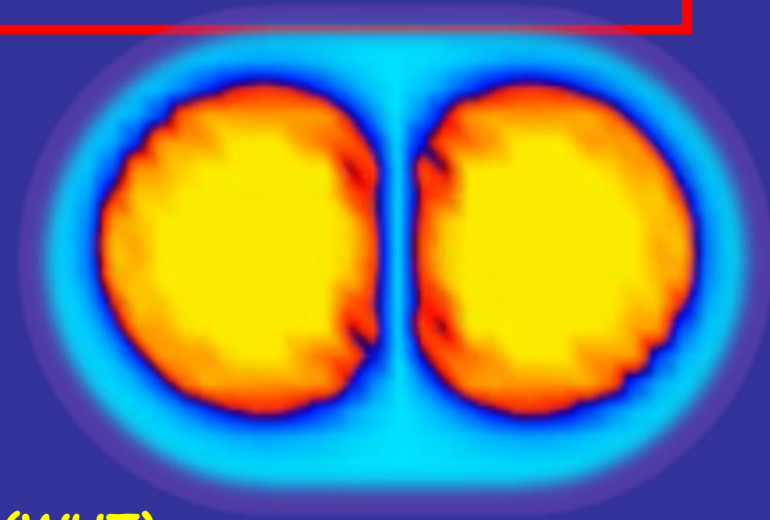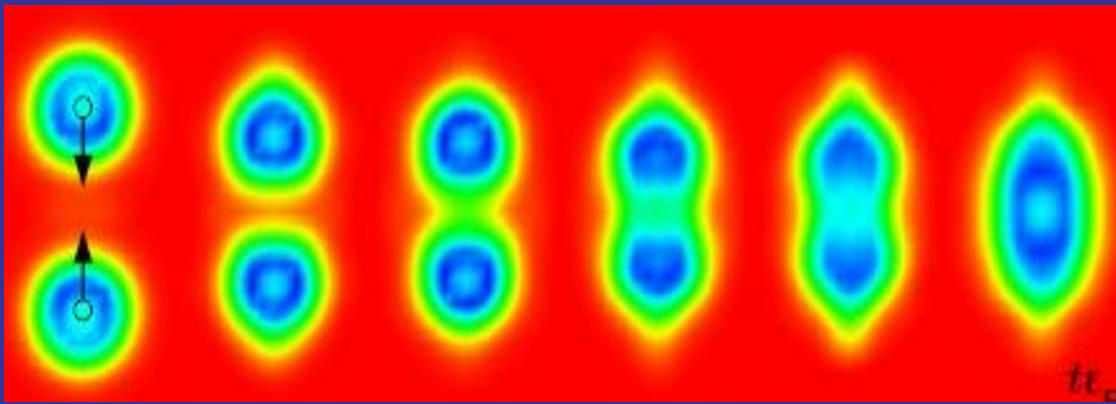# *Solitonic excitation in heavy ion collision and ultracold atomic gases.*

Piotr Magierski
Warsaw University of Technology (WUT)
University of Washington (UW)

Collaborators:

Matthew Barton (WUT)
Andrzej Makowski (WUT)
Kazuyuki Sekizawa (Tokyo Inst. of Techn.)
Buğra Tüzemen (WUT)
Gabriel Wlazłowski (WUT)
Tomasz Zawiślak (WUT)

OUTLINE:

- Solitonic excitations in nuclear collisions and ultracold Fermi gases.

- Metastable Larkin-Ovchinnikov droplets (ferrons) in spin imbalanced Fermi gas

# What do we know about pairing correlations in atomic nuclei?

Odd-even mass staggering gives us estimate of the pairing strength (unfortunately obscured by polarization effects)

$$|\Delta| \approx \frac{12}{\sqrt{A}} \ \text{MeV}$$
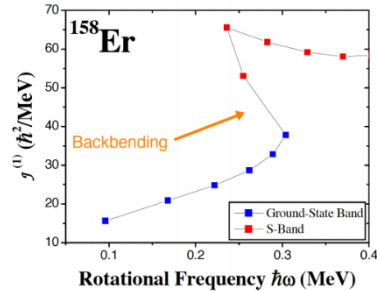
High spin experimental data: backbending of moments of inertia produced by the alignment of the correlated nucleon pair is a sensitive function of pairing correlations.

A. Johnson, H. Ryde, S.A. Hjorth,
Nucl. Phys. A179, 753 (1972)

Last but not least: modeling of large amplitude nuclear motion require to include pairing correlations.
e.g. Nuclear fission at low energies
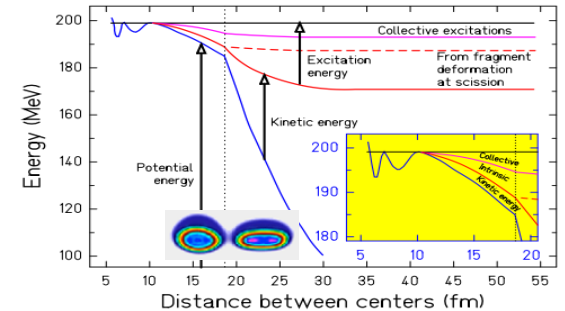
From Schmidt & Jurado:Phys.Rev.C83:061601,2011

Can we probe the pairing field phase in nuclei?

Nuclear Josephson junction: enhacement of neutron pair transfer in nuclear collision

V.I. Gol'danskii, A.I. Larkin JETP 53, 1032 (1967)
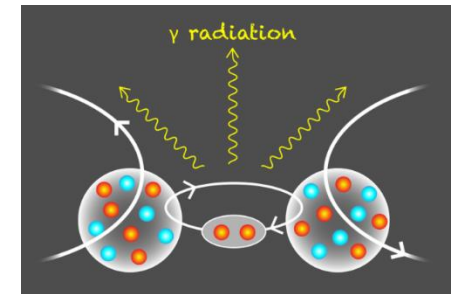K. Dietrich, Phys.Lett. B32, 428 (1970)

(experimental data are not conclusive)

