Nambu-Covariant Many-Body Theory

A Green's functions framework for superfluid systems

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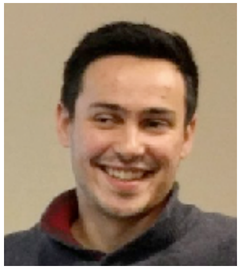
Universitat de Barcelona
&
Department of Physics
University of Surrey

ECT* Workshop
Nuclear Physics Meets Condensed Matter
19 July 2021
ECT*/Online



M. Drissi
UNIVERSITY OF SURREY
TRIUMF

arxiv:2107xx+ arxiv:2107yy

Outlook

1. **Motivation:** why superfluidity in neutron stars?

2. Some **previous** results

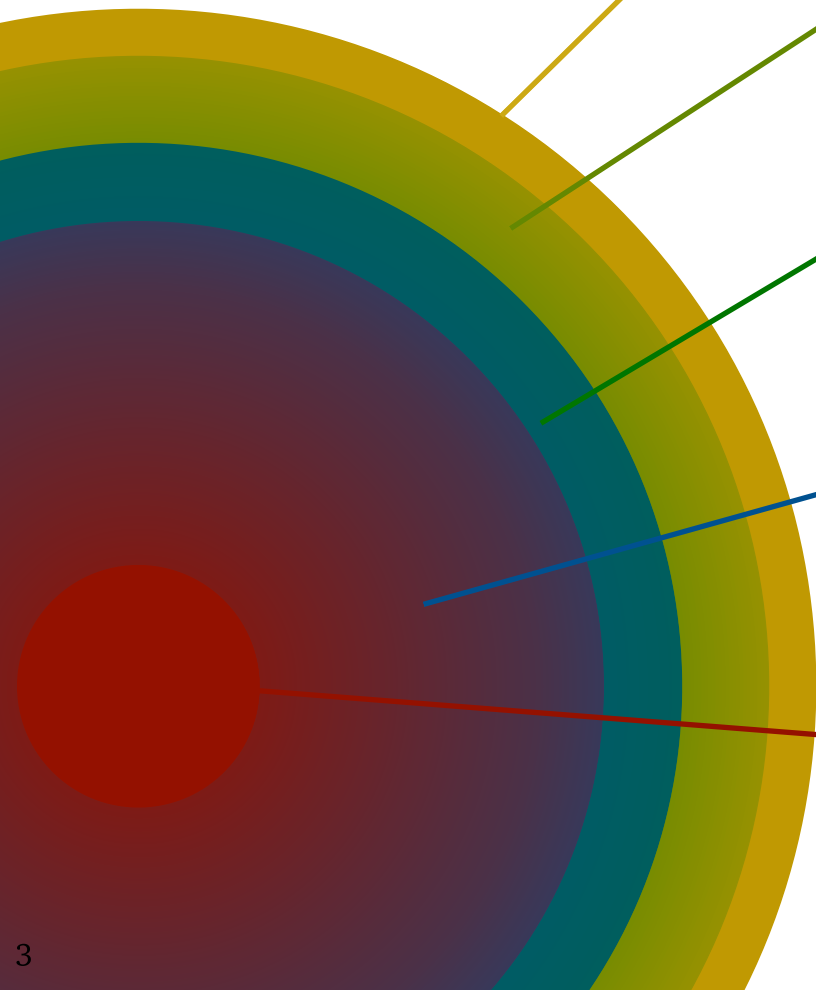
3. **Nambu-Covariant Perturbation Theory**

4. **Nambu-Covariant Self-Consistent Green's Functions**

Stellar corpse CSI

$$R \approx 10 \text{ km}$$

$$M \approx (1-2)M_{\odot}$$



Atmosphere (1-10 m)
Ionized gas, x-ray emission

Outer crust (0.3-0.5km)
Ions, electrons

Inner crust (1-2 km)
Nuclear lattice/pasta

Outer core ($\sim$9 km)
Neutron liquid, % of protons and e-

Inner core ($\sim$0-3 km)
Unknown - quark gluon plasma?

Neutron star structure

Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m + 4\pi p r^3)(\epsilon + p)}{r(r - 2Gm/c^2)}$$

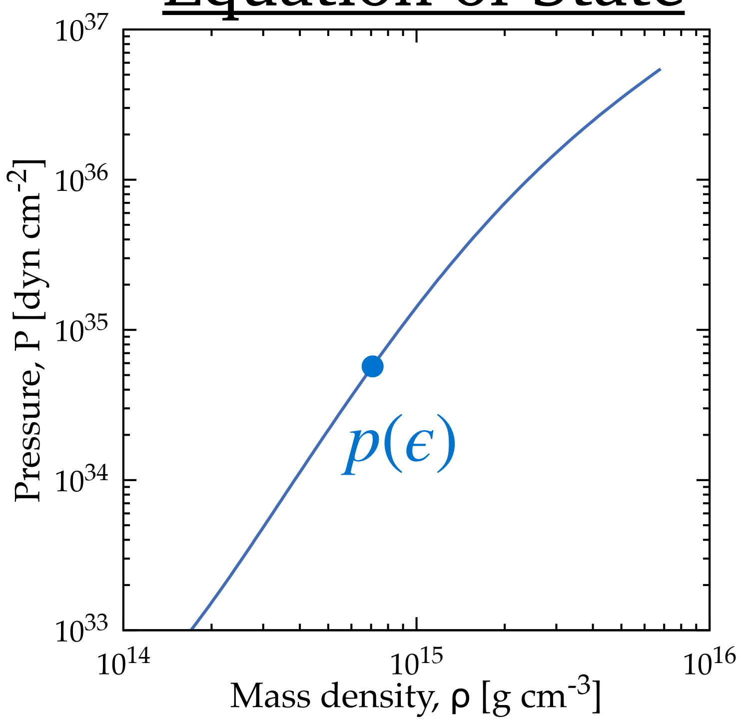
$$\frac{dm}{dr} = \frac{4\pi}{c^2}\epsilon r^2$$

+

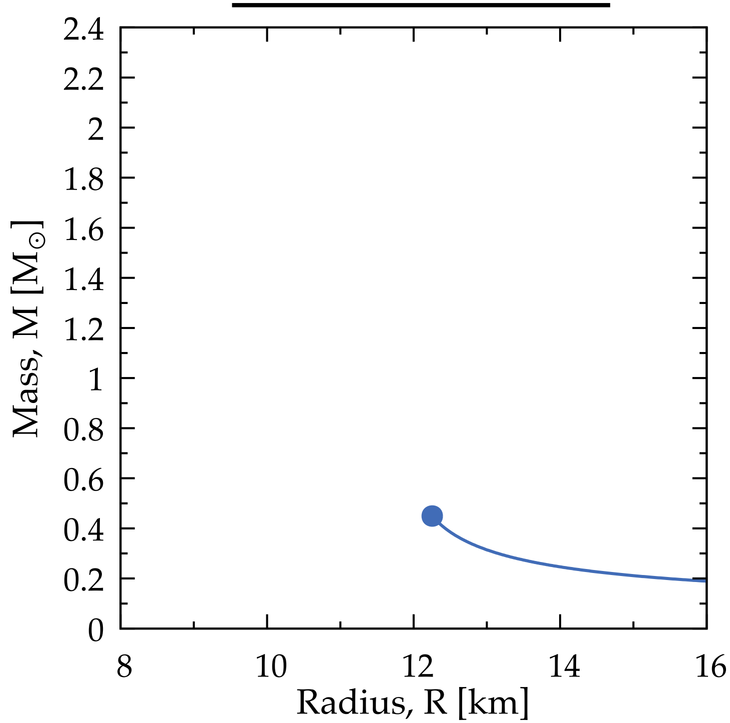
EoS

$$p \equiv p(\epsilon)$$

Equation of State



Mass-Radius



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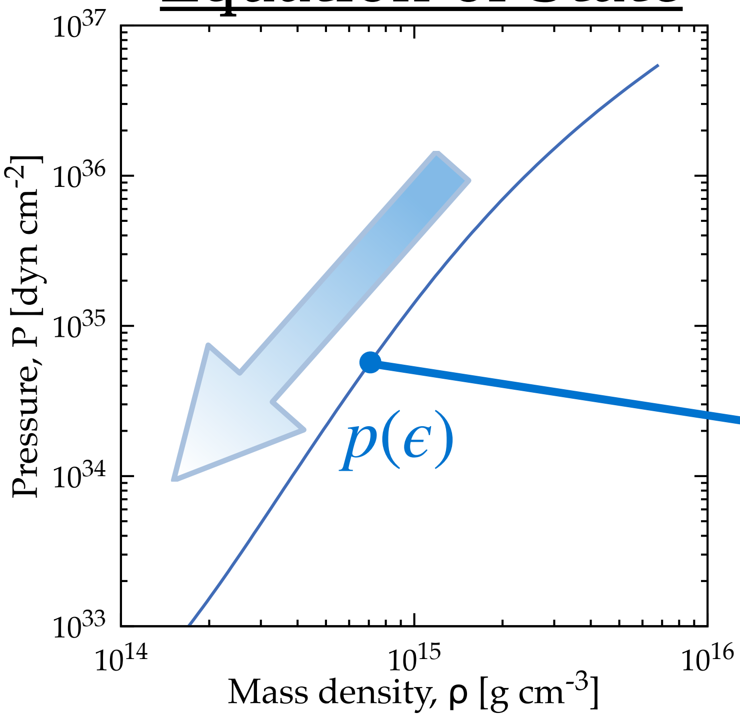

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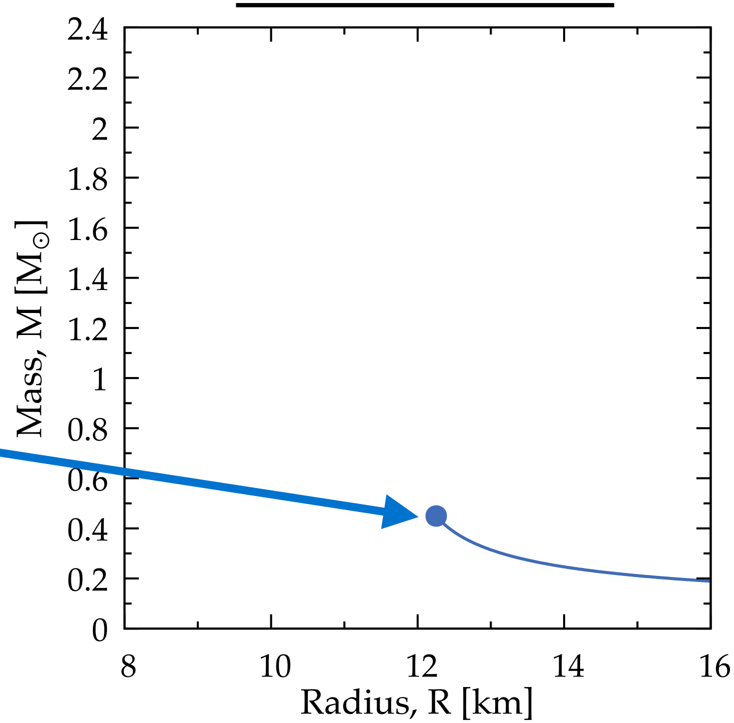

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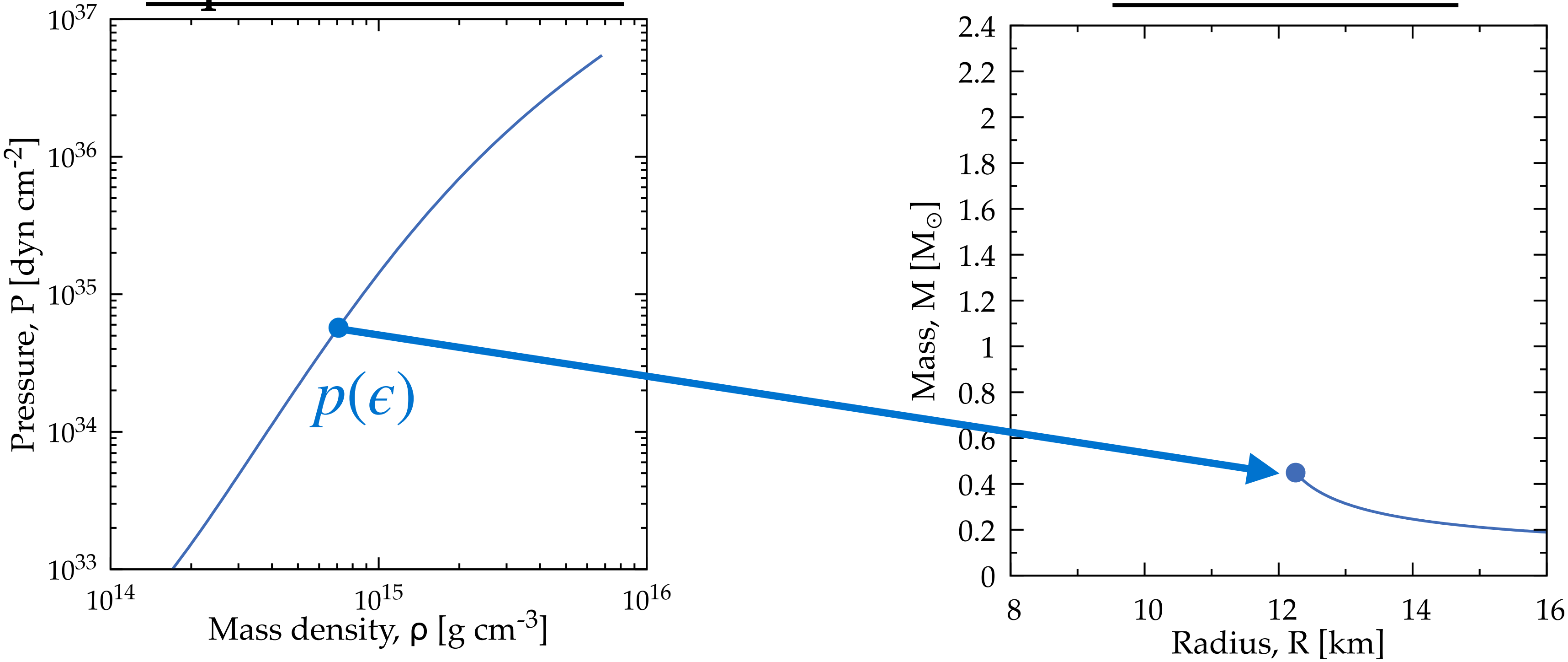

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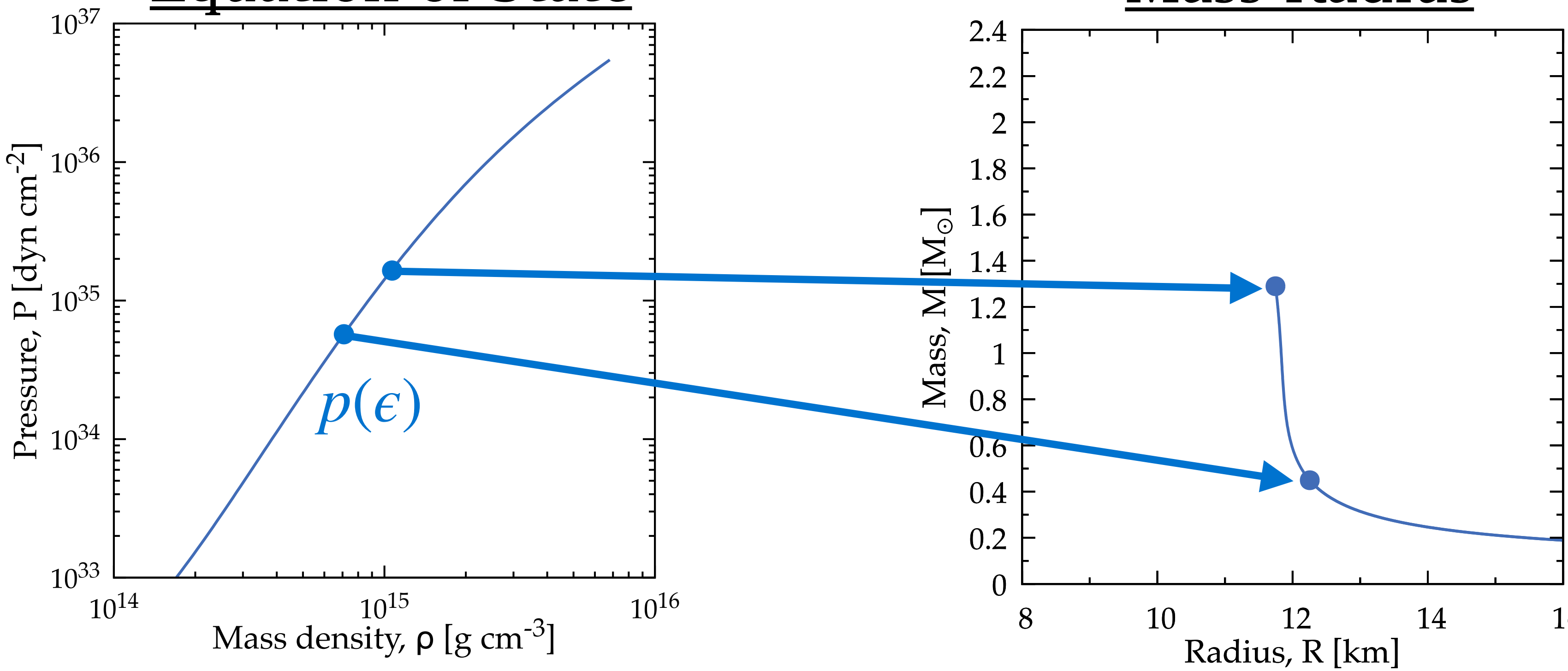

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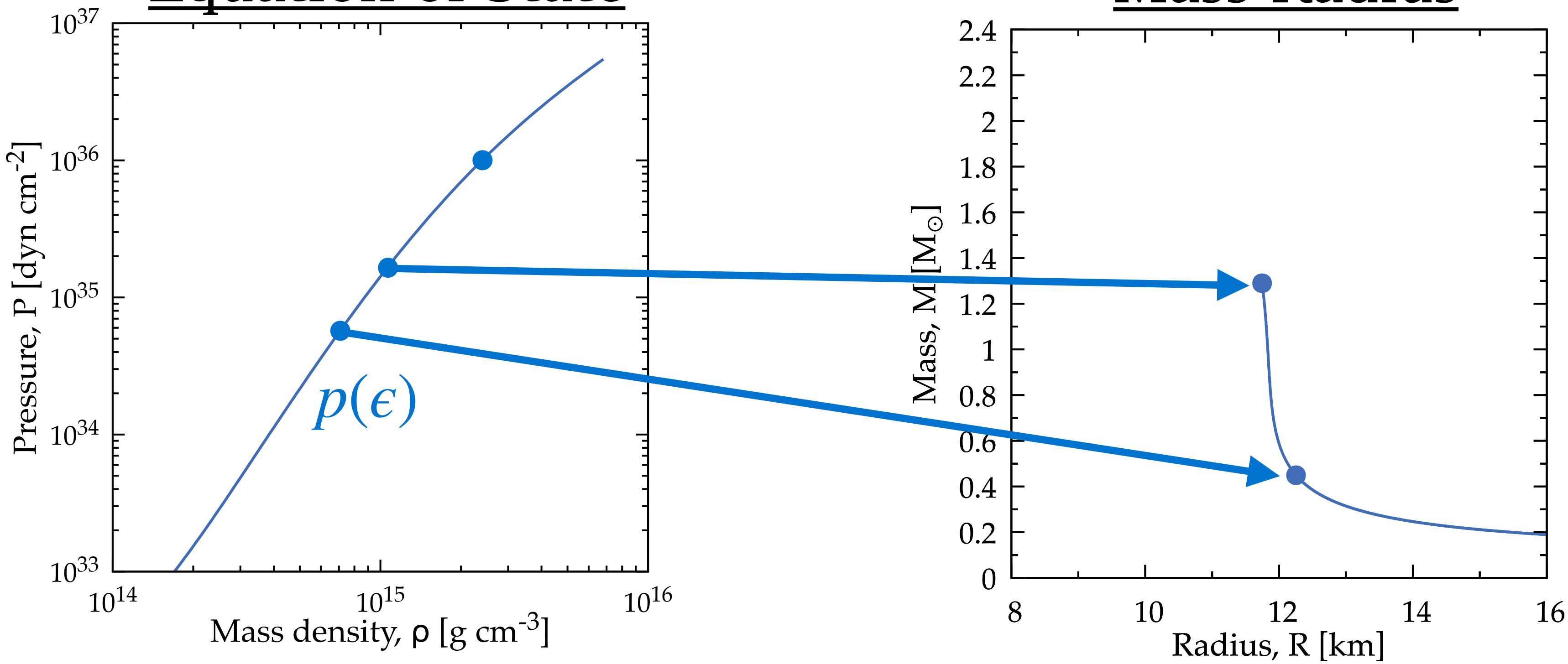

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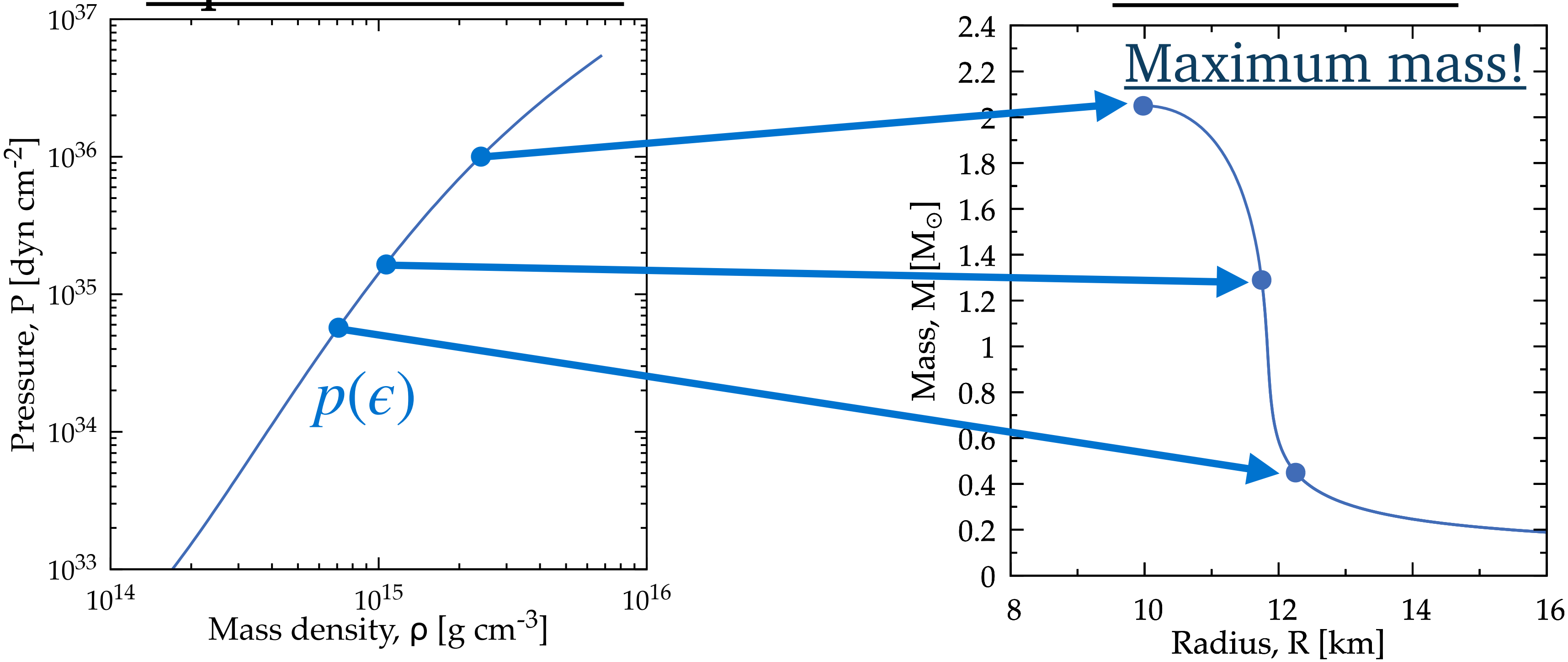
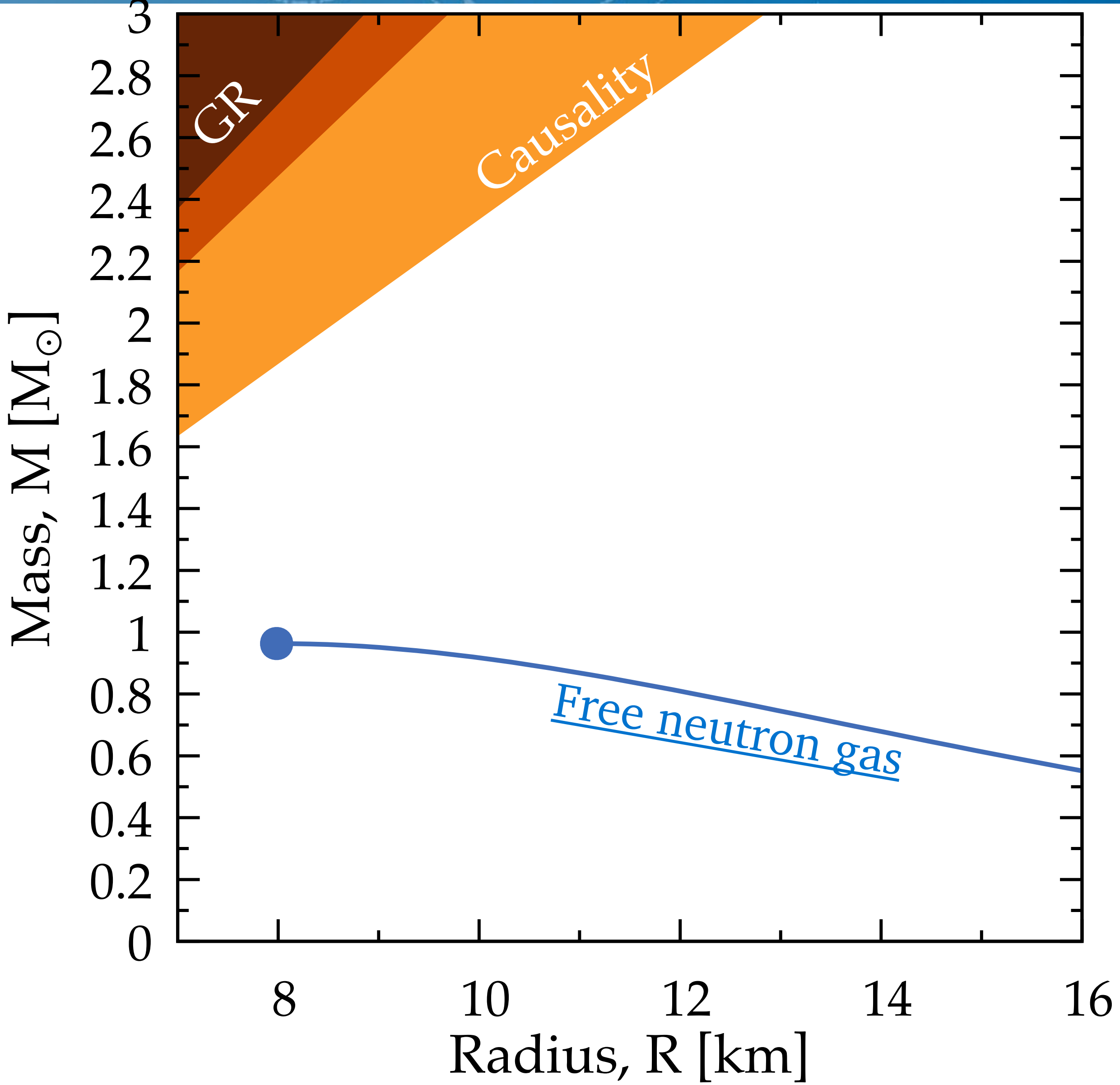


How important is our job?



