Un-nuclear Physics: conformal symmetry in nuclear reactions

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Outline

- Universality, Schrödinger symmetry and the unitary limit
- Nuclear reactions with neutrons
- Summary and outlook

In collaboration with **Dam Thanh Son** (University of Chicago)

Reference: HWH, D.T. Son, arXiv:2103.12610

Universality

Universality: Physical systems with different short-distance behavior exhibit identical behavior at large distances

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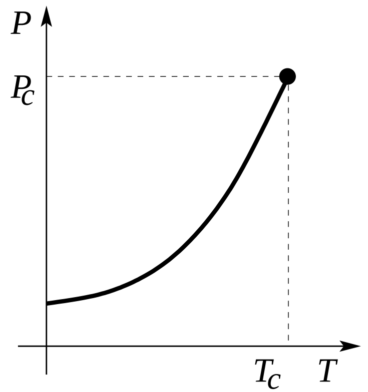
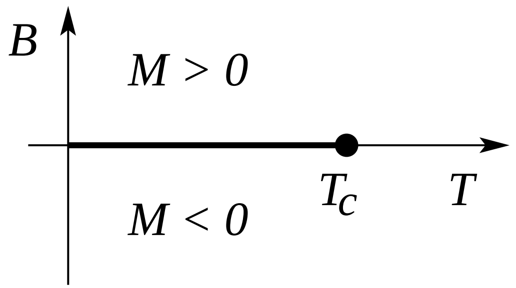
- Condensed matter systems near critical point





$$\rho_{liq/gas}(T) - \rho_c \longrightarrow \pm A(T_c - T)^\beta$$

liquid-gas system

$$M_0(T) \longrightarrow A'(T_c - T)^\beta$$

Ferromagnet (one easy axis)

- Universality class determines critical exponents: $\beta = 0.325$
- Scale invariance (often conformal invariance)

Unparticle physics

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- (Relativistic) Unparticle (Georgi, Phys. Rev. Lett. **98**, 221601 (2007))
  - field in relativistic conformal field theory
  - hidden conformal symmetry sector beyond Standard model (weakly coupled)
  - no evidence at LHC so far
    (CMS Coll., EPJC **75**, 235 (2015), PRD **93**, 052011, JHEP **03**, 061 (2017))
- (Non-relativistic) un-nucleus (HWH, Son, arXiv:2103.12610)
  - non-relativistic analog of Georgi's unparticle
  - field ψ in non-relativistic conformal field theory
    (cf. Nishida, Son, Phys. Rev. D **76**, 086004 (2007))
  - ψ characterized by mass M and scaling dimension Δ
  - free field has $\Delta = 3/2 \quad \Longleftrightarrow \quad$ mass dimension
    $\Rightarrow$ lowest possible value (unitarity)

Schrödinger symmetry

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- Non-relativistic conformal symmetry: Schrödinger symmetry
  - Galilei symmetry
    - space + time translations
    - rotations
    - Galilei boosts
  - Scale transformations

$$\boldsymbol{x} \to e^{\lambda}\boldsymbol{x}, \qquad t \to e^{2\lambda}t, \qquad \psi \to e^{-\lambda D}\psi$$

  - Special conformal transformations

$$\boldsymbol{x} \to \frac{\boldsymbol{x}}{1+\lambda' t}, \qquad t \to \frac{t}{1+\lambda' t}, \qquad \psi \to \psi' = \dots$$

- 12 Parameters
- Generators: H, $\boldsymbol{P}$, $\boldsymbol{L}$, $\boldsymbol{K}$, D, C, satisfy Schrödinger algebra

Unitary limit

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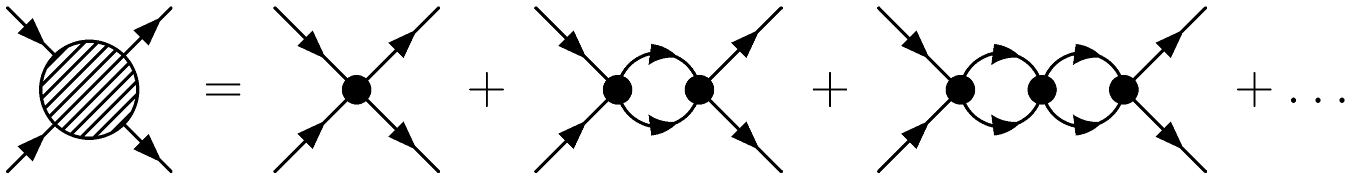
- Spin-1/2 Fermions with zero-range interactions ($|a| \gg r_e$)