Recent attempt: particle oscillatory transfer (AC Josephson junction)
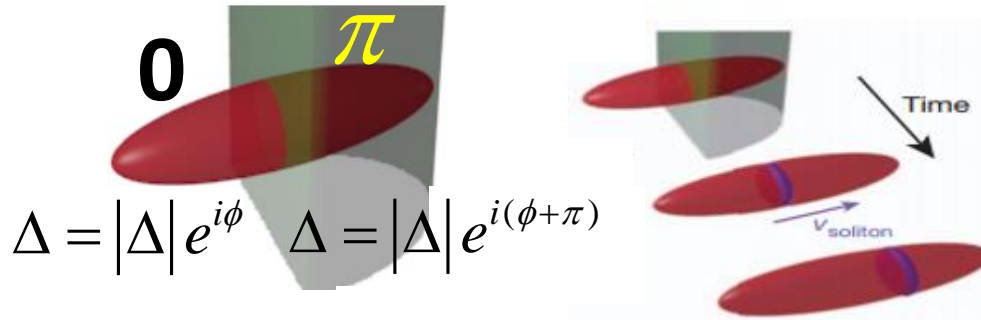C.Potel,F.Barranco,E.Vigezzi, R.A. Broglia, Phys.Rev. C103, L021601(2021)
surprising agreement of gamma spectra with experiment!
(Although just one reaction:^{116}Sn+ ^{60}Ni has been studied)
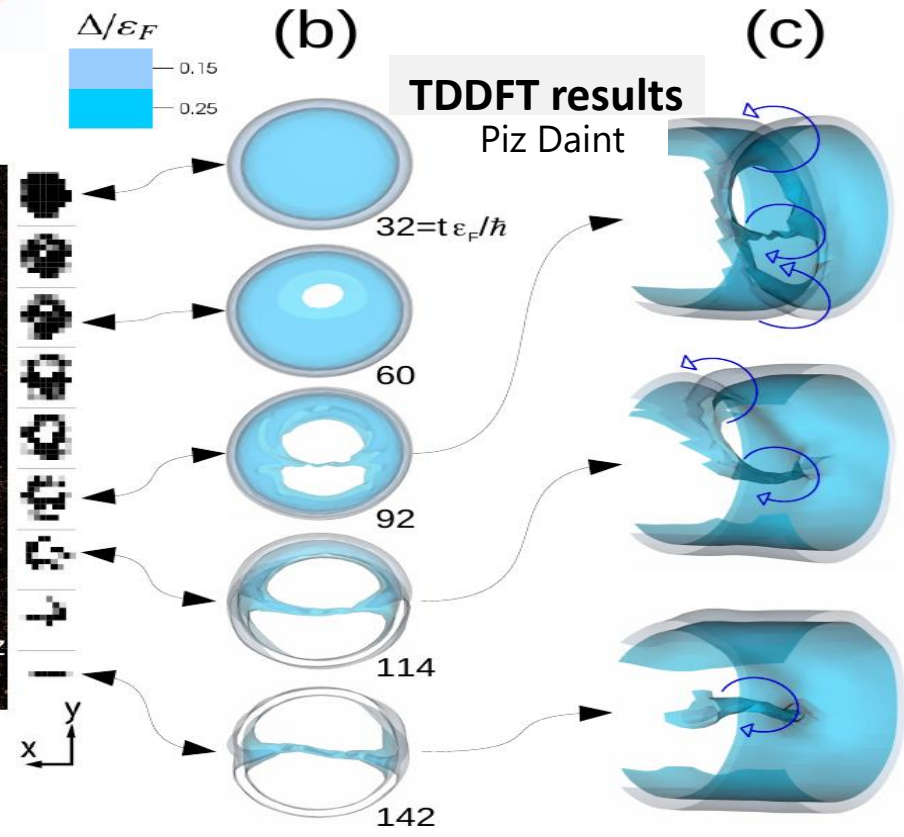

From P.M. , Physics 14,27(2021)

Can we observe a similar effect in nuclear systems as in atomic Fermi gas?

0 π

$$\Delta = |\Delta| e^{i\phi} \quad \Delta = |\Delta| e^{i(\phi+\pi)}$$

Time

$v_{soliton}$

Series of MIT experiments:
Nature 499, 426 (2013);
PRL 113, 065301 (2014);
PRL 116, 045304 (2016);
→ observation of decay
of a dark soliton into a vortex line

MIT experiment
Phys. Rev. Lett. 116, 045304 (2016)

Δ/ε_F **(b)** **(c)**

0.15
0.25

TDDFT results
Piz Daint

$32 = t\varepsilon_F/\hbar$
60
92
114
142
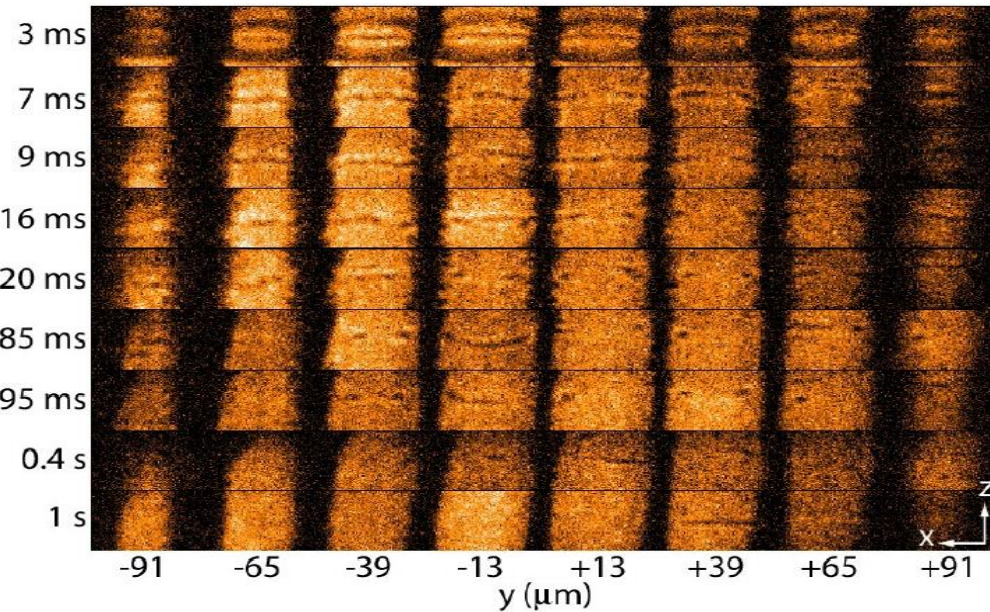
Decay of solitonic excitation (pairing nodal structure) generates a sequence of topological excitations: **dark soliton-> Phi soliton -> vortex ring -> vortex line** reproduced by TDDFT

Solving time-dependent problem for superfluids...