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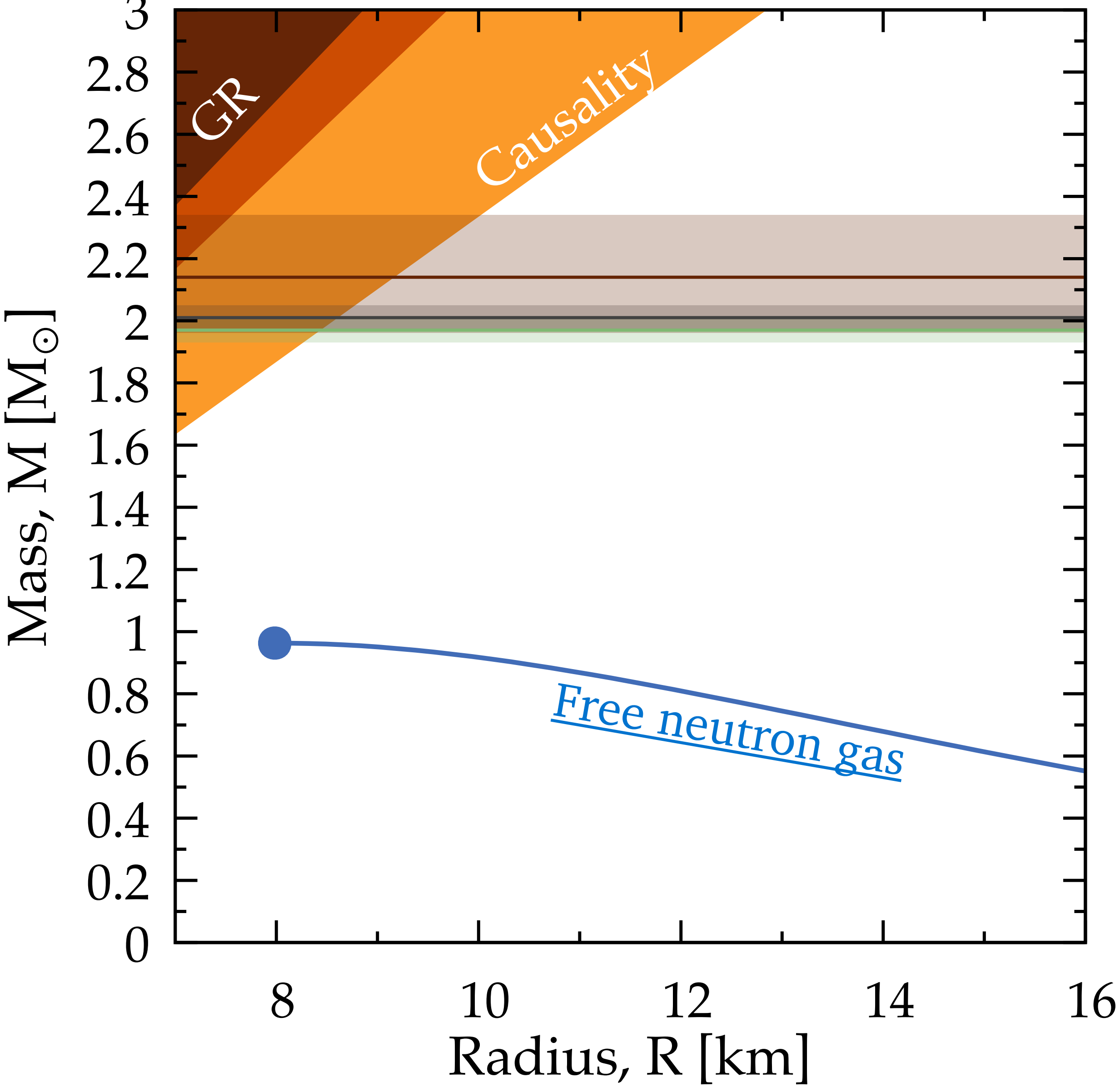

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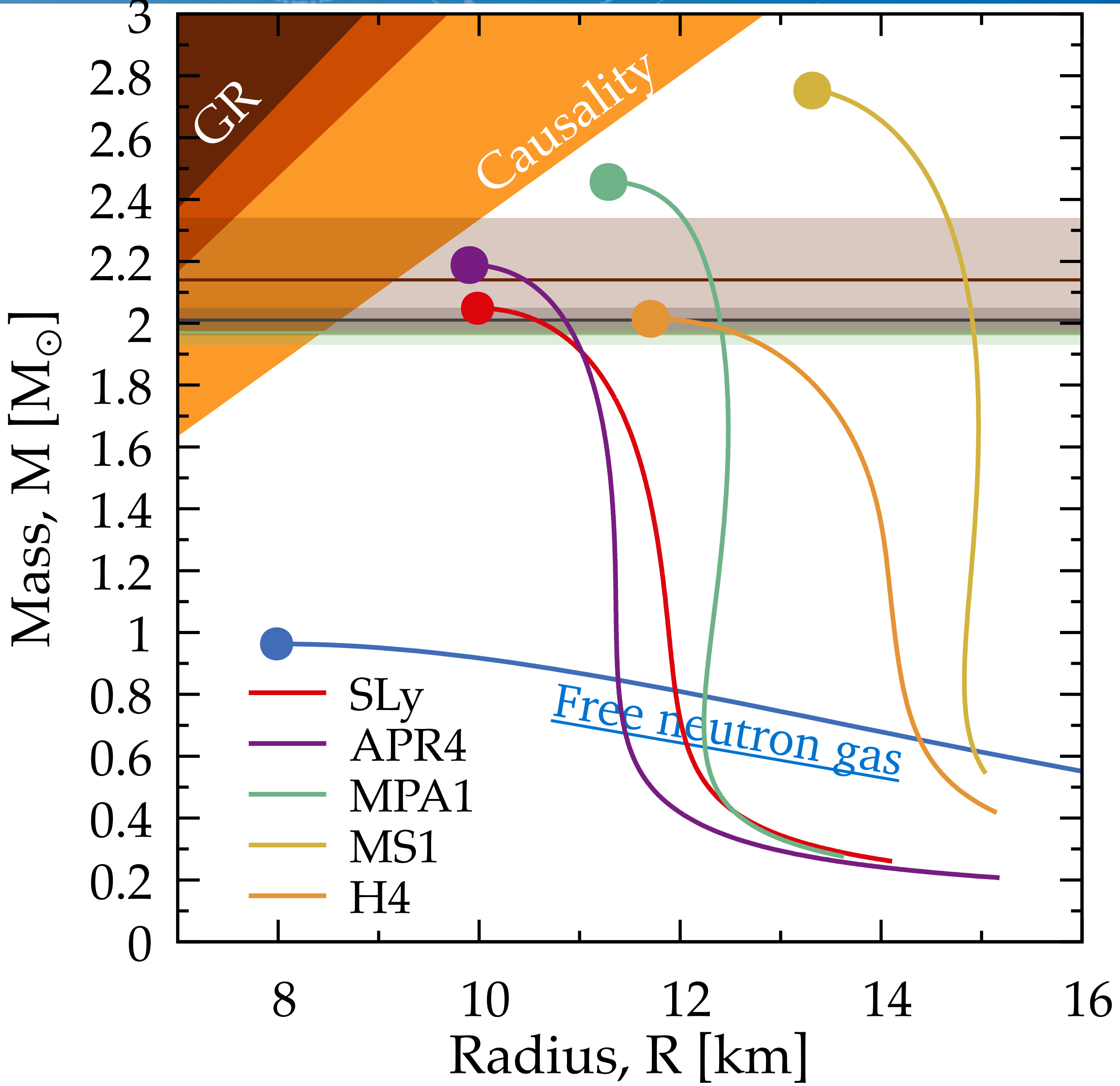

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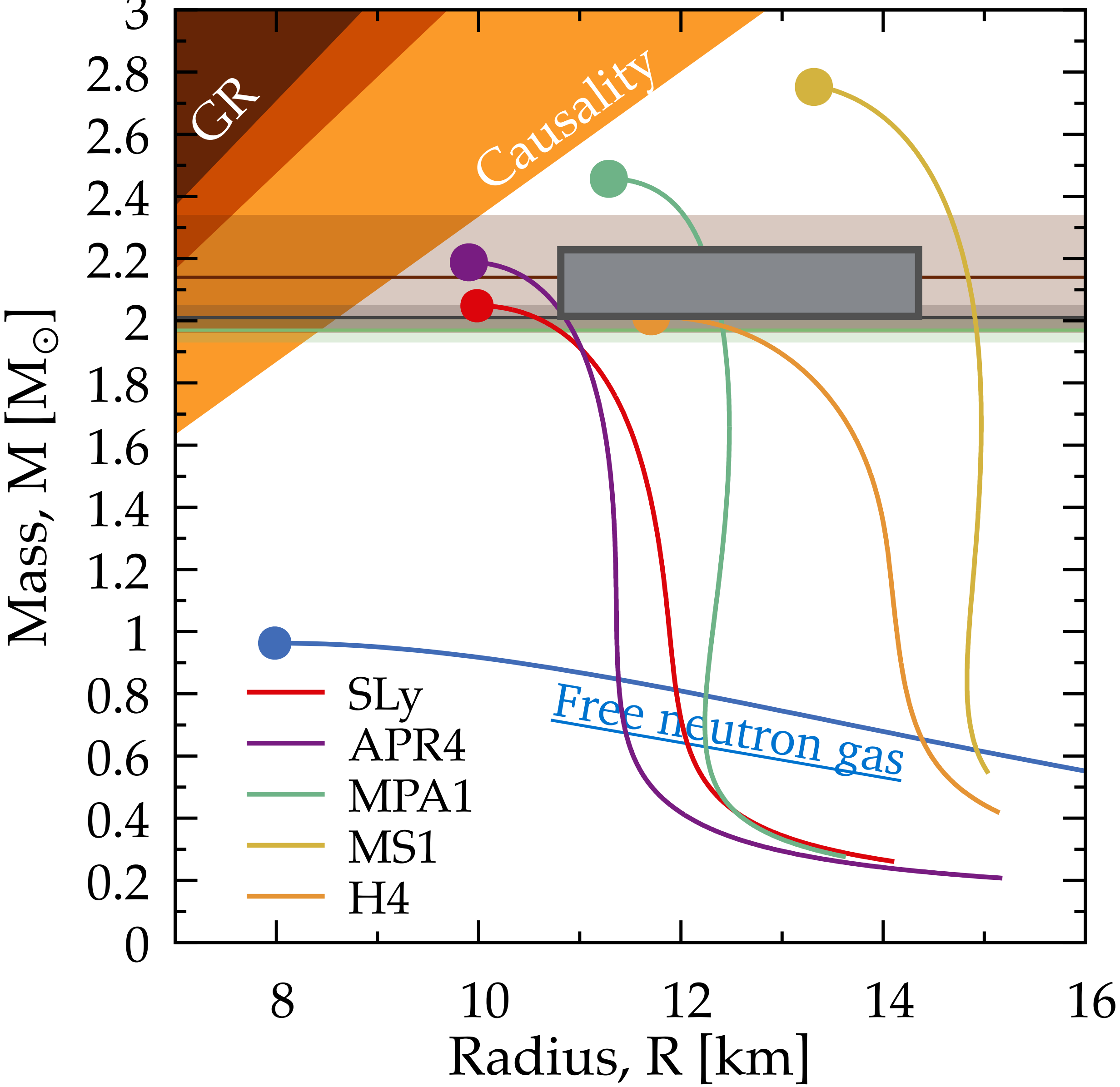

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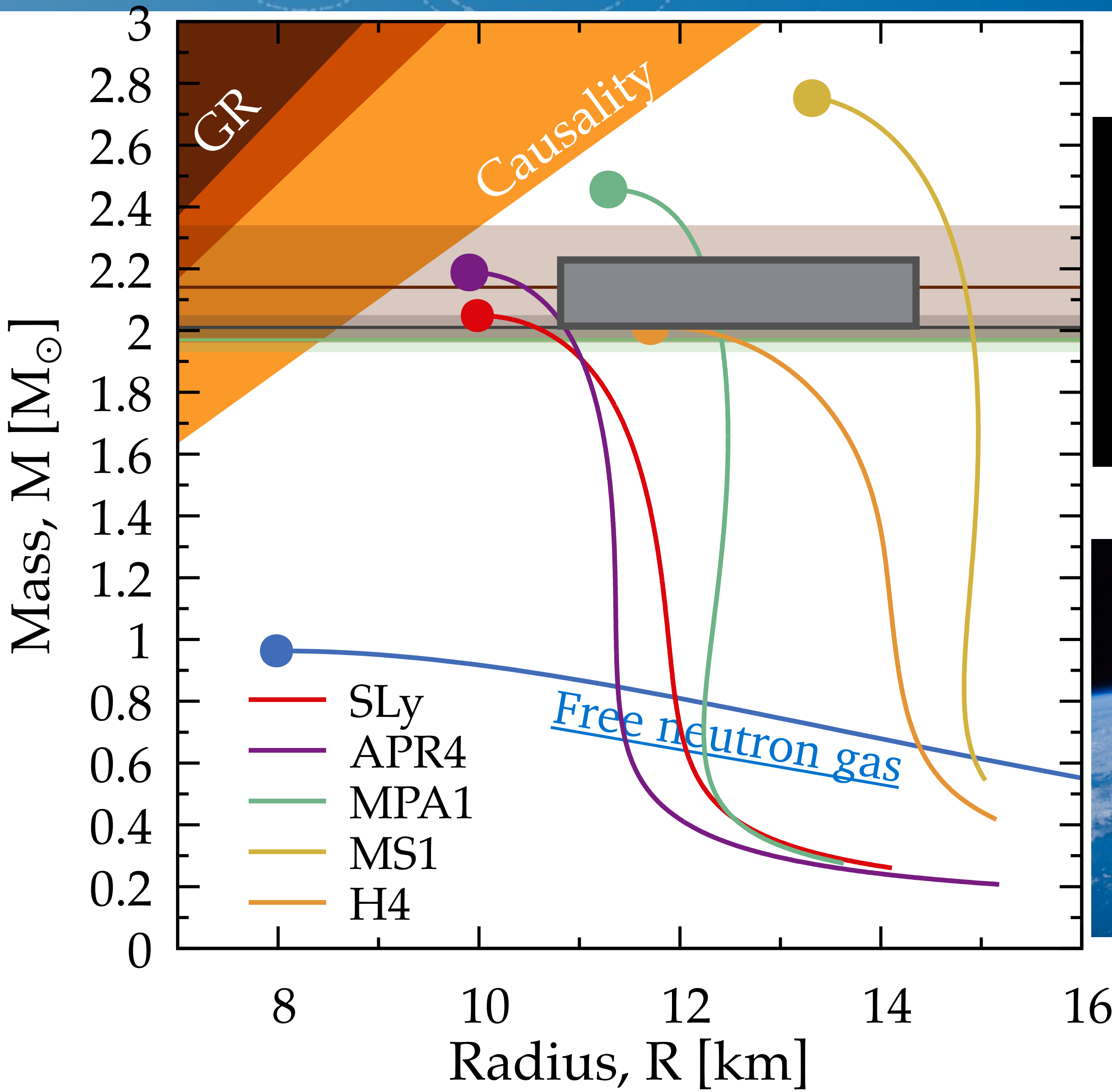
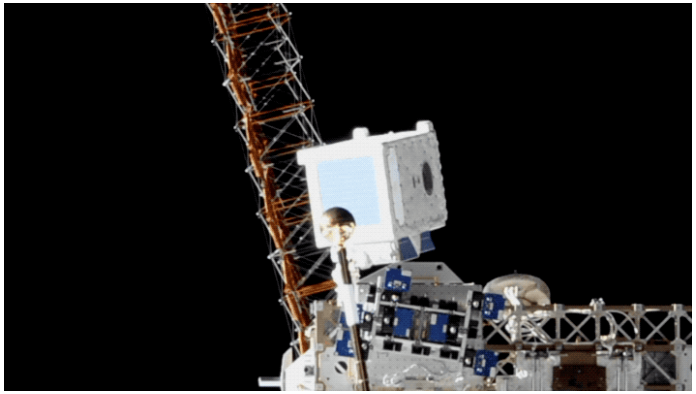
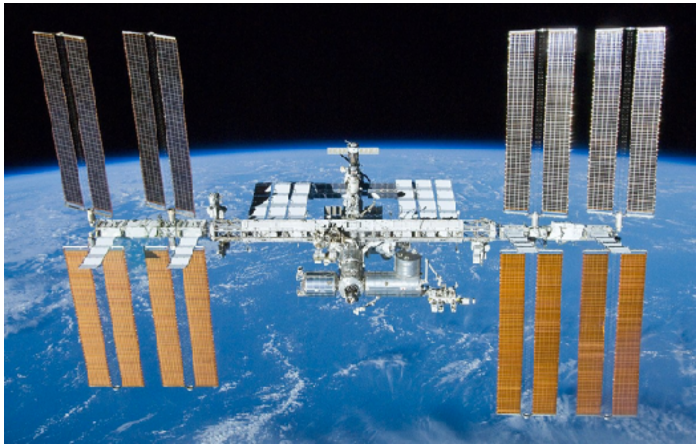


NICER

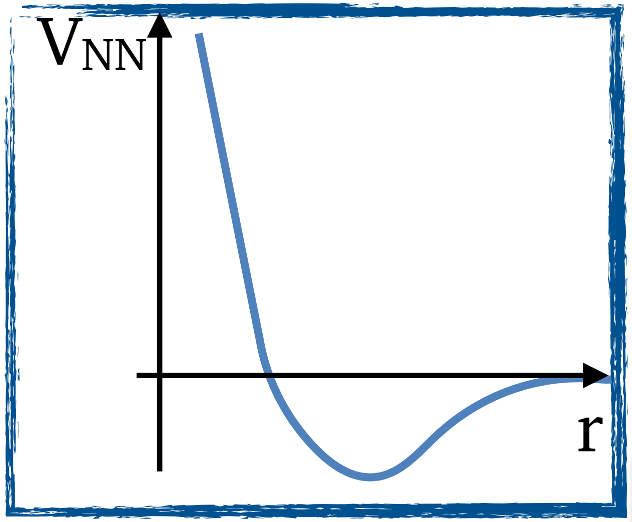




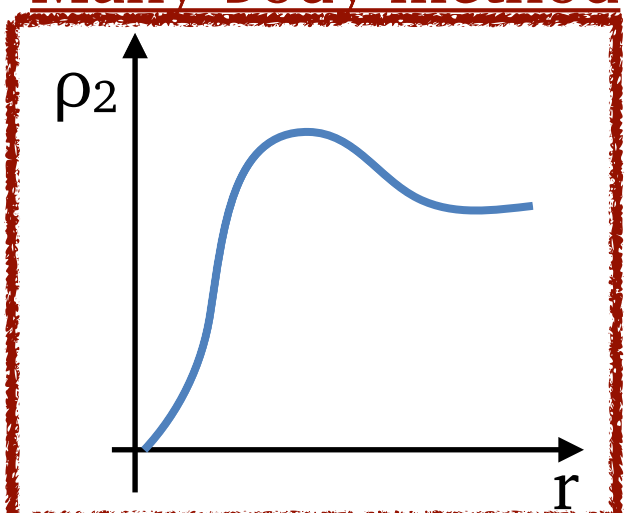
NICER Collaboration
Miller et al. arXiv:2105.06979
Riley et al. arXiv:2105.06981
Raaijmakers et al. arXiv:2105.06981
https://heasarc.gsfc.nasa.gov/docs/nicer/

Nuclear predictions 19xx style

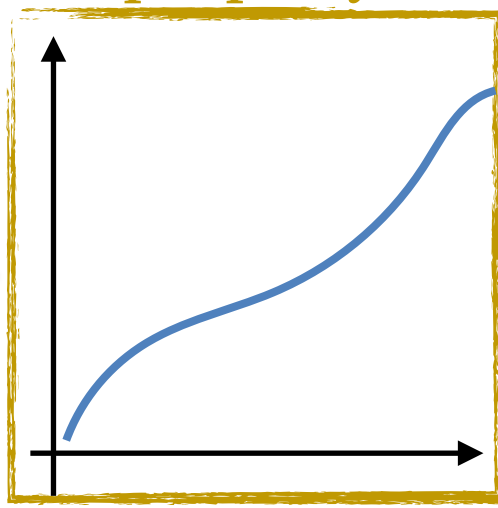
Hamiltonian



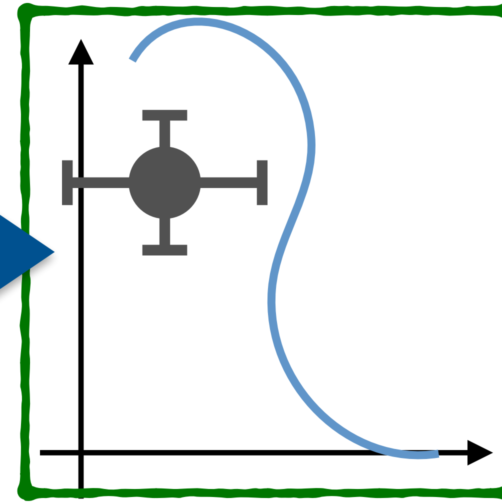
Many-body method

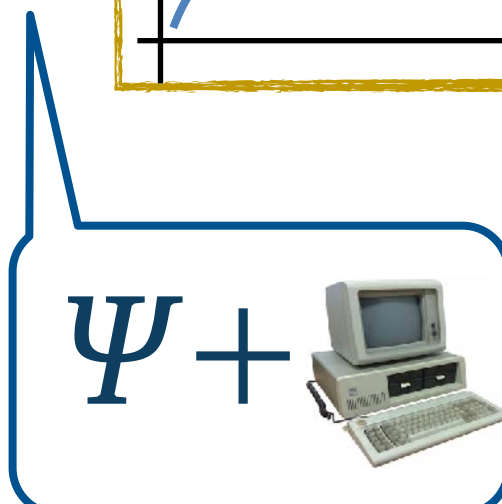


Astronuclear property



Neutron star observations

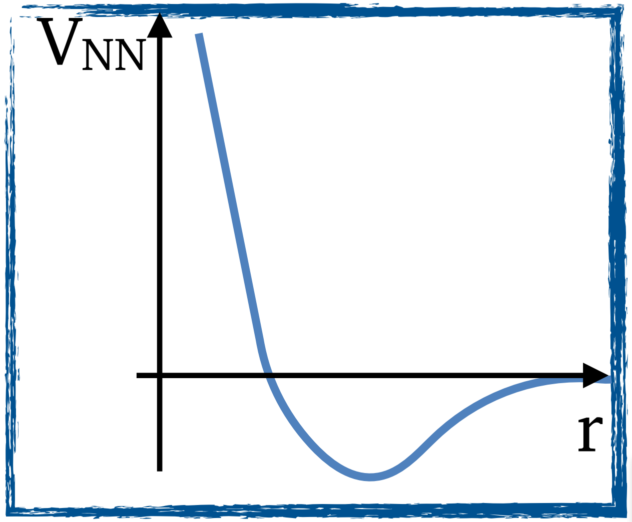




Astro
"black box"

