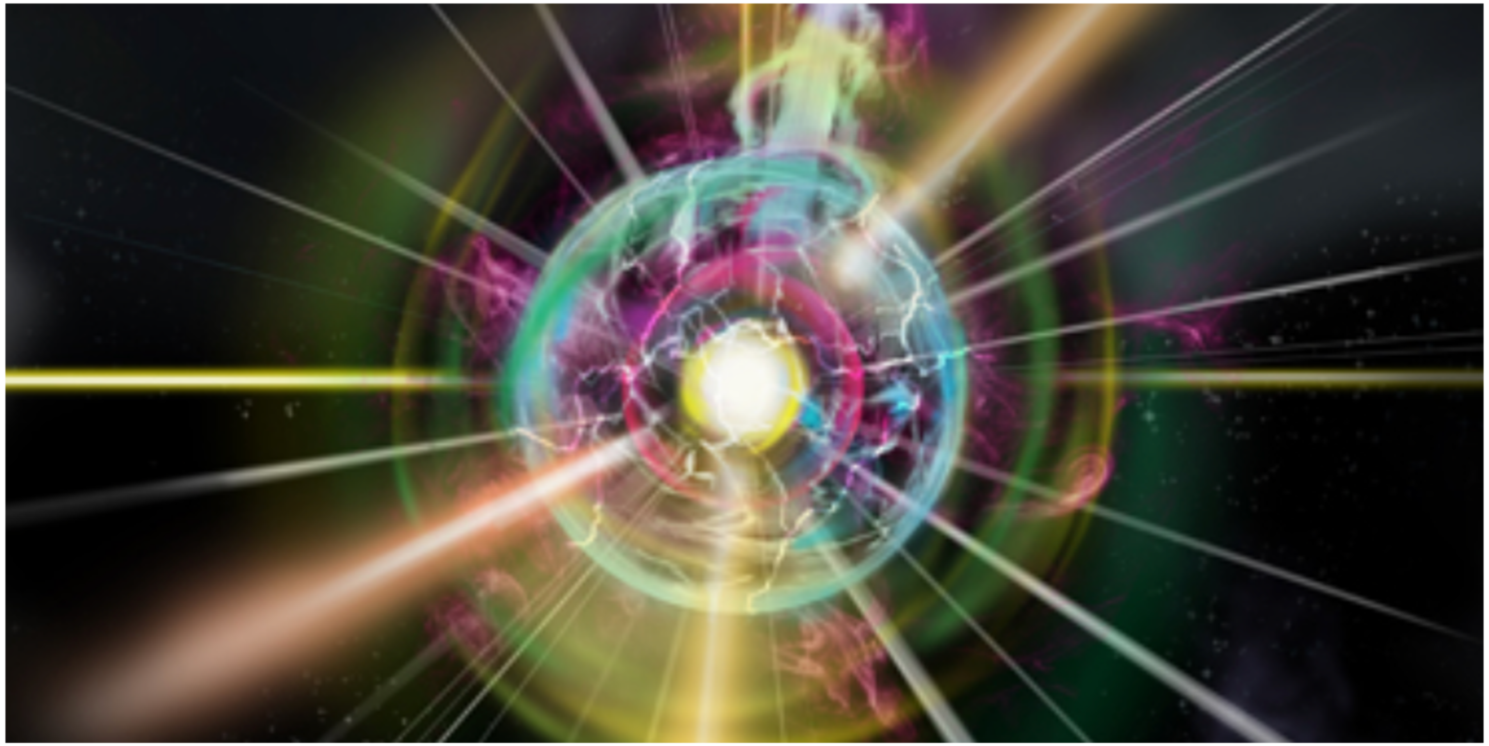
Rotations of odd-mass nuclei: magnetic monopoles and Berry phases

Nuclear Physics Meets Condensed Matter, virtual ECT* workshop, 21 July 2021

(image credit: Heikka Valja)



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Rotational excitations of atomic nuclei are well understood

- Odd-mass nuclei are usually described in the particle-rotor model, where a nucleon with spin $\vec{K}$ is coupled to a rotor with spin $\vec{R}$ to total spin $\vec{I} = \vec{R} + \vec{K}$

- Rotor Hamiltonian

$$H = \sum_{k=x',y',z'} \frac{R_k^2}{2C_k}$$

leads to particle-rotor Hamiltonian

$$H_{\rm rot} = \sum_{k=x',y',z'} \frac{(I_k - K_k)^2}{2C_k}$$

- Q: How can this be understood in a Lagrangian approach?
- Q: How can one systematically improve such a description?

