Soft-gluon effective coupling

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LFC21

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FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS

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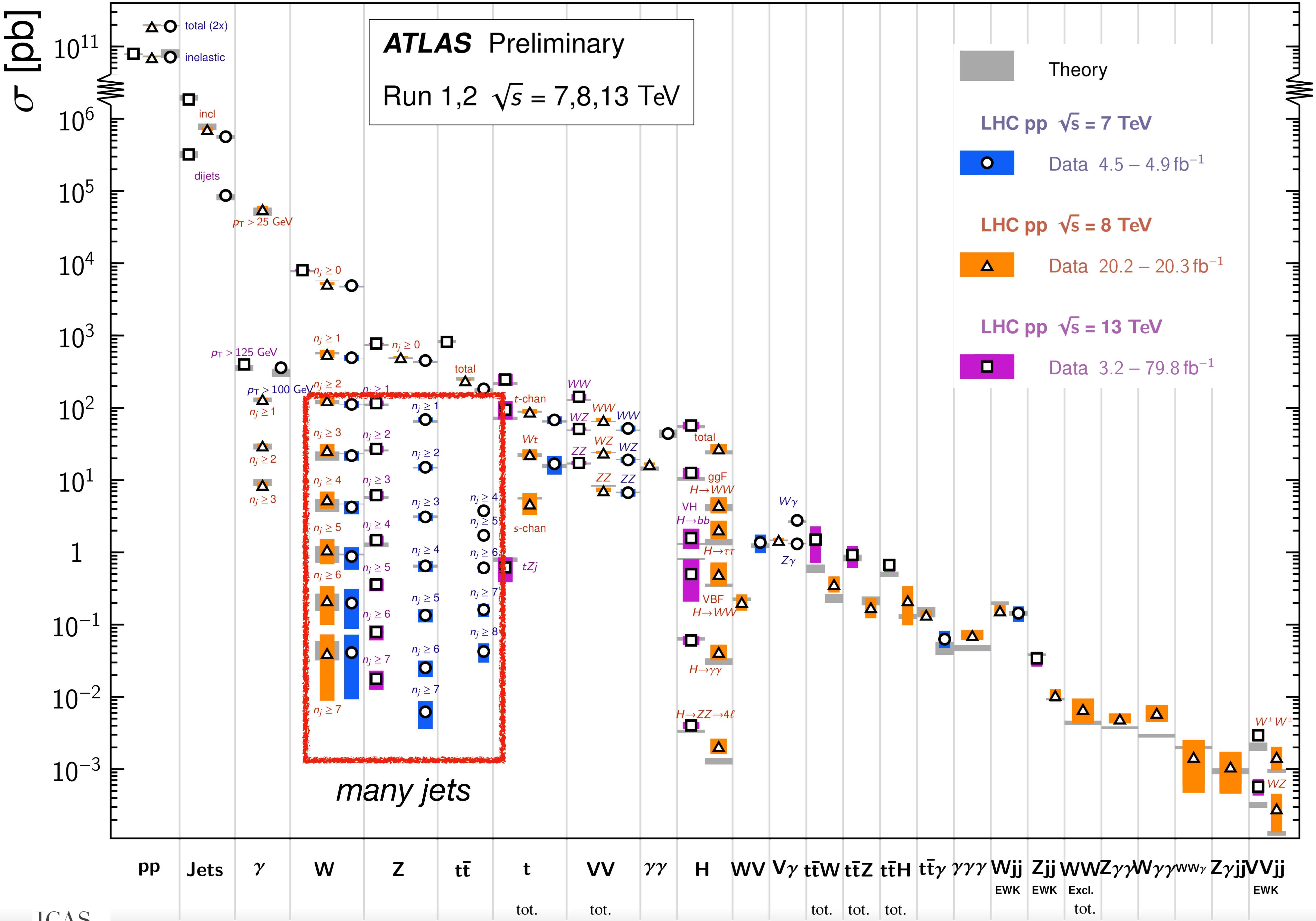
Outline

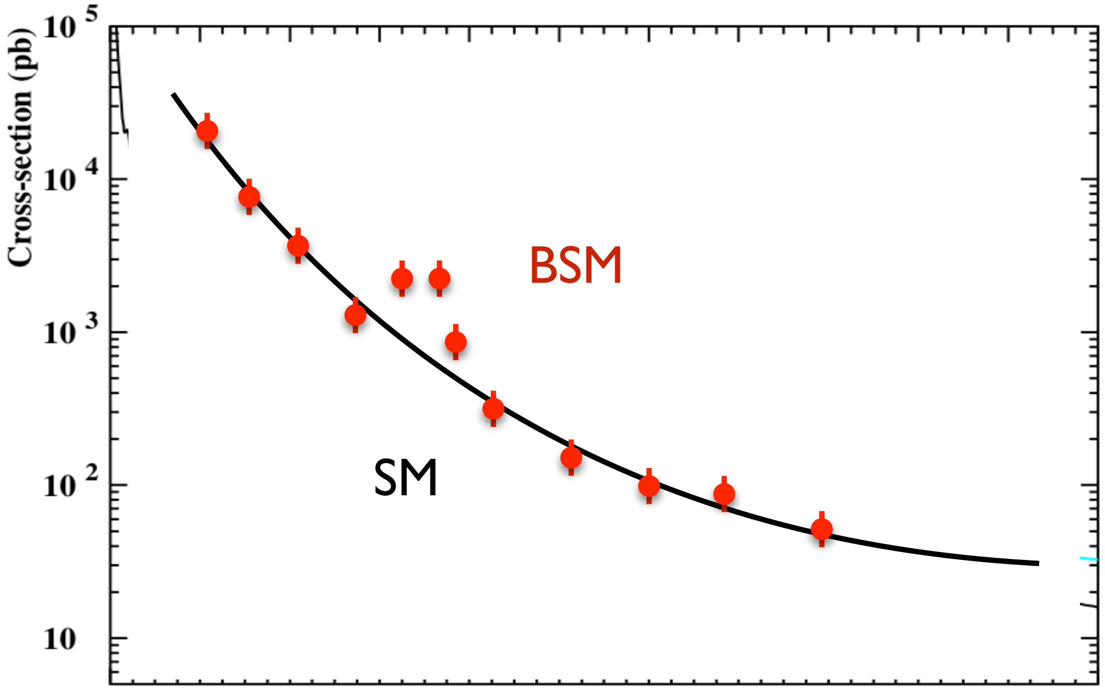
- Intro pQCD
- Resummation
- Soft-gluon effective coupling
- NLO, NNLO and beyond
- Conformal relation (all orders)
- Conclusions

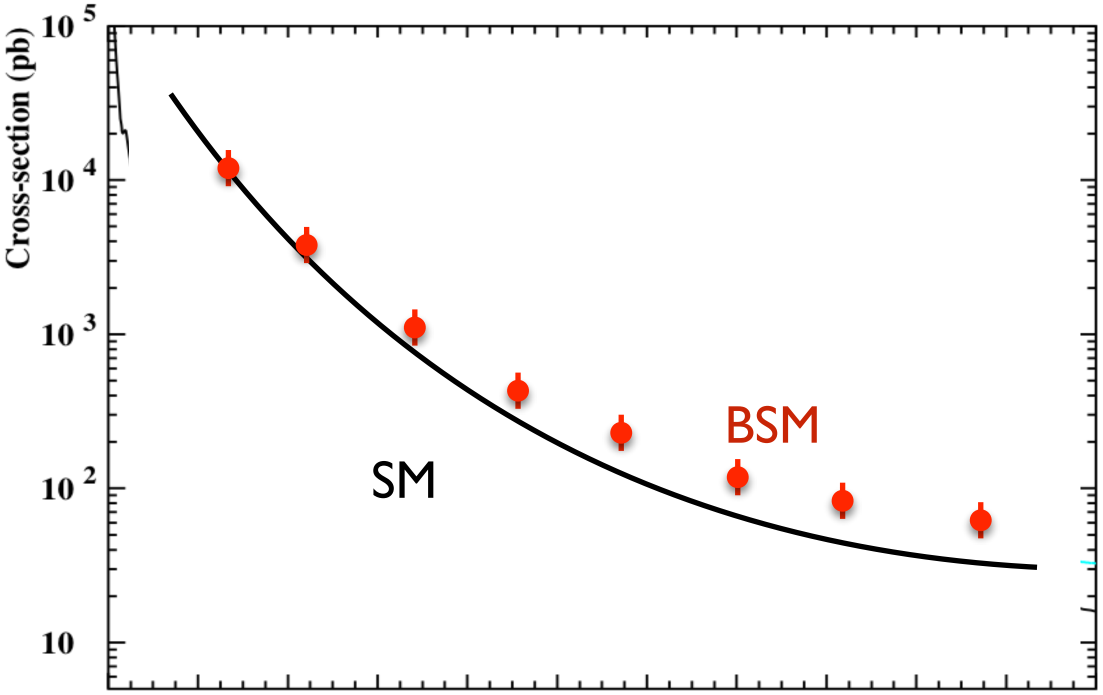
Standard Model Production Cross Section Measurements

