Fractional quantum Hall effect in a relativistic QFT
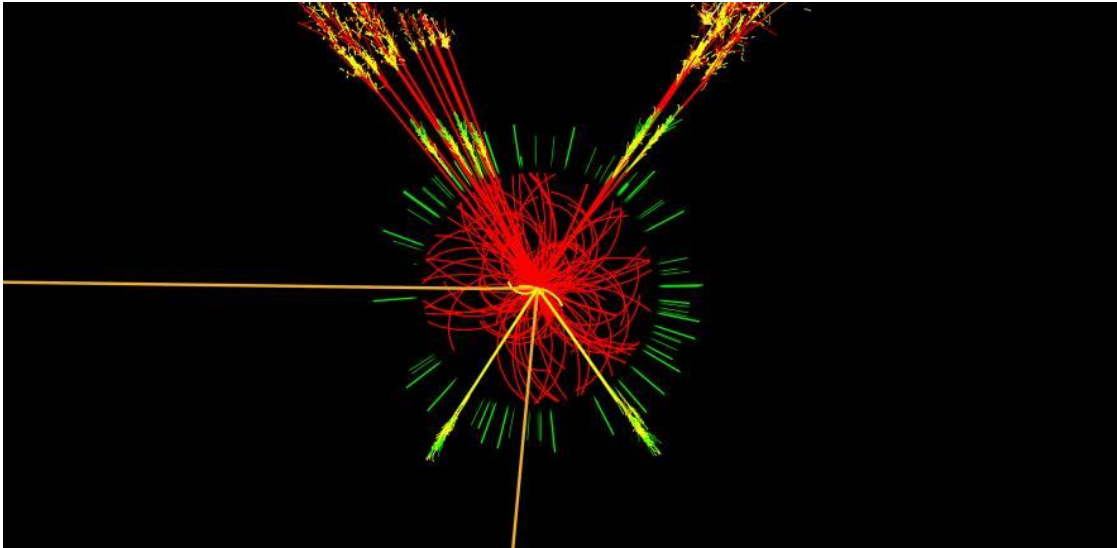

Figure courtesy: Nationalmaglab
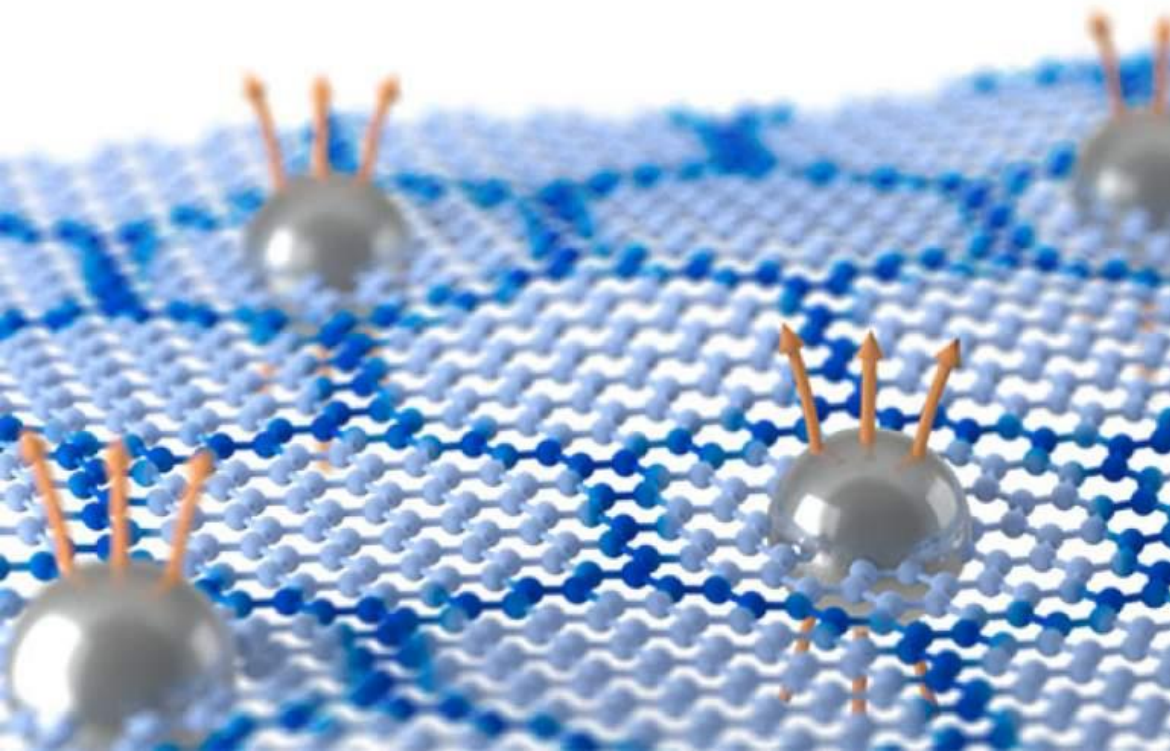
Srimoyee Sen,
Iowa State University
In collaboration with David B. Kaplan

Nuclear Physics meets Condensed Matter: symmetry, topology, and gauge, 2021

Questions of principle that span different areas of physics

Courtesy: U of Cambridge

Courtesy: sciencealert.com

Anderson-Higgs mechanism

Questions of principle that span different areas of physics (phase transitions)

Topological phases and metallic superconductors.