Effective field theories for heavy nuclei

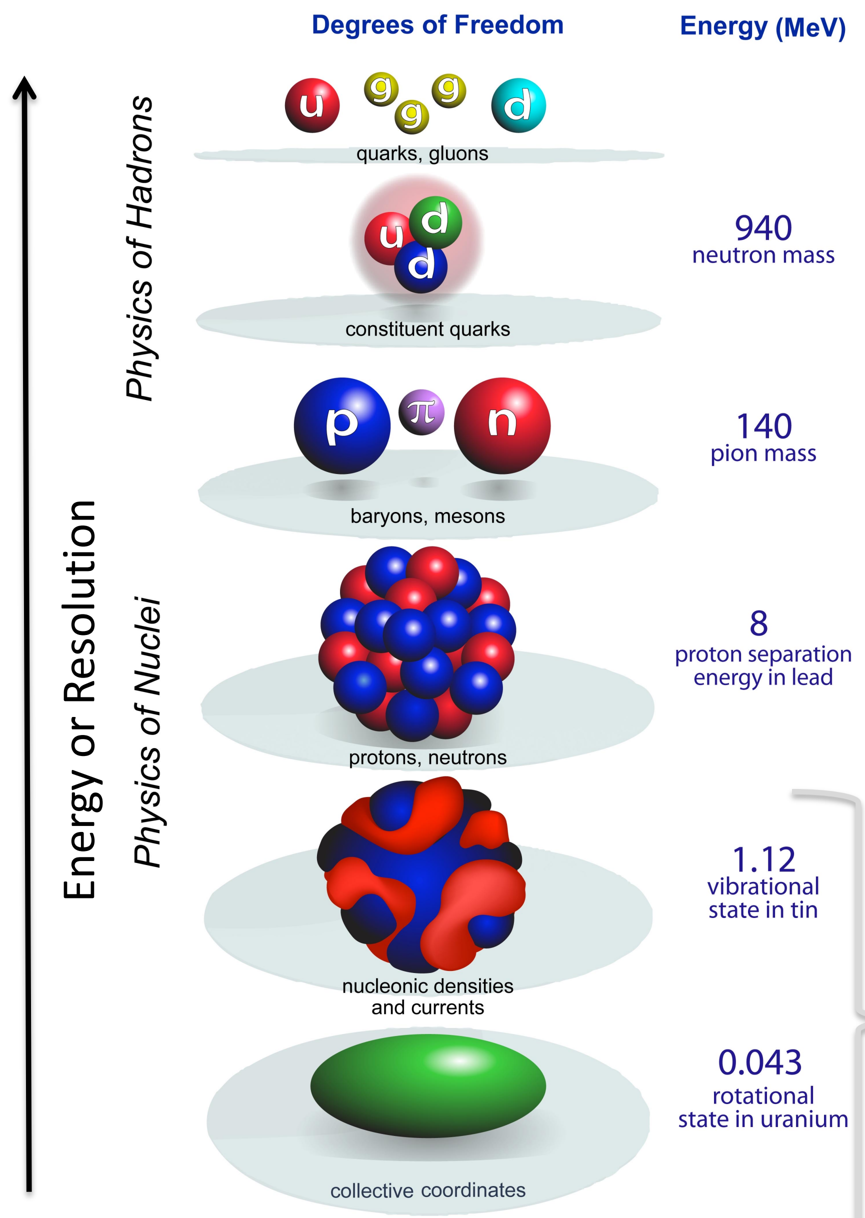
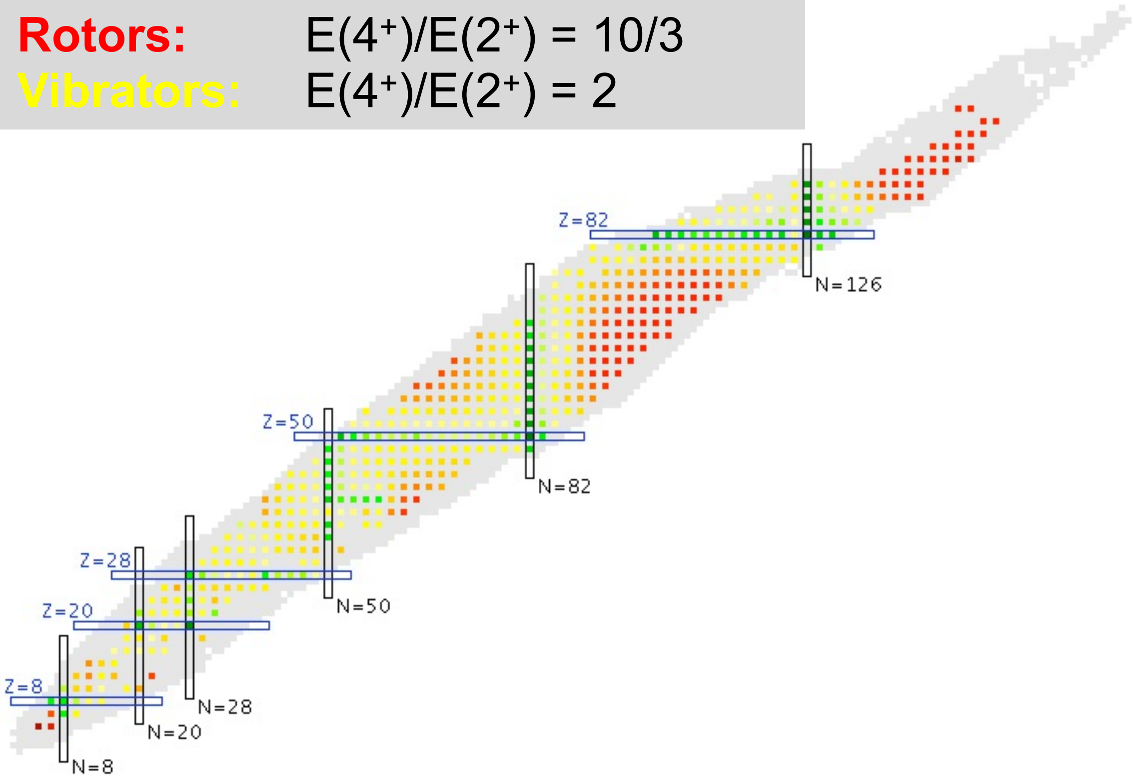


Fig.: Bertsch, Dean, Nazarewicz (2007)

Rotors: $E(4^+)/E(2^+) = 10/3$

Vibrators: $E(4^+)/E(2^+) = 2$



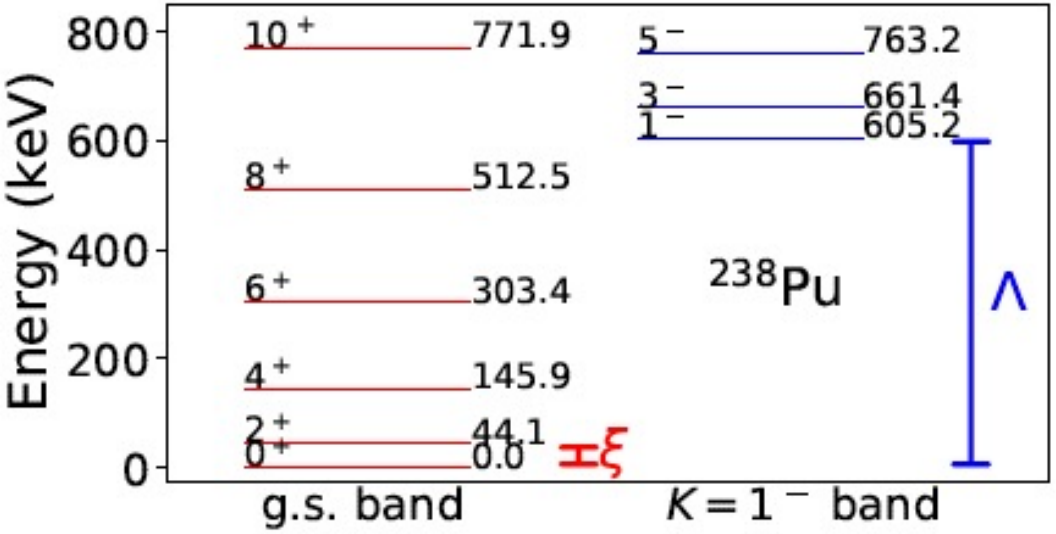
Vibrators: EFT based on linear (Wigner/Weyl) realization [Coello Pérez & TP 2015; 2016; Coello Pérez, Menéndez & Schwenk 2018]

Rotors: EFTs based on non-linear realization of SO(3)

Axially symmetric nuclei: [TP 2011; TP & Weidenmüller 2014; Coello Pérez & TP 2015; TP & Weidenmüller arXiv:2005.11865; Alnamlah, Coello Pérez, Phillips arXiv:2011.01083]

Triaxial deformation: [Chen, Kaiser, Meißner, Meng 2017; 2018; 2020]

