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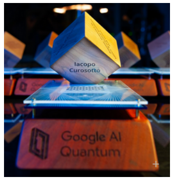
Photons for Quantum Simulation



Quantum Hall fluids of atoms and of light

Iacopo Carusotto

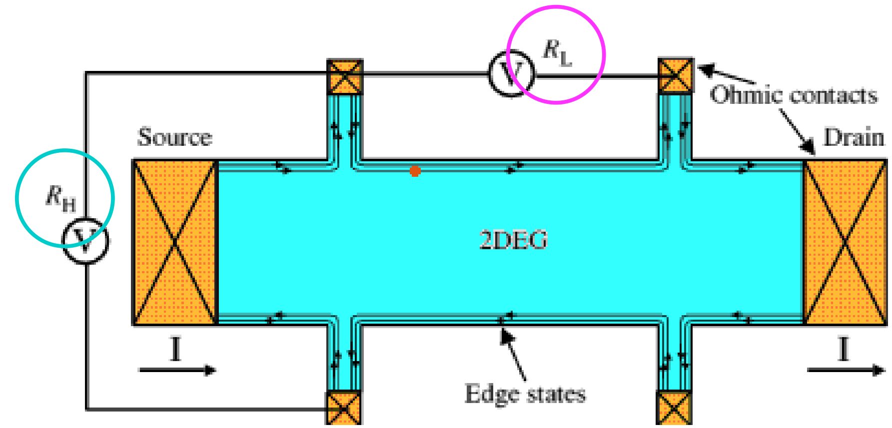
INO-CNR BEC Center and Università di Trento, Italy

Fractional Quantum Hall effect

Thin and extremely clean 2D electron gas

Measure longitudinal and transverse resisitivity:

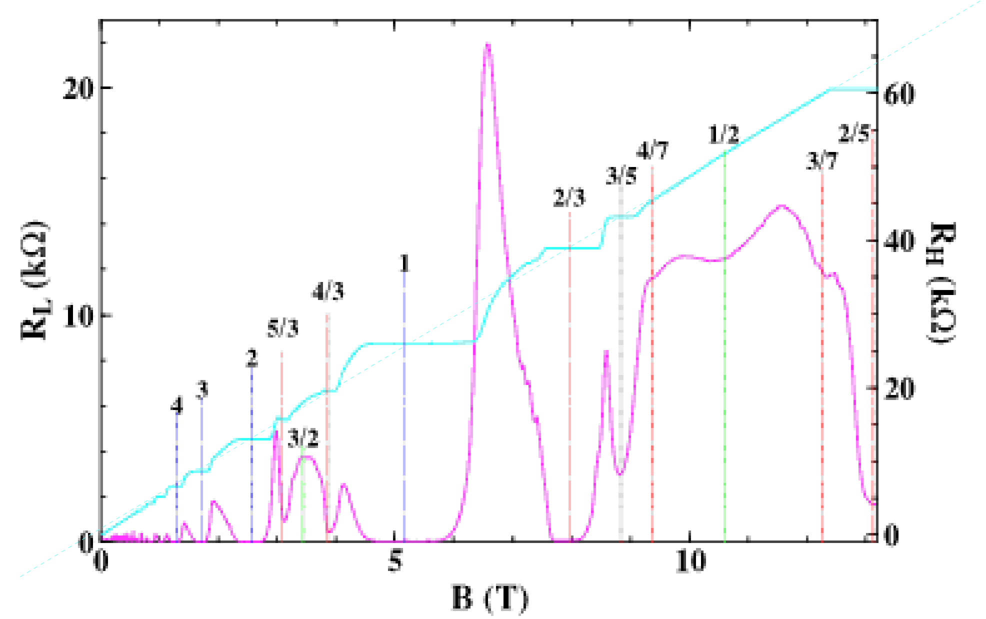
- Textbook Hall effect: $R_T \sim B$
- Expt → Intriguing features @ rational $1/\nu = B/B_0$
  - R_L drops to zero
  - R_H shows plateaux



Effect benefits (!!!) of (moderate) disorder,
which sets extension of plateaux

Integer ν → single electron physics,
signature of band topology

Fractional ν → strongly correlated fluid
topological phase of matter
(non-)Abelian anyonic excitations
topological quantum computing



This talk: Is this physics specific to electrons? What about FQH in quantum fluids of light?

Nobel prizes: Von Klitzing (1985); Laughlin, Stoermer, Tsui (1998)

In a nutshell:

- *How to make neutral particles such as photons to feel a Lorentz force?*
- *Can this be used to study topological effects ?*
- *What about integer/fractional quantum Hall states ?*

Linear Topological Optics Part 1

- *What about nonlinear optics and laser operation in topological models?*
- *Technological applications*

Nonlinear Topological Optics Part 2

- *Can one generate quantum many-body states ?*
- *What physics to be probed with them? E.g. anyons?*

Quantum nonlinear Topological Optics Part 3

Part 1:

a brief journey through the early days of topological photonics

REVIEWS OF MODERN PHYSICS, VOLUME 91, JANUARY–MARCH 2019

Topological photonics

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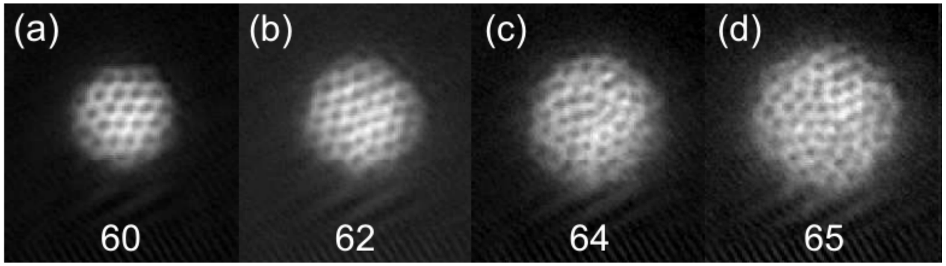
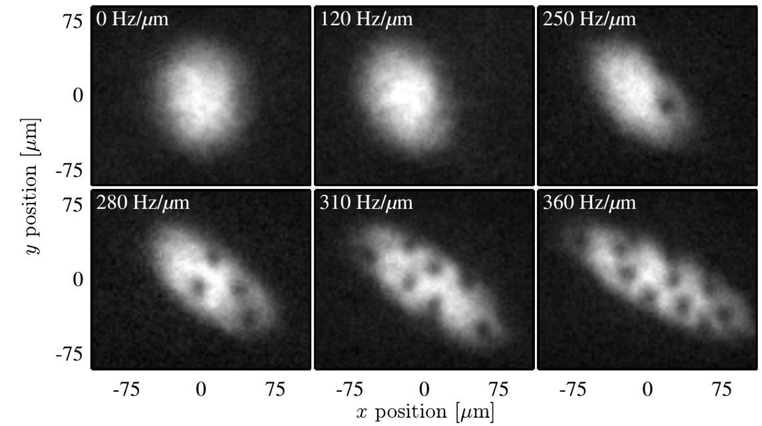
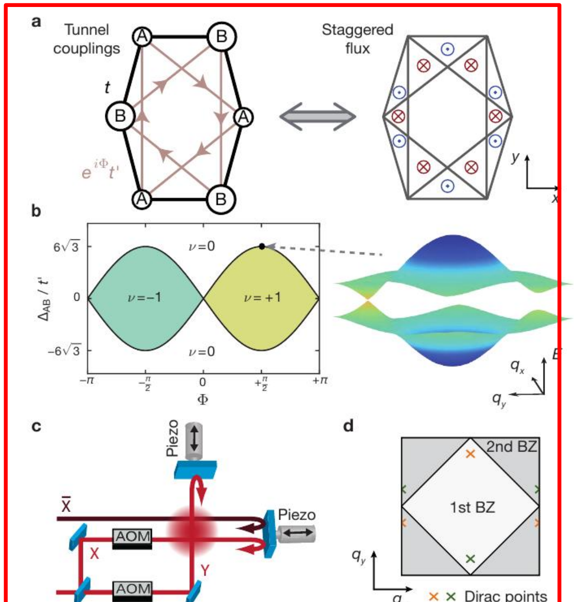
Prehistory: Synthetic gauge fields for atoms



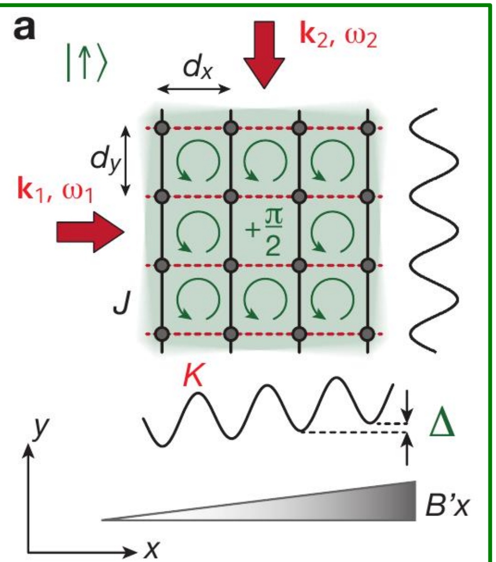
Coriolis force in rotating frame: vortices in atomic gas
Bretin et al., PRL 92, 050403 (2004)



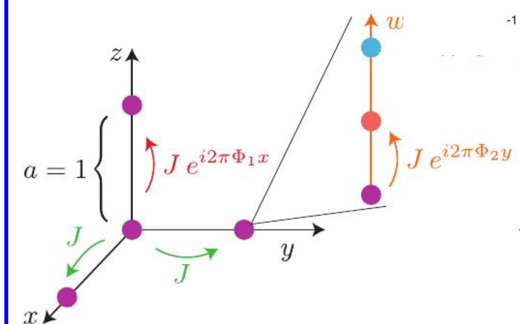
Synthetic gauge field by Raman + magnetic field
Lin et al. Nature 471, 83 (2011)

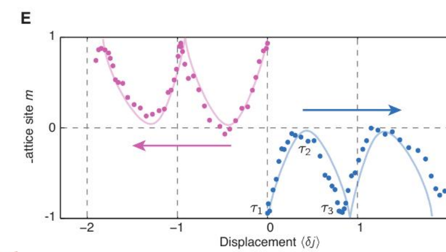


Shaken lattice
Haldane model for atoms
Jotzu et al., Nature 515, 237 (2014)

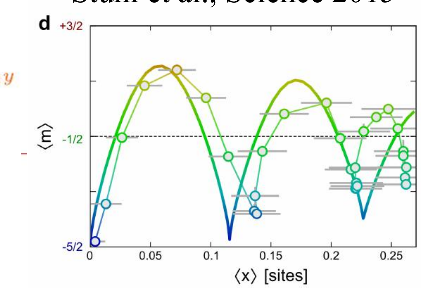


Laser-assisted hopping
2D Harper-Hofstadter model
Aidelsburger et al., PRL 111, 185301 (2013)





Stuhl et al., Science 2015



Mancini et al., Science 2015.

Synthetic dimensions (so far 1+1D = 2D)
Chiral edge states in synthetic Hall ribbons

2008-9 – The birth of *topological photonics* (th)

PRL **100,** 013904 (2008) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2008

Possible Realization of Directional Optical Waveguides in Photonic Crystals with Broken Time-Reversal Symmetry

F. D. M. Haldane and S. Raghu*

Department of Physics, Princeton University, Princeton, New Jersey 08544-0708, USA

(Received 23 March 2005; revised manuscript received 30 May 2007; published 10 January 2008)

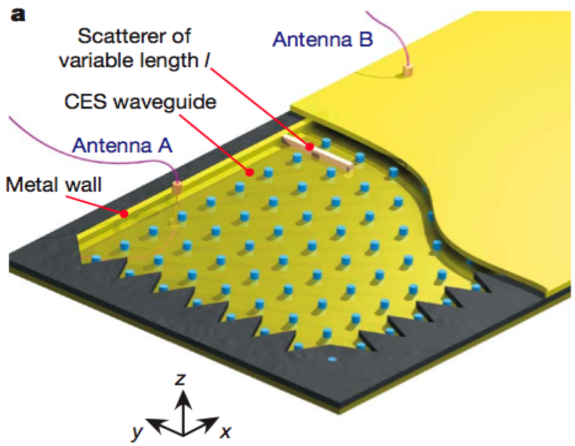
We show how, in principle, to construct analogs of quantum Hall edge states in "photonic crystals" made with nonreciprocal (Faraday-effect) media. These form "one-way waveguides" that allow electromagnetic energy to flow in one direction only.

