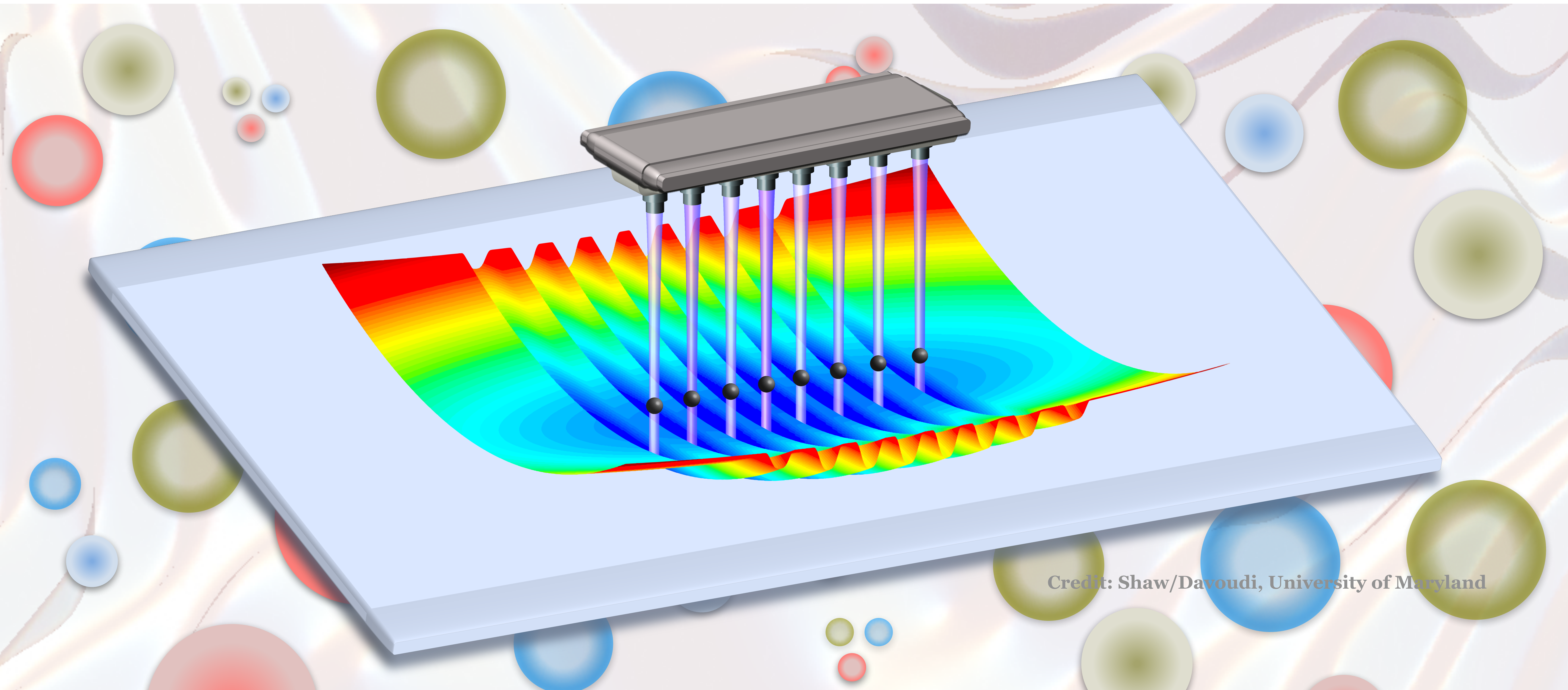
QUANTUM FIELD THEORIES WITH TRAPPED-ION QUANTUM SIMULATORS?

ZOHREH DAVOUDI

UNIVERSITY OF MARYLAND, COLLEGE PARK



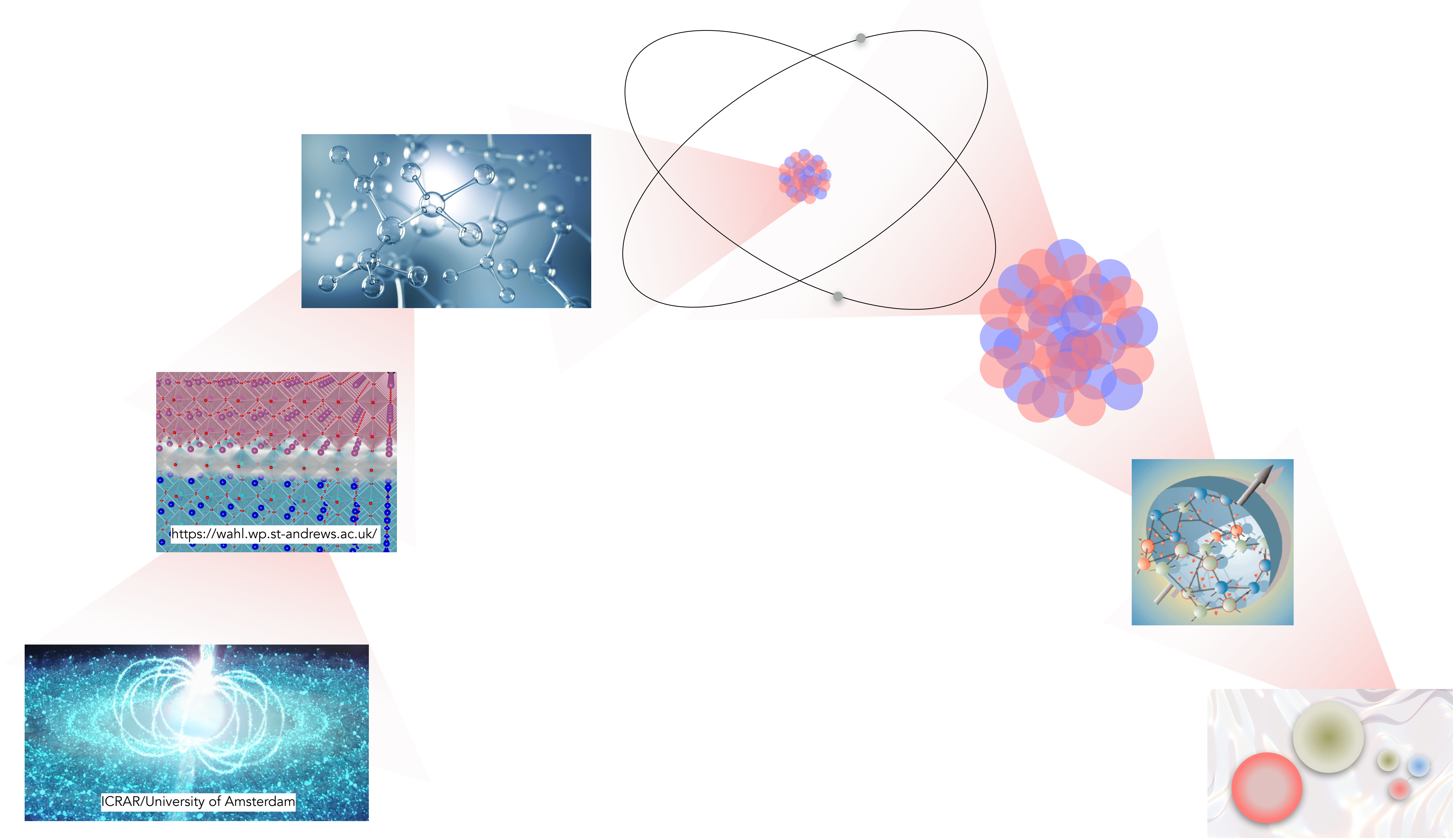
Credit: Shaw/Davoudi, University of Maryland

ECT* (virtual) workshop
Nuclear Physics meets Condensed Matter
July 19-21, 2021

TO PUT THINGS IN
THE CONTEXT…

QUANTUM SIMULATION FOR NUCLEAR PHYSICS: WHAT IT IMPLIES.

Quantum simulation amounts to leveraging a quantum system that can be controlled to study another quantum system that is more elusive, experimentally or computationally.



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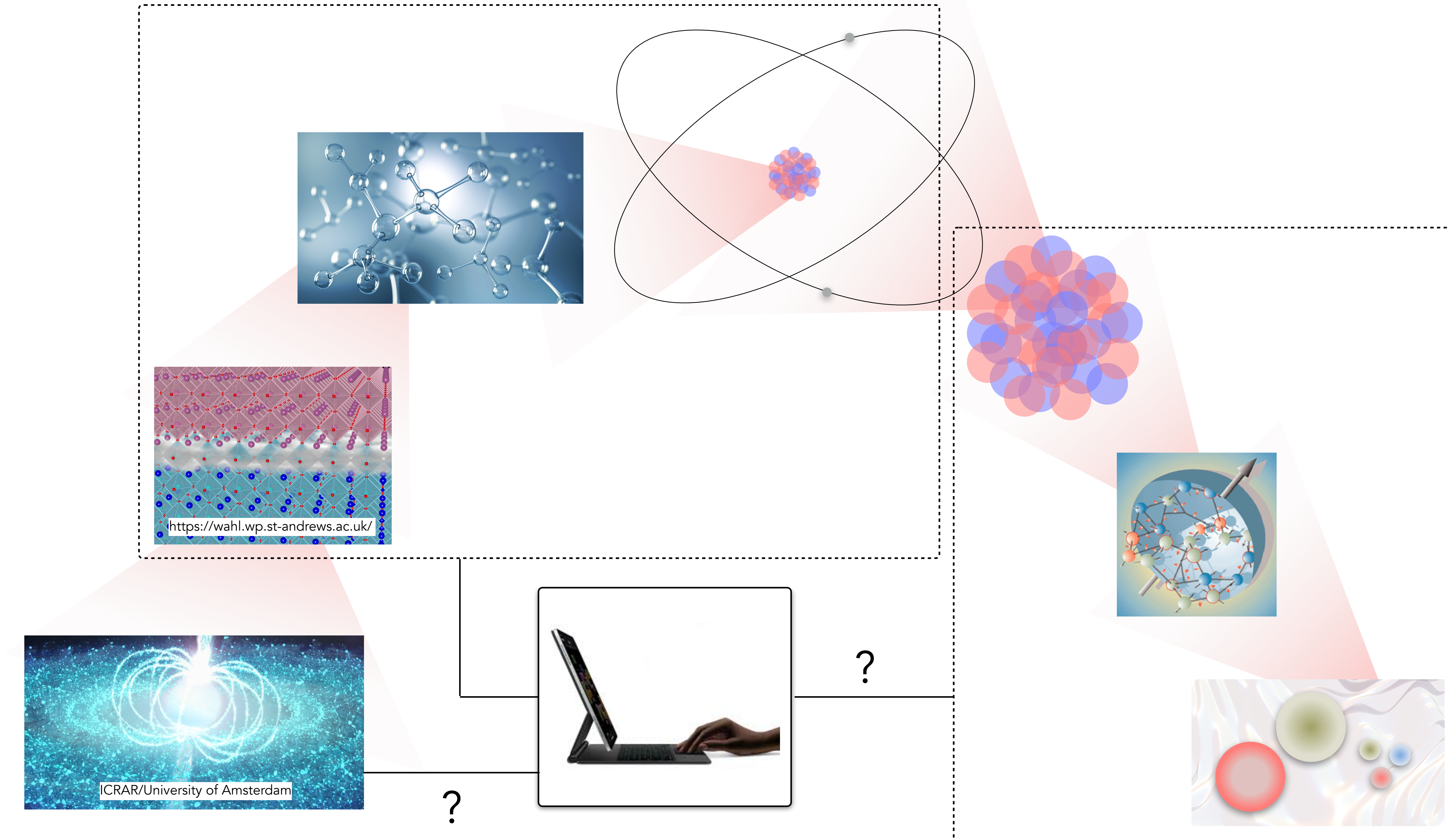