The real-time dynamics is given by equations, which are formally equivalent to the Time-Dependent HFB (TDHFB) or Time-Dependent Bogolubov-de Gennes (TDBdG) equations

$$h \sim f_1(n, \nu, \ldots)\nabla^2 + \boldsymbol{f}_2(n, \nu, \ldots) \cdot \nabla + f_3(n, \nu, \ldots)$$

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{n,a}(\boldsymbol{r},t) \\ u_{n,b}(\boldsymbol{r},t) \\ v_{n,a}(\boldsymbol{r},t) \\ v_{n,b}(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_a(\boldsymbol{r},t) & 0 & 0 & \Delta(\boldsymbol{r},t) \\ 0 & h_b(\boldsymbol{r},t) & -\Delta(\boldsymbol{r},t) & 0 \\ 0 & -\Delta^*(\boldsymbol{r},t) & -h_a^*(\boldsymbol{r},t) & 0 \\ \Delta^*(\boldsymbol{r},t) & 0 & 0 & -h_b^*(\boldsymbol{r},t) \end{pmatrix}\begin{pmatrix} u_{n,a}(\boldsymbol{r},t) \\ u_{n,b}(\boldsymbol{r},t) \\ v_{n,a}(\boldsymbol{r},t) \\ v_{n,b}(\boldsymbol{r},t) \end{pmatrix}$$

We explicitly track fermionic degrees of freedom!

where *h* and Δ depends on "densities":

$$n_\sigma(\boldsymbol{r},t) = \sum_{E_n < E_c} |v_{n,\sigma}(\boldsymbol{r},t)|^2, \qquad \tau_\sigma(\boldsymbol{r},t) = \sum_{E_n < E_c} |\nabla v_{n,\sigma}(\boldsymbol{r},t)|^2,$$

$$\nu(\boldsymbol{r},t) = \sum_{E_n < E_c} u_{n,\uparrow}(\boldsymbol{r},t)v_{n,\downarrow}^*(\boldsymbol{r},t), \qquad j_\sigma(\boldsymbol{r},t) = \sum_{E_n < E_c} \text{Im}[v_{n,\sigma}^*(\boldsymbol{r},t)\nabla v_{n,\sigma}(\boldsymbol{r},t)],$$

huge number of nonlinear coupled 3D Partial Differential Equations
(in practice $n=1,2,\ldots, 10^5 - 10^6$)

- P. M., *Nuclear Reactions and Superfluid Time Dependent Density Functional Theory,* Frontiers in Nuclear and Particle Physics, vol. 2, 57 (2019)
- A. Bulgac, *Time-Dependent Density Functional Theory and Real-Time Dynamics of Fermi Superfluids,* Ann. Rev. Nucl. Part. Sci. 63, 97 (2013)
- A. Bulgac, M.M. Forbes, P. M., Lecture Notes in Physics, Vol. 836, Chap. 9, p.305-373 (2012)

Present computing capabilities:
- ▶ full 3D (unconstrained) superfluid dynamics
- ▶ spatial mesh up to 100^3
- ▶ max. number of particles of the order of 10^4
- ▶ up to 10^6 time steps

  (for cold atomic systems – time scale: a few ms
   for nuclei – time scale: 100 zs)

Warsaw University of Technology | W-SLDA Toolkit

https://wslda.fizyka.pw.edu.pl

W-SLDA Toolkit

Self-consistent solver of mathematical problems which have structure formally equivalent to Bogoliubov-de Gennes equations.

static problems: st-wslda

$$\begin{pmatrix} h_a(\boldsymbol{r}) - \mu_a & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & -h_b^*(\boldsymbol{r}) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix} = E_n \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix}$$
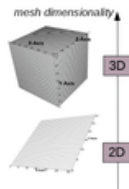
time-dependent problems: td-wslda

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_a(\boldsymbol{r},t) - \mu_a & \Delta(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & -h_b^*(\boldsymbol{r},t) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix}$$

Speed-up calculations by exploiting High Performance Computing

Functionals for studies of BCS and unitary regimes

Dimensionalities of problems: 3D, 2D and 1D
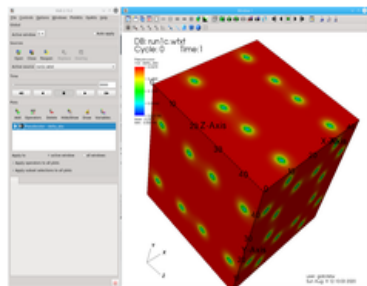
mesh dimensionality

3D

2D

W-SLDA allows to solve problems

in 3D without any symmetry restrictions: $\Psi = \varphi(x,y,z)$

in 2D with translational invariance along z direction: $\Psi =$

in 1D with translational invariance along y and z direction

Integration with VisIt: visualization, animation and analysis tool

W-SLDA is integrated with open-source VisIt tool. It allows for:

visualizing 3D, 2D and 1D results,

data processing,

creating animations for time-dependent simulations.

Contributors

Project leader

Gabriel Wlazłowski
🏛 Warsaw University of Technology, Faculty of Physics
👤 Main developer of the W-SLDA Toolkit

Theory expertise

Aurel Bulgac
🏛 Department of Physics, University of Washington
👤 Aurel Bulgac derived Superfluid Local Density Approximation (SLDA) equations for cold atoms, presently implemented in W-SLDA Toolkit. He also supervised implementation of core algorithms of td-wslda codes.

Piotr Magierski
🏛 Warsaw University of Technology, Faculty of Physics
👤 Piotr Magierski contributed to development of Superfluid Local Density Approximation (SLDA) method.

Michael McNeil Forbes
🏛 Department of Physics and Astronomy, Washington State University
👤 Michael Forbes together with Aurel Bulgac developed Asymmetric Superfluid Local Density Approximation (ASLDA) for spin-imbalanced atomic gases.

HPC expertise

Kenneth J. Roche
🏛 Pacific Northwest National Laboratory
👤 Kenneth J. Roche supervised parallelization process (MPI + CUDA) of main engine of td-wslda codes.

Maciej Marchwiany
🏛 Interdisciplinary Centre for Mathematical and Computational Modelling (ICM)
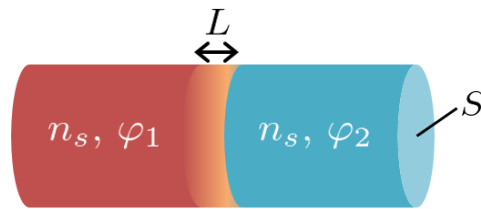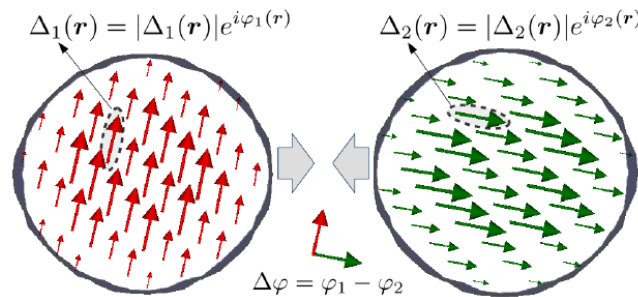👤 Implementation of parallel I/O in td-wslda codes (2016-2018)

Nuclear collisions

Collisions of superfluid nuclei having <u>different phases</u> of the <u>pairing fields</u>
<u>The main questions are:</u>
-how a possible solitonic structure can be manifested in nuclear system?
-what observable effect it may have on heavy ion reaction:
  kinetic energy distribution of fragments, capture cross section, etc.?