Status: July 2018

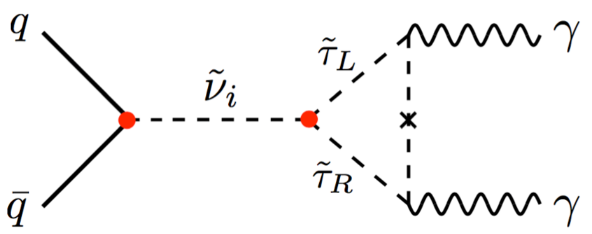


There should be Physics BSM: search is DRIVEN BY EXPERIMENTS now

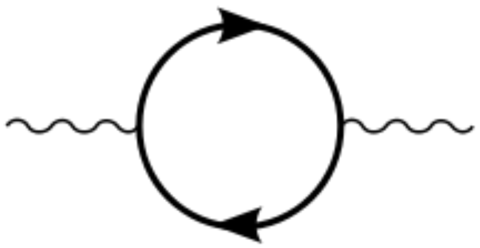




Search for
new *states*
Resonances
“Descriptive TH”

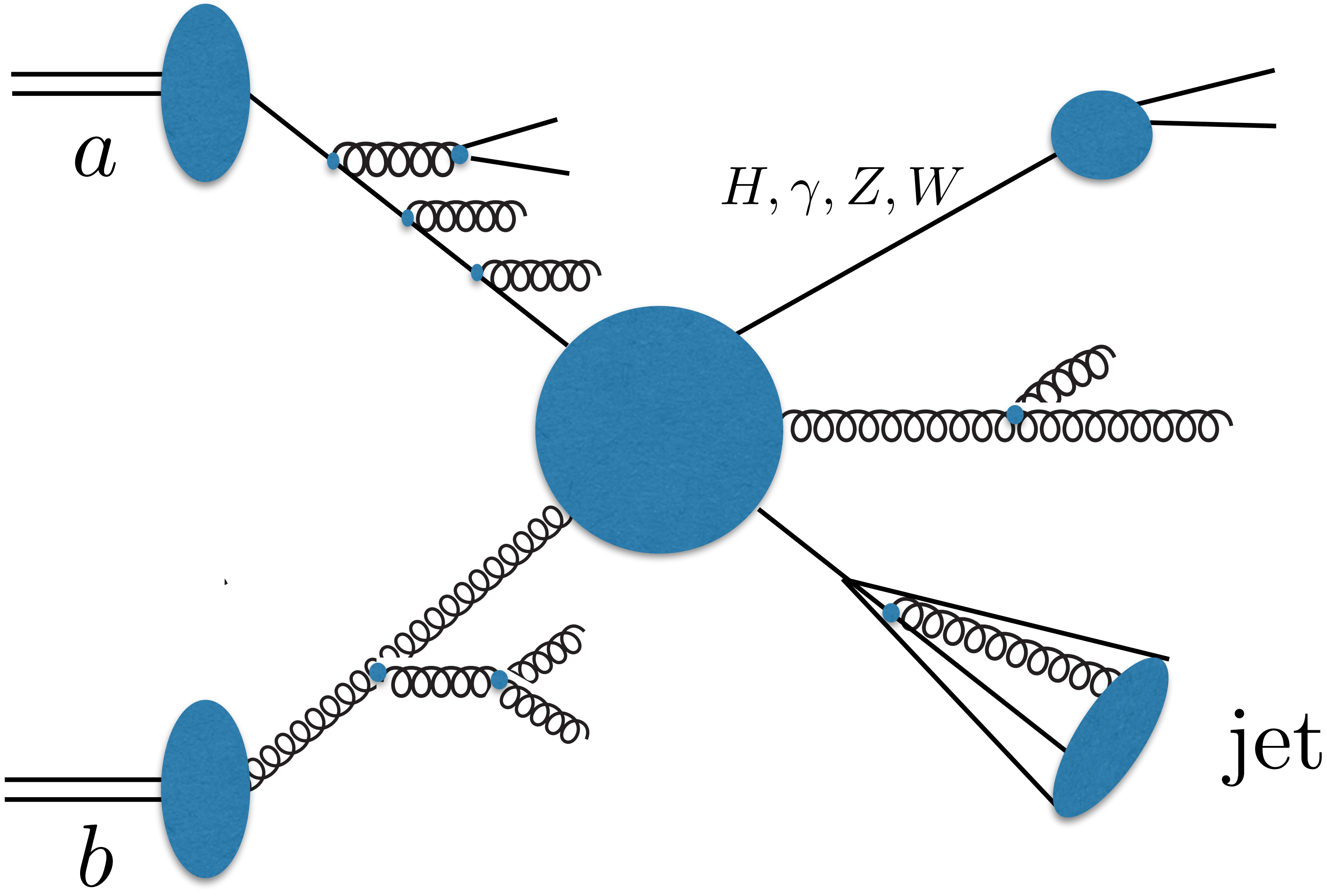


Search for new
interactions
Deviations from TH
“Precision TH”



- Need for precision ~ 1% EXP-TH accuracy (HIGGS very relevant)

- In the LHC era, QCD is everywhere!



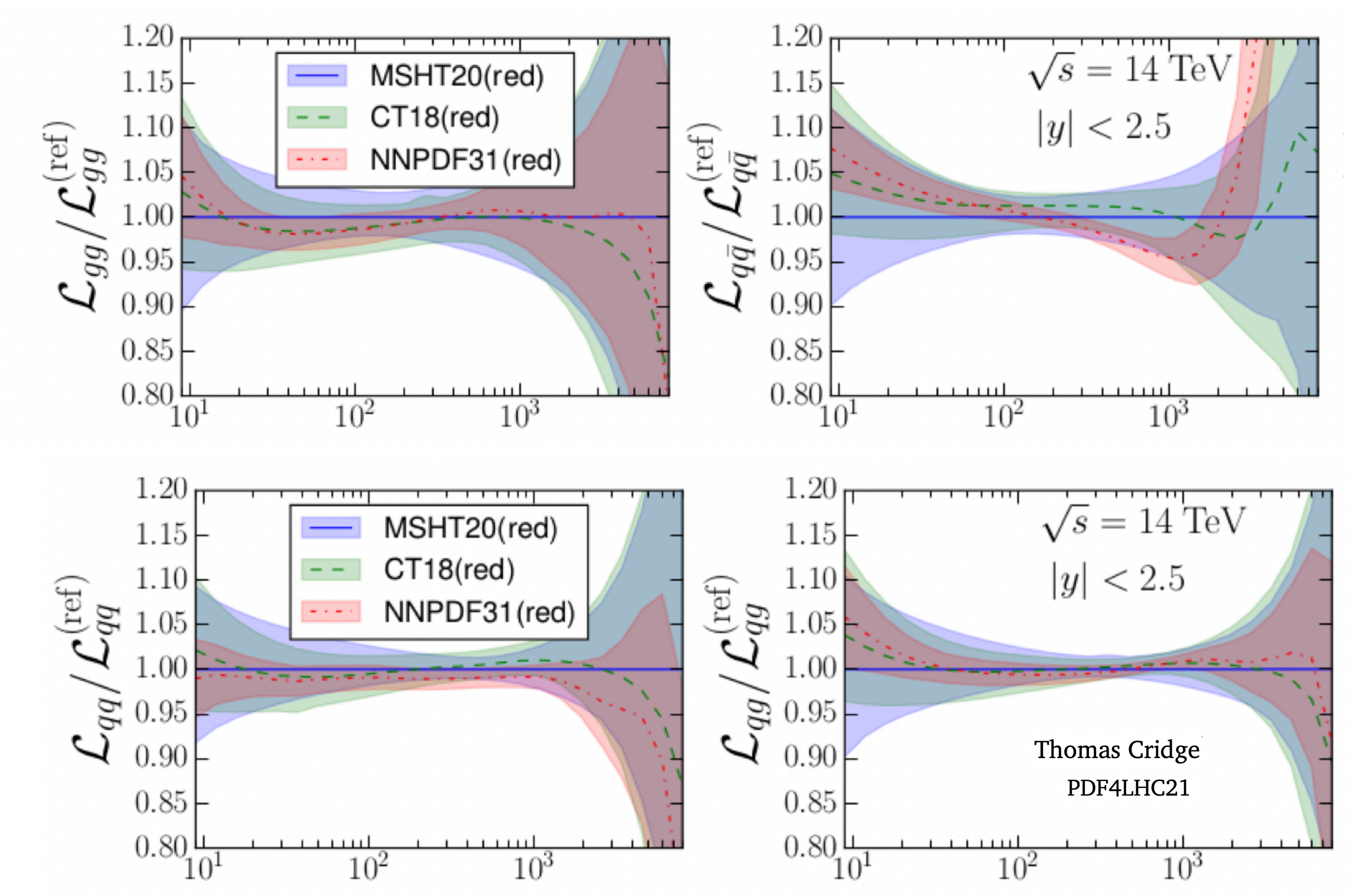
non-perturbative parton distributions

$$d\sigma = \sum_{ab} \int dx_a \int dx_b \, f_a(x_a, \mu_F^2) f_b(x_b, \mu_F^2) \times d\hat{\sigma}_{ab}(x_a, x_b, Q^2, \alpha_s(\mu_R^2)) \boxed{+\mathcal{O}\left(\left(\frac{\Lambda}{Q}\right)^m\right)}$$