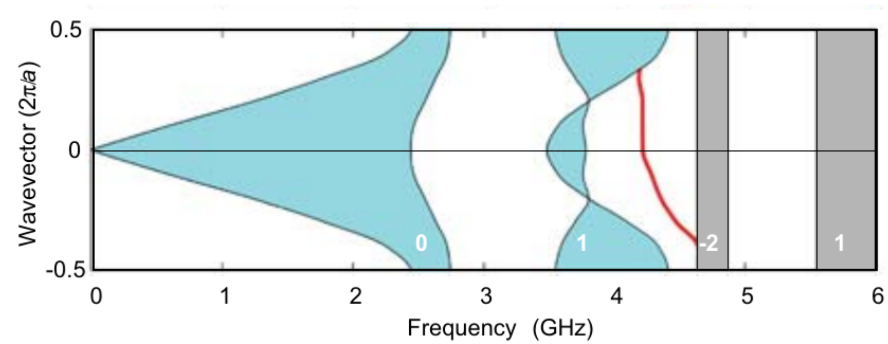
DOI: 10.1103/PhysRevLett.100.013904 PACS numbers: 42.70.Qs, 03.65.Vf

- IQH depends on geometrical properties of Bloch band states
  - Berry connection $\mathcal{A}_n^a = \frac{\langle u_n|\boldsymbol{B}_0(\omega_n)|\nabla_k^a u_n\rangle - \langle \nabla_k^a u_n|\boldsymbol{B}_0(\omega_n)|u_n\rangle}{2i\langle u_n|\boldsymbol{B}_0(\omega_n)|u_n\rangle}$
  - Berry curvature $\mathcal{F}_n^{ab}(\boldsymbol{k}) = \nabla_k^a \mathcal{A}_n^b - \nabla_k^b \mathcal{A}_n^a$
  - Integer-valued Chern number $C_n^{(1)}(\Sigma) = \frac{1}{2\pi}\iint_\Sigma dk_a \wedge dk_b \mathcal{F}_n^{ab}.$
  - C_n fixes transverse conductivity σ_H and number of edge states (bulk-boundary correspondence)

- Haldane-Raghu → IQH not specific to fermionic electrons
  - Complex band structures can be realized for photons in periodic structures, aka photonic crystals
  - Need to break T-reversal to have $C_n \neq 0$ → include magnetic elements
  - σ_H not directly defined, but chiral edge states give one-way waveguide on the edge

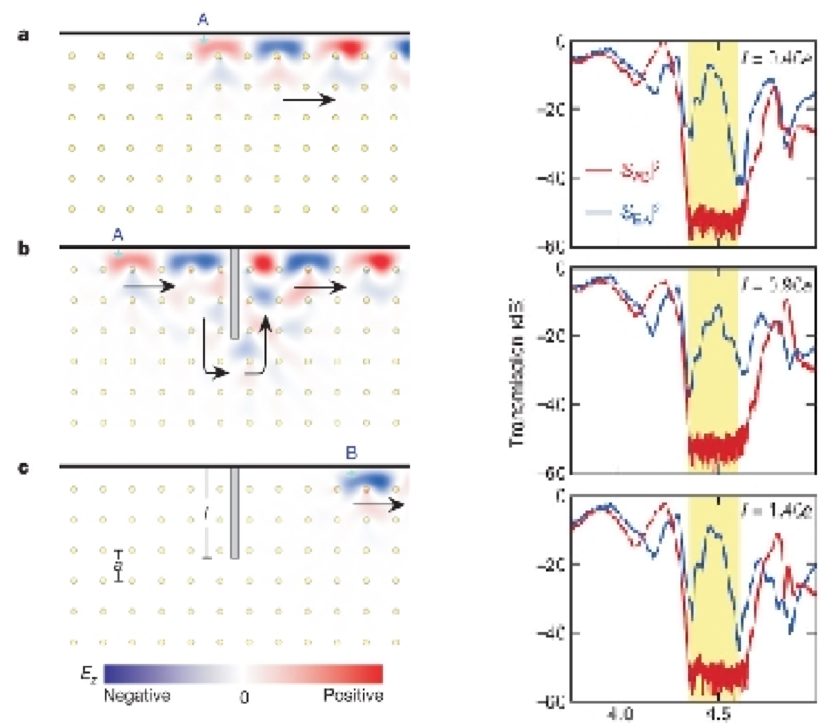
2008-9 – The birth of *topological photonics* (expt)





Magneto-optical photonic crystals for μ-waves

- T-reversal broken by magnetic elements
- Band wih non-trivial Chern number:
  → chiral edge states within gaps



Experiment:

- measure transmission from antenna to receiver
- only in one direction → unidirectional propagation
- immune to back-scattering by defects → topologically protected

Z Wang, Y Chong, JD Joannopoulos, M Soljačić, *Observation of unidirectional backscattering-immune topological electromagnetic states*, Nature 461, 772 (2009)

2013 - Harper-Hofstadter & Haldane models for visible photons

Goal:

- avoid the need of magnetic materials
- scale up to visible light where optical nonlinearities and quantum emitters easily accessible

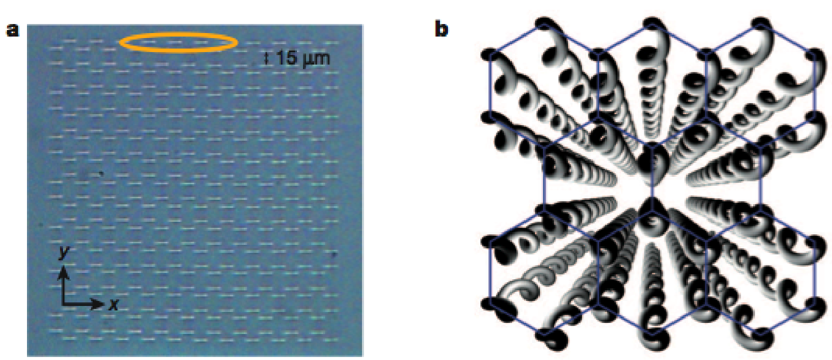
Many proposals: geometrical phases (Umucalilar), opto-mechanics (Rabl), ...

2D lattice of coupled cavities with tunneling phase

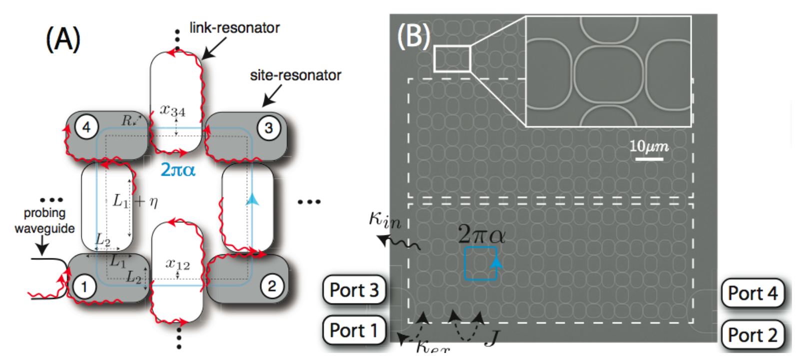
$$H = \sum_i \hbar\omega_o \hat{a}_i^\dagger \hat{a}_i - \hbar J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j e^{i\phi_{ij}} + \sum_i \left[\hbar F_i(t)\, \hat{a}_i^\dagger + \text{h.c.} \right]$$

Experiments along these lines:

- Floquet bands in helically deformed honeycomb waveguide lattices → Rechtsman/Szameit/Segev
- silicon ring cavities → Hafezi/Taylor (JQI)
- electronic circuits with lumped elements → J. Simon (Chicago)
- strained honeycomb lattice for polaritons → A. Amo/J.Bloch (C2N)



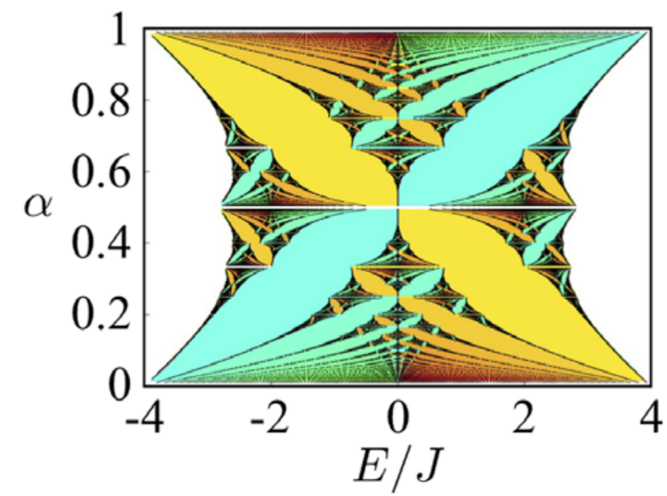
Rechtsman, Plotnik, et al., Nature 496, 196 (2013)



Hafezi et al.,Nat. Phot. **7**, 1001 (2013)

2013 - Imaging chiral edge states

2D square lattice of coupled resonators
at large magnetic flux



Eigenstates organize in bulk Hofstadter bands

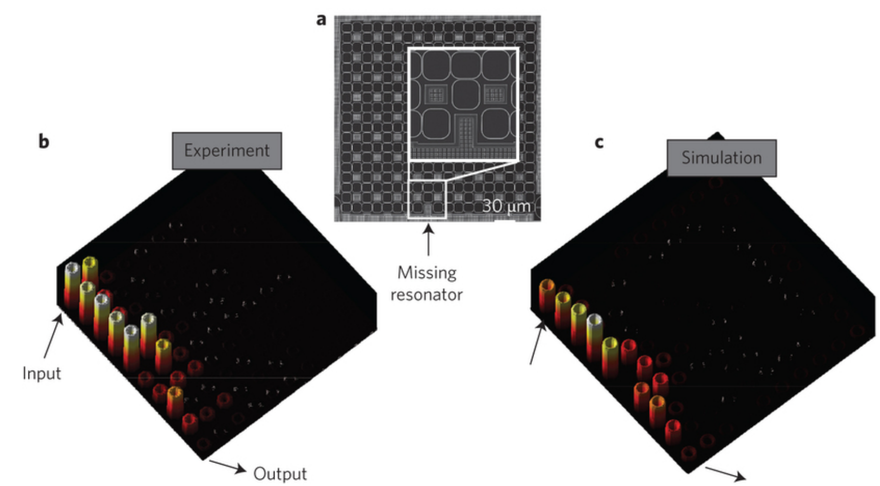
- Berry connection in k-space: $A_{n,k} = i\langle u_{n,k} | \nabla_k u_{n,k} \rangle$
- Berry curvature $\Omega_n(\mathbf{k}) = i(\langle \partial_{k_x} u_{n,\mathbf{k}} | \partial_{k_y} u_{n,\mathbf{k}} \rangle - \langle \partial_{k_y} u_{n,\mathbf{k}} | \partial_{k_x} u_{n,\mathbf{k}} \rangle)$
- Chern number $C_n = \frac{1}{2\pi} \int_{\mathrm{BZ}} d^2k \Omega_n(k_x, k_y),$

Bulk-edge correspondance:

$A_{n,k}$ has non-trivial Chern number $C_n \neq 0$

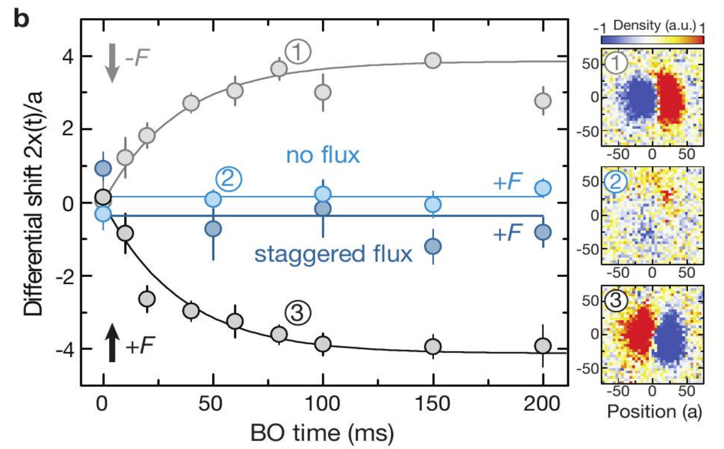
→ chiral edge states within gaps

- unidirectional propagation
- (almost) immune to scattering by defects



Hafezi et al.,Nat. Phot. **7**, 1001 (2013)
Similar images for Haifa expt

How to observe geometrical & topological properties of bulk ?