$$\mathcal{L}_{BF} = \frac{i}{2\pi} \epsilon^{\mu\nu\rho} B_\mu \partial_\nu A_\rho \text{ : Topological field theory}$$
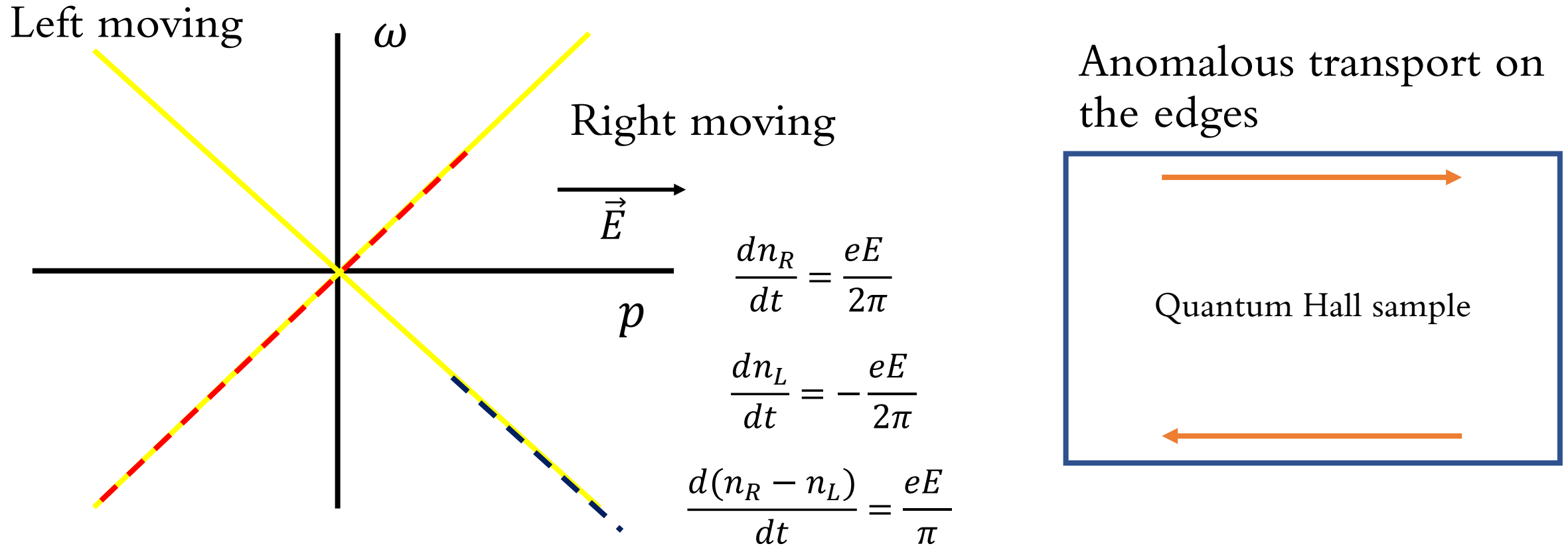
T. H. Hansson, Vadim Oganesyan, S. L. Sondhi

Courtesy: sciencealert.com
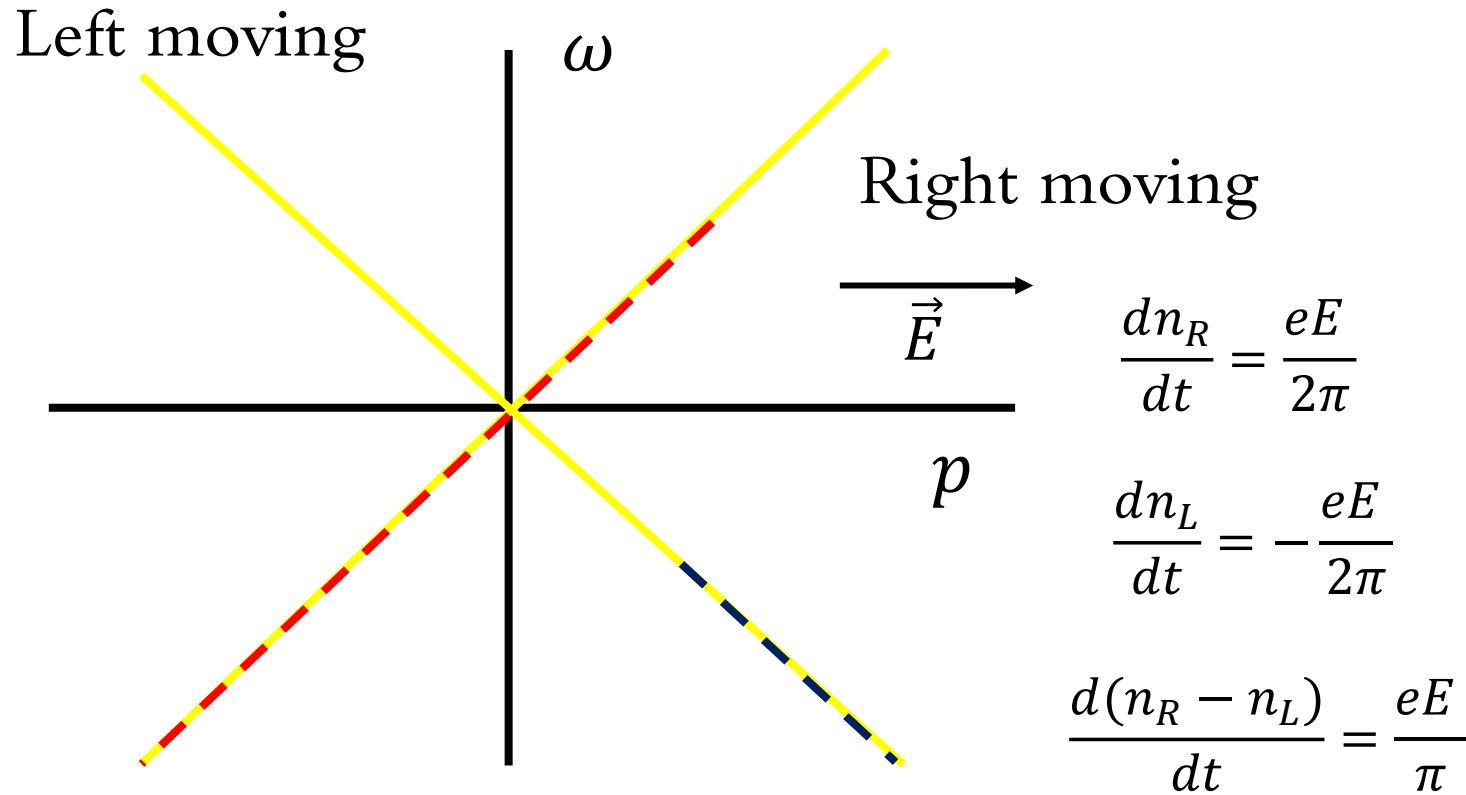
Intrinsic topological order and topological QFTs.

Insights into quark-Hadron phase transitions ?