perturbative partonic cross-section

- Require precision for perturbative and non-perturbative contribution

PDF4LHC21



Thomas Cridge
PDF4LHC21

central values and uncertainties ~ at the few percent level

The perturbative toolkit for precision at colliders

EW

Automation

LO, NLO

NNLO, N^{3}LO…

PS

Resummation

Resummation

- QCD based on convergence of perturbative expansion

$$\sigma = \mathcal{C}_0 + \alpha_s \mathcal{C}_1 + \alpha_s^2 \mathcal{C}_2 + \alpha_s^3 \mathcal{C}_3 + \ldots$$

requires $\alpha_s \ll 1 \quad , \quad \mathcal{C}_n \sim \mathcal{O}(1)$

In the boundaries of phase space → soft and collinear emission

unbalance cancellation of infrared singularities
between real and virtual contributions

- Convergence spoiled when two scales are very different

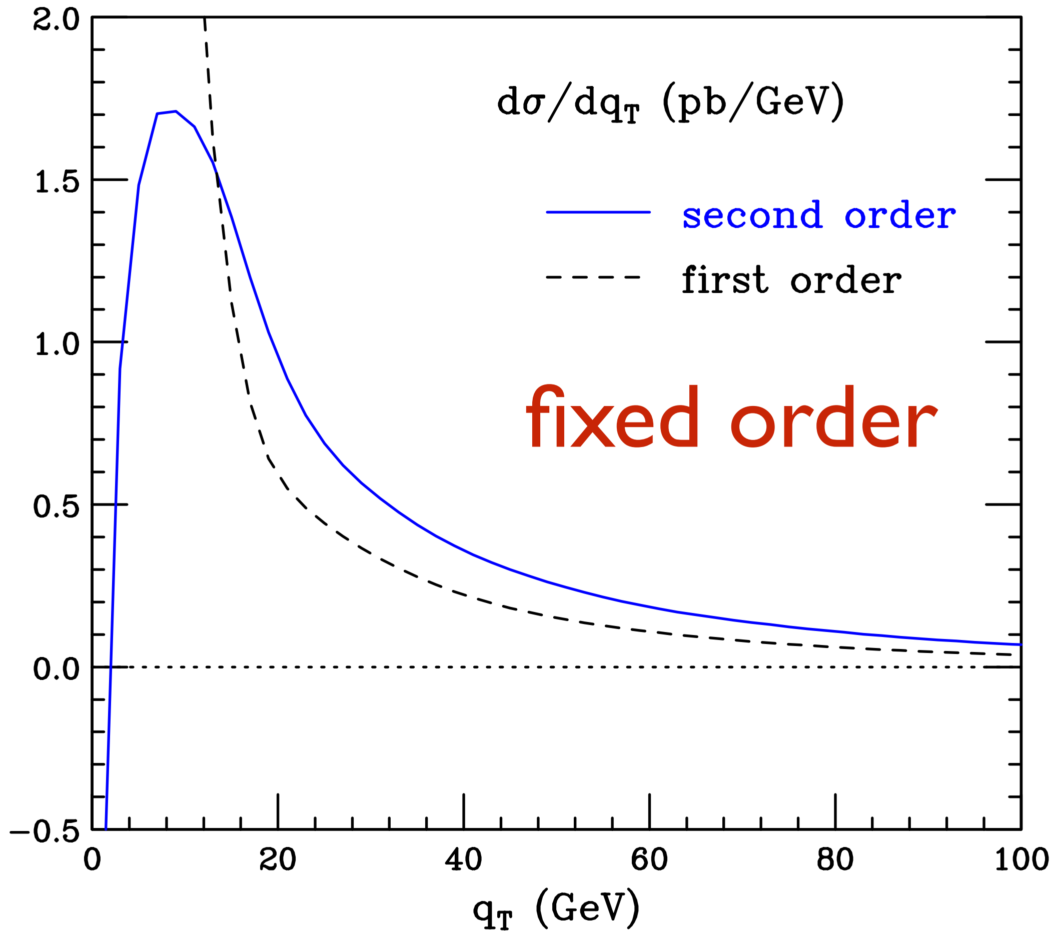
$$L = |\log \frac{E_1}{E_2}| \gg 1 \qquad \mathcal{C}_m \sim L^n \quad n \sim 2m$$

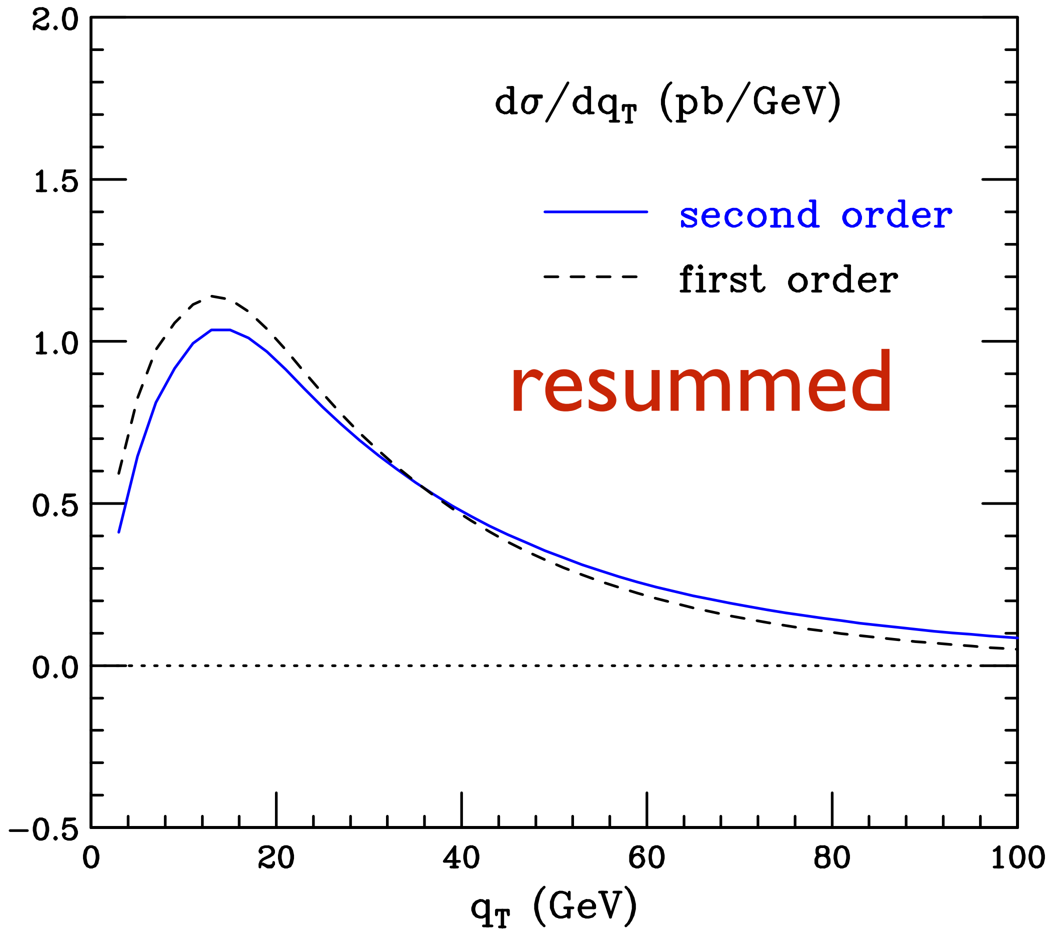
low transverse momentum	$\log \frac{q_T}{Q}$	DY, Higgs
threshold	$\log \left(1 - \frac{Q^2}{\hat{s}}\right)$	Higgs, HQ
high energy	$\log x$	DIS BFKL

- Need to be resumed to some logarithmic accuracy to improve convergence of perturbative expansion