Scales in heavy deformed nuclei

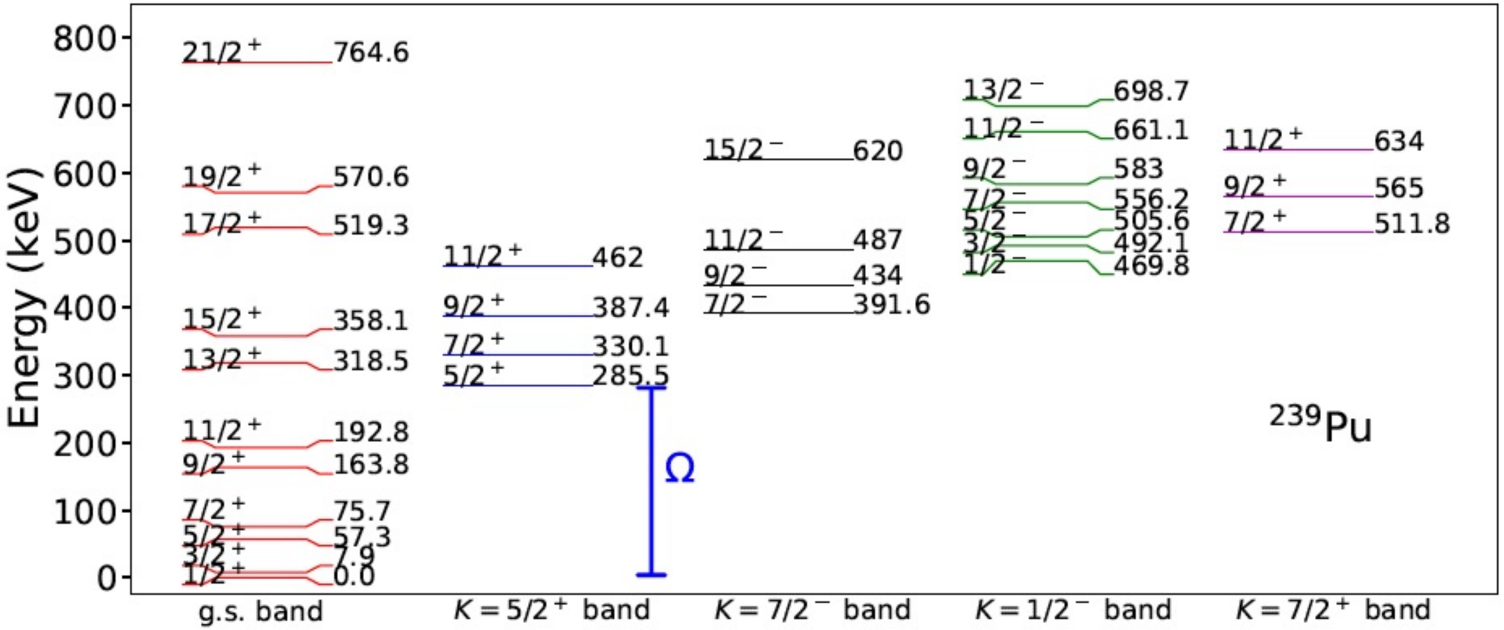


ξ = the small energy scale of interest

Λ = breakdown scale

Ω = single-particle scale of fermion

EFT exploits the small ratio $\frac{\xi}{\Lambda} \ll 1$



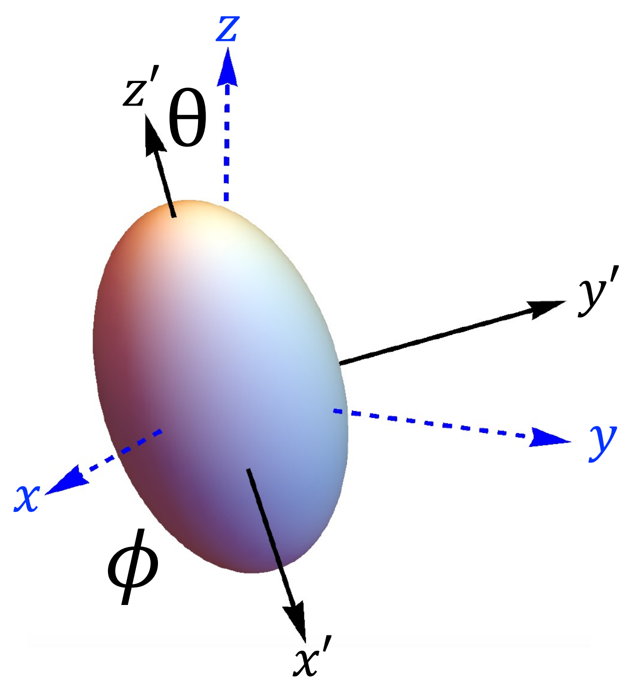
Effective Field Theory for Deformed Nuclei in a Nutshell 1

The EFT for deformed nuclei works at lowest resolution; expansion in small parameter $\frac{\xi}{\Lambda} \ll 1$

- In the EFT for deformed nuclei we deal with emergent symmetry breaking from $SO(3) \to SO(2)$, and the degrees of freedom (θ, ϕ) are combined in the radial unit vector

$$\vec{e}_r(\theta,\phi) = (\cos\phi \sin\theta\,,\ \sin\phi\sin\theta\,,\ \cos\theta)^T$$

which parametrizes the coset $SO(3)/SO(2) \sim S^2$, i.e. the unit sphere.

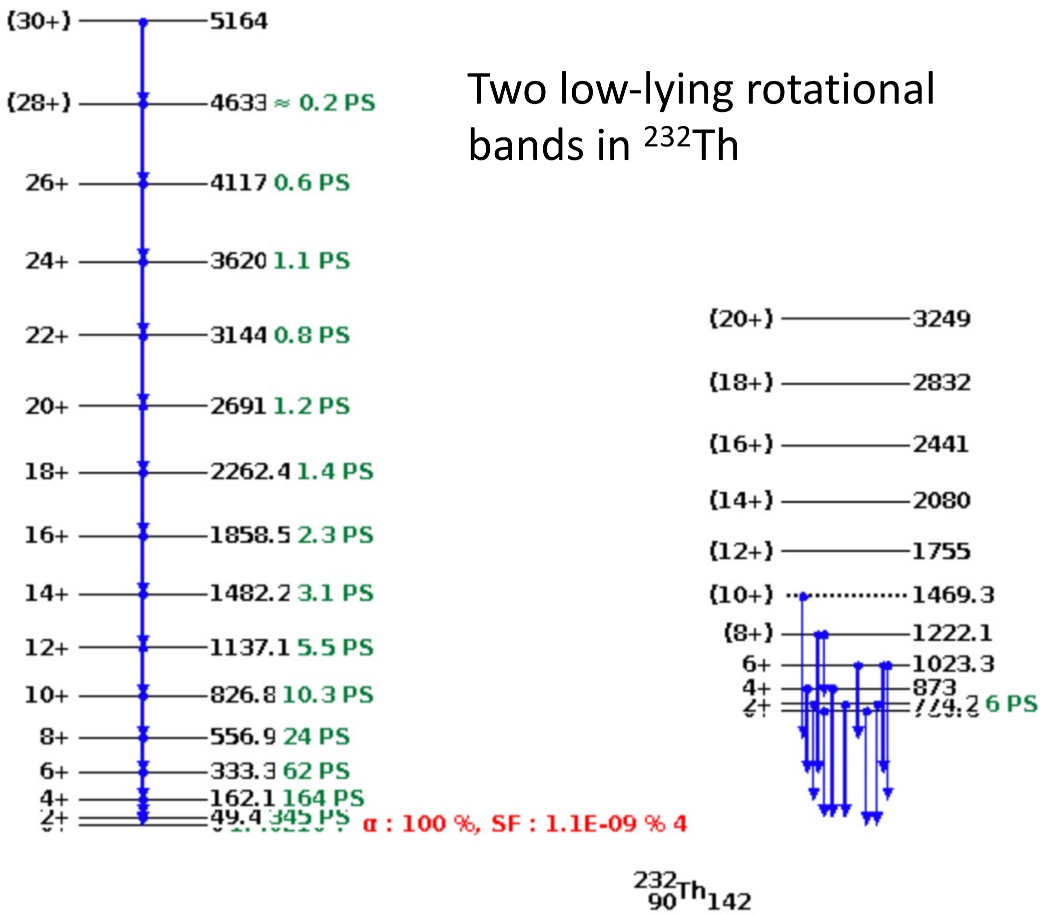


- Nambu-Goldstone bosons appear with derivatives only;
  simplest Lagrangian is that of a rotor $L = \frac{C_0}{2}\left(\frac{d}{dt}\,\vec{e}_r(\theta,\phi)\right)^2$
- Hamiltonian $H = \frac{J(J+1)}{2C_0}$
- EFT = expansion in powers of $\left(\frac{d}{dt}\,\vec{e}_r(\theta,\phi)\right)^2$

