitous background to SM, Higgs measurements and new physics searches



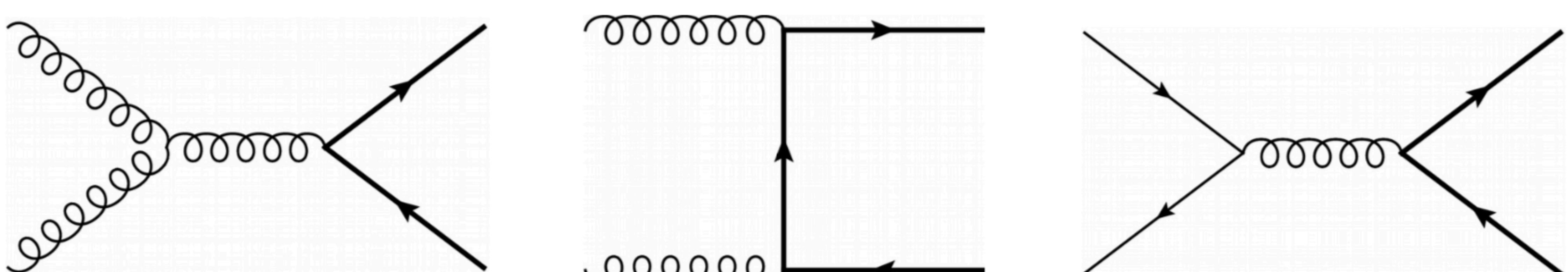



# Introduction

Main source of top-quark event at hadron colliders is $t\bar{t}$ production

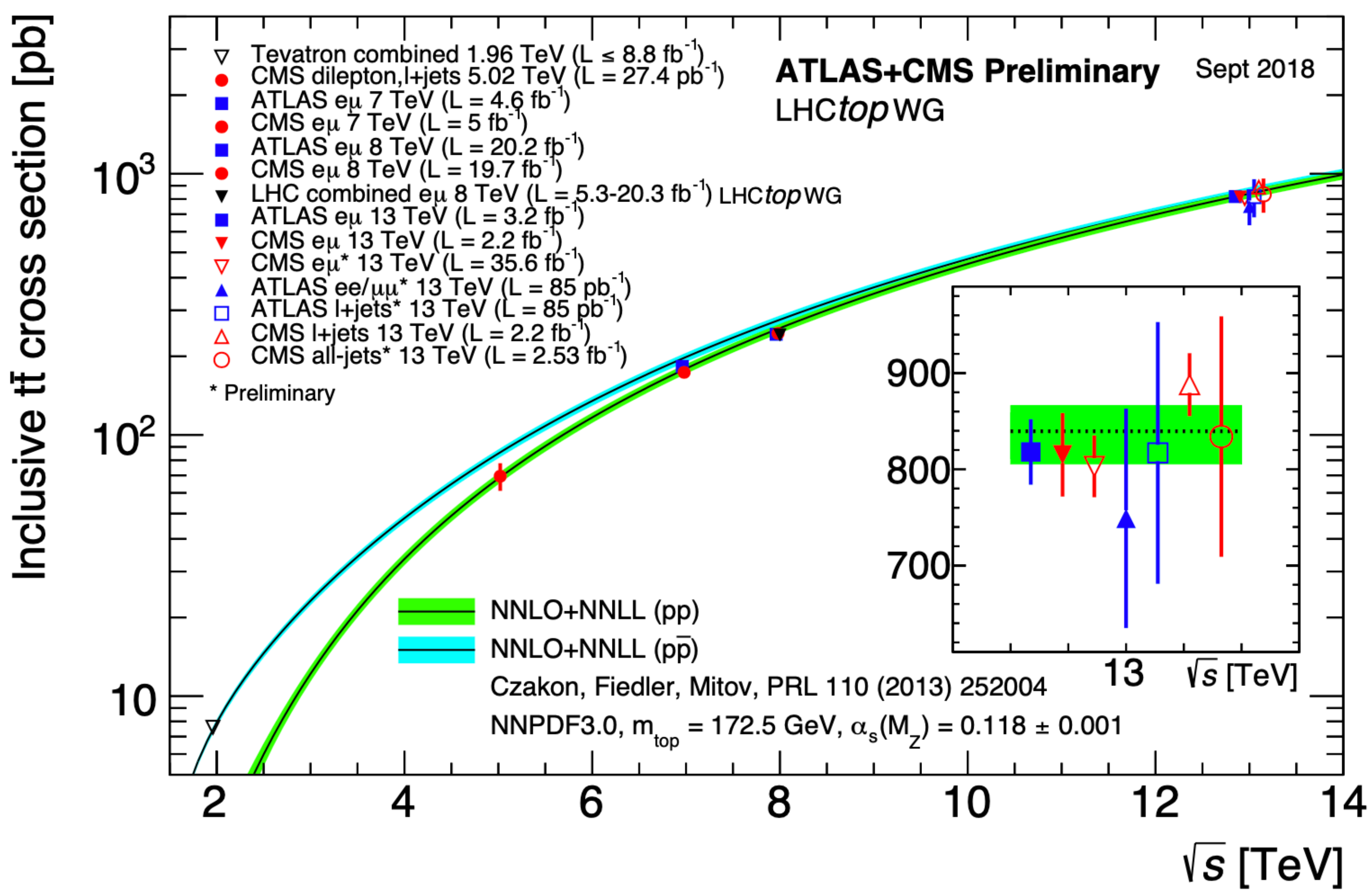



About 15 $t\bar{t}$ events per second at the LHC !



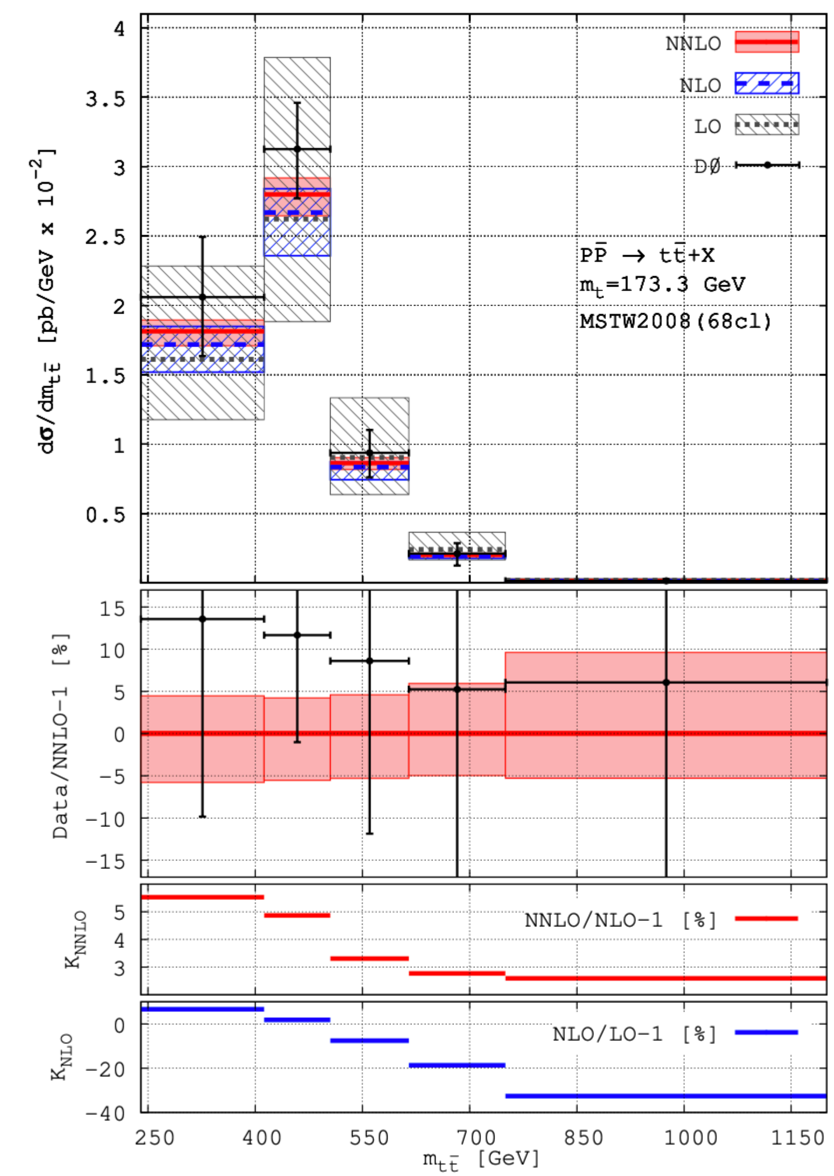
gg contribution dominant
at the LHC (85% at LO)

# Theoretical status




Inclusive cross section known at NNLO+NNLL in QCD

Bärnreuther, Czakon, Mitov (2012)
Czakon, Mitov (2012)
Czakon, Fiedler, Mitov (2013)
Czakon, Fiedler, Heymes, Mitov (2015,2016)




NNLO calculation extended to some differential distributions

Czakon, Fiedler, Heymes, Mitov (2015,2017)

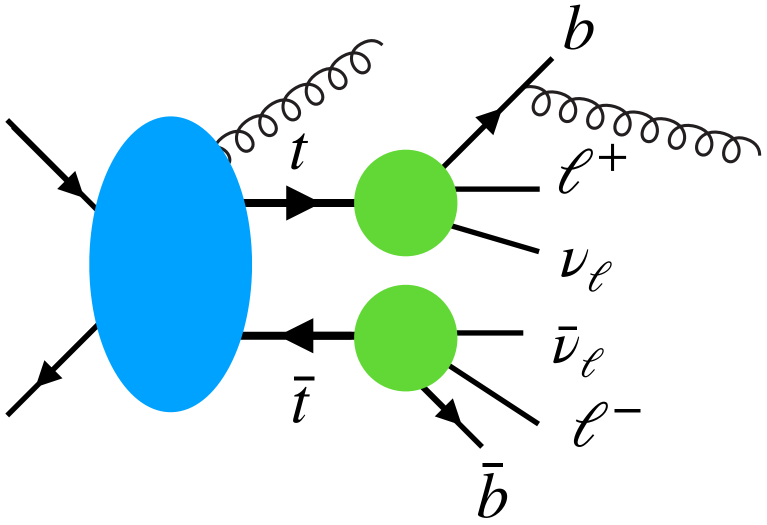
# Top decay

- Narrow width approximation

  - based on the limit $\Gamma_t/M_t \to 0$

  - considerable simplifications from the factorisation of production and decay

  - treatment of spin-correlations possible




Melnikov, Schulze (2009)

- Off-shell calculations

  - consider the complete process, say $pp \to b\bar{b}l\nu l\nu + X$

  - challenges come from high-multiplicity phase space and interferences between production and decay

# Full NLO QCD and EW calculations

- Off-shell effects through complete process $pp \to b\bar{b}l\nu l\nu + X$ in NLO QCD

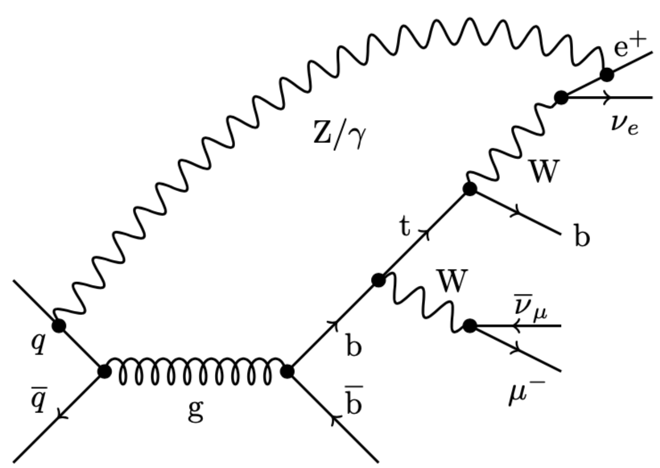
Bevilacqua, Czakon, van Hameren, Papadopoulos, Worek (2010)
Denner, Dittmaier, Kallweit, Pozzorini (2012)
Heinrich et al (2013)

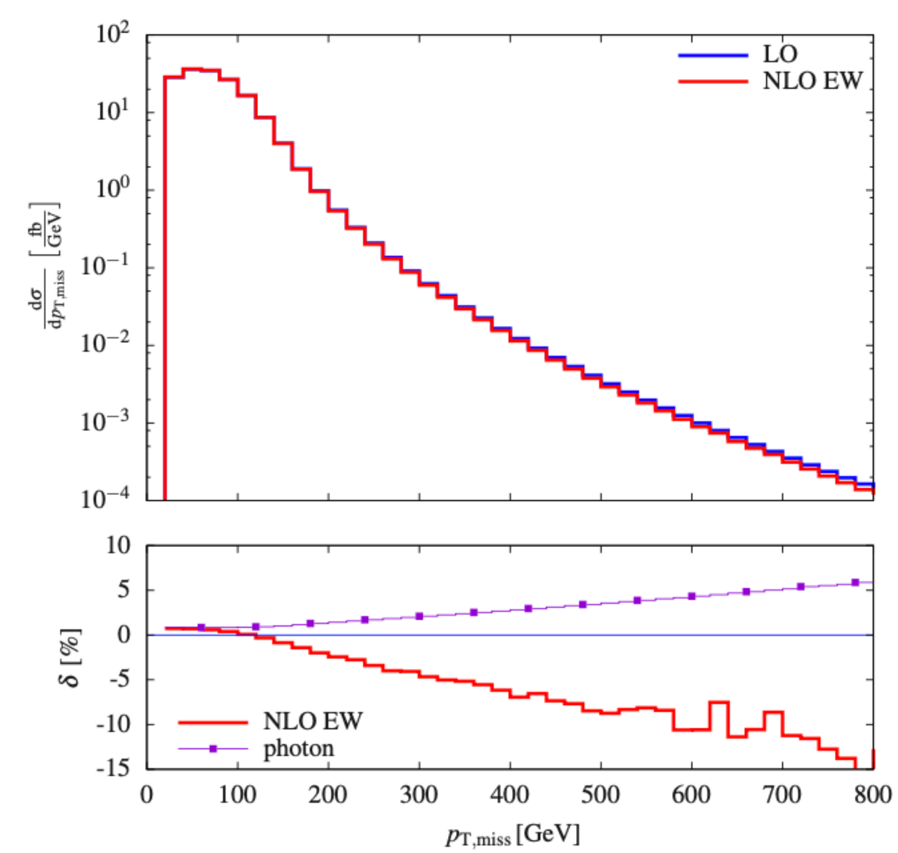
- Unified $t\bar{t}$ and $Wt$ with massive b-quarks

Cascioli, Kallweit, Maierhöfer, Pozzorini (2013)
Frederix (2013)