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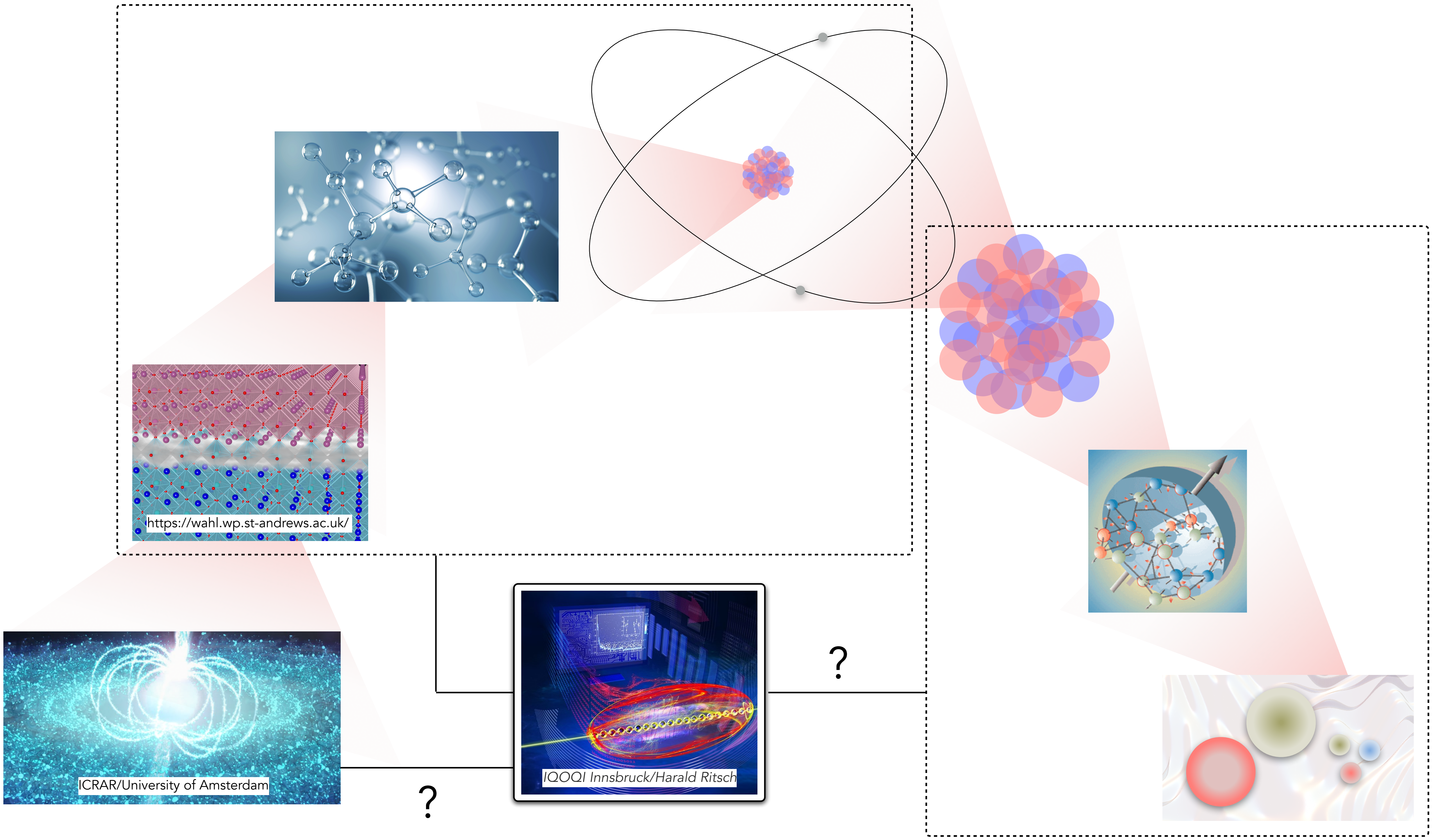
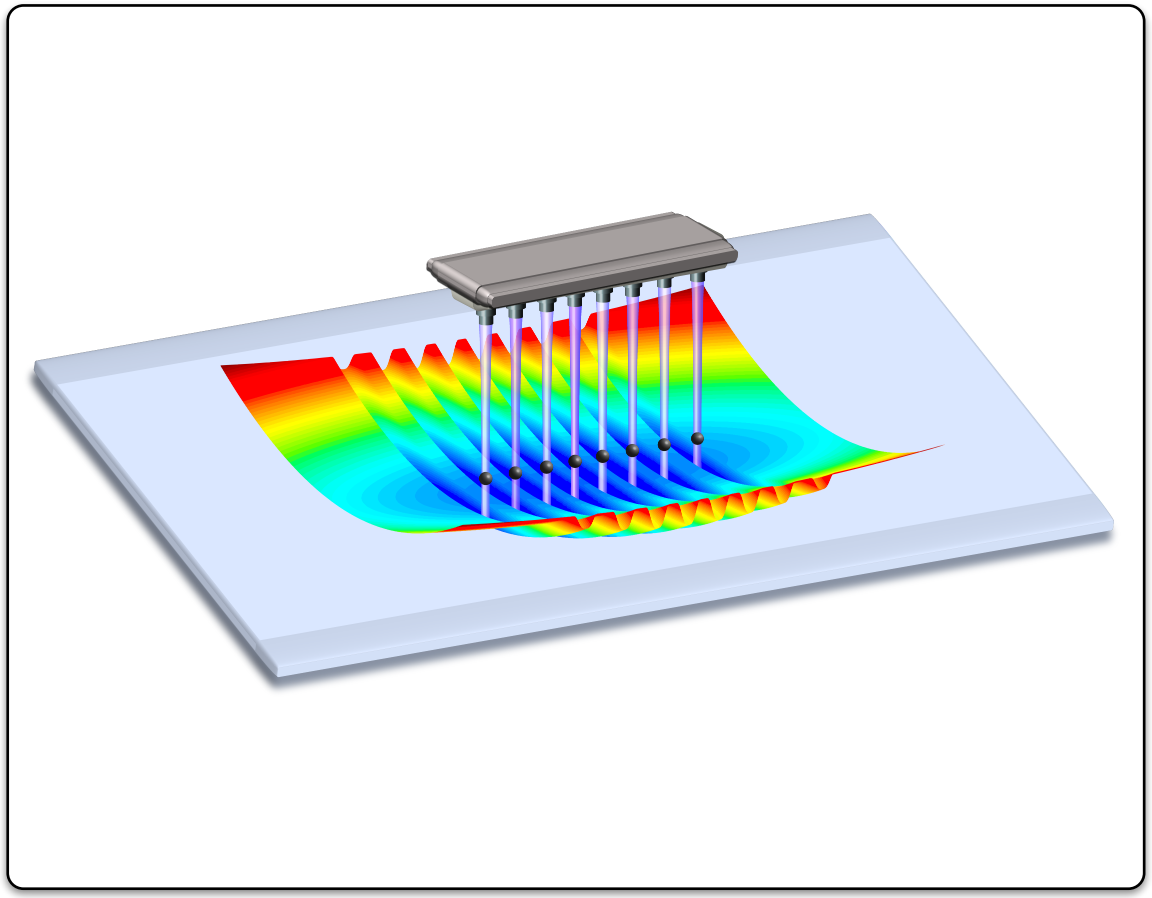
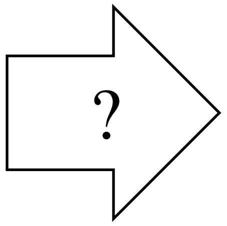


THE END GOAL FPR US: QUANTUM SIMULATION OF QUANTUM CHROMODYNAMICS!

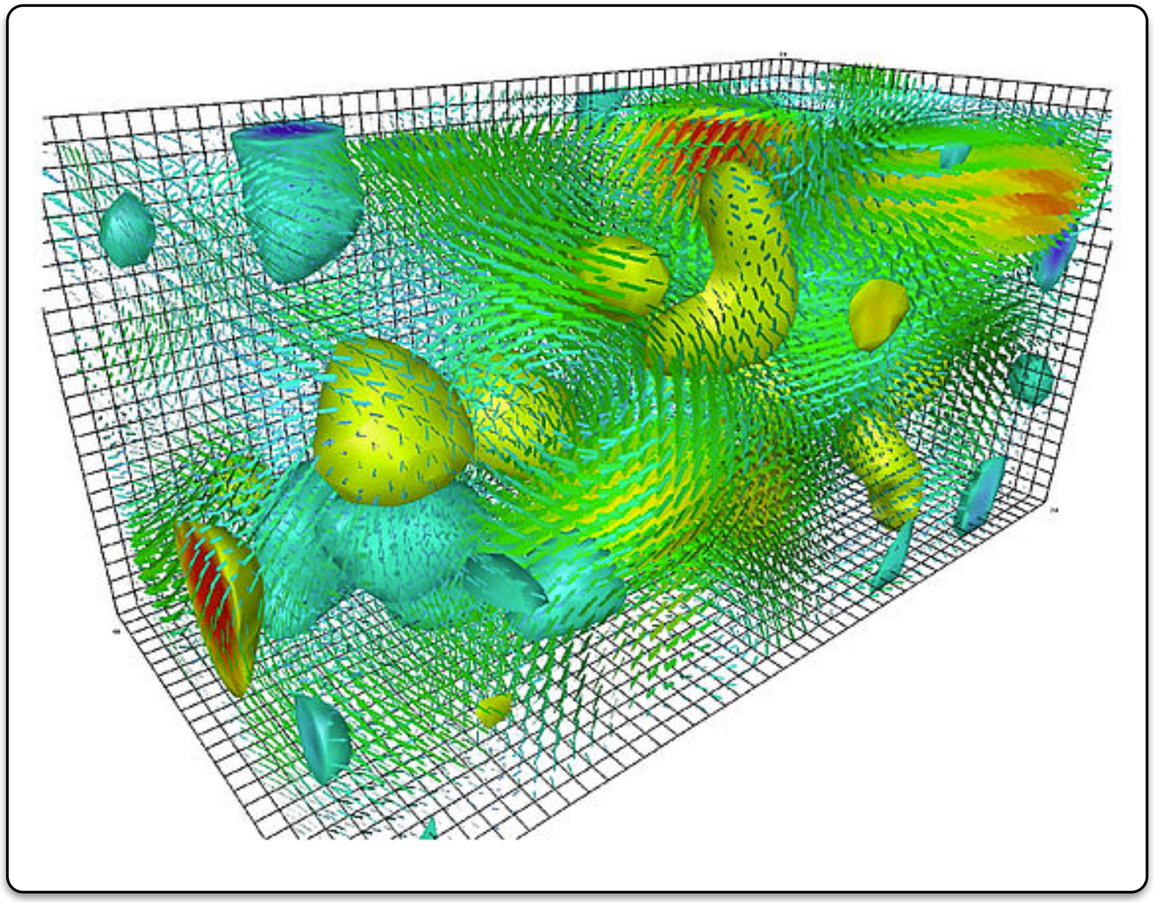
A controlled quantum system



CREDIT: ANDREW SHAW, UNIVERSITY OF MARYLAND



Strong-interaction physics



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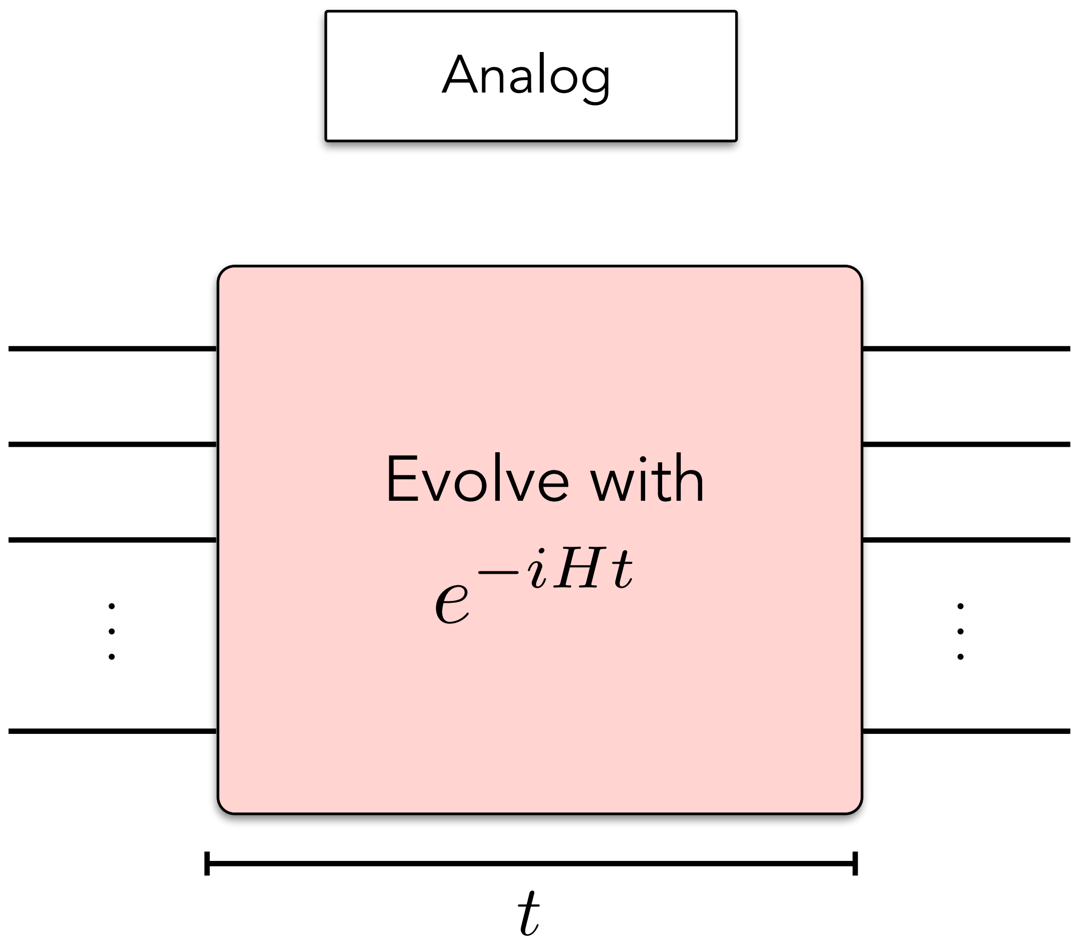
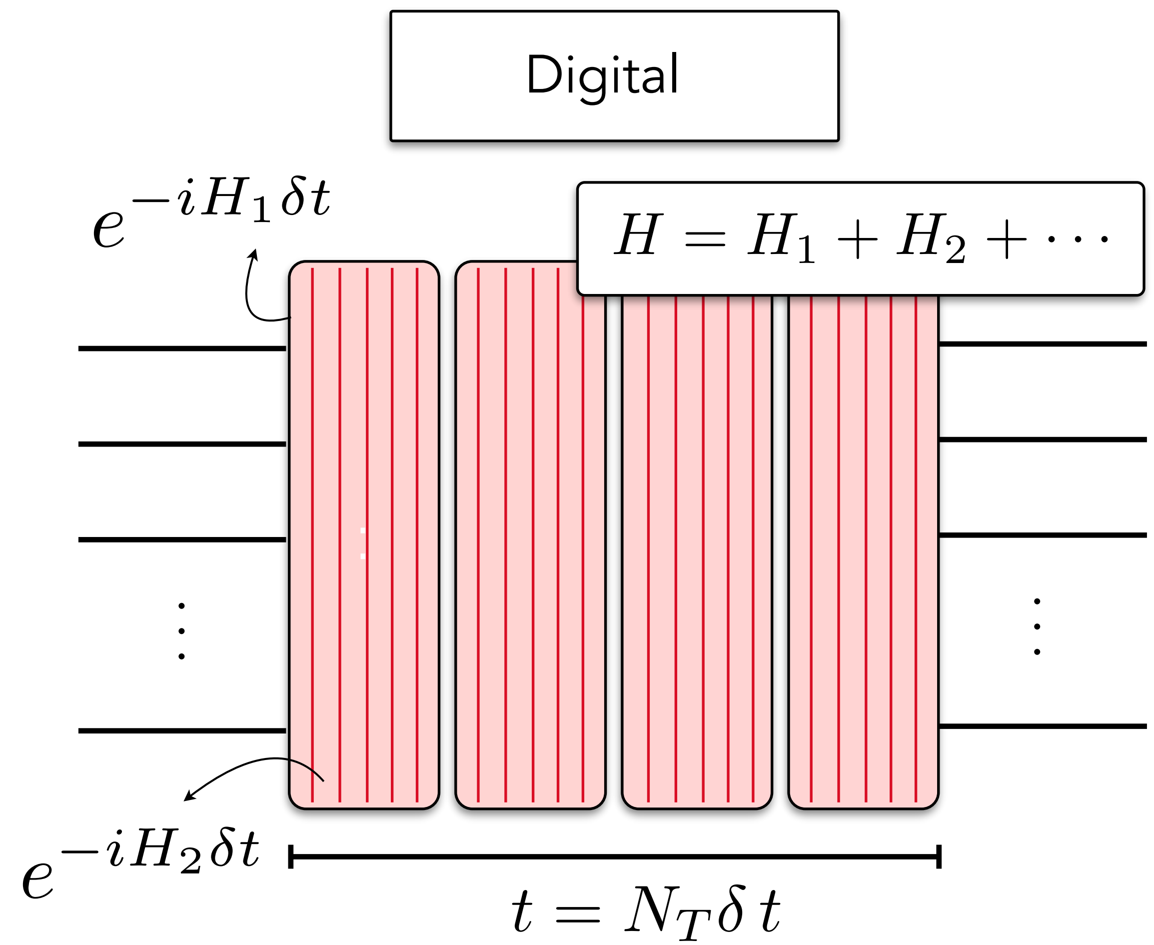
DIFFERENT APPROACHES TO QUANTUM SIMULATION