Phys.Rev.D 102 (2020) 10, 105021, *Phys.Rev.D* 100 (2019) 3, 034015, Collaborators: Aleksey Cherman, Larry Yaffe, Theo Jacobson

Ideas that span different areas of physics

Left moving

Right moving

Anomalous transport on the edges

$$\frac{dn_R}{dt} = \frac{eE}{2\pi}$$

$$\frac{dn_L}{dt} = -\frac{eE}{2\pi}$$

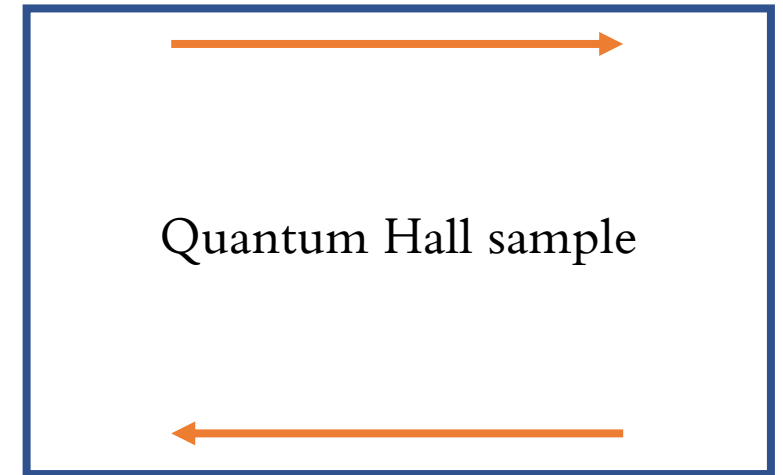$$\frac{d(n_R - n_L)}{dt} = \frac{eE}{\pi}$$

ω

p

$\vec{E}$

Quantum Hall sample

Chiral anomaly and the quantum Hall effect and lattice quantum field theory

Ideas that span different areas of physics

Left moving

Right moving

ω

p

$\vec{E}$

$$\frac{dn_R}{dt} = \frac{eE}{2\pi}$$

$$\frac{dn_L}{dt} = -\frac{eE}{2\pi}$$

$$\frac{d(n_R - n_L)}{dt} = \frac{eE}{\pi}$$

Anomalous transport on the edges

Quantum Hall sample

Chiral anomaly and the quantum Hall effect

Let's focus on this.

Anomaly and Quantum Hall Effect(QHE)
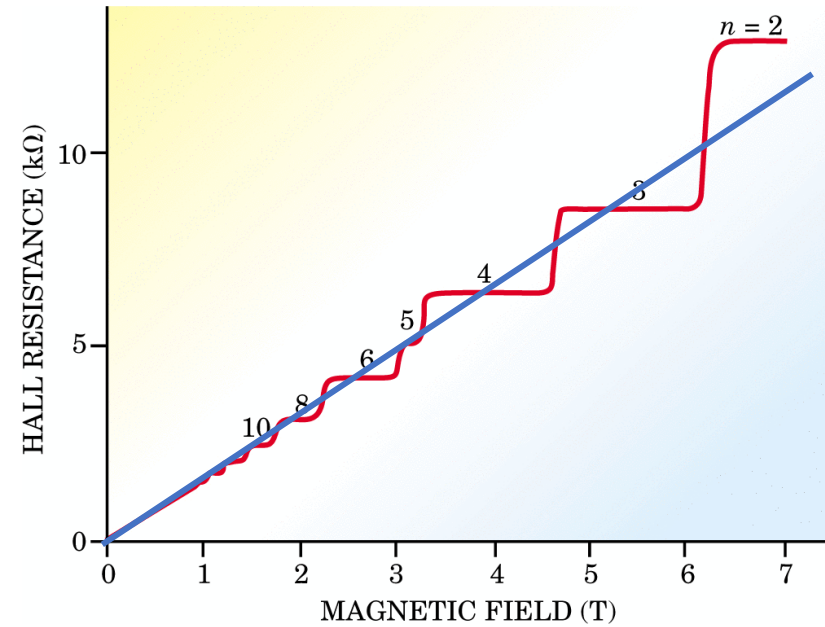
current

current

current

$\vec{E}$

Left mover

Right mover
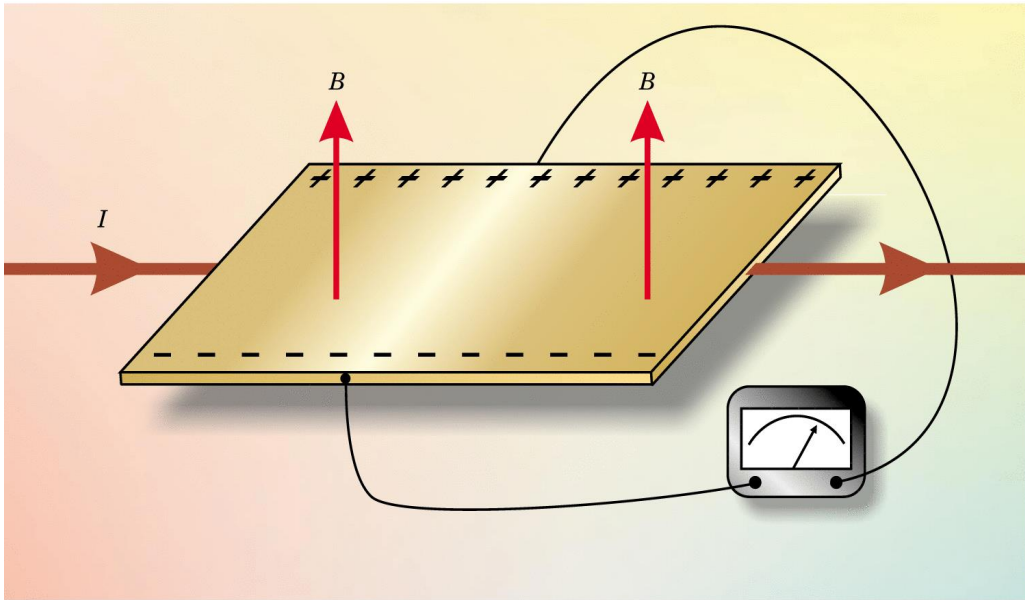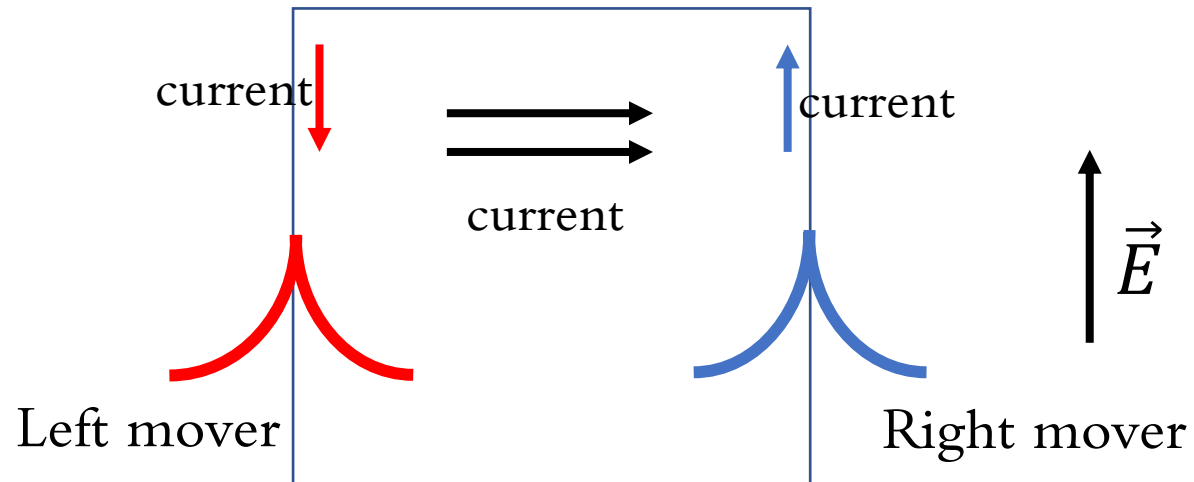
Integer (k) quantum Hall effect