Clearly, we cannot control phases of the pairing field in nuclear experiments and the possible signal need to be extracted after averaging over the phase difference.
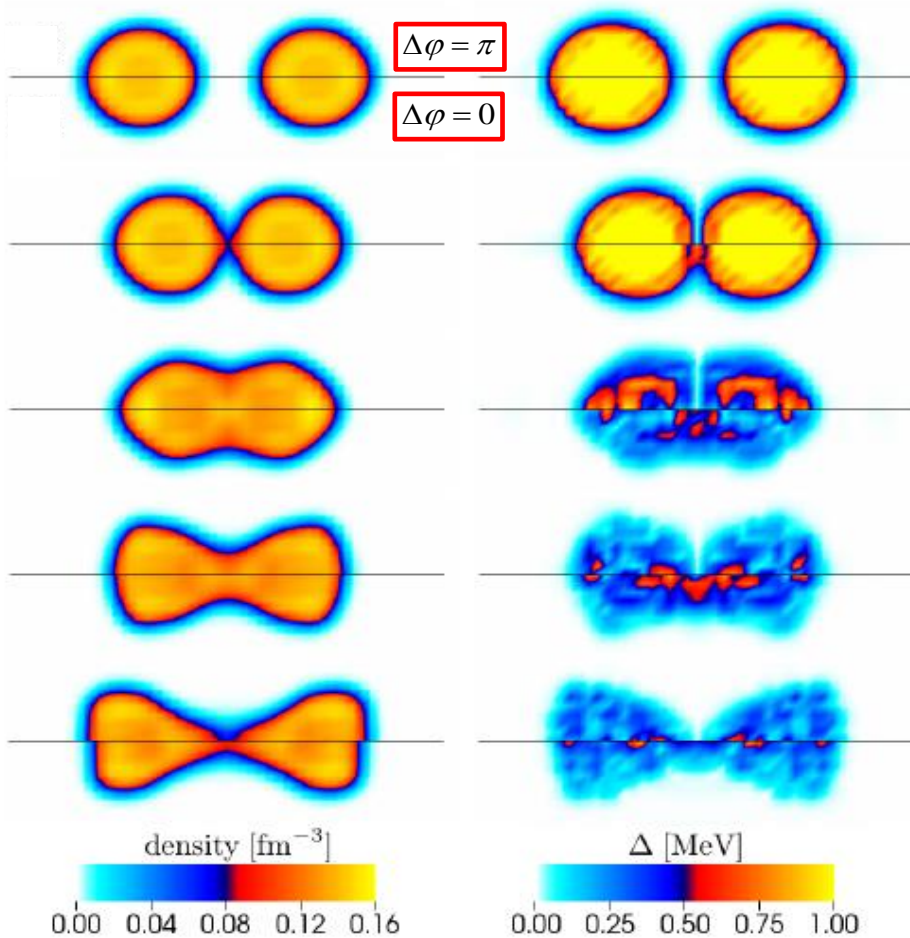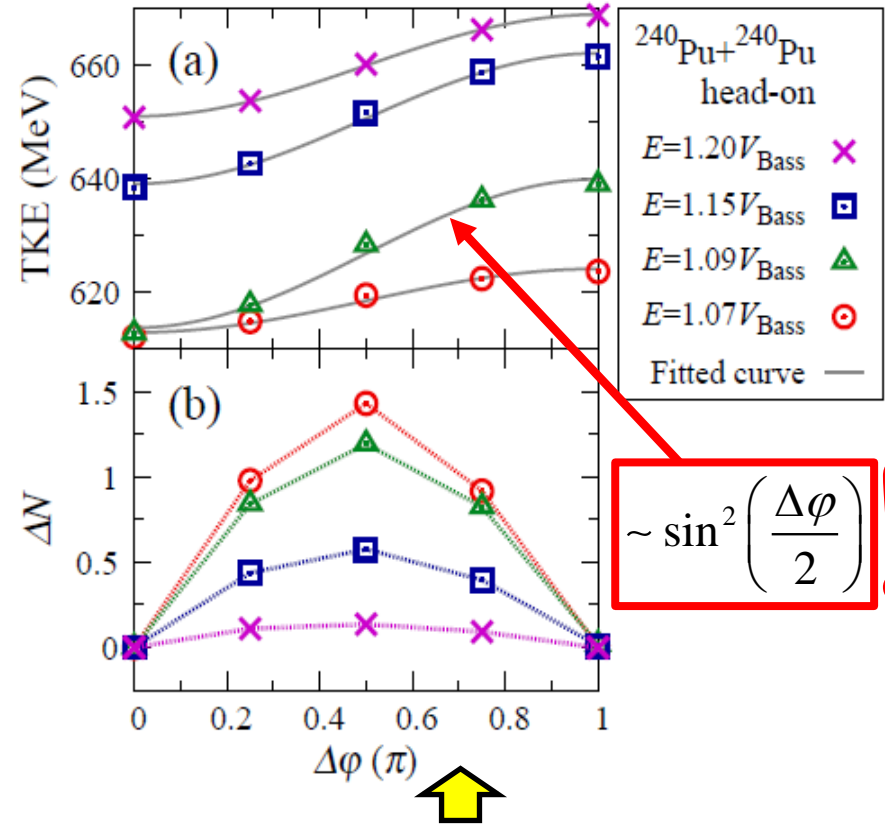
$$\Delta_1(\boldsymbol{r}) = |\Delta_1(\boldsymbol{r})|e^{i\varphi_1(\boldsymbol{r})} \qquad \Delta_2(\boldsymbol{r}) = |\Delta_2(\boldsymbol{r})|e^{i\varphi_2(\boldsymbol{r})}$$

$$\Delta\varphi = \varphi_1 - \varphi_2$$

From Ginzburg-Landau (G-L) approach:

$n_s, \varphi_1 \qquad n_s, \varphi_2 \qquad S$

L

$$E_j = \frac{S}{L}\frac{\hbar^2}{2m}n_s \sin^2 \frac{\Delta\varphi}{2}$$

$\Delta\varphi \ (\equiv \varphi_1 - \varphi_2)$

For typical values characteristic for two medium nuclei: $E_j \approx 30 MeV$

^{240}Pu+^{240}Pu

Total kinetic energy of the fragments (TKE)

$\Delta\varphi = \pi$

$\Delta\varphi = 0$

density [fm^{-3}]

0.00 0.04 0.08 0.12 0.16

Δ [MeV]

0.00 0.25 0.50 0.75 1.00

(a)

TKE (MeV)

660

640

620

^{240}Pu+^{240}Pu
head-on

$E=1.20V_{\text{Bass}}$ ✕
$E=1.15V_{\text{Bass}}$ ▣
$E=1.09V_{\text{Bass}}$ △
$E=1.07V_{\text{Bass}}$ ⊙

Fitted curve ——

$\sim \sin^2\left(\dfrac{\Delta\varphi}{2}\right)$ ❗

(b)

ΔN

1.5

1

0.5

0

0   0.2   0.4   0.6   0.8   1

$\Delta\varphi\ (\pi)$

Average particle transfer between fragments.