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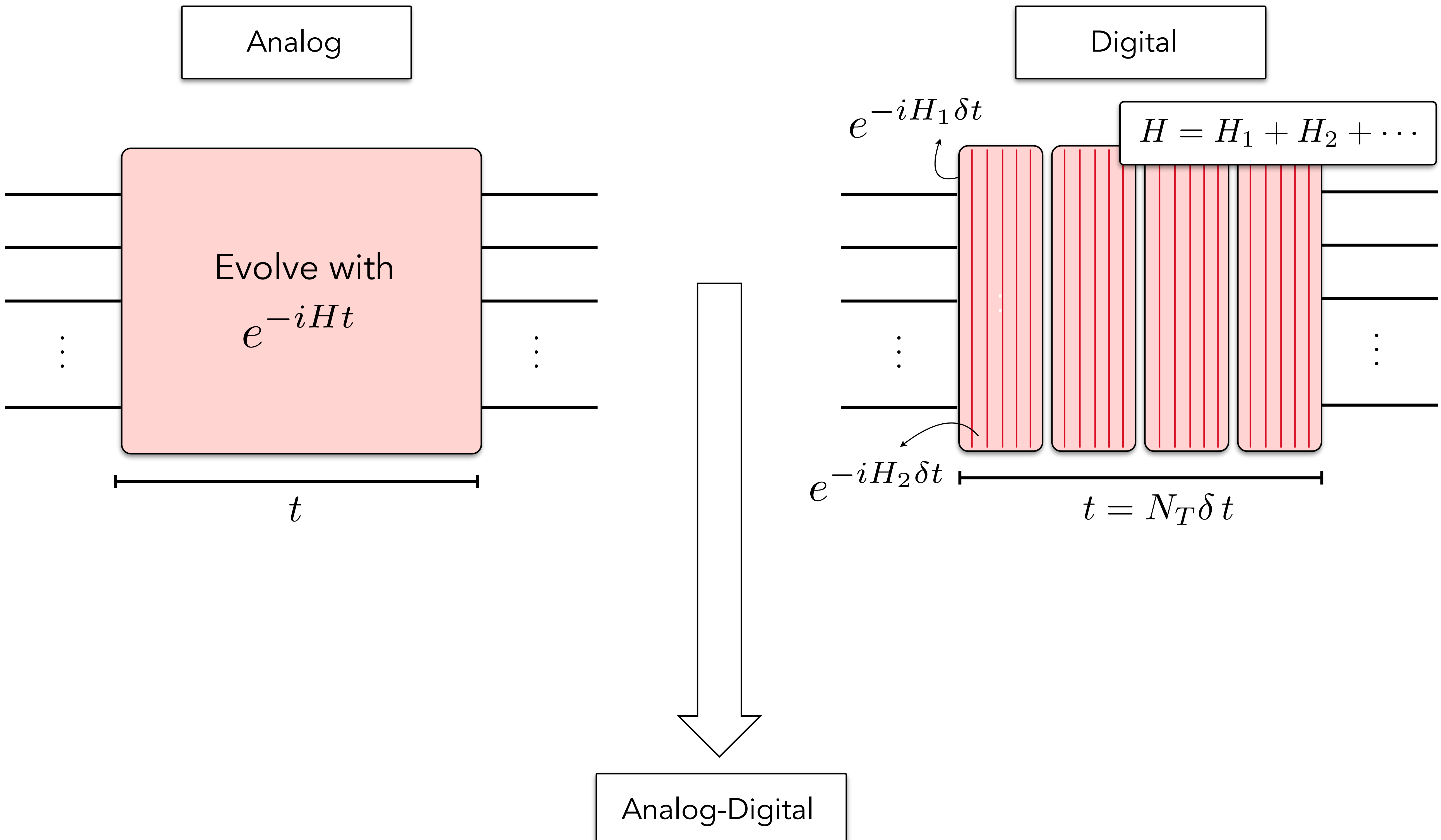
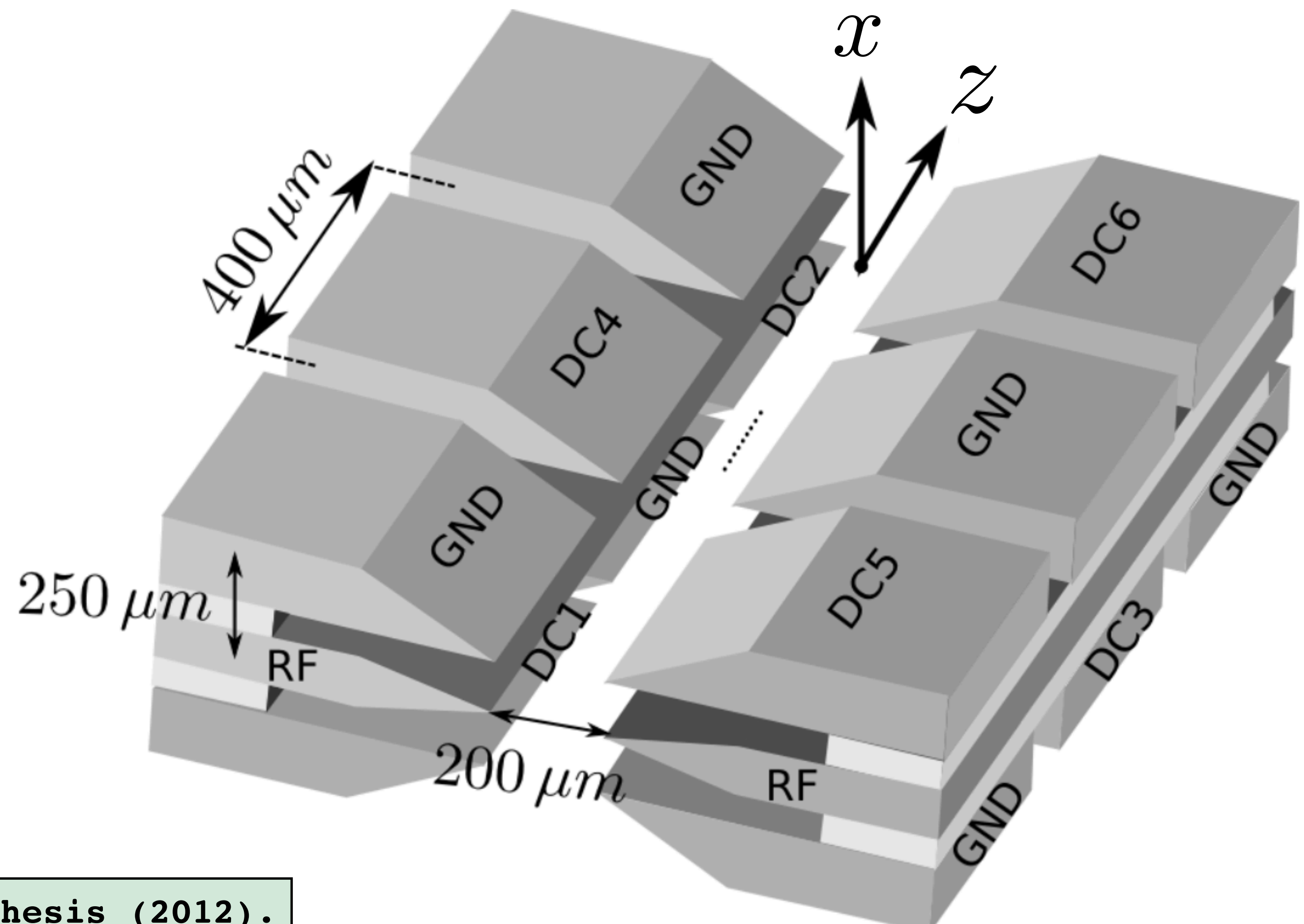
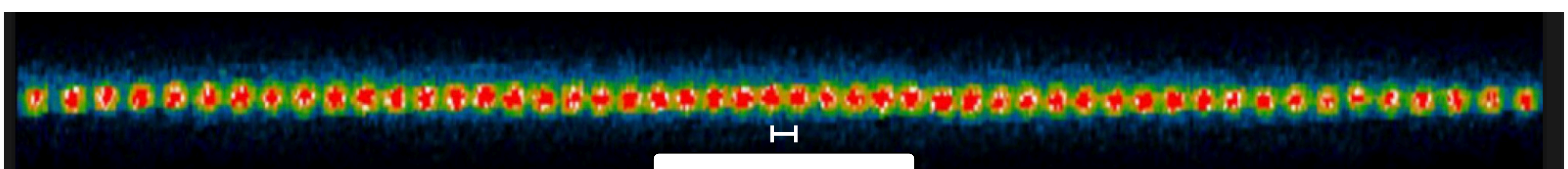


A QUICK TOUR TO THE UNDERLYING PHYSICS OF TRAPPED-ION SIMULATORS

A RADIO-FREQUENCY PAUL TRAP:



Islam, UMD PhD Thesis (2012).

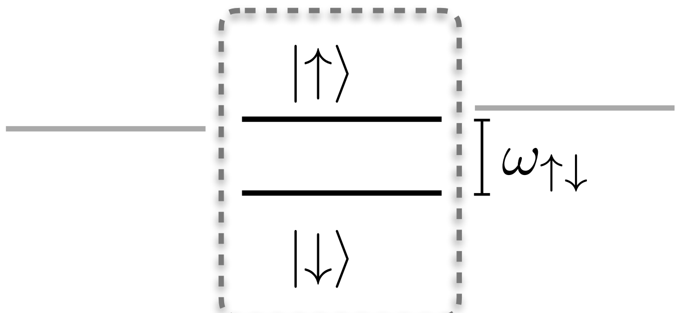


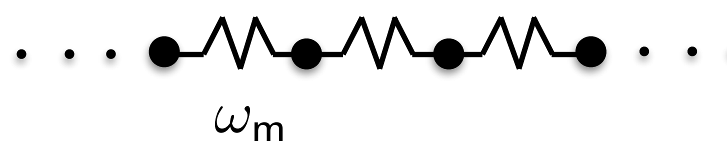
$\sim$ few μm

UMD, Monroe Lab.

ION-LASER HAMILTONIAN

$$H_{\text{free}} = \frac{\omega_{\uparrow\downarrow}}{2}\sum_{i=1}^{N}\sigma_i^z + \sum_{\mathsf{m}=1}^{3N}\omega_{\mathsf{m}}(a_{\mathsf{m}}^{\dagger}a_{\mathsf{m}} + \frac{1}{2})$$





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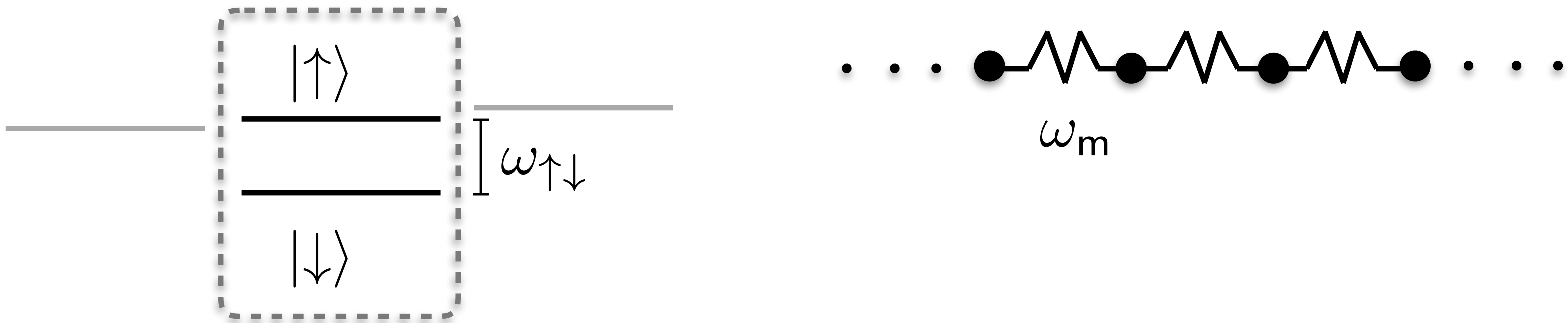
$$H_{\text{free}} = \frac{\omega_{\uparrow\downarrow}}{2}\sum_{i=1}^{N}\sigma_i^z + \sum_{\text{m}=1}^{3N}\omega_{\text{m}}\left(a_{\text{m}}^\dagger a_{\text{m}} + \frac{1}{2}\right)$$



$$H_{\text{int}} = -\mu\cdot\mathbf{E} = \sum_{i=1}^{N}\sum_{L=1}^{n_L}\Omega_L e^{-i\Delta\omega_L t + i\Delta\varphi_L + i\Delta\boldsymbol{k}_L\cdot\Delta\boldsymbol{r}_i}\left(\alpha_0\mathbb{I}_i + \alpha_1\sigma_i^x + \alpha_2\sigma_i^y + \alpha_3\sigma_i^z\right) + \text{h.c.}$$

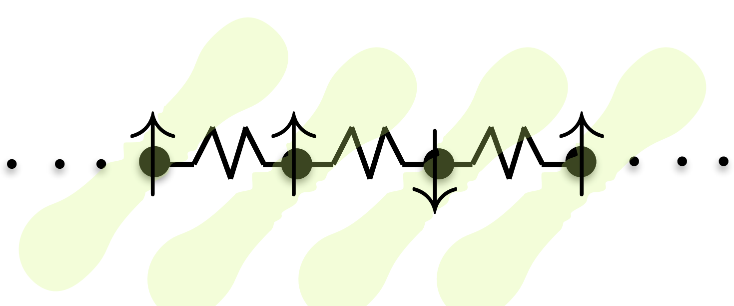
Rabi frequency (Ω_L)

Beatnote frequency ($\Delta\omega_L$)

Phase ($\Delta\varphi_L$)

Momentum vector ($\Delta\boldsymbol{k}_L$)

Depends on trap characteristics ($\Delta\boldsymbol{r}_i$)



Wineland et al, J.Res.Natl.Inst.Stand.Tech. 103 (1998) 259, Schneider et al, Rep. Prog. Phys. 75 024401 (2012)

INTERACTION-PICTURE HAMILTONIAN

$H_I =$

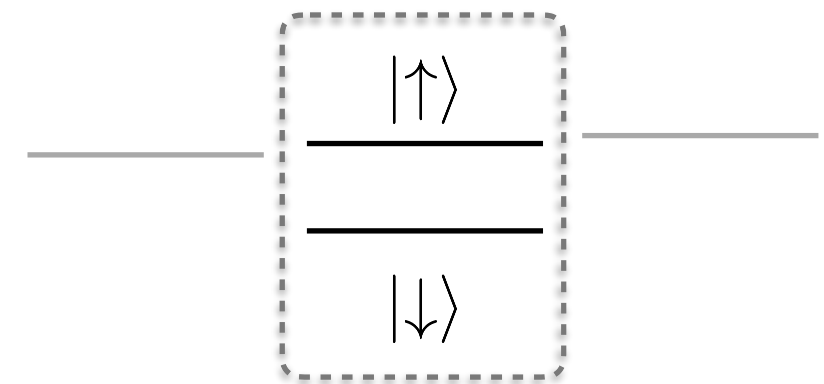
A time-dependent Hamiltonian

Wineland et al, J.Res.Natl.Inst.Stand.Tech. 103 (1998) 259, Schneider et al, Rep. Prog. Phys. 75 024401 (2012)

INTERACTION-PICTURE HAMILTONIAN

$$H_I = \qquad (\alpha_0 \mathbb{I} + \alpha_1 \sigma_x^{(i)} + \alpha_2 \sigma_y^{(i)} + \alpha_3 \sigma_z^{(i)}) \Big]$$

Acts on the internal states of each ion; the pseudo-spins

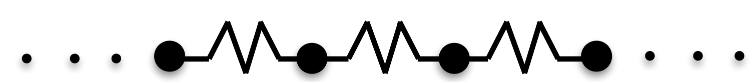


INTERACTION-PICTURE HAMILTONIAN

Wineland et al, J.Res.Natl.Inst.Stand.Tech. 103 (1998) 259, Schneider et al, Rep. Prog. Phys. 75 024401 (2012)

$$H_I = \left(e^{i\sum_{m=1}^{3N}\eta_m^{(i)}(a_m e^{-i\omega_m t}+a_m^\dagger e^{i\omega_m t})}\right)\left(\alpha_0\mathbb{I}+\alpha_1\sigma_x^{(i)}+\alpha_2\sigma_y^{(i)}+\alpha_3\sigma_z^{(i)}\right)\Big]$$

Quantize $e^{i\mathbf{k}^{(i)}\cdot\mathbf{x}^{(i)}}$ operator in terms of the normal modes of the motion of the chain.