Low energy EFT in the bulk: Chern-Simons theory of level k in $2 + 1$ dimensions.

$$S_{CS} = \frac{k}{4\pi} \int d^3x \; \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$

Integer quantized

Magnetic field breaks parity

Hall conductivity $= \dfrac{k\, e^2}{2\pi}$ $\longleftarrow$ $J_i = -\dfrac{k}{2\pi} \epsilon_{ij} E_j$

Current transverse to the electric field (and magnetic field)

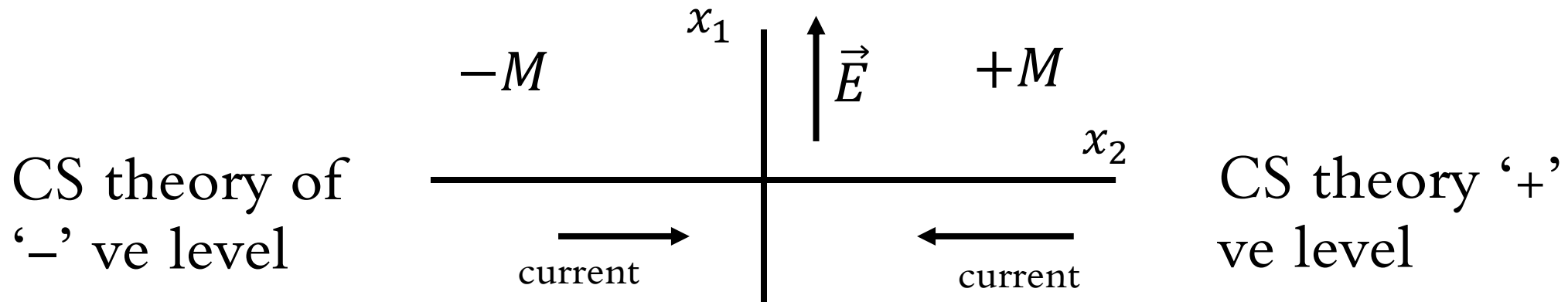Current conservation works due to k edge modes on the boundary $\longleftarrow$ Integer quantized

Relativistic QFT parallel for QHE (Callan-Harvey 1985)

Don't need magnetic fields.

Take abelian gauge theory coupled to k Heavy Dirac fermion in $2 + 1$ D.

Consider a domain wall in the fermion mass.

CS theory of '$-$' ve level

$-M$ x_1 $\vec{E}$ $+M$ x_2

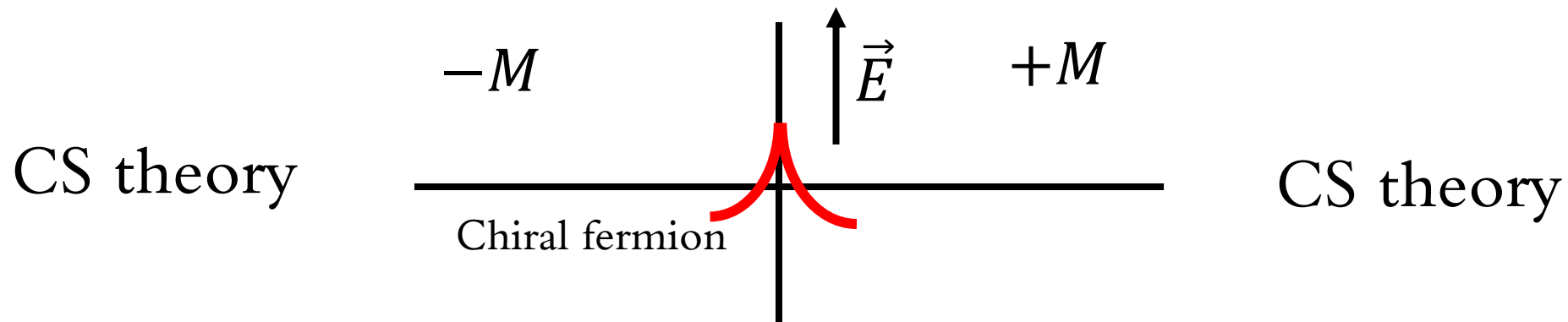current   current

CS theory '+' ve level

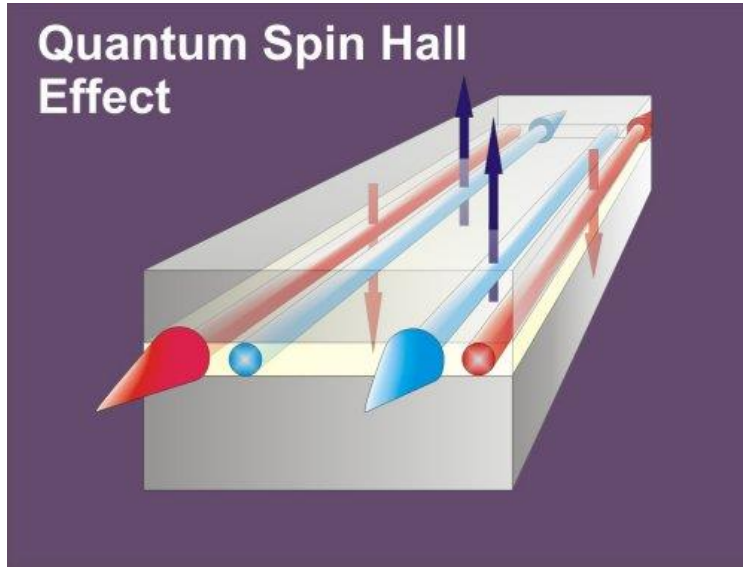# Relativistic QFT parallel for QHE (Callan-Harvey 1985)

Solve the Dirac equation in the domain wall background.