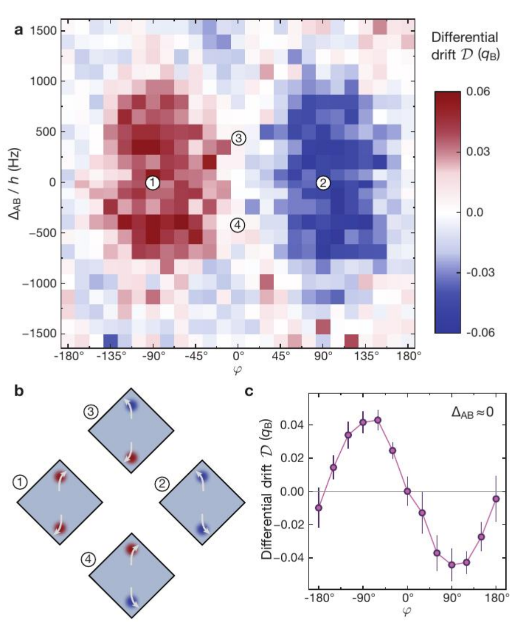
Aidelsburger et al. Nat. Phys. 11, 162 (2015)

Semiclass. EoM: $\hbar\dot{\mathbf{k}}_c(t) = e\mathbf{E}\,,$

$$\hbar\dot{\mathbf{r}}_c(t) = \nabla_{\mathbf{k}}\mathcal{E}_{n,\mathbf{k}} - e\mathbf{E}\times\mathbf{\Omega}_n(\mathbf{k})$$

Berry curvature → sort of **k**-space magnetic field
Lateral displacement analogous to Lorentz force

Depending on band filling:
Anomalous vs. Integer Quantum Hall effect

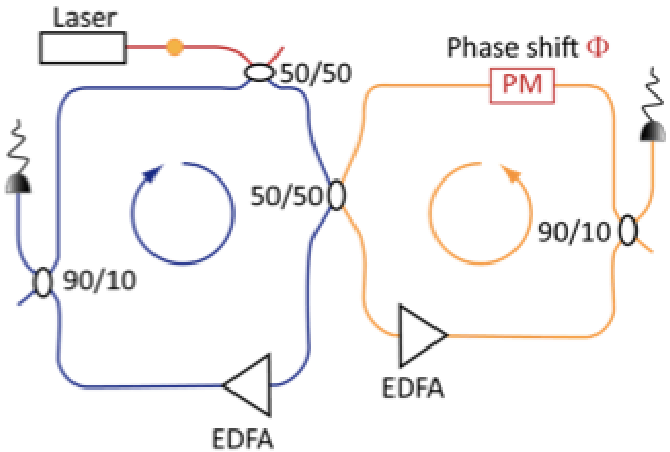


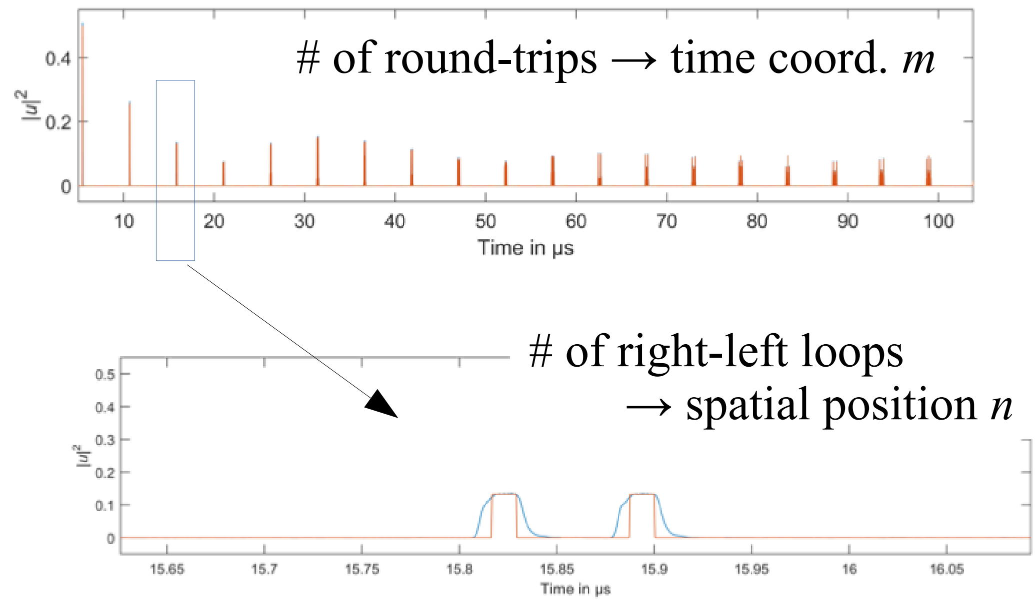
Jotzu et al., Nature 515, 237-240 (2014)

Several experiments with atoms

An old concept, see e.g. review in Xiao-Chang-Niu, RMP 82, 1959 (2010).
First proposals for atoms: Dudarev, IC et al. PRL 92, 153005 (2004)
Price-Cooper, PRA 83, 033620 (2012)

2016 - Experimental mapping of Berry curvature



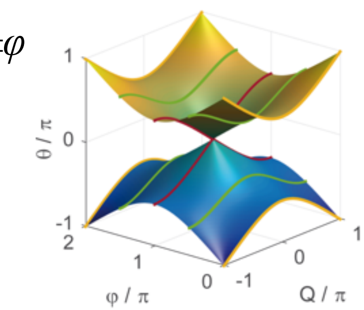


Optical mesh lattice:

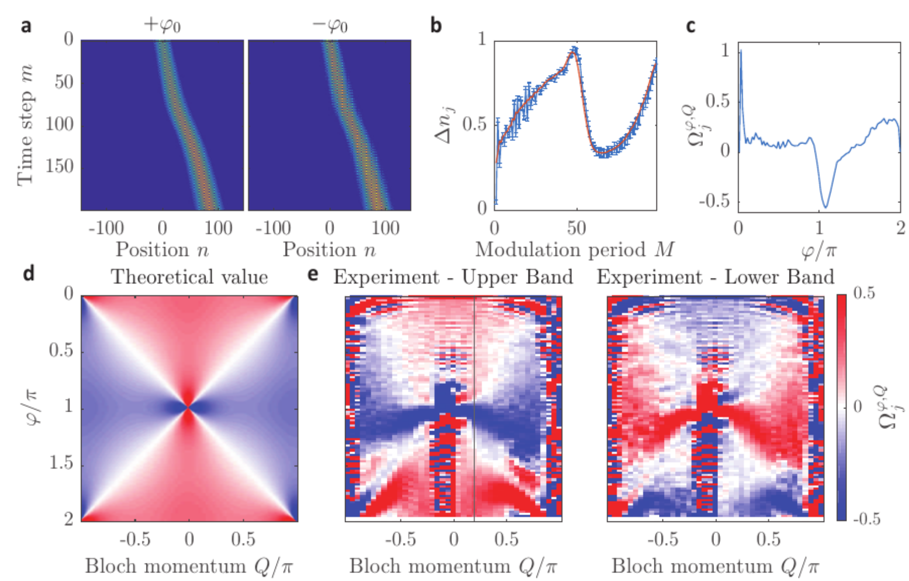
- Pair of optical fibers coupled at beam splitter
- Pulse arrival time → space-time position

- Periodic temporal modulation of $\Phi(m)=\pm\varphi$
  1D Floquet band structure $\theta(Q,\varphi)$,
  φ considered as 2nd dim
  Berry curvature

$$\Omega_j^{\varphi,Q} = \frac{\partial}{\partial\varphi}\langle\psi_j|i\frac{\partial}{\partial Q}|\psi_j\rangle - \frac{\partial}{\partial Q}\langle\psi_j|i\frac{\partial}{\partial\varphi}|\psi_j\rangle$$



- Geometrical charge pumping if φ adiabatically varied
- Look at lateral displacement along n at all times m
  → reconstruct Berry curvature $\Omega_j^{(\varphi,Q)}$ in whole FBZ



Wimmer, Price, IC, Peschel, Nat. Phys. 2017

Cold atoms → state tomography (Fläschner et al., Science '16)
Polaritons → anomalous Hall effect via spin-orbit (Gianfrate et al. Nature '20)

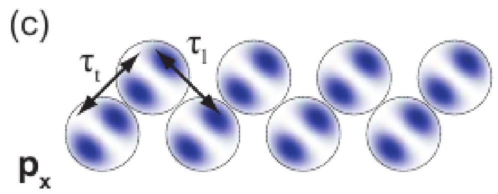
Part 2:

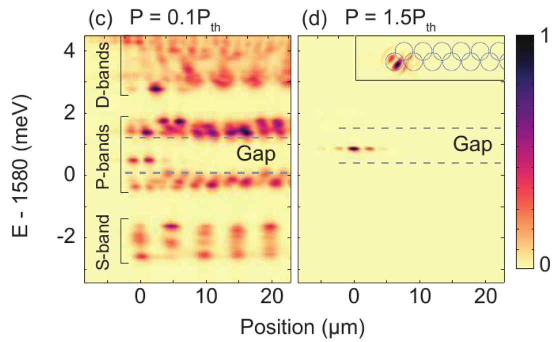
Topological lasing

a.k.a. non-equilibrium BEC in chiral edge state

2017 – Topological lasing

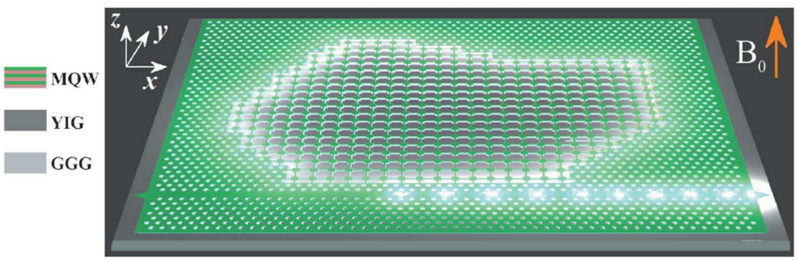
What happens if one adds gain to a topological model ?

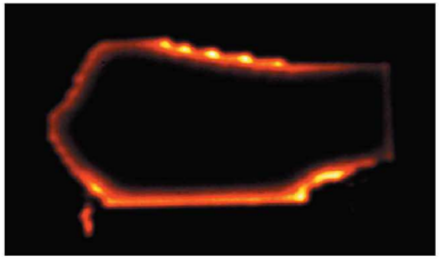


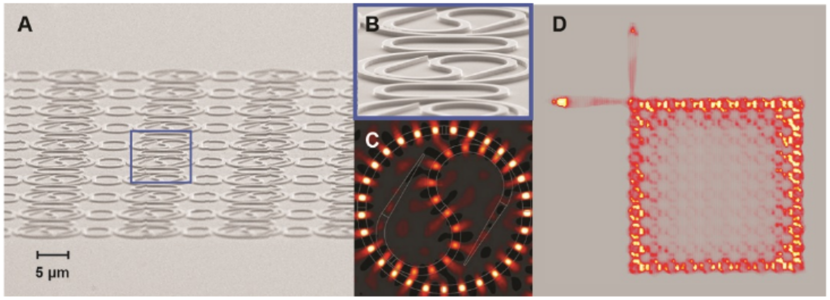


St. Jean, et al., Nat. Phot. '17
System: 1D SSH array of micropillar cavities for exciton-polaritons under incoherent pump

Bahari et al., Science 2017
System: 2D photonic crystal slab, amplification by QWs, magnetic field to break T



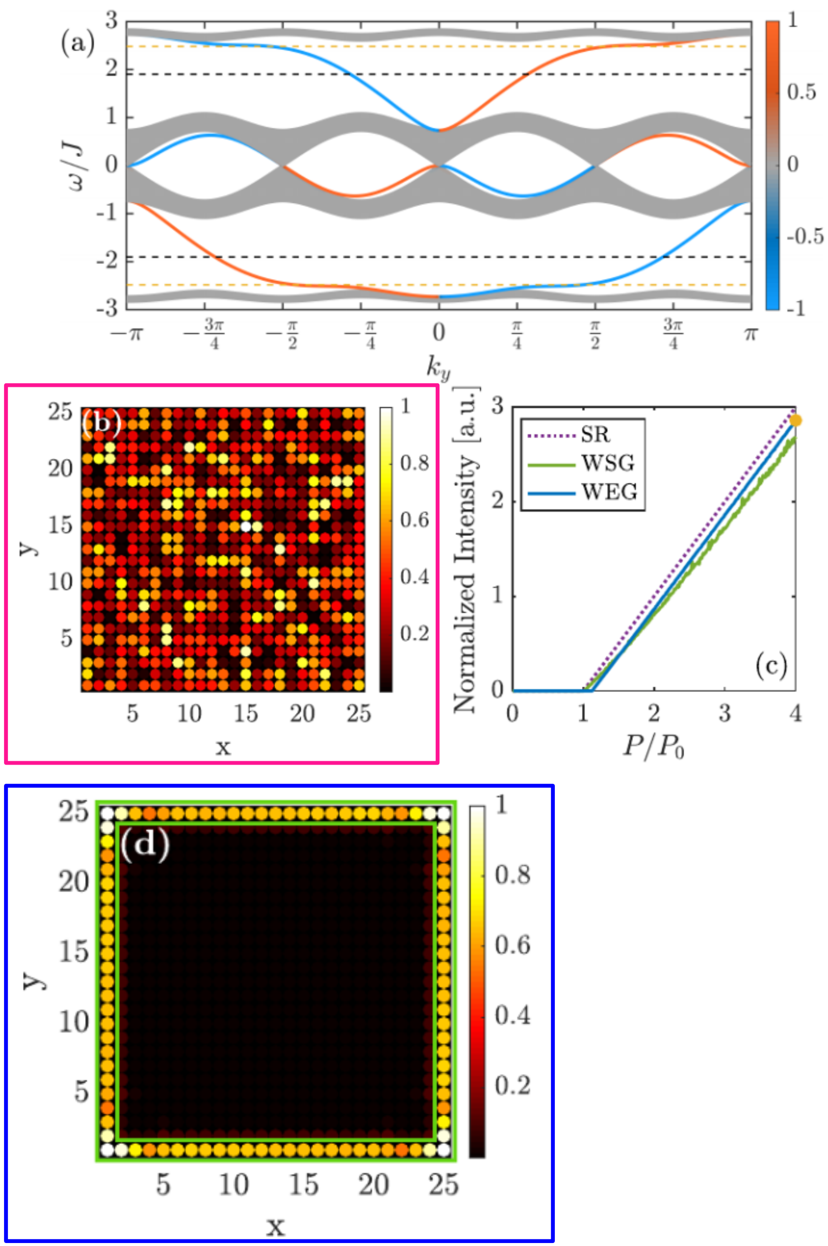




Bandres et al., Science 2018
System: array of Si-based ring resonators with optically pumped III-V amplifier layer.
Tai-Ji shape to break inversion symmetry

Early theoretical work by Conti & Pilozzi, Solnyshkov, Nalitov & Malpuech.
Other expts: Khajavikhan's group, PRL 2018...