Wineland et al, J.Res.Natl.Inst.Stand.Tech. 103 (1998) 259, Schneider et al, Rep. Prog. Phys. 75 024401 (2012)

INTERACTION-PICTURE HAMILTONIAN

$$H_I = \sum_{i=1}^{N} \left[\left(\sum_{I=1}^{n_L} \frac{1}{2} \Omega_I^{(i)} e^{-i(\omega_I - \omega_{\uparrow\downarrow})t + i\phi_I^{(i)}} \right) \left(e^{i \sum_{m=1}^{3N} \eta_m^{(i)} (a_m e^{-i\omega_m t} + a_m^\dagger e^{i\omega_m t})} \right) (\alpha_0 \mathbb{I} + \alpha_1 \sigma_x^{(i)} + \alpha_2 \sigma_y^{(i)} + \alpha_3 \sigma_z^{(i)}) \right]$$

Depends on intensity and phases of the lasers (global or individual).

INTERACTION-PICTURE HAMILTONIAN

Wineland et al, J.Res.Natl.Inst.Stand.Tech. 103 (1998) 259, Schneider et al, Rep. Prog. Phys. 75 024401 (2012)

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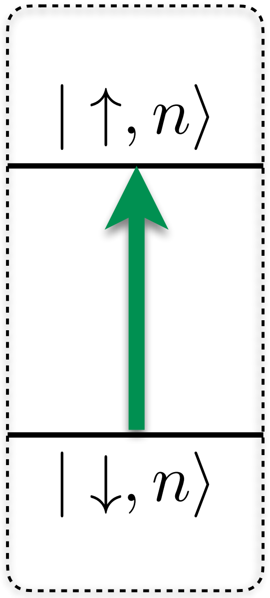
$\mathcal{O}(\eta^0)$:

$$H_{carr} = -\frac{\Omega}{2} \left(\sigma^+ e^{-i\phi} + \sigma^- e^{i\phi} \right)$$

One-qubit operations



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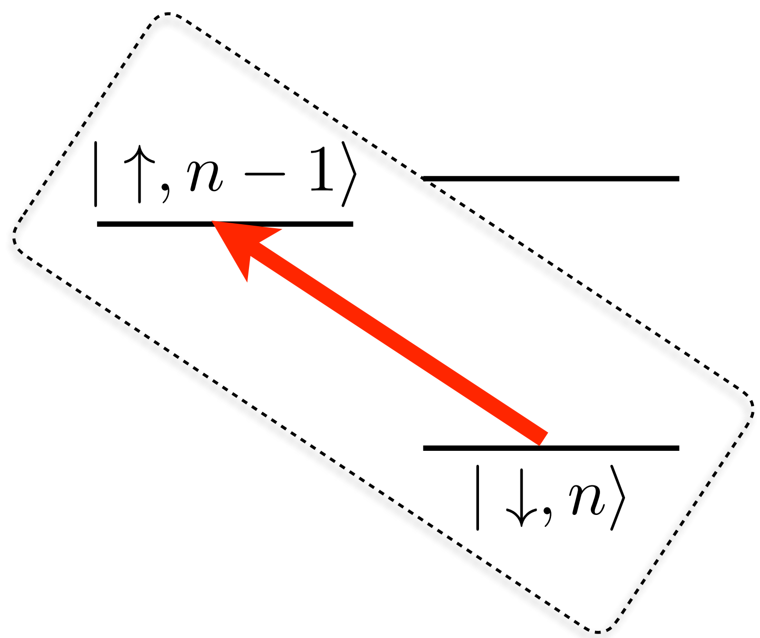
$\mathcal{O}(\eta^1)$:

$$H_{rsb} \approx \frac{i}{2} \eta \Omega \left[a^\dagger \sigma^- e^{i\phi} - a \sigma^+ e^{-i\phi} \right]$$

Spin-phonon transitions



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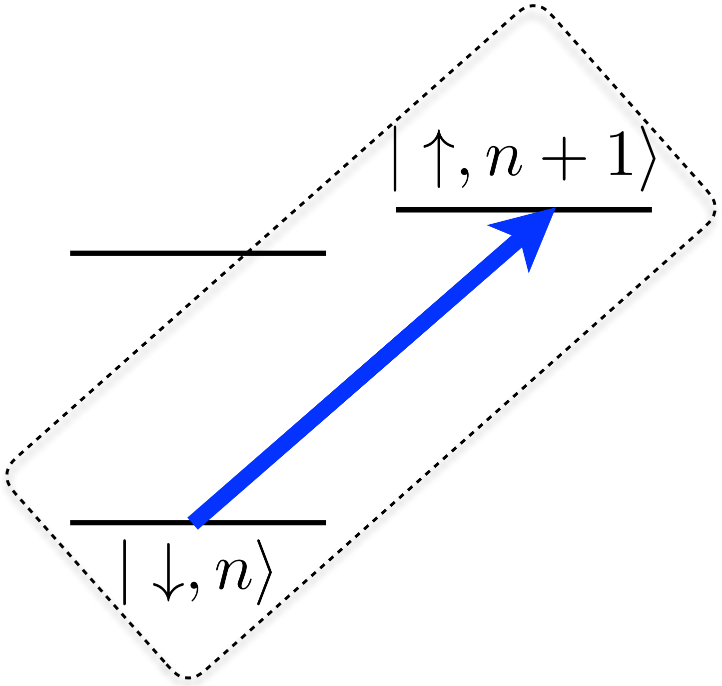
$$H_{bsb} \approx \frac{i}{2} \eta \Omega \left[a \sigma^- e^{i\phi} - a^\dagger \sigma^+ e^{-i\phi} \right]$$

Spin-phonon transitions



TWO-QUBIT ENTANGLING OPERATION

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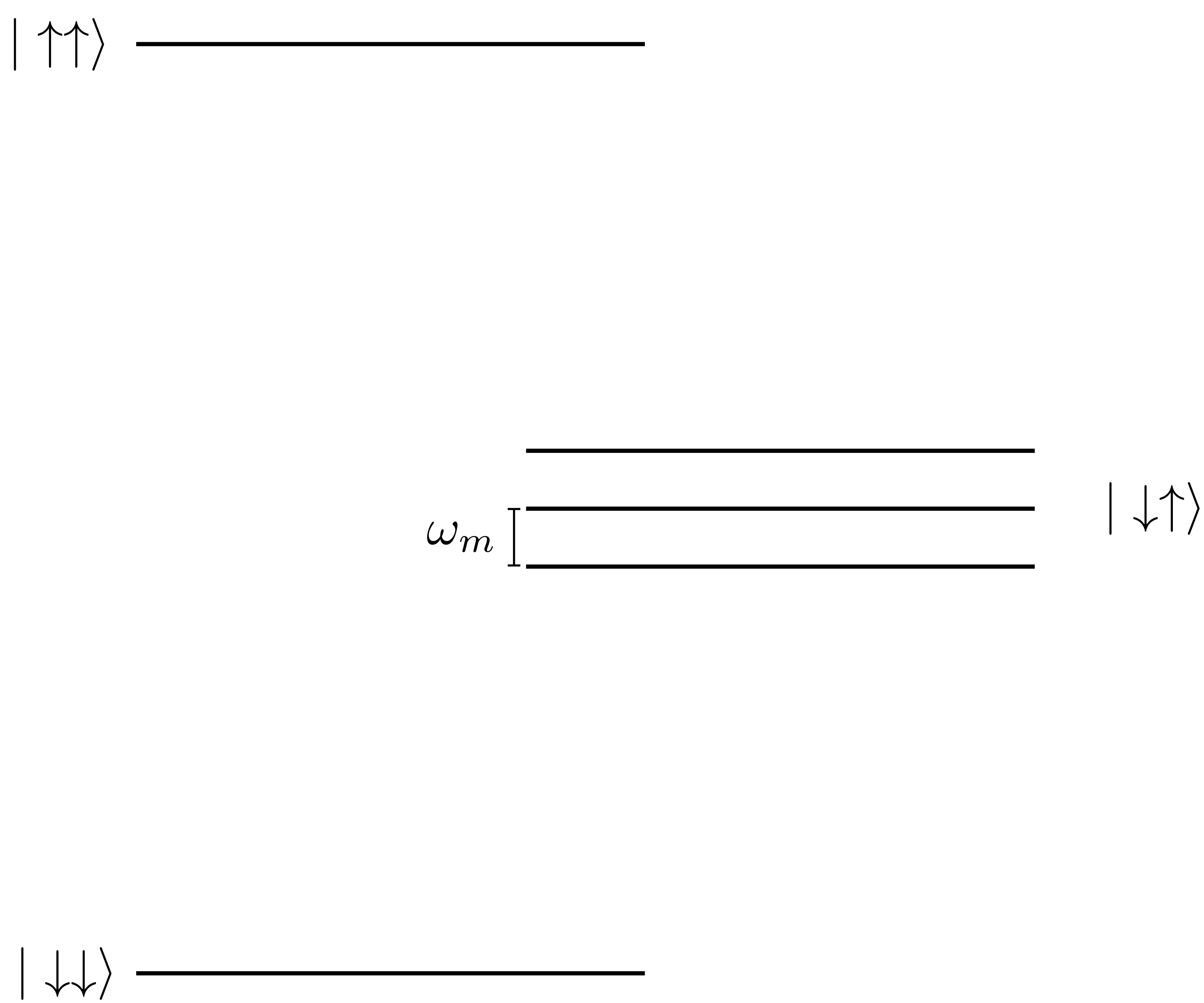
Adiabatic elimination technique and the use of sideband transitions effectively couples two spins



Cirac and Zoller, Phys.Rev.Lett.74, 4091 (1995),
Sorenson and Molmer, Phys. Rev. A 62, 022311 (2000)

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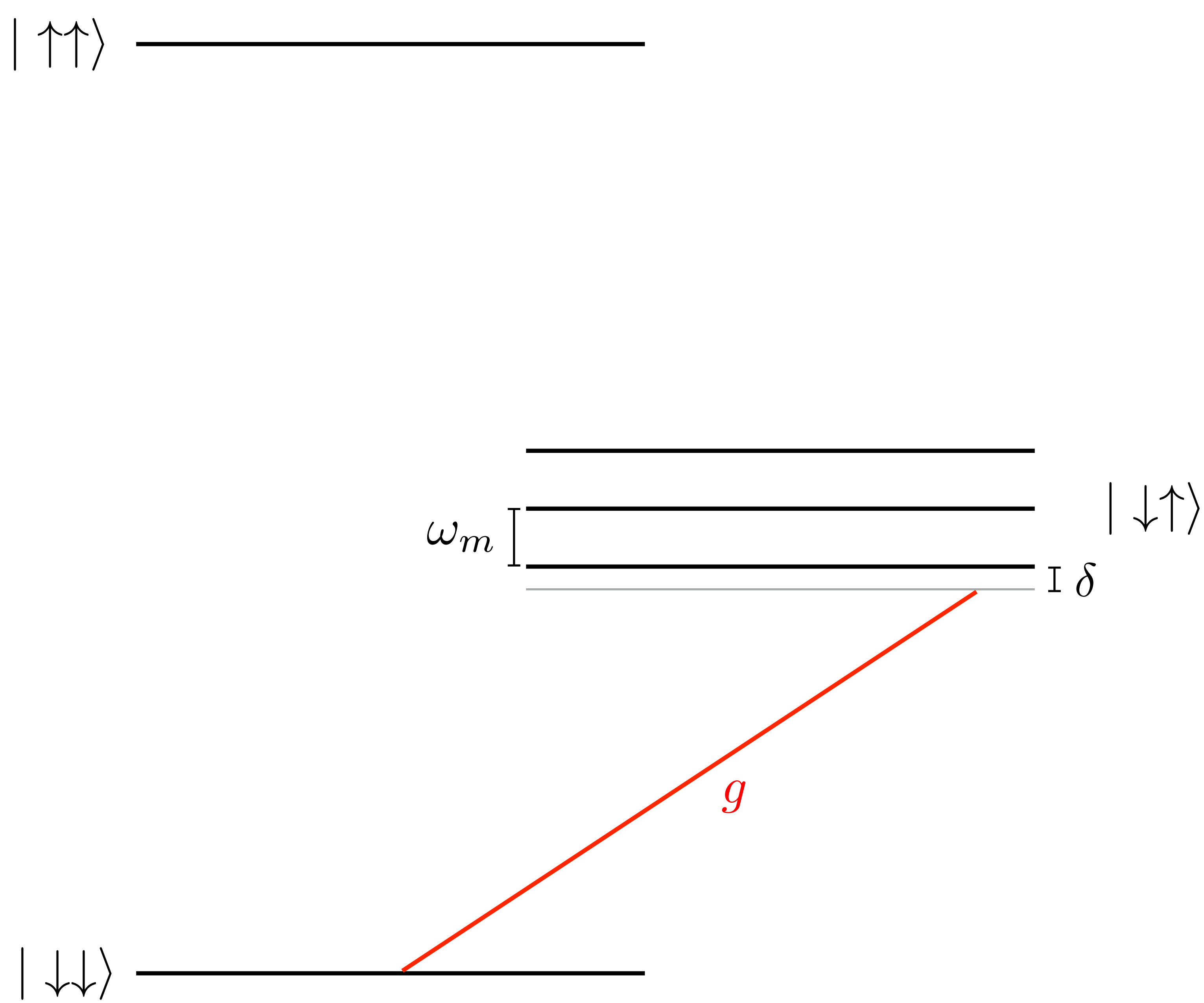