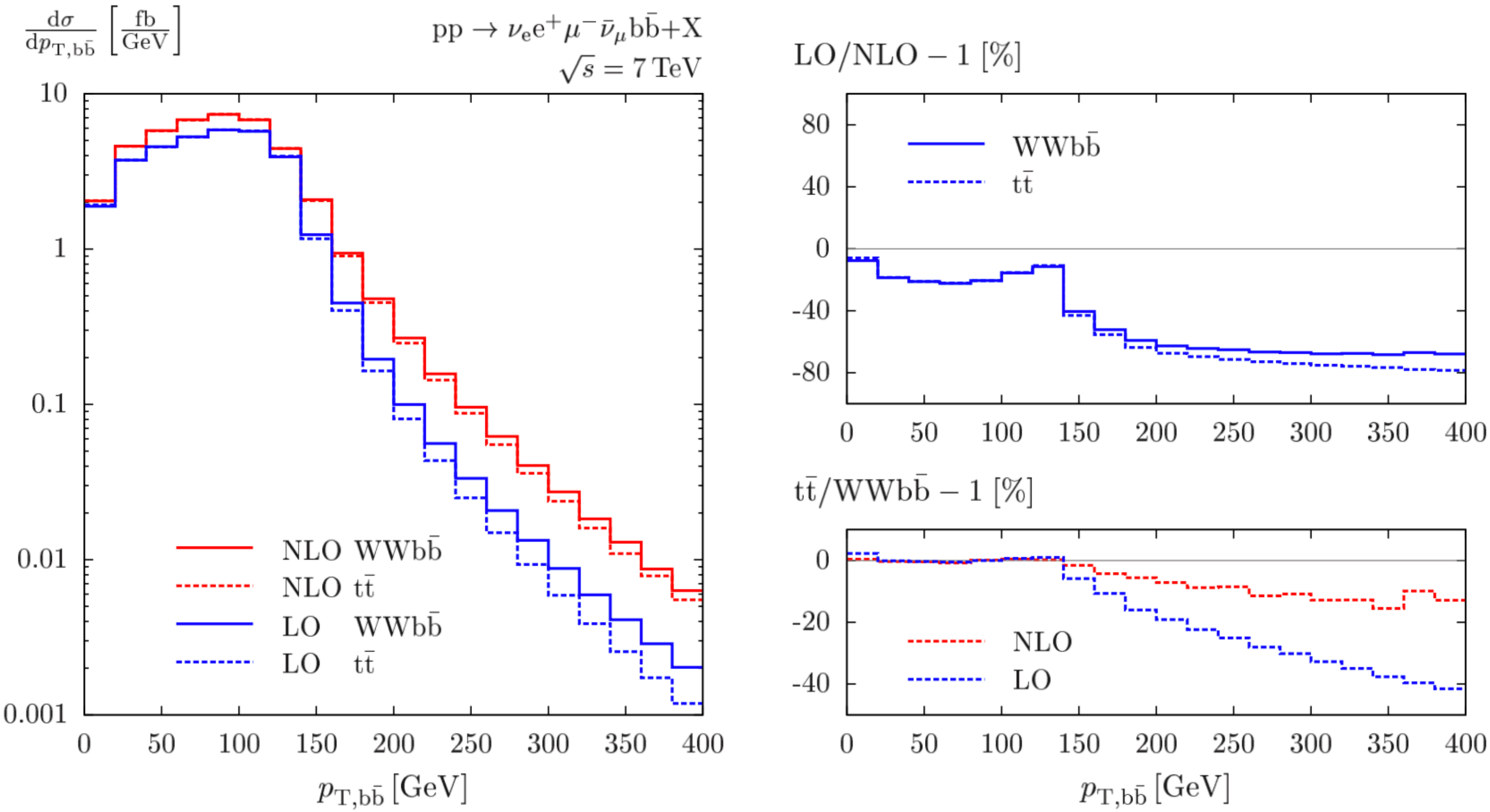
- NLO EW corrections to full $b\bar{b}l\nu l\nu$

Denner and Pellen (2016)



# Finite-width effects




Pozzorini et al. (2012)

Finite-width effects of the top quark are typically small but more important than those of the W

# A new NNLO calculation

# Why a new NNLO calculation ?

- Very complex calculation, only one group able to complete it till recently

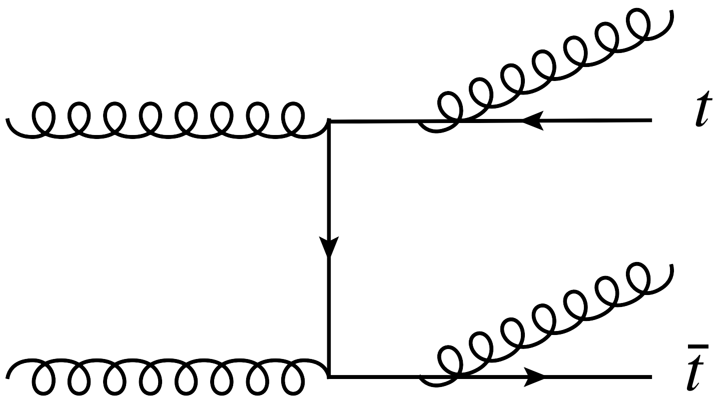
Bärnreuther, Czakon, Mitov (2012)
Czakon, Mitov (2012)
Czakon, Fiedler, Mitov (2013)
Czakon, Fiedler, Heymes, Mitov (2015,2016)

- Experience shows that NNLO calculations are difficult and that an independent check is always useful

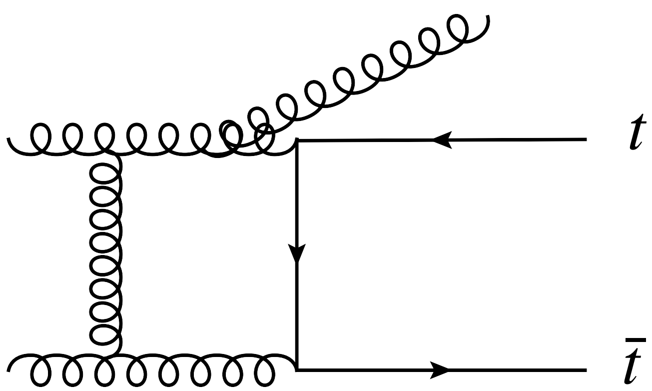
  - Drell-Yan — Hamberg, Matsuura, Van Neerven (1991); Harlander, Kilgore (2000)
  - $e^+ e^- \to 3$ jets — Gehrmann-De Ridder, Gehrmann, Glover, Heinrich (2008) ; Weinzierl (2008)
  - Diphoton hadroproduction — Catani, Cieri, Ferrera, de Florian, MG (2012); Campbell, Ellis, Williams (2016)
  - Higgs production in VBF — Cacciari et al. (2015); Cruz-Martinez, Gehrmann, Glover, Huss (2018)
  - Higgs+jet(s) — Boughezal et al (2015); Caola, Melnikov et al (2015); Chen, Gehrmann, Glover, Jaquier (2015)

- No public parton level event generator was available

# NNLO: building blocks

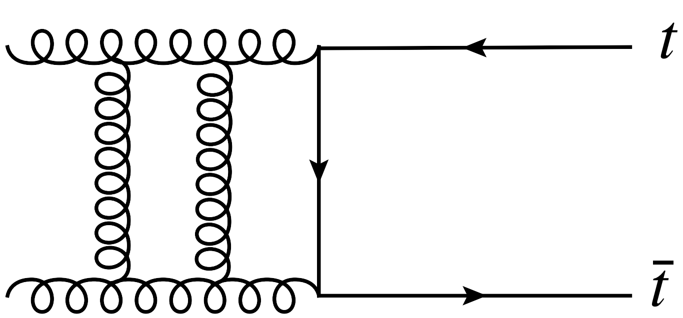



Tree-level amplitudes with two additional partons




One-loop amplitudes with one additional parton
(to be evaluated in unresolved regions where instabilities may arise)

All the three contributions separately divergent !




Two-loop amplitudes → currently the major bottleneck (new class of functions, charting new territory…)

+ one-loop squared amplitudes

Crucial to keep the calculation fully differential: corrections for fiducial and inclusive rates may be significantly different

# NNLO: building blocks

- Tree-level amplitudes with two additional partons and one-loop amplitudes with one additional parton are the same entering the computation of $t\bar{t}$+jet

Dittmaier, Uwer, Weinzierl (2007,2008)

→ Nowadays they can be obtained with automatic generators like Openloops, Recola….

- The one loop squared contribution is known

Korner, Merebashvili, Rogal (2008)
Anastasiou, Aybat (2008)
Kniehl, Merebashvili, Korner, M. Rogal (2008)

- Two-loop amplitude only available numerically

Czakon (2008)
Barnreuther, Czakon, and Fiedler (2013)

Recent progress in the computation of non-planar master integrals suggests that the analytic calculation can be completed soon

Bonciani et al (2019)
Gehrmann et al (2019)

All the contributions in principle available but separately divergent !

→ **Subtraction scheme needed !**

# Handling IR singularities

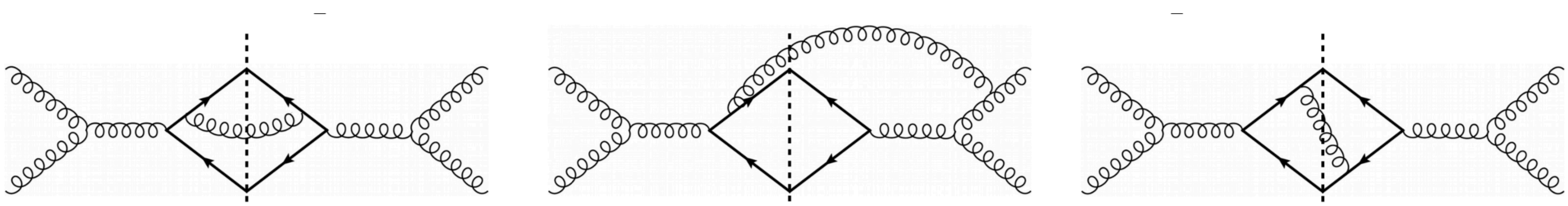
We use the well established $q_T$ subtraction formalism

Catani, MG (2007)

The NNLO QCD ingredients for the production of a colourless final state are fully sufficient to deal with initial state radiation

In the case of heavy-quark ($Q$) production additional soft singularities appear that need to be taken into account

Catani, MG (2011)
Catani, Cieri, de Florian, Ferrera, MG (2012,2013)



$$d\sigma^{Q\bar{Q}}_{(N)NLO} = \mathcal{H}^{Q\bar{Q}}_{(N)NLO} \otimes d\sigma^{Q\bar{Q}}_{LO} + \left[ d\sigma^{Q\bar{Q}+\text{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right]$$

Catani, Devoto, Kallweit, Mazzitelli, Sargsyan, MG (2019)

Catani, Devoto, Kallweit, Mazzitelli, MG (2020)

# Implementation in MATRIX

As for the other NNLO calculations in MATRIX all spin and colour correlated tree-level and one loop amplitudes are obtained with **Openloops**

→ **Excellent numerical stability in IR singular regions**

Four parton tree-level colour correlations are computed analytically

Real-virtual contributions cross checked with **Recola**

The calculation is now fully implemented into the MATRIX framework

Automatic evaluation of scale uncertainties

Cross sections at 0.1 % accuracy computable with O(1000) CPU days

# Inclusive results

Use NNPDF3.1 NNLO PDFs and $M_t$=173.3 GeV

| $\sigma_{\rm NNLO}$ [pb] | MATRIX | TOP++ |
| --- | --- | --- |
| 8 TeV | $238.5(2)^{+3.9\%}_{-6.3\%}$ | $238.6^{+4.0\%}_{-6.3\%}$ |
| 13 TeV | $794.0(8)^{+3.5\%}_{-5.7\%}$ | $794.0^{+3.5\%}_{-5.7\%}$ |
| 100 TeV | $35215(74)^{+2.8\%}_{-4.7\%}$ | $35216^{+2.9\%}_{-4.8\%}$ |

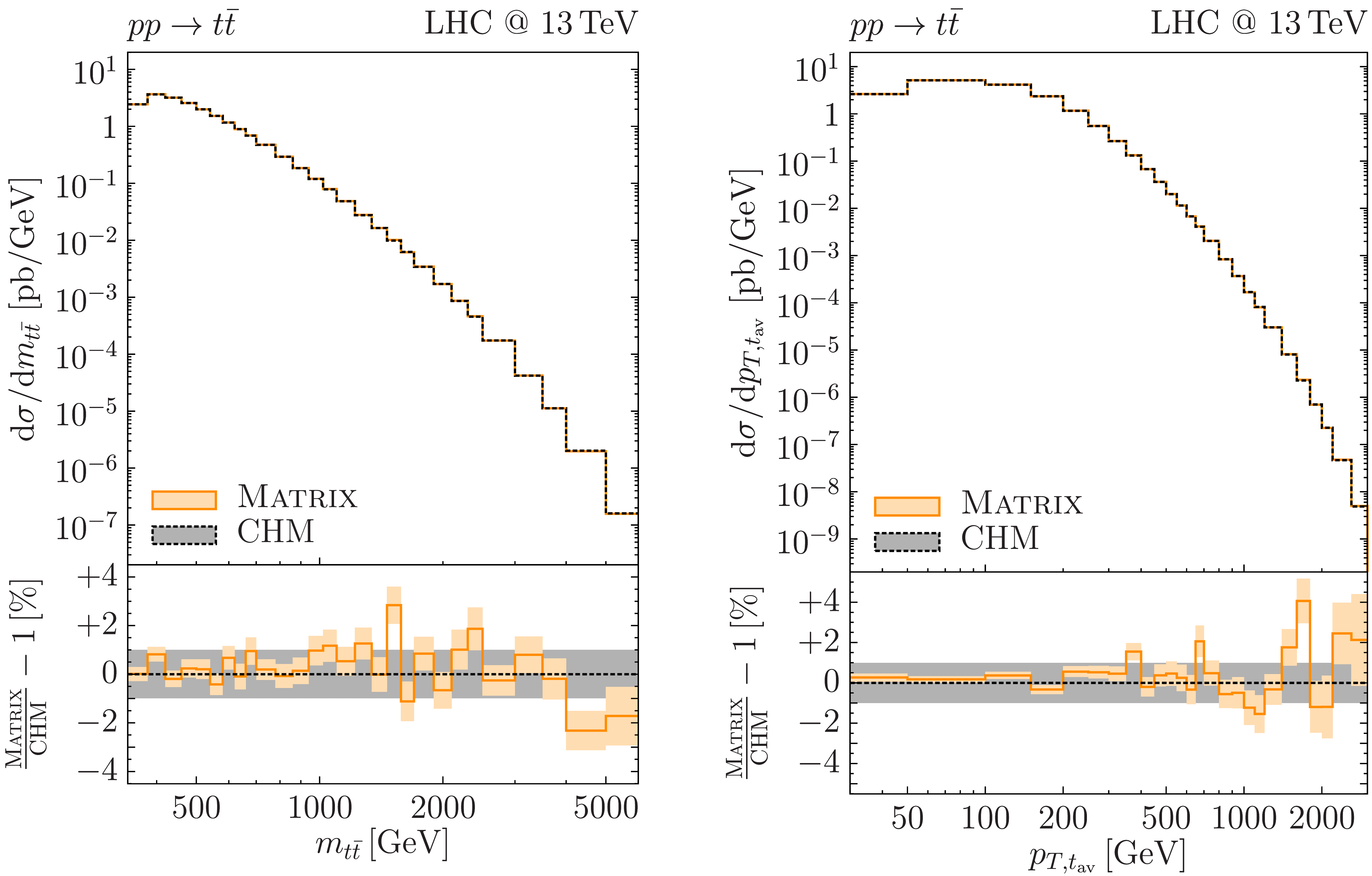
statistical+systematic uncertainties

Excellent agreement with Top++ !