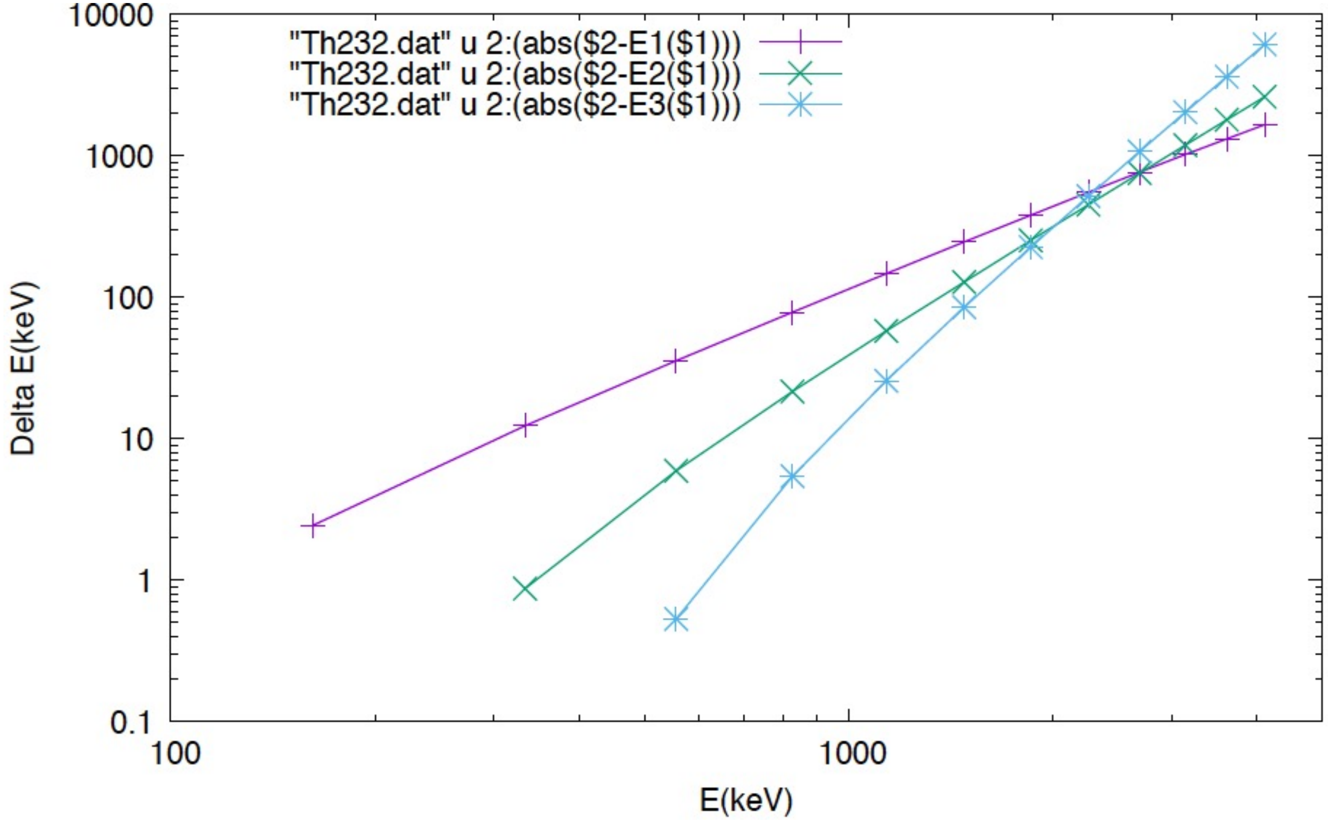
[TP 2011; TP & Weidenmüller 2014, 2015; Coello Pérez & TP 2015, 2016; Chen, Kaiser, Meißner & Meng 2017, 2018, 2020; Alnamlah, Coello Pérez & Phillips 2020]

^{232}Th as an example

Two low-lying rotational bands in ^{232}Th



"Lepage plot," → P. Lepage, arXiv:nucl-th/9706029



Q1: What is the low-energy scale ξ?

A1: $\xi \approx 50$ keV

Q2: What is the breakdown scale Λ?