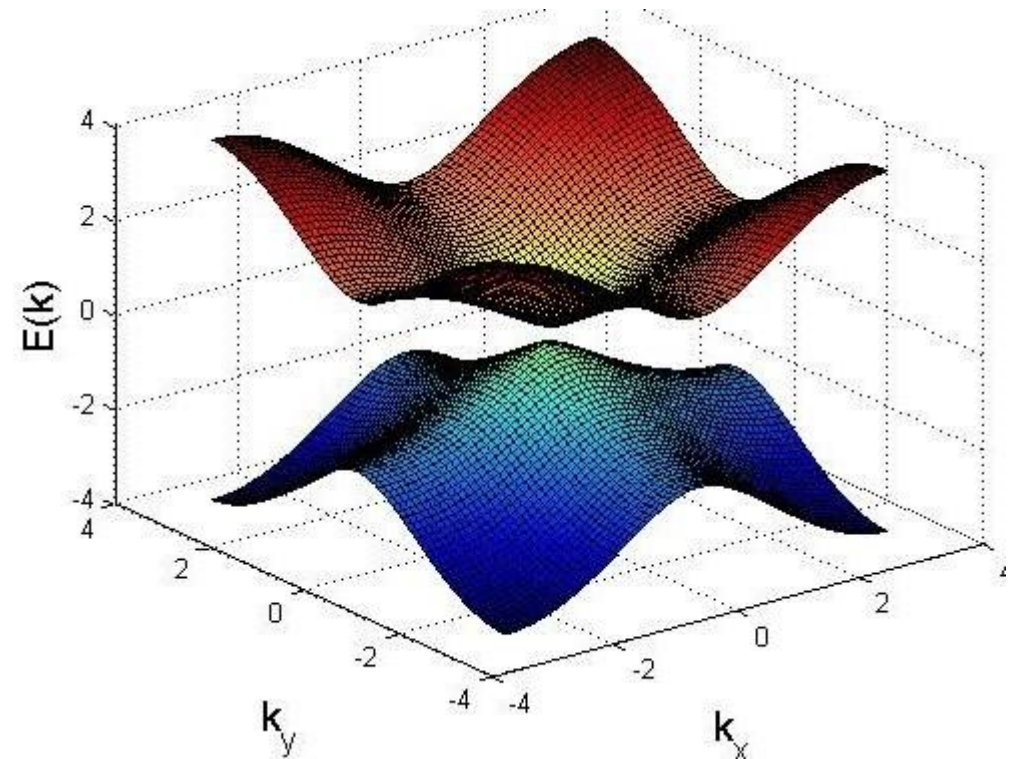Chiral fermions stuck on the $1 + 1$ D wall. Appears anomalous.

Not so when embedded in higher dimension like here.

Many other examples :

Spin Hall effect and DWF in anomaly free rep.

Chern insulators and lattice DWF.

DBK, PLB 288(1992) 342. K Jansen, PLB 288 (1992) 348. CL Kane, EJ. Mele PRL 95(2005) 226801. Golterman, Jansen, Kaplan, PLB 301 (1993) 219.

UV complete IR topological phenomena

- The hope is that we will discover new IR phenomena for topological materials in the process.

- In fact, Chern insulators first appeared in lattice gauge theory.. but went unnoticed.

- New topological phenomena in nuclear/HEP ?

Hope of cross pollination.

How about a UV completion of fractional quantum Hall effect (FQHE) ?

Let's first list the features of FQHE.

The setup is similar to IQHE.

fraction

But you have fractional currents.

$$J_i = -\frac{\nu}{2\pi}\epsilon_{ij}E_j$$

As if you have fractional charges on the edges.

What kind of UV completion do we want ?

- We are in 2+1 D.

- Want Lorentz invariance.

- We don't want to use magnetic field to break parity: so we will stick with heavy fermions.

- Want the fermions to be integer charged.. i.e. fractional charge should emerge in the IR.

- A perturbative UV completion.

Take inspiration from FQHE EFT.

The EFT looks like

Integer

$$S = \int d^3x \left(\frac{1}{2\pi} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu a_\rho - \frac{m}{4\pi} \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho \right)$$

Use the equations of motion for a, (same as integrating out a)

$$a^\mu = \frac{A^\mu}{m}$$

Take inspiration from FQHE EFT.

Substitute back in the action such that after integrating out a, we are left with

$$S = \int d^3x \left(\frac{1}{4\pi m} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho \right)$$

Fraction

The current is :

$$J_i = -\frac{1}{2\pi m} \epsilon_{ij} E_j$$

Take inspiration from FQHE EFT.

We will discuss more phenomenology later.

But for now we want to design a UV completion for the two blocks:

$$L = \int d^3x \left(\frac{1}{2\pi} \boxed{\epsilon^{\mu\nu\rho} A_\mu \partial_\nu a_\rho} - \frac{m}{4\pi} \boxed{\epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho} \right)$$

Mixing term   Quadratic term: level m CS term

Recap UV completion of level m CS theory in 2+1 D

$$\mathcal{L} = \int d^3x \left(\frac{m}{2\pi} \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho \right)$$

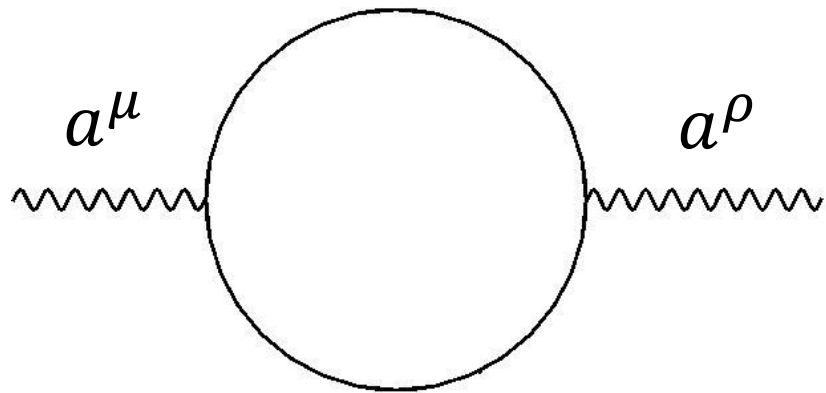How do you arrive at this ?