We find that the quantitative impact of the two-loop amplitude is extremely small (0.1% of the full NNLO cross section at 13 TeV)

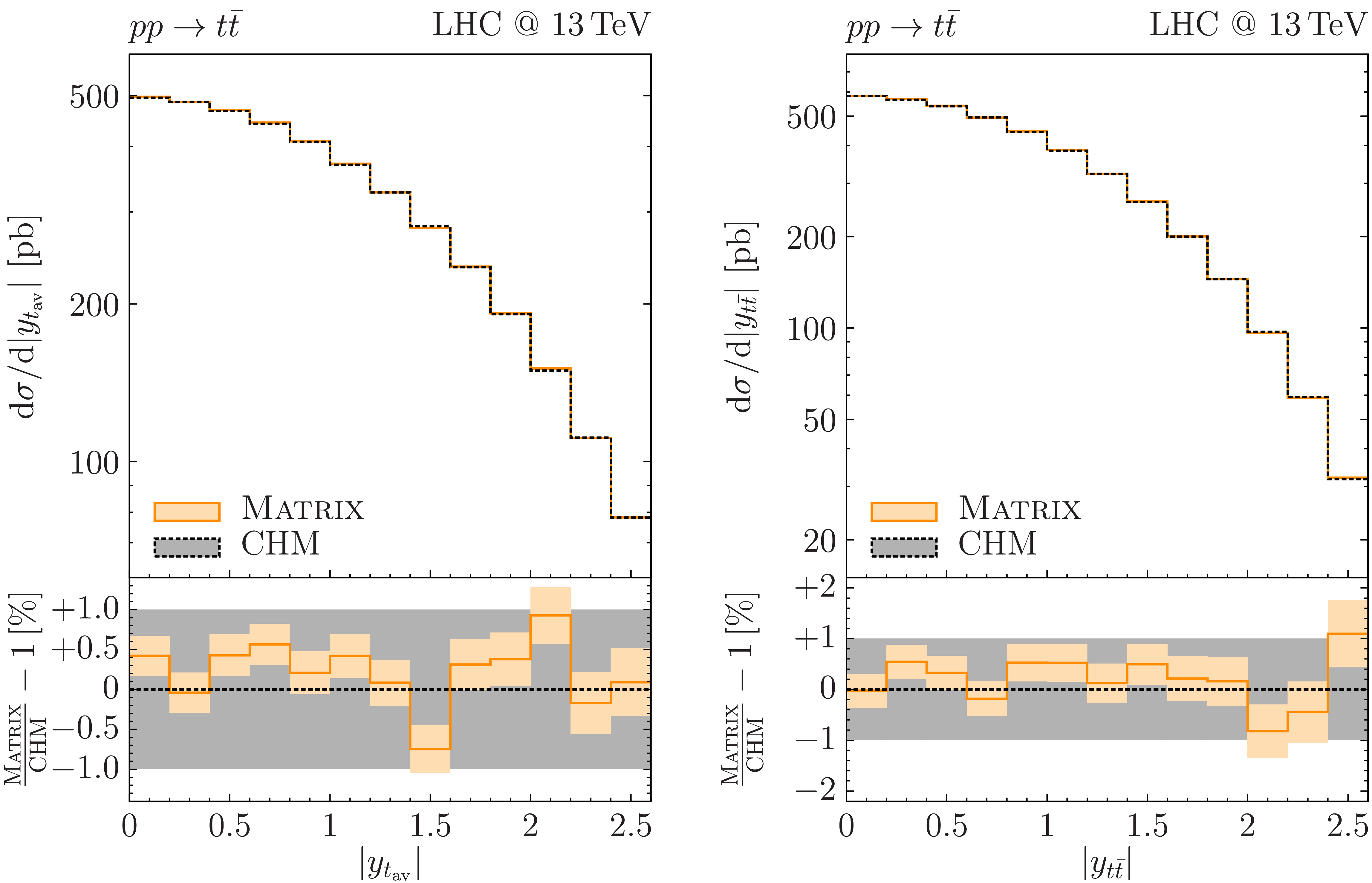
→ (Almost) completely independent computation !

# Going differential: validation




Excellent agreement even in extreme kinematical regions

# Going differential: validation




Excellent agreement even in extreme kinematical regions

# Going differential: results

LO, NLO and NNLO predictions obtained using NNPDF3.1 PDFs with $\alpha_S(m_Z)=0.118$ at the corresponding order

CMS data of CMS-TOP-17-002 in the lepton+jets channel

Extrapolation to parton level in the inclusive phase space

→ Our calculation is carried out without cuts

To compare with data we multiply our absolute predictions by 0.438 (semileptonic BR of the $t\bar{t}$ pair) times 2/3 (only electrons and muons)

# The choice of scales

Perturbative results depend on the choice of the renormalisation and factorisation scales $\mu_R$ and $\mu_F$

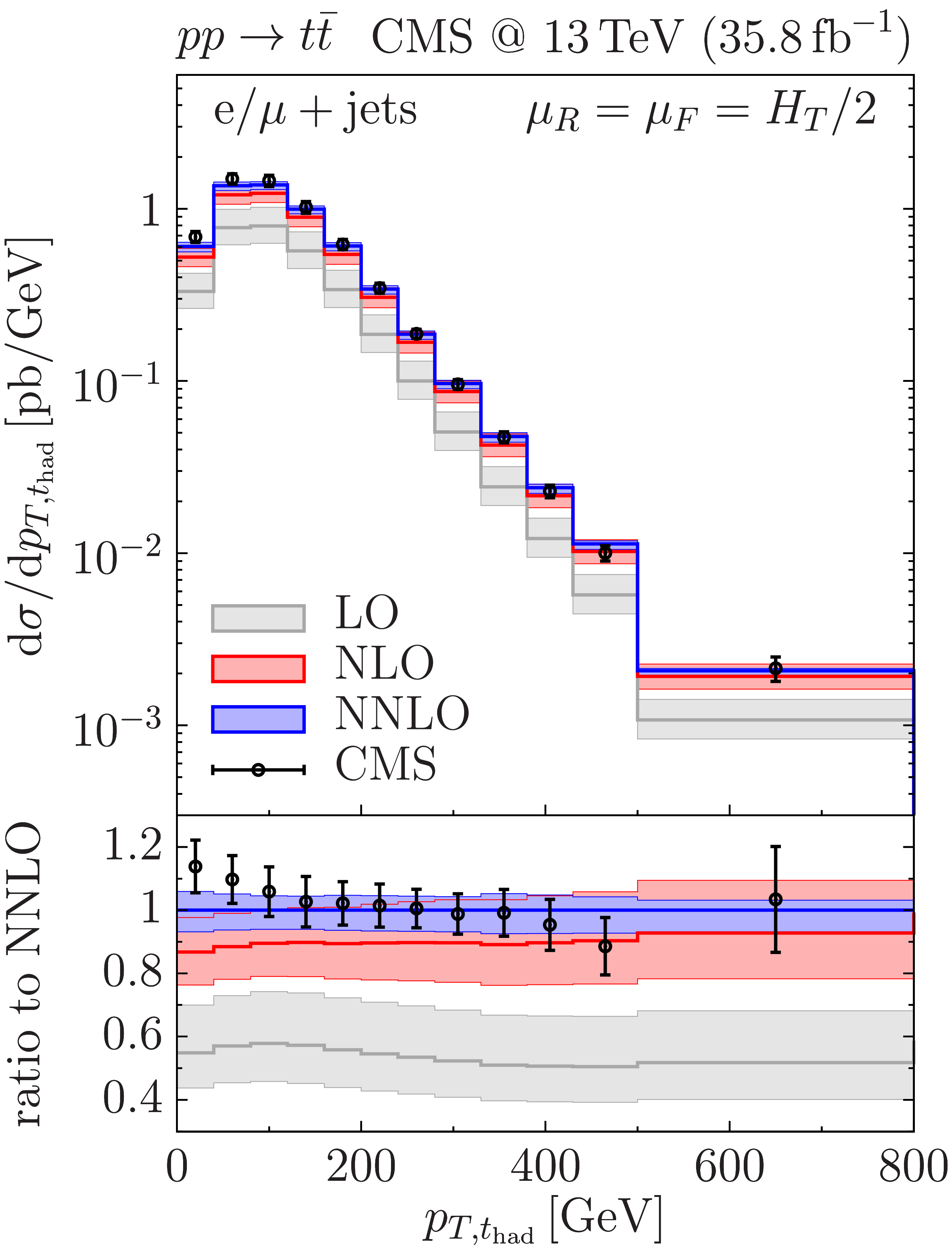
These scales should be chosen of the order of the characteristic hard scale

- Total cross section: the hard scale is the top mass $m_t$
- The same can be said for the rapidity distributions
- Invariant mass distribution: $m_{tt}$
- Tranverse momentum distributions: $m_T$

A dynamical central scale like $\mu_0 = H_T/2 = (m_{T,t} + m_{T,\bar{t}})/2$ turns out to be a good approximation of all these characteristic scales

Scale uncertainties: $\mu_0/2 < \mu_F, \mu_R < 2\mu_0$ $\quad$ $0.5 < \mu_F/\mu_R < 2$

# Single-differential distributions

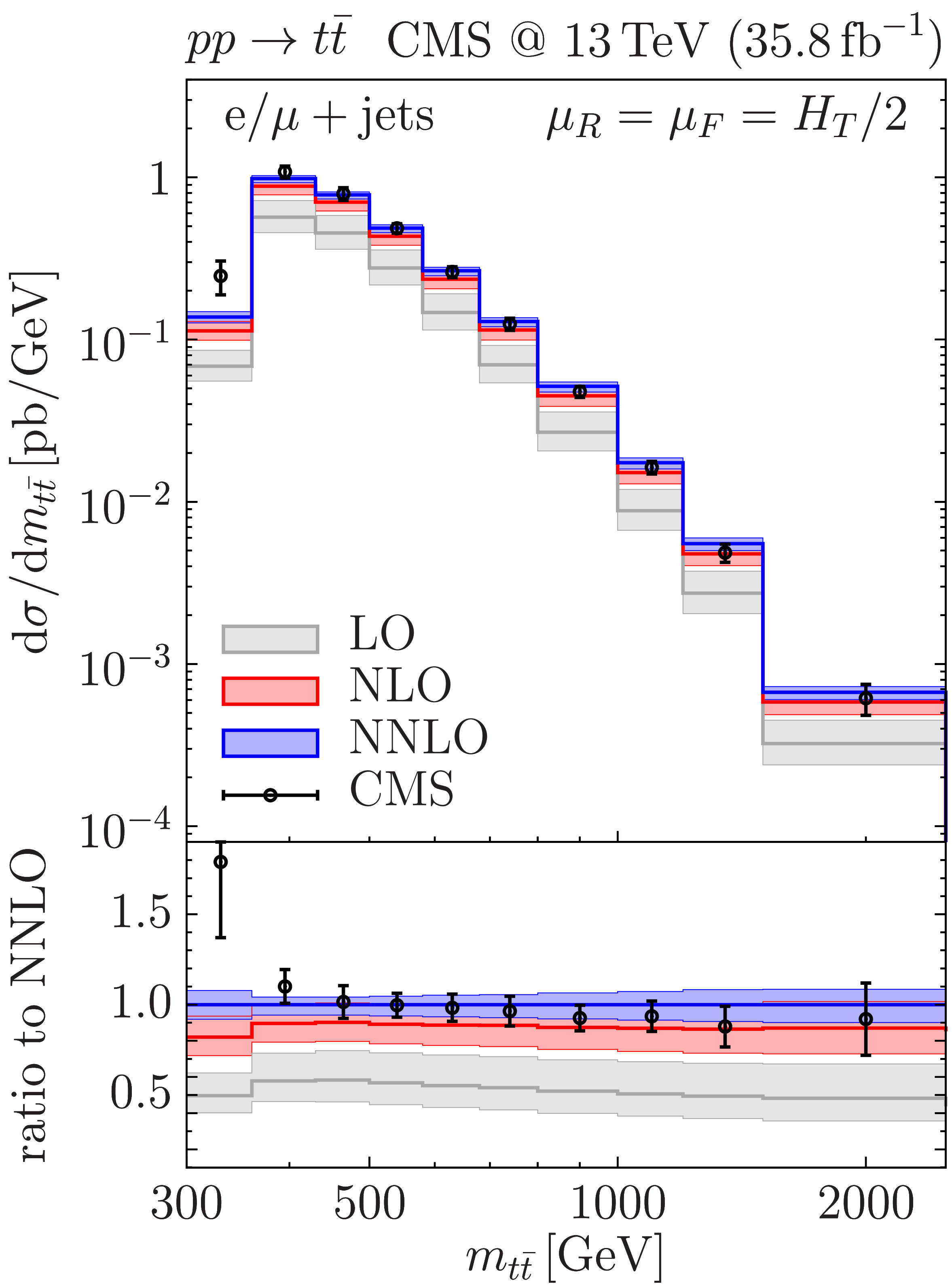



As noted in various previous analyses the measured $p_T$ distribution is slightly softer than the NNLO prediction