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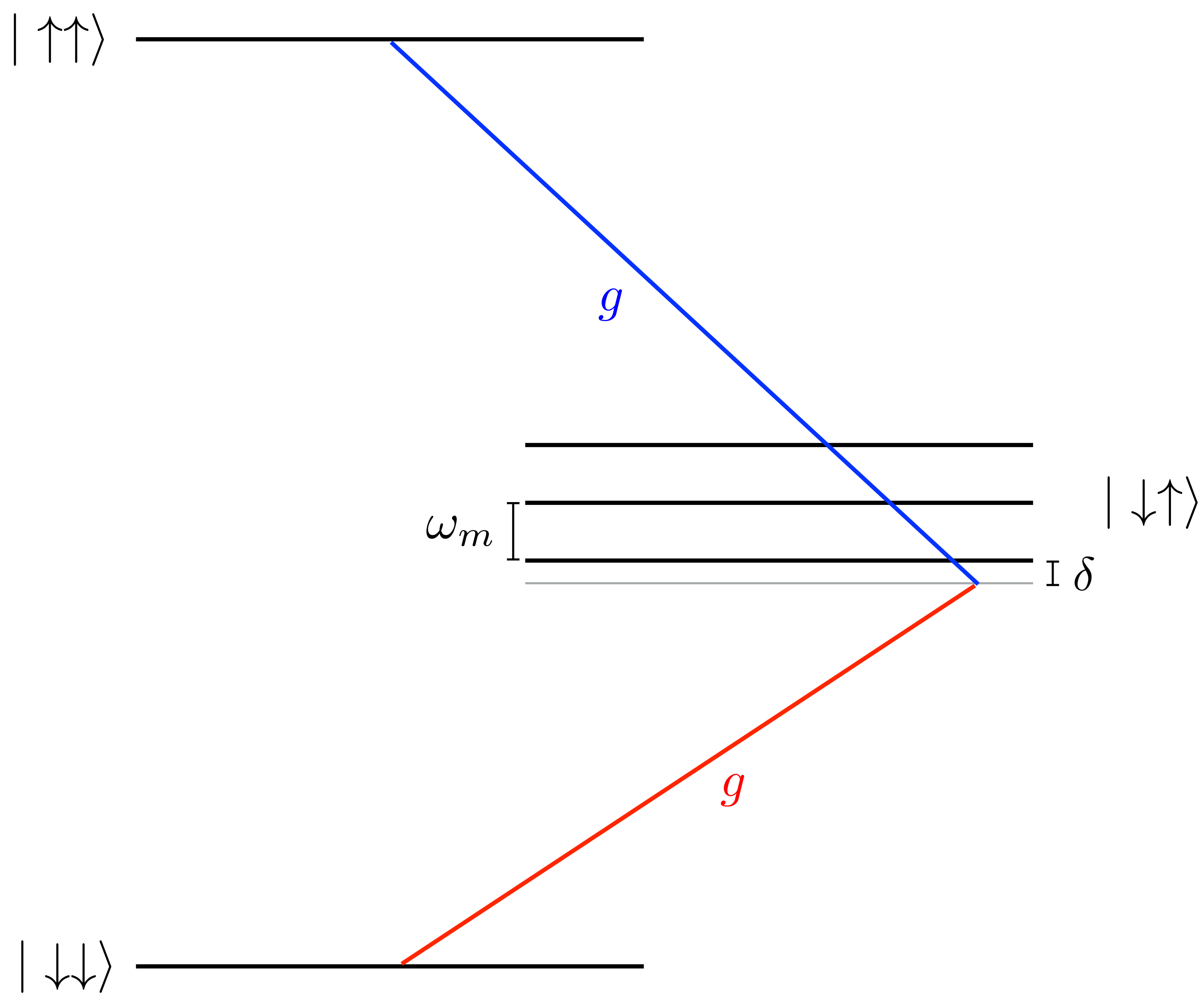

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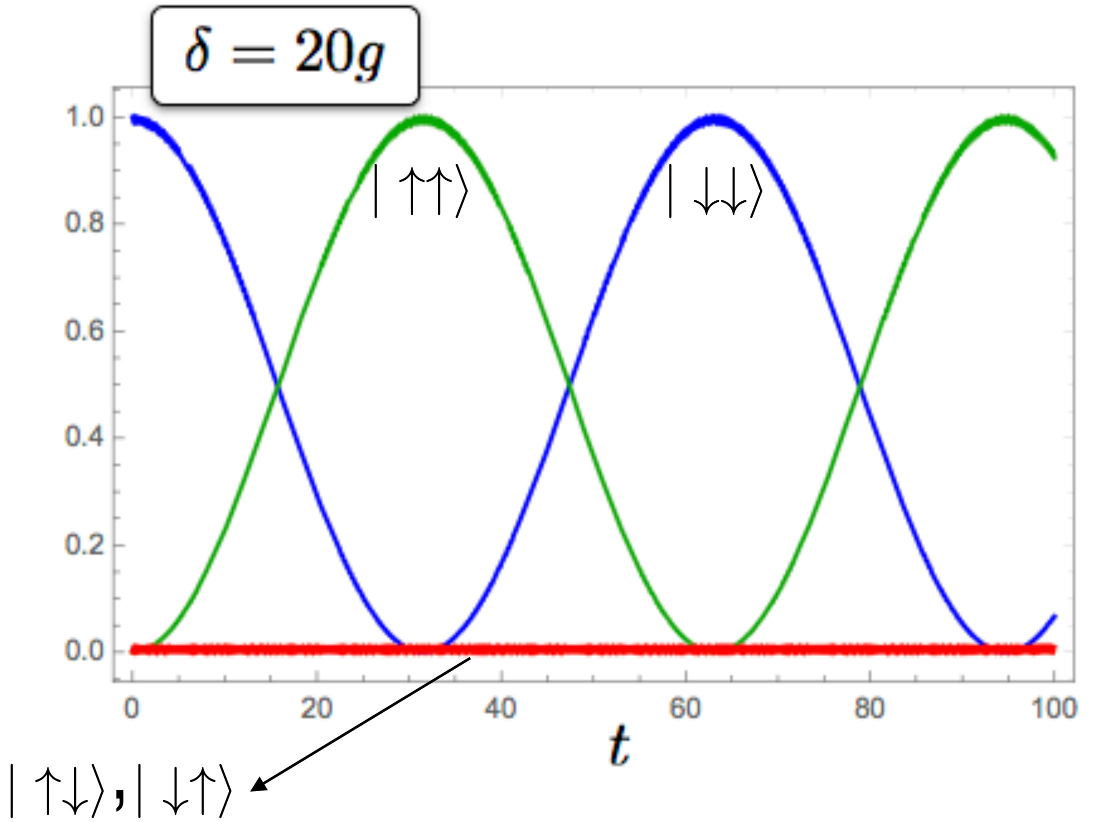

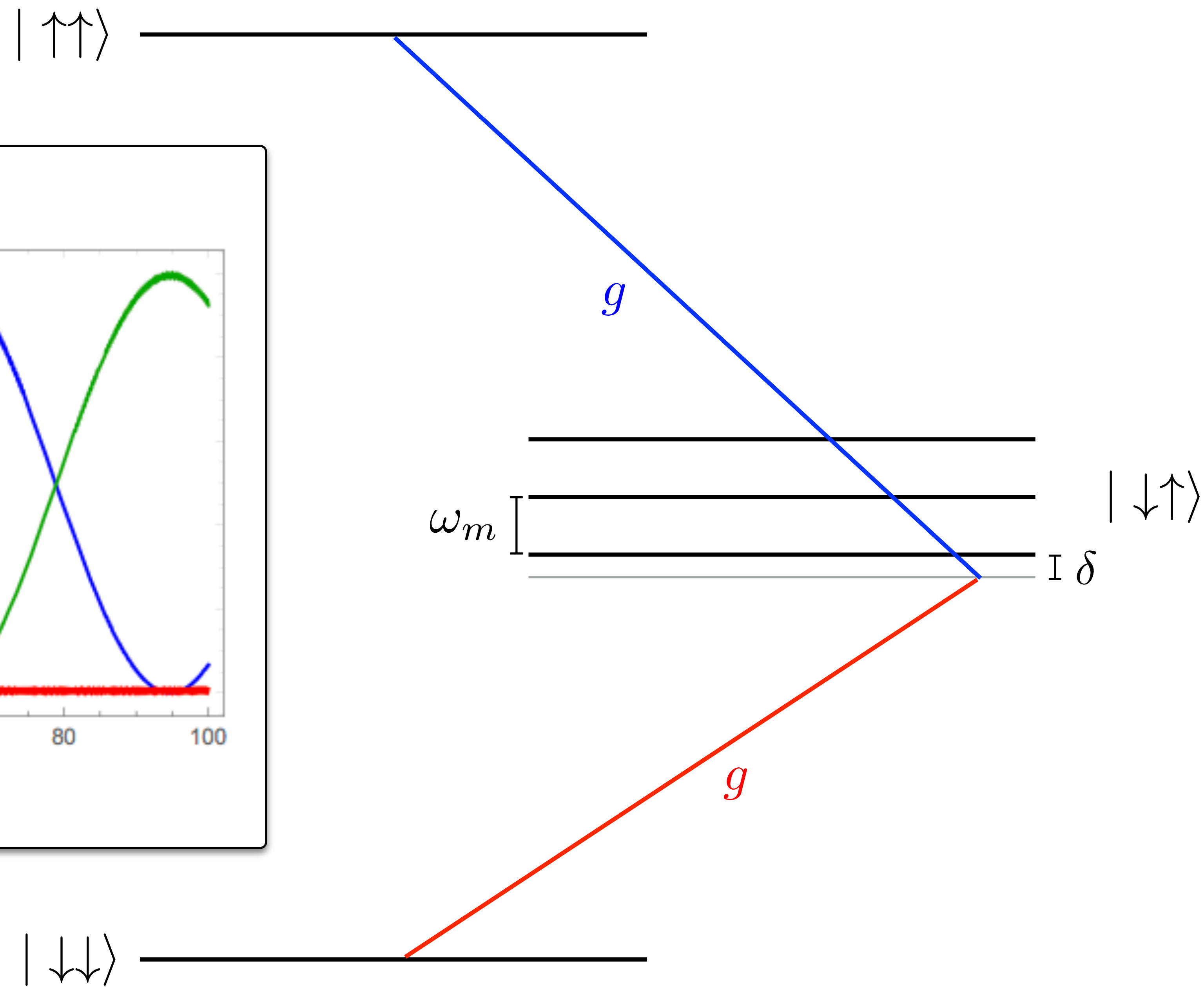

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TWO-QUBIT ENTANGLING OPERATION

Wineland et al, J.Res.Natl.Inst.Stand.Tech. 103 (1998) 259, Schneider et al, Rep. Prog. Phys. 75 024401 (2012)

The physics is simple, and is derived from a Magnus expansion of the time-evolution operator:

$$U(t,0) = \mathcal{T} e^{-i\int_0^t H_I(t_1)dt_1}$$

$$= e^{-i\int_0^t dt_1 H_I(t_1) - \frac{1}{2}\int_0^t dt_2 \int_0^{t_2} dt_1 [H_I(t_2), H_I(t_1)] + \cdots}$$

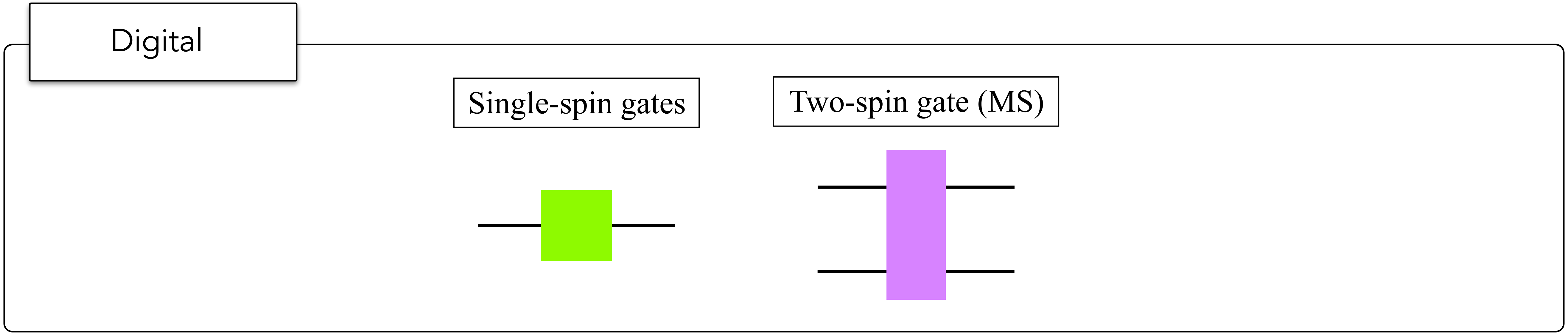
$$\approx e^{-iH_{\mathrm{eff}}t}$$

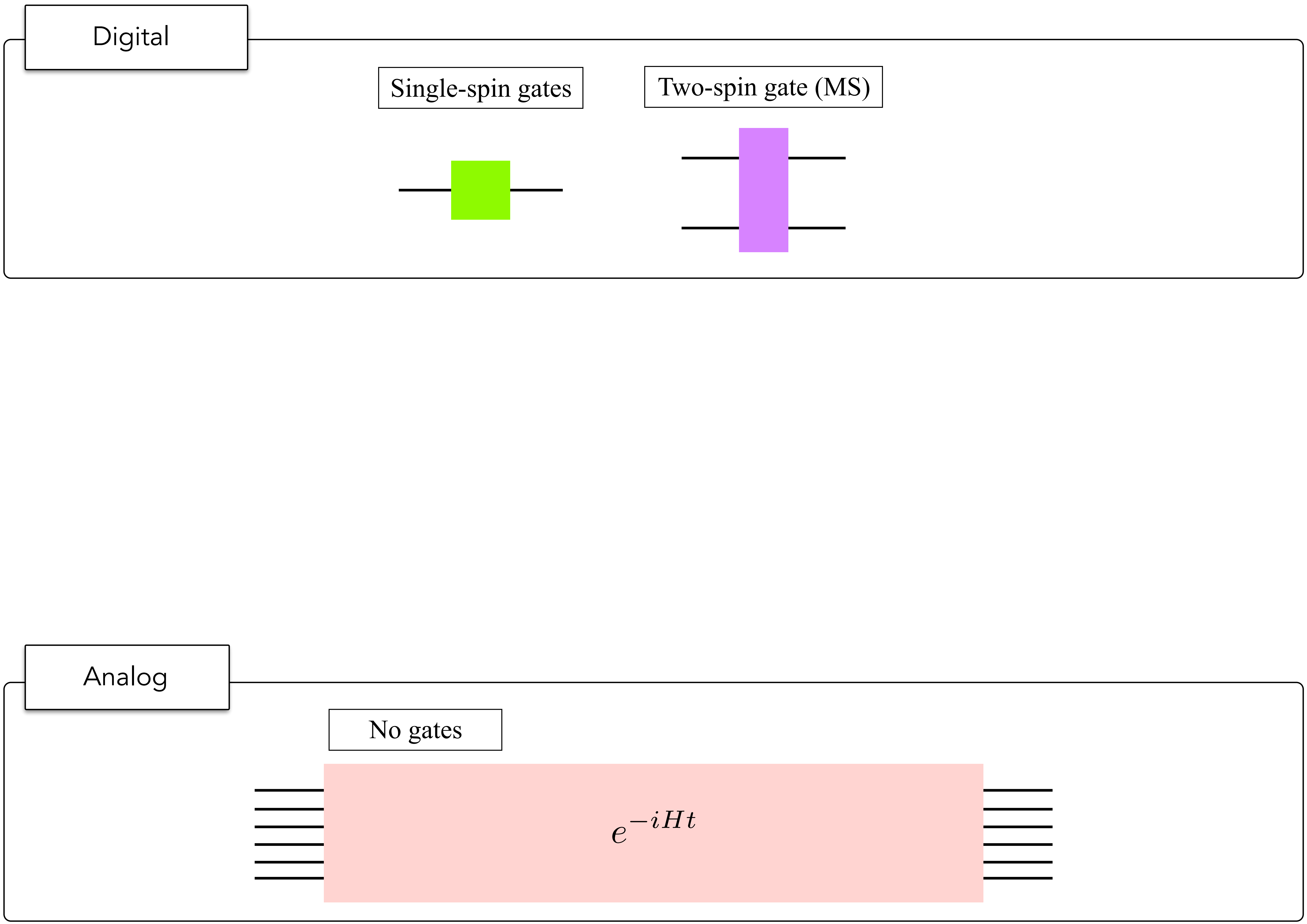
Only an approximation (off-resonant terms, lower and higher order terms, etc. still present in dynamics but can be suppressed).