A2: (from Lepage plot) $\Lambda \approx 2400$ keV

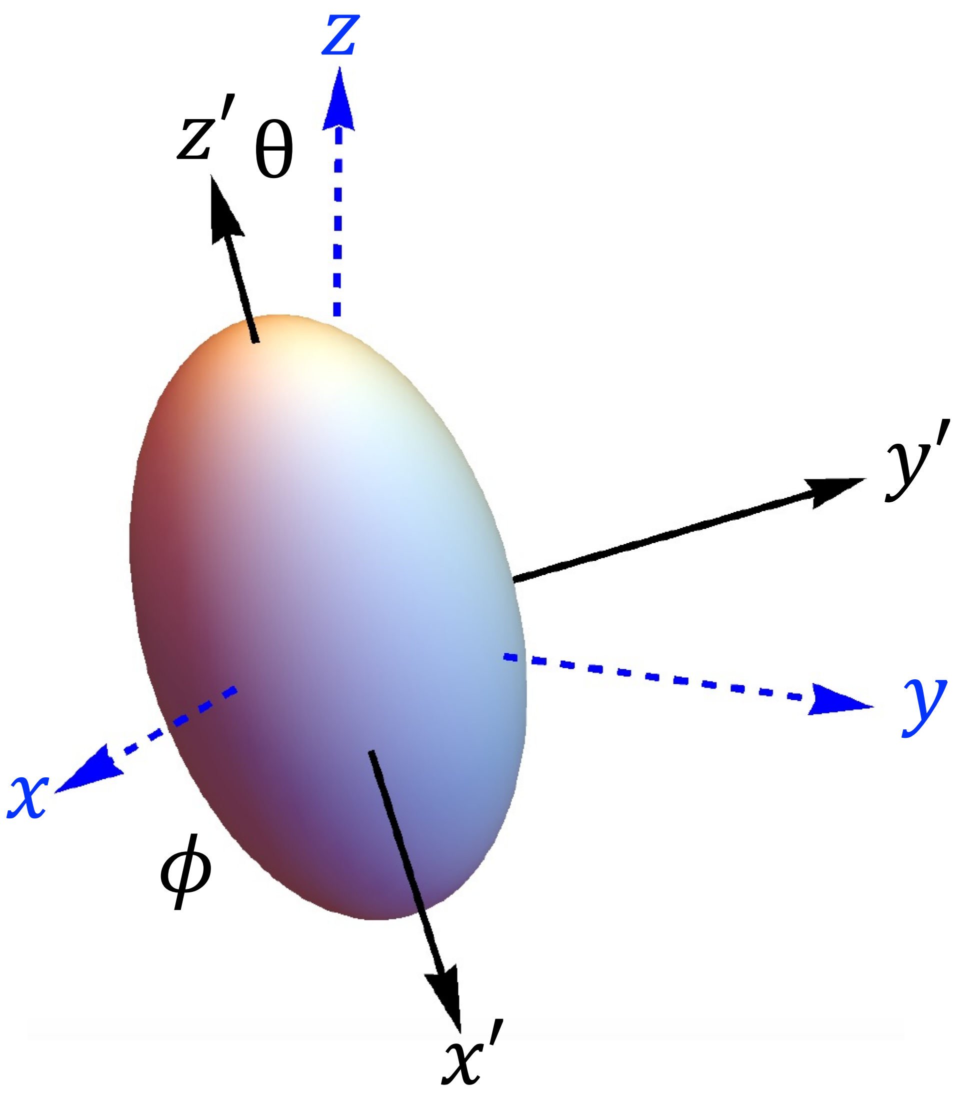
LO: $E_{LO} = aI(I+1)$

NLO: $E_{NLO} = aI(I+1) + b[I(I+1)]^2$

NNLO: $E_{NNLO} = aI(I+1) + b[I(I+1)]^2 + c[I(I+1)]^3$

→ Alnamlah, Coello Pérez, Phillips arXiv:2011.01083

Axially symmetric even-even nucleus



- Effective field theory: Nonlinear realization of $SO(3)$ in case of spontaneous symmetry breaking down to axial $SO(2)$ [Weinberg 1968, Callan, Coleman, Wess & Zumino 1969]: Degrees of freedom parameterize the unit sphere, i.e. the coset $SO(3)/SO(2) \sim S^2$
- Traditional NP: We have an axially symmetric rotor, and its orientation is in direction of the angles (θ, ϕ).
- Berry: The body-fixed system is only defined up to rotations around the body-fixed symmetry (z') axis → gauge freedom

Effective Field Theory for Deformed Nuclei in a Nutshell 2

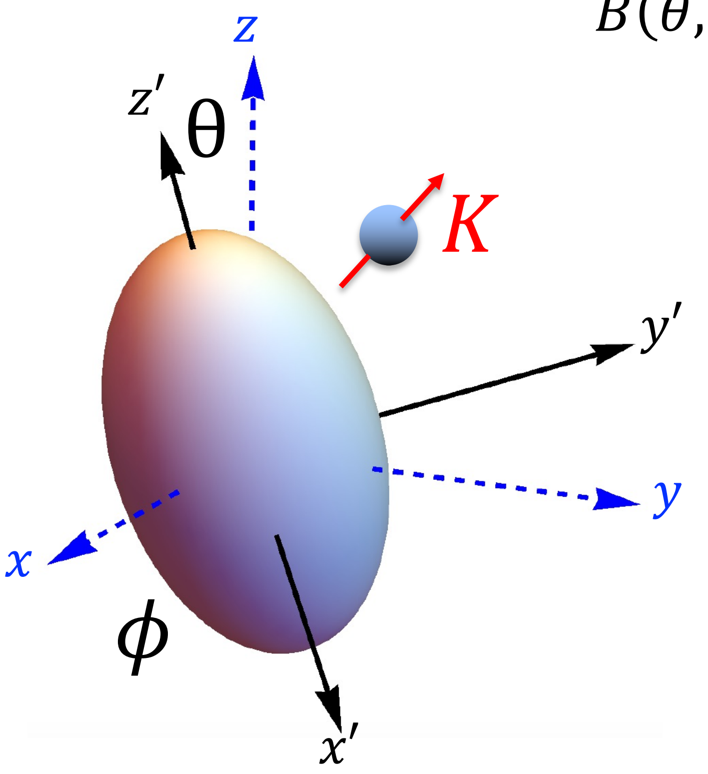
Nucleon (with spin $\vec{K}$) is a fast degrees of freedom; adiabatic motion generates gauge potentials

$$\vec{A}(\theta,\phi) = \left(\vec{e}_r(\theta,\phi)\cdot\vec{K}\right)\cot\theta\ \vec{e}_\phi(\theta,\phi)$$

$$\vec{A}(\theta,\phi) = g\vec{e}_r(\theta,\phi)\times\vec{K}$$

Corresponding "magnetic" fields (or Berry curvatures) are spherically symmetric monopoles

$$\vec{B}(\theta,\phi) = \vec{\nabla}\times\vec{A}(\theta,\phi) - i\vec{A}(\theta,\phi)\times\vec{A}(\theta,\phi)$$
$$= (g^2-1)\left(\vec{e}_r(\theta,\phi)\cdot\vec{K}\right)\vec{e}_r(\theta,\phi)$$

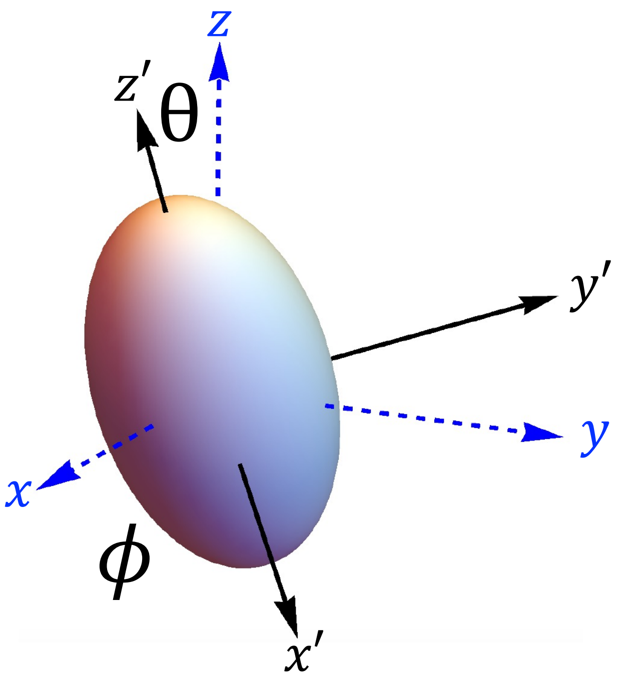


- The coupling to the rotor is via $\vec{A}(\theta,\phi)\cdot\frac{d}{dt}\vec{e}_r(\theta,\phi)$
- Lagrangian $L = \frac{C_0}{2}\left(\frac{d}{dt}\vec{e}_r(\theta,\phi)\right)^2 + \vec{A}(\theta,\phi)\cdot\frac{d}{dt}\vec{e}_r(\theta,\phi)$

[TP & Weidenmüller Phys. Rev. C 102, 044324 (2020); arXiv:2005.11865]

Gauge freedom

In an axially-symmetric nucleus, the body-fixed coordinate system is arbitrary with respect to rotations around the symmetry axis. A gauge function $\gamma = \gamma(\theta, \phi)$ can be introduced that specifies the angle between different coordinate systems [Littlejohn & Reinsch, Rev. Mod. Phys. 1997]



Deformed nuclei

$$\vec{A}(\theta, \phi) = K_{z'} \cot\theta \;\; \vec{e}_{\phi}(\theta, \phi)$$

Wigner $D^{I}_{MK_{z'}}(\phi, \theta, 0)$ functions are solutions

Wu Yang monopole

$$\vec{A}(\theta, \phi) = K_{z'} \frac{\cos\theta \pm 1}{\sin\theta} \vec{e}_{\phi}(\theta, \phi)$$

Monopole harmonics are solutions

Gauge function:

$$\gamma(\theta, \phi) = \pm\phi$$

→ T. Dray, “A unified treatment of Wigner D function, spin weighted spherical harmonics, and monopole harmonics,” J. Math. Phys. 27, 781 (1986)]

→ Solutions are $D^{I}_{MK_{z'}}(\phi, \theta, \gamma(\theta, \phi))$

Comments on rotational invariance

- The Abelian gauge potential is not invariant under rotations:

$$\vec{A}(\theta,\phi) = \left(\vec{e}_r(\theta,\phi)\cdot\vec{K}\right)\cot\theta\ \ \vec{e}_\phi(\theta,\phi)$$

- After a rotation, a gauge transformation can be used to bring the potential back into its original form [Fierz 1944].