Perturbative prediction relatively stable when going from NLO to NNLO

Data and theory are consistent within uncertainties

# Single-differential distributions



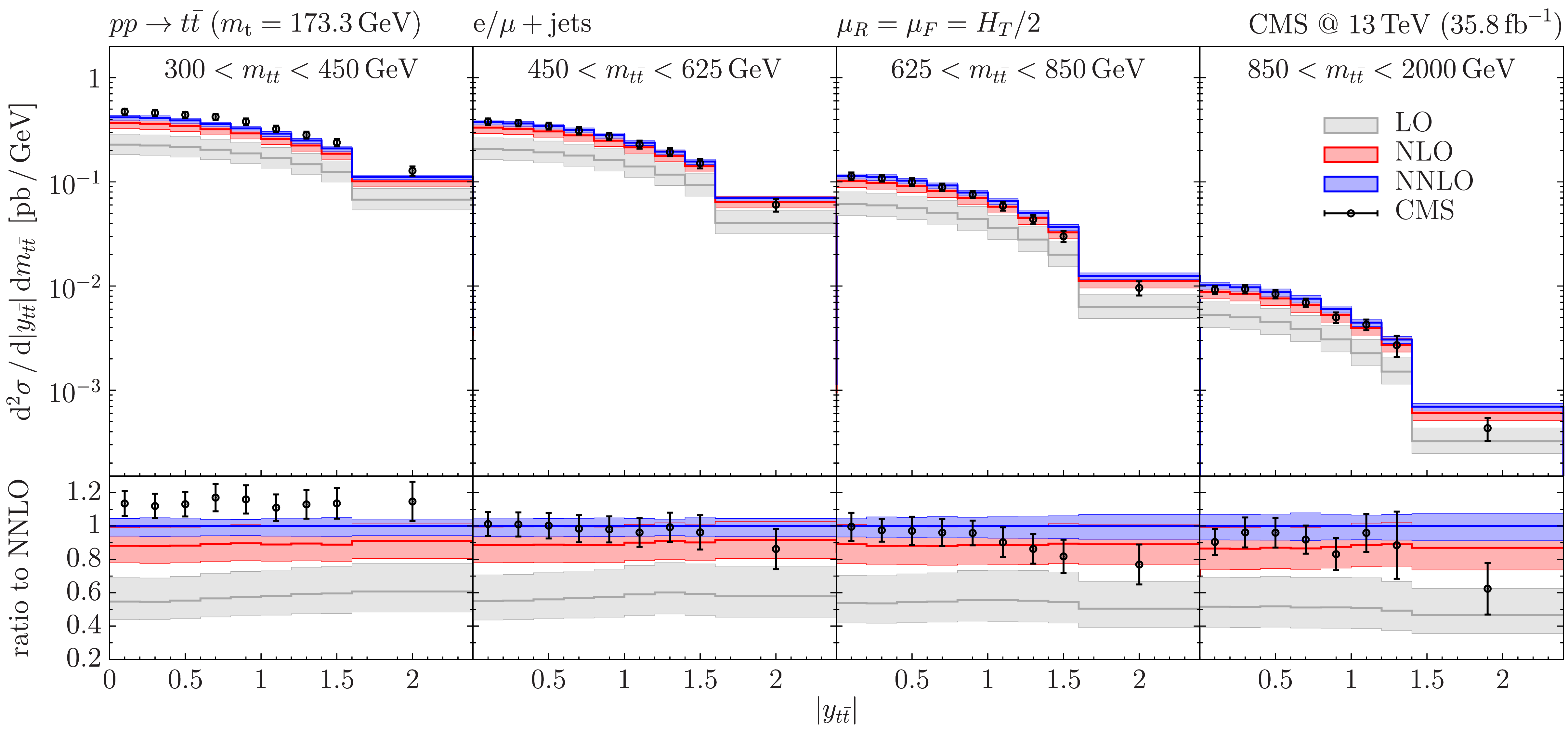

Good description of the data except in the first bin

Issues in extrapolation ? Smaller $m_t$ ?

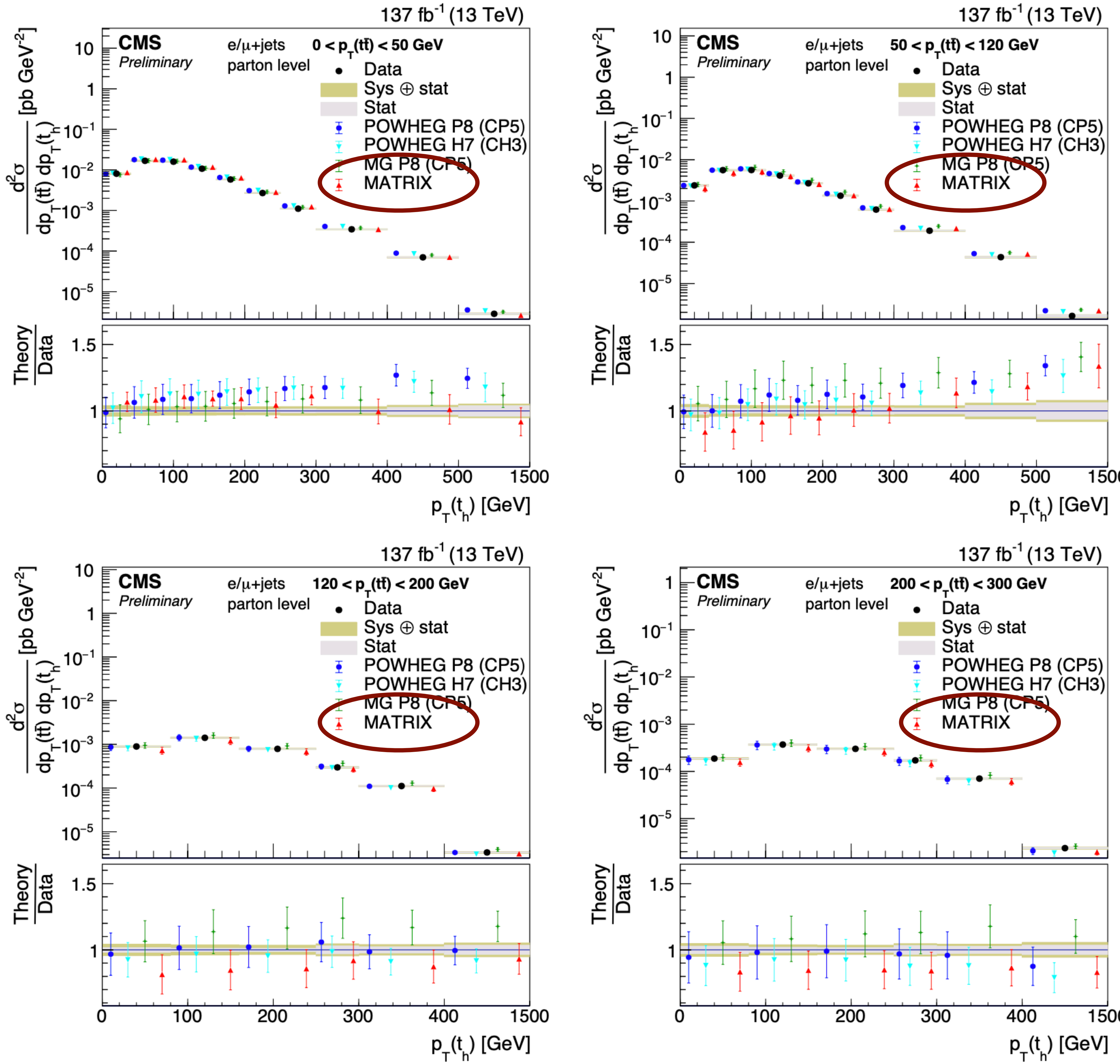
A smaller $m_t$ (just by about 2 GeV) leads to a higher theoretical prediction in this bin and to small changes at higher $m_{tt}$

CMS-TOP-18-004: leptonic channel: a fit with the same PDFs leads to $m_t$=170.81 ± 0.68 GeV

# Double-differential distributions



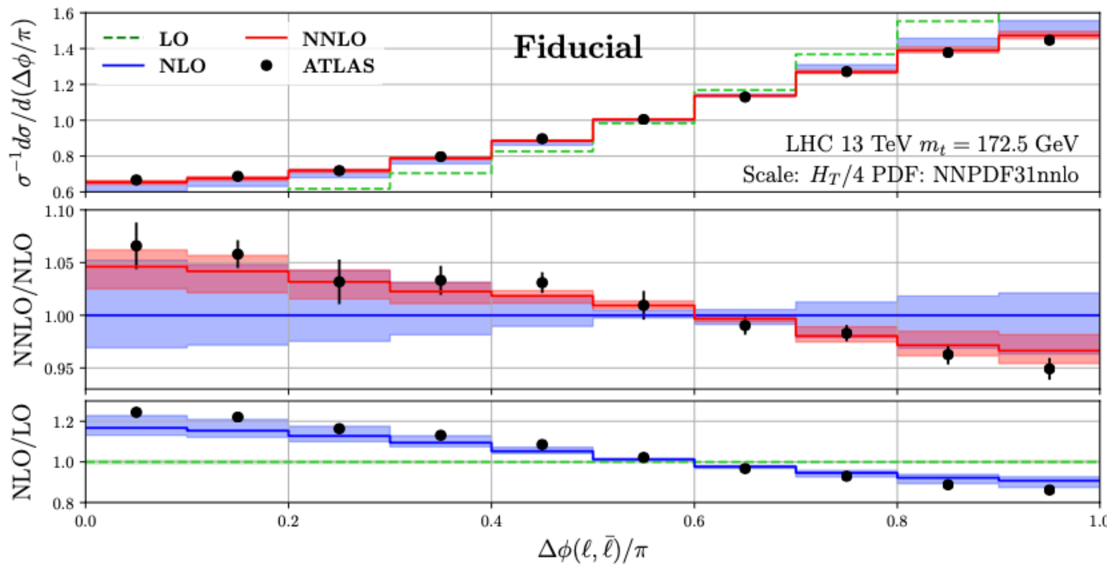

The first $m_{tt}$ interval now extends up to 450 GeV ➡ better agreement with the data

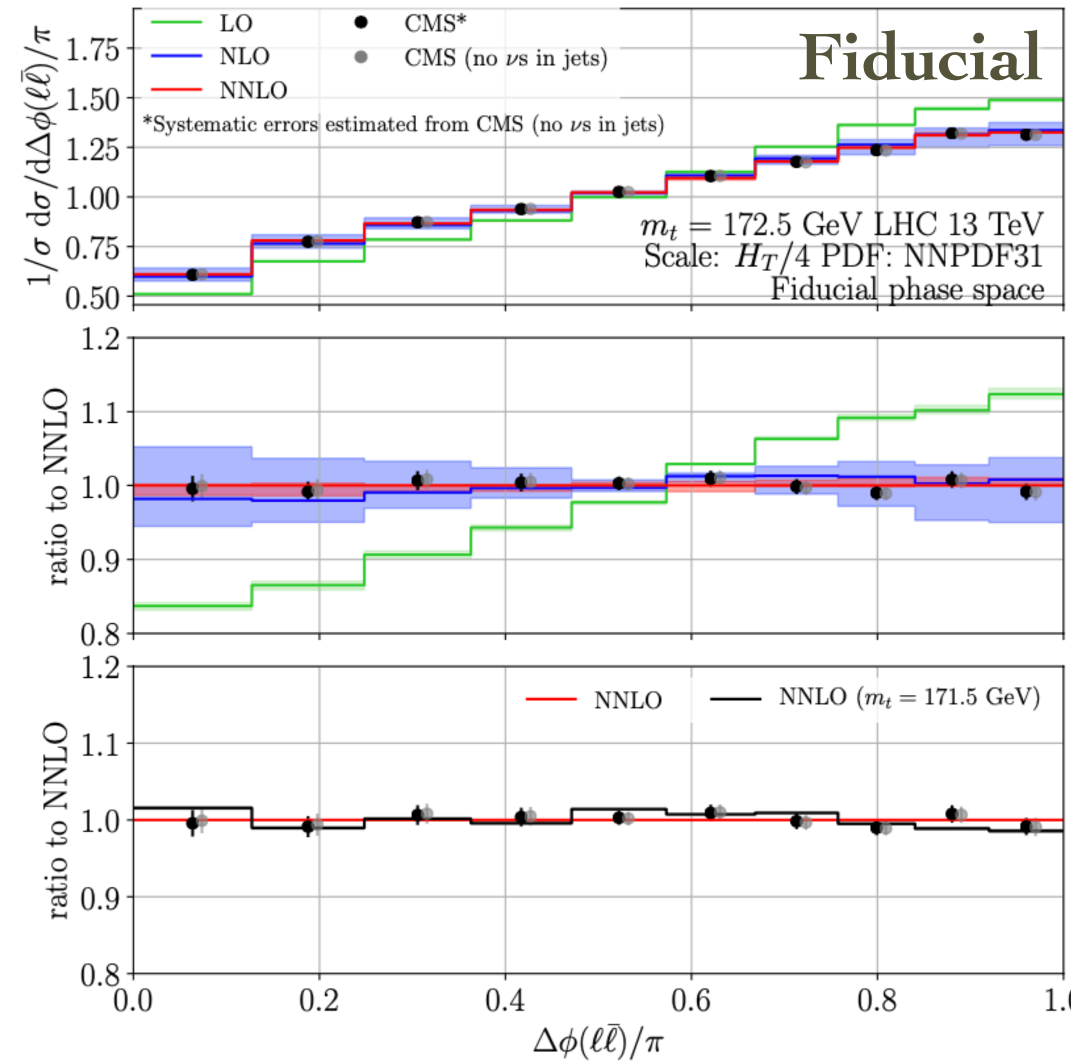


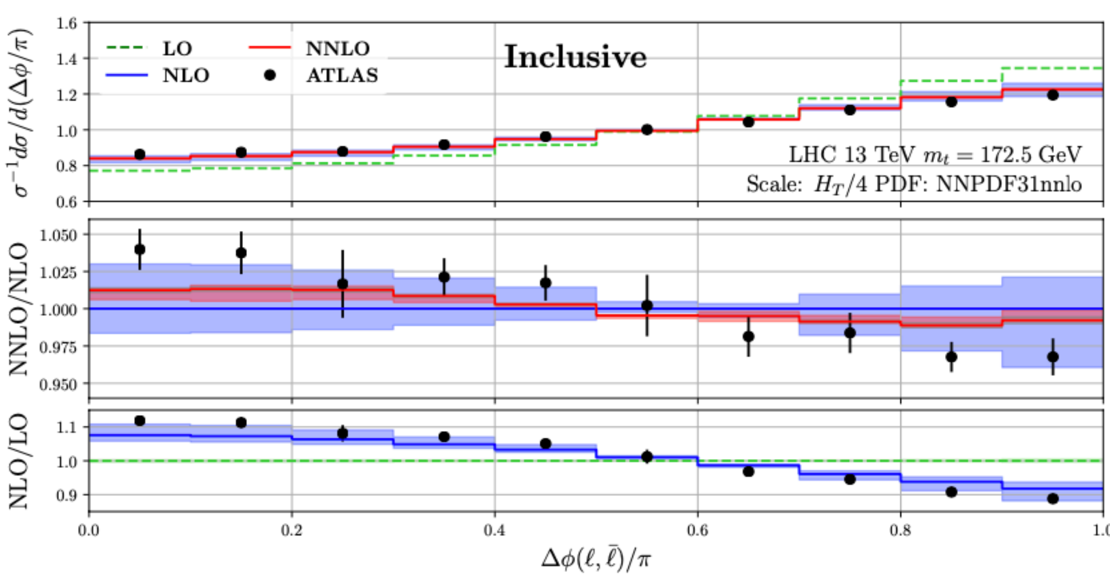
137 fb$^{-1}$ (13 TeV)
CMS
Preliminary
e/μ+jets
parton level
0 < $p_T(t\bar{t})$ < 50 GeV
50 < $p_T(t\bar{t})$ < 120 GeV
120 < $p_T(t\bar{t})$ < 200 GeV
200 < $p_T(t\bar{t})$ < 300 GeV
Data
Sys ⊕ stat
Stat
POWHEG P8 (CP5)
POWHEG H7 (CH3)
MG P8 (CP5)
MATRIX
$\frac{d^2\sigma}{dp_T(t\bar{t})\,dp_T(t_h)}$ [pb GeV$^{-2}$]
$\frac{\text{Theory}}{\text{Data}}$
$p_T(t_h)$ [GeV]