Nuclear predictions 19xx style

Hamiltonian



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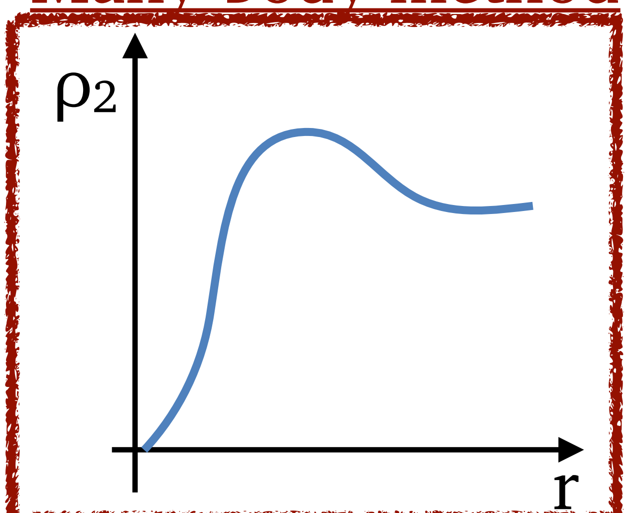

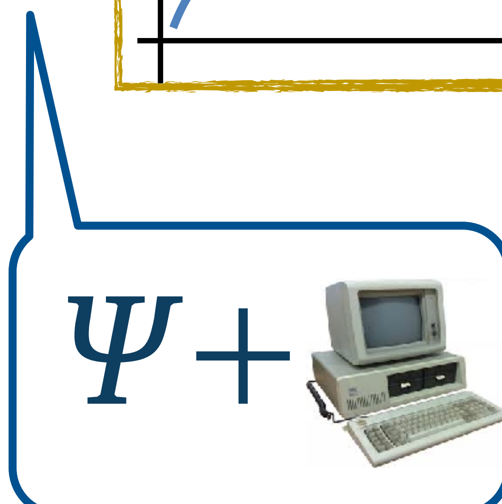

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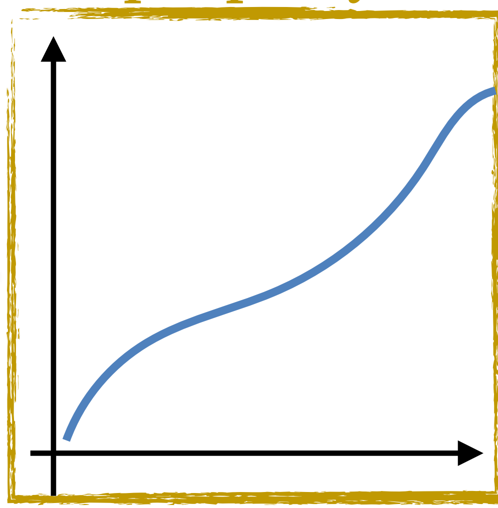

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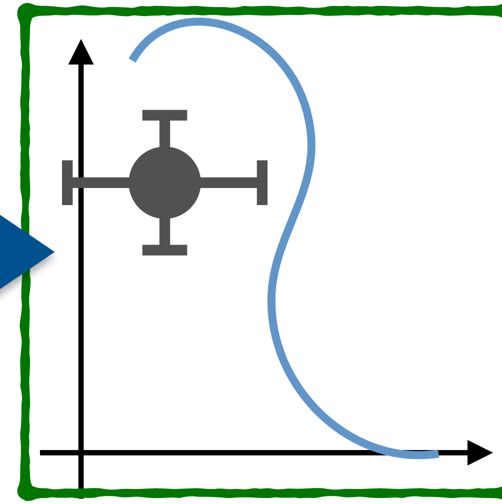

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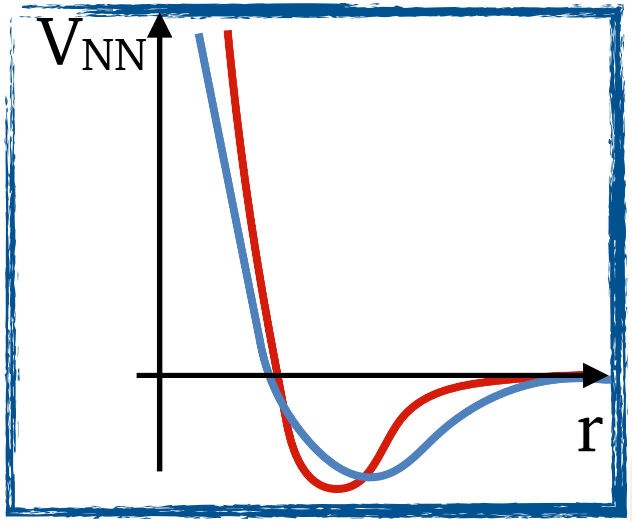

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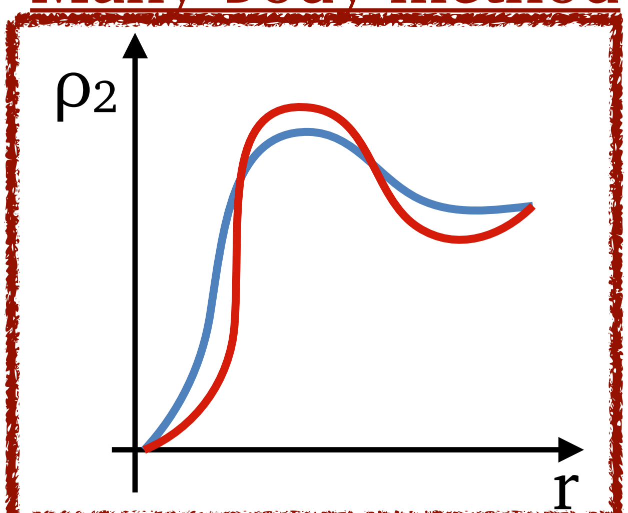

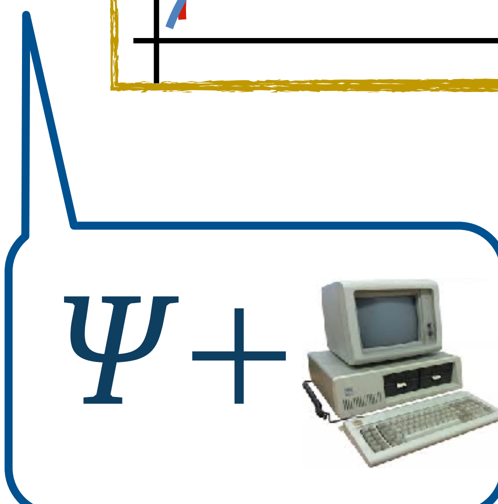

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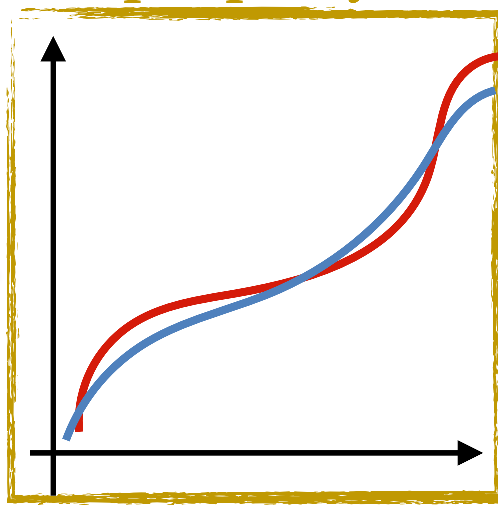

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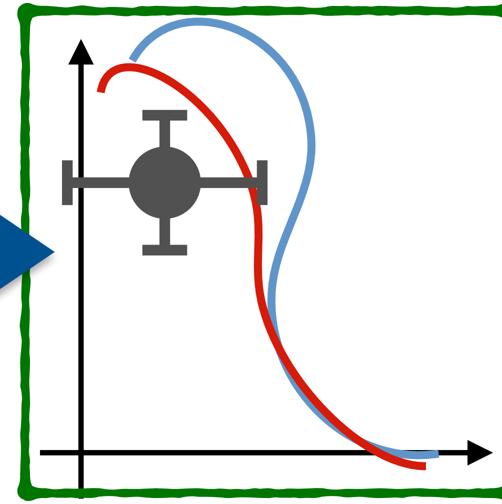

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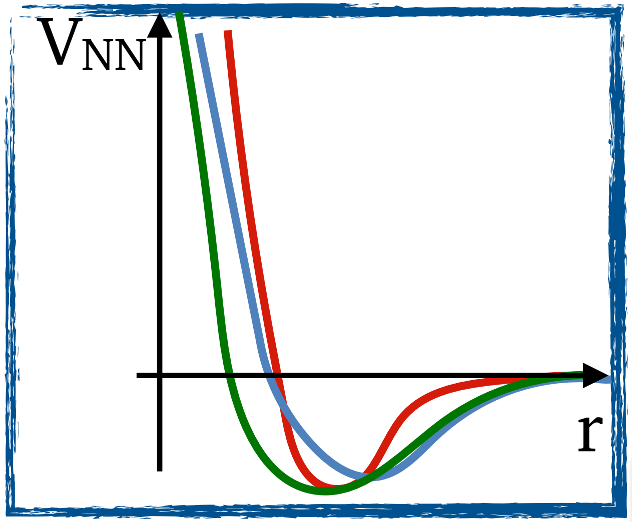

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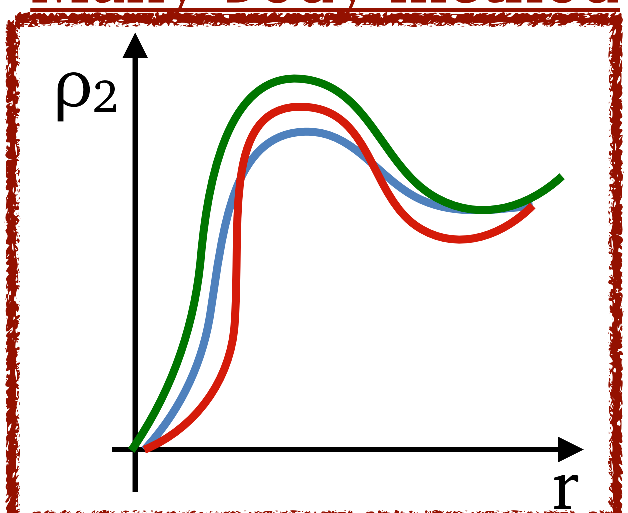

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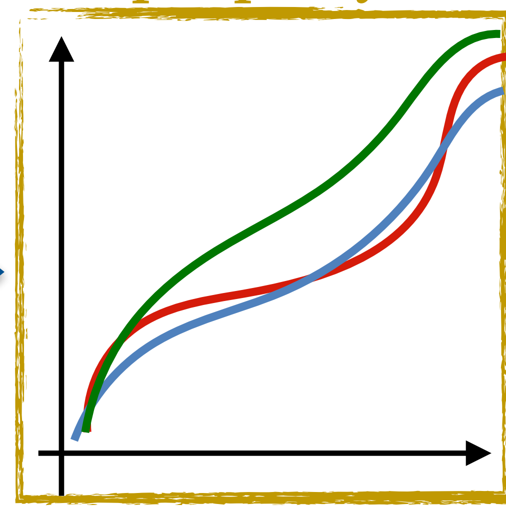

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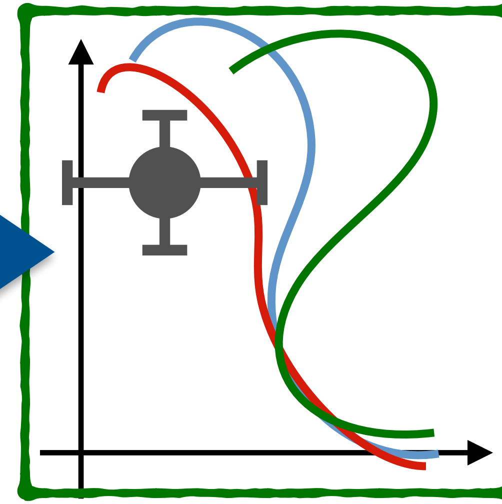
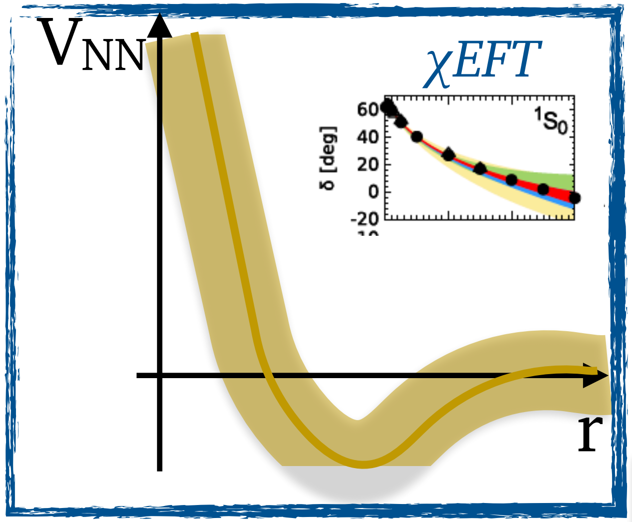


Ψ +

Astro "black box"

Nuclear error quantification

Hamiltonian



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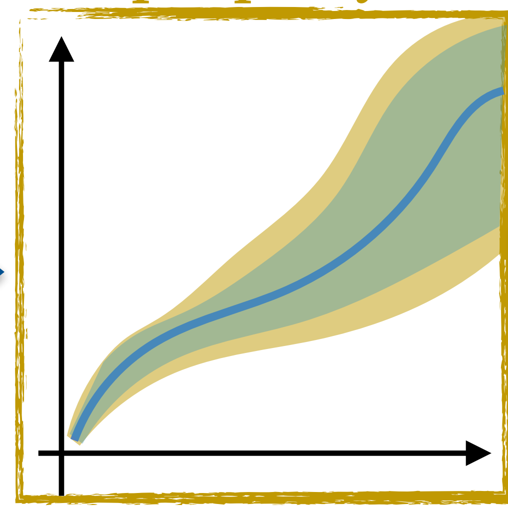

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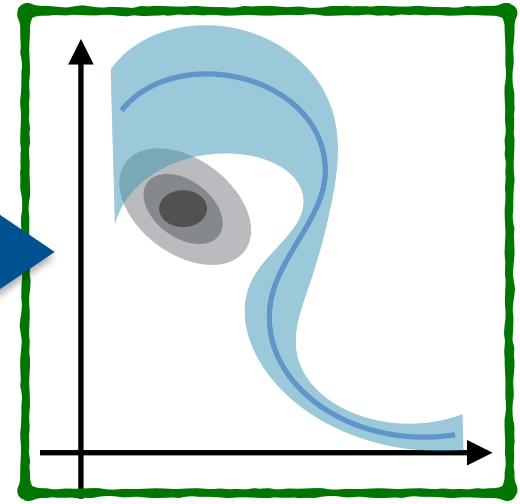

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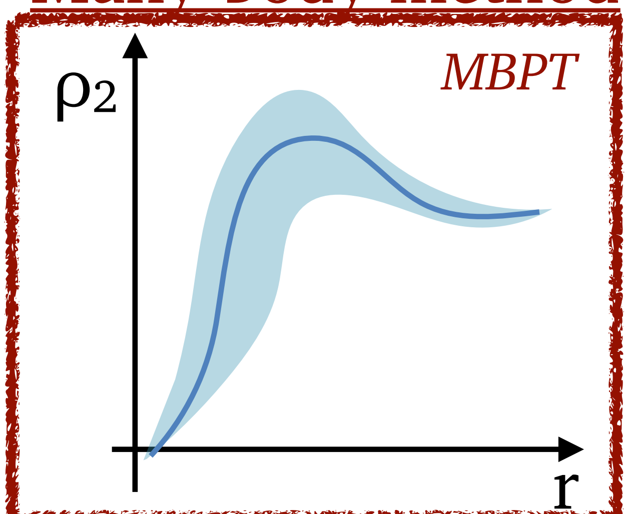

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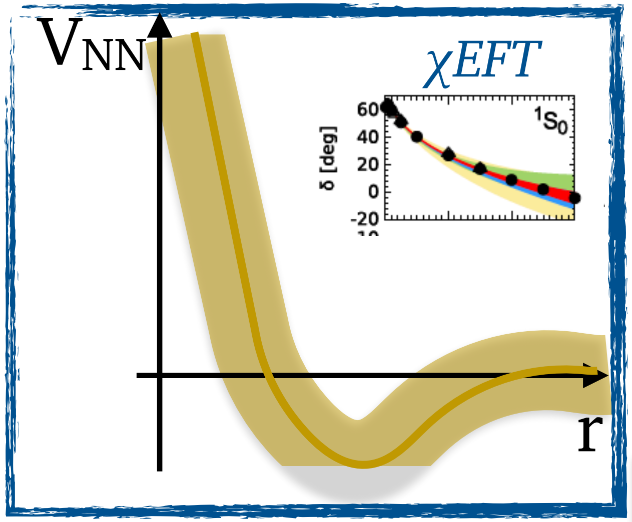
<u>Hamiltonian</u>



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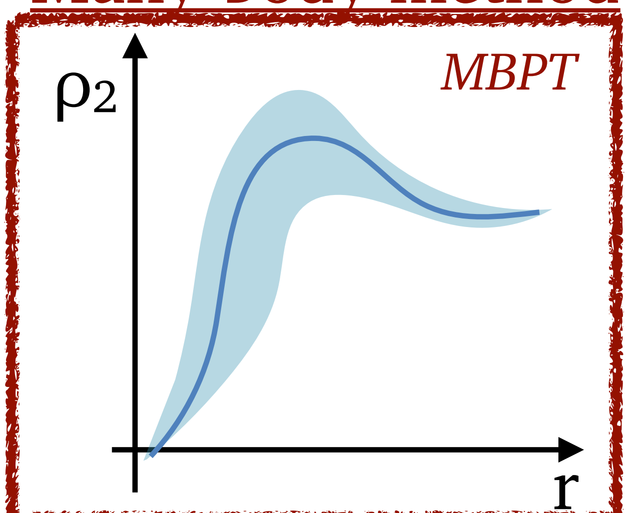

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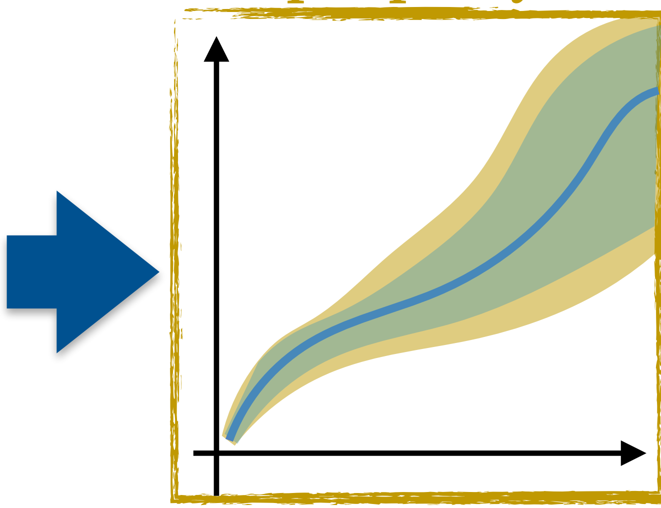

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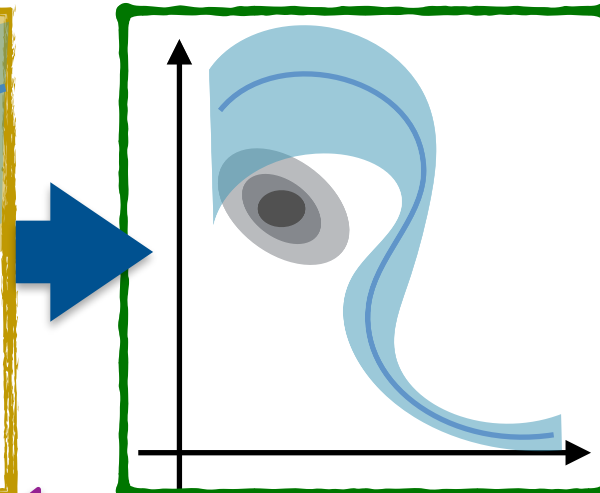

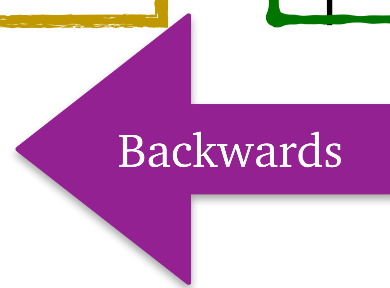

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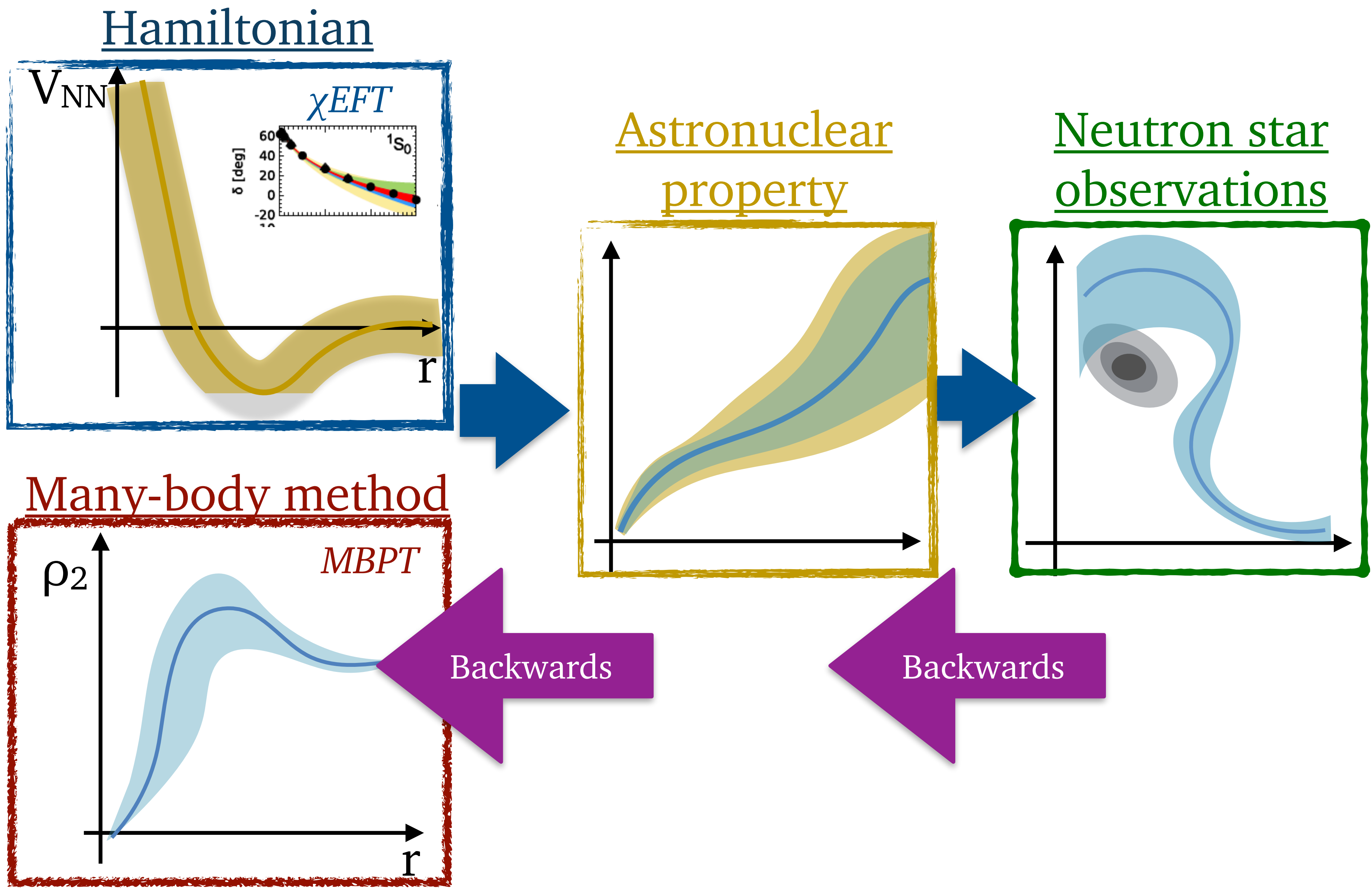