Take an abelian gauge field a^μ coupled to m flavors of massive fermions of charge 1 and mass $M > 0$. (2+1 D)

$$S = \int d^3x \left(-\frac{1}{4g^2} f^2 + \sum_{i=1,..m} \overline{\psi_i} \{ i\gamma^\mu D_\mu - M \} \psi_i \right)$$

Recap UV completion of level m CS theory in 2+1 D

Coupling constant g^2 is dimensionful.

Integrate out the fermions in the bulk when $g^2 < M$.

$$S_{CS} = \frac{l}{4\pi} \int d^3x \; \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho$$
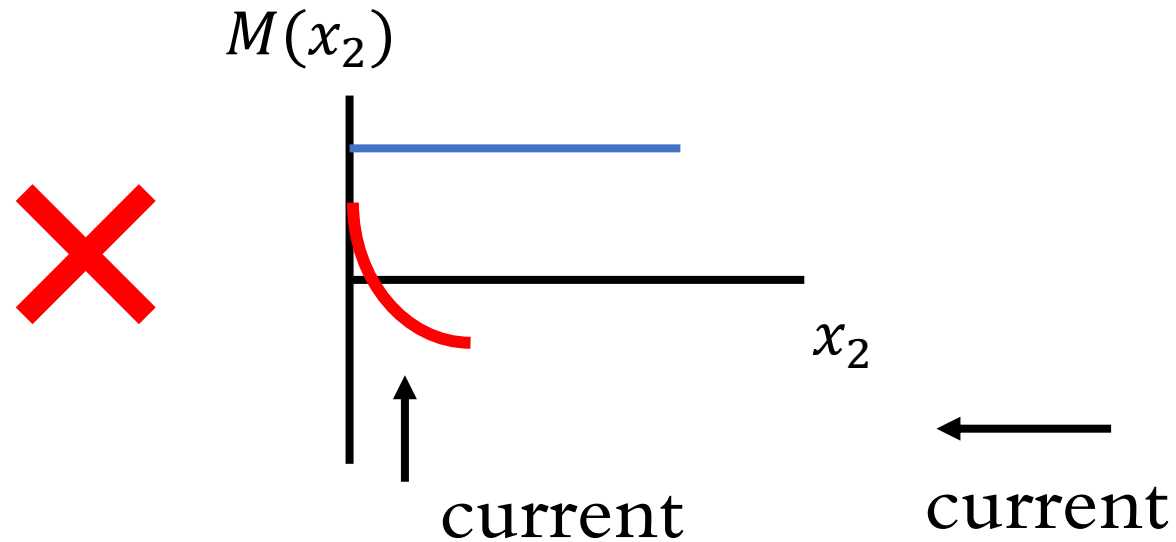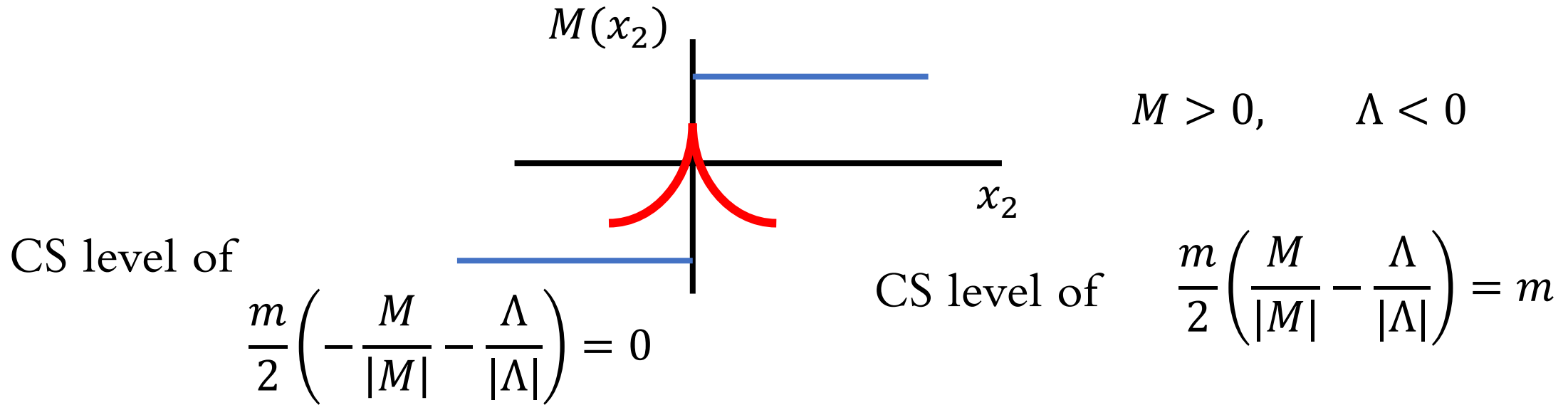
Pauli Villars mass

$$l = \frac{m}{2}\left(\frac{M}{|M|} - \frac{\Lambda}{|\Lambda|}\right) = m, \qquad \text{for} \quad M > 0, \Lambda < 0$$

$$S = \frac{m}{4\pi} \int d^3x \; \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho - \int d^3x \frac{f^{\mu\nu} f_{\mu\nu}}{4g^2}$$

Forget in the long wavelength low energy limit

a^μ a^ρ

Domain wall in fermion mass

$M(x_2)$

x_2

$M > 0, \qquad \Lambda < 0$

CS level of

$$\frac{m}{2}\left(-\frac{M}{|M|}-\frac{\Lambda}{|\Lambda|}\right)=0$$

CS level of $\quad \dfrac{m}{2}\left(\dfrac{M}{|M|}-\dfrac{\Lambda}{|\Lambda|}\right)=m$

$M(x_2)$

✖

x_2

current   current

Back to FQHE: UV completion

We will need two $U(1)$ gauge fields. Call these A and Z.