Topological lasing in 2D models: basic features



Topologically trivial system:

- pumping many cavities gives complicate many-mode emission
- hard to preserve coherence and fully exploit gain when gain distributed on many sites to increase emission power
- serious technological problem for high-power semiconductor laser applications

Topological system:

- 2D Topolaser operation into edge mode when edge only is pumped (WEG)
  - Chiral propagation immune to disorder
  - Efficient single mode lasing with high slope efficiency

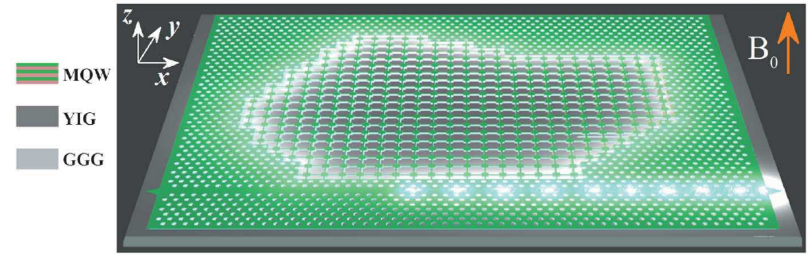
Seems to work, but even more exciting physics...

Figures from M. Seclì's Msc thesis @ UniTN, 2017 and Seclì *et al.*, Phys. Rev. Research 2019
Same results in Harari et al., Science 2018; See also work by Pilozzi-Conti and by Kovanis-Longhi

Coherence of topolaser emission (I)

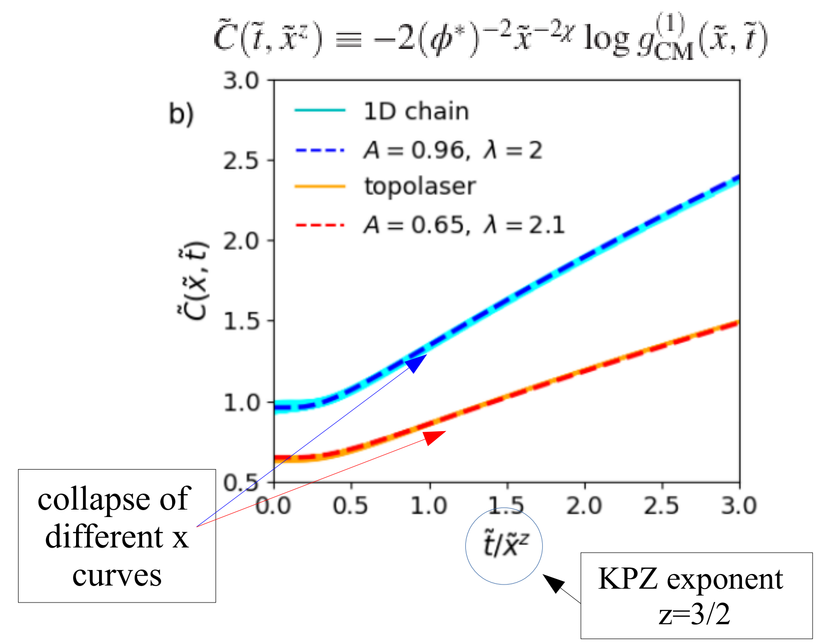
Important fundamental & applied questions:

- What are ultimate limitations of coherence?
- How robust is coherence to disorder?
- What advantage over standard lasers?



Laser operation in spatially extended system:

- Linearized theory not enough, crucial role of nonlinearities
- Kardar-Parisi-Zhang model of non-equilibrium stat mech (Altman/Diehl, Gladilin/Wouters, Canet/Minguzzi)
- spatio-temporal scaling properties of phase-coherence



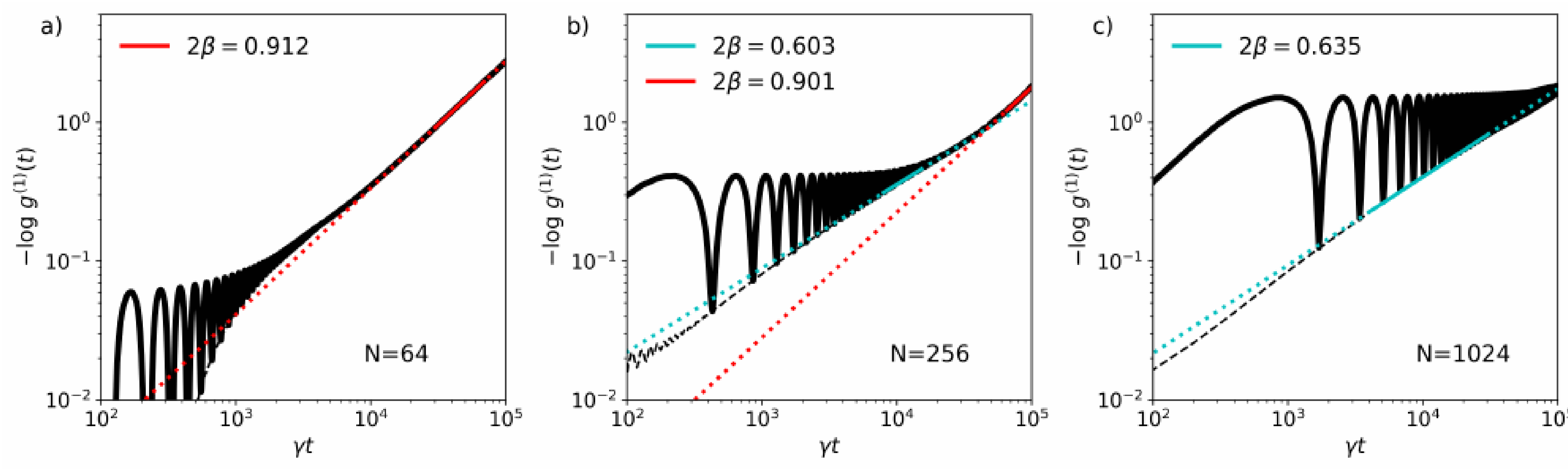
Topological laser:

- One-dimensional edge state gives effective 1D dynamics
- KPZ spatio-temporal scaling of $g^{(1)}(x,t)$
- Periodic boundary conditions around device

Teaser: experimental evidence of KPZ using polariton quasi-condensates @ C2N
Fontaine *et al.*, to appear soon!

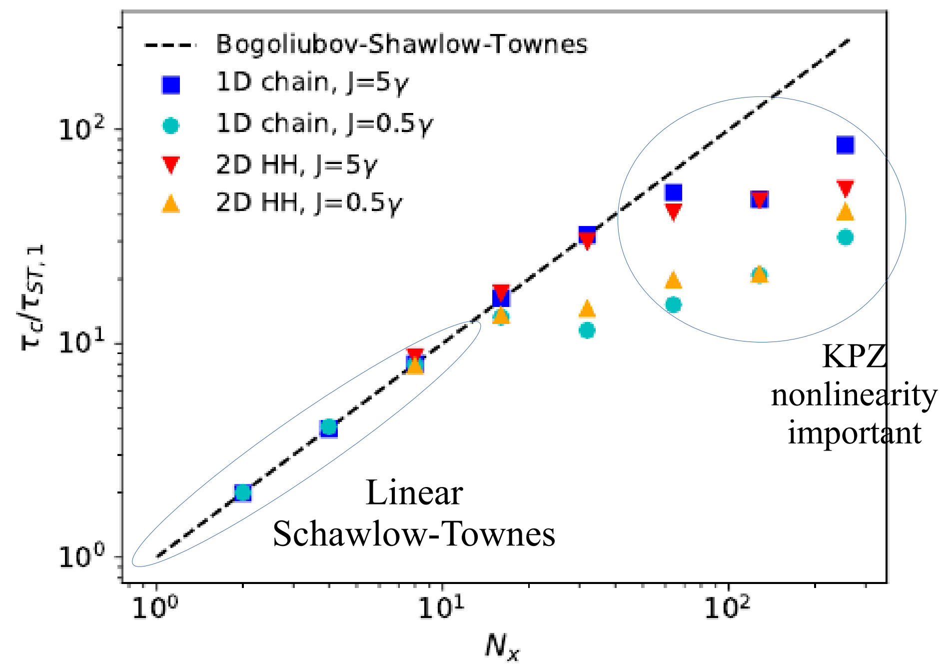
I. Amelio and IC, PRX 10, 041060 (2020)

Coherence of topolaser emission (II)



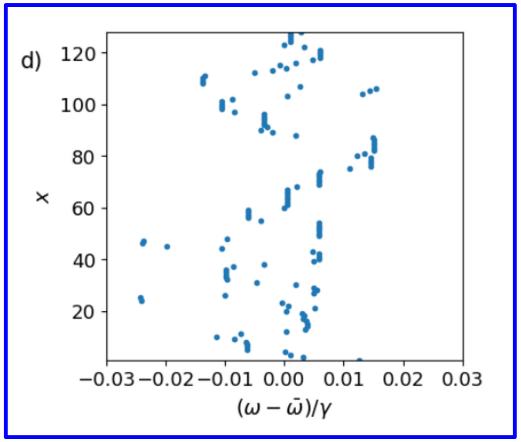
Coherence of laser emission:

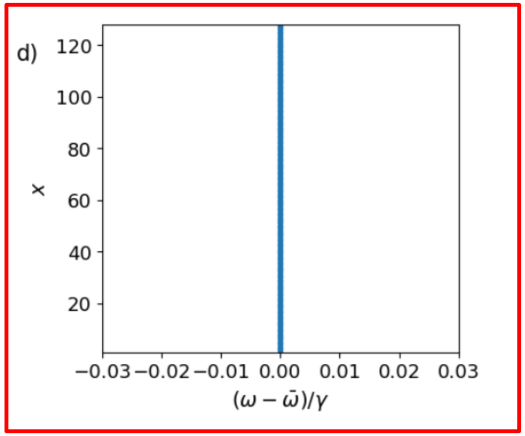
- Physical system necessarily finite
  → crossover from stretched-exp to exp decay
  at long times for given size N_x
- crossover from Schawlow-Townes τ_c to KPZ
  at long times for increasing size N_x
- imposes fundamental limitation to τ_c
- physics similar to 1D chains, but...

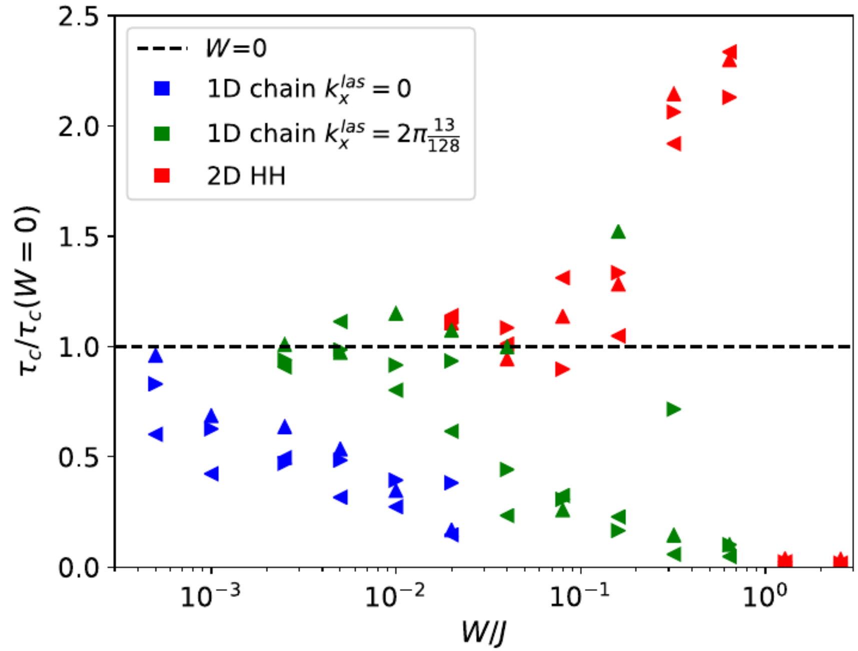


I. Amelio and IC, *Theory of the coherence of topological lasers*, PRX 10, 041060 (2020)

Coherence of topolaser emission (III)







In the presence of static disorder:

- Non-Topological: weak disorder suppresses temporal coherence (mode fragmentation, multimode emission, localization, etc.)
- Topological: robust spatio-temporal coherence, chiral propagation travels through/around defects without backscattering.

Technologically important in (semiconductor) laser technology:

Allows to phase lock many individual lasers → strong intensity and high coherence

Next steps: extend theory to Class-B lasers. Control instabilities and maintain single-mode emission

Fundamental question → effect of convective/absolute instability on coherence properties

I. Amelio and IC, *Theory of the coherence of topological lasers*, PRX 10, 041060 (2020)

Part 3:

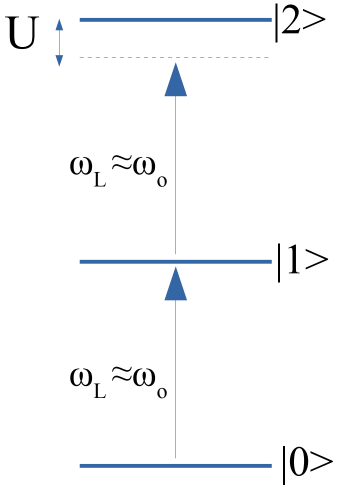
Strongly interacting photon fluids

from photon blockade to
Mott insulator states
and quantum Hall fluids

Photon blockade

Driven-dissipative Bose-Hubbard model:

$$H_0 = \sum_i \hbar\omega_o \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j\rangle} \hat{b}_i^\dagger \hat{b}_j + \hbar\frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) + \sum_i F_i(t)\,\hat{b}_i + h.c.$$

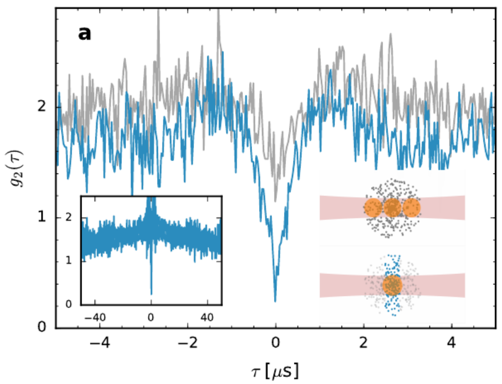


- Array of single-mode cavities at ω_o , tunnel coupling J , losses γ
- Polariton interactions: on-site interaction U due to optical nonlinearity
- If   U >> γ & J, coherent pump resonant with 0→1, but not with 1→2.