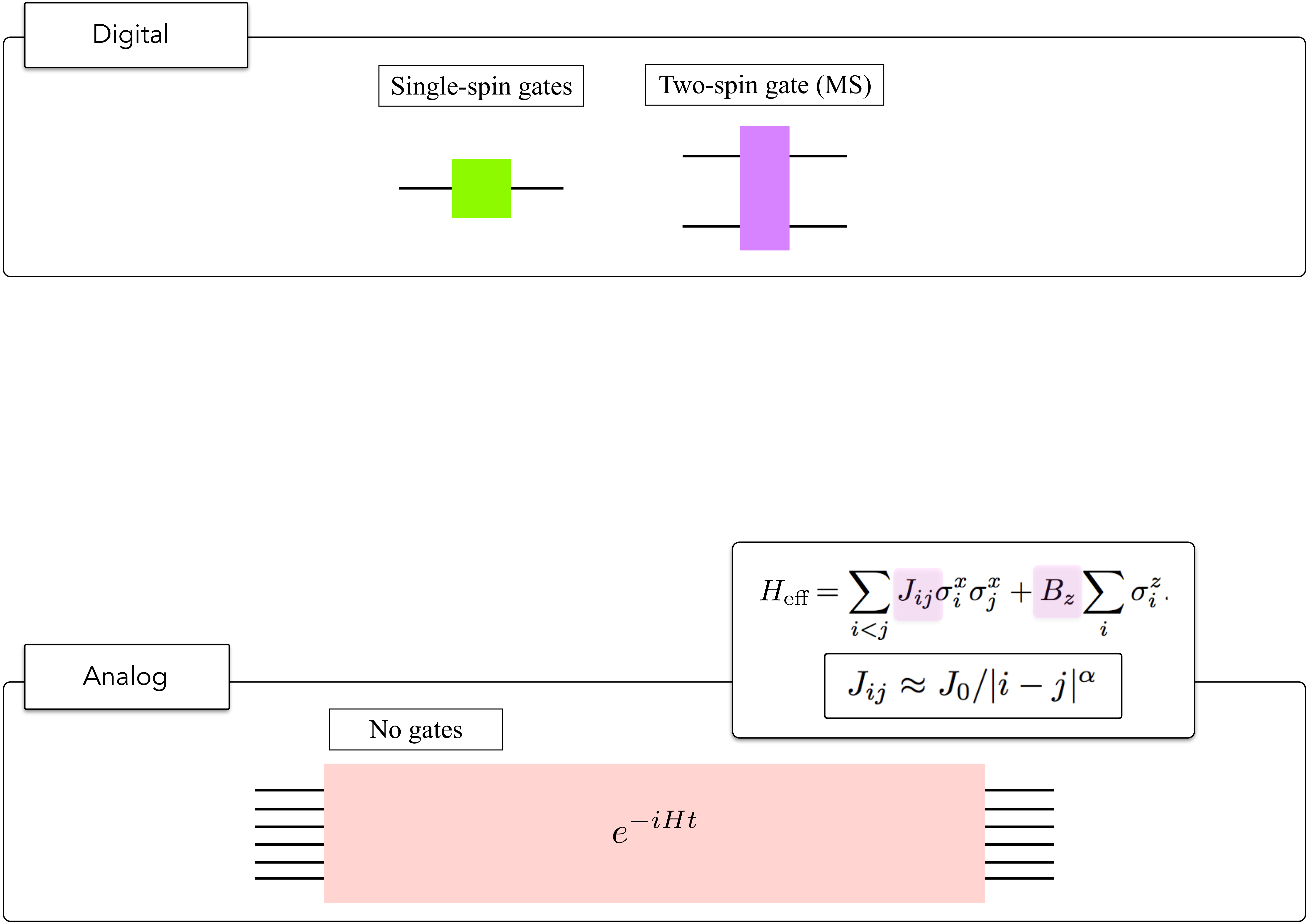
An effective Ising Hamiltonian $H_{\mathrm{eff}} \propto \sigma_x^{(i)} \otimes \sigma_x^{(j)}$ requires:

$$\eta\Omega/\delta \ll 1$$

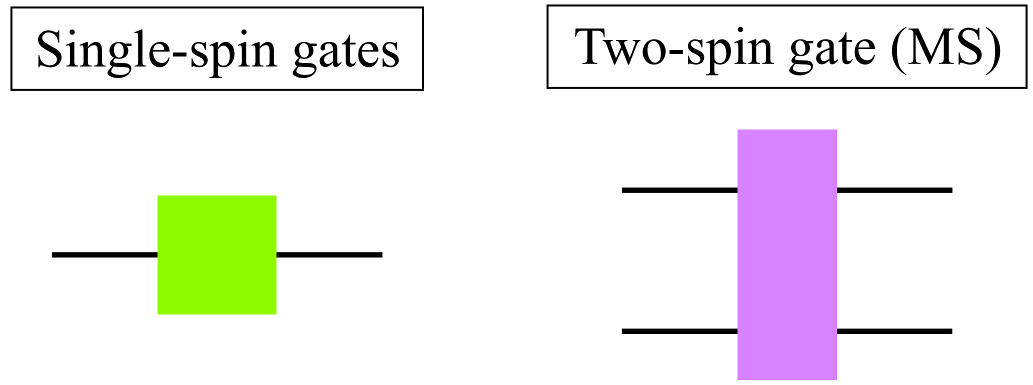
DIGITAL, ANALOG, AND HYBRID MODES OF TRAPPED-ION SIMULATORS


Digital
Single-spin gates
Two-spin gate (MS)


Digital
Single-spin gates
Two-spin gate (MS)
Analog
No gates
e^{-iHt}


Digital
Single-spin gates
Two-spin gate (MS)
Analog
No gates
$H_{\mathrm{eff}} = \sum_{i<j} J_{ij}\sigma_i^x\sigma_j^x + B_z \sum_i \sigma_i^z.$
$J_{ij} \approx J_0/|i-j|^\alpha$
e^{-iHt}

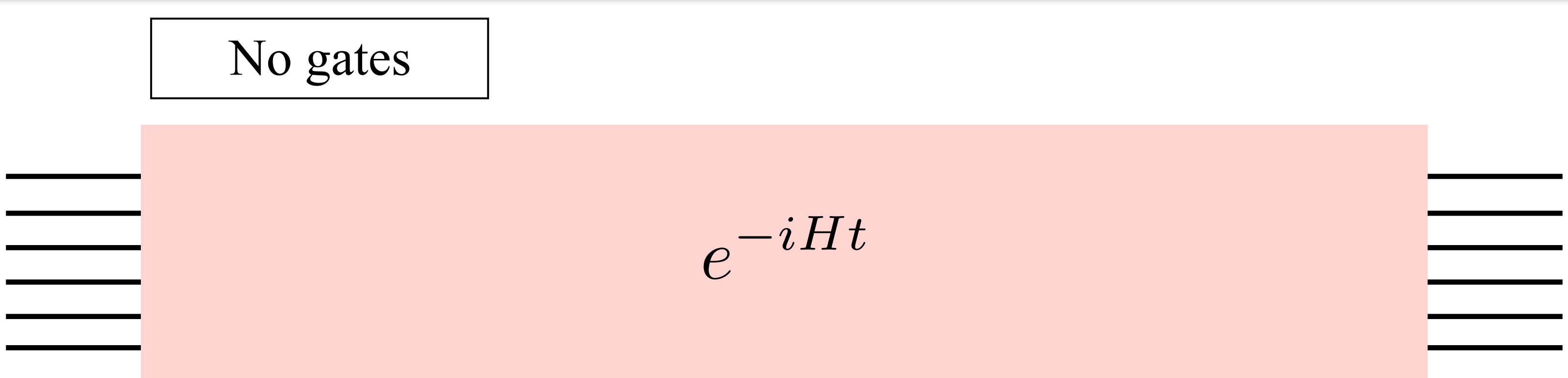
Digital



ZD, Hafezi, Monroe, Pagano, Seif, Shaw, Phys. Rev. Research, 2, 023015 (2020), arXiv: 1908.03210 [quant-ph].

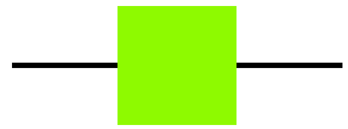
$$H_{\text{eff}} = \sum_{\substack{i,j \\ j<i}} \left[J_{i,j}^{(xx)} \sigma_x^{(i)} \otimes \sigma_x^{(j)} + J_{i,j}^{(yy)} \sigma_y^{(i)} \otimes \sigma_y^{(j)} + J_{i,j}^{(zz)} \sigma_z^{(i)} \otimes \sigma_z^{(j)} \right] - \frac{1}{2} \sum_{i=1}^{N} B_z^{(i)} \sigma_z^{(i)}$$

Analog

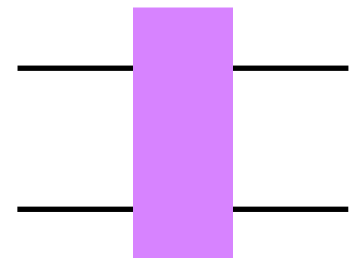


Digital

Single-spin gates



Two-spin gate (MS)



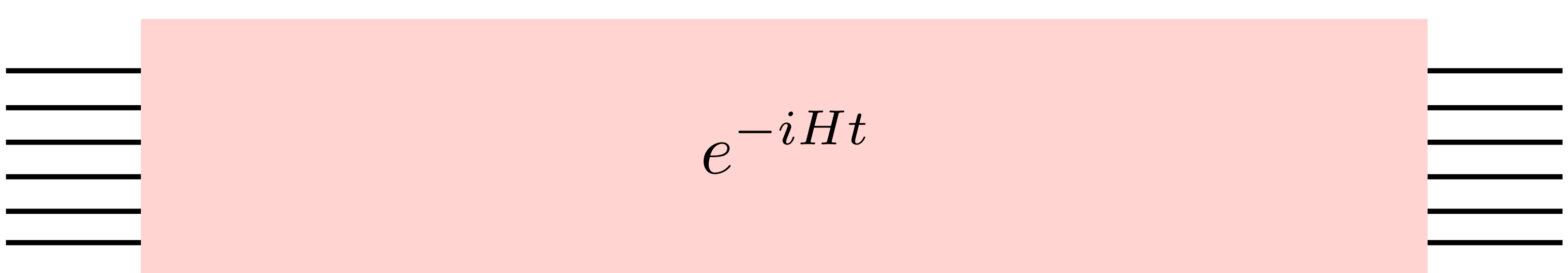
Andrade, ZD, Grass, Hafezi, Pagano, Seif, arXiv: 2107.xxxx [quant-ph].

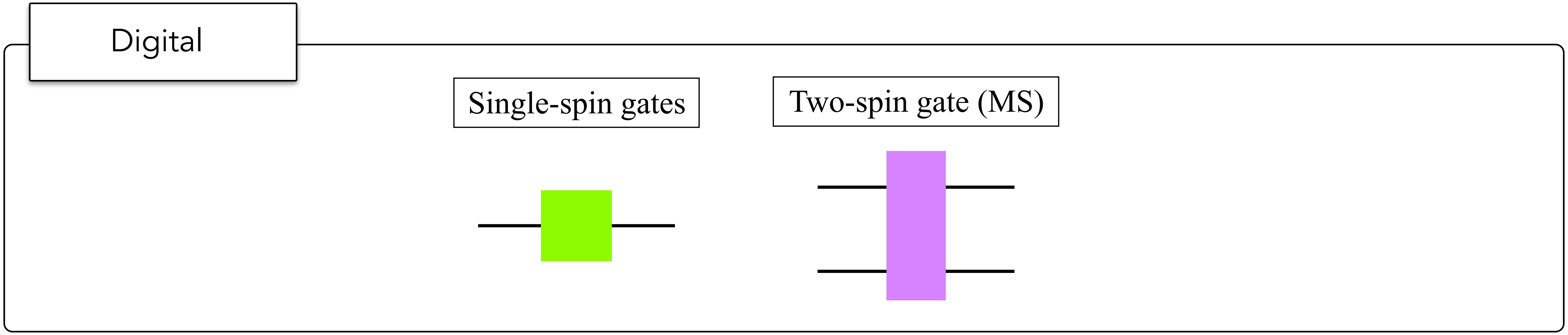
See also: Bermudez et al, Pays.Rev.A79, 060303 R (2009).