The anticipated mapping: $A \rightarrow A$ and $Z \rightarrow a$ in

$$L = \int d^3x \left(\frac{1}{2\pi} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu a_\rho - \frac{m}{4\pi} \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho \right)$$

There are two coupling constants e and g corresponding to the two gauge fields.
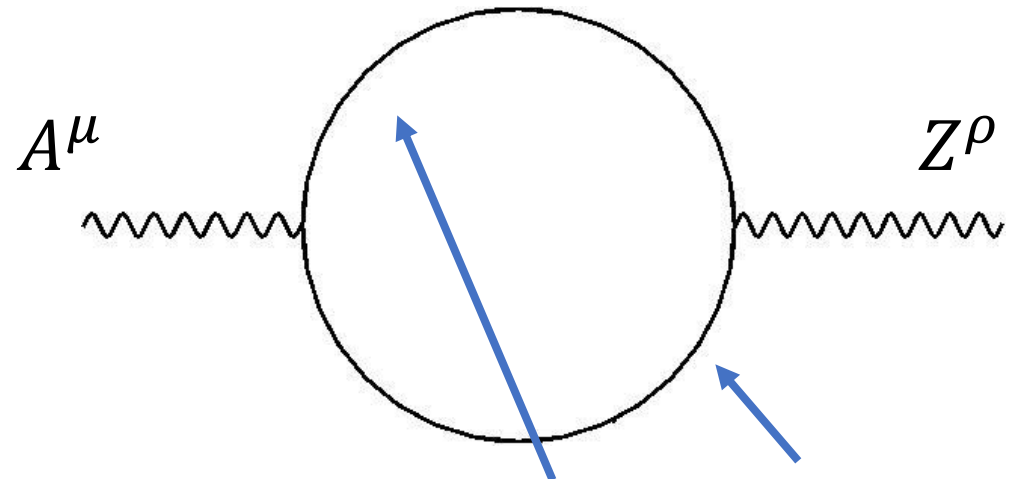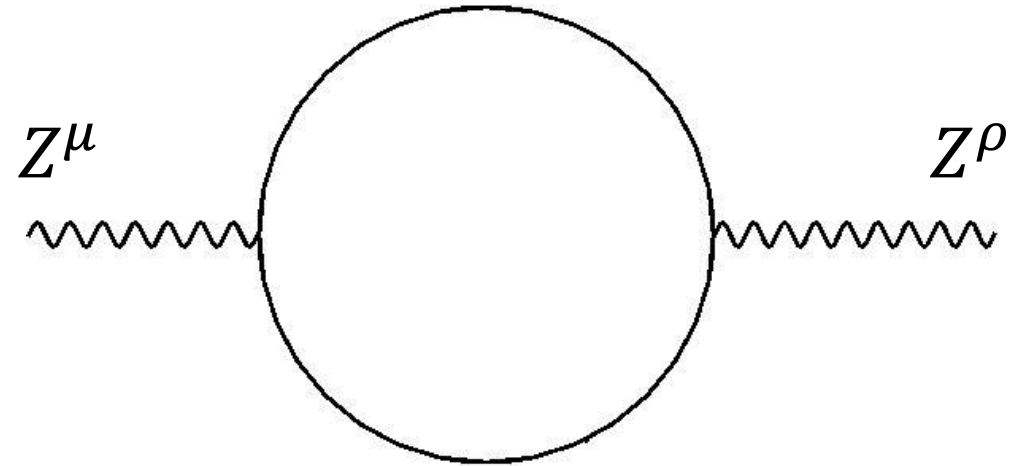
Mixing term

We know how to get
a quadratic CS term.

$$S_{CS} = \frac{l}{4\pi} \int d^3x \, \epsilon^{\mu\nu\rho} Z_\mu \partial_\nu Z_\rho$$

To get the mixed CS term
we need a diagram:

$$S_{CS} \sim \frac{\#}{4\pi} \int d^3x \, \epsilon^{\mu\nu\rho} A_\mu \partial_\nu Z_\rho$$

Z^μ Z^ρ

A^μ Z^ρ

Fermions charged under both A and Z

Fermions (remember we want integer charges)

We will need a few different species of fermions of multiple flavors charged under the two gauge fields :

Call them ψ, χ, ω.

q_A= charge under the gauge field A

q_Z= charge under the gauge field Z

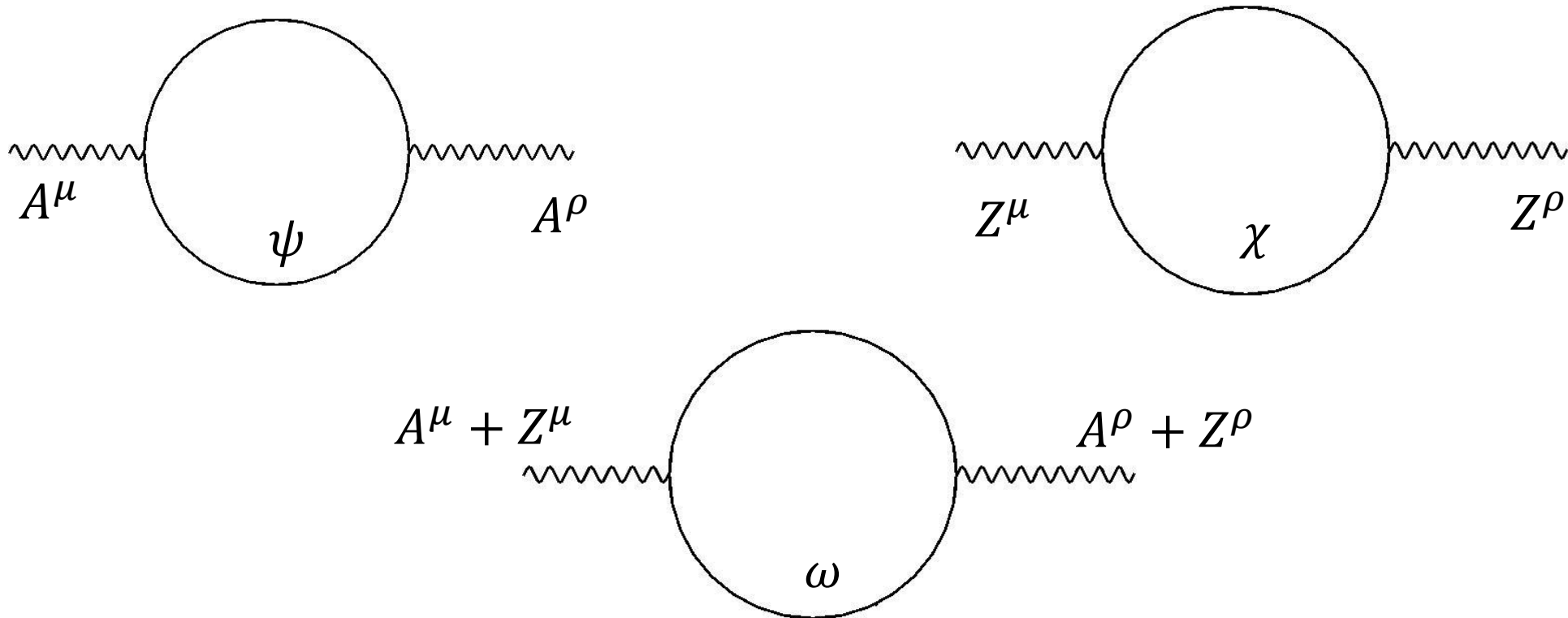|	n_{flavor}	q_A	q_Z
ψ	n_ψ	1	0
χ	n_χ	0	1
ω	n_ω	1	1