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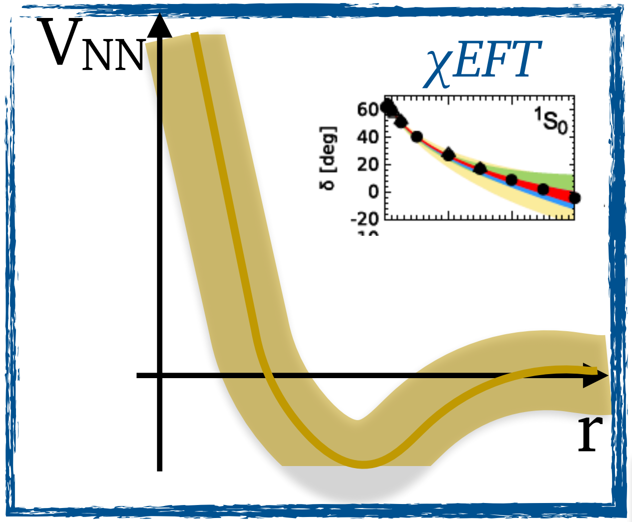

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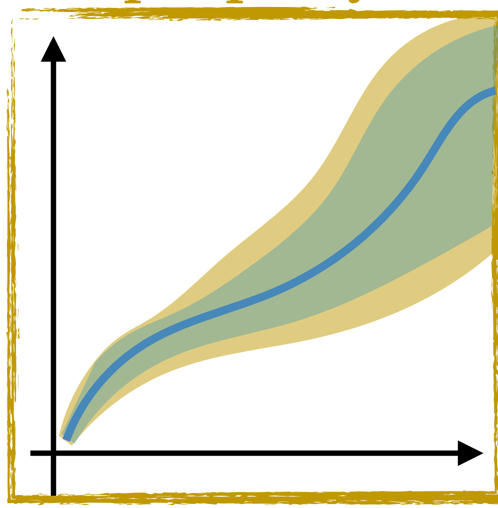

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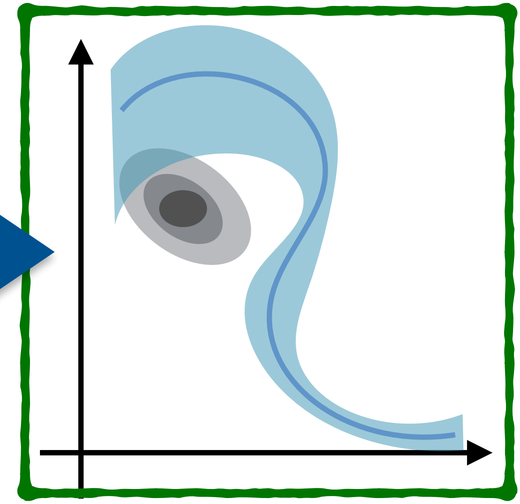

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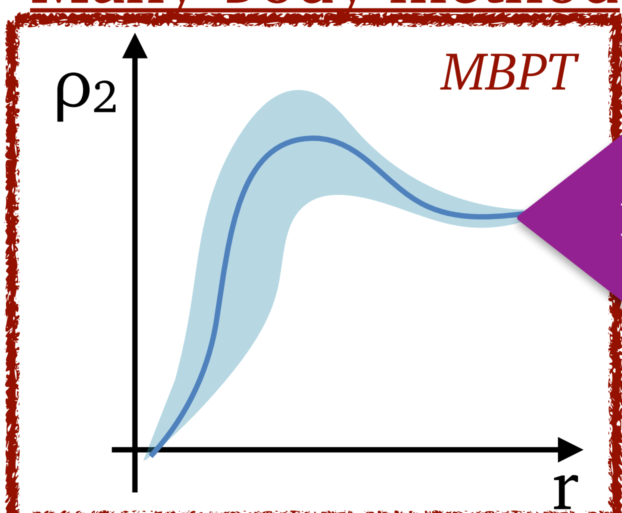

Backwards

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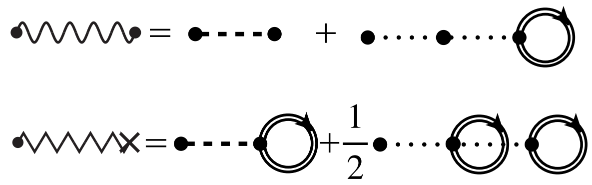
Normal phase (EoS) ✔

Superfluid phase (gaps) ✘

Self-Consistent Green's Functions

(ρ, T)

2N & 3N forces



Carbone, Rios & Polls PRC **88** 044302 (2013);
PRC **90**, 054322 (2014);
Carbone PhD Thesis

Self-Consistent Green's Functions

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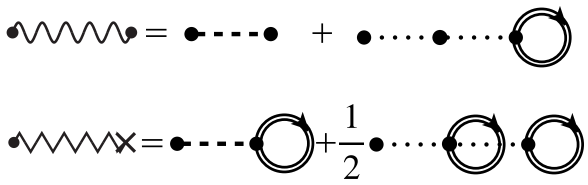
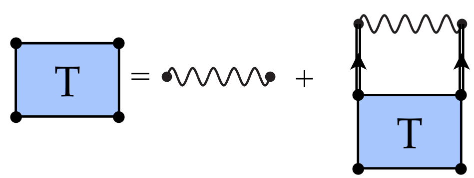


In-medium interaction



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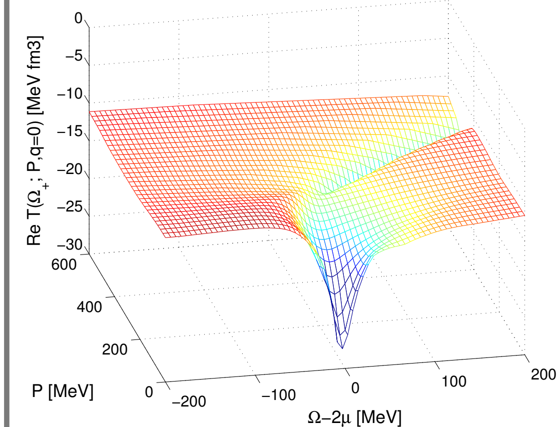
T-matrix at *T*=5 MeV



Ramos, Polls & Dickhoff, NPA **503** 1 (1989)
Alm *et al.*, PRC **53** 2181 (1996)
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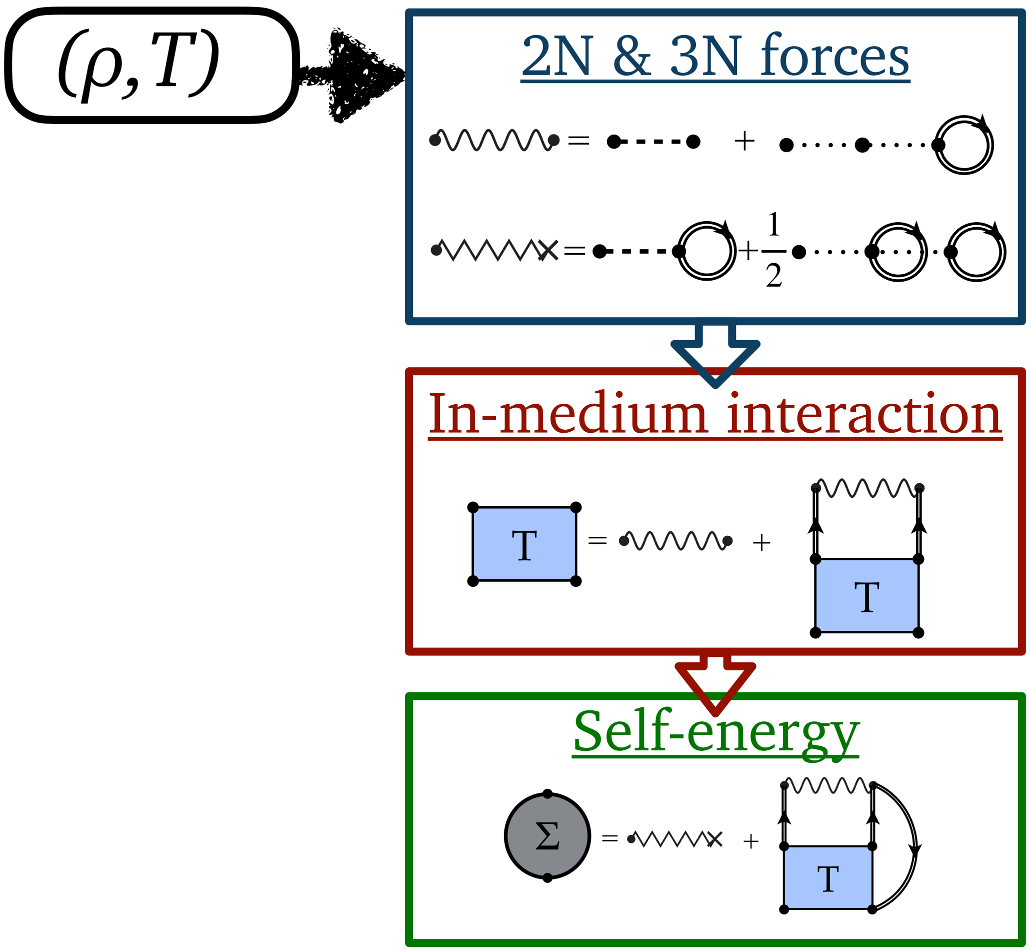
Self-Consistent Green's Functions



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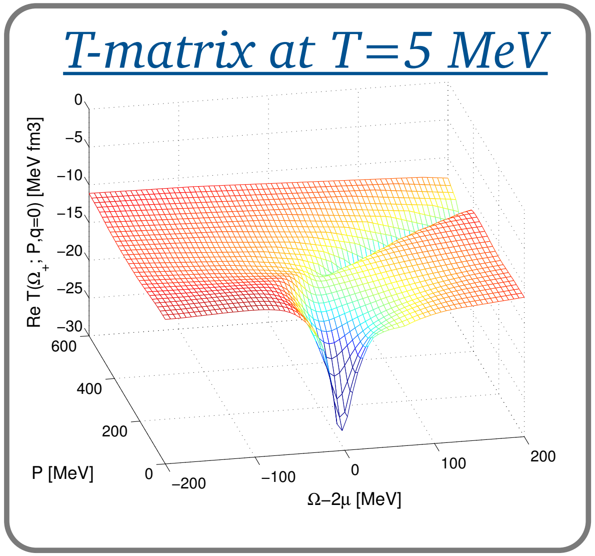

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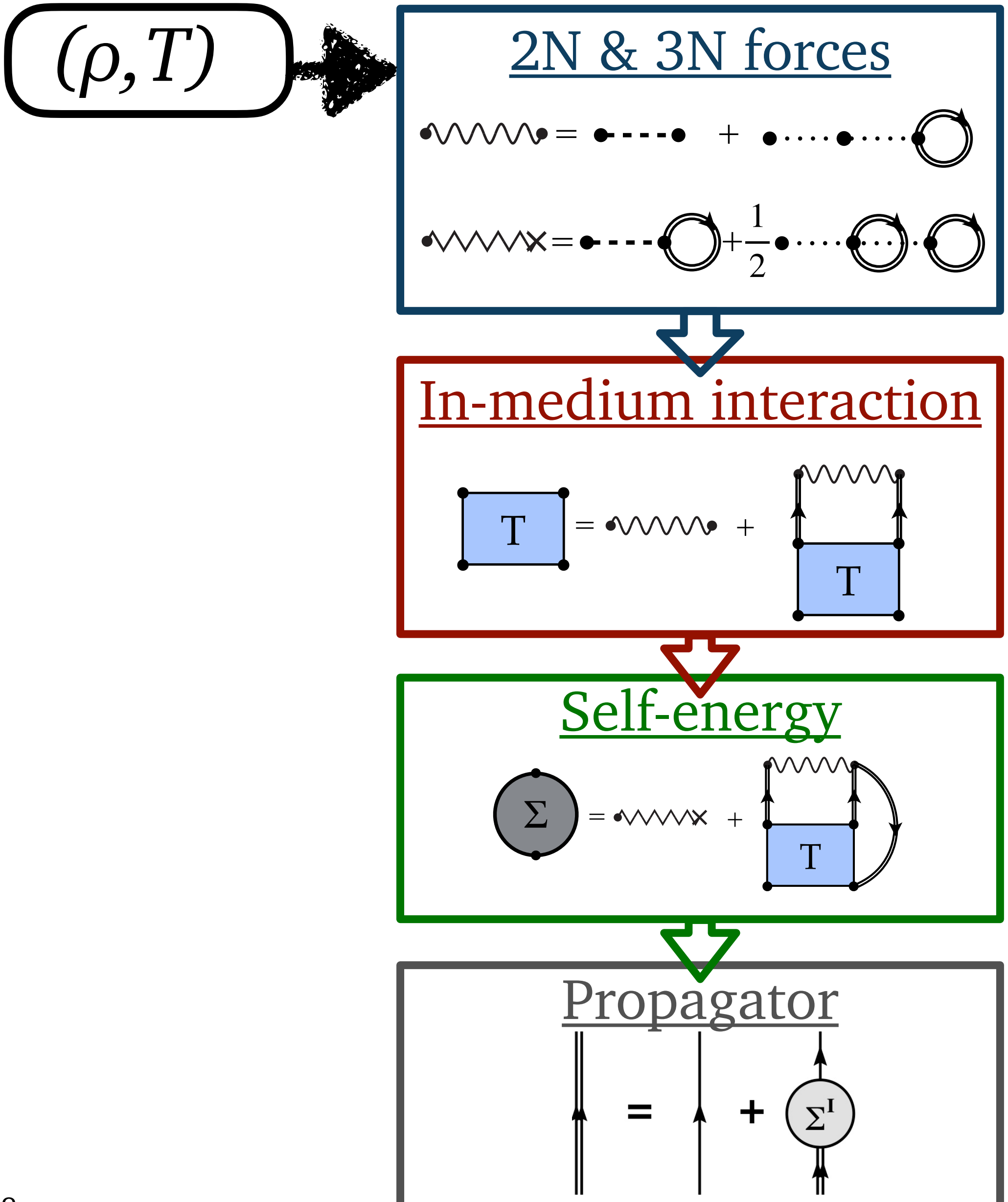
Self-Consistent Green's Functions



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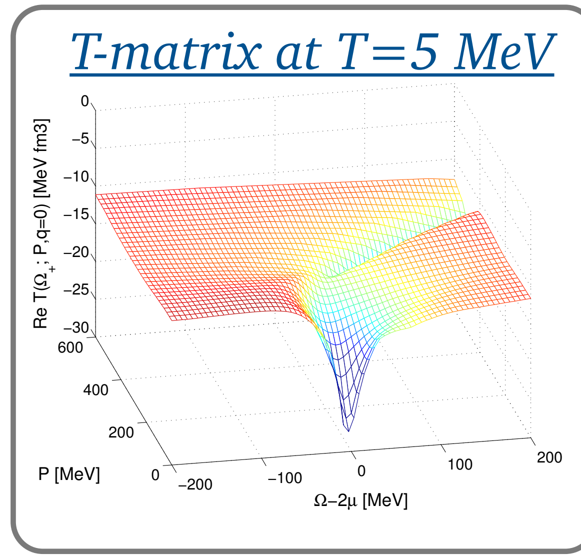

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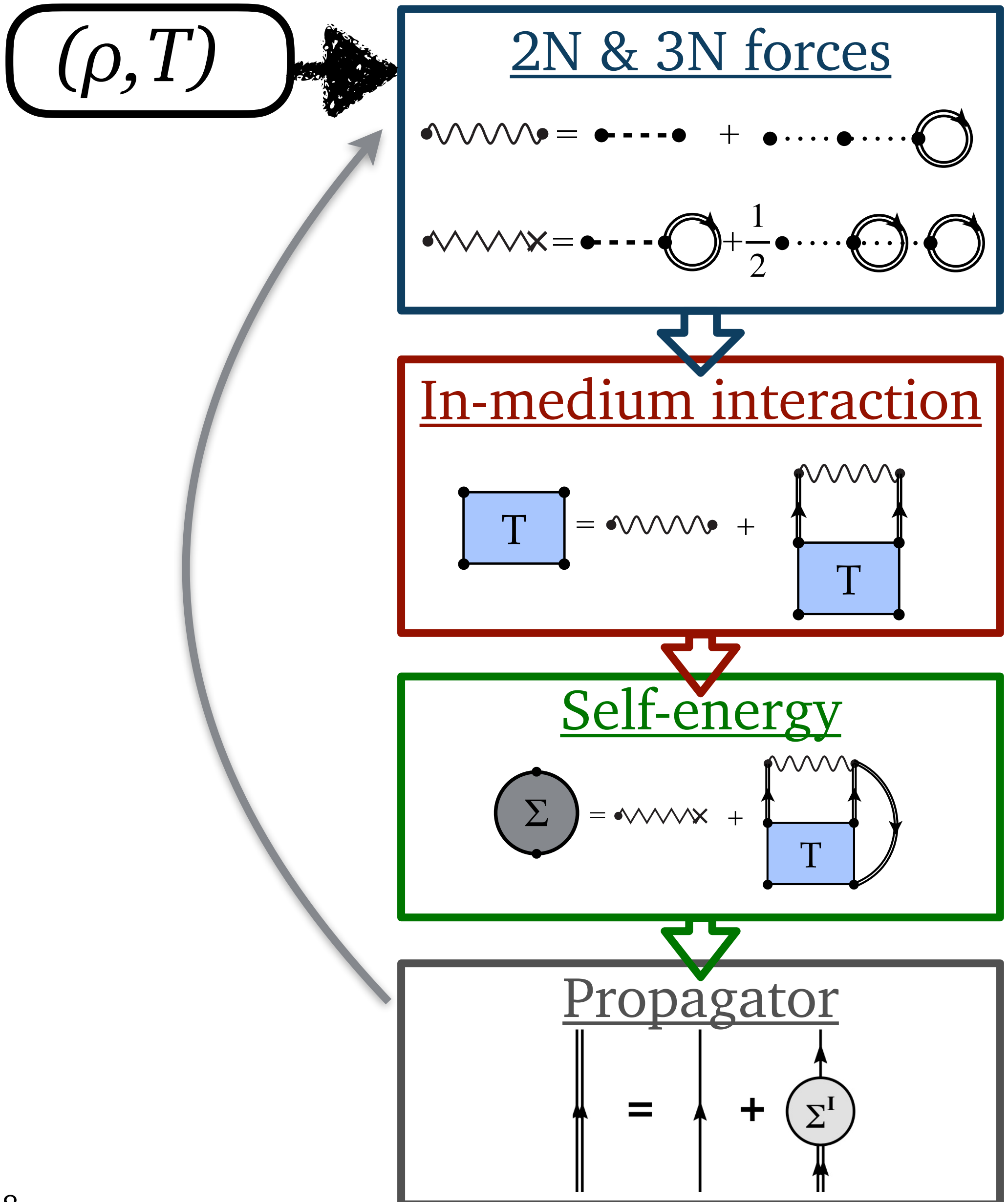
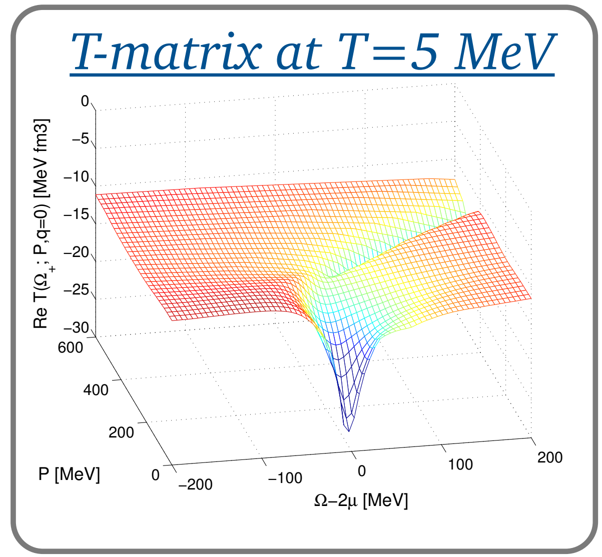
Rios & Soma PRL **108** 012501 (2012)

Self-Consistent Green's Functions



Carbone, Rios & Polls PRC **88** 044302 (2013);
PRC **90**, 054322 (2014);
Carbone PhD Thesis



Ramos, Polls & Dickhoff, NPA **503** 1 (1989)
Alm *et al.*, PRC **53** 2181 (1996)
Dewulf *et al.*, PRL **90** 152501 (2003)
Frick & Muther, PRC **68** 034310 (2003)
Rios, PhD Thesis, U. Barcelona (2007)
Soma & Bozek, PRC **78** 054003 (2008)
Rios & Soma PRL **108** 012501 (2012)

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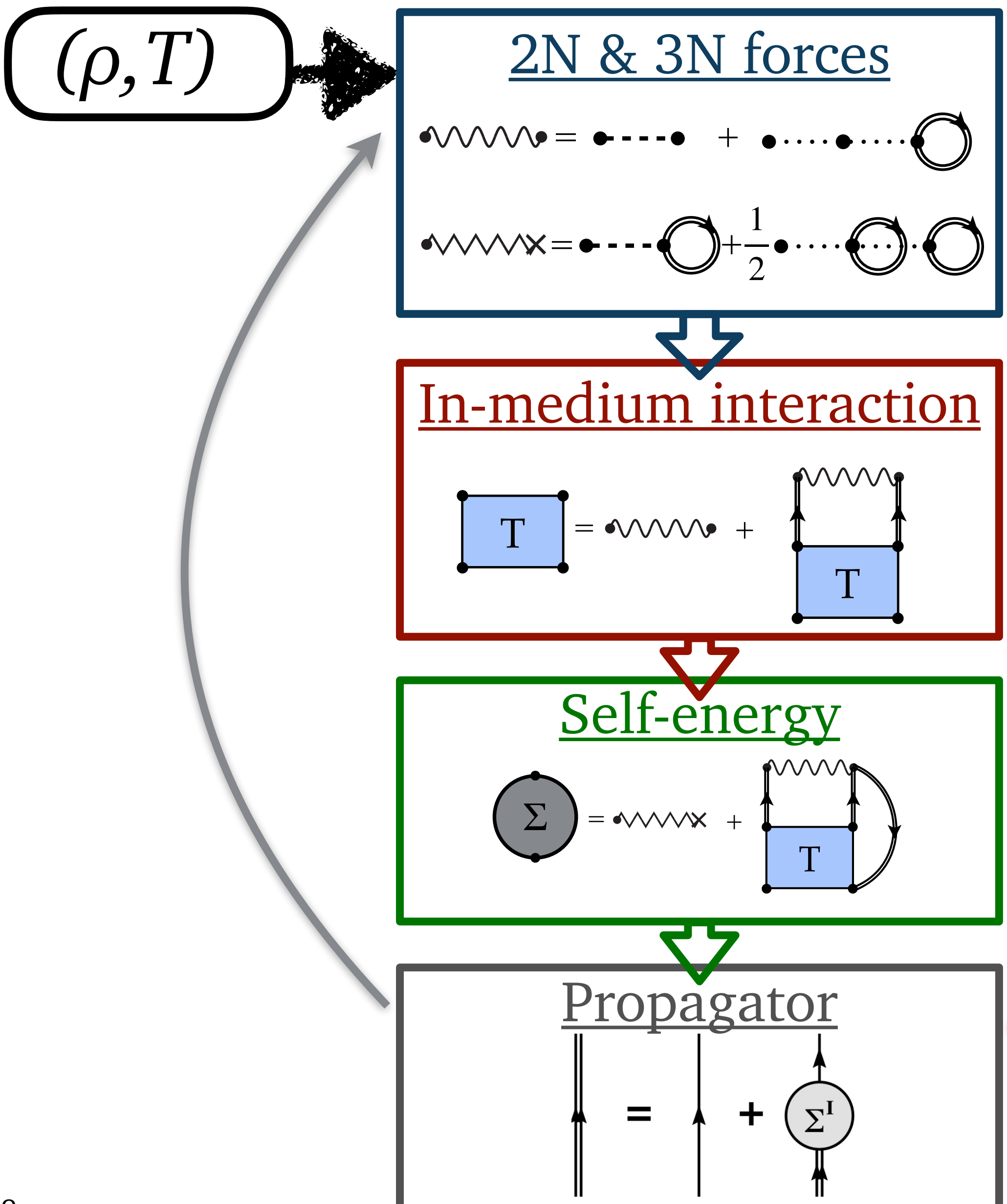