Creation of the solitonic structure between colliding nuclei prevents energy transfer to internal degrees of freedom and consequently enhances the kinetic energy of outgoing fragments.
Surprisingly, the gauge angle dependence from the G-L approach is perfectly well reproduced in the kinetic energies of outgoing fragments!

Effective barrier height for fusion as a function of the phase difference

^{90}Zr+^{90}Zr head-on

30 MeV

Results with Fayans functional
(no spin-orbit term)
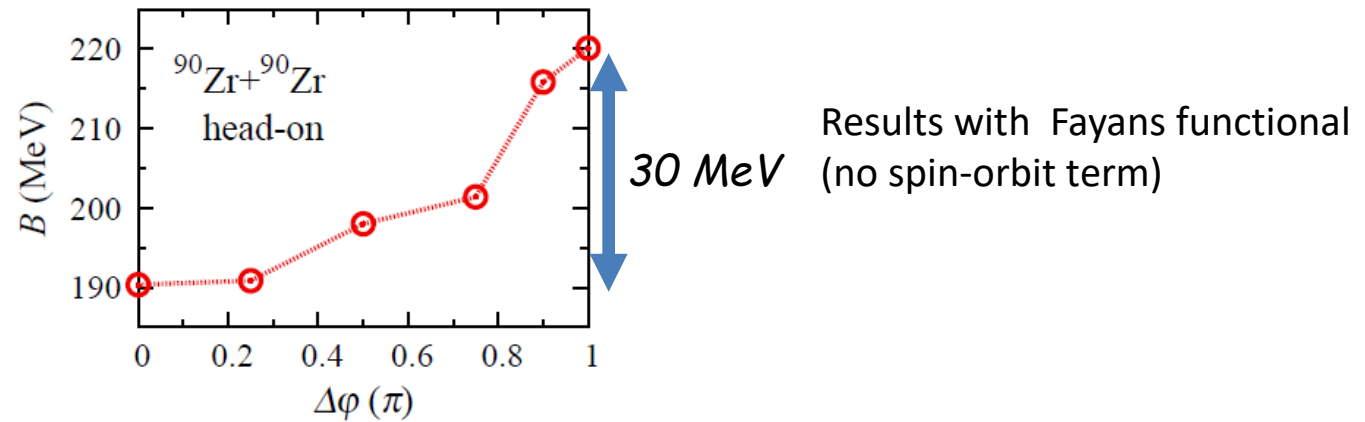
What is an average extra energy needed for the capture?

$$E_{extra} = \frac{1}{\pi} \int_0^{\pi} \left(B(\Delta\varphi) - V_{Bass} \right) d\left(\Delta\varphi \right) \approx 10 MeV$$

The effect is found (within TDDFT) to be of the order of <u>30MeV</u> for medium nuclei and occur for <u>energies up to 20-30% of the barrier height</u>.

P. M., K. Sekizawa, G. Wlazłowski, Phys. Rev. Lett. 119 042501 (2017)

G. Scamps, Phys. Rev. C 97, 044611 (2018): barrier fluctuations extracted from experimental data indicate that the effect exists although is weaker than predicted by TDDFT

|	^{90}Zr	^{96}Zr	
	$\Delta_n = 0.0 MeV$ $\Delta_p = 0.09 MeV$	$\Delta_n = 0.73 MeV$ $\Delta_p = 0.93 MeV$	$\Delta_n = 1.98 MeV$ $\Delta_p = 0.32 MeV$
$E_{min}(0)$ (MeV)	184	180	179
$E_{min}(\pi)$ (MeV)	184	186	185

Recent results with SkM* functional:
Minimum energy needed for capture.
M. Barton et al.

Stabilizing nodal structures in spin-imbalanced atomic Fermi gas

Larkin-Ovchinnikov: $\Delta(r) \sim cos(qr)$

Fulde-Ferrell: $\Delta(r) \sim \exp(iqr)$

A.I. Larkin and Y. N. Ovchinnikov, Sov. Phys. JETP 20, 762 (1965)
P. Fulde and R. A. Ferrell, Phys. Rev. 135, A550 (1964)

$$E \sim [n_a g(x)]^{5/3}$$

$$x = n_b/n_a$$
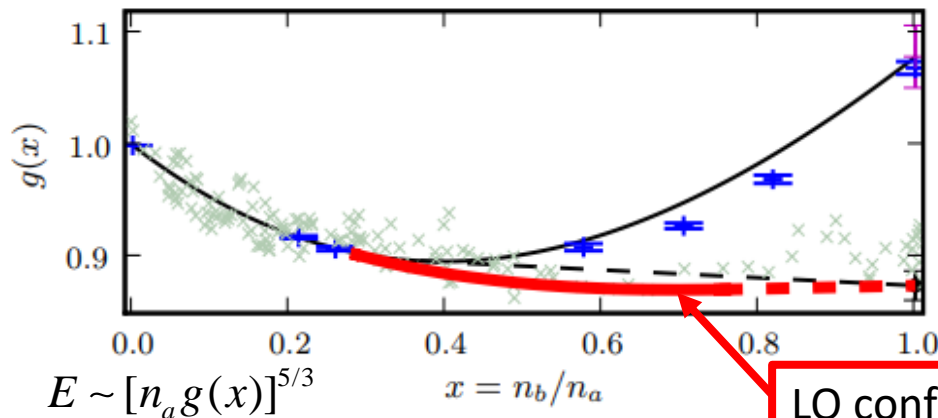
Bulgac & Forbes have shown, within DFT, that Larkin-Ovchinnikov (LO) phase may exist in the unitary Fermi gas (UFG)