Photon blockade → Effectively impenetrable photons

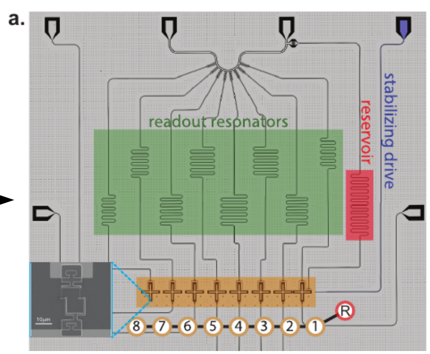
Opposite regime than non-interacting photons of Maxwell's eqs.

Single-cavity blockade observed in many platforms since the 2000s,
present challenge → scale up to many-cavity geometry

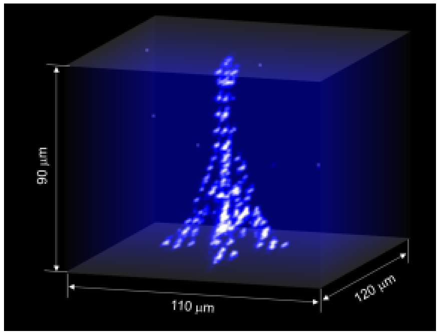


Polariton blockade via Rydberg-EIT

Circuit QED device



Fluid of spin excitations in lattice of Rydberg atoms. (Broways, Lukin,...)



Photon blockade + synthetic gauge field = FQHE for light

Bose-Hubbard model: $$H_0 = \sum_i \hbar\omega_\circ \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j\rangle} \hat{b}_i^\dagger \hat{b}_j e^{i\varphi_{ij}} + \hbar\frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1)$$

gauge field gives phase in hopping terms

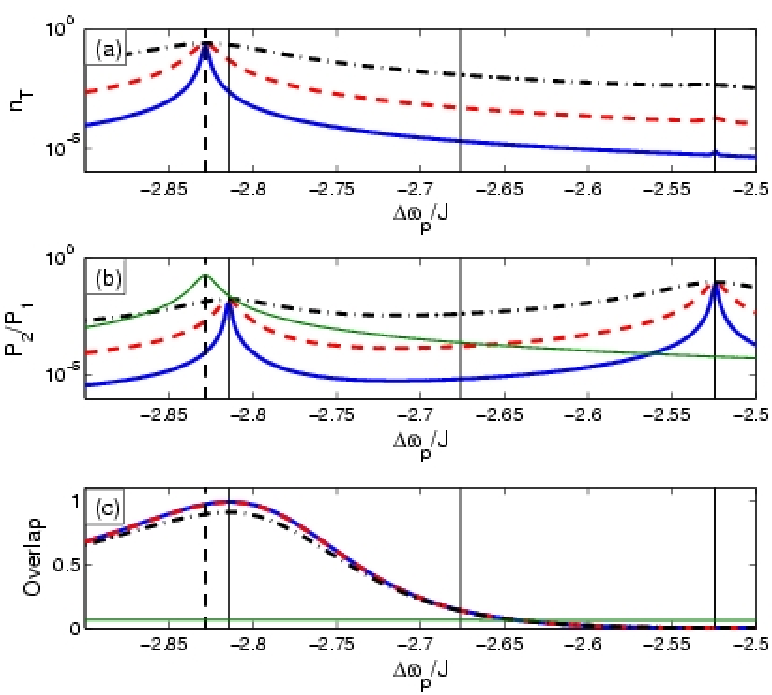
with usual coherent drive and dissipation → look for non-equil. steady state

Transmission spectra:

- peaks correspond to many-body states
- comparison with eigenstates of H_0
- good overlap with Laughlin wf (with PBC)

$$\psi_l(z_1, ..., z_N) = \mathcal{N}_L F_{\mathrm{CM}}^{(l)}(Z) e^{-\pi\alpha \sum_i y_i^2} \times \prod_{i<j}^{N} \left(\vartheta \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} \left(\frac{z_i - z_j}{L} \middle| i \right) \right)^2$$

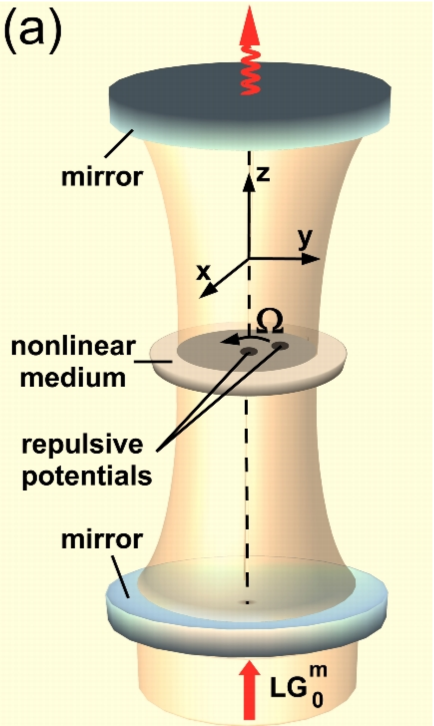
- no need for adiabatic following, etc....



R. O. Umucalilar and IC, *Fractional quantum Hall states of photons in an array of dissipative coupled cavities*, PRL 108, 206809 (2012)
See also related work by Cho, Angelakis, Bose, PRL 2008; Hafezi et al. NJP 2013; arXiv:1308.0225

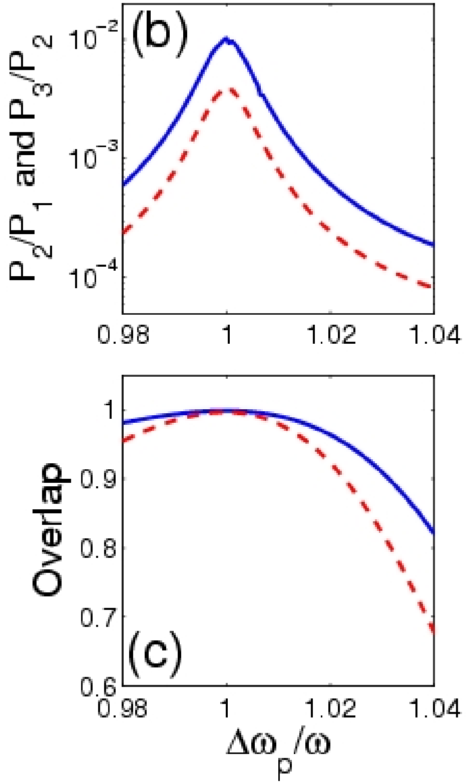
Continuous space FQH physics

Single cylindrical cavity. No need for cavity array



same form → Coriolis $F_c = -2m\Omega \times v$

→ Lorentz $F_L = e\, v \times B$



Photon gas injected by Laguerre-Gauss pump

with finite orbital angular momentum

Strong repuls. interact., e.g. layer of Rydberg atoms

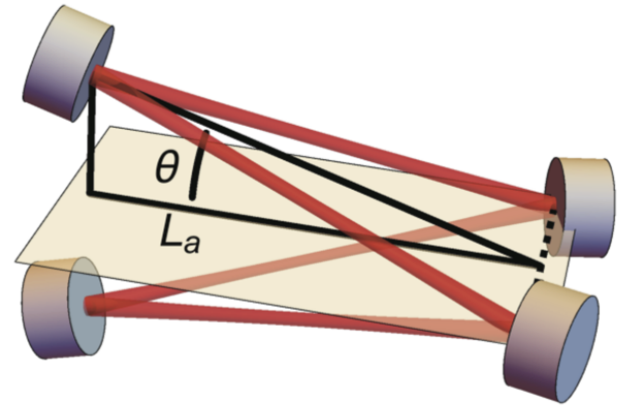
Resonant peak in transmission due to Laughlin state:

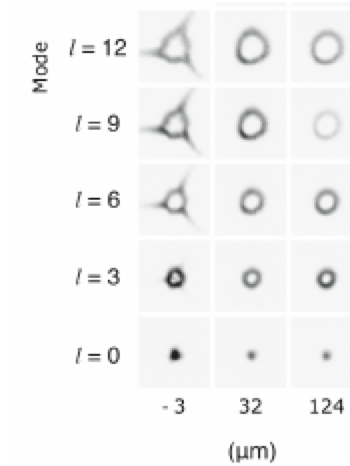
$$\psi(z_1, \ldots, z_N) = e^{-\sum_i |z_i|^2/2} \prod_{i<j} (z_i - z_j)^2$$

R. O. Umucalilar and IC, *Anyonic braiding phases in a rotating strongly correlated photon gas*, arXiv:1210.3070

Experiment @ Chicago

A far smarter design



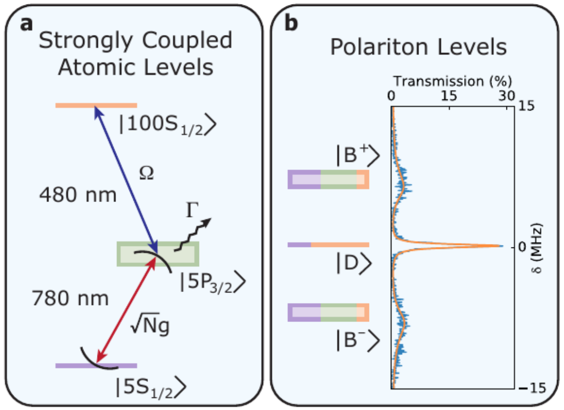


Non-planar ring cavity:

- Parallel transport→ synthetic B
- Landau levels for photons observed

Crucial advantages:

- Narrow frequency range relevant
- Integrated with Rydberg-EIT reinforced nonlinearities



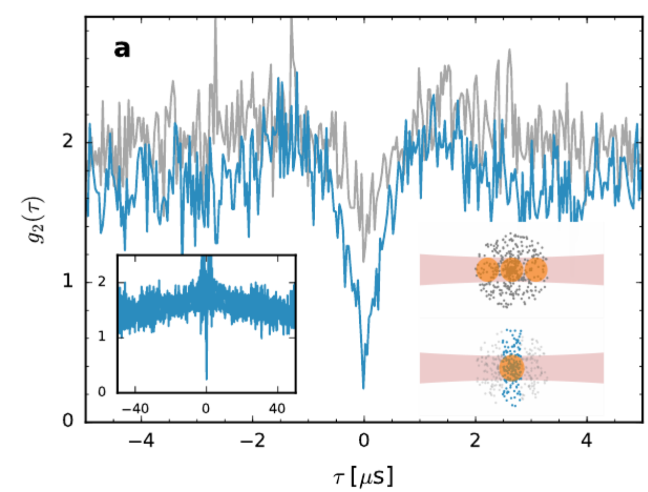
Polariton blockade on lowest (0,0) mode

- Equivalent to $\Delta_{\text{Laughlin}} > \gamma$

Easiest strategy for Laughlin

- Coherent pumping→ multi-photon peaks to few-body states
- Laughlin state → quantum correlations between orbital modes

(Umucalilar-Wouters-IC, PRA 2014)



Breaking news: 2-photon Laughlin state realized
(Clark et al., Nature 2020)

Figures from J. Simon's group @ U. Chicago
Schine et al., Nature 2016; Jia et al. 1705.07475

Experiment @ Chicago (II)

PHYSICAL REVIEW A **89**, 023803 (2014)

Probing few-particle Laughlin states of photons via correlation measurements

R. O. Umucalılar* and M. Wouters
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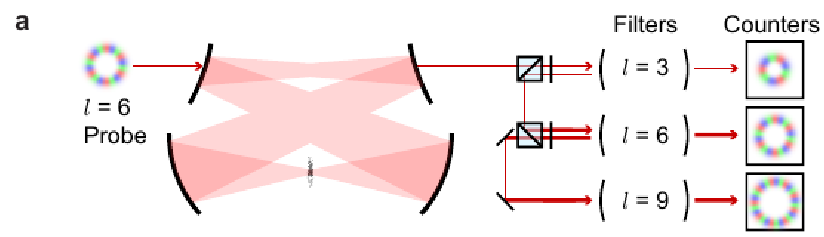
(Received 29 November 2013; published 5 February 2014)