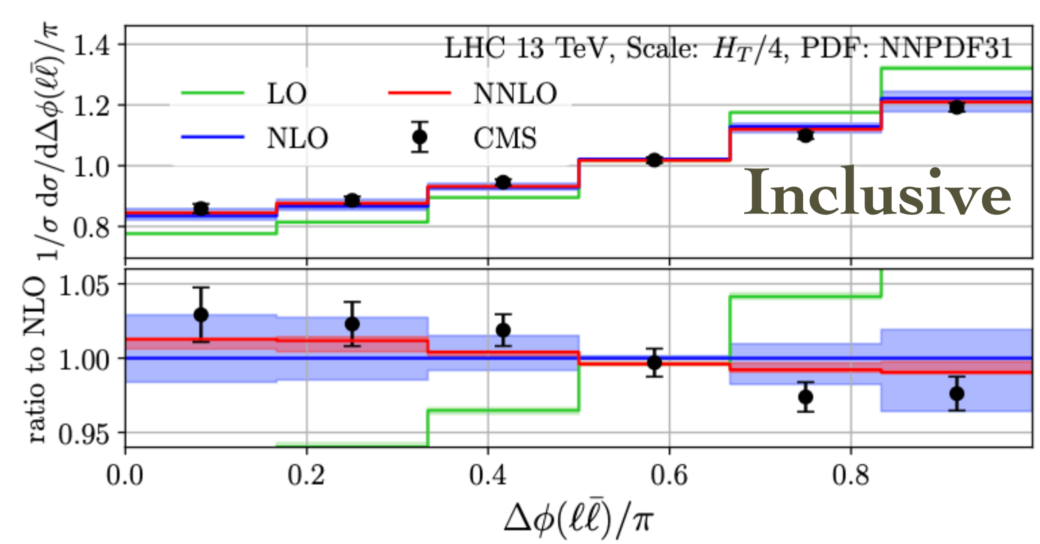
# The frontier: NNLO with decays (NWA)

Czakon et al. (2020)




QCD predictions describe the $\Delta\phi(\ell\bar{\ell})$ well in the fiducial region while some tension exists when the comparison is done in the inclusive phase space

➡ Effect of extrapolation relevant at this level of precision

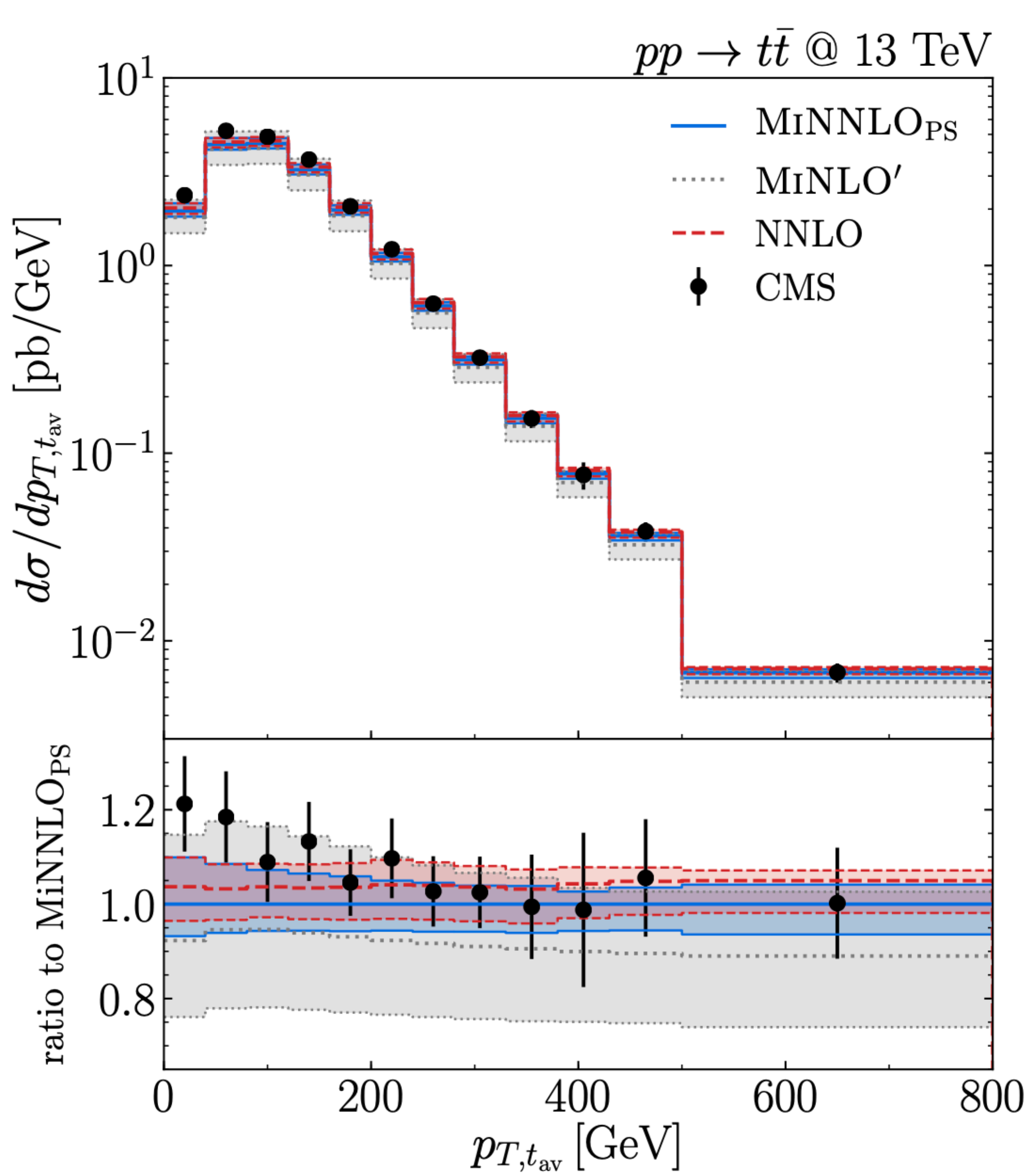
# NNLOPS

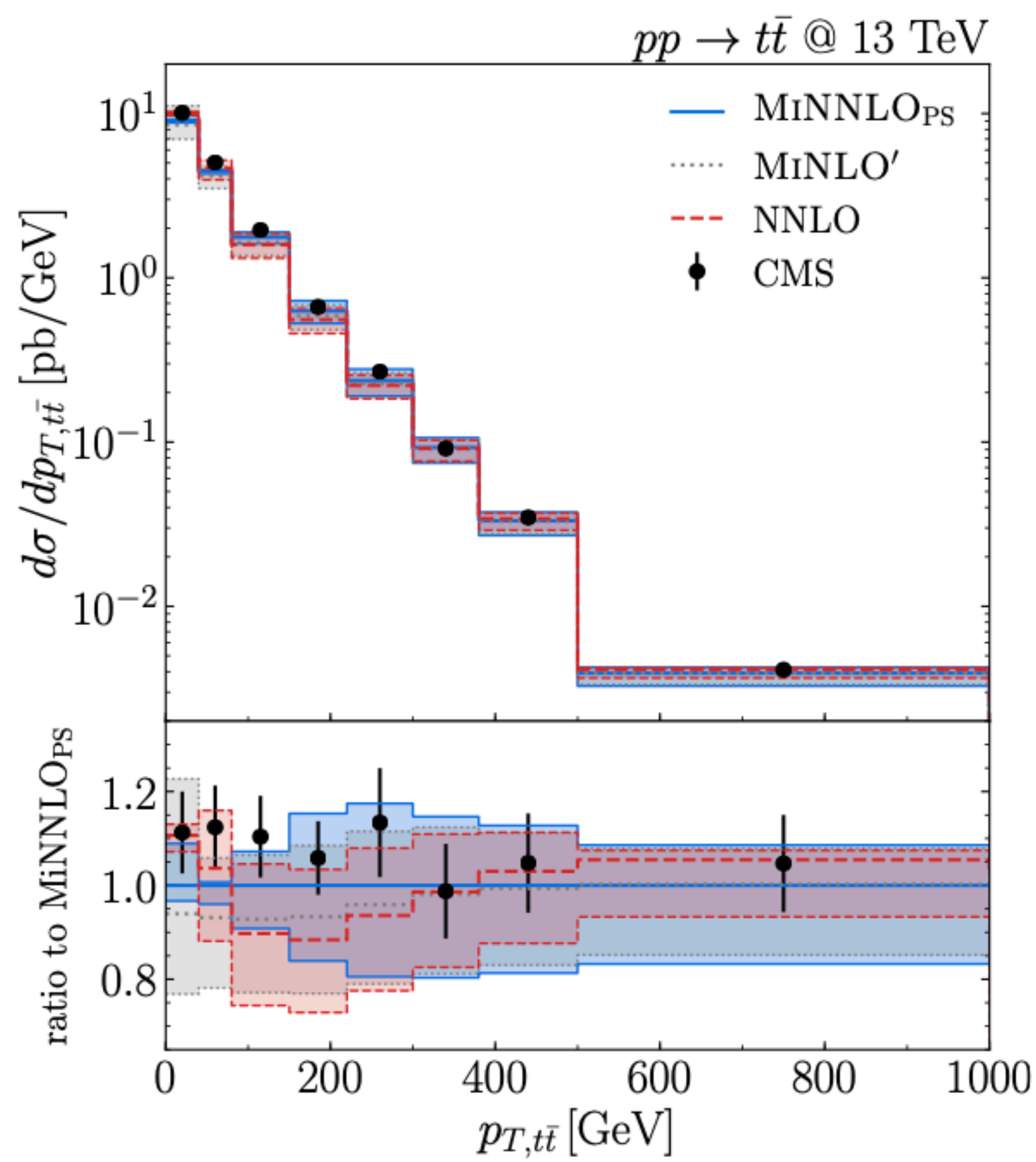
Mazzitelli, Monni, Nason, Re,
Wiesemann, Zanderighi (2020)

NNLO calculation recently deployed into the first NNLO calculation matched to parton shower for this process

All-order radiative contributions implemented through the shower using the MiNNLOPS method

Monni, Nason, Re, Wiesemann,
Zanderighi (2019)
Monni, Re, Wiesemann (2020)