LO configuration – supersolid state

A. Bulgac, M.M.Forbes, Phys. Rev. Lett. 101,215301 (2008)
See also review of mean-field theories : Radzihovsky,Sheehy, Rep.Prog. Phys.73,076501(2010)

Trapping ultracold atoms in a <u>uniform potential</u> recently become possible:

The problem:
In the trap the volume where LOFF phase may be created is relatively small .
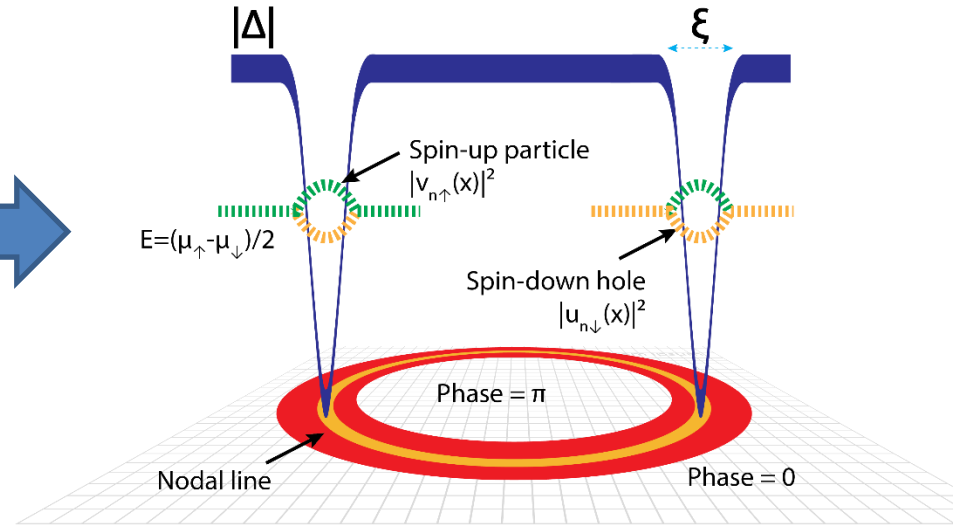
unless
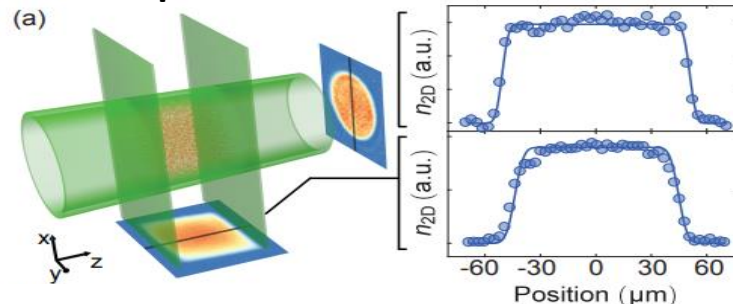
B. Mukherjee etal. Phys. Rev. Lett. 118, 123401 (2017)

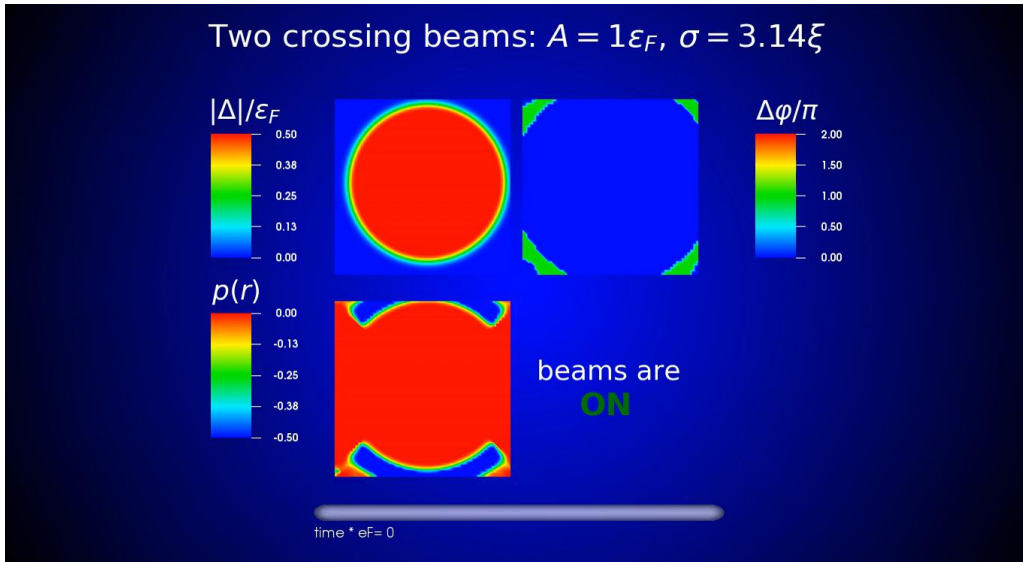# Creating Larkin-Ovchinnikov droplet (ferron) dynamically in unitary Fermi gas

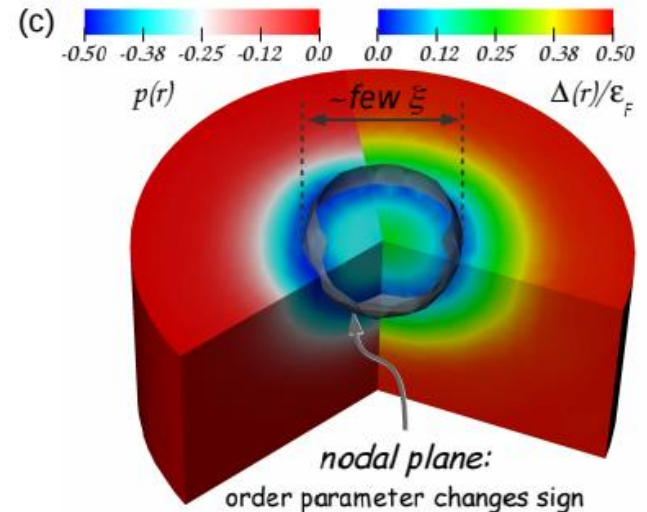Spin-selective potential applied locally leads to Cooper pair breaking
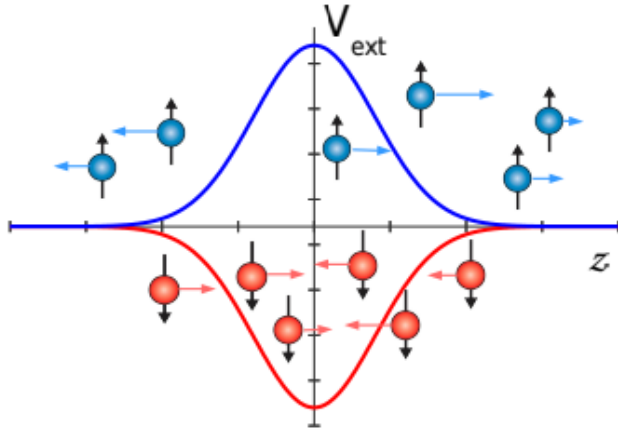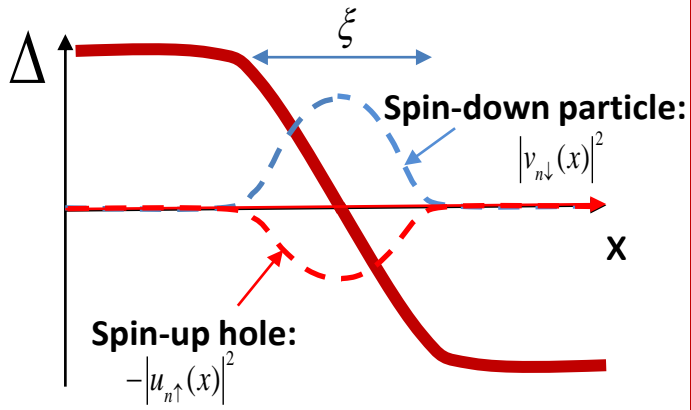
Pairing field nodal structure