- Renormalization group equation: $$\Lambda \frac{d}{d\Lambda} \tilde{g}_2 = \tilde{g}_2 (1 + \tilde{g}_2)$$
- Two fixed points:

  – $\tilde{g}_2 = 0 \;\Leftrightarrow\; a = 0 \quad \Longrightarrow$ no interaction

  – $\tilde{g}_2 = -1 \;\Leftrightarrow\; 1/a = 0 \quad \Longrightarrow$ unitary limit

  $\Longrightarrow$ conformal/Schrödinger symmetry

  (Mehen, Stewart, Wise, PLB **474**, 145 (2000); Nishida, Son, PRD **76**, 086004 (2007); ...)

- Neutrons: $a \approx -18.6$ fm, $r_e \approx 2.8$ fm

  $\Rightarrow$ neutrons are approximately conformal

Conformal field theory

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- Two-point function of primary field operator $\mathcal{U}$ ("un-nucleus")

$$G_{\mathcal{U}}(t,\mathbf{x}) = -i\langle T\mathcal{U}(t,\mathbf{x})\mathcal{U}^\dagger(0,\mathbf{0})\rangle = C\,\frac{\theta(t)}{(it)^\Delta}\exp\left(\frac{iMx^2}{2t}\right)$$

- Determined by symmetry up to overall constant C
- Two-point function in momentum space

$$G_{\mathcal{U}}(\omega,\boldsymbol{p}) = -C\left(\frac{2\pi}{M}\right)^{3/2}\Gamma\left(\frac{5}{2}-\Delta\right)\left(\frac{p^2}{2M}-\omega\right)^{\Delta-\frac{5}{2}}$$

  - pole only for $\Delta = 3/2$ (free field)
  - cut for $\Delta > 3/2$
- General un-nucleus (unparticle) does not behave like a particle

Conformal field theory

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- Imaginary part of propagator

$$\operatorname{Im} G_{\mathcal{U}}(\omega, \boldsymbol{p}) \sim \begin{cases} \delta\left(\omega - \frac{p^2}{2M}\right), & \Delta = \frac{3}{2}\,, \\ \left(\omega - \frac{p^2}{2M}\right)^{\Delta - \frac{5}{2}} \theta\left(\omega - \frac{p^2}{2M}\right), & \Delta > \frac{3}{2} \end{cases}$$

- Examples of un-nuclei
  - free field: $\mathcal{U} = \psi, \quad M = m_\psi, \quad \Delta = 3/2$
  - N free fields: $\mathcal{U} = \psi_1 \ldots \psi_N, \quad M = N m_\psi, \quad \Delta = 3N/2$
  - N interacting fields: $\mathcal{U} = \psi_1 \ldots \psi_N, \quad M = N m_\psi, \quad \Delta > 3/2$
- In our case: un-nucleus is strongly interacting multi-neutron state with

$$\underbrace{1/(ma^2)}_{0.1\ \text{MeV}} \ll E_n^{cms} \ll \underbrace{1/(mr_e^2)}_{5\ \text{MeV}}$$

Scaling dimension

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- How to calculate scaling dimension Δ?

(1) Δ can be obtained from field theory calculation

(2) Δ can be obtained from operator state correspondence

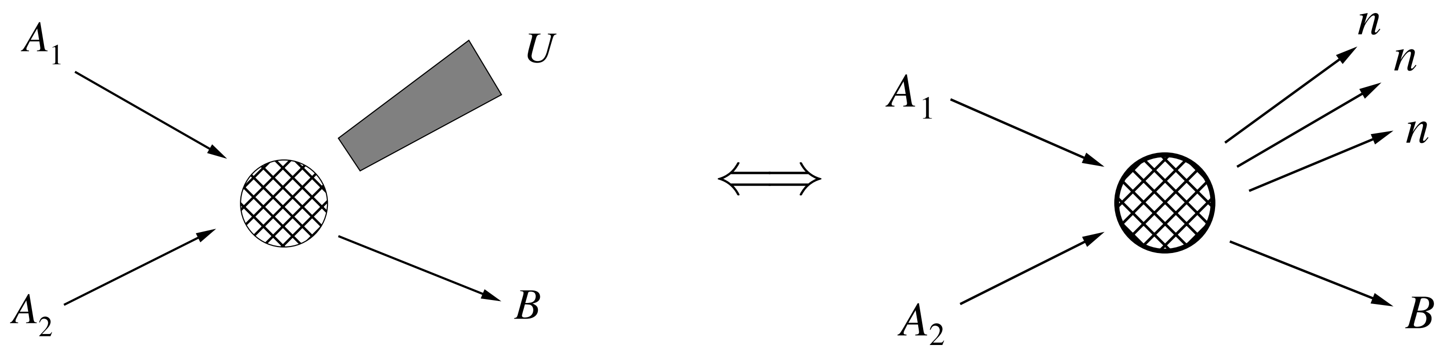
Δ of primary operator = (Energy of state in HO)/$\hbar\omega$

(Nishida, Son, Phys. Rev. D **76**, 086004 (2007))

N	S	L	$\mathcal{O}$	Δ
2	0	0	$\psi_1\psi_2$	2
3	1/2	1	$\psi_1\psi_2\nabla_j\psi_2$	4.27272
3	1/2	0	$\psi_1\nabla_j\psi_2\nabla_j\psi_2$	4.66622
4	0	0	$\psi_1\psi_2\nabla_j\psi_1\nabla_j\psi_2$	5.07(1)
5	1/2	1	$\ldots$	7.6(1)

Reactions with neutrons

- Application: High-energy nuclear reaction with final state neutrons



$$E_{\text{kin}} = (M_{A_1} + M_{A_2} - M_B - M_{\mathcal{U}})c^2 + \frac{p_{A_1}^2}{2M_{A_1}} + \frac{p_{A_2}^2}{2M_{A_2}} = E_B + E_{\mathcal{U}}$$

- Assumption: energy scale of primary reaction $\gg E_{\mathcal{U}} - \frac{\boldsymbol{p}^2}{2M_{\mathcal{U}}} = E_n^{cms}$
- Factorization: $$\frac{d\sigma}{dE} \sim |\mathcal{M}_{primary}|^2 \operatorname{Im} G_{\mathcal{U}}(E_{\mathcal{U}}, \boldsymbol{p})$$
- Reproduces Watson-Migdal treatment of FSI for $2n$

  (Watson, Phys. Rev. **88**, 1163 (1952); Migdal, Sov. Phys. JETP **1**, 2 (1955))

Reactions with neutrons

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- Two ways to do experiments

  (a) detect recoil particle B

$$\frac{d\sigma}{dE} \sim (E_0 - E_B)^{\Delta - 5/2}, \qquad E_0 = (1 + M_B/M_{\mathcal{U}})^{-1} E_{\text{kin}}$$

  (b) detect all final state particles including neutrons

$$\frac{d\sigma}{dE} \sim (E_n^{cms})^{\Delta - 5/2}$$

- Consistent with previous experiments for $^3\text{H}(\pi^-, \gamma)3n$

  (Miller et al., Nucl. Phys. A **343**, 347 (1980))

- Two few events in recent tetraneutron experiment: $^4\text{He}(^8\text{He}, ^8\text{Be})4n$

  (Kisamori et al., Phys. Rev. Lett. **116**, 052501 (2016))

Reaction calculations

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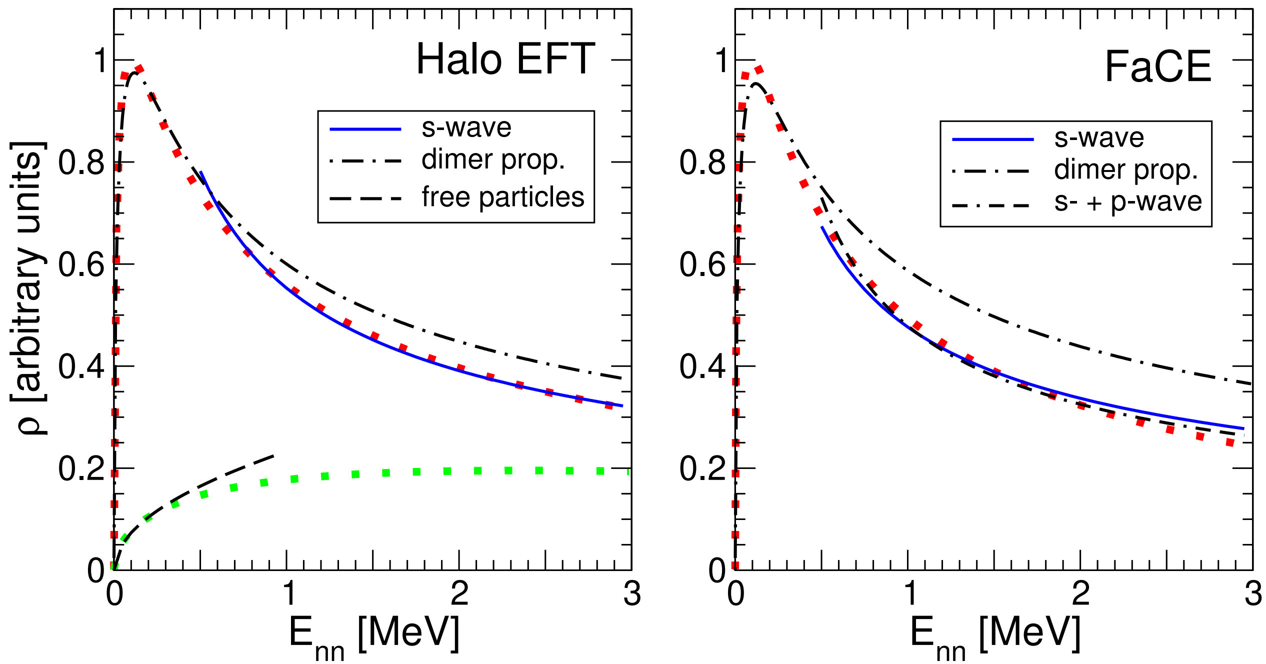
- Two-neutron spectrum for $^{6}\mathrm{He}(p,p\alpha)2n$ (Göbel et al., arXiv:2103.03224)

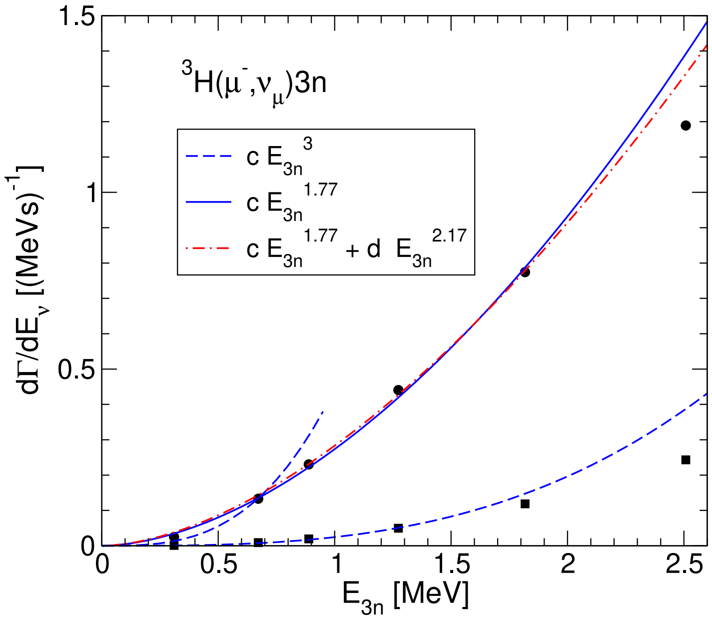


- Can be understood from dimer propagator ($\Delta = 2$)