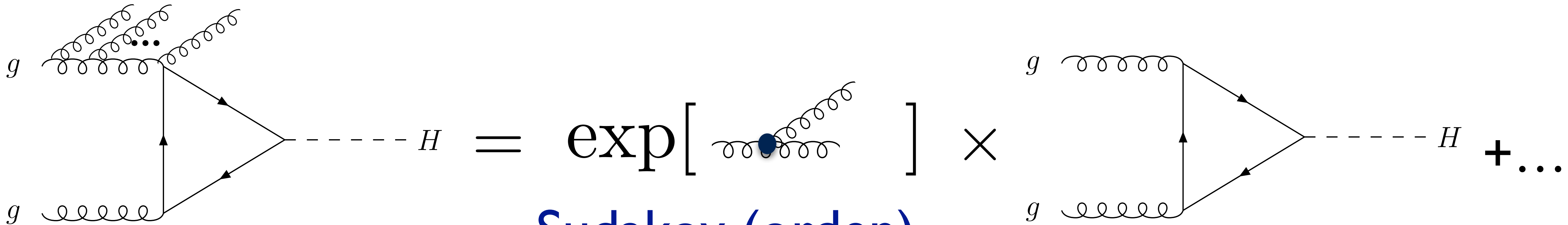
Higgs transverse momentum

$$\alpha_s^n \log^{2n} \frac{q_T}{M_H}$$





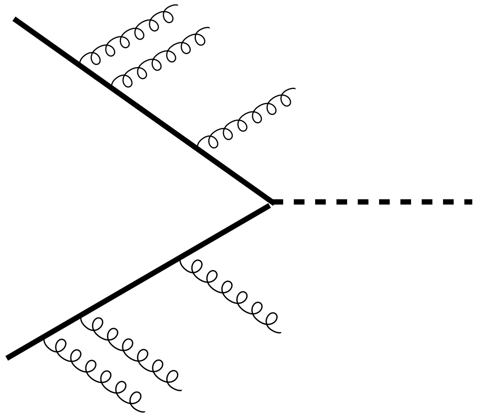
- Sum large log. corrections to all orders: restore convergence



Sudakov (orden)

‣ Threshold resummation (invariant mass M)

$c\bar{c} \to F$



DY $q\bar{q}$

Higgs gg

$$\log\left(1-\frac{M^2}{\hat{s}}\right) \equiv \log(1-z)$$

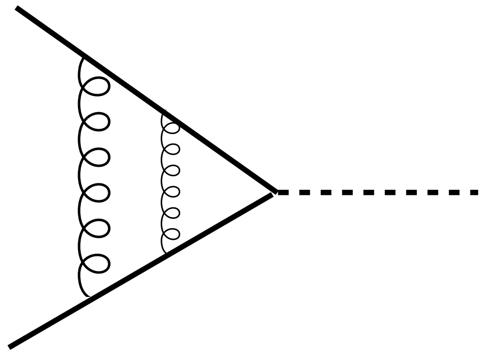
- resummed partonic cross-section in Mellin space (N)

$$\int_0^1 dz\; z^{N-1} \quad ➡ \quad \log N$$

$$\hat{\sigma}^{F(\text{res})}_{i\bar{i},N}\left(M^2;\alpha_S\left(M^2\right)\right) = \sigma^{(0)}_{i\bar{i}\to F}\left(M^2;\alpha_S\left(M^2\right)\right) C^{\text{th}}_{i\bar{i}\to F}\left(\alpha_S\left(M^2\right)\right) \Delta_{i,N}\left(M^2\right)$$

‣ Hard factor (Process dependent and free of logs)

$$C_{i\to F}\left(\alpha_S\right) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n C^{(n)}_{i\to F}$$



- from virtual corrections and non-log soft

- Sudakov form factor (all Logs)

$$\Delta_{i,N}\left(M^2\right) = \exp\left\{\int_0^1 dz \frac{z^{N-1}-1}{1-z}\left[2\int_{\mu_F^2}^{(1-z)^2M^2}\frac{dq^2}{q^2}A_i\left(\alpha_S\left(q^2\right)\right)+D_i\left(\alpha_S\left((1-z)^2M^2\right)\right)\right]\right\}$$

- process independent (and free of logs)

$$A_i\left(\alpha_S\right) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n A_i^{(n)}$$ soft-collinear emission

$$D_i\left(\alpha_S\right) = \sum_{n=2}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n D_i^{(n)}$$ soft non-collinear radiation

$$P_{ii}\left(\alpha_S; z\right) = \frac{1}{1-z}A_i\left(\alpha_S\right) + \ldots$$

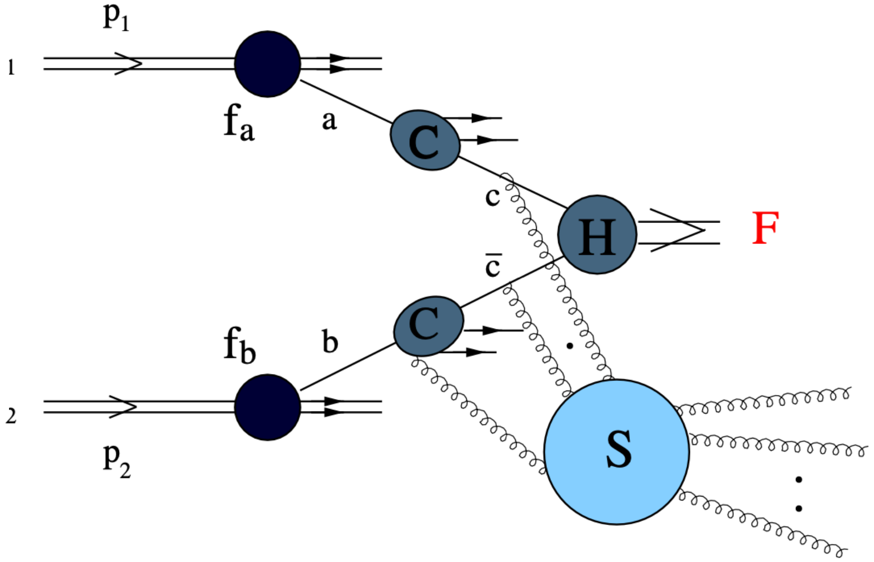
soft limit of
splitting function
cusp anomalous
dimension

factorization of
collinear divergences
Scheme dependent
$\overline{MS}$

q_T resummation

- Production of a system with large invariant mass Q and q_T

$$\frac{d\sigma}{dq_T^2 dQ^2 d\phi} = \sum_{a,b,c} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty db \frac{b}{2} J_0(bq_T) \frac{d\sigma^{(LO)}_{c\bar{c}}}{d\phi} H_c^F(\alpha_s(Q^2))$$
$$\delta(Q^2 - x_1 x_2 s) \; (f_{a/h_1} \otimes C_{ca}) \left(x_1, \frac{b_0^2}{b^2}\right) \; (f_{b/h_2} \otimes C_{\bar{c}b}) \left(x_2, \frac{b_0^2}{b^2}\right) S_c(Q,b)$$



- Impact parameter b space (Fourier)

$$S_c(Q,b) = \exp\left\{ -\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} \left[A_c\left(\alpha_s\left(q^2\right)\right) \ln\frac{Q^2}{q^2} + B_c\left(\alpha_s\left(q^2\right)\right) \right] \right\}$$

$$A_i\left(\alpha_S\right) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n A_i^{(n)}$$ soft-collinear emission

$$B_i\left(\alpha_S\right) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n B_i^{(n)}$$ collinear radiation (non-soft)

$$H_i^F\left(\alpha_S\right) = 1 + \sum_{n=2}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n H_i^{F(n)}$$ Hard contribution (process dependent)

- In both A drives soft-collinear emission
- And they are the same up to second order But different origin!

$$\Delta_{i,N}\left(M^2\right) = \exp\left\{\int_0^1 dz\frac{z^{N-1}-1}{1-z}\left[2\int_{\boxed{\mu_F^2}}^{(1-z)^2M^2}\frac{dq^2}{q^2}A_i\left(\alpha_S\left(q^2\right)\right)+D_i\left(\alpha_S\left((1-z)^2M^2\right)\right)\right]\right\}$$

$\overline{MS}$ factorization (subtraction) of collinear singularities : scheme dependent

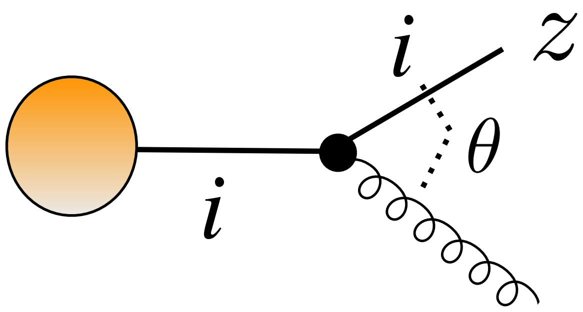
$$S_c(Q,b) = \exp\left\{-\int_{\boxed{b_0^2/b^2}}^{Q^2}\frac{dq^2}{q^2}\left[A_c\left(\alpha_s\left(q^2\right)\right)\ln\frac{Q^2}{q^2}+B_c\left(\alpha_s\left(q^2\right)\right)\right]\right\}$$