Generation of *ferron* in the unitary regime

Two crossing beams: $A = 1\varepsilon_F$, $\sigma = 3.14\xi$

Ferron structure

Andreev states and stability of pairing nodal points

Spin-down particle:
$$\left|v_{n\downarrow}(x)\right|^2$$

Spin-up hole:
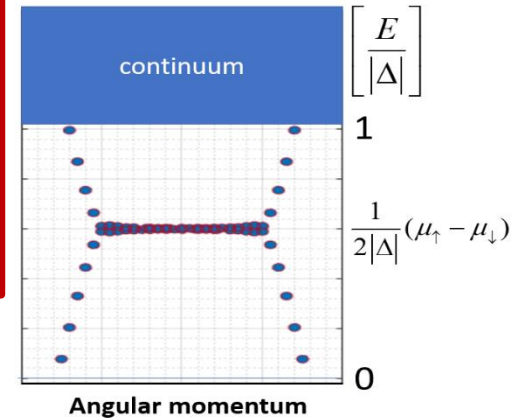$$-\left|u_{n\uparrow}(x)\right|^2$$

Due to quasiparticle scattering the localized Andreev states appear at the nodal point. These states induce local spin-polarization

BdG in the Andreev approx. ($\Delta \ll k_F^2$)

$$\begin{bmatrix} -2ik_F \dfrac{d}{dx} & \Delta(x) \\ \Delta^*(x) & 2ik_F \dfrac{d}{dx} \end{bmatrix} \begin{bmatrix} u_{n\uparrow}(x) \\ v_{n\downarrow}(x) \end{bmatrix} = E_n \begin{bmatrix} u_{n\uparrow}(x) \\ v_{n\downarrow}(x) \end{bmatrix}$$

Schematic spectrum of subgap states for ferron

Angular momentum

Non-central collision of two impurities

Surprisingly, the nodal structure remains stable even during collisions

P. M., B. Tüzemen, G. Wlazłowski, Phys. Rev. A100, 033613 (2019)

The velocities of impurites are about 30% of the velocity of sound.
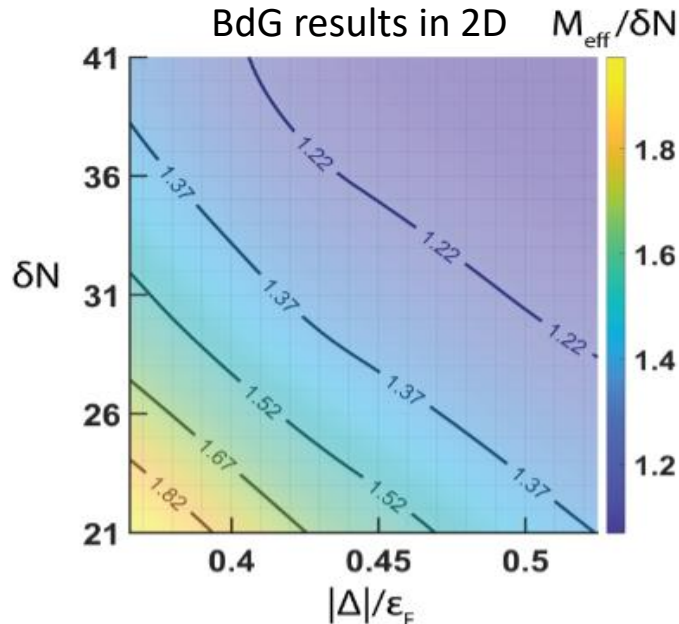
Ferron as a composite particle

$$M_{eff} = \lim_{q \to 0} \frac{\left| \int d^3r \left(j_\uparrow + j_\downarrow \right) \right|}{|q|}$$

$$M_{eff} = M_{pol} + \delta M = \left(N_\uparrow - N_\downarrow \right) + \delta M$$

<u>Scales with surface.</u>
Proportional to the excess spin-up particles residing at the nodal surface/line.

<u>Scales with volume.</u>
Hydrodynamic contribution related to the flow generated by moving impurity.