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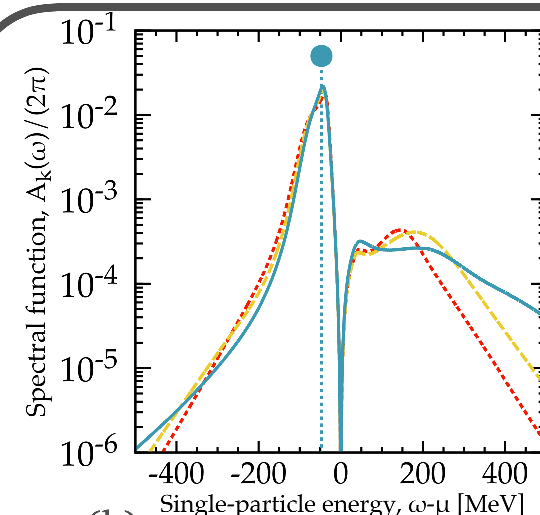
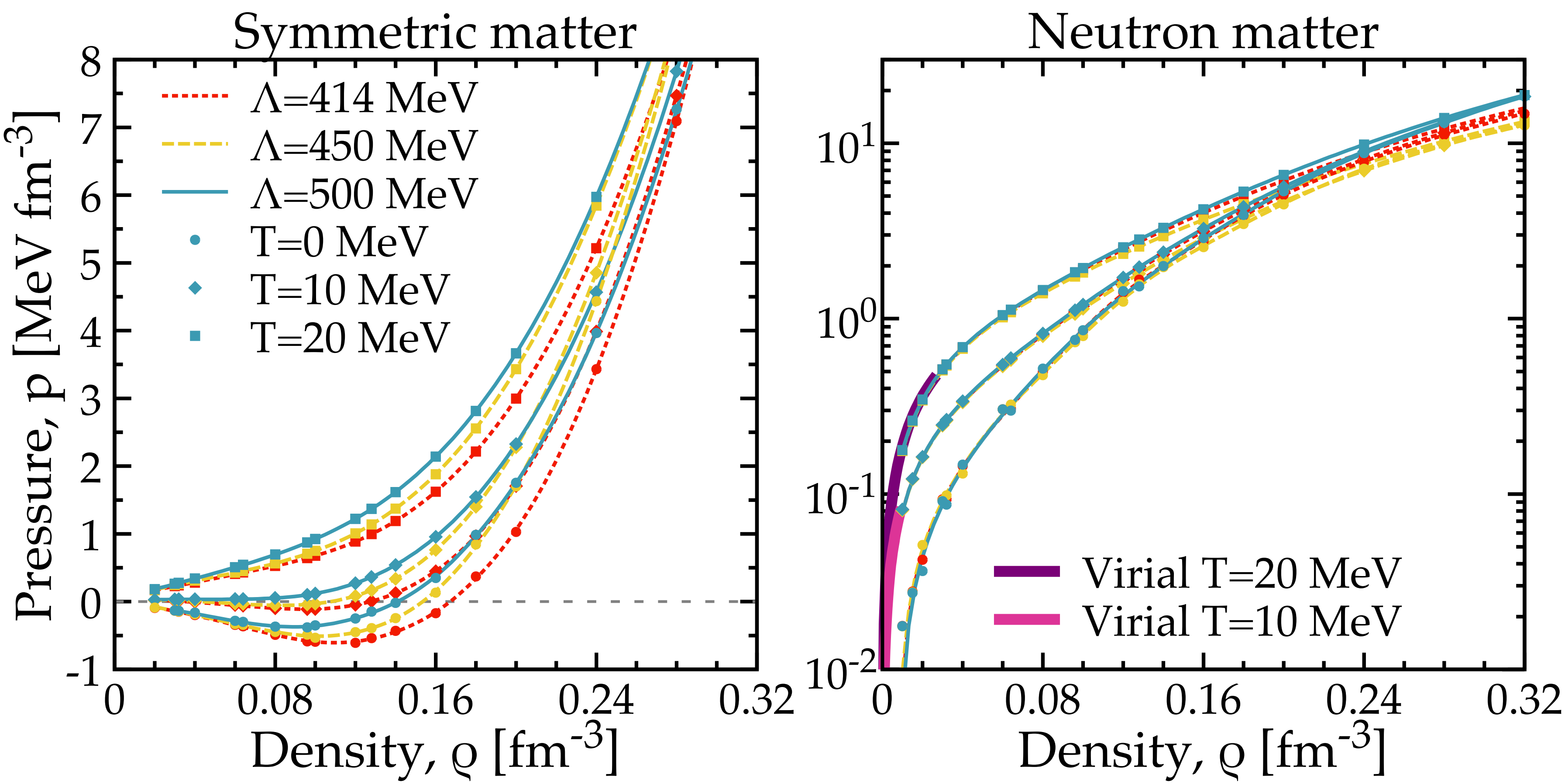


n(k)
Thermodynamics & EoS
Transport

Ramos, Polls & Dickhoff, NPA **503** 1 (1989)
Alm *et al.*, PRC **53** 2181 (1996)
Dewulf *et al.*, PRL **90** 152501 (2003)
Frick & Muther, PRC **68** 034310 (2003)
Rios, PhD Thesis, U. Barcelona (2007)
Soma & Bozek, PRC **78** 054003 (2008)
Rios & Soma PRL **108** 012501 (2012)

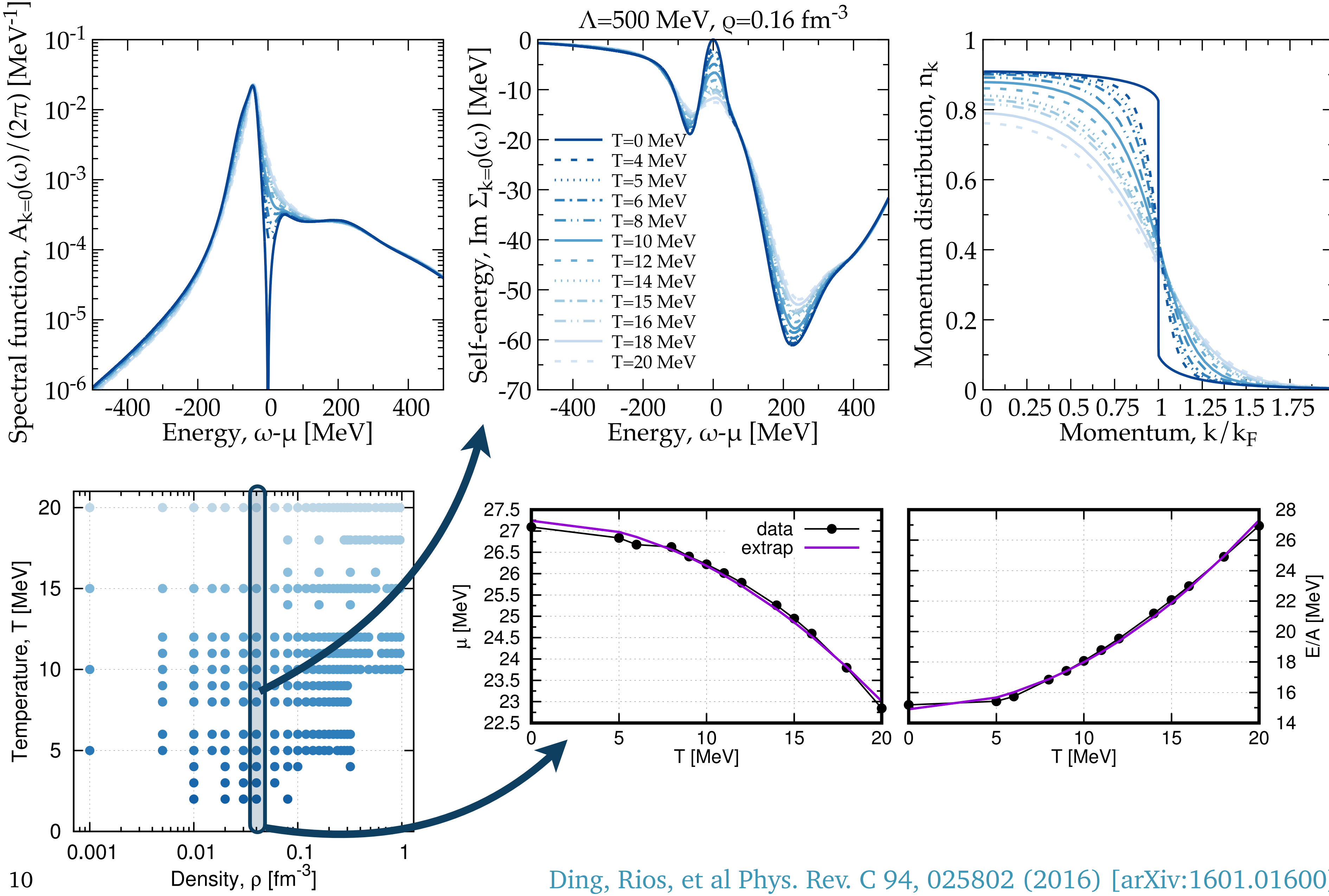
EoS at finite temperature



- Relevant & **necessary** for astro simulations

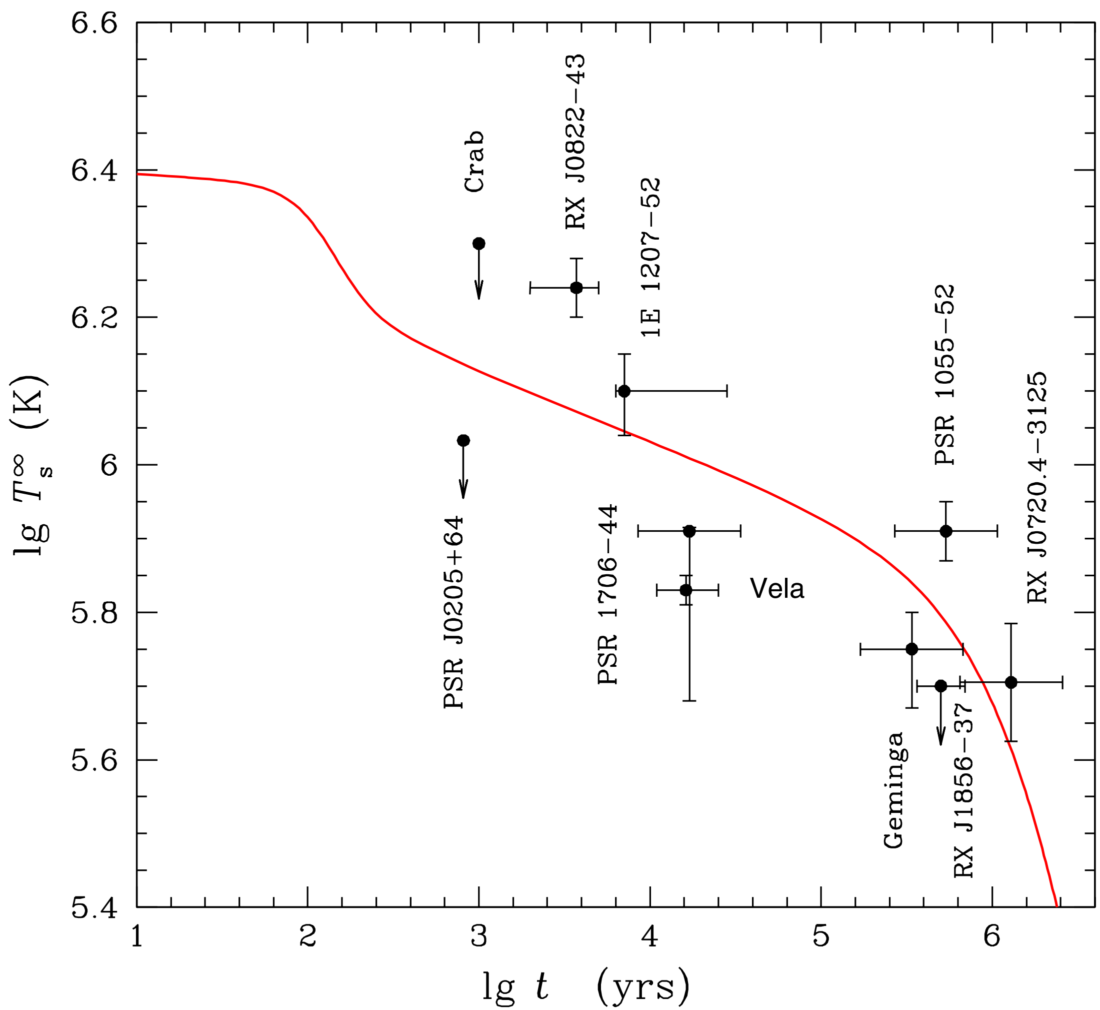
[Rios, Frontiers Physics fphy.2020.00387 (2020)]
[[arXiv:2006.10610]]

Zero temperature extrapolation



Ding, Rios, et al Phys. Rev. C 94, 025802 (2016) [arXiv:1601.01600]

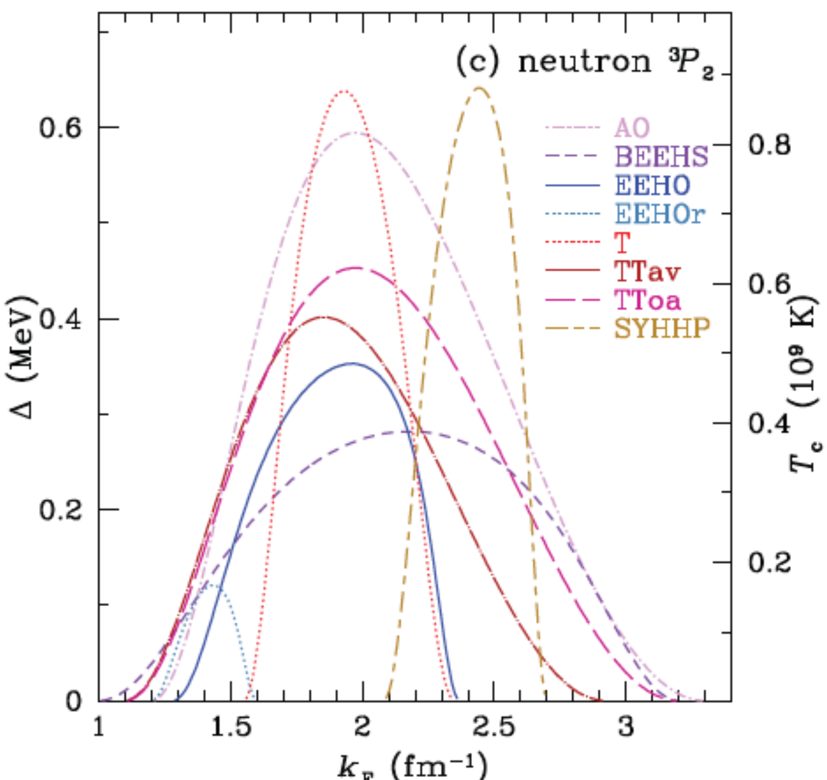
Cooling curve of neutron stars



- Observational data available for a handful of NS
- Sensitive to **interior** physics (mostly **pairing**)

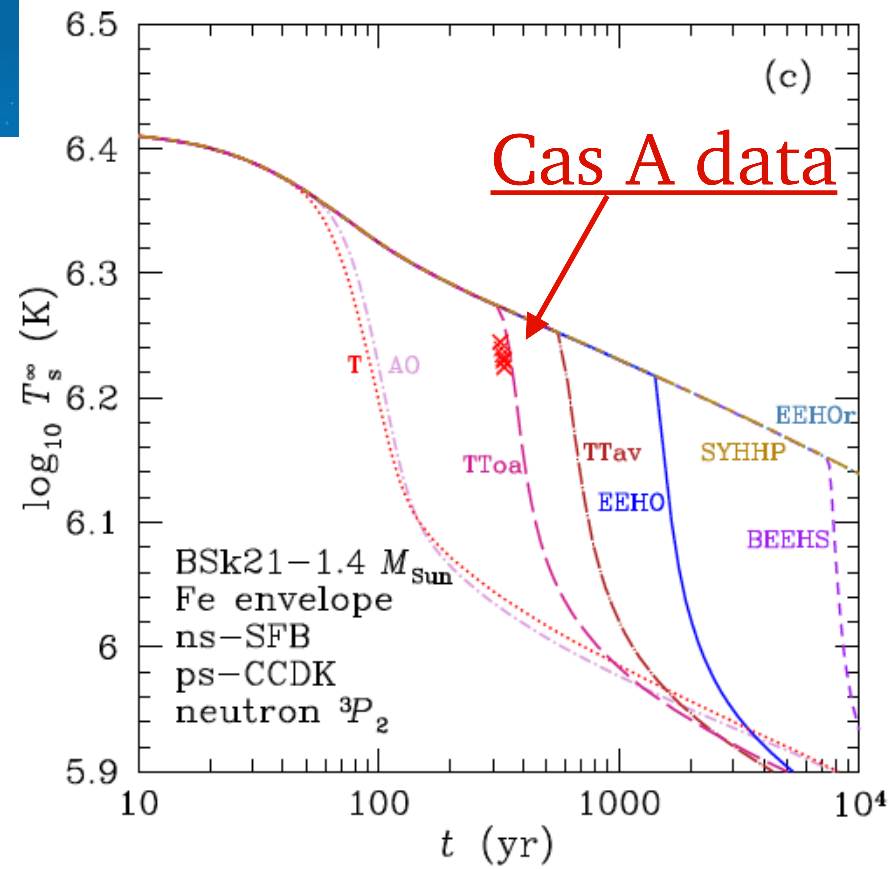
Yakovlev & Pethick,ARAA **42** 169 (2004)

Cooling of CasA



Ho, *et al.*, PRC **91** 015806 (2015)

Page, *et al.*, *PRL* **106** 081101 (2011)



Ingredients

(a) Mass of pulsar

(b) EoS (determines radius)

(c) Internal composition

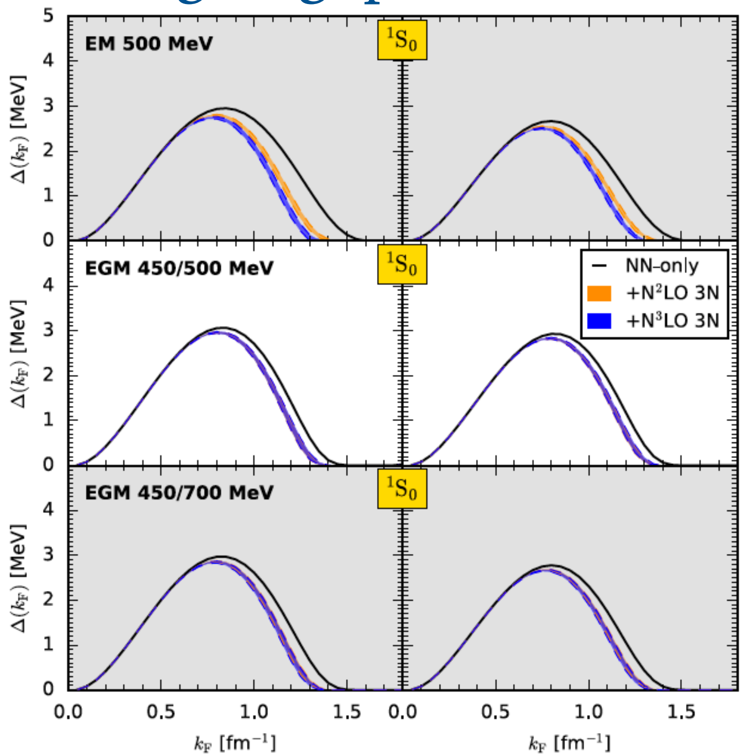
(d) **Pairing gaps** ($^1S_0$ & $^3PF_2$ channels)

(e) Atmosphere composition

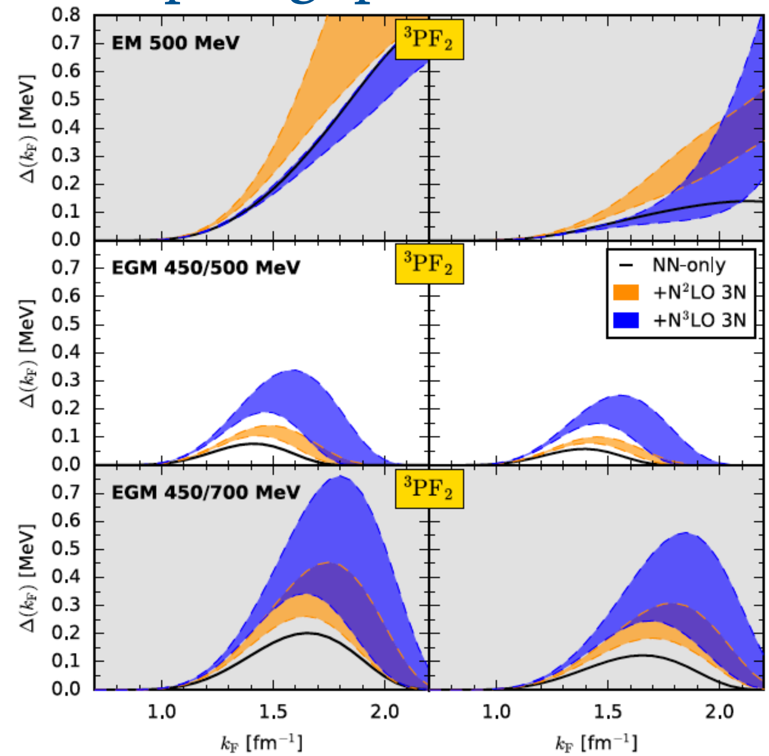
Name	Process	Emissivity (erg cm^{-3} s^{-1})
Modified Urca (neutron branch)	$n+n \to n+p+e^- + \bar{\nu}_e$ $n+p+e^- \to n+n+\nu_e$	$\sim 2\times10^{21}\, R T_9^8$
Modified Urca (proton branch)	$p+n \to p+p+e^- + \bar{\nu}_e$ $p+p+e^- \to p+n+\nu_e$	$\sim 10^{21}\, R T_9^8$
Bremsstrahlungs	$n+n \to n+n+\nu+\bar{\nu}$ $n+p \to n+p+\nu+\bar{\nu}$ $p+p \to p+p+\nu+\bar{\nu}$	$\sim 10^{19}\, R T_9^8$
Cooper pair	$n+n \to [nn]+\nu+\bar{\nu}$ $p+p \to [pp]+\nu+\bar{\nu}$	$\sim 5\times10^{21}\, R T_9^7$ $\sim 5\times10^{19}\, R T_9^7$
Direct Urca (nucleons)	$n \to p+e^- + \bar{\nu}_e$ $p+e^- \to n+\nu_e$	$\sim 10^{27}\, R T_9^6$

BCS+HF gaps in neutron matter

Singlet gaps with 3NF



Triplet gaps with 3NF



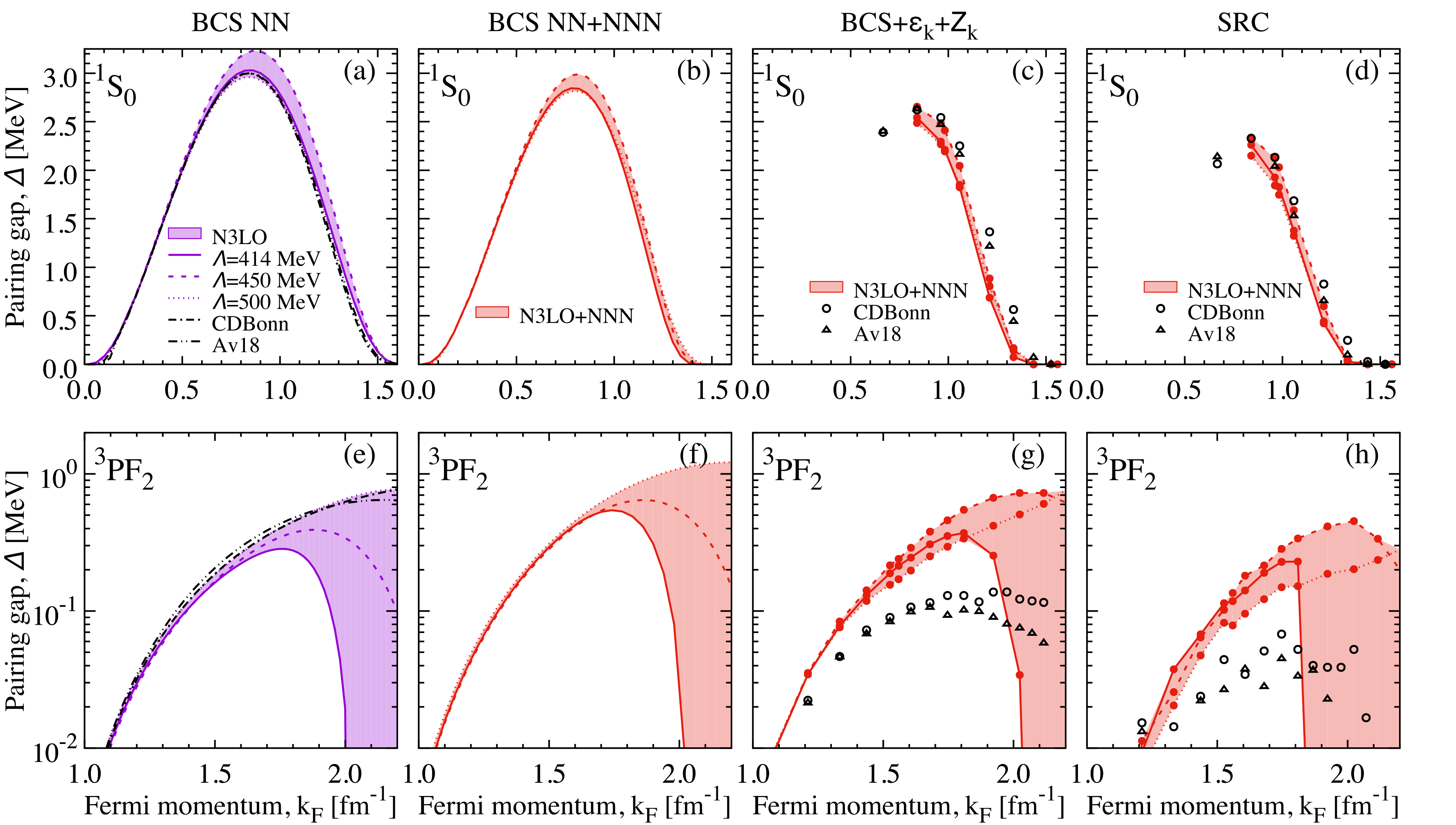
BCS equation

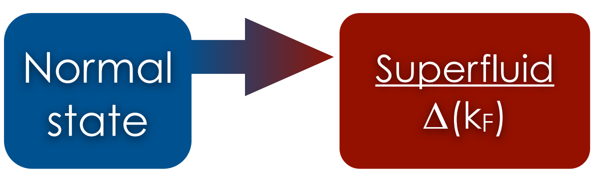
$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'} \quad + \quad \begin{aligned} \chi_k &= \varepsilon_k - \mu \\ \varepsilon_k &= \frac{k^2}{2m} + U(k) - \mu \end{aligned}$$