NO factorization of singularities : regularized by transverse momentum

- Equality up to $\mathcal{O}(\alpha_s^2)$ not accidental: up to this order cut-off on q_T^2 is essentially equivalent to $\overline{MS}$ factorization! Not true at higher orders

Soft-gluon effective coupling

- Another way to resum large logs (and more) → Monte Carlo/PS



collinear branching driven by splitting function

$$P_{ii}\left(\alpha_S;z\right)=\frac{1}{1-z}A_i\left(\alpha_S\right)+\ldots$$

soft and collinear emission (DL accuracy)

$$dw_i^{DL}=\frac{\alpha_S}{2\pi}P_{ii}(z)\,dz\frac{d\theta^2}{\theta^2}\simeq C_i\frac{\alpha_S}{\pi}\frac{dz}{1-z}\frac{dq_T^2}{q_T^2}$$

intensity of soft-gluon radiation

double logarithmic spectrum

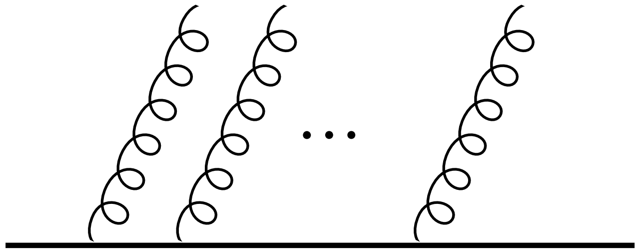
- Intensity of soft-gluon radiation at LO given by $C_i\frac{\alpha_S}{\pi}$ $\quad C_i=C_F(q),C_A(g)$

lowest order soft-gluon effective coupling

- The resummation of soft-collinear terms at LL achieved by

$$C_i\frac{\alpha_S}{\pi} \to C_i\frac{\alpha_S\left(q_T^2\right)}{\pi}$$

coupling evaluated at q_T (resum)



- The resummation of soft-collinear terms at NLL achieved by (MC@NLL)

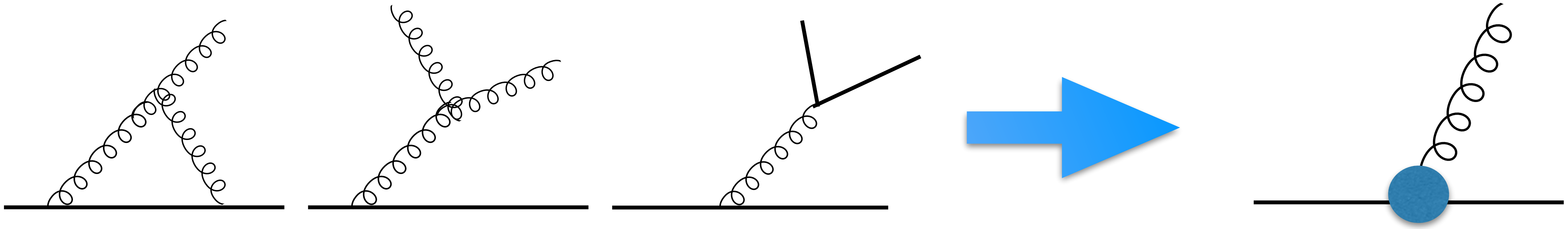
$$C_i\frac{\alpha_S}{\pi} \to \mathscr{A}_i^{CMW}\left(\alpha_S\left(q_T^2\right)\right) = C_i\frac{\alpha_S^{CMW}\left(q_T^2\right)}{\pi} = C_i\frac{\alpha_S\left(q_T^2\right)}{\pi}\left(1+\frac{\alpha_S\left(q_T^2\right)}{2\pi}K\right)$$

Catani, Marchesini, Webber (1991)

soft effective coupling at NLL

$$K=\left(\frac{67}{18}-\frac{\pi^2}{6}\right)C_A-\frac{5}{9}n_F$$

NLL



- Used in Monte Carlo parton shower (partial inclusion of NLL terms)
- Dispersive approach to power corrections
- Analytical resummation

- Up to 2-loops, the soft-gluon effective coupling is still given by the cusp anomalous dimension

$$A_i^{(1)} = C_i, \quad A_i^{(2)} = \frac{1}{2} C_i \left[C_A \left(\frac{67}{18} - \frac{1}{6}\pi^2 \right) - \frac{5}{9} N_f \right] \equiv \frac{1}{2} C_i K$$

- Higher orders of cusp known

$$A_i^{(3)} = C_i \left[\left(\frac{245}{96} - \frac{67}{216}\pi^2 + \frac{11}{720}\pi^4 + \frac{11}{24}\zeta_3 \right) C_A^2 + \left(-\frac{209}{432} + \frac{5}{108}\pi^2 - \frac{7}{12}\zeta_3 \right) C_A n_F + \left(-\frac{55}{96} + \frac{1}{2}\zeta_3 \right) C_F n_F - \frac{1}{108} n_F^2 \right]$$

Moch, Vermaseren, Vogt (2004)

$A_i^{(4)}$ known

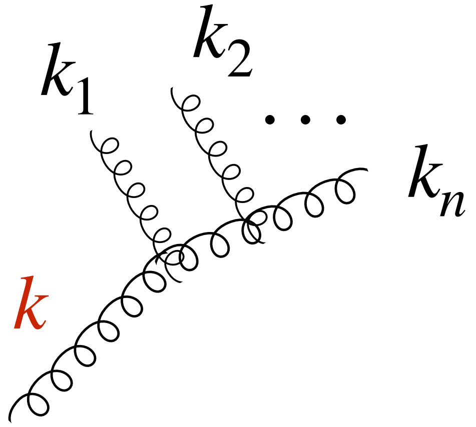
Moch, Ruijl, Ueda, Vermaseren, Vogt (2018)
Henn, Korchemsky, Mistlberger (2019)
von Manteuffel, Panzer, Schabinger (2020)

- But, cusp=soft coupling beyond 2-loops? Can no hold in general

  - Need all order definition for soft effective coupling

- all-order definition provided in terms of a *web* Banfi, El-Menoufi, Monni (2018)

- Probability of correlated emission of an arbitrary number of soft-collinear partons with total momentum k in $d = 4 - 2\epsilon$ dimensions



$$w\left(k;\epsilon\right) = \mathcal{N}\sum_{n=1}^{\infty} S(n)\int\left(\prod_{i=1}^{n}\left[dk_i\right]\right)\tilde{M}_s^2\left(k_1,\ldots,k_n\right)\delta^{(d)}\left(k - \sum_i k_i\right)$$

$$M_s^2(k_1) = \tilde{M}_s^2(k_1)$$

$$M_s^2(k_1,k_2) = \left[\tilde{M}_s^2(k_1)\tilde{M}_s^2(k_2)\right]_{\mathrm{sym}} + \tilde{M}_s^2(k_1,k_2)$$

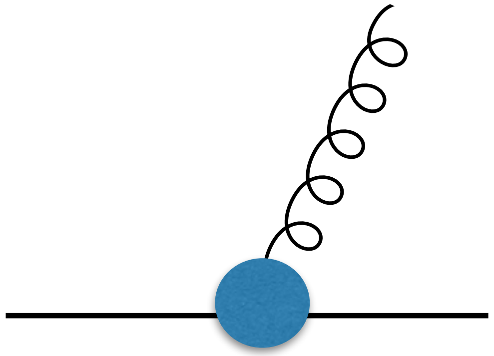
$$M_s^2(k_1,k_2,k_3) = \left[\tilde{M}_s^2(k_1)\tilde{M}_s^2(k_2)\tilde{M}_s^2(k_3)\right]_{\mathrm{sym}} + \left[\left(\tilde{M}_s^2(k_1)\tilde{M}_s^2(k_2,k_3)\right)_{\mathrm{sym}} + (1\leftrightarrow 2) + (1\leftrightarrow 3)\right] + \tilde{M}_s^2(k_1,k_2,k_3)$$

$$\tilde{M}_s^2(k_1,\ldots,k_n) = \tilde{M}_{s,0}^2(k_1,\ldots,k_n) + \frac{\alpha_s(\mu_R)}{2\pi}\tilde{M}_{s,1}^2(k_1,\ldots,k_n) + \ldots$$

The web is boost invariant: depends only on k_T and $m_T^2 = k_T^2 + k^2$

It is finite, can be computed in 4 dimensions, but better keep d

- At lowest order (and for the soft-effective coupling) gluon is on-shell $m_T^2 = k_T^2$ $(k^2 = 0)$
  - But in general k_T and $m_T^2 = k_T^2 + k^2$