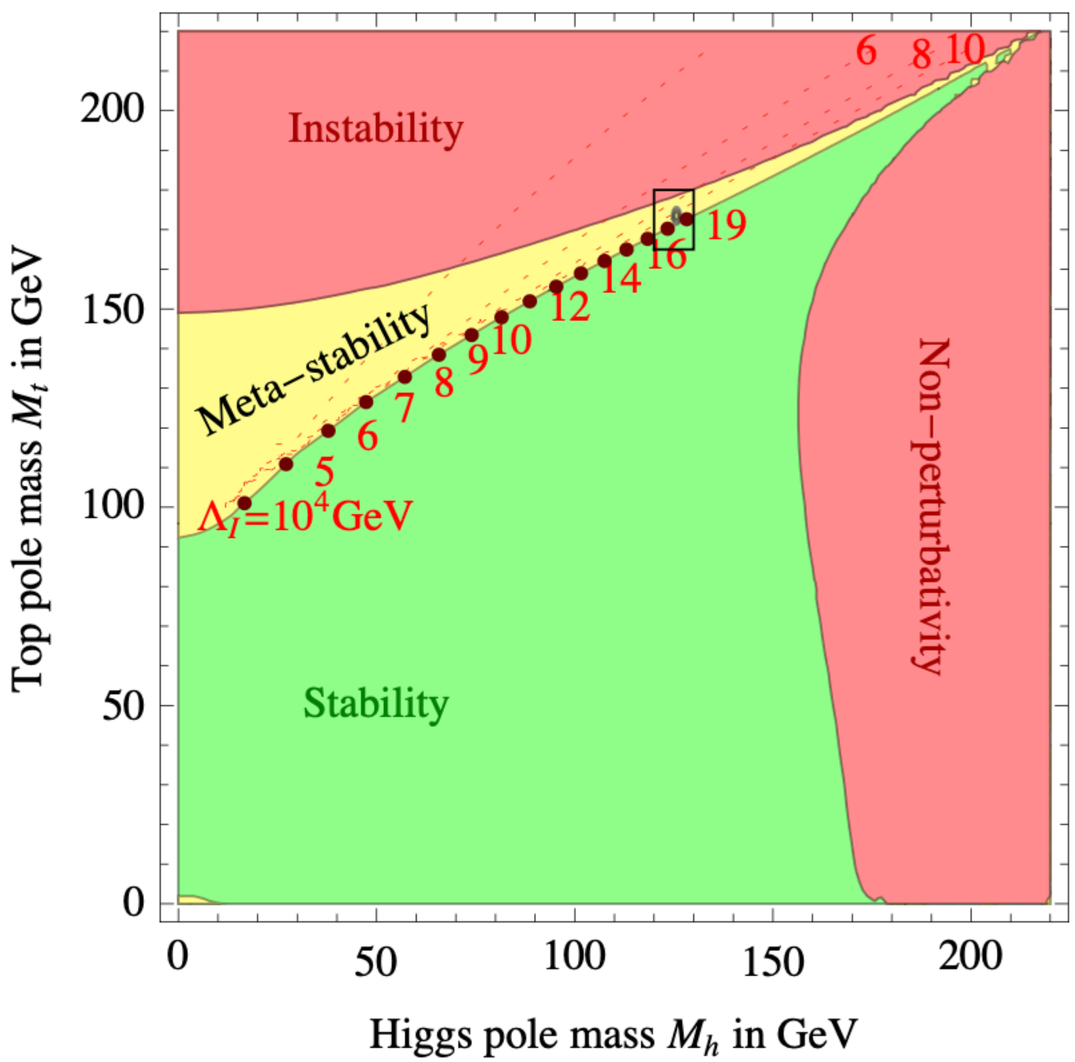


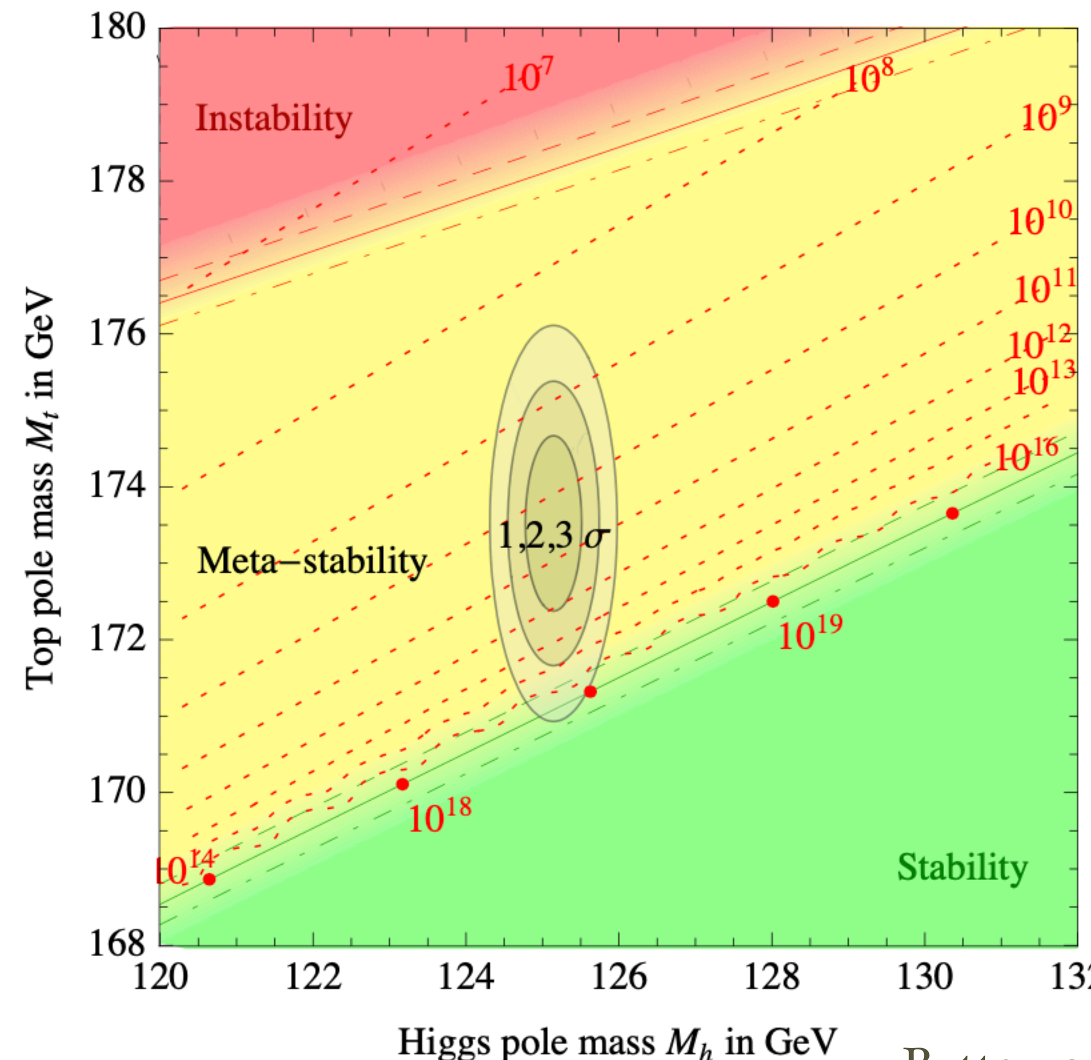



# Top mass

# Top mass

The top mass is a fundamental parameter of the SM




Buttazzo et al (2013)

With the current values of the top and Higgs masses the vacuum is metastable (assuming no new physics up to the Planck scale)

# Top mass

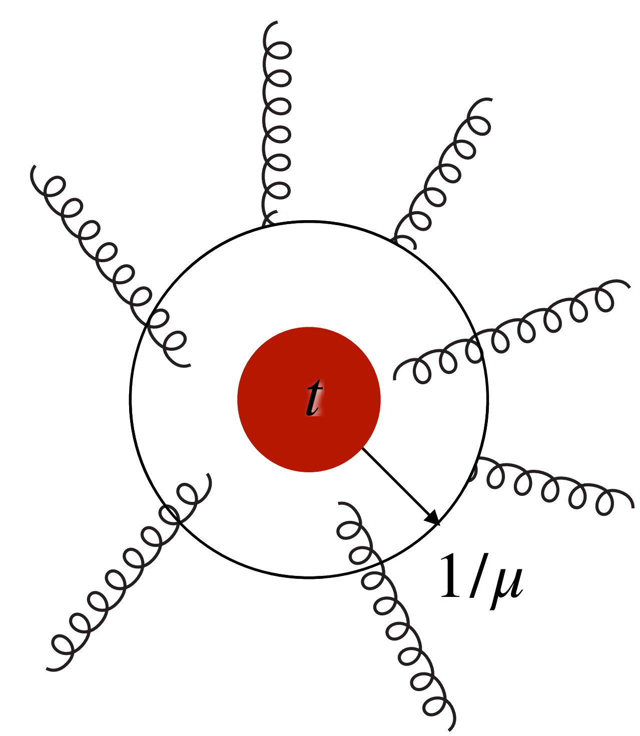
Direct measurements of the top mass are those in which the top quark is "observed" through its decay products

Recall: the top quark is so heavy that it decays before strong interactions come into play to form bound states

→ The top quark can be viewed as a physical though unstable particle

Pole mass is the natural definition but….

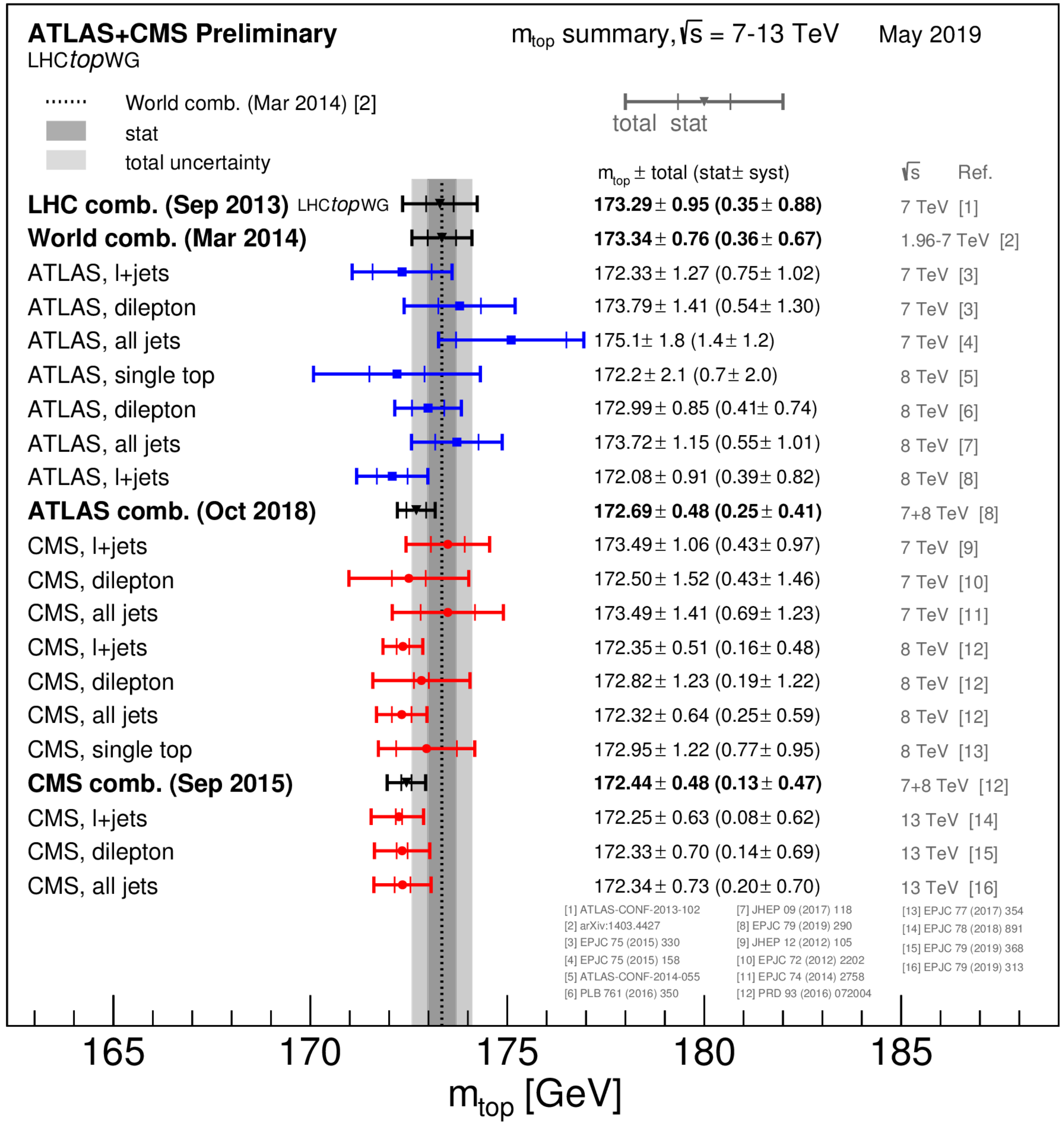
The quark mass is also given by the accompanying gluon field




The pole mass definition includes the surrounding gluon radiation up to infinite distances

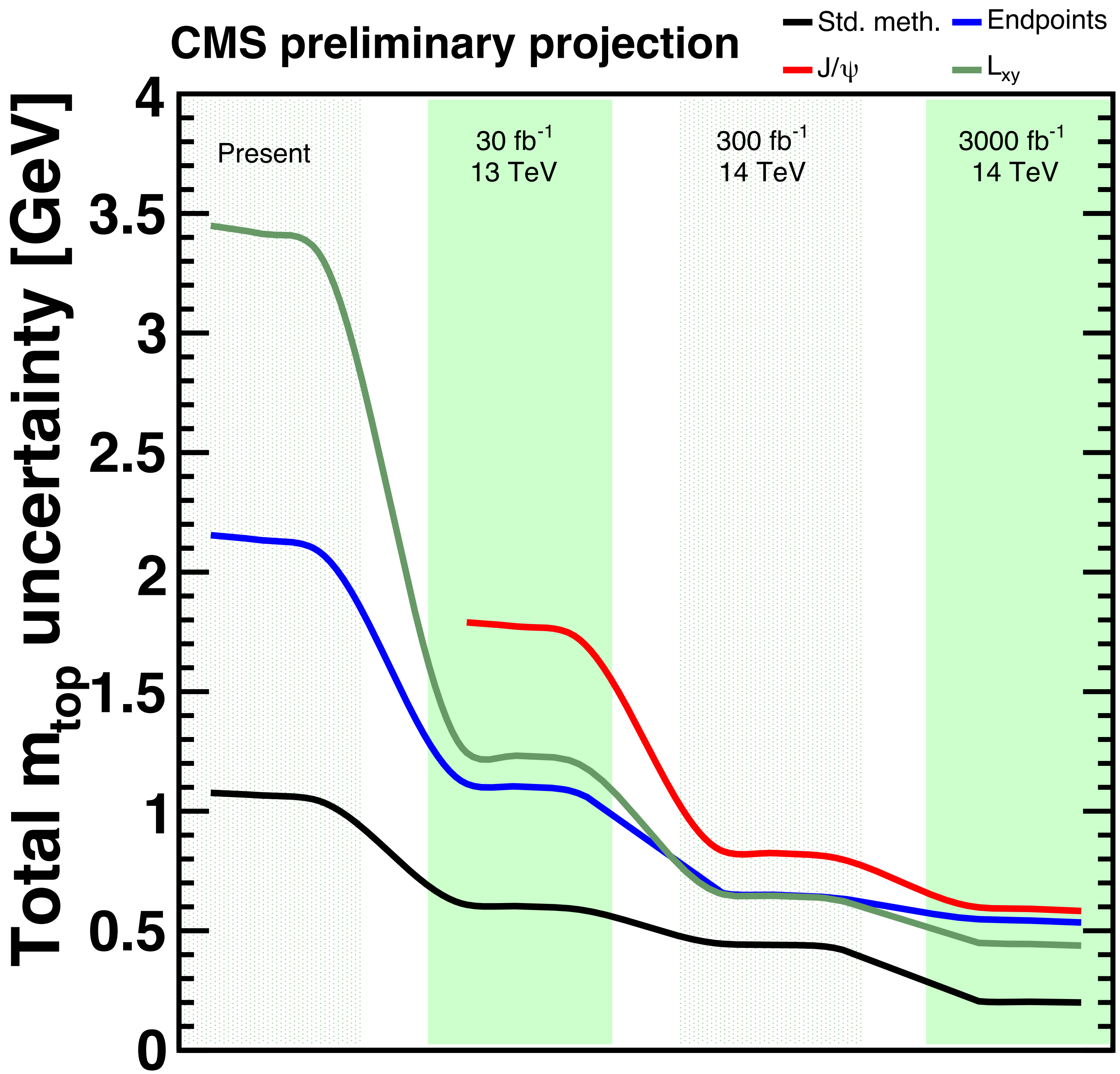
Alternatively one can keep just the contributions at distances below $1/\mu$ or at scales $E > \mu$ (short distance mass)

# Top mass




Direct measurements can be considered as measurements of the pole mass

# Top mass




Direct measurements can be considered as measurements of the pole mass

# Top mass

More precisely: the top mass must be properly defined by renormalisation of related UV divergences: in the pole scheme such procedure fixes the pole of the quark propagator, at any order in perturbation theory, to the same value $M_t$

In the $\overline{\text{MS}}$ scheme the renormalised mass $m_t(\mu_m)$ is defined by subtracting UV divergences in dimensional regularization, and, therefore, the pole of the quark propagator receives corrections at any order in perturbation theory