The hierarchy of scales

We will want the following hierarchy of scales

$$|\Lambda| > |M| > g^2 > e^2$$

I want to integrate out the fermions first in a weakly coupled regime to end up with CS theory.

So, clearly, I need fermion masses to be large compared to other physical scales $|\Lambda| > |M| > g^2,\ e^2$

Integrating out the fermions: $(|M| \gg e^2, g^2)$

$$\longrightarrow \quad \frac{\epsilon^{\alpha\beta\gamma}}{4\pi}\left(n_\psi\, A_\alpha \partial_\beta A_\gamma + n_\chi Z_\alpha \partial_\beta Z_\gamma + n_\omega (A_\alpha + Z_\alpha)\partial_\beta\left(A_\gamma + Z_\gamma\right)\right)$$

Next, we anticipate integrating out the Z gauge field

Hierarchy of scales

This is the same as using the EOM for the a gauge field earlier.

What enables the integrating out of the gauge field ?

Note the gauge field kinetic terms in a CS + Maxwell theory

$$L_Z = \frac{Z_{\mu\nu}Z^{\mu\nu}}{4g^2} + k\,\epsilon^{\alpha\beta\gamma}Z_\alpha\partial_\beta Z_\gamma$$

There is a pole at $\sim k\,g^2$. So, the gauge theory is gapped.

Hierarchy of scales

The Z gauge field mass $\sim g^2$.

The A gauge field mass $\sim e^2$.

We want to integrate out the Z gauge field.

So, $e^2 < g^2$.

Integrating the Z gauge field out

The Z dependent part of the Lagrangian:

$$\mathcal{L} = \frac{1}{2}\left[\left(Z - \frac{n_\omega}{2\pi}A\Delta Q^{-1}\right)Q\left(Z - \frac{n_\omega}{2\pi}Q^{-1}\Delta A\right) - \left(\frac{n_\omega}{2\pi}\right)^2 A\Delta Q^{-1}\Delta A\right]$$

$$Q = -\frac{n_\chi + n_\omega}{2\pi}\Delta + \boxed{\frac{\Xi}{\xi^2}} \quad\longleftarrow\quad \text{Gauge fixing}$$

$$\Delta^{\alpha\beta} = \epsilon^{\alpha\beta\gamma}\partial_\gamma \,, \Xi^{\alpha\beta} = \partial_\alpha\partial_\beta, \quad Q^{-1} = \frac{2\pi}{n_\chi + n_\omega}\frac{\Delta}{\partial^2} + \frac{\xi\Xi}{(\partial^2)^2}$$

Integrating the Z gauge field out: the EFT

$$\Delta Q^{-1}\Delta = -\frac{2\pi\Delta}{n_\chi + n_\omega}$$

$$\mathcal{L} = \frac{1}{2}\left[\left(Z - \frac{n_\omega}{2\pi}A\Delta Q^{-1}\right)Q\left(Z - \frac{n_\omega}{2\pi}Q^{-1}\Delta A\right) - \left(\frac{n_\omega}{2\pi}\right)^2 A\Delta Q^{-1}\Delta A\right]$$

$$\mathcal{L}_{EFT} = -\frac{1}{4e^2}F^2 + \nu\frac{\epsilon^{\alpha\beta\gamma}}{4\pi}A_\alpha\partial_\beta A_\gamma + \cdots$$

$$\nu = \left(n_\psi + \frac{n_\chi n_\omega}{n_\chi + n_\omega}\right)$$
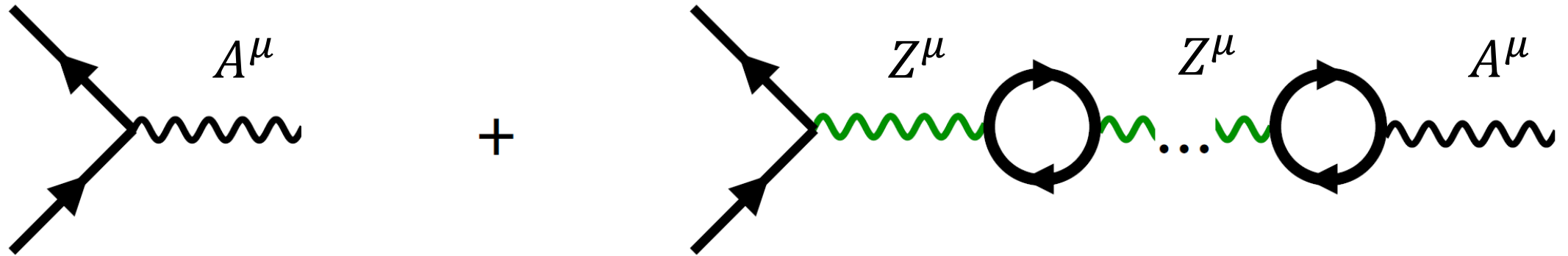
Edge modes

Consider a half plane

Solve Dirac equation to get edge modes on the half plane boundary.

These zero modes have integer charges under the gauge fields.

How do you conserve the fractional current then ?

Edge modes

After integrating out the Z gauge field, the edge modes get fractional charge

The fractional charges

$$q'_\psi = 1, \quad q'_\chi = -\frac{n_\omega}{n_\chi + n_\omega}, \quad q'_\omega = 1 - \frac{n_\omega}{n_\chi + n_\omega}$$

Current conservation works out with these fractional charges.

Summary and open questions

We have a relativistic UV completion for the FQHE.

What about nonabelian quantum Hall states ?

Topological insulators in $3 + 1$ dimensions ?

Gauge theories in symmetry broken phases ?