- **Error estimates** from nuclear force (chiral expansion) ✓
- **Many-body uncertainty?** ✗

Drischler, Kruger, Hebeler, Schwenk, *Phys Rev C* **95** 024302 (2017) [arXiv:1610.05213]

Beyond-BCS in neutron matter: SRC





Rios, Dickhoff, Polls, JLTP **189**, 234 (2017)
[arXiv:1707.04140]
Rios, Dickhoff, Polls, et al PRC **94**, 025802 (2016)
[arxiv:1601.01600]

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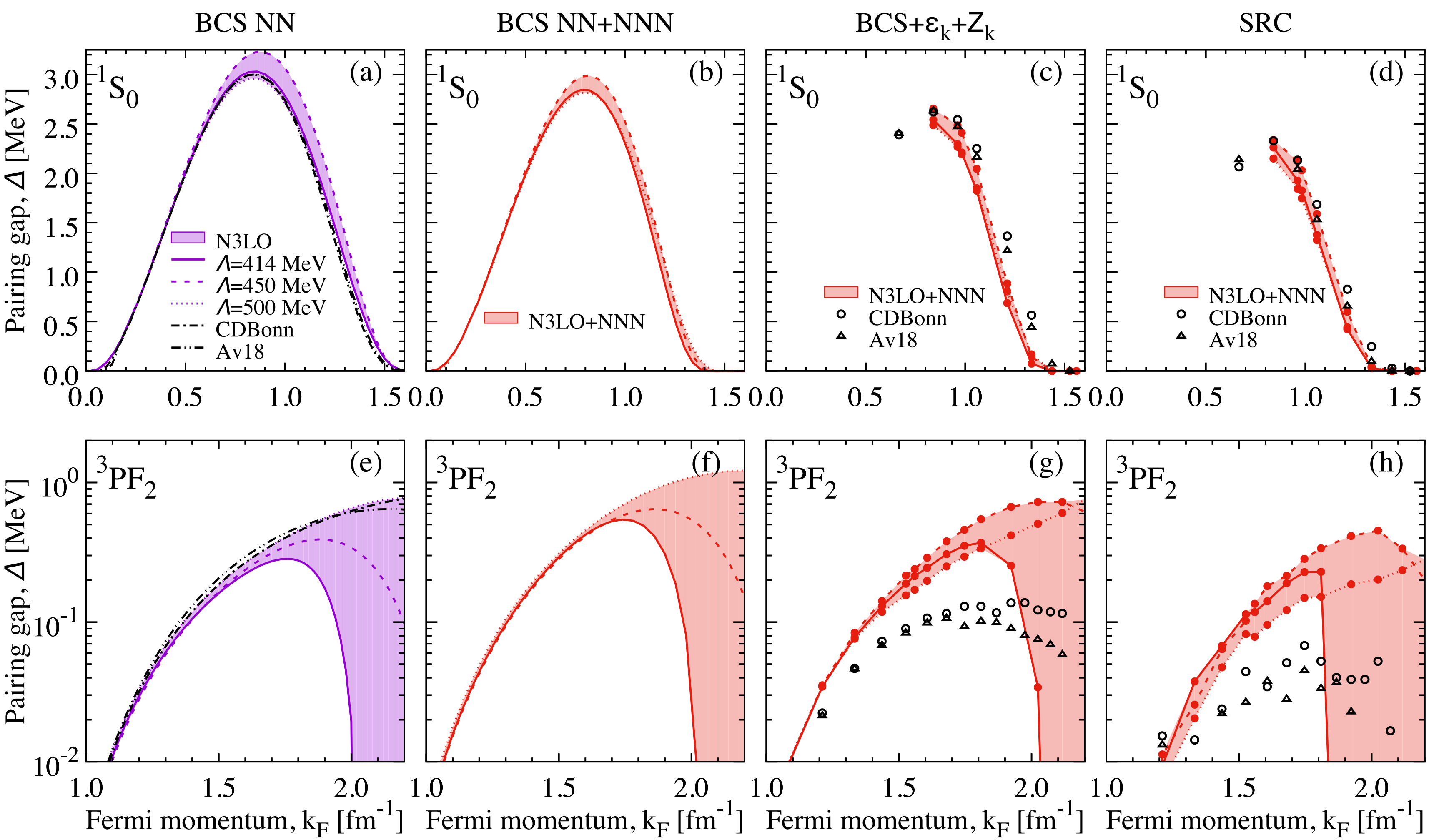
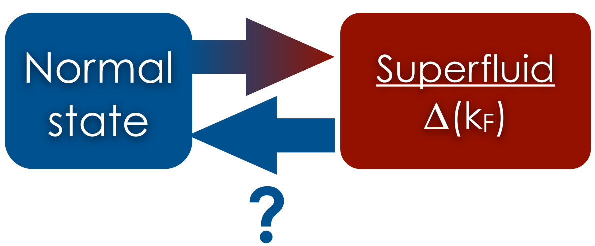
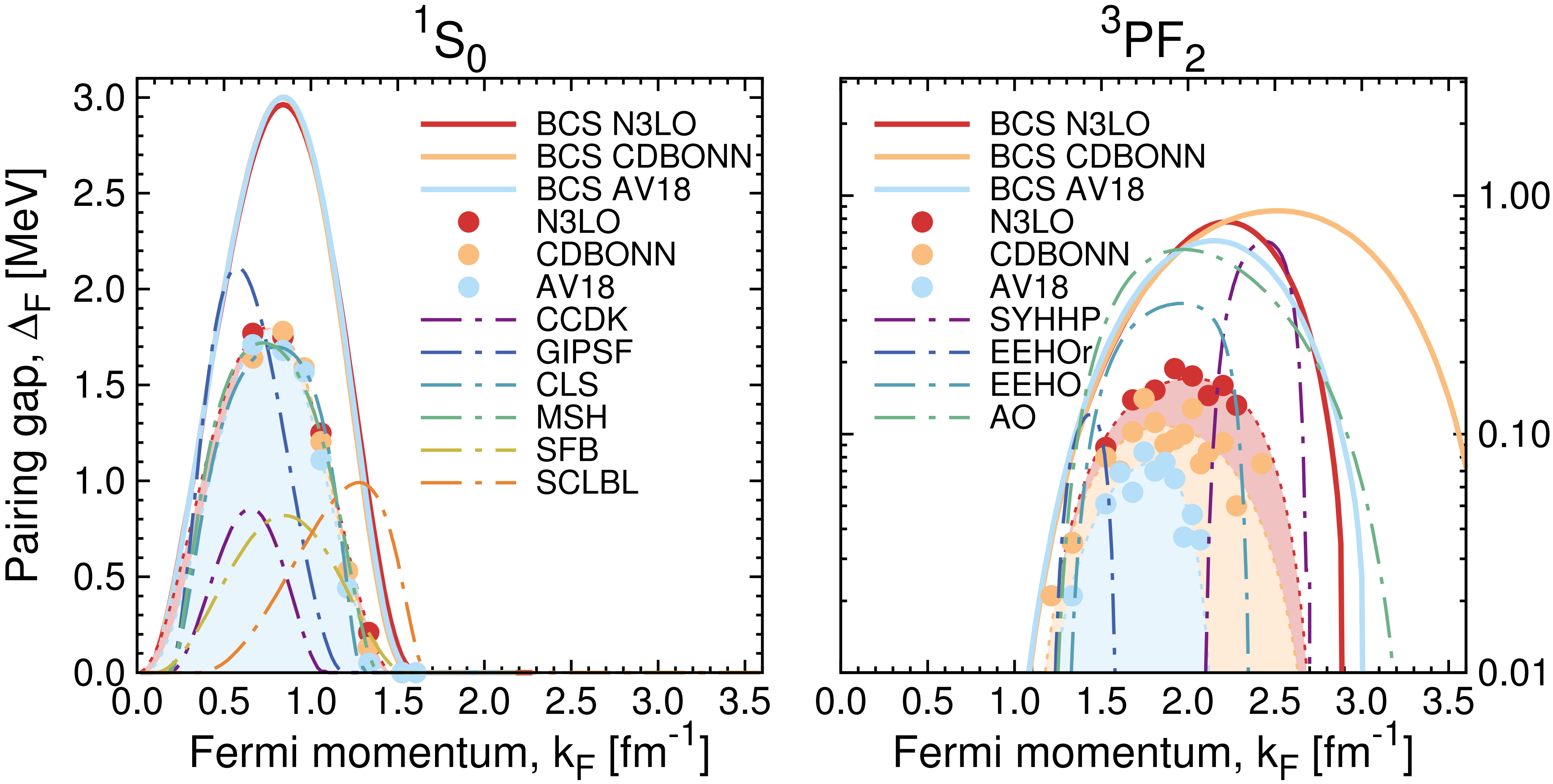


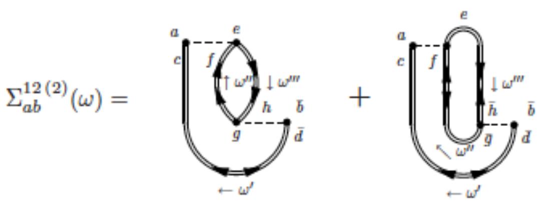


Rios, Dickhoff, Polls, JLTP **189**, 234 (2017)
[arXiv:1707.04140]
Rios, Dickhoff, Polls, et al PRC **94**, 025802 (2016)
[arxiv:1601.01600]

Beyond-BCS pairing: overview



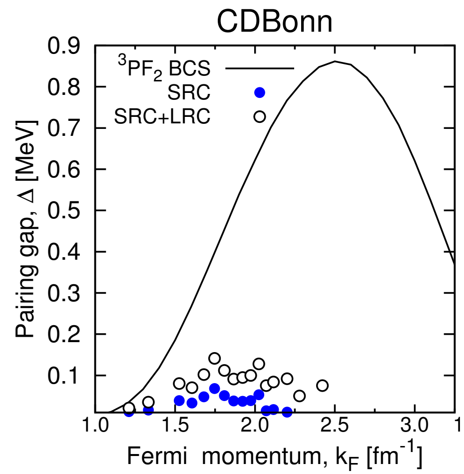
How to go beyond BCS?



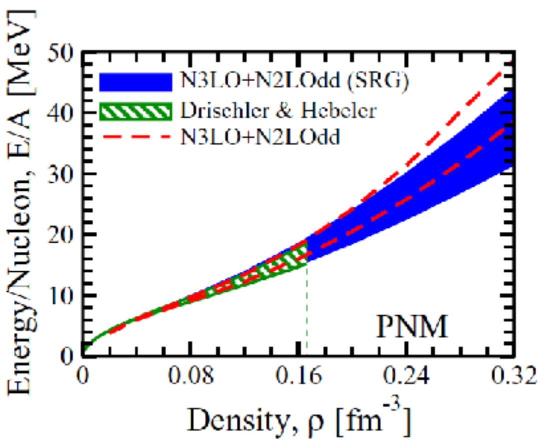
Normal state

Superfluid $\Delta(k_F)$

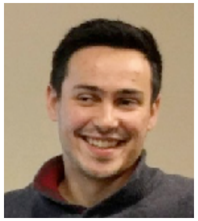
+



+



- **Existing frameworks** difficult to generalise
- **Nambu-covariant SCGF technique**
  - Symmetry breaking ✔
  - Finite temperature ✔
  - Systematic expansion w diagrams ✔
  - 3 nucleon forces ✔



M. Drissi

Nambu-Covariant Perturbation Theory

Nambu fields

[Anderson, 1958] [Nambu, 1960]

- $\mathscr{B}$ and $\bar{\mathscr{B}}$ ≡ orthonormal bases $|b\rangle \to |\bar{b}\rangle$
- Let $\bar{\cdot}$ be the involution $(\bar{1} = 2, \bar{2} = 1)$
- Define $\mu \equiv (b, g)$ and $\bar{\mu} \equiv (\bar{b}, \bar{g})$ where $g \in \{1,2\}$
- Then Nambu fields are defined as

$$\mathrm{A}^{\mu} \equiv \mathrm{A}^{(b,g)} \equiv \begin{pmatrix} a_b \\ a^{\dagger}_{\bar{b}} \end{pmatrix}_g$$

$$\mathrm{A}^{\dagger}_{\mu} \equiv \mathrm{A}^{\dagger}_{(b,g)} \equiv \begin{pmatrix} a^{\dagger}_b & a_{\bar{b}} \end{pmatrix}_g$$

- Canonical anticommutation relation

$$\{\mathrm{A}^{\mu}, \mathrm{A}^{\nu}\} = \delta_{\mu\bar{\nu}} \quad , \quad \{\mathrm{A}^{\dagger}_{\mu}, \mathrm{A}^{\dagger}_{\nu}\} = \delta_{\mu\bar{\nu}} \quad , \quad \{\mathrm{A}^{\mu}, \mathrm{A}^{\dagger}_{\nu}\} = \delta_{\mu\nu}$$

Operators

- Operators as polynomial of Nambu fields

$$O \equiv \sum_{\mu_1 \dots \mu_{2k}} o^{\mu_1 \dots \mu_k}{}_{\mu_{k+1} \dots \mu_{2k}} \mathrm{A}^{\dagger}_{\mu_1} \dots \mathrm{A}^{\dagger}_{\mu_k} \mathrm{A}^{\mu_{k+1}} \dots \mathrm{A}^{\mu_{2k}}$$

$$O \equiv \sum_{\mu_1 \dots \mu_{2k}} o_{\mu_1 \dots \mu_{2k}} \mathrm{A}^{\mu_1} \dots \mathrm{A}^{\mu_{2k}}$$

$$O \equiv \sum_{\mu_1 \dots \mu_{2k}} o^{\mu_1 \dots \mu_{2k}} \mathrm{A}^{\dagger}_{\mu_1} \dots \mathrm{A}^{\dagger}_{\mu_{2k}}$$

Tensor definitions

- Let $\mathscr{W}$ a unitary Bogoliubov transformation

$$\mathrm{B}^{\mu} = \sum_{\nu} (\mathscr{W}^{\dagger})^{\mu}{}_{\nu} \mathrm{A}^{\nu}$$

$$\mathrm{B}^{\dagger}_{\mu} = \sum_{\nu} \mathscr{W}^{\nu}{}_{\mu} \mathrm{A}^{\dagger}_{\nu}$$

- Definition: *(p,q)*-tensor is multi-dim array s.t.

$$t'^{\mu_1 \dots \mu_p}{}_{\nu_1 \dots \nu_q} \equiv \sum_{\kappa_1 \dots \kappa_p} \sum_{\lambda_1 \dots \lambda_q} (\mathscr{W}^{\dagger})^{\mu_1}{}_{\kappa_1} \dots (\mathscr{W}^{\dagger})^{\mu_p}{}_{\kappa_p} \, t^{\kappa_1 \dots \kappa_p}{}_{\lambda_1 \dots \lambda_q} (\mathscr{W})^{\lambda_1}{}_{\nu_1} \dots (\mathscr{W})^{\lambda_q}{}_{\nu_q}$$

- *p contravariant* & *q covariant* indices

Metric tensor

- Definition: (0,2)-, (1,1)-, (2,0)-tensors

$$\left.\begin{aligned} g_{\mu\nu} &\equiv \delta_{\mu\bar{\nu}} \\ g^{\mu}{}_{\nu} &\equiv \delta_{\mu\nu} \\ g^{\mu\nu} &\equiv \delta_{\mu\bar{\nu}} \end{aligned}\right\} + \text{transform like a tensor}$$

- Raising/lowering indices of a tensor:

$$o_{\mu_1 \dots \mu_{2k}} = \sum_{\alpha_1 \dots \alpha_k} g_{\mu_1 \alpha_1} \dots g_{\mu_k \alpha_k} \, o^{\alpha_1 \dots \alpha_k}{}_{\mu_{k+1} \dots \mu_{2k}}$$

Why Bogoliubov tensor algebra?

Tensor product: $r^{\mu_1}{}_{\nu_1}{}^{\mu_2\mu_3} = s^{\mu_1}{}_{\nu_1} \, t^{\mu_2\mu_3}$

Tensor contraction: $r^{\mu}{}_{\nu} = \sum_{\alpha} s^{\mu}{}_{\alpha} \, t^{\alpha}{}_{\nu}$

- Co(contra-)variance under Bogoliubov transforms provide **invariant** expressions in any basis
- Potential to **optimise** the extended basis
- **Tensor-network** structure becomes **transparent**
- Leads to **diagrammatic** expansion (à la de Dominicis-Martin or Haussmann)
- Other **formalisms** through specific basis or metric

Perturbative expansion

Hamiltonian partitioning

$$\Omega = \Omega_0 + \Omega_1$$

$$\Omega_0 = \frac{1}{2}\sum_{\mu\nu} U_{\mu\nu} \mathrm{A}^{\mu}\mathrm{A}^{\nu}$$

$$\Omega_1 = \sum_{k=1}^{n} \frac{1}{(2k)!} \sum_{\mu_1\ldots\mu_{2k}} v^{(k)}_{\mu_1\ldots\mu_{2k}} \mathrm{A}^{\mu_1}\ldots\mathrm{A}^{\mu_{2k}}$$

Covariant **k-body** vertices

Green's functions

- Contravariant **k-body** Green's function

$$(-1)^k \mathcal{G}^{\mu_1\cdots\mu_{2k}}(\tau_1,\ldots,\tau_{2k}) \equiv \left\langle \mathrm{T}\left[\mathrm{A}^{\mu_1}(\tau_1)\ \ldots\ \mathrm{A}^{\mu_{2k}}(\tau_{2k})\right]\right\rangle$$

with $\langle\,.\,\rangle = \mathrm{Tr}\left(\,.\,\rho\right)$ and $\rho \equiv \dfrac{e^{-\beta\Omega}}{\mathrm{Tr}\left(e^{-\beta\Omega}\right)}$

- Unperturbed case: $\Omega \longleftrightarrow \Omega_0$

Expansion

- Interaction picture expression

$$(-1)^k \mathcal{G}^{\mu_1\cdots\mu_{2k}}(\tau_1,\ldots,\tau_{2k}) = \frac{\left\langle \mathrm{T}\left[e^{-\int_0^\beta \mathrm{d}s\ \Omega_1(s)}\ \mathrm{A}^{\mu_1}(\tau_1)\ \ldots\ \mathrm{A}^{\mu_{2k}}(\tau_{2k})\right]\right\rangle_0}{\left\langle \mathrm{T}e^{-\int_0^\tau \mathrm{d}s\ \Omega_1(s)}\right\rangle_0}$$

- Perturbative expansions

$$\left\langle \mathrm{T}\left[e^{-\int_0^\beta \mathrm{d}s\ \Omega_1(s)}\ \mathrm{A}^{\mu_1}(\tau_1)\ \ldots\ \mathrm{A}^{\mu_{2k}}(\tau_{2k})\right]\right\rangle_0 =$$

$$\sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \int_0^\beta \mathrm{d}\tau_1' \ldots \int_0^\beta \mathrm{d}\tau_n' \left\langle \mathrm{T}\left[\Omega_1(\tau_1')\ldots\Omega_1(\tau_n')\ \mathrm{A}^{\mu_1}(\tau_1)\ \ldots\ \mathrm{A}^{\mu_{2k}}(\tau_{2k})\right]\right\rangle_0$$

- Statistical Wick theorem + Linked-cluster theorem

$\Rightarrow$ Feynman diagrammatics *almost* as usual

- We provide a set of **Feynman rules**
- Also rules to evaluate **Matsubara sums**
- **Simpler** expressions than in other approaches (Gorkov Green's functions or BMPT

Drissi, Rios & Barbieri, Paper I, arxiv:2107xx

Feynman diagram building blocks

Formulations

- **Time-independent partitioning**
  - Time representation
  - **Energy** representation

Fourier Transform

- **Matsubara frequencies**

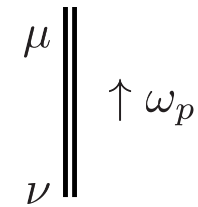
$$\mathscr{G}^{\mu\nu}(\omega_p) \equiv \int_0^\beta \mathrm{d}\tau \; e^{i\omega_p \tau} \; \mathscr{G}^{\mu\nu}(\tau)$$

$$\mathscr{G}^{\mu\nu}(\tau) = \frac{1}{\beta} \sum_p \; e^{-i\omega_p \tau} \; \mathscr{G}^{\mu\nu}(\omega_p)$$

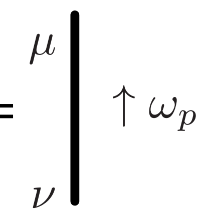
- **Time-dependent partitioning** (not today!)

Propagators

$$-\mathscr{G}^{\mu\nu}(\omega_p) =$$



$$-\left(\mathscr{G}^{(0)}\right)^{\mu\nu}(\omega_p) =$$



Fully antisymmetric vertex

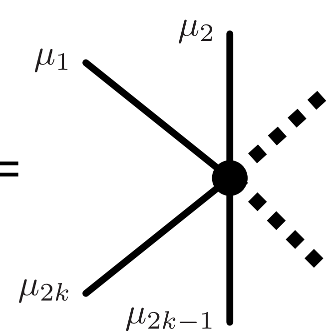
- **Definition**

$$v^{(k)}_{[\mu_1 \, \mu_2 \, \dots \, \mu_{2k-1} \, \mu_{2k}]} \equiv \frac{1}{(2k)!} \sum_{\sigma \in S_{2k}} \epsilon(\sigma) \; v^{(k)}_{\mu_{\sigma(1)} \, \mu_{\sigma(2)} \, \dots \, \mu_{\sigma(2k-1)} \, \mu_{\sigma(2k)}}$$

- **Antisymmetrisation** defines a new *(0,2k)*-tensor
- ***Not*** the case in a *mixed* representation

k-body vertex

$$v^{(k)}_{[\mu_1 \, \mu_2 \, \dots \, \mu_{2k-1} \, \mu_{2k}]} =$$



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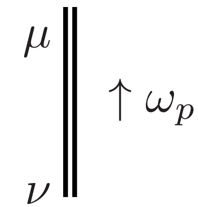

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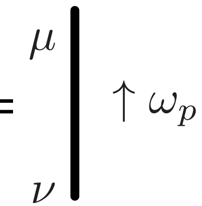

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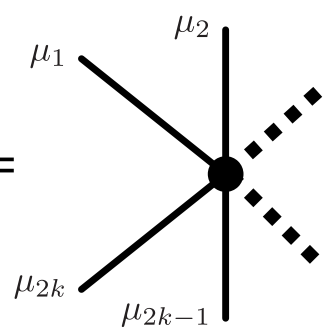
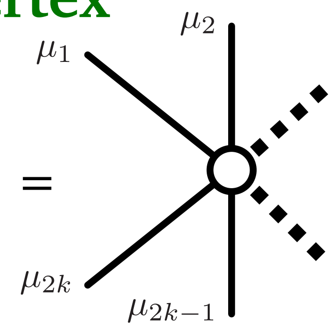


Drissi, Rios & Barbieri, Paper I, arxiv:2107xx

Why antisymmetric vertices?