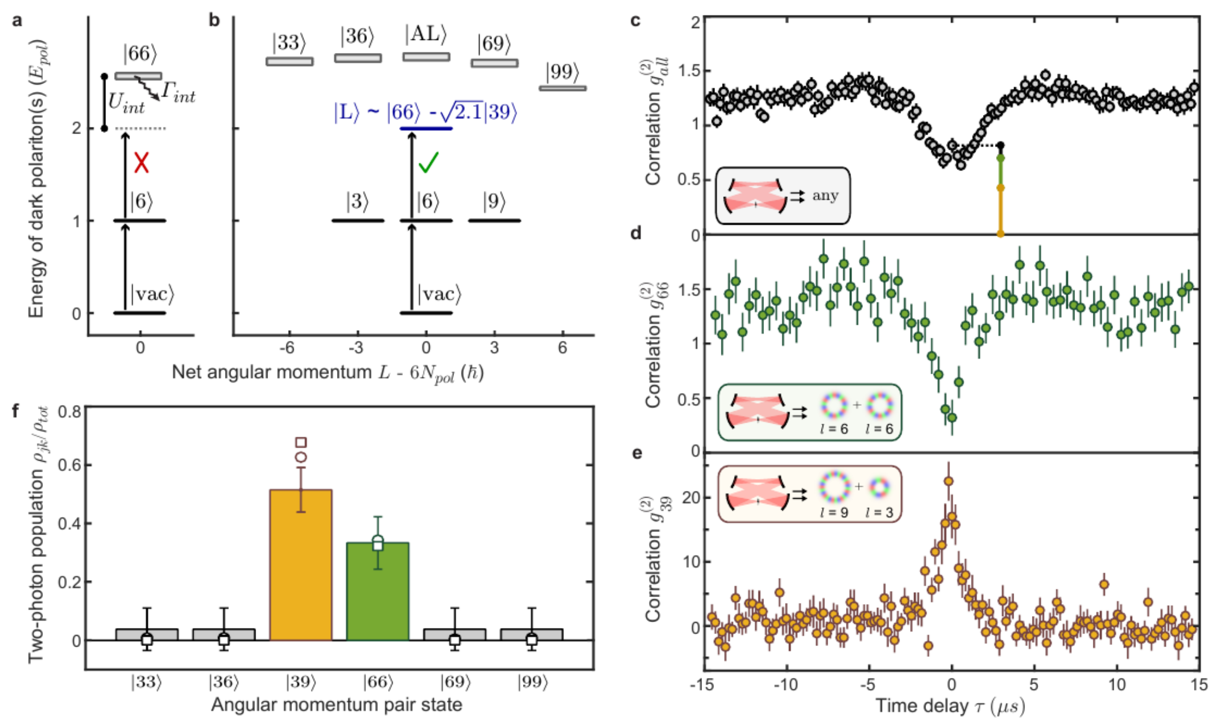
We propose methods to create and observe Laughlin-like states of photons in a strongly nonlinear optical cavity. Such states of strongly interacting photons can be prepared by pumping the cavity with a Laguerre-Gauss beam, which has a well-defined orbital angular momentum per photon. The Laughlin-like states appear as sharp resonances in the particle-number-resolved transmission spectrum. Power spectrum and second-order correlation function measurements yield unambiguous signatures of these few-particle strongly correlated states.



Quantum optical tricks to highlight generation of two-photon Laughlin state

<u>Challenge:</u> scale up to larger number of particles

Coherent pump scheme scales very bad with N for topological states



L. W. Clark, N. Schine, C. Baum, N. Jia, J. Simon, *Observation of Laughlin states made of light*, Nature 2020

What about large FQH fluids?

Coherent pump:

- Able to selectively generate few-body states
- Limited by (exponentially) decreasing matrix element for larger systems

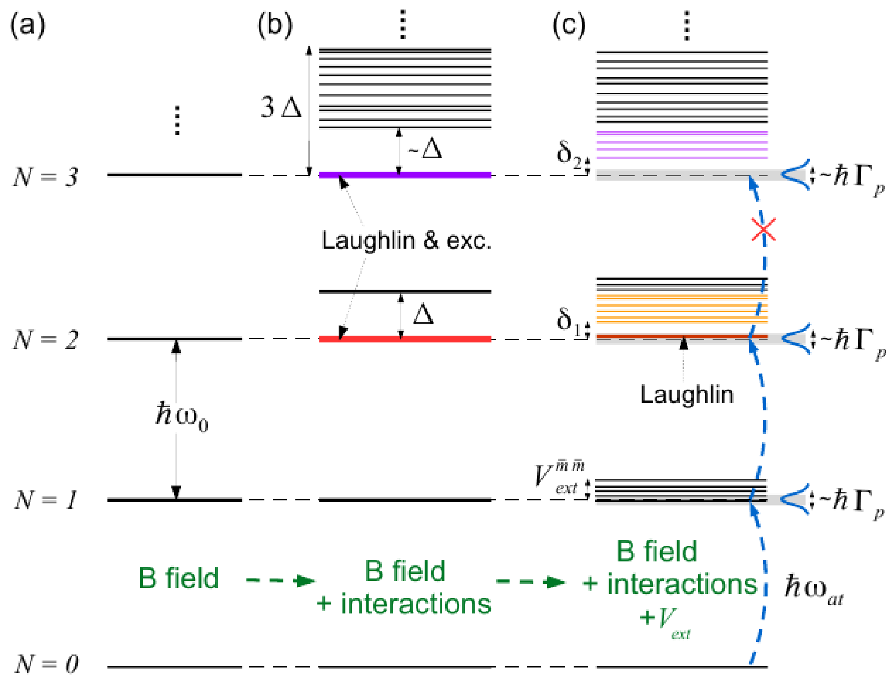
Frequency-dependent incoherent pump:

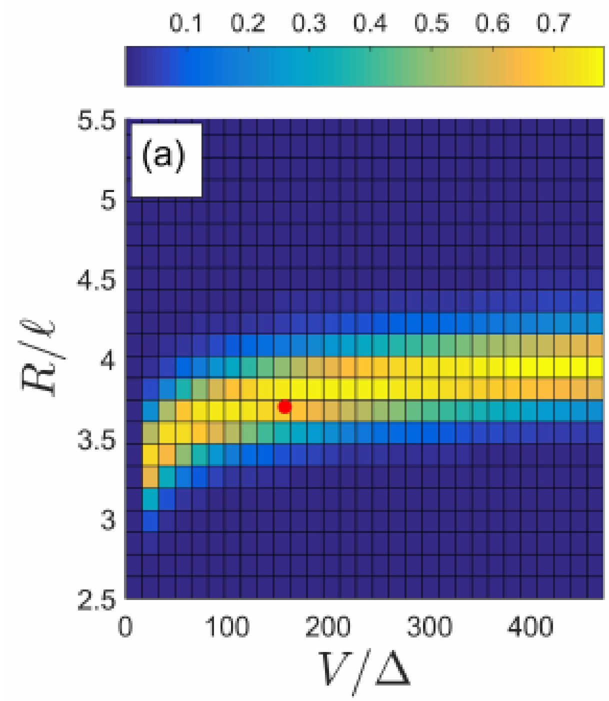
- Interactions → many-body gap Δ
- Edge excitations not gapped. Hard-wall confinement gives small δ
- Non-Markovianity blocks excitation to higher states

Calculations only possible for small systems:

- Large overlap with Laughlin states
- Excitations localized mostly on edge

Open question: what are ultimate limitations of this pumping method?





R. O. Umucalilar and IC, *Generation and spectroscopic signatures of a fractional quantum Hall liquid of photons in an incoherently pumped optical cavity*, PRA 2017

R. O. Umucalilar, J. Simon, IC, *Autonomous stabilization of photonic Laughlin states through angular momentum potentials*, arXiv:2105.06751

Part 3.2:

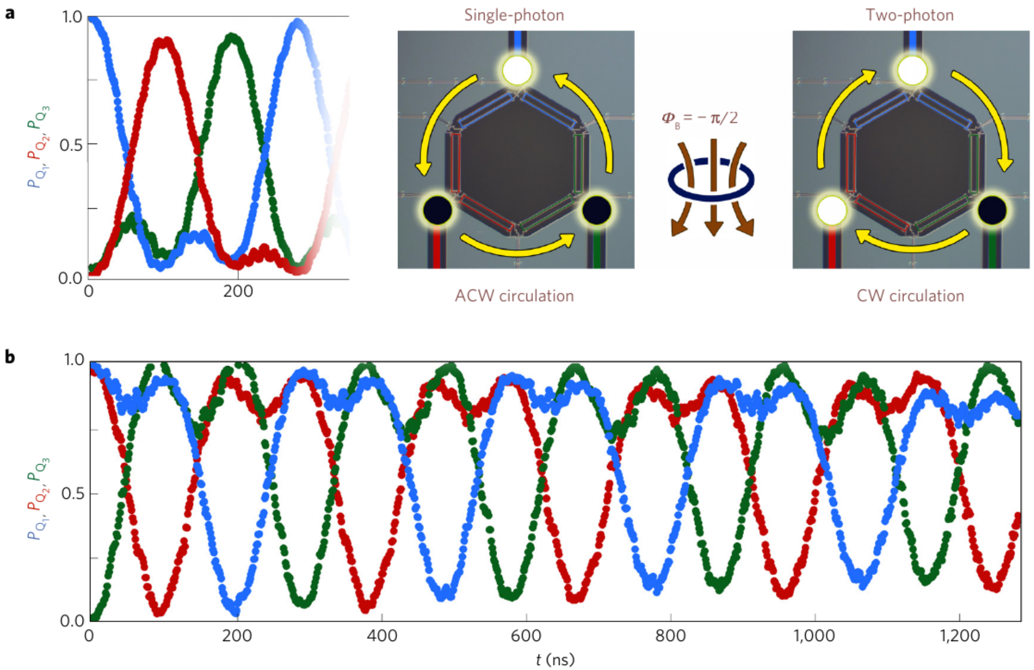
How to probe the dynamics of FQH states?

How to observe anyonic statistics of quasi-hole excitations?

Conservative dynamics in circuit-QED experiment:
interplay of strong interactions & synthetic magnetic field

Ring-shaped array of qubits in a superconductor-based circuit-QED platform

- Transmon qubit: two-level system
  → Impenetrable microwave photons
- Time-modulation of couplings
  → synthetic gauge field



Roushan et al., Nat. Phys. 2016

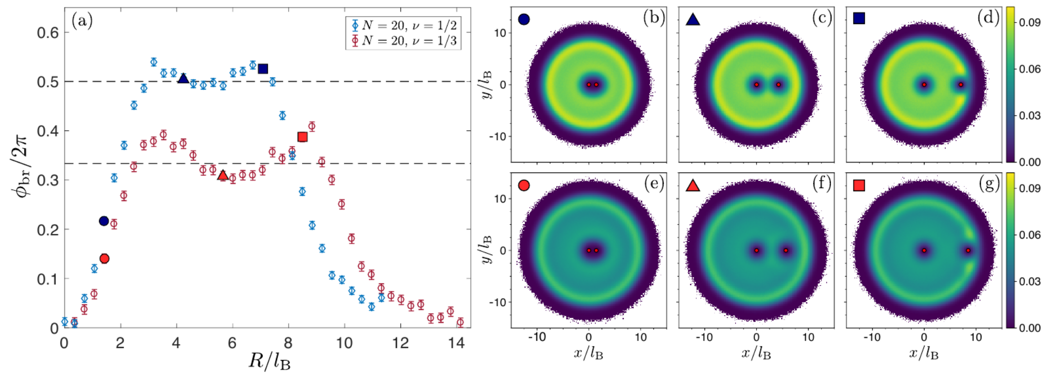
- Independently initialize sites
- Follow unitary evolution until bosons lost
  (microwave photons → long lifetime)
- Monitor site occupation in time

"Many"-body effect:

two-photon state → opposite rotation compared to one-photon state

(similar to cold-atom experiment in Greiner's lab: Tai et al., Nature 2017)

Observing anyonic statistics via time-of-flight measurements



Braiding phase → Berry phase when two quasi-holes are moved around each other

$$\varphi_{\mathrm{B}}(R) = i\oint_R \langle\Psi(\theta)|\partial_\theta|\Psi(\theta)\rangle d\theta.$$

Braiding operation can be generated by rotations, so braiding phase related to L_z

$$\varphi_{\mathrm{B}}(R) = \frac{1}{\hbar}\oint_R \langle\Psi(\theta)|L_z|\Psi(\theta)\rangle d\theta = \frac{2\pi}{\hbar}\langle L_z\rangle$$

Self-similar expansion of lowest-Landau-levels → L_z can be measured in time-of-flight via size of the expanding cloud

$$\langle r^2\rangle_{\mathrm{tof}} = \frac{1}{N}\left(\frac{\hbar t}{\sqrt{2}Ml_B}\right)^2\left(\frac{\langle L_z\rangle}{\hbar} + N\right) = \left(\frac{\hbar t}{2Ml_B^2}\right)^2\langle r^2\rangle$$

Can be applied to both cold atoms or to fluids of light looking at far-field emission pattern
Difficulty → small angular momentum difference of QH compared to total L_z

Umucalilar, Macaluso et al., *Observing anyonic statistics via time-of-flight measurements*, PRL (2018)

Quasi-Hole structure vs. anyon statistics (I)

- Compare (two) single quasi-holes and
  overlapping pair of quasi-holes:

$$\frac{\varphi_{\rm br}}{2\pi} = \frac{1}{\hbar}\left[\langle \hat{L}_z\rangle_{|\eta_1|=|\eta_2|} - \langle \hat{L}_z\rangle_{\eta_1=\eta_2}\right].$$

- Relates to difference of density profiles:

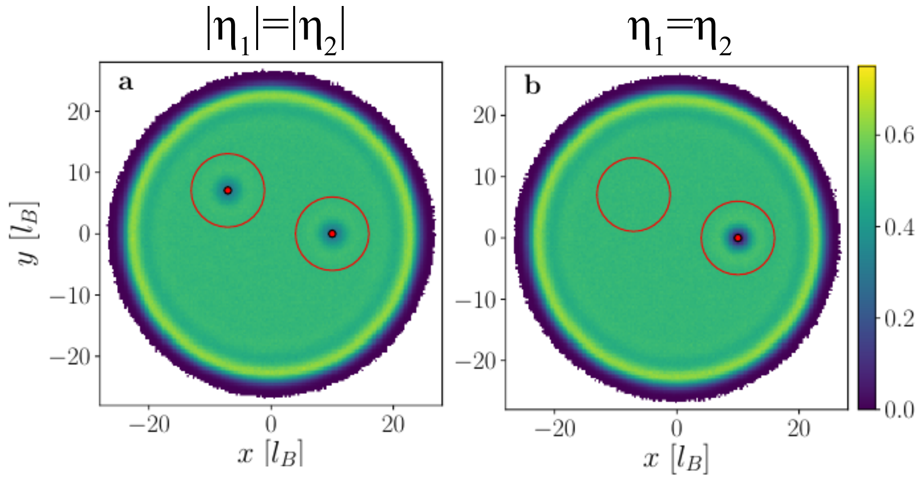
$$\frac{\varphi_{\rm br}}{2\pi} = \frac{N}{2l_B^2}\left[\langle r^2\rangle_{|\eta_1|=|\eta_2|} - \langle r^2\rangle_{\eta_1=\eta_2}\right],$$

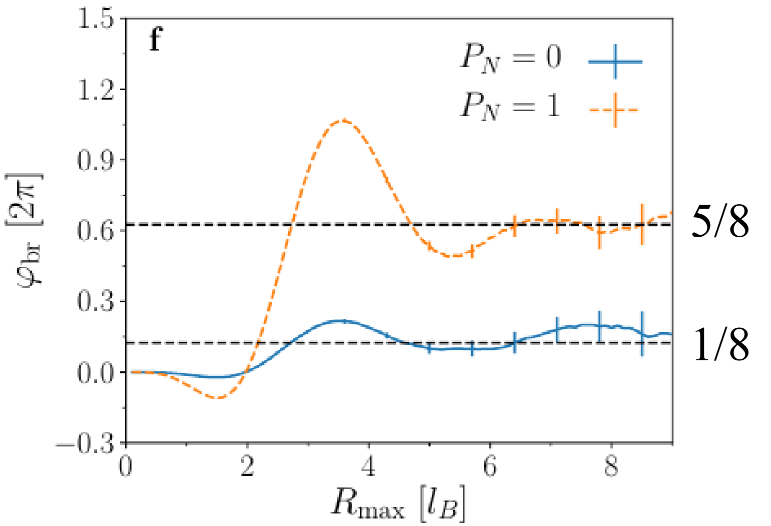
- Incompressibility → external region unaffected

- Statistics inferred from local density difference around QH core, i.e. variance of density depletion

- Insensitive to spurious excitation of
  (ungapped) edge states

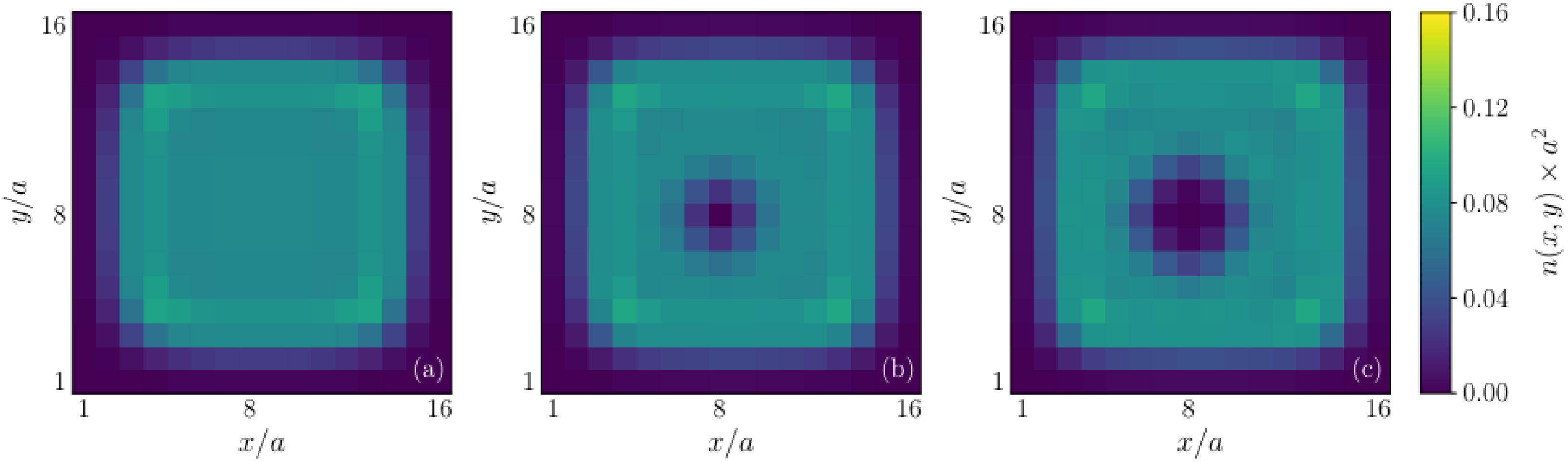
- Numerical calculation using Moore-Read wavefunction allows to distinguish fusion channels of even/odd
  total particle number





E. Macaluso, T. Comparin, L. Mazza, IC, *Fusion channels of non-Abelian anyons from angular-momentum and density-profile measurements*, PRL 2019

Quasi-Hole structure vs. anyon statistics (II)



N	L	α	Ω/t
12	16	0.15	1×10^{-3}

Discrete lattice model → Harper-Hofstadter-Bose-Hubbard

Ground state using Tree-Tensor-Network ansatz

- experimentally realistic "large" system
- open boundary conditions with harmonic trap
- repulsive potentials to pin quasi-holes

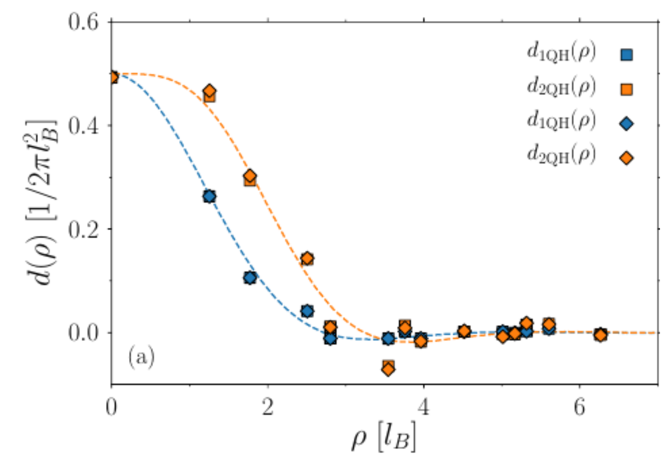
Apply discretized version of braiding phase formula

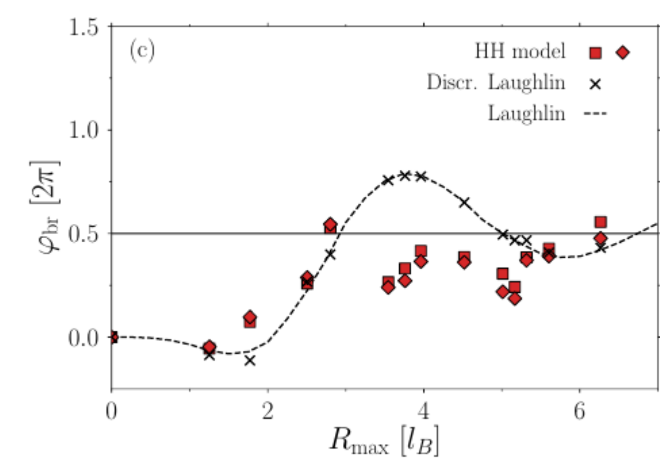
$$\frac{\varphi_{\rm br}}{2\pi} = \frac{N}{2l_B^2}\left[\langle r^2\rangle_{|\eta_1|=|\eta_2|} - \langle r^2\rangle_{\eta_1=\eta_2}\right],$$

to physical ground state wavefunction

→ Accurate reconstruction of anyonic statistics

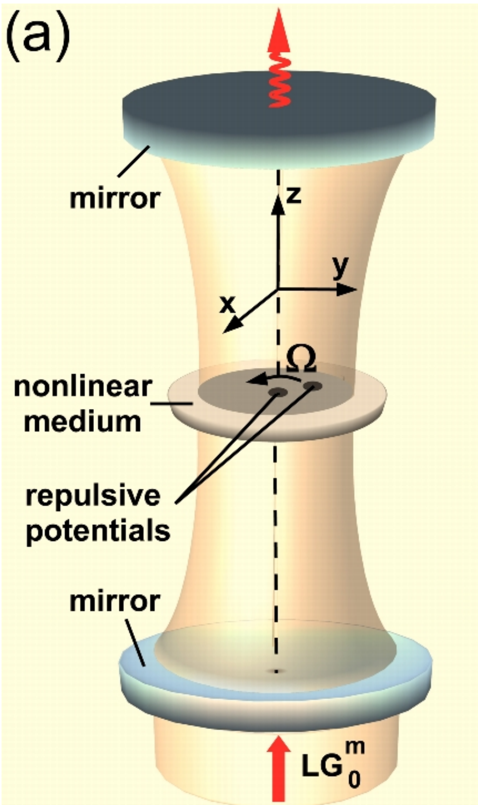
→ Experiment accessible in state-of-the-art circuit-QED systems





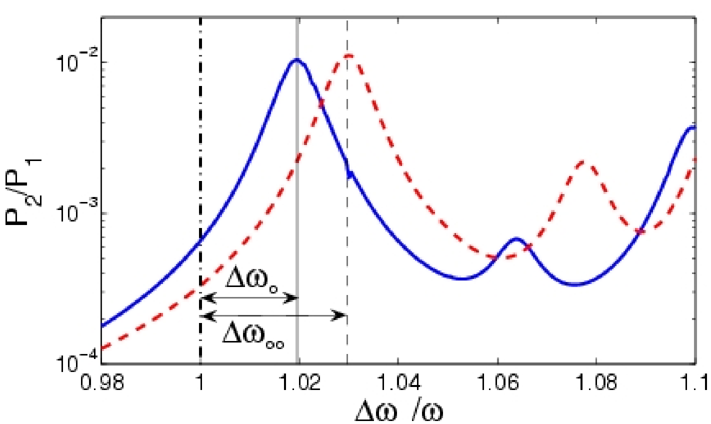
E. Macaluso *et al.*, *Charge and statistics of lattice quasiholes from density measurements: a Tree Tensor Network study,* Phys. Rev. Research (2020)

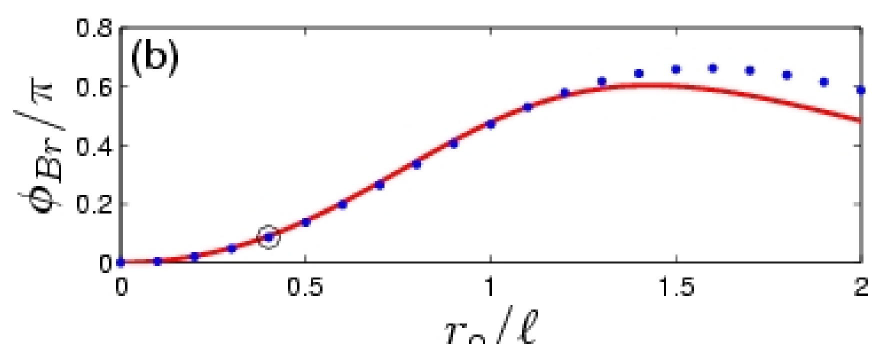
Optical signatures of the anyonic braiding phase



- LG pump to create and maintain quantum Hall liquid
- Localized repulsive potentials in trap:
  → create quasi-hole excitation in quantum Hall liquid
  → position of holes adiabatically braided in space
- Anyonic statistics of quasi-hole: many-body Berry phase ϕ_{Br} when positions swapped during braiding
- Berry phase extracted from shift of transmission resonance while repulsive potential moved with period T_{rot} along circle

$$\phi_{Br} \equiv (\Delta\omega_{oo} - \Delta\omega_{o})\, T_{rot} \quad [2\pi]$$



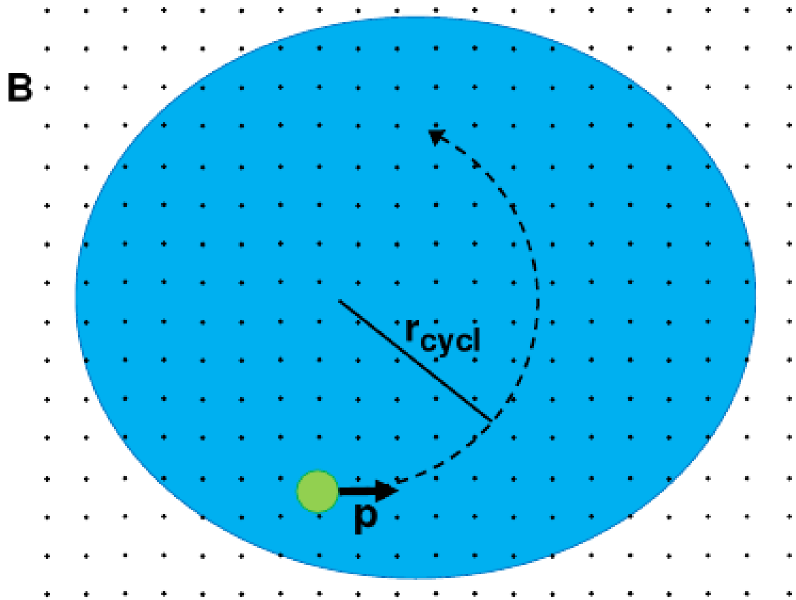


R. O. Umucalilar and IC, *Anyonic braiding phases in a rotating strongly correlated photon gas*, arXiv:1210.3070

Quantum mechanics of anyons (I) – single particle

Laughlin wavefunction of Fractional Quantum Hall:

- quasi-holes → no E_{kin}, no independent life
- dressed by heavy impurity → anyonic molecule
- full-fledged mechanical degree of freedom



Born-Oppenheimer approx:

- Heavy impurity→ slow Degree of Freedom
- Light FQH particles → fast DoF

$$H_{\rm eff} = \frac{\left[-i\nabla_{\mathbf{R}} - (Q - \nu q)\,\mathbf{A}(\mathbf{R})\right]^2}{2\mathcal{M}}$$

- Mass $\mathcal{M}$ → M (impurity) + QH dragging effect
- Impurity & FQH particles feel (Synth-)B,
  so synth-Charge → Q (impurity) – ν q (QH)

Cyclotron orbit → fractional charge and BO mass correction

A. Muñoz de las Heras, E. Macaluso, IC, Phys. Rev. X 10, 041058 (2020)

Quantum mechanics of anyons (II) – two particles

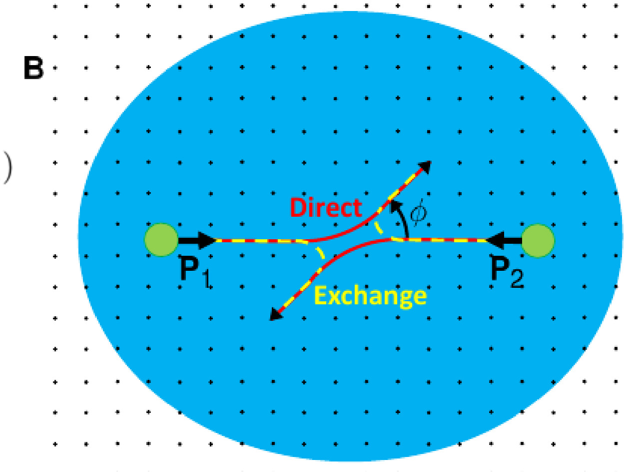
Each particle → attached flux

$$\mathcal{A}_j(\mathbf{R}) = \mathcal{A}_{\mathrm{q}}(\mathbf{R}_j) + \mathcal{A}_{\mathrm{stat},j}(\mathbf{R}) = \frac{\mathcal{B}_{\mathrm{q}}}{2}\mathbf{u}_{\mathrm{z}} \times \mathbf{R}_j + (-1)^j \frac{\nu}{R_{\mathrm{rel}}^2}\mathbf{u}_{\mathrm{z}} \times \mathbf{R}_{\mathrm{rel}}$$

Relative motion:

- inter-particle potential
- statistical A_{rel} due to attached flux

$$H_{\mathrm{rel}} = \frac{[\mathbf{P}_{\mathrm{rel}} + \mathbf{A}_{\mathrm{rel}}(\mathbf{R}_{\mathrm{rel}})]^2}{2\mathcal{M}_{\mathrm{rel}}} + V_{\mathrm{ii}}(R_{\mathrm{rel}})$$



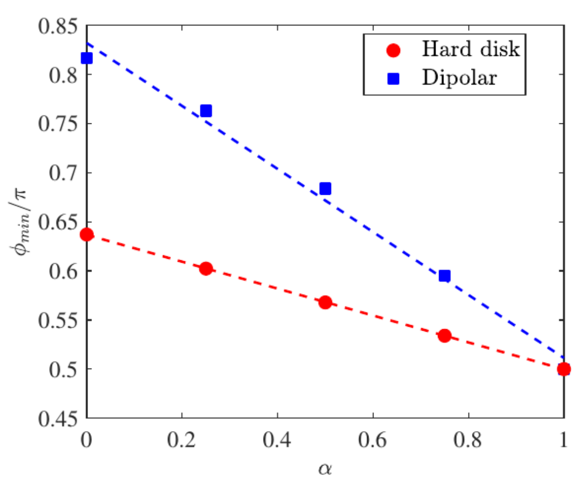
2-body scattering: interference of direct & exchange

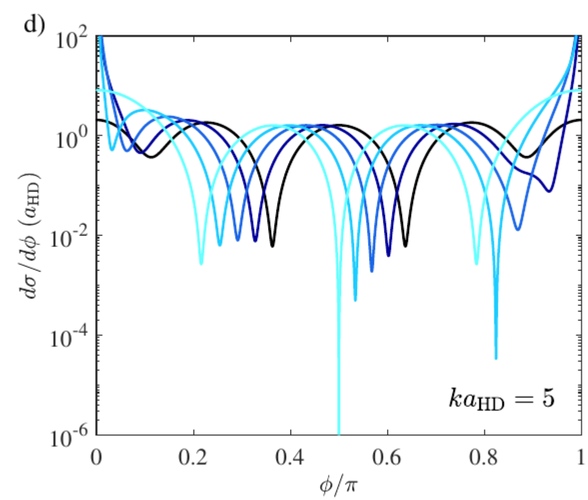
- fringes in differential cross section
- fringe position depends on attached flux, i.e. fractional statistics

Measures fractional statistics

Scheme works best with polar molecules (heavy + long-range interactions) in atoms (light FQH gas)

Work in progress: extend to fluids of light, e.g. Rydberg polaritons





A. Muñoz de las Heras, E. Macaluso, IC, Phys. Rev. X 10, 041058 (2020)

Conclusions and perspectives

1-body magnetic and topological effects for photons in synthetic gauge field:

- Unidirectional and topologically protected edge states (2009-)
- Geometrical properties of bulk & anomalous current (2016-)

Topological lasing (archetypal example of nonlinear topo-optics phenomenon):

- Experimental observation of laser operation into topological edge mode (2017-)
- Theoretical studies of semiclassical field profile and dynamical stability (2018-)
- Coherence properties of topolaser: KPZ physics and robustness against disorder (2020-)

→ a unique platform to study quantum effects in non-equilibrium statistical mechanics

First steps in strongly correlated many-body physics:

- Photon blockade in many platforms: CQED with atoms and solids, circuit-QED, Rydberg atoms,…
- Mott-insulator → recent experimental observation @ Chicago
- Chain of strongly interacting bosons in synthetic gauge field
  → recent experimental observation @ GoogleLabs!
- Few-body Laughlin states → first annoucement by J. Simon, Apr. '19

→ challenge: scale up to macroscopic fluids: exotic properties of fractional quantum Hall fluids !

If you wish to know more...

BEC
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REVIEWS OF MODERN PHYSICS, VOLUME 85, JANUARY–MARCH 2013

Quantum fluids of light

Iacopo Carusotto*
INO-CNR BEC Center and Dipartimento di Fisica, Università di Trento, I-38123 Povo, Italy

Cristiano Ciuti†
Laboratoire Matériaux et Phénomènes Qu...
Bâtiment Condorcet, 10 rue Alice Domon ...

IC, C. Ciuti, RMP **85**, 299 (2013)

Come and visit us in Trento,
we are open!
(of course following COVID rules)

nature physics | FOCUS | REVIEW ARTICLE
https://doi.org/10.1038/s41567-020-0815-y

Check for updates

Photonic materials in circuit quantum electrodynamics

Iacopo Carusotto[1], Andrew A. Houck[2], Alicia J. Kollár[3,4], Pedram Roushan[5], David I. Schuster[6,7] and Jonathan Simon[6,7]

Review article on Nature Physics (2020)

REVIEWS OF MODERN PHYSICS, VOLUME 91

Topological photonics

Review article arXiv:1802.04173 by Ozawa, Price, Amo, Goldman, Hafezi, Lu, Rechtsman, Schuster, Simon, Zilberberg, IC, RMP **91**, 015006 (2019)

We acknowledge generous financial support from:

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European Union funding
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PhoQuS
Photons for Quantum Simulation

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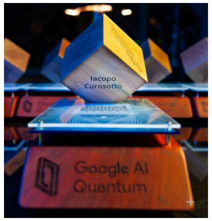

PhD & Post-Doc positions soon available

Contact:
iacopo.carusotto@unitn.it

Save the date
July 3-8 (TBC), 2022
"Quantum Fluids of Light and Matter" Summer School
in Varenna

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SUMMER COURSES ~~2020~~ ~~2021~~ 2022

VILLA MONASTERO – VARENNA, LAKE COMO

Course 208
FOUNDATIONS OF COSMIC RAY ASTROPHYSICS
25 - 30 June

Topics:

- High-energy Cosmic Rays and gamma-radiation
- Particle acceleration in Pulsars and PWNe
- Charged particles in turbulent fields and Cosmic Ray transport
- Cosmic Ray propagation in extragalactic space and secondary messengers
- Particle acceleration at shocks and in turbulence
- Phenomenological models of galactic Cosmic Ray transport
- Star Formation Regions and Cosmic Rays
- The microphysics of Cosmic Ray instabilities
- Interactions of Cosmic Rays with matter and radiation
- Magnetic reconnection
- Future facilities in high-energy astrophysics
- Basics of Cosmic Ray Feedback

Lecturers:

Felix Aharonian – DIAS, Dublin (Ireland)
Elena Amato – INAF, Arcetri Osservatorio, Firenze (Italy)
Pasquale Blasi – GSSI and INFN, L'Aquila (Italy)
Denise Boncioli – Università and INFN, L'Aquila, (Italy)
Damiano Caprioli – The University of Chicago (USA)
Carmelo Evoli – GSSI and INFN, L'Aquila (Italy)
Stefano Gabici – APC, AstroParticule et Cosmologie, Paris (France)
Alexandre Marcowith – Université Montpellier and CNRS/IN2P3, Montpellier (France)
Pasquale Dario Serpico – LAPTh, Université Grenoble Alpes, Annecy (France)
Lorenzo Sironi – Columbia University, New York (USA)
Emma de Ona Wilhelmi – ICE, Barcelona (Spain) and DESY, Hamburg (Germany)
Ellen Zweibel – University of Wisconsin-Madison (USA)

Directors:

Felix Aharonian – Dublin Institute of Advanced Studies (Ireland)

Course 209
QUANTUM FLUIDS OF LIGHT AND MATTER - QFLM2020
3 – 8 July

Topics:

- Basics of quantum gases
- Quantum fluids of light
- Topological matter
- Topological photonics
- Strongly correlated open quantum systems
- Quantum trajectories and quantum jumps in quantum optics
- Quantum optics with Rydberg atoms
- Circuit QED
- Optics of strongly correlated electron gases
- History of nonlinear optics

Lecturers:

Hannes Bernien – University of Chicago (USA)
Jacqueline Bloch – Centre de Nanosciences et de Nanotechnologies, Palaiseau (France)
Howard Carmichael – University of Auckland (New Zealand)
Steven Girvin – Yale University, New Haven (USA) + Shruti Puri
Atac Imamoglu – ETH Zurich (Switzerland)
Luigi Lugiato – Universita' dell'Insubria, Como (Italy)
Tomoki Ozawa – RIKEN, Saitama (Japan)
Nicolas Regnault – Laboratoire de Physique, Ecole Normale Superieure Paris, CNRS (France) and Princeton University (USA)
Leticia Tarruell – ICFO, The Institute of Photonic Sciences, Castelldefels, Barcelona (Spain)

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Iacopo Carusotto – INO-CNR BEC Center, Povo TN (Italy)
Cristiano Ciuti – Laboratoire Matériaux et Phénomènes Quantiques, Université de Paris (France)

Course 210
MULTIMODAL AND NANOSCALE OPTICAL MICROSCOPY
11 – 16 July

Topics:

- Multimodal optical microscopy
- Fluorescence microscopy
- Non linear optical microscopy
- Label free
- Mueller matrix optical microscopy
- Brillouin microscopy
- Polarization microscopy
- Three-dimensional microscopy
- F methods in microscopy (FRAP FLIM FRET FCS)
- Lifetime fluorescence
- Super resolution
- Phototoxicity and photodamage
- Optical and magnetic trapping
- Image formation
- Inverse problems
- Bioimage analysis

Lecturers:

Sara Abrahamsson – Jack Baskin School of Engineering UC Santa Cruz (USA)
Francisco Balzarotti – Max Planck Institute for Biophysical Chemistry, Göttingen (Germany)
Sophie Brasselet – Institut Fresnel Domaine Universitaire St Jerome, Marseille (France)
Julien Colombelli – Institute for Research in Biomedicine - IRB Barcelona, Barcelona Institute of Science and Technology, Barcelona (Spain)
Elisa Ferrando-May – University of Konstanz (Germany)
Laura Finzi – Emory College of Arts and Sciences, Atlanta (USA)
Luca Lanzanò – Nanoscopy IIT, Erzelli Labs, Genova (Italy)
Davide Mazza – Experimental Imaging Center Ospedale San Raffaele, Milano (Italy)