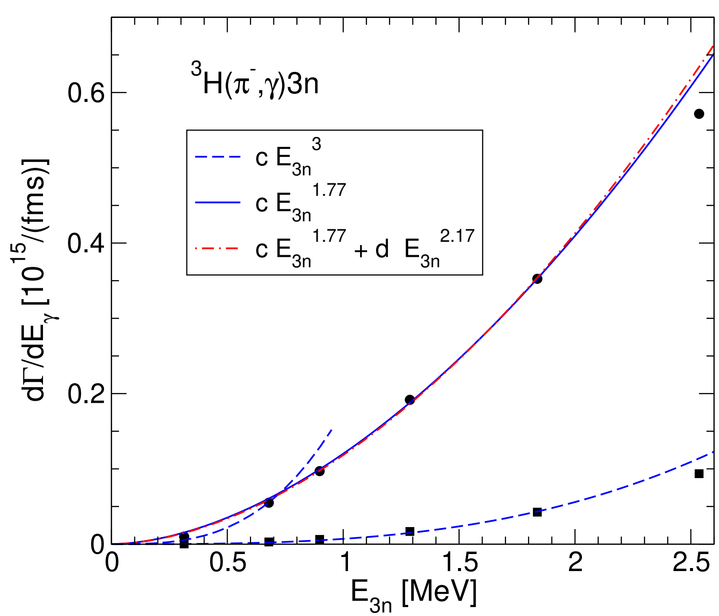
$$G_d(E_{nn}, \mathbf{0}) \sim \frac{1}{1/a + i\sqrt{mE_{nn}}} \quad \Rightarrow \quad \mathrm{Im}\, G_d(E_{nn}, \mathbf{0}) \sim \frac{\sqrt{E_{nn}}}{(ma^2)^{-1} + E_{nn}}$$

Reaction calculations

- Radiative muon/pion capture on the triton (AV18 + UIX)



Golak et al., PRC **98**, 054001 (2018)



Golak et al., PRC **94**, 054001 (2016)

- Un-nucleus behavior prediction

$$\frac{d\Gamma}{dE} \sim (E_{3n})^{4.27272-5/2} \sim (E_{3n})^{1.77272}, \qquad 0.1\ \mathrm{MeV} \ll E_{3n} \ll 5\ \mathrm{MeV}$$

New experiments

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- New experiments in complete kinematics at RIBF/RIKEN
- Measurement of a_{nn} in $^{6}\mathrm{He}(p, p\alpha)2n$
  (T. Aumann et al., NP2012-SAMURAI55R1 (2020))
  - un-nucleus behavior prediction

$$\frac{d\rho}{dE} \sim (E_{2n})^{2-5/2} \sim (E_{2n})^{-0.5}, \qquad 0.1\ \mathrm{MeV} \ll E_{2n} \ll 5\ \mathrm{MeV}$$

- Search for tetraneutron resonances in $^{8}\mathrm{He}(p, p\alpha)4n$
  (S. Paschalis et al., NP1406-SAMURAI19R1 (2014))
  - un-nucleus behavior prediction

$$\frac{d\rho}{dE} \sim (E_{4n})^{5.07-5/2} \sim (E_{4n})^{2.57}, \qquad 0.1\ \mathrm{MeV} \ll E_{4n} \ll 5\ \mathrm{MeV}$$

Summary and Outlook

- Universality in the unitary limit

  $\Rightarrow$ (approximate) conformal symmetry
  $\Rightarrow$ power law behavior of observables determined by Δ

- Application to high-energy nuclear reactions with neutrons
- Model-independent constraints on nuclear reactions
- Connection between reactions & properties of trapped particles

Summary and Outlook

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- Universality in the unitary limit

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- Application to high-energy nuclear reactions with neutrons
- Model-independent constraints on nuclear reactions
- Connection between reactions & properties of trapped particles
- Other applications & extensions
  - Two-component Fermions in ultracold atom physics
  - Neutral charm mesons (Braaten, HWH, arXiv:2107.02831)
  - Systems with the Efimov effect?

    $\Rightarrow$ bosonic atoms, nucleons, α particles
    $\Rightarrow$ complex scaling dimensions
    $\Rightarrow$ scale symmetry broken