- (The non-Abelian gauge potential clearly is invariant: $\vec{A}(\theta,\phi) = g\vec{e}_r(\theta,\phi)\times\vec{K}$)

Impact on the gauge potentials on spectra

- The Abelian gauge potential is leading order; (yields unremarkable shift of rotational band)

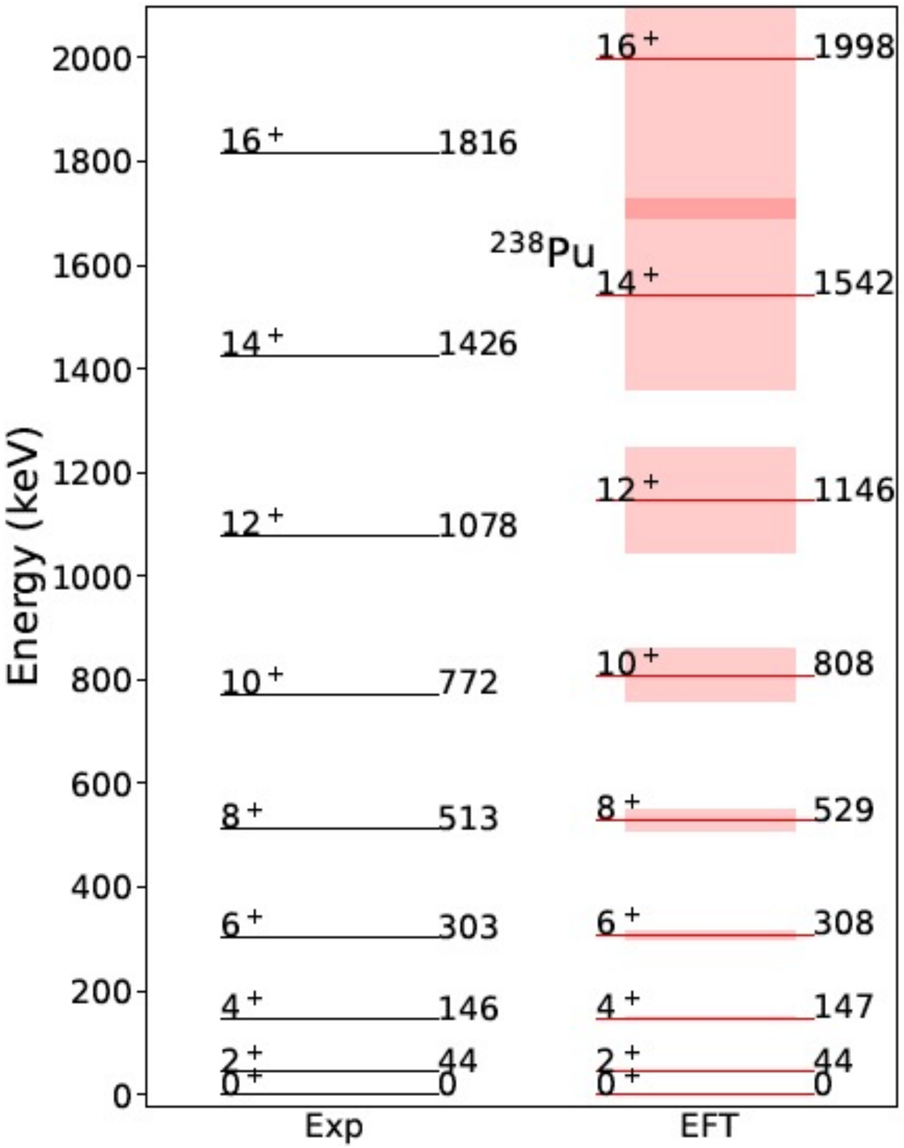
$$E(I,K) = a\,[I(I+1) - K^2]$$

- The non-Abelian gauge potential connects states that differ by one unit of K.
  - Visible impact in $|K| = 1/2$ bands (with substates $K = \pm 1/2$)

$$E(I,K) = a[I(I+1) - K^2] - 2ag\delta_{|K|}^{\frac{1}{2}}(-1)^{I+\frac{1}{2}}\left(I + \frac{1}{2}\right)$$

  - Can also impact rotational bands that are close in energy and differ in their band-head spins by one unit of K [Kerman 1956]

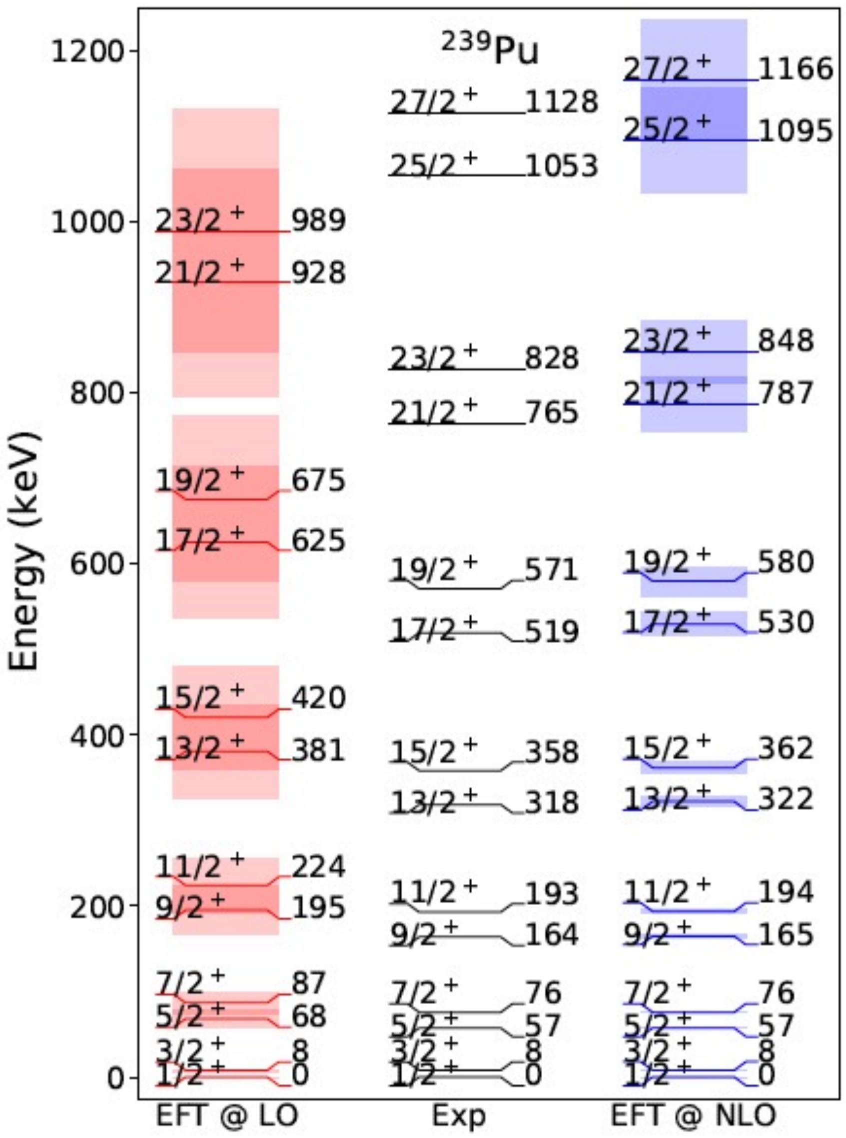
^{239}Pu as a neutron coupled to ^{238}Pu: finite $K = 1/2$ has Abelian and non-Abelian gauge potentials



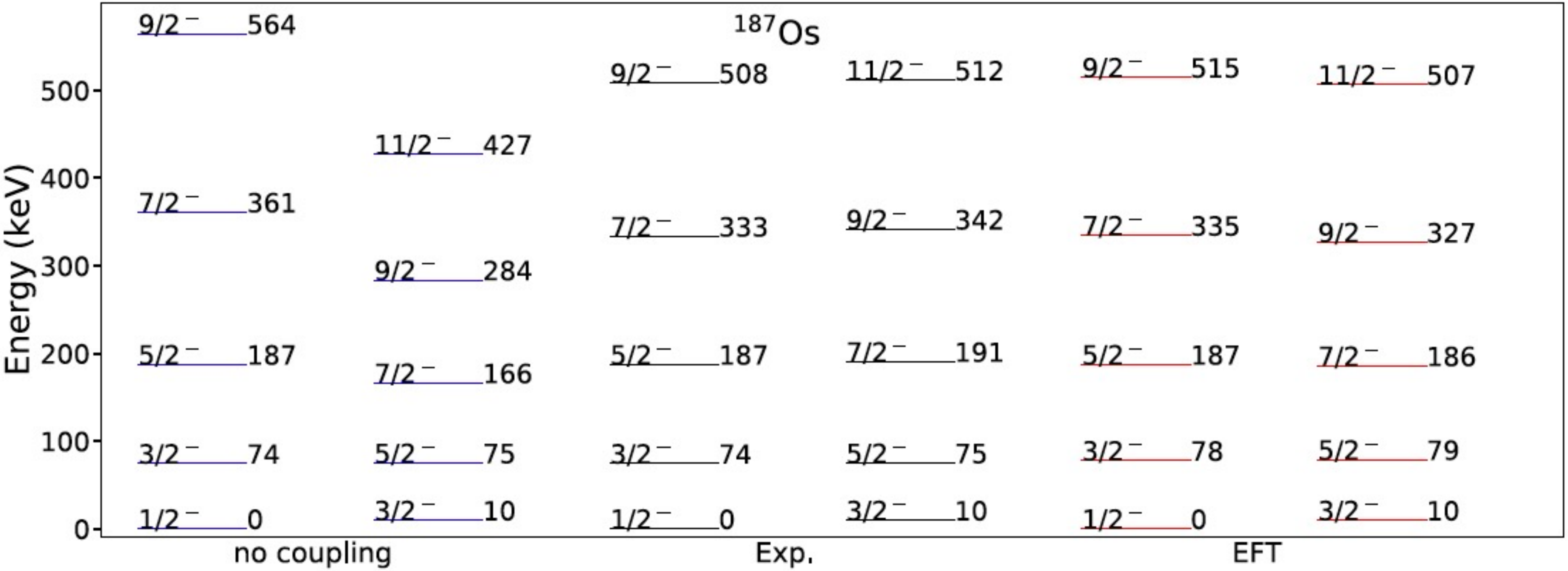
Uncertainty estimates based on power counting

Leading order: Take moment of inertia (MOI) from ^{238}Pu and adjust decoupling coefficient

Next-to-leading order: re-adjust MOI for ^{239}Pu



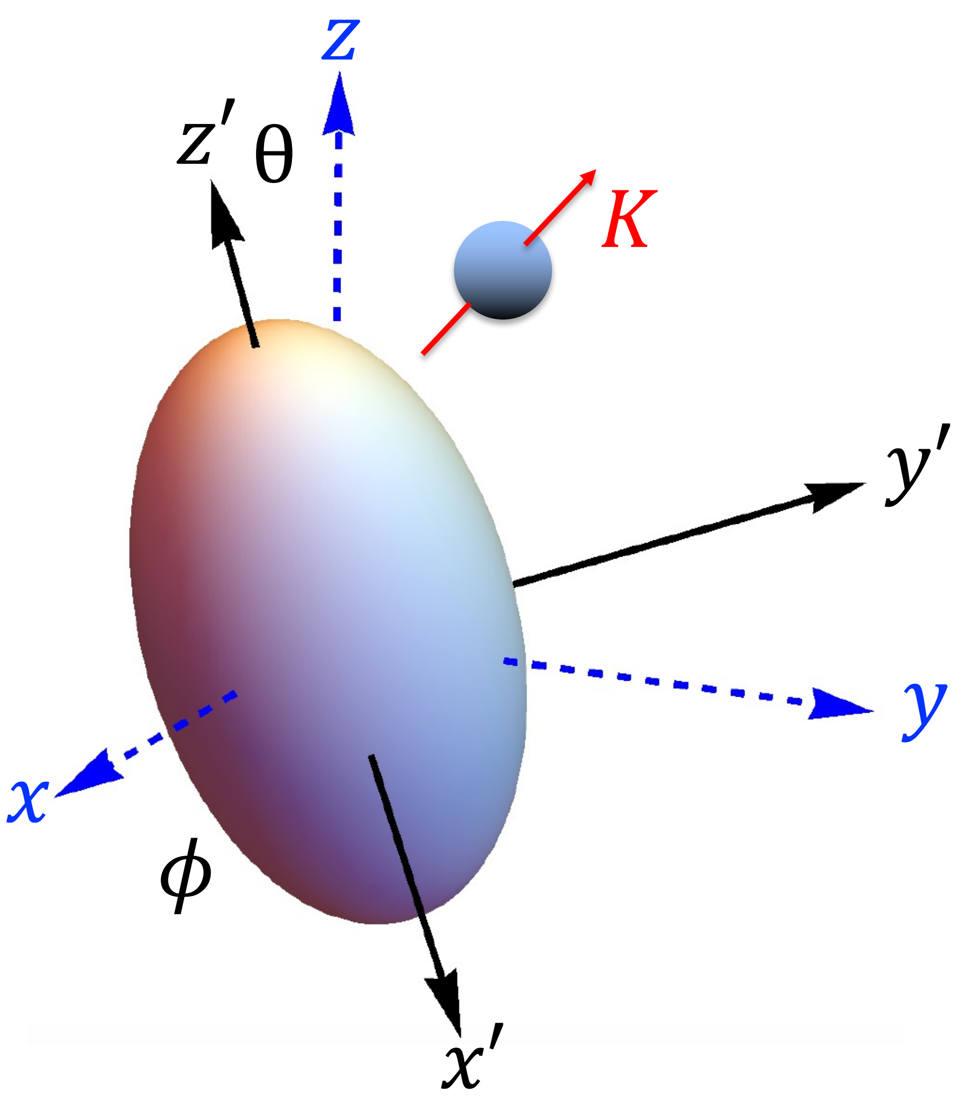
Impact of non-Abelian gauge potential: Coupling of rotational bands that differ by one unit in spin in ^{178}Os



Malmskog et al 1971;
Morgen et al 1973;
Sodan et al 1975

(Uncertainty estimates for the EFT are about 7%)

Odd nucleon coupled to even-even rotor



- Effective field theory: Non-linear realization of broken $SO(3)$: The dynamics of the odd nucleon is defined in the body-fixed system; it introduces a covariant derivative.

- Traditional NP: This is the “strong” coupling limit; Coriolis forces appear in the co-rotating body-fixed system

- Berry: The nucleon is much faster than the rotor. The adiabatic approximation introduces gauge potentials

Gauge potentials, Berry phases, and Coriolis forces

Different interpretations of the velocity–dependent rotor–nucleon couplings

1. **Coriolis forces** enter in rotating frames: Velocity-dependent forces are present in rotating nuclei [Bohr, Kerman, Mottelson, Nilsson 1950s].

2. **Molecular Aharonov-Bohm effect**: In rotating molecules, the nuclei are slow (and the electrons are fast), and the adiabatic decoupling (à la Born Oppenheimer) introduces Berry phases and gauge potentials [Mead & Truhlar 1979; Wilczek & Zee 1984; Kuratsuji & Iida 1985; Nazarewicz 1996].

3. **Covariant derivative**: In presence of spontaneous symmetry breaking, the rotational symmetry is realized non-linearly for the rotor's degrees of freedom. This introduces a covariant derivative $iD \equiv i\partial_t + \mathbf{v} \cdot A$ [Weinberg 1968; Callan, Coleman, Wess & Zumino 1969].

4. **Gauge invariance**: The ambiguities in defining a body-fixed frame, i.e. separating rotational and intrinsic degrees of freedom, imply a gauge invariance [Littlejohn & Reinsch 1997]. In our case: ambiguities regarding rotations around the *z'* axis.

[Leutwyler 1994; Roman & Soto 1999; Hofmann 1999; Chandrasekharan et al. 2008; Brauner 2010; …]

Nuclear Physics meets Condensed Matter

1. The odd-mass deformed rotor is equivalent to a particle on a sphere subject to Abelian and non-Abelian gauge potentials that are of the monopole type. The same Lagrangian governs the Quantum Hall Effect, see [B. Estienne, S. M. Haaker, and K. Schoutens, Particles in non-Abelian gauge potentials: Landau problem and insertion of non-Abelian flux, New J. Phys. 13, 045012 (2011)]. Relation between angular momentum projection and flux quanta $K_{z'} \leftrightarrow N_\phi$

2. The EFT for deformed nuclei [TP & Weidenmüller 2014] is an adaptation of a similar EFT for (anti)ferromagnets to the finite-system case [Leutwyler 1994; Roman & Soto 1999; Hofmann 1999; Chandrasekharan et al. 2008; Brauner 2010].

Falling Cat Problem

Q: How does a cat change its orientation, i.e. its angular momentum, without an external torque?

A: Changes in its shape (intrinsic degrees of freedom) induce a change in the external orientation.

Q: What does this has to do with odd-mass deformed nuclei?

A: In both cases, non-Abelian gauge potentials arise that describe the internal dynamics and couple it to the overall orientation. (In the nucleus, the odd nucleon causes the internal dynamics.)

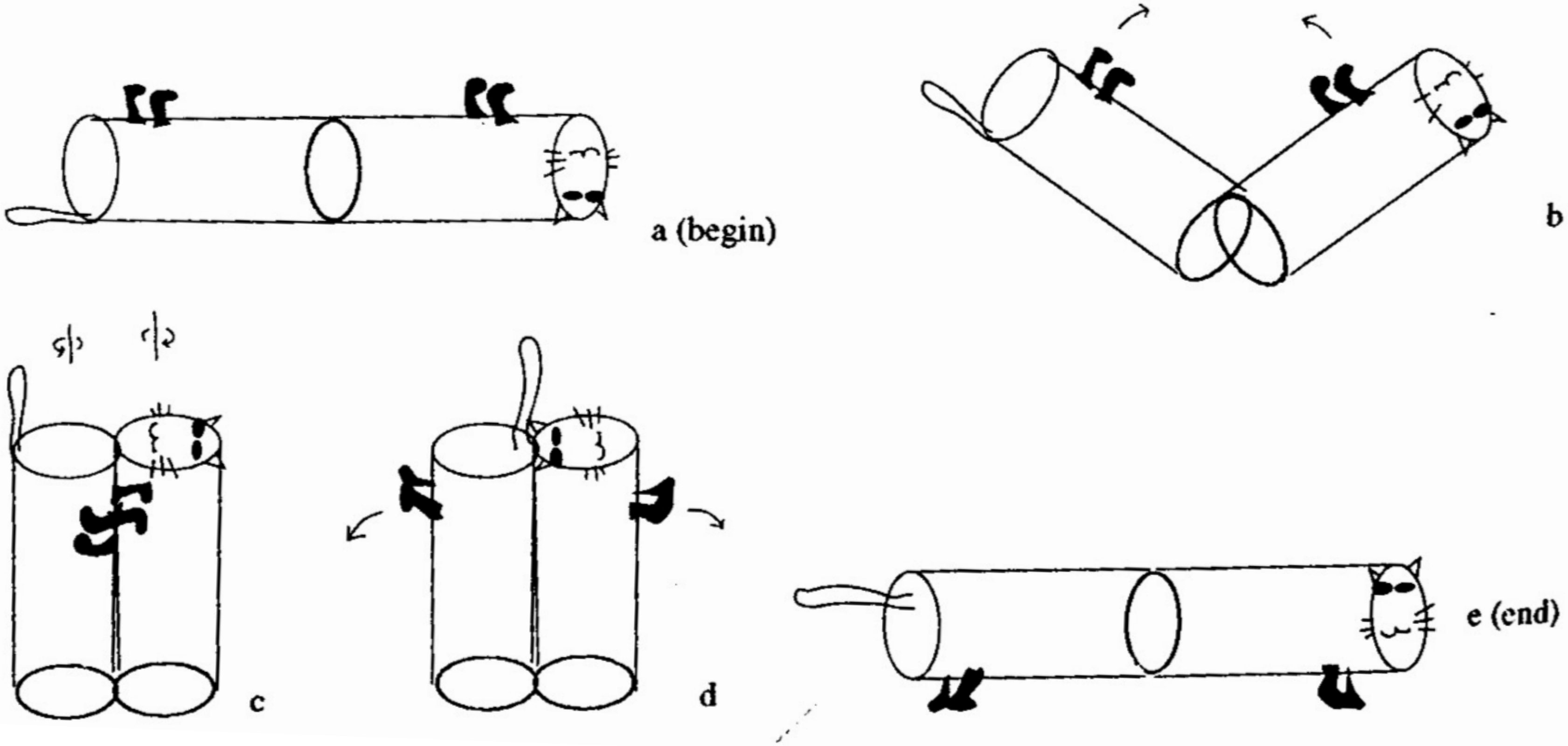
→ Gauge theory of deformable bodies

Shapere & Wilczek, *Geometric Phases in Physics* (1989); Littlejohn & Reinsch, Rev. Mod. Phys. (1997)

“Gauge theory of the falling cat,” Montgomery (1993)

6. Some Specific Reorientations and Steering Strategies

6.1. A Cartoon. Probably the simplest path resulting in the cat flip is the one depicted below.



“Bend, twist, unbend” makes a closed loop in internal configuration space while leading to a rotation.

Summary

- Develop effective theory for emergent symmetry breaking guided by standard approach in spontaneous symmetry breaking
- Lowest-resolution EFT in nuclear physics
- Systematically improvable approach
  - Re-discovers venerable models
  - Gives uncertainty estimates
- Odd nuclei naturally introduce gauge potentials and Berry phases
  - Abelian and non-Abelian gauge potentials generate monopole fields for the rotor
  - The fast nucleon adiabatically follows the slow rotor; only its spin projection onto the rotor's symmetry axis matters
  - These relate odd-mass deformed nuclei to falling cats