Different renormalisation schemes are perturbatively related

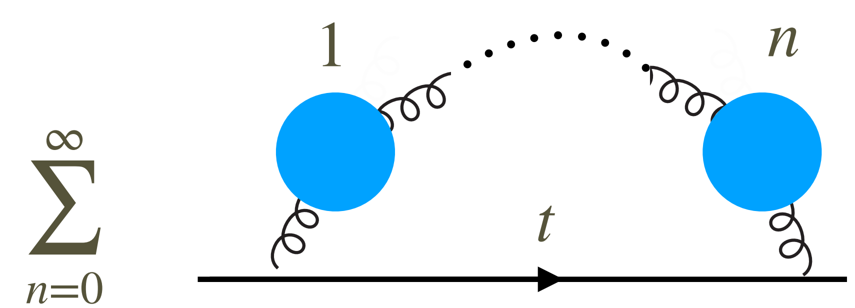
$$M_t = m_t(\mu_m)\, d(m_t(\mu_m), \mu_m) = m_t(\mu_m) \left(1 + \sum_{k=1}^{\infty} \left(\frac{\alpha_S(\mu_m)}{\pi}\right)^k d^{(k)}(\mu_m)\right)$$

coefficients $d^{(k)}$ known for $k \leq 4$

Chetyrkin, Steinhauser (1999); Melnikov, Ritbergen (1999)
Marquard et al (2016)

# Pole mass ambiguity

The pole mass $M_t$ is affected by a renormalon ambiguity




In short: the coefficients of the perturbative series are factorially divergent and the series is not Borel summable

→ this leads to an ambiguity of order $\Lambda_{QCD}$ in the pole mass

Recent estimates of the ambiguity range from 110 to 250 MeV

Beneke, Marquard, Nason, Steinhauser (2016)
Hoang, C. Lepenik, and M. Preisser (2017)

This is of the order (or below) the accuracy that can be reasonably achieved in the top-mass measurement at the LHC

# The $\overline{\text{MS}}$ mass

The $\overline{\text{MS}}$ mass depends on arbitrary renormalization scale $\mu_m$ (similarly to the QCD coupling $\alpha_S(\mu_R)$) and such scale dependence is perturbatively computable

$$\frac{d \ln m_t(\mu_m)}{d \ln \mu_m^2} = - \sum_{k=0}^{\infty} \gamma_k \left( \frac{\alpha_S(\mu_m)}{\pi} \right)^{k+1}$$

Note: scale dependence of $\overline{\text{MS}}$ mass much slower than $\alpha_S$

$$\frac{d \ln m_t(\mu)}{d \ln \mu} \sim \frac{1}{2} \frac{d \ln \alpha_S(\mu)}{d \ln \mu} \qquad \text{at LO}$$

The $\overline{\text{MS}}$ mass can be specified by fixing a reference scale + RG evolution

Customary scale $\bar{m}_t$ defined such that $m_t(\bar{m}_t) = \bar{m}_t$ (no special meaning !)

Typical values: $M_t = 173$ GeV $\longleftrightarrow$ $\bar{m}_t = 164$ GeV ($\sim 10$ GeV difference)

[ Note: at scale $\mu_m = \bar{m}_t/2$ ➡ $m_t(\mu_m) = M_t + \mathcal{O}(1 \text{ GeV})$ , simply because at this scale $d^{(1)} \sim 0$ ]

Two main consequences of scale dependence of $\overline{\text{MS}}$ mass

- perturbative QCD predictions unavoidably depend on $\mu_m$ (in addition to renormalization scale $\mu_R$ from $\alpha_S(\mu_R)$ and factorization scale $\mu_F$ from PDFs)
- $\mu_m$ can possibly be set to a scale very different from $M_t \sim \bar{m}_t$ to embody ("resum") higher-order corrections ➡ running mass effects

# From pole to $\overline{\text{MS}}$ predictions

Start from on-shell cross section $\sigma(M_t, X)$ (total or differential) with pole mass $M_t$

e.g. up to NNLO $$\sigma_{\text{NNLO}}(\alpha_S(\mu_R), \mu_R, \mu_F; M_t; X) = \sum_{i=0}^{2} \left(\frac{\alpha_S(\mu_R)}{\pi}\right)^{i+2} \sigma^{(i)}(M_t; \mu_R, \mu_F; X)$$

Perform all-order replacement $M_t \to m_t(\mu_m)$ and define $\overline{\text{MS}}$ scheme cross section through the all-order equality

$$\bar{\sigma}(\alpha_S(\mu_R), \mu_R, \mu_F; \mu_m, m_t(\mu_m); X) = \sigma(\alpha_S(\mu_R), \mu_R, \mu_F; M_t = m_t(\mu_m)\, d(m_t(\mu_m), \mu_m); X)$$

$\overline{\text{MS}}$ scheme

Pole scheme

Expand in $\alpha_S(\mu_R)$ (e.g. up to NNLO) at fixed $m_t(\mu_R)$:

$$\bar{\sigma}_{\text{NNLO}}(\alpha_S(\mu_R), \mu_R, \mu_F; \mu_m, m_t(\mu_m); X) = \sum_{i=0}^{2} \left(\frac{\alpha_S(\mu_R)}{\pi}\right)^{i+2} \bar{\sigma}^{(i)}(m_t(\mu_m); \mu_m, \mu_R, \mu_F; X)$$

**within this formulation, pole scheme and $\overline{\text{MS}}$ scheme results are formally equivalent to all orders in $\alpha_S$ but different if expanded at fixed orders**

# Explicit expressions up to NNLO

Langenfeld, Moch, Uwer (2009)
Dowling, Moch (2014)

$$\bar{\sigma}^{(0)}(m_t(\mu_m);\mu_F;X) = \left[\sigma^{(0)}(m;\mu_F;X)\right]_{m=m_t(\mu_m)}$$

At NLO: 1st derivative of the LO

$$\bar{\sigma}^{(1)}(m_t(\mu_m);\mu_m,\mu_R,\mu_F;X) = \left[\sigma^{(1)}(m;\mu_R,\mu_F;X) + d^{(1)}(\mu_m)\, m\, \partial_m \sigma^{(0)}(m;\mu_F;X)\right]_{m=m_t(\mu_m)}$$

At NNLO: 1st derivative of the NLO and 2nd derivative of the LO

$$\begin{aligned}\bar{\sigma}^{(2)}(m_t(\mu_m);\mu_m,\mu_R,\mu_F;X) = \Bigg[&\sigma^{(2)}(m;\mu_R,\mu_F;X) \\ &+ m\Bigg(d^{(1)}(\mu_m)\,\partial_m\sigma^{(1)}(m;\mu_R,\mu_F;X) + \frac{1}{2}\left(d^{(1)}(\mu_m)\right)^2 m\,\partial_m^2\sigma^{(0)}(m;\mu_F;X) \\ &+ d^{(2)}(\mu_m)\,\partial_m\sigma^{(0)}(m;\mu_F;X) + \beta_0\, d^{(1)}(\mu_m)\ln\left(\frac{\mu_R^2}{\mu_m^2}\right)\partial_m\sigma^{(0)}(m;\mu_F;X)\Bigg)\Bigg]_{m=m_t(\mu_m)}\end{aligned}$$

The results depend on renormalization coefficients $d^{(k)}$, perturbative terms $\sigma^{(k)}$ of on-shell cross section and their mass derivatives $\partial_m^n \sigma^{(k)}$

Note that: the mass derivatives can be very sizeable and spoil the perturbative convergence of the $\overline{\text{MS}}$ cross section $\bar{\sigma}$ (see e.g. the invariant mass of $t\bar{t}$ pair close to its threshold region )

- **General expectations**

at low orders, $\sigma$ and $\bar{\sigma}$ can give consistent (within scale uncertainties) results (differences can be larger for observables close to kinematical thresholds for $t\bar{t}$ on-shell production)

at higher orders, $\sigma$ and $\bar{\sigma}$ can be quantitatively very similar

→ equivalent perturbative description

- **Setup**

Our results depend on 3 auxiliary scales $\mu_i = \{\mu_R, \mu_F, \mu_m\}$ independently varied by a factor of two around central $\mu_0$ :

$\mu_i = \xi_i \mu_0$, $\xi_i = \{1/2, 1, 2\}$ with constraints $\mu_i/\mu_j \leq 2$

→ 15-point scale variation in $\overline{\rm MS}$ scheme
( customary 7-point in pole scheme with 2 auxiliary scales )

We compare pole scheme and $\overline{\rm MS}$ scheme by setting

- pole scheme: $M_t = 173.3$ GeV and use $\mu_0 = M_t$

- $\overline{\rm MS}$ scheme: $\overline{m}_t = 163.7$ GeV (mass evolution at NNLO) and use $\mu_0 = \overline{m}_t$
(varying $\mu_m$ with $0.5 < \mu_m/\mu_0 < 2 \longrightarrow 155 \text{ GeV} < m_t(\mu_m) < 173 \text{ GeV}$ )

We use NNPDF31 and $\sqrt{s} = 13$ TeV

# Results: total cross section

| scheme | pole | $\overline{\text{MS}}$ | | | |
|---|---|---|---|---|---|
| variation | 7-point | 15-point | $\mu_m = \mu_0$ | $\mu_{R/F} = \mu_0$ | $\mu_{R/F} = \mu_m$ |
| LO (pb) | $478.9\ ^{+29.6\%}_{-21.4\%}$ | $625.7\ ^{+29.4\%}_{-21.9\%}$ | $^{+29.4\%}_{-21.3\%}$ | $^{+24.7\%}_{-21.9\%}$ | $^{+1.5\%}_{-1.5\%}$ |
| NLO (pb) | $726.9\ ^{+11.7\%}_{-11.9\%}$ | $826.4\ ^{+7.6\%}_{-9.7\%}$ | $^{+7.6\%}_{-9.6\%}$ | $^{+5.6\%}_{-9.7\%}$ | $^{+1.2\%}_{-1.2\%}$ |
| NNLO (pb) | $794.0\ ^{+3.5\%}_{-5.7\%}$ | $833.8\ ^{+0.5\%}_{-3.1\%}$ | $^{+0.4\%}_{-2.9\%}$ | $^{+0.3\%}_{-3.1\%}$ | $^{+0.0\%}_{-0.3\%}$ |

- order-by-order consistency of the results and very similar at NNLO
- $\overline{\text{MS}}$ typically higher at central scale and with smaller uncertainties at (N)NLO
  [ $\mu_R$ and $\mu_m$ dependences have similar size but opposite sign (cancellations) ]
- $\overline{\text{MS}}$ results have faster apparent convergence

$$\frac{\text{NLO}}{\text{LO}} = 1.52 \text{ (pole)},\ 1.32\ (\overline{\text{MS}}) \qquad \frac{\text{NNLO}}{\text{NLO}} = 1.09 \text{ (pole)},\ 1.01\ (\overline{\text{MS}})$$

first noticed by Langenfeld, Moch, Uwer (2009)

- Technical explanation: at LO the $\overline{\text{MS}}$ cross section is obtained by evaluating the pole cross section with $\bar{m}_t = 163.7$ GeV and is thus much larger than the pole cross section; at NLO there is a further negative effect from $\partial_m \sigma^{(0)}$

# Results: total cross section

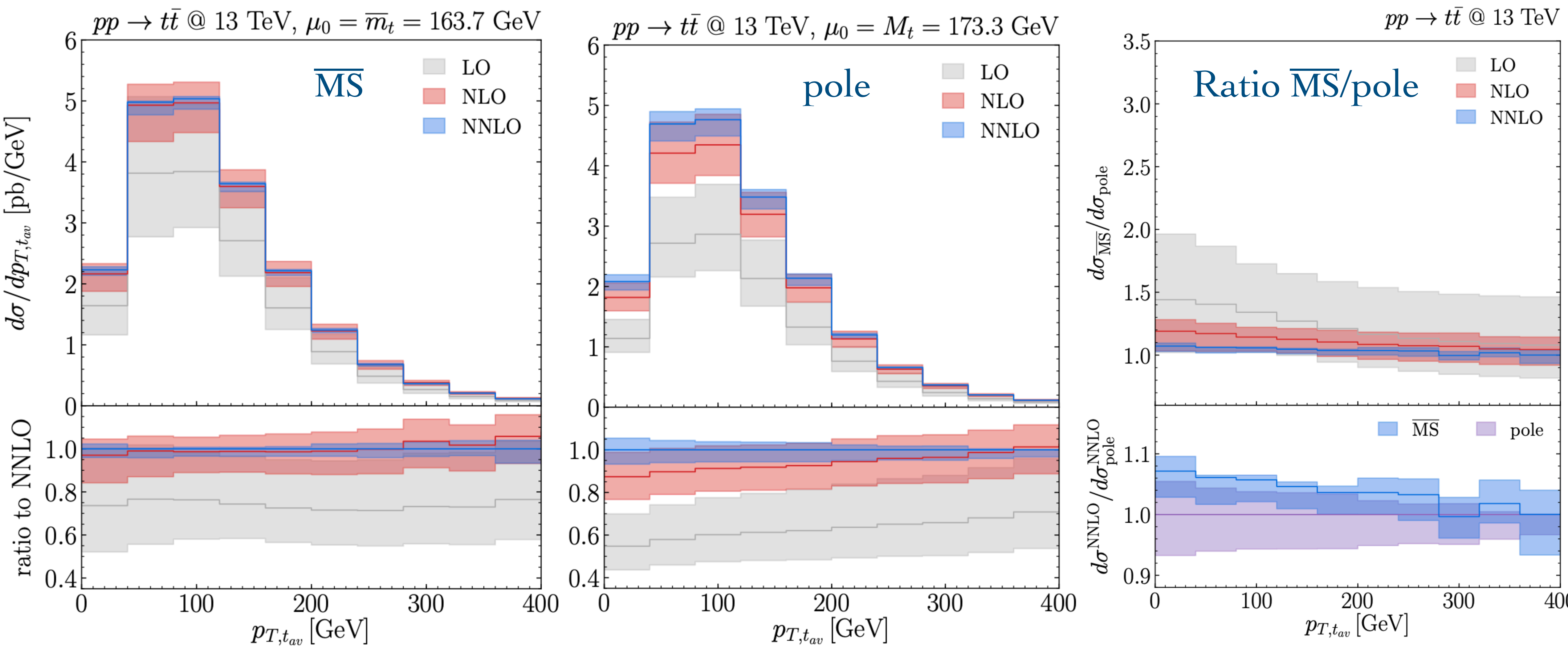
| scheme | pole | $\overline{\text{MS}}$ | $\overline{\text{MS}}$ | pole |
|---|---|---|---|---|
| central scale choice | $\mu_{R/F} = M_t$ | $\mu_{R/F} = \overline{m}_t$<br>$\mu_m = \overline{m}_t/2$ | $\mu_{R/F} = \overline{m}_t$<br>$\mu_m = \overline{m}_t$ | $\mu_{R/F} = M_t/2$ |
| LO (pb) | 478.9 | 488.9 | 625.7 | 619.8 |
| NLO (pb) | 726.9 | 746.4 | 826.4 | 811.4 |
| NNLO (pb) | 794.0 | 808.0 | 833.8 | 822.4 |