Effective mass
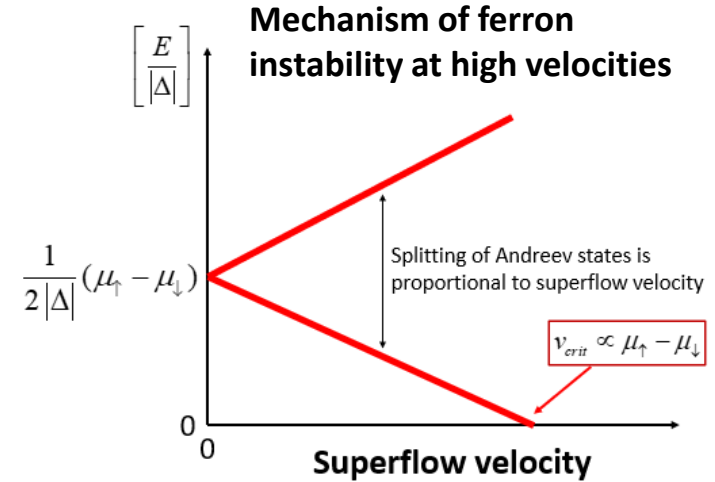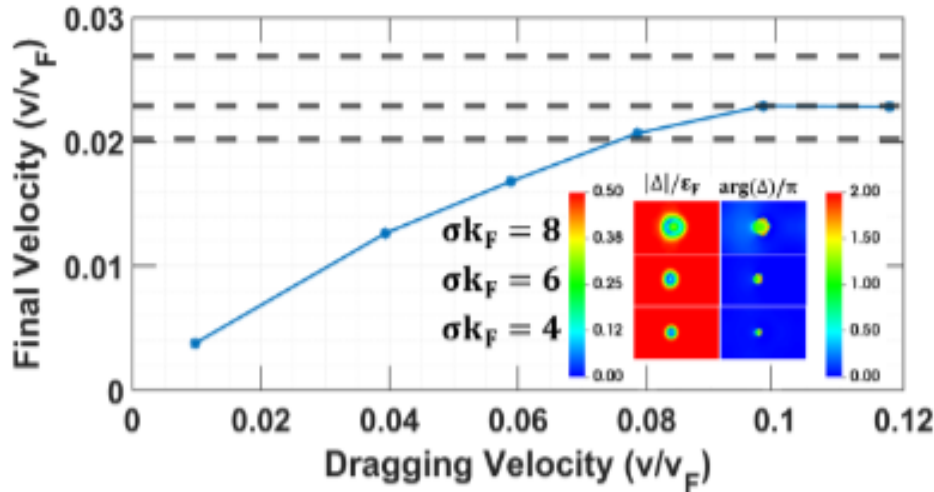BdG results in 2D $M_{eff}/\delta N$

$$\delta N = N_\uparrow - N_\downarrow$$

Ferron size is related to the spin polarization:

$$R \sim \delta N \quad \text{in 2D}$$

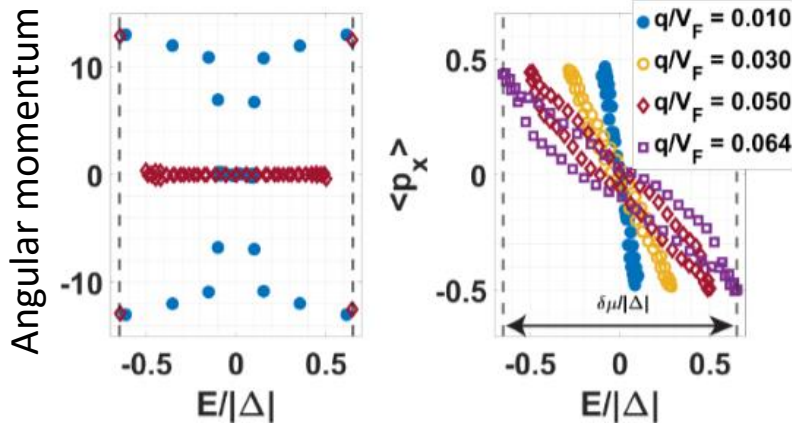$$R \sim \sqrt{\delta N} \quad \text{in 3D}$$

P. M., B. Tüzemen, G. Wlazłowski, arXiv:2102.04833

Ferron critical velocity

Peculiarity of ferron dynamics: there is a limiting velocity propotional to its polarization.

Mechanism of ferron instability at high velocities

$$\frac{1}{2|\Delta|}(\mu_\uparrow - \mu_\downarrow)$$

Splitting of Andreev states is proportional to superflow velocity

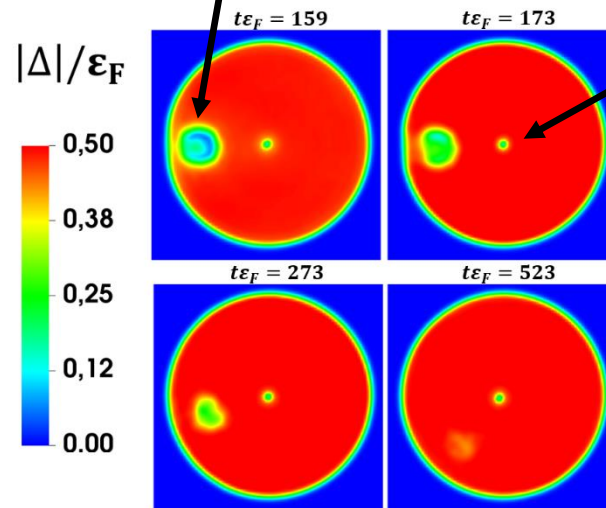$v_{crit} \propto \mu_\uparrow - \mu_\downarrow$
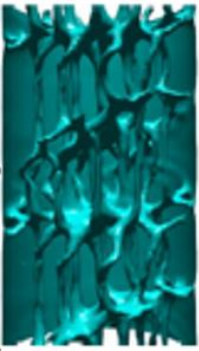
Andreev states for ferron
BdG calculations in 2D

Consequence:

Instability of *ferron* in the vicinity of *quantum vortex*

P. M.,
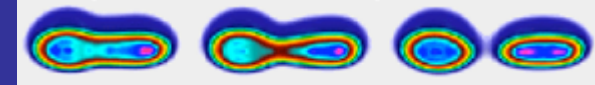B. Tüzemen,
G. Wlazłowski,
arXiv:2102.04833

Quantum turbulence

K. Hossain (WSU)
M.M. Forbes (WSU)
K. Kobuszewski (WUT)
S. Sarkar (WSU)
G. Wlazłowski (WUT)

Vortex dynamics in neutron star crust

N. Chamel (ULB)
D. Pęcak (WUT)
G. Wlazłowski (WUT)
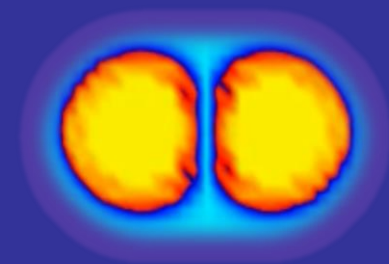
Nuclear fission

I. Abdurrahman (UW)
A. Bulgac (UW)
K. Goodbey (UW)
I. Stetcu (LANL)

Josephson junction in atomic Fermi gases - dissipative effects

N. Proukakis (NU)
M. Tylutki (WUT)
G. Wlazłowski (WUT)
K. Xhani (LENS & NU)

Nonequilibrium superfluidity in Fermi systems

Nuclear collisions

M. Barton (WUT)
A. Makowski (WUT)
K. Sekizawa (Tokyo I.)
G. Wlazłowski (WUT)

Collisions of vortex-antivortex pairs

A. Barresi (WUT)
A. Boulet (WUT)
G. Wlazłowski (WUT)
and LENS exp. group

Spin-imbalanced Fermi gases

B. Tuzemen (WUT)
G. Wlazłowski (WUT)
T. Zawiślak (WUT)