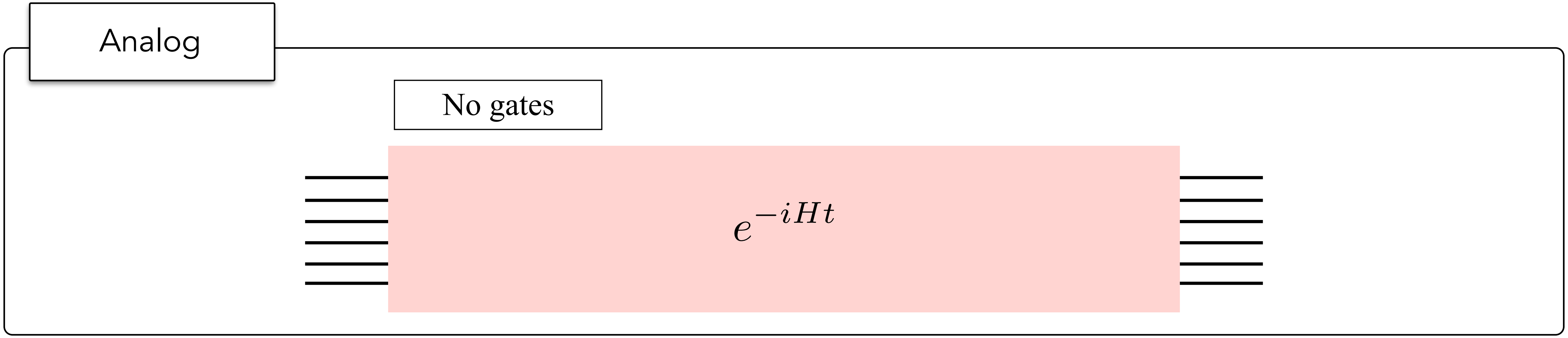
Analog

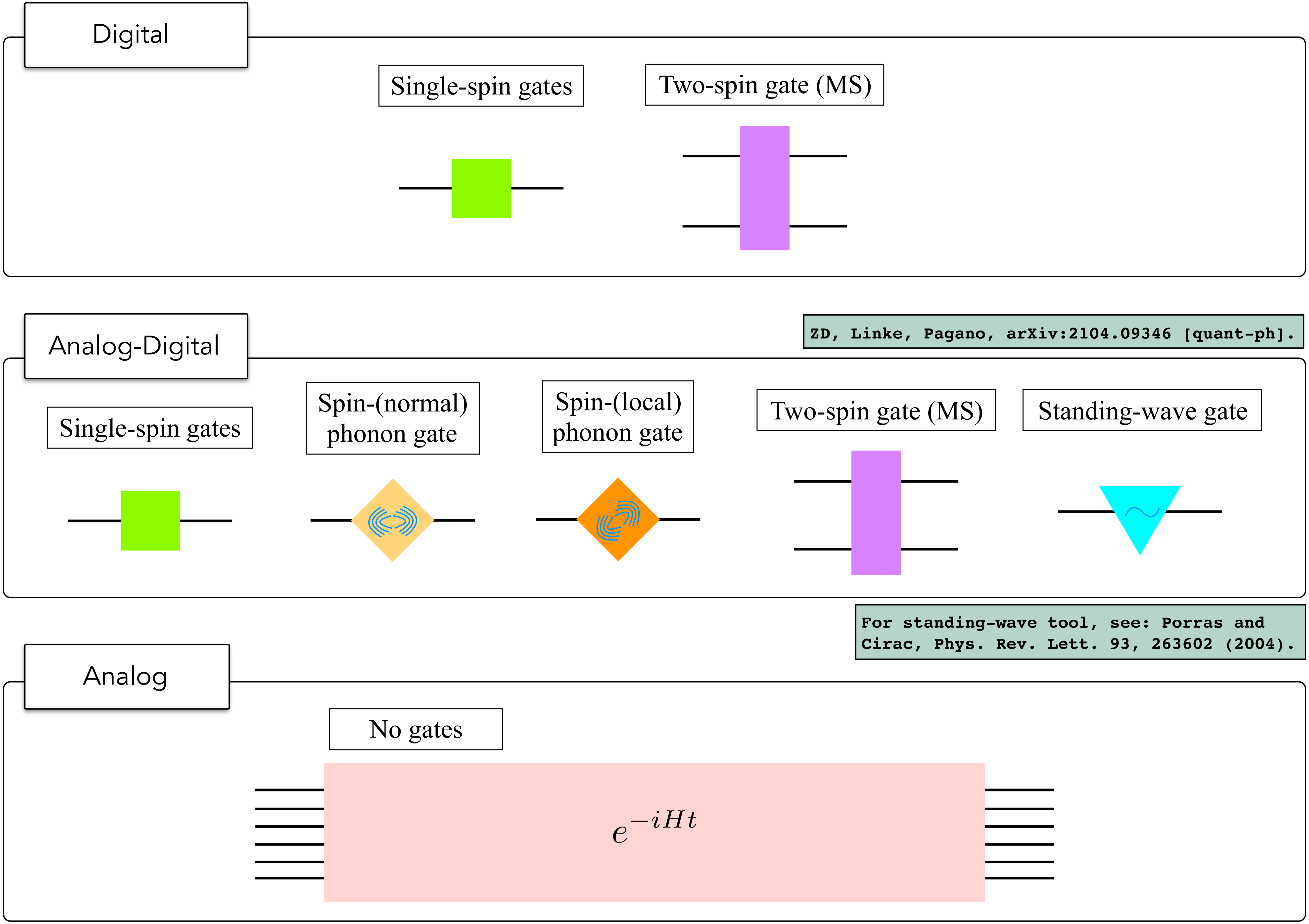
$$H_{\mathrm{eff}} = \sum_i J_i^{(\sigma)} \sigma_z^{(i)} + \sum_{i,j} J_{i,j}^{(\sigma\sigma)} \sigma_+^{(i)} \otimes \sigma_+^{(j)} + \sum_{i,j,k} J_{i,j,k}^{(\sigma\sigma\sigma)} \sigma_+^{(i)} \otimes \sigma_+^{(j)} \otimes \sigma_+^{(k)} + \mathrm{h.c.}$$

No gates




Digital
Single-spin gates
Two-spin gate (MS)


Analog
No gates
e^{-iHt}

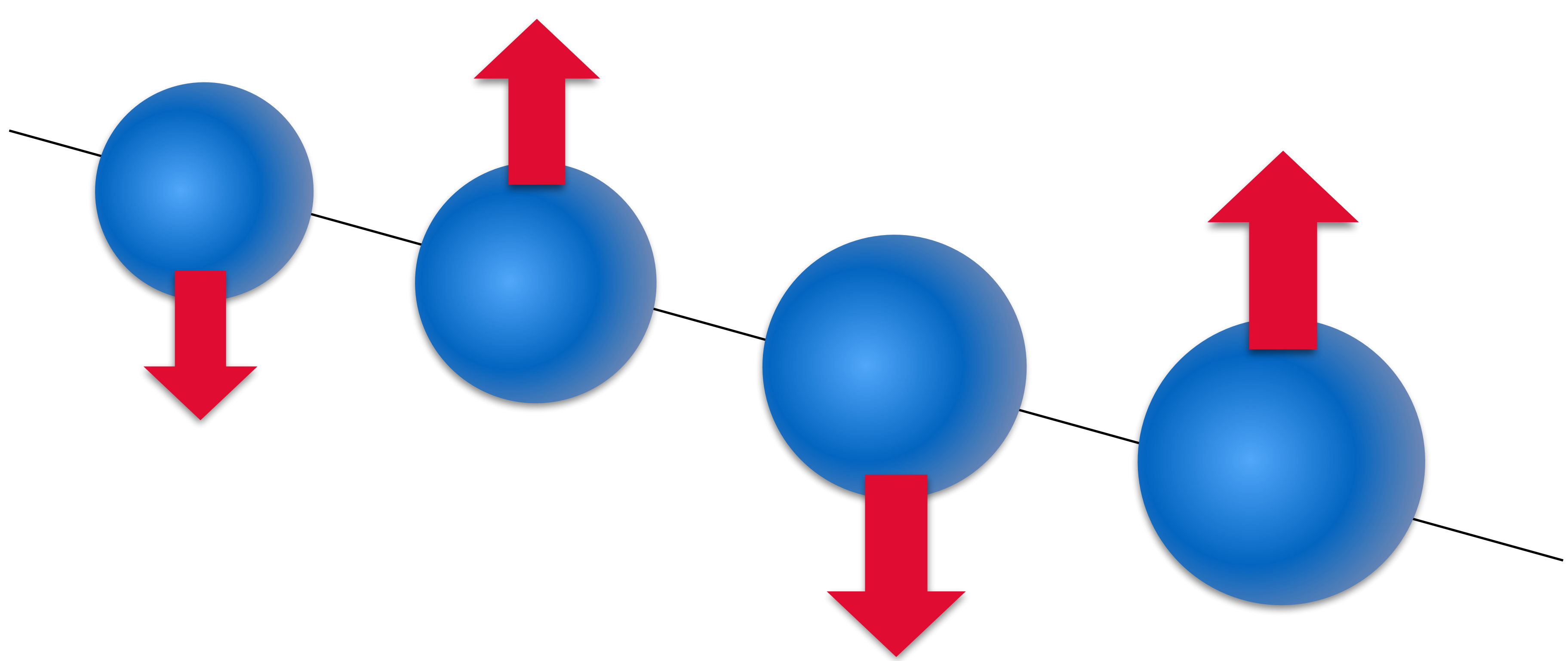

Digital
Single-spin gates
Two-spin gate (MS)
Analog-Digital
ZD, Linke, Pagano, arXiv:2104.09346 [quant-ph].
Single-spin gates
Spin-(normal) phonon gate
Spin-(local) phonon gate
Two-spin gate (MS)
Standing-wave gate
For standing-wave tool, see: Porras and Cirac, Phys. Rev. Lett. 93, 263602 (2004).
Analog
No gates
e^{-iHt}

A LATTICE GAUGE THEORY EXAMPLE STUDIED WITHIN EACH MODE OF THE (ENHANCED) TRAPPED-ION SIMULATOR

Very interesting work such as:
Lamata et al, Phys. Rev. Lett. 98 253005 (2007), Gerritsma et al, Nature 463, 68 (2010), Casanova et al, Phys. Rev. Lett. 107, 260501 (2011), and Bermudez et al, Phys. Rev. X 7, 041012 (2017) will not be covered here.

LATTICE SCHWINGER MODEL: A TESTBED FOR QUANTUM SIMULATION OF LATTICE GAUGE THEORIES

$$H = -ix \sum_{n=1}^{N-1} \left[\psi_n^\dagger U_n \psi_{n+1} - \text{h.c.}\right] + \sum_{n=1}^{N-1} E_n^2 + \mu \sum_{n=1}^{N} (-1)^n \psi_n^\dagger \psi_n$$



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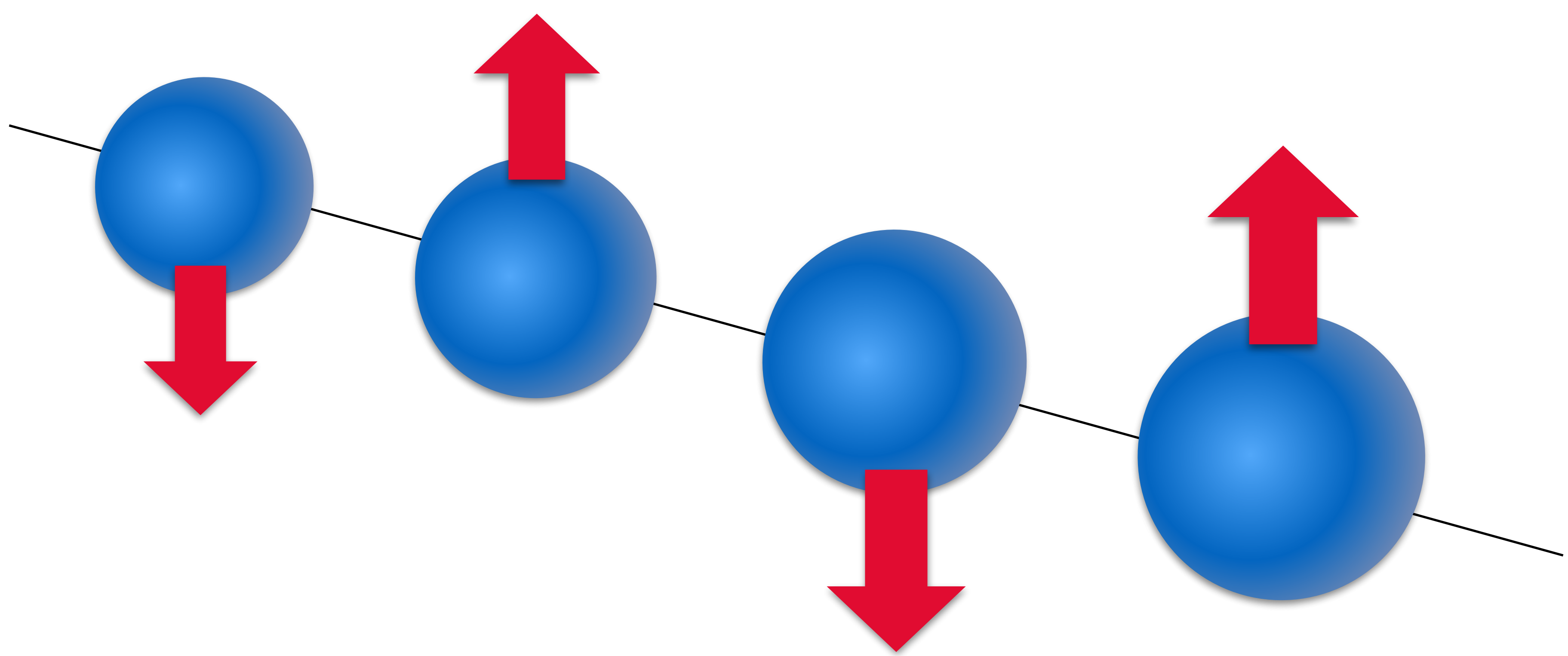
A gauge transformation plus Gauss's law with OBCs: ⇩

Martinez et al, Nature 534, 516 EP (2016).

$$H = x \sum_{n=1}^{N-1} \left[\sigma_+^{(n)} \sigma_-^{(n+1)} + \sigma_+^{(n+1)} \sigma_-^{(n)}\right] + \sum_{n=1}^{N-1} \left[\epsilon_0 + \frac{1}{2} \sum_{m=1}^{n} \left(\sigma_z^{(m)} + (-1)^m\right)\right]^2 + \frac{\mu}{2} \sum_{n=1}^{N} (-1)^n \sigma_Z^{(n)}.$$