Such apparent convergence strongly depends on the choice of the central scale $\mu_0$

- Slower: $\overline{\text{MS}}$ scheme ( $\mu_{0,m} = \overline{m}_t/2$) and pole scheme ( $\mu_0 = M_t$ ) behave similarly

- Faster: $\overline{\text{MS}}$ scheme ( $\mu_{0,m} = \overline{m}_t$) and pole scheme ( $\mu_0 = M_t/2$ ) behave similarly

# Results: differential distributions



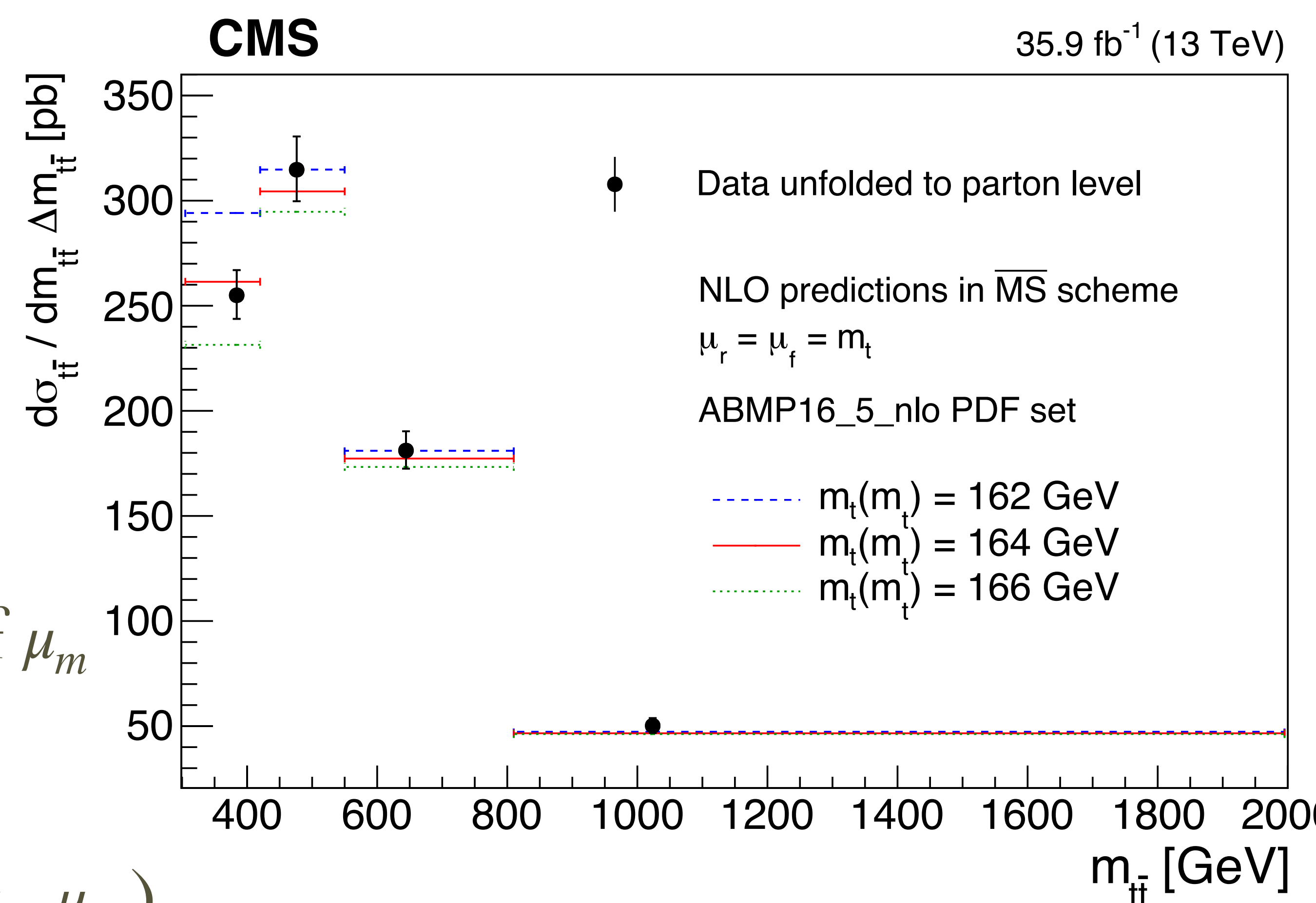

comparison pole scheme ( $\mu_0 = M_t$ ) vs. $\overline{\text{MS}}$ scheme ( $\mu_0 = \overline{m}_t$ )

- overall features similar to those for total cross sections: at NNLO shape differences are quite small and within scale uncertainties

→ the results in the two schemes behave similarly at (sufficiently) high order

# The invariant-mass distribution

CMS measured the $m_{t\bar{t}}$ distribution and compare their data with an NLO calculation with fixed $\overline{\mathrm{MS}}$ mass $\bar{m}_t$ (i.e. $\mu_m = \bar{m}_t$ in all bins) and fit value of $\bar{m}_t$ to data in each bin*




Studying running-mass effects requires using a running (bin-dependent) value of $\mu_m$

Two different options for central scale $\mu_0$:

- FIXED mass : set $\mu_0 = \bar{m}_t$ ( for $\mu_m, \mu_R, \mu_F$ )
  [ NNLO extension of CMS NLO calculation ]
- RUNNING mass : set $\mu_0 \simeq m_{t\bar{t}}/2$ ( for $\mu_m, \mu_R, \mu_F$ )
  ( i.e. $m_t(m_{t\bar{t}}/2)$ is bin-dependent and it varies by about 10 GeV :
  from $m_t \sim 160$ GeV in 1-st. bin $\rightarrow$ to $m_t \sim 150$ GeV in 4-th. bin )

Setup: ABMP16 PDFs (as done by CMS) and $\bar{m}_t = 161.6$ GeV as extracted at NNLO by CMS from the same data with the same PDFs

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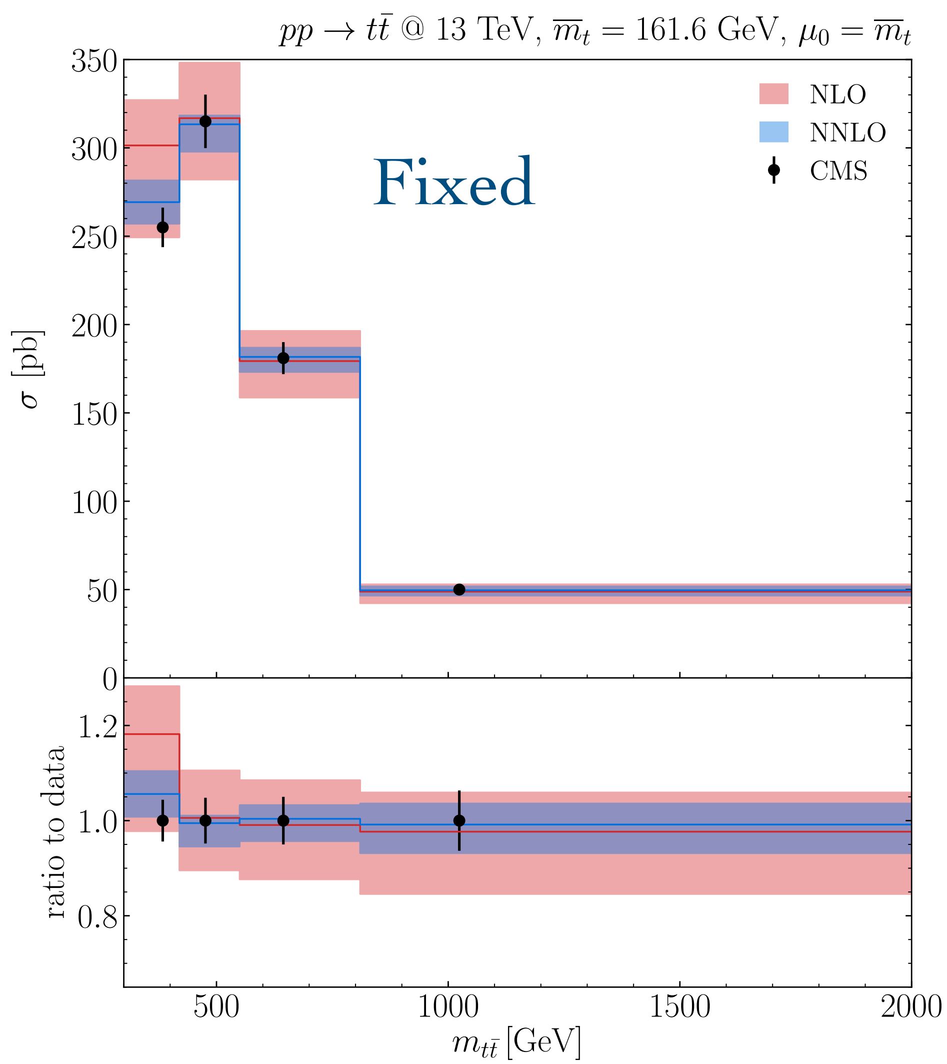
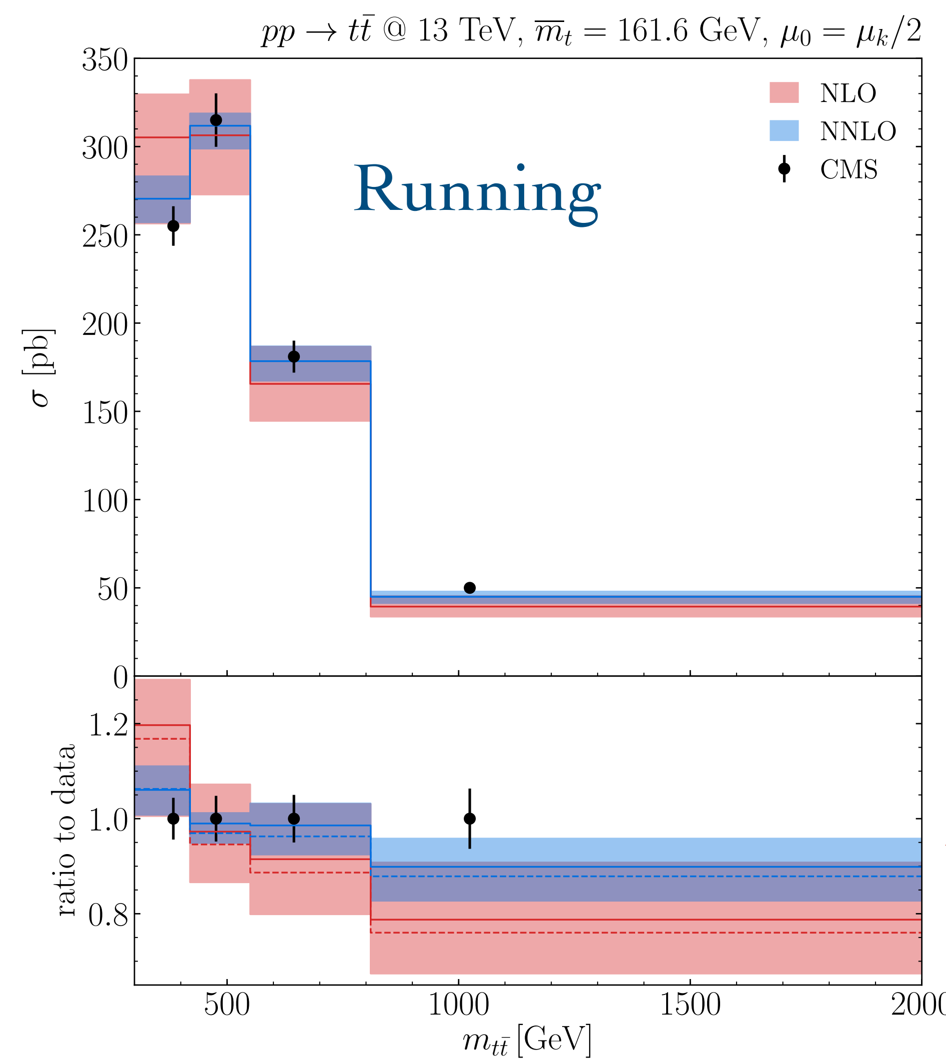
*Updated for Top2020 workshop but still neglecting scale uncertainties

# Running mass effects




$\mu_0 \simeq m_{t\bar{t}}/2$

dashed lines: results with $\mu_R = \mu_F \simeq m_{t\bar{t}}/2$ but keeping $\mu_m = \bar{m}_t$

- practically ("by definition") no theory differences at low $m_{t\bar{t}}$
- differences at high $m_{t\bar{t}}$ are small and mainly driven by running of $\alpha_S$ and PDFs

**NNLO corrections lead to reduced theoretical uncertainties and to an improved agreement with data but no significant sensitivity to running mass effects**

Note: very high invariant masses $m_{t\bar{t}} \gg M_t$ a resummation of soft and collinear effects would be needed

Ahrens et al (2010; Ferroglia et al (2012); Czakon et al (2018)

# Summary

- I have quickly reviewed the status of theoretical predictions for top-quark production at hadron colliders

- I have presented a new computation of heavy-quark production at NNLO

- NNLO results for the inclusive cross section and multi differential distributions: excellent agreement with Top++ and with independent results by Czakon and collaborators

  → Absolutely non-trivial check given that the computations are carried out with two completely independent methods

- The numerical program is available to ATLAS and CMS and we plan to implement it in the next MATRIX release

- Theoretical tools are continuously improving to match the precision of the experimental data

# Summary

- The top mass is a fundamental parameter in the SM: when the precision will approach $\mathcal{O}(\Lambda_{QCD})$ top mass measurements at hadron colliders pose deep theoretical issues on our understanding of QCD in its non-perturbative regime

- We have extended our NNLO computation to the $\overline{\rm MS}$ scheme for the top mass: this is obtained from a formal reorganisation of the perturbative expansion

- Perturbative predictions in such scheme depend on three scales: we have used a 15-point scale variations to assess perturbative uncertainties

- The $\overline{\rm MS}$ results show an apparent faster convergence with respect to the results in the pole scheme: this however strongly depends on the central scale choice

- Shape differences between the pole and $\overline{\rm MS}$ scheme results are reduced by the inclusion of the high-order contributions, and they are quite small at NNLO

- First study of running mass effects ( $m_t(\mu_m)$ with $\mu_m \sim m_{t\bar{t}}/2$ ) for the invariant-mass distribution of $t\bar{t}$ pair in region up to $m_{t\bar{t}} \sim 1$ TeV

  → No significant sensitivity to running mass effects