Un-symmetrised vertex

$$v^{(k)}_{\mu_1\,\mu_2\,\dots\,\mu_{2k-1}\,\mu_{2k}} =$$



Antisymmetrized vertex

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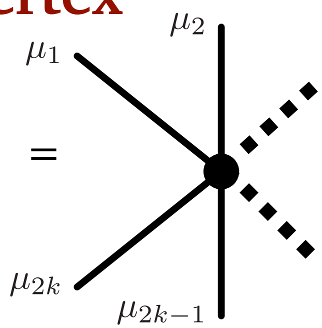

Drissi, Rios & Barbieri, Paper I, arxiv:2107xx

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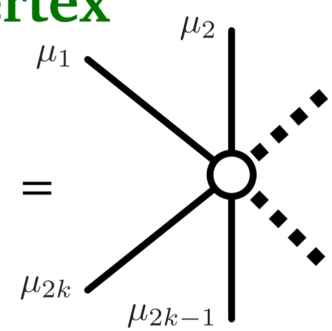

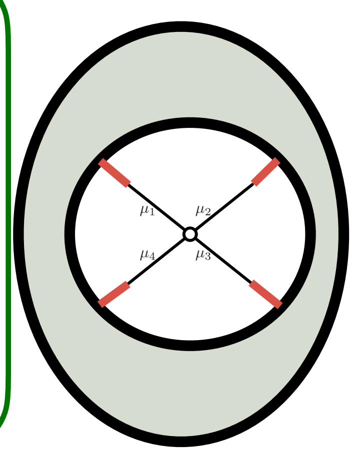

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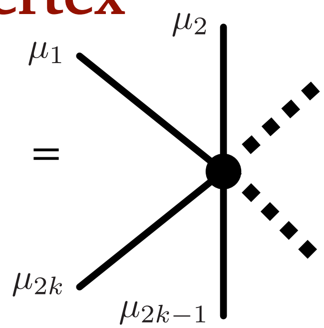

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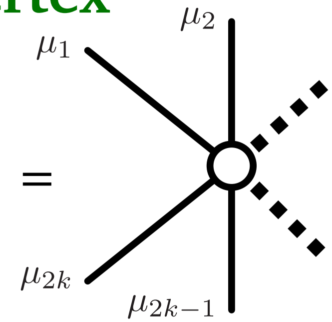

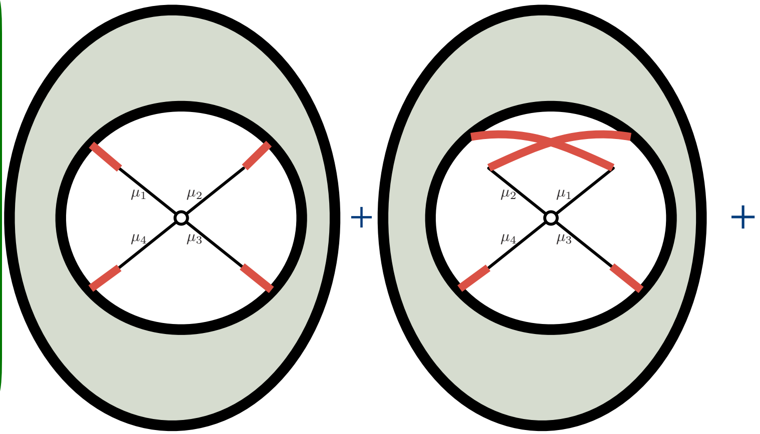

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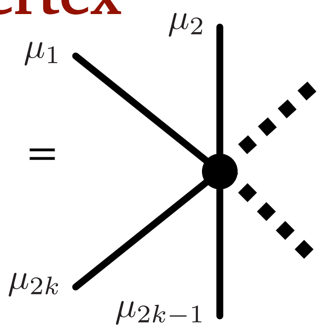
$$v^{(k)}_{[\mu_1\,\mu_2\,\dots\,\mu_{2k-1}\,\mu_{2k}]} =$$

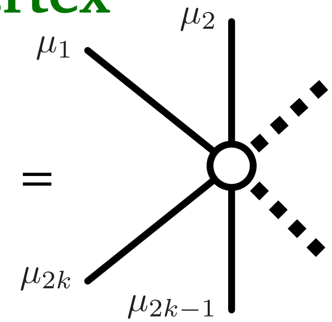


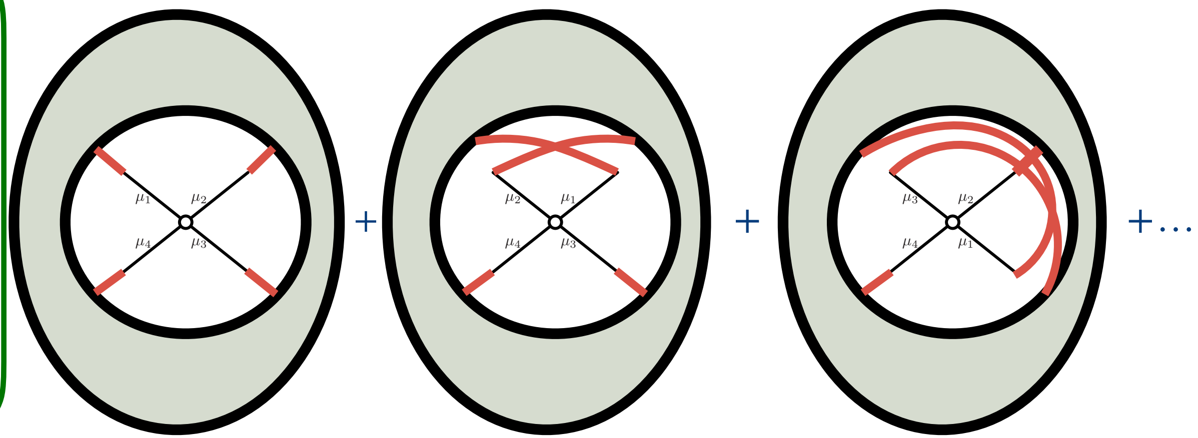
Drissi, Rios & Barbieri, Paper I, arxiv:2107xx

Why antisymmetric vertices?

Un-symmetrised vertex

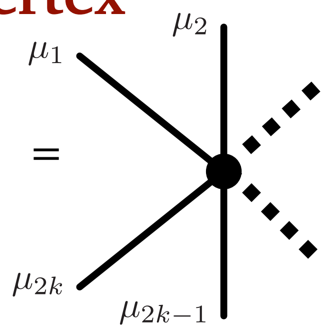
$$v^{(k)}_{\mu_1\,\mu_2\,\dots\,\mu_{2k-1}\,\mu_{2k}} =$$





Antisymmetrized vertex

$$v^{(k)}_{[\mu_1\,\mu_2\,\dots\,\mu_{2k-1}\,\mu_{2k}]} =$$



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Why antisymmetric vertices?

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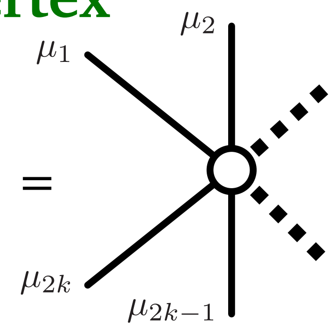
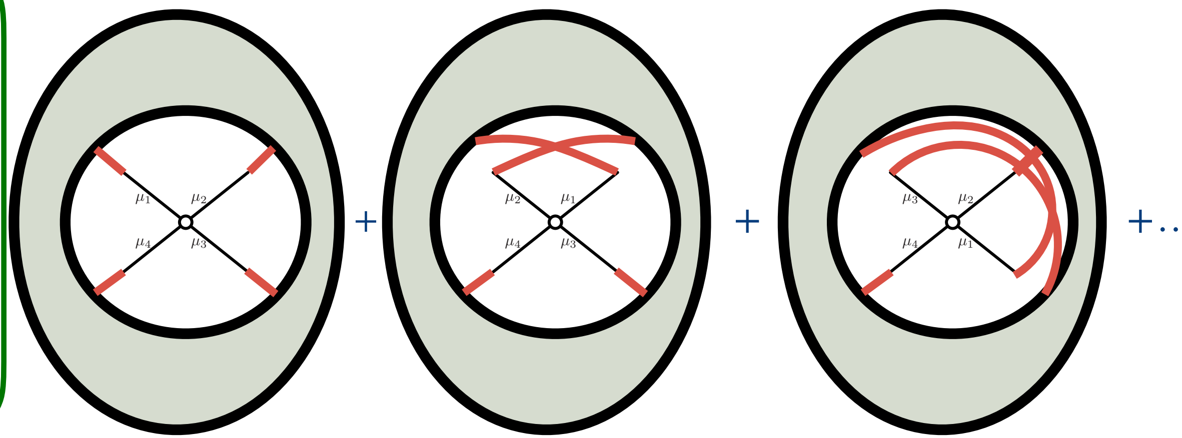
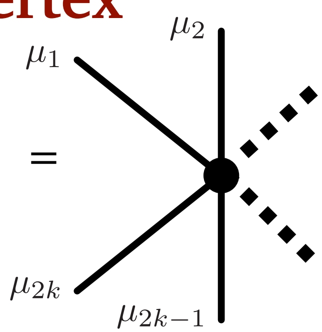




Antisymmetrized vertex

$$v^{(k)}_{[\mu_1\,\mu_2\,\dots\,\mu_{2k-1}\,\mu_{2k}]} =$$



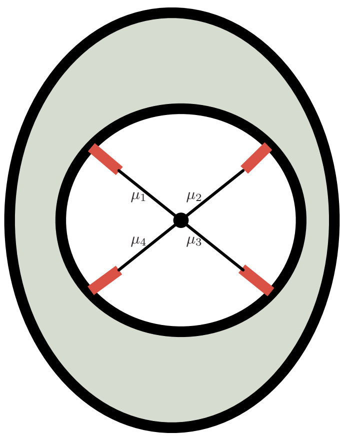


Diagram factorisation

- Derivations rely on
  - Wick theorem $\Rightarrow$ sum over pairing
  - Sum over single-particle and Nambu indices

➡ Extends Hugenholtz antisymmetrisation

- Antisym is a **one-off pre-computing** cost

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Perturbative expansion

Order n graphical rules

- Draw all topologically distinct **connected unlabelled** diagrams
  - with ***2k* external** legs
  - with ***n* vertices** (for order n contributions)

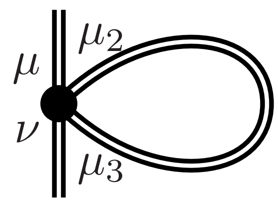
Feynman rules

1. Label vertices from *1* to n
   - S is the number of vertex labels permutations leaving the diagram invariant
2. For each line multiply by $-\left(\mathscr{G}^{(0)}\right)^{\mu\nu}(\omega_e)$
3. For each k-body vertex multiply by $v^{(k)}_{[\mu_1\ \mu_2\ \dots\ \mu_{2k-1}\ \mu_{2k}]}$
4. Sum over each internal μ index and each independent ω_e frequency
5. Multiply by $\dfrac{(-1)^{n+L}}{S \times 2^T \ \prod_{l=2}^{l\mathrm{max}} (l!)^m}$

Gaudin rules

- These simplify **Matsubara sums**
- Require **spanning trees**

Tadpoles are exceptional



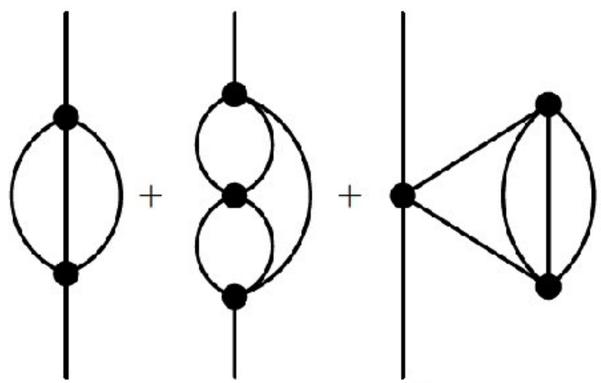
$$I_{\mu\nu} = \sum_{\mu_2\dots\mu_{2k-1}} \frac{(-1)^k}{2^{k-1}(k-1)!}\, v^{(k=2)}_{[\mu\ \dot{\mu}_2\ \dot{\mu}_3\ \nu]}$$
$$\times \frac{1}{\beta}\sum_{\omega_e} -\mathscr{G}^{\mu_2\mu_3}(\omega_e)e^{-i\omega_e\eta_p}$$

- **Partially antisymmetrized** vertices needed:

$$v^{(k)}_{[\mu_1\dots\dot{\mu}_x\dots\dot{\mu}_y\dots\mu_{2k}]} \equiv \frac{2^p p!}{(2k)!}\sum_{\sigma\in S_{2k}/S_2^p\times S_p} \epsilon(\sigma)\ v^{(k)}_{\mu_{\sigma(1)}\dots\dot{\mu}_x\dots\dot{\mu}_y\dots\mu_{\sigma(2k)}}$$

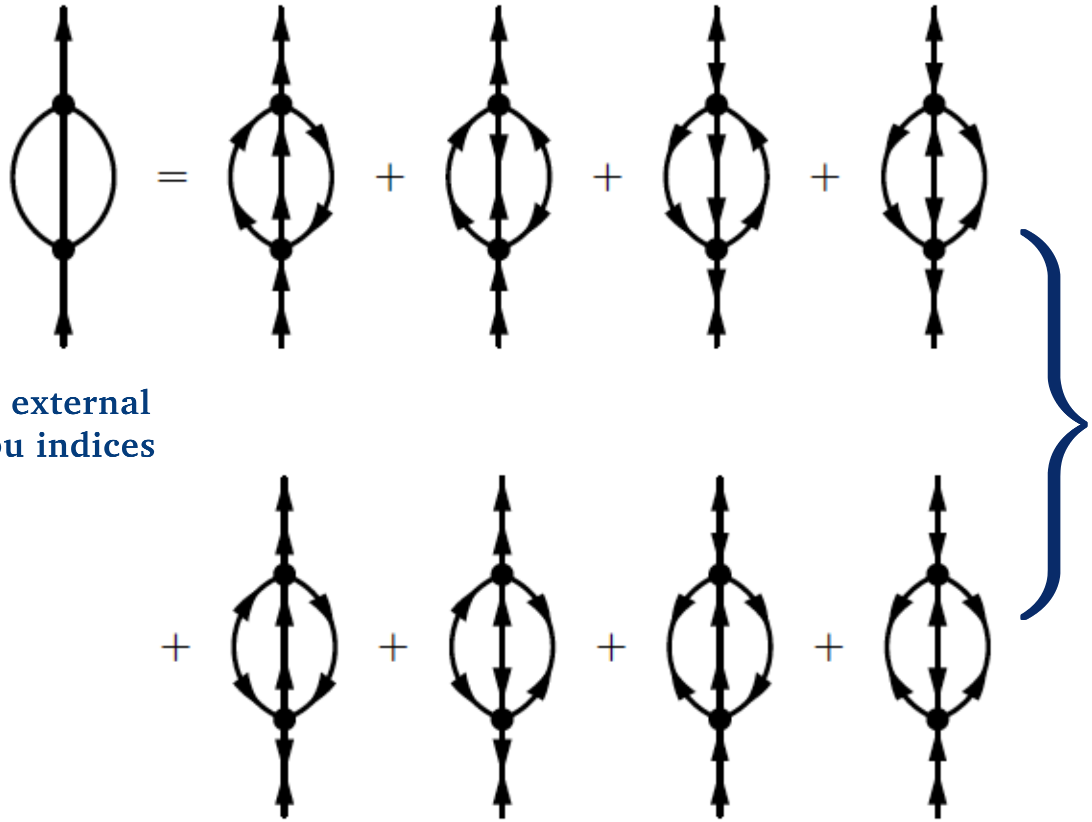
- p internal lines are **fixed**
- ***k*-body generalisation** works

HFB partitioning 3rd order



$\mathscr{G}_2$ $\qquad$ $\mathscr{G}_3$ $\qquad$ $\mathscr{G}_3'$

Connection with Gorkov



Fixed external Nambu indices

Contribution to a **normal** component of the 2-body GF

- Changes of **single-particle basis** bring **admixtures** in normal & anomalous components…
- but leave **Nambu-convariant diagrams invariant**!

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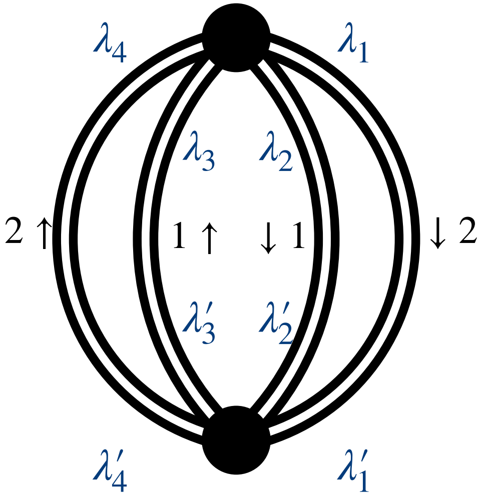
Basic example



$\mathscr{A}_4$	$\mathscr{A}_3$	$\mathscr{A}_2$	$\mathscr{A}_1$
$\dfrac{-f(\epsilon_3)\, f(-\epsilon_2)\, f(-\epsilon_1)}{\epsilon_4+\epsilon_3-\epsilon_2-\epsilon_1}$	$\dfrac{f(-\epsilon_4)\, f(-\epsilon_2)\, f(-\epsilon_1)}{\epsilon_4+\epsilon_3-\epsilon_2-\epsilon_1}$	$\dfrac{f(-\epsilon_4)\, f(-\epsilon_3)\, f(-\epsilon_1)}{-\epsilon_4-\epsilon_3+\epsilon_2+\epsilon_1}$	$\dfrac{f(-\epsilon_4)\, f(-\epsilon_3)\big(-f(\epsilon_2)\big)}{-\epsilon_4-\epsilon_3+\epsilon_2+\epsilon_1}$

$$\boxed{\begin{aligned} I = \frac{1}{48} \sum_{\substack{\lambda_1\lambda_2\lambda_3\lambda_4 \\ \lambda'_1\lambda'_2\lambda'_3\lambda'_4}} v^{(2)}_{[\lambda_1\lambda_2\lambda_3\lambda_4]} v^{(2)}_{[\lambda'_4\lambda'_3\lambda'_2\lambda'_1]} & \int_{-\infty}^{+\infty} \frac{\mathrm{d}\epsilon_1}{2\pi}\frac{\mathrm{d}\epsilon_2}{2\pi}\frac{\mathrm{d}\epsilon_3}{2\pi}\frac{\mathrm{d}\epsilon_4}{2\pi}\, S^{\lambda'_1\lambda_1}(\epsilon_1)\; S^{\lambda'_2\lambda_2}(\epsilon_2)\; S^{\lambda_3\lambda'_3}(\epsilon_3)\; S^{\lambda_4\lambda'_4}(\epsilon_4) \\ & \times \frac{f(\epsilon_1)\, f(\epsilon_2)\, f(-\epsilon_3)\, f(-\epsilon_4) \;-\; f(-\epsilon_1)\, f(-\epsilon_2)\, f(\epsilon_3)\, f(\epsilon_4)}{\epsilon_1+\epsilon_2-\epsilon_3-\epsilon_4} \end{aligned}}$$

Spectral function

$$\mathscr{G}^{\mu\nu}(\omega_p) = \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega'}{2\pi}\, \frac{S^{\mu\nu}(\omega')}{i\omega_p - \omega'}$$

- Lehmann representation

$$S^{\mu\nu}(\omega) \equiv \frac{1}{Z} \sum_{m,n} \langle \Psi_m | \mathrm{A}^\mu | \Psi_n \rangle \langle \Psi_n | \mathrm{A}^\nu | \Psi_m \rangle$$
$$\times\, e^{-\beta\Omega_m} \left(1 + e^{-\beta\omega}\right)\; (2\pi)\; \delta\left(\Omega_n - \Omega_m - \omega\right)$$

Properties

- Symmetries
  - **Hermiticity:** $\left(S^\mu{}_\nu(\omega)\right)^* = S^\nu{}_\mu(\omega)$
  - **Antisymmetry:** $S^\mu{}_\nu(\omega) = S_\nu{}^\mu(-\omega) \left(\neq S^\nu{}_\mu(-\omega) \right)$
- Positive bounds
  - $S(\omega) \succ 0 \iff$ each principal minor is strictly positive

Self-consistent Green's function resummation

Dyson equation

- <u>Partitioning considered</u>

$$\Omega = \underbrace{\frac{1}{2!}\sum_{\mu\nu} U_{\mu\nu}\; \mathrm{A}^{\mu}\mathrm{A}^{\nu}}_{\Omega_0} + \underbrace{\frac{1}{4!}\sum_{\alpha\beta\gamma\delta} v^{(2)}_{\alpha\beta\gamma\delta}\; \mathrm{A}^{\alpha}\mathrm{A}^{\beta}\mathrm{A}^{\gamma}\mathrm{A}^{\delta}}_{\Omega_1}$$

- <u>Dyson equation</u>

$$\mathcal{G}^{\mu\nu}(\omega_n) = \mathcal{G}^{(0)\mu\nu}(\omega_n) + \sum_{\lambda_1\lambda_2} \mathcal{G}^{(0)\mu\lambda_1}(\omega_n)\; \Sigma_{\lambda_1\lambda_2}(\omega_n)\; \mathcal{G}^{\lambda_2\nu}(\omega_n)$$

Diagrammatic expansion of $\Sigma_{\mu\nu}(\omega_n)$

- <u>with unperturbed propagators</u>

$$\Sigma_{\mu\nu}(\omega_n) = \frac{\mathcal{J}_{\mu\nu}(\omega_n) - \mathcal{J}_{\nu\mu}(-\omega_n)}{2}$$

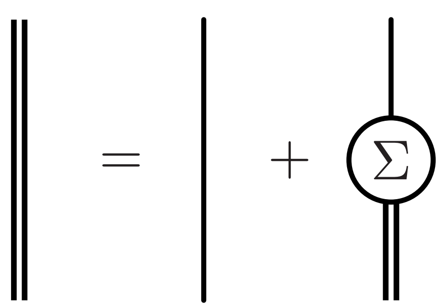
$$\mathcal{J}_{\mu\nu}(\omega_n) = \sum \text{1PI diagrams with } \mathcal{G}^{(0)}$$

- <u>with self-consistent propagators</u>

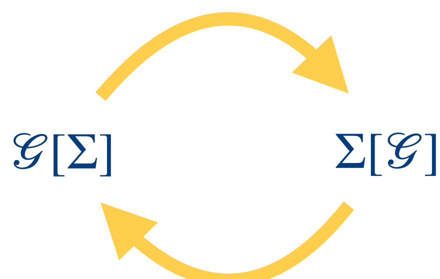
$$\Sigma_{\mu\nu}(\omega_n) = \frac{\mathcal{J}_{\mu\nu}(\omega_n) - \mathcal{J}_{\nu\mu}(-\omega_n)}{2}$$

$$\mathcal{J}_{\mu\nu}(\omega_n) = \sum \text{2PI diagrams with } \mathcal{G} \;\Big(= \mathcal{J}_{\mu\nu}(\omega_n)\Big)$$

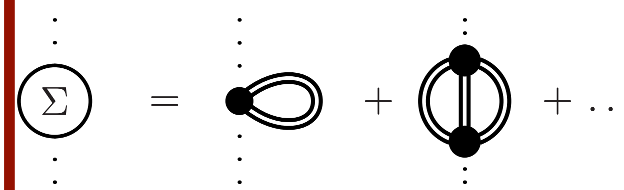
Diagrammatic representation



SCGF cycle



Self-energy expression



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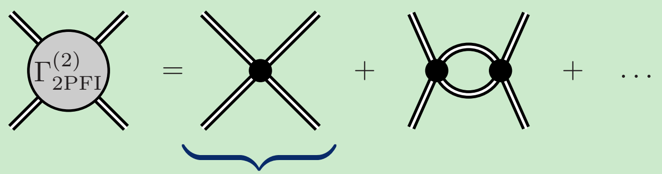
T-matrix: ladder's rung

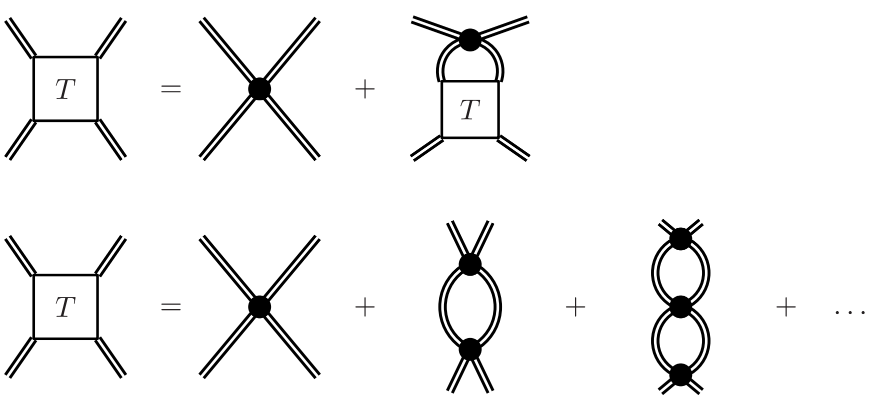
Approximations on $\Gamma^{(2)}_{\mathbf{2PFI}}$

- Sum of all possible rungs



Ladder approximation

T-matrix $\equiv \Gamma^{(2)}$ in ladder approximation



Ladder approximation

- Analytic/Retarded/Advanced/Sp function $\Rightarrow$ as usual
- T-matrix equation

$$T_{MN}(Z) \;=\; V^{(2)}_{MN} \;+\; \frac{1}{2} \sum_{LL'} V^{(2)}_{ML}\, \Pi^{LL'}(Z)\, T_{L'N}(Z)$$

where $V^{(2)}_{MN} \equiv v^{(2)}_{[\mu_1\mu_2\nu_1\nu_2]}$, $M \equiv (\mu_1, \mu_2)$ & $N \equiv (\nu_1, \nu_2)$