- given the two variables, propose two definitions for soft-coupling

$$\widetilde{\mathscr{A}}_{T,i}\left(\alpha_S\left(\mu^2\right);\epsilon\right) = \frac{1}{2}\mu^2\int_0^\infty dm_T^2\, dk_T^2\, \delta\left(\mu^2 - k_T^2\right) w_i(k;\epsilon)$$

defined at fixed value of k_T

Banfi, El-Menoufi, Monni (2018)

suitable for q_T-related observables

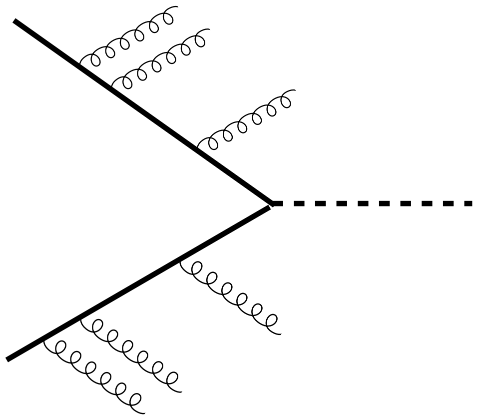
$$\widetilde{\mathscr{A}}_{0,i}\left(\alpha_S\left(\mu^2\right);\epsilon\right) = \frac{1}{2}\mu^2\int_0^\infty dm_T^2\, dk_T^2\, \delta\left(\mu^2 - m_T^2\right) w_i(k;\epsilon)$$

defined at fixed value of m_T

suitable for threshold-related observables

- can take limit $\epsilon \to 0$ to obtain the physical couplings
- But convenient to keep *D*-dimensional definition $\widetilde{\mathscr{A}}_i^{(n)}(\epsilon) = \mathscr{A}_i^{(n)} + \sum_{k=1}^{\infty} \epsilon^k \widetilde{\mathscr{A}}_i^{(n;k)}$

- How to compute it? Use soft factorization formula

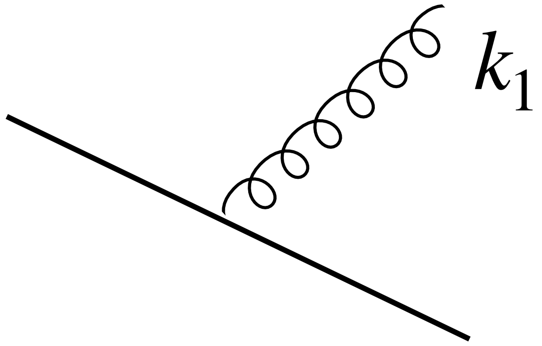


$$\left|\mathcal{M}_{i\bar{i}}\left(p_1,p_2;k_1,\ldots k_N\right)\right|^2 \simeq \left|\mathcal{M}_{i\bar{i}}\left(p_1,p_2\right)\right|^2 \left|J_i\left(k_1,\cdots,k_N\right)\right|^2$$

Born ME

squared soft-parton current

- Lowest order:



soft current $$\tilde{M}_s^2\left(k_1\right) = |J_i(k_1)|^2_{LO} = 8\pi^2 \frac{p_1\cdot p_2}{p_1\cdot k_1 p_2\cdot k_1} C_i \frac{\alpha_s^0\mu_0^{2\epsilon}}{\pi}$$

Web $$w_{i,LO}(k,\epsilon) = \frac{2}{m_T^2} C_i \frac{\alpha_s(\mu_R^2)}{\pi}\,\delta(m_T^2-k_T^2)\left(\frac{\mu_R^2}{k_T^2}\right)^{\epsilon} c(\epsilon)$$

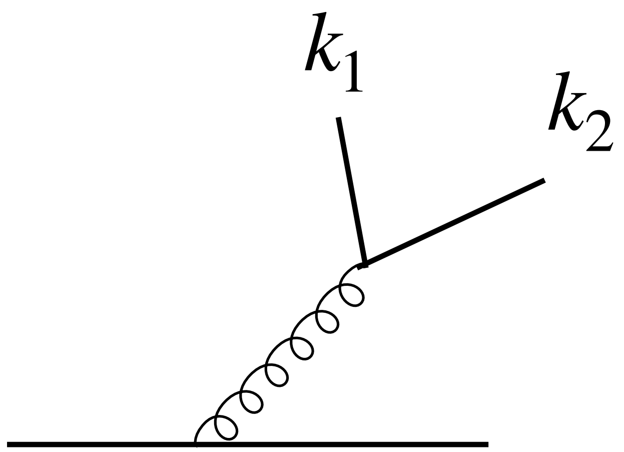
- at lowest order both couplings agree $w_i(k;\epsilon)\sim\delta\left(k^2\right)=\delta\left(m_T^2-k_T^2\right)$

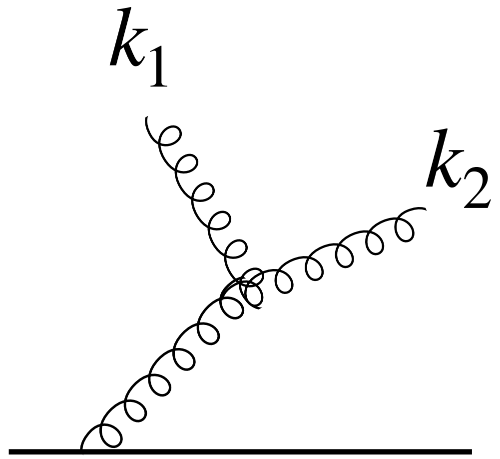
$$\widetilde{\mathcal{A}}^{(1)}_{T,i}(\epsilon) = \widetilde{\mathcal{A}}^{(1)}_{0,i}(\epsilon) = C_i c(\epsilon) \qquad c(\epsilon)\equiv \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} = 1-\frac{\pi^2}{12}\epsilon^2 - \frac{1}{3}\zeta_3\epsilon^3+\mathcal{O}\left(\epsilon^4\right)$$

no ϵ term

- we computed both couplings at α_s^2 (all orders in ϵ)

Real contributions



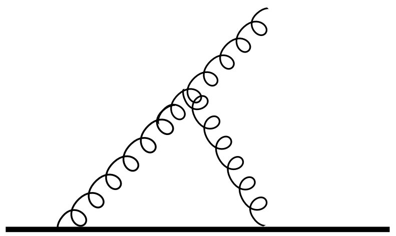


$$\mathcal{I}_{ij}(q_1,q_2) = \frac{(p_i\cdot q_1)\,(p_j\cdot q_2)+(p_j\cdot q_1)\,(p_i\cdot q_2)-(p_i\cdot p_j)\,(q_1\cdot q_2)}{(q_1\cdot q_2)^2\,[p_i\cdot(q_1+q_2)]\,[p_j\cdot(q_1+q_2)]}$$

$$\begin{aligned}
\mathcal{S}_{ij}(q_1,q_2) &= \frac{(1-\epsilon)}{(q_1\cdot q_2)^2}\,\frac{p_i\cdot q_1\,p_j\cdot\,q_2+p_i\cdot q_2\,p_j\cdot\,q_1}{p_i\cdot(q_1+q_2)\,p_j\cdot(q_1+q_2)} \\
&- \frac{(p_i\cdot p_j)^2}{2p_i\cdot q_1\;p_j\cdot q_2\;p_i\cdot q_2\;p_j\cdot q_1}\left[2-\frac{p_i\cdot q_1\,p_j\cdot\,q_2+p_i\cdot q_2\,p_j\cdot\,q_1}{p_i\cdot(q_1+q_2)\,p_j\cdot(q_1+q_2)}\right] \\
&+ \frac{p_i\cdot p_j}{2q_1\cdot q_2}\left[\frac{2}{p_i\cdot q_1\,p_j\cdot\,q_2}+\frac{2}{p_j\cdot q_1\,p_i\cdot\,q_2}\right. \\
&- \left.\frac{1}{p_i\cdot(q_1+q_2)\,p_j\cdot(q_1+q_2)}\left(4+\frac{(p_i\cdot q_1\,p_j\cdot\,q_2+p_i\cdot q_2\,p_j\cdot\,q_1)^2}{p_i\cdot q_1\;p_j\cdot q_2\;p_i\cdot q_2\;p_j\cdot q_1}\right)\right]
\end{aligned}$$

$\int dPS_2$ all poles from integration

Virtual contributions



$$\tilde{M}^2_{s,1}(k) = -\tilde{M}^2_{s,0}(k)C_A\,\frac{1}{\epsilon^2}\,\frac{\Gamma^4(1-\epsilon)\Gamma^3(1+\epsilon)}{\Gamma^2(1-2\epsilon)\Gamma(1+2\epsilon)}\left(\frac{4\pi\mu_R^2}{k_t^2}\right)^{\epsilon}$$