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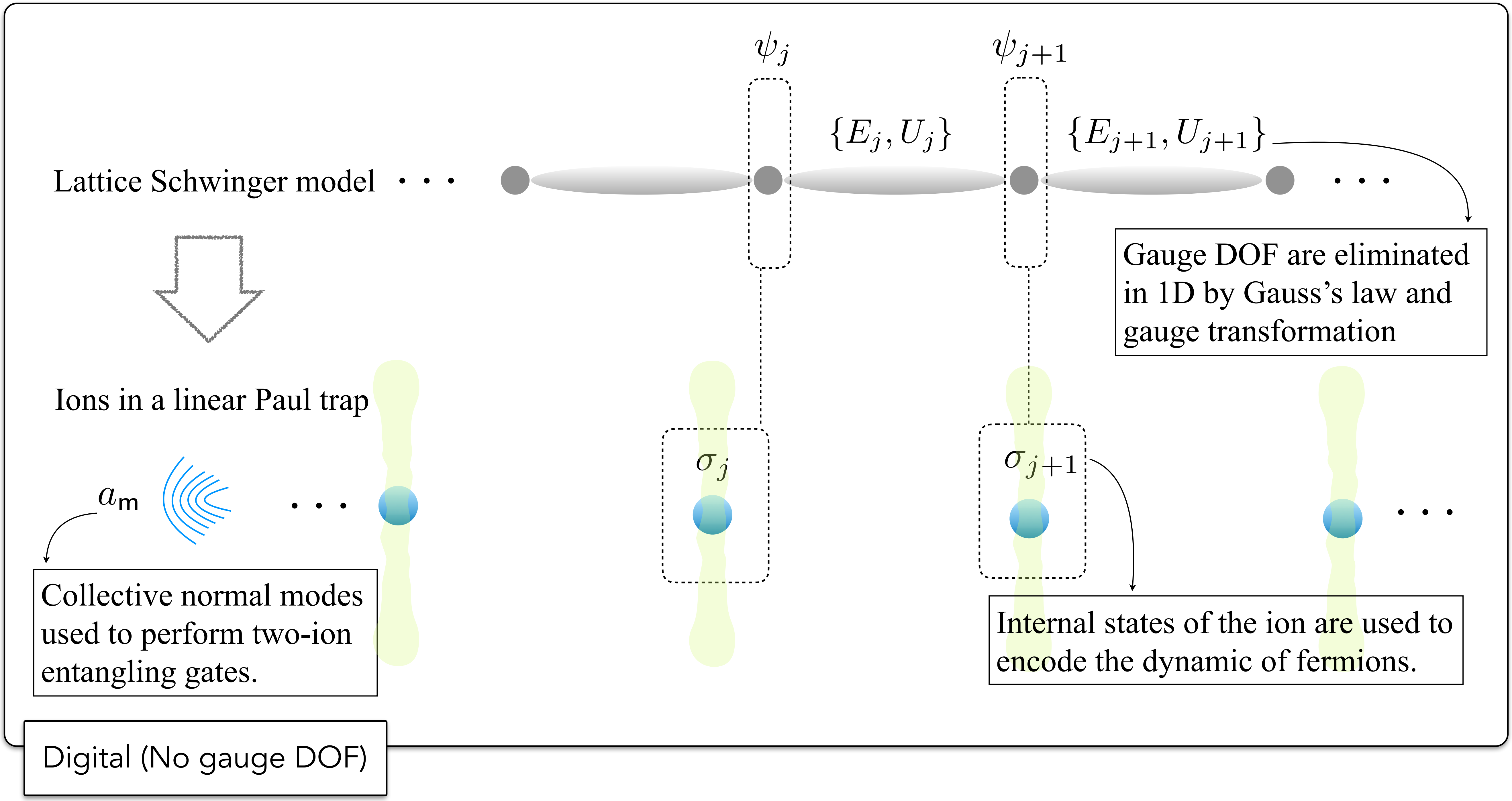
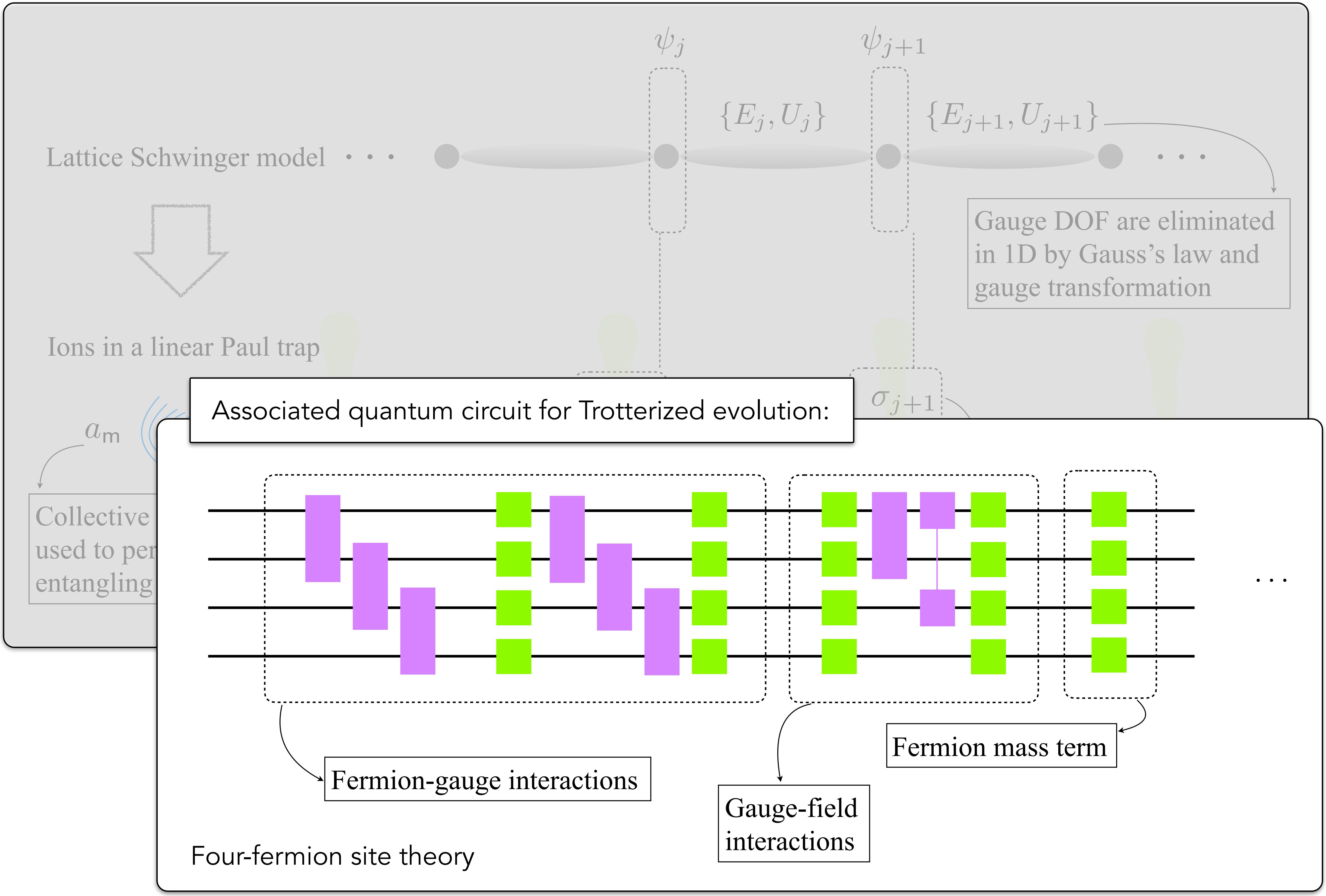
Nearest neighbor spin-spin interactions

Long range spin-spin interactions plus an effective magnetic field

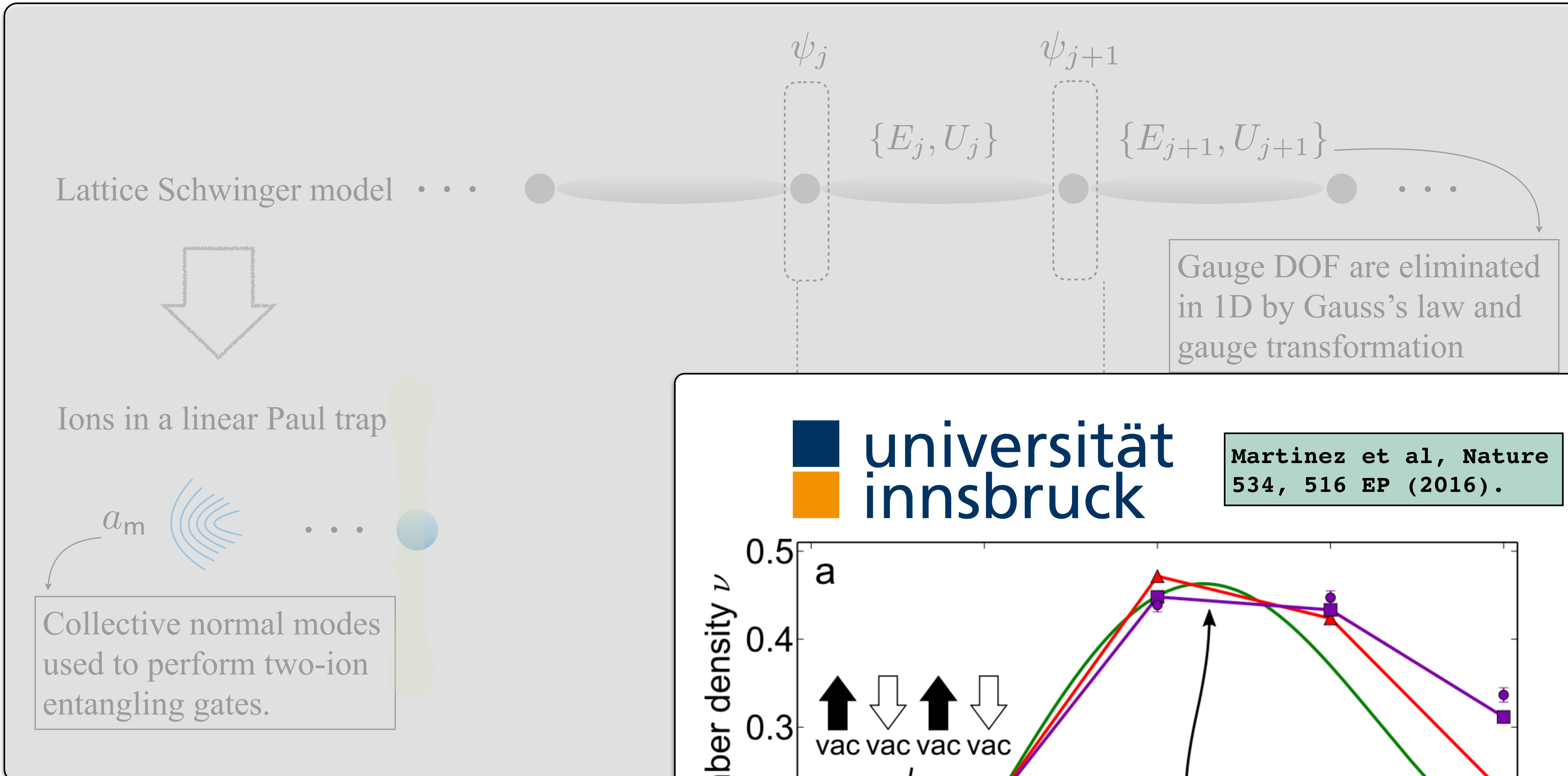
An effective magnetic field



$$H = x \sum_{n=1}^{N-1} \left[\sigma_+^{(n)} \sigma_-^{(n+1)} + \sigma_+^{(n+1)} \sigma_-^{(n)} \right] + \sum_{n=1}^{N-1} \left[\epsilon_0 + \frac{1}{2} \sum_{m=1}^{n} \left(\sigma_z^{(m)} + (-1)^m \right) \right]^2 + \frac{\mu}{2} \sum_{n=1}^{N} (-1)^n \sigma_Z^{(n)}.$$



ψ_j
ψ_{j+1}
$\{E_j, U_j\}$
$\{E_{j+1}, U_{j+1}\}$
Lattice Schwinger model
Gauge DOF are eliminated in 1D by Gauss's law and gauge transformation
Ions in a linear Paul trap
σ_{j+1}
a_m
Collective
used to per
entangling
Associated quantum circuit for Trotterized evolution:
Fermion-gauge interactions
Gauge-field interactions
Fermion mass term
Four-fermion site theory



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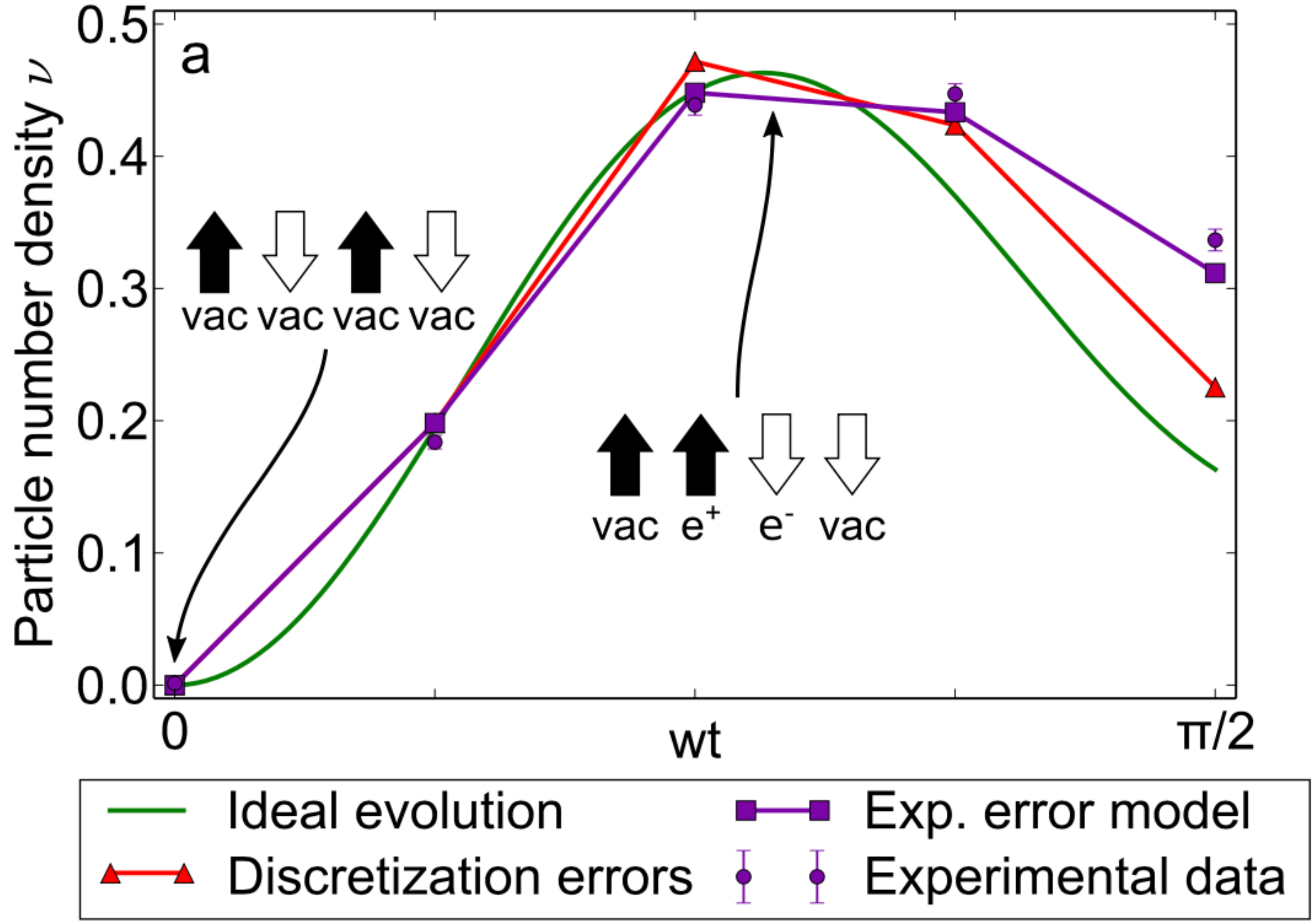