Solving the ladder

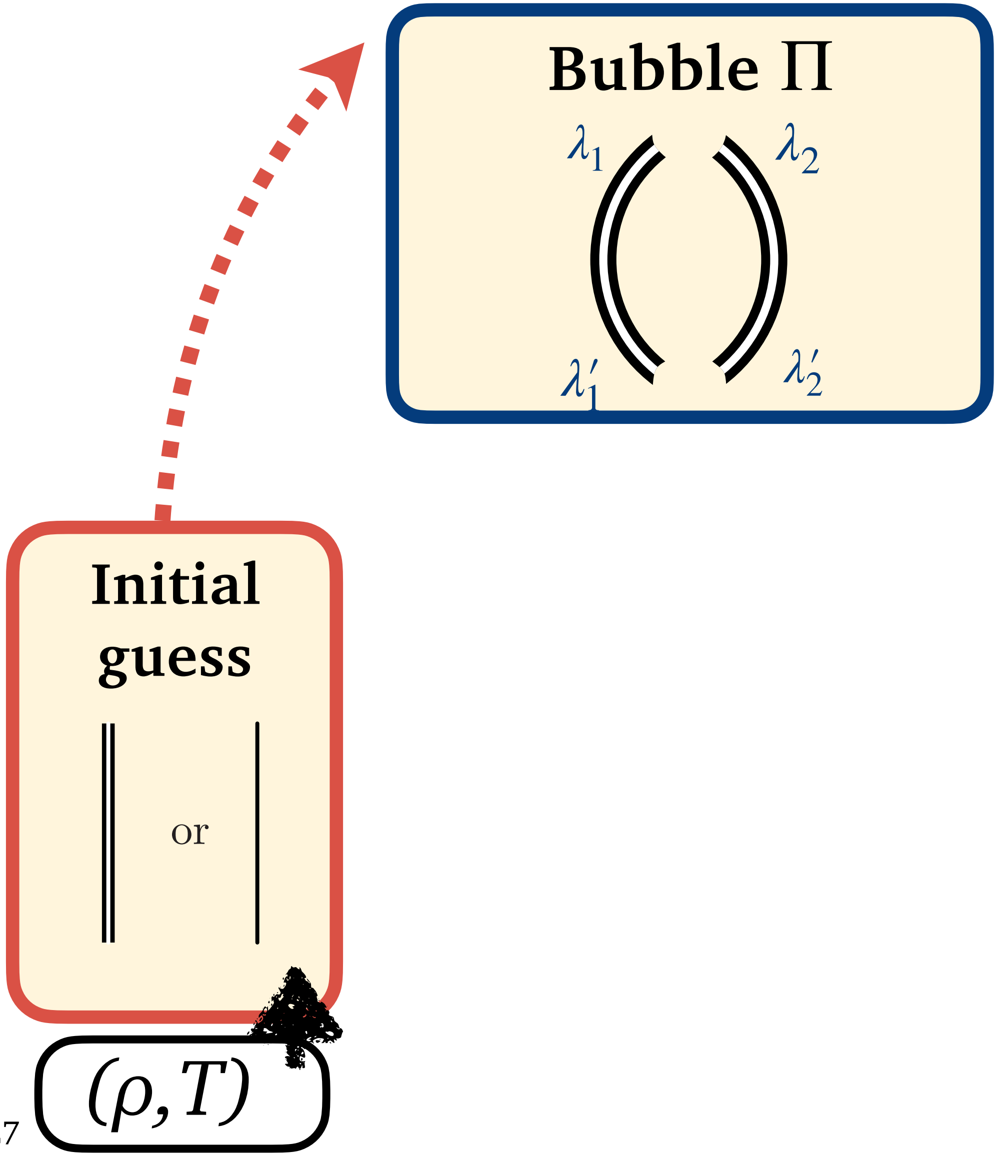
- Spectral representation

$$T_{MN}(Z) \equiv \underbrace{V^{(2)}_{MN}}_{\text{Instantaneous part}} + \underbrace{\int_{-\infty}^{+\infty} \frac{d\Omega}{2\pi} \frac{\mathcal{T}_{MN}(\Omega)}{Z - \Omega}}_{T^C(Z) \equiv \text{continuous part}}$$

- Solution

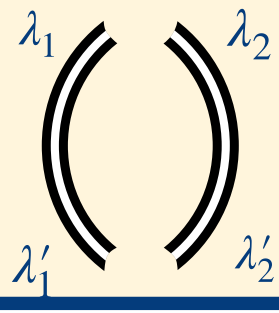
$$\mathcal{T}(\Omega) = iV^{(2)} \left\{ \left(gg - \frac{1}{2}\Pi^R(\Omega)V^{(2)} \right)^{-1} - \left(gg - \frac{1}{2}\Pi^A(\Omega)V^{(2)} \right)^{-1} \right\}$$

Nambu-Covariant Ladders

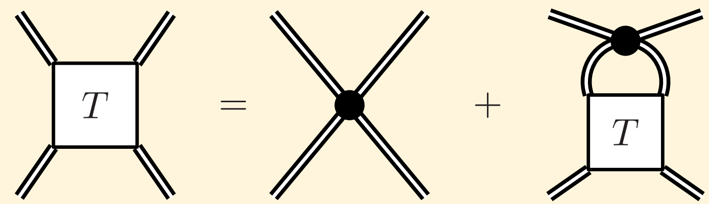


Nambu-Covariant Ladders

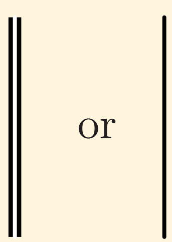
Bubble Π



T-matrix



Initial guess



(ρ, T)

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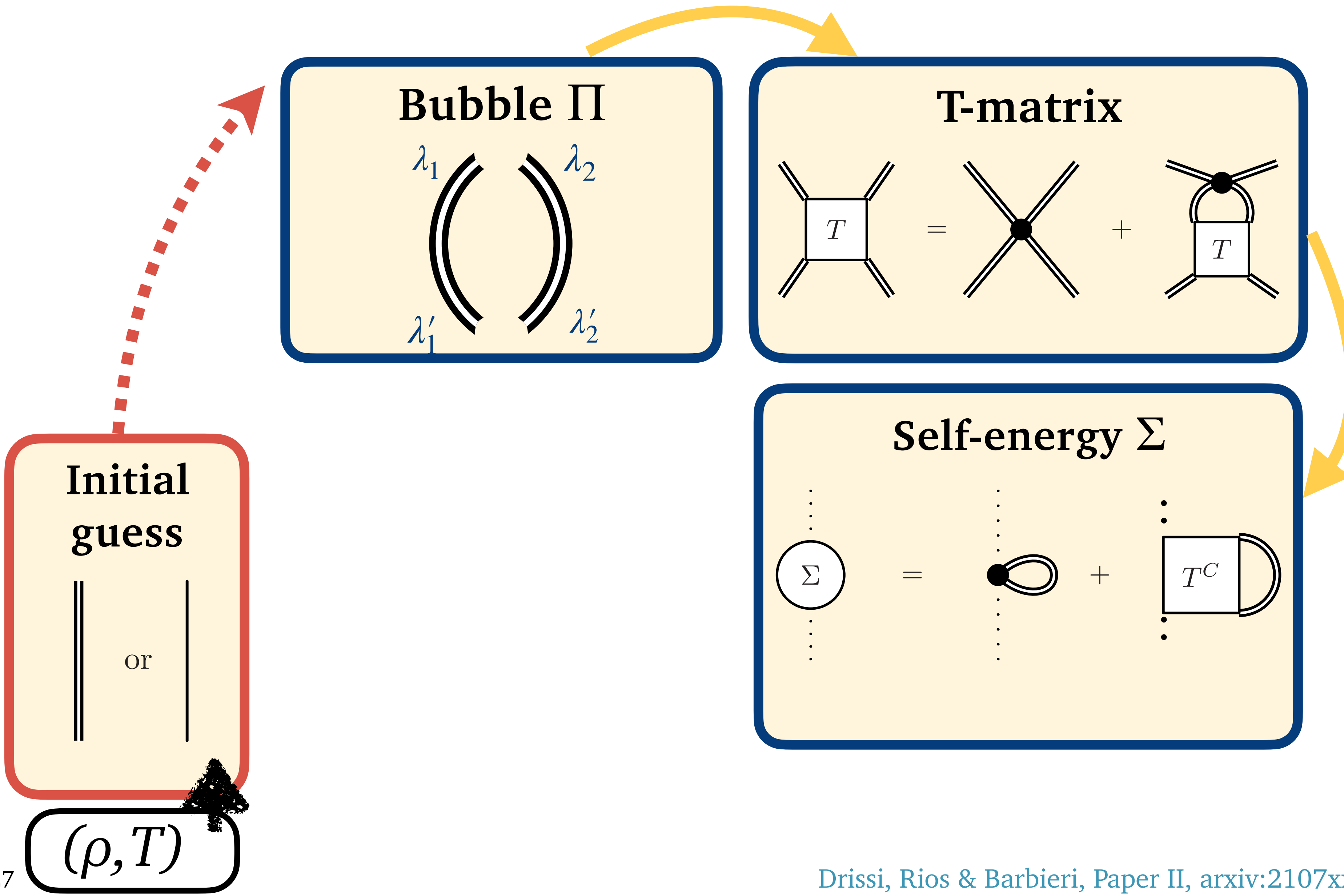
Nambu-Covariant Ladders



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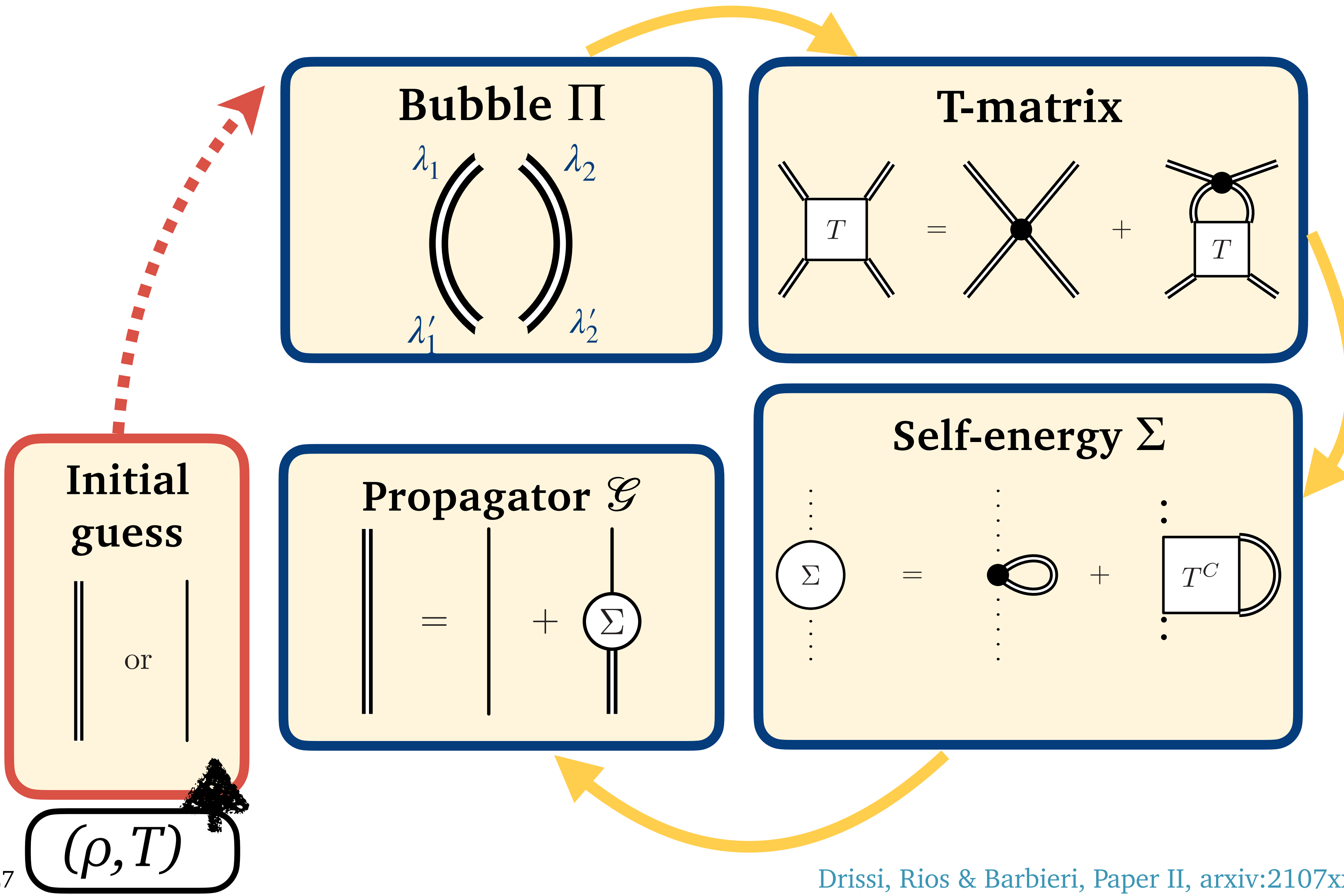
Nambu-Covariant Ladders



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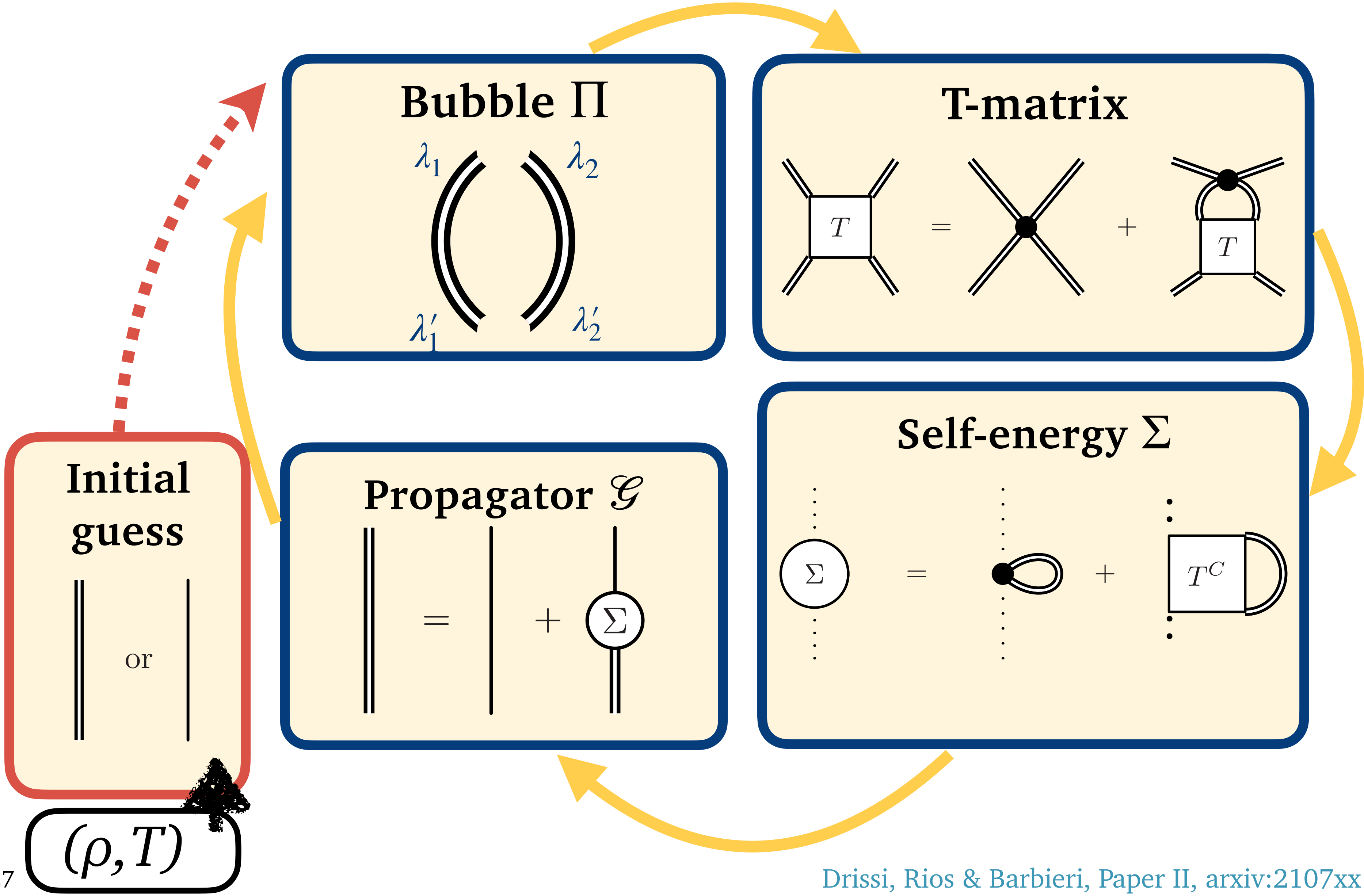

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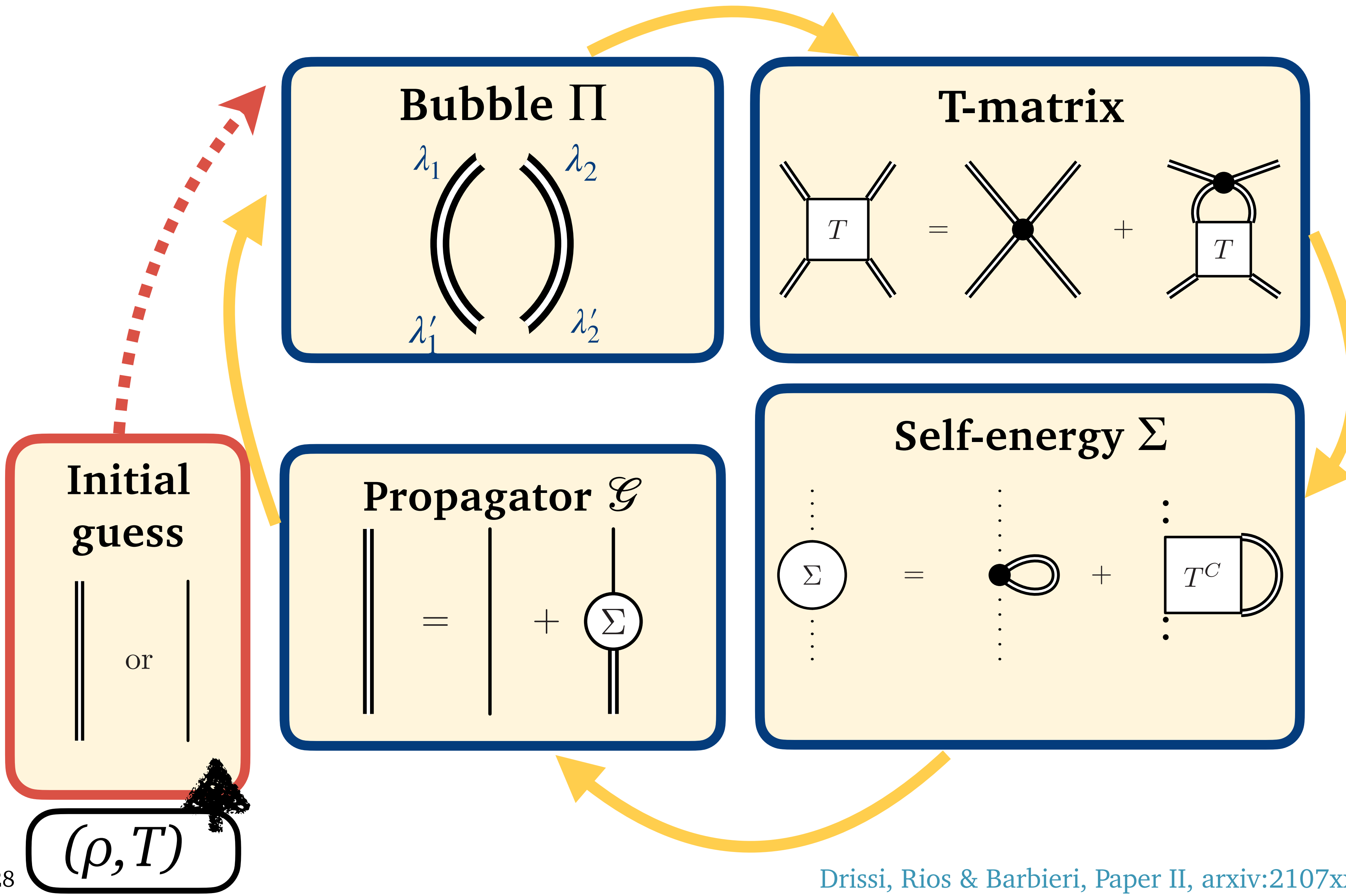

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Nambu-Covariant Ladders

Bubble Π

$$P^{MN}(\Omega) = \frac{1}{b(\Omega)} \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{2\pi} S^{\mu_1\nu_1}(\omega)\, f(\omega) \times S^{\mu_2\nu_2}(\Omega-\omega)\, f(\Omega-\omega)$$

T-matrix

$$\mathcal{T}(\Omega) = i V^{(2)} \left\{ \left(gg - \frac{1}{2}\Pi^R(\Omega) V^{(2)} \right)^{-1} - \left(gg - \frac{1}{2}\Pi^A(\Omega) V^{(2)} \right)^{-1} \right\}$$

Self-energy Σ

$$\Gamma_{\mu\nu}(\omega) = -\frac{1}{3} \sum_{\lambda_1\lambda_2} \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega'}{2\pi} \left[f(\omega') + b(\omega'-\omega) \right] \times \mathcal{T}_{\mu\lambda_1\lambda_2\nu}(\omega-\omega')\, S^{\lambda_1\lambda_2}(\omega')$$

$$\Sigma^{\infty}_{\mu\nu} = \frac{1}{2} \sum_{\mu_2\mu_3} v^{(2)}_{[\mu\dot{\mu}_2\dot{\mu}_3\nu]} \int_{-\infty}^{+\infty} \frac{\mathrm{d}\epsilon}{2\pi} f(-\epsilon)\, S^{\mu_2\mu_3}(\epsilon)$$

Propagator $\mathcal{G}$

$$S(\omega) = i\left(\omega\, g - \left(U + \Sigma^R(\omega) \right) \right)^{-1} - i\left(\omega\, g - \left(U + \Sigma^A(\omega) \right) \right)^{-1}$$

Initial guess

Unperturbed $S^{(0)}(\omega)$

/

Refined initial $S(\omega)$

(ρ, T)

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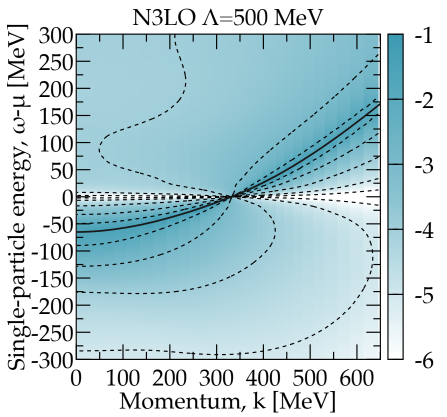
Conclusions

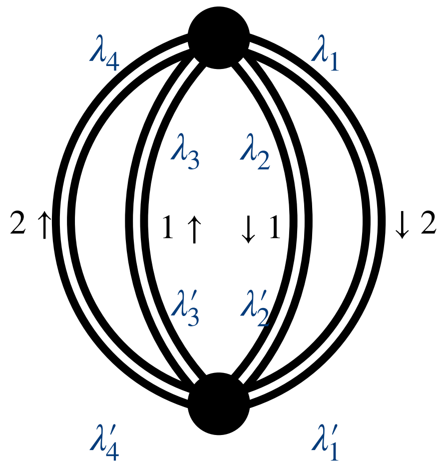
1) **Normal-phase** SCGF

- **Spectral strength** is available
- EoS & thermodynamics too!



2) **Nambu-covariant** SCGFs

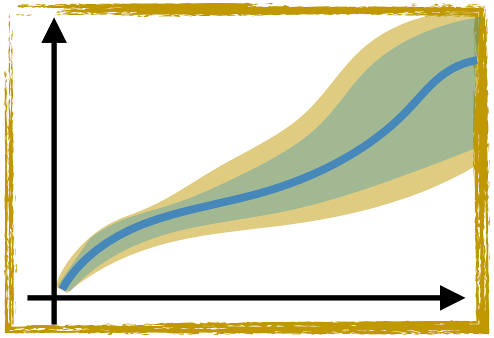
- Formally relevant
- Perturbative expansion simplified
- Allows for different approximation schemes

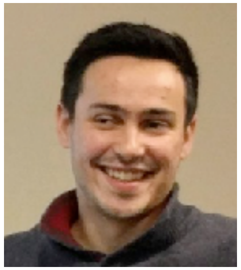


Next:

Numerical implementation

Uncertainties in predictions?





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TRIUMF

(@TRIUMF from 10/2021)

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W. H. Dickhoff (St Louis) + A. Polls

Thank you!

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