$\int dPS_1$ trivial, all poles from 1 loop soft-current

- we computed both couplings at α_s^2 (all orders in ϵ)
- web is the same, differences from "kinematic" integration

$$\widetilde{\mathcal{A}}^{(2)}_{T,i}(\epsilon) = C_i \left\{ -\frac{c(\epsilon)\,(11C_A - 2n_F)}{12\,\epsilon} + \frac{c(2\epsilon)\,\pi}{\sin(\pi\epsilon)} \frac{[C_A(11-7\epsilon) - 2\,n_F(1-\epsilon)]}{4(3-2\epsilon)(1-2\epsilon)} + \frac{C_A\,c(2\epsilon)\,h(\epsilon)\,\pi}{2\sin(\pi\epsilon)} - \frac{C_A\,c(2\epsilon)\,\pi^2}{2\sin^2(\pi\epsilon)} \left(\frac{2-\sin^2(\pi\epsilon)}{\cos(\pi\epsilon)} - \frac{2\sin(\pi\epsilon)}{\pi\epsilon} \right) \right\}$$

where $h(\epsilon) = \gamma_E + \psi(1-\epsilon) + 2\,\psi(1+2\epsilon) - 2\,\psi(1+\epsilon)$

$$\widetilde{\mathcal{A}}^{(2)}_{0,i}(\epsilon) = C_i \left\{ -\frac{c(\epsilon)\,(11C_A - 2n_F)}{12\,\epsilon} + \frac{c^2(2\epsilon)}{\epsilon\,c^2(\epsilon)} \frac{[C_A(11-7\epsilon) - 2\,n_F(1-\epsilon)]}{4(3-2\epsilon)(1-2\epsilon)} + \frac{C_A\,c^2(2\epsilon)\,r(\epsilon)}{2(1-2\epsilon)\,c^2(\epsilon)} - \frac{C_A\,c(2\epsilon)}{2\,\epsilon^2} \left(\frac{(\pi\epsilon)^2\cos(\pi\epsilon)}{\sin^2(\pi\epsilon)} + \frac{\pi\,\epsilon}{\sin(\pi\epsilon)} - \frac{2\,c(2\epsilon)}{c^2(\epsilon)} \right) \right\}$$

where $r(\epsilon) = \frac{2}{1+\epsilon}\ {}_3F_2(1,1,1-\epsilon;2-2\epsilon,2+\epsilon;1) - \frac{1}{1-\epsilon}\ {}_3F_2(1,1,1-\epsilon;2-2\epsilon,2-\epsilon;1)$

- Expanding to $\mathcal{O}(\epsilon^2)$

$$\widetilde{\mathcal{A}}_{T,i}^{(2)}(\epsilon) = A_i^{(2)} + \epsilon C_i \left[C_A \left(\frac{101}{27} - \frac{11\pi^2}{144} - \frac{7\zeta_3}{2} \right) + n_F \left(\frac{\pi^2}{72} - \frac{14}{27} \right) \right]$$

$$+\epsilon^2 C_i \left[C_A \left(\frac{607}{81} - \frac{67\pi^2}{216} - \frac{77\zeta_3}{36} - \frac{7\pi^4}{120} \right) + n_F \left(\frac{5\pi^2}{108} - \frac{82}{81} + \frac{7\zeta_3}{18} \right) \right] + \mathcal{O}\left(\epsilon^3\right)$$

$$\widetilde{\mathcal{A}}_{0,i}^{(2)}(\epsilon) = A_i^{(2)} + \epsilon C_i \left[C_A \left(\frac{101}{27} - \frac{55\pi^2}{144} - \frac{7\zeta_3}{2} \right) + n_F \left(\frac{5\pi^2}{72} - \frac{14}{27} \right) \right]$$

$$+\epsilon^2 C_i \left[C_A \left(\frac{607}{81} - \frac{67\pi^2}{72} - \frac{143\zeta_3}{36} - \frac{\pi^4}{36} \right) + n_F \left(\frac{5\pi^2}{36} - \frac{82}{81} + \frac{13\zeta_3}{18} \right) \right] + \mathcal{O}\left(\epsilon^3\right)$$

- agree with cusp anomalous dimension in 4 dimensions
  - dominated by $k^2 \simeq 0$ and equal at first order
  - ϵ terms different (phase space integration)

$$\Delta = \frac{\pi^2}{3} \pi \beta_0 \epsilon + \ldots$$

- Why interested in *D*-dimensional expression?

$$\boxed{\widetilde{\mathscr{A}}_{T,i}\left(\alpha_S;\epsilon=\beta\left(\alpha_S\right)\right)=\widetilde{\mathscr{A}}_{0,i}\left(\alpha_S;\epsilon=\beta\left(\alpha_S\right)\right)=A_i\left(\alpha_S\right)}$$

- 'D-dimensional' $\beta = 0$ conformal point $\epsilon = \beta(\alpha_s)$

in QCD $$\frac{d\ln\alpha_s(\mu^2)}{d\ln\mu^2}=-\epsilon+\beta(\alpha_s(\mu^2))$$ $$\beta=-(\beta_0\alpha_s+\beta_1\alpha_s^2+\ldots)$$

- Compute Sudakov (threshold resummation) using soft-coupling
- Use dimensional regularisation (collinear divergences)
- Perform factorisation in $\overline{MS}$ scheme : identify cusp
- Compare to usual Sudakov from factor

$$A_i\left(\alpha_S\left(\mu_F^2\right)\right)=\frac{d}{d\ln\mu_F^2}\mathscr{P}_\epsilon\left\{\int_0^{\mu_F^2}\frac{dq_T^2}{q_T^2}\widetilde{\mathscr{A}}_i\left(\alpha_S\left(q_T^2\right);\epsilon\right)\right\}$$

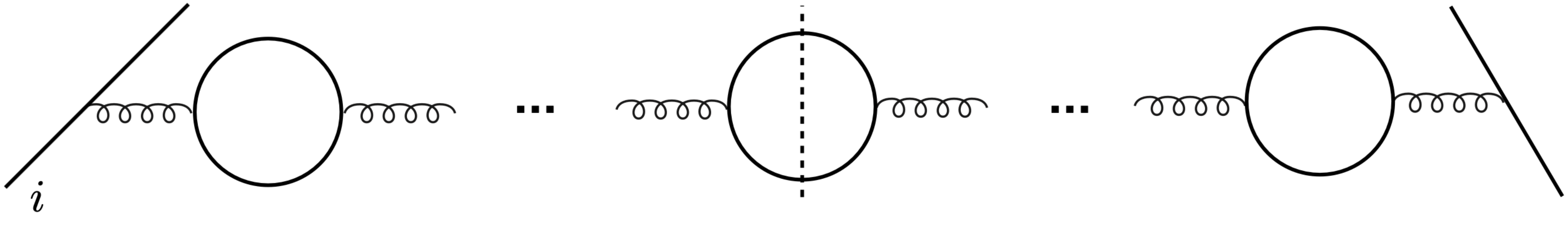
Conformal relation

- Explicit check for n_F leading terms to all orders

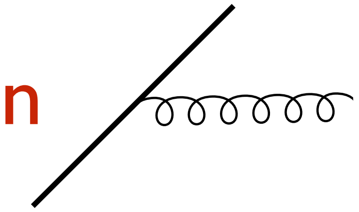
Catani, deF, Devoto, Grazzini, Mazzitelli (to appear)

- n_F terms can appear from:

(renormalized) bubble insertions



+ LO Renormalization



$$\times \frac{1}{1+\frac{\beta_0}{\epsilon}\alpha_s}$$

web: Both can be rearranged as geometric series (keep only n_F in beta)

- find all-order expression and sum the series ✓

Beneke, Braun (1995)

$$\widetilde{\mathscr{A}}_{T,i}\left(\alpha_S;\epsilon=-\beta_0\alpha_S\right)=\widetilde{\mathscr{A}}_{0,i}\left(\alpha_S;\epsilon=-\beta_0\alpha_S\right)=\boxed{A_i=C_i\frac{\alpha_S}{\pi}\frac{\Gamma(4+2\beta_0\alpha_S)}{6\Gamma(1-\beta_0\alpha_S)\,\Gamma^2(2+\beta_0\alpha_S)\,\Gamma(1+\beta_0\alpha_S)}}$$

- Exploit conformal relation to higher orders

$$\widetilde{\mathscr{A}}_{T,i}\left(\alpha_S;\epsilon=\beta\left(\alpha_S\right)\right)=\widetilde{\mathscr{A}}_{0,i}\left(\alpha_S;\epsilon=\beta\left(\alpha_S\right)\right)=A_i\left(\alpha_S\right)$$

$$\beta=-\left(\beta_0\alpha_s+\beta_1\alpha_s^2+\dots\right)$$

- First and second order…

$$\widetilde{\mathscr{A}}_i(\alpha_s;\epsilon)=\left(\frac{\alpha_s}{\pi}\right)\left(A_i^{(1)}+\epsilon\widetilde{\mathscr{A}}_i^{(1;1)}\right)+\left(\frac{\alpha_s}{\pi}\right)^2\widetilde{\mathscr{A}}_i^{(2;0)}+\dots$$

  - set $\epsilon=-\beta_0\,\alpha_s$ ➡ $A_i^{(2)}=\widetilde{\mathscr{A}}_i^{(2;0)}-\left(\beta_0\pi\right)\widetilde{\mathscr{A}}_i^{(1;1)}$

- Reason why for 1st and 2nd order they agree to cusp anomalous dimension

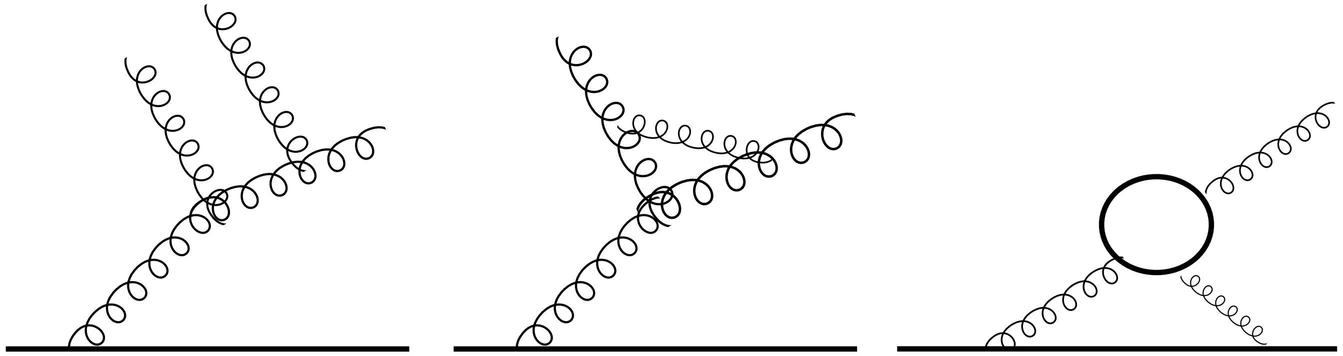
$$\widetilde{\mathscr{A}}_{T,i}^{(1)}(\epsilon)=\widetilde{\mathscr{A}}_{0,i}^{(1)}(\epsilon)=C_i c(\epsilon)\qquad c(\epsilon)\equiv\frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}=1-\frac{\pi^2}{12}\epsilon^2-\frac{1}{3}\zeta_3\epsilon^3+\mathcal{O}\left(\epsilon^4\right)$$

no ϵ term $\qquad \widetilde{\mathscr{A}}_i^{(1;1)}=0$

- Exploit conformal relation to higher orders

$$\widetilde{\mathscr{A}}_{T,i}\left(\alpha_{\mathrm{S}};\epsilon=\beta\left(\alpha_{\mathrm{S}}\right)\right)=\widetilde{\mathscr{A}}_{0,i}\left(\alpha_{\mathrm{S}};\epsilon=\beta\left(\alpha_{\mathrm{S}}\right)\right)=A_i\left(\alpha_{\mathrm{S}}\right)$$

- An explicit third order computation would demand evaluation of



  - triple soft current (Born)
  - double soft current (one loop)
  - single soft current (two loops)

- But, expanding conformal equation to third order one directly obtains

$$\mathscr{A}_i^{(3)}=A_i^{(3)}-\left(\beta_0\pi\right)^2\widetilde{\mathscr{A}}_i^{(1;2)}+\left(\beta_0\pi\right)\widetilde{\mathscr{A}}_i^{(2;1)}$$

3-loop soft effective coupling (4-*D*) is simply given by the cusp anomalous + ϵ^n terms to (*D*-dimensional) second order (different for each definition)

- **q_T resummation**

$$\mathscr{A}_{T,i}^{(3)} = A_i^{(3)} + C_i\left(\beta_0\pi\right)^2\frac{\pi^2}{12} + C_i\left(\beta_0\pi\right)\left[C_A\left(\frac{101}{27} - \frac{11\pi^2}{144} - \frac{7\zeta_3}{2}\right) + n_F\left(\frac{\pi^2}{72} - \frac{14}{27}\right)\right]$$

- Agrees with previous result and 3rd order coefficient for q_T resummation

Banfi, El-Menoufi, Monni (2018) Becher, Neubert (2011)

- **Threshold resummation (new!)**

$$\mathscr{A}_{0,i}^{(3)} = A_i^{(3)} + C_i\left(\beta_0\pi\right)^2\frac{\pi^2}{12} + C_i\left(\beta_0\pi\right)\left[C_A\left(\frac{101}{27} - \frac{55\pi^2}{144} - \frac{7\zeta_3}{2}\right) + n_F\left(\frac{5\pi^2}{72} - \frac{14}{27}\right)\right]$$

- Provides soft-gluon effective coupling for threshold resummation at 3rd order
- Full Soft-collinear contributions (DY-like)

$$\Delta = C_i\frac{\pi^2}{3}(\pi\beta_0)^2$$

- Comparing to Sudakov form factor at 4th order (threshold resummation)

$$\mathscr{A}_{0,i}^{(4)} = A_i^{(4)} + C_i \Bigg\{ C_A^3 \left(\frac{121\pi^2\zeta_3}{288} - \frac{21755\zeta_3}{864} + \frac{33\zeta_5}{4} + \frac{33\zeta_5}{17280} - \frac{41525\pi^2}{15552} + \frac{3761815}{186624} \right)$$

$$+ C_A^2 n_F \left(-\frac{11\pi^2\zeta_3}{144} + \frac{6407\zeta_3}{864} - \frac{3\zeta_5}{2} - \frac{11\pi^4}{432} + \frac{9605\pi^2}{7776} - \frac{15593}{1944} \right)$$

$$+ C_A C_F n_F \left(\frac{17\zeta_3}{9} + \frac{11\pi^4}{1440} + \frac{55\pi^2}{576} - \frac{7351}{2304} \right) + C_A n_F^2 \left(-\frac{179\zeta_3}{432} + \frac{13\pi^4}{4320} - \frac{695\pi^2}{3888} \right)$$

$$+ C_F n_F^2 \left(-\frac{19\zeta_3}{72} - \frac{\pi^4}{720} - \frac{5\pi^2}{288} + \frac{215}{384} \right) + n_F^3 \left(-\frac{\zeta_3}{108} + \frac{5\pi^2}{648} - \frac{29}{1458} \right) \Bigg\}$$

only possible for $\mathscr{A}_0$

$A_i^{(4)}$ known

Moch, Ruijl, Ueda, Vermaseren, Vogt (2018)
Henn, Korchemsky, Mistlberger (2019)
von Manteuffel, Panzer, Schabinger (2020)

- violates Casimir scaling

- 4th order soft effective coupling (threshold related)

$$\mathscr{A}_{0,i}\left(\alpha_S\right) = C_i \frac{\alpha_S}{\pi} \left[1 + 0.54973\alpha_S - 1.7157\alpha_S^2 - \left(5.980\,\delta_{iq} + 6.236\,\delta_{ig}\right)\alpha_S^3 + \mathcal{O}\left(\alpha_S^4\right) \right]$$

Full Soft-collinear radiation from hard partons

- Extra term satisfies Casimir scaling and dominates numerically over cusp

- Processes involving several hard partons more complicated (MC)
  - need to account for (extra) soft wide-angle emission and collinear radiation

- But soft coupling $\mathscr{A}_i$ not affected
  - intensity of soft-collinear radiation from parton i
  - exactly known to 4th order (for threshold related)

- All information for (threshold) resummation is contained in the d-dimensional version of the soft-coupling

Conclusions

- Definition(s) of Soft-gluon effective coupling at higher orders
- Explicit results for NNLO
- N^{3}LO soft-coupling for threshold related observables
- Conformal relation (all orders)

$$\widetilde{\mathscr{A}}_{T,i}\left(\alpha_{\mathrm{S}};\epsilon=\beta\left(\alpha_{\mathrm{S}}\right)\right)=\widetilde{\mathscr{A}}_{0,i}\left(\alpha_{\mathrm{S}};\epsilon=\beta\left(\alpha_{\mathrm{S}}\right)\right)=A_i\left(\alpha_{\mathrm{S}}\right)$$

- Towards improving the precision of PS : precision QCD for LHC and future colliders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danke
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teşekkür ederim
Баярлалаа
спасибо
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tack
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tapadh leat
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nanni
nandri
thank you
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obrigada
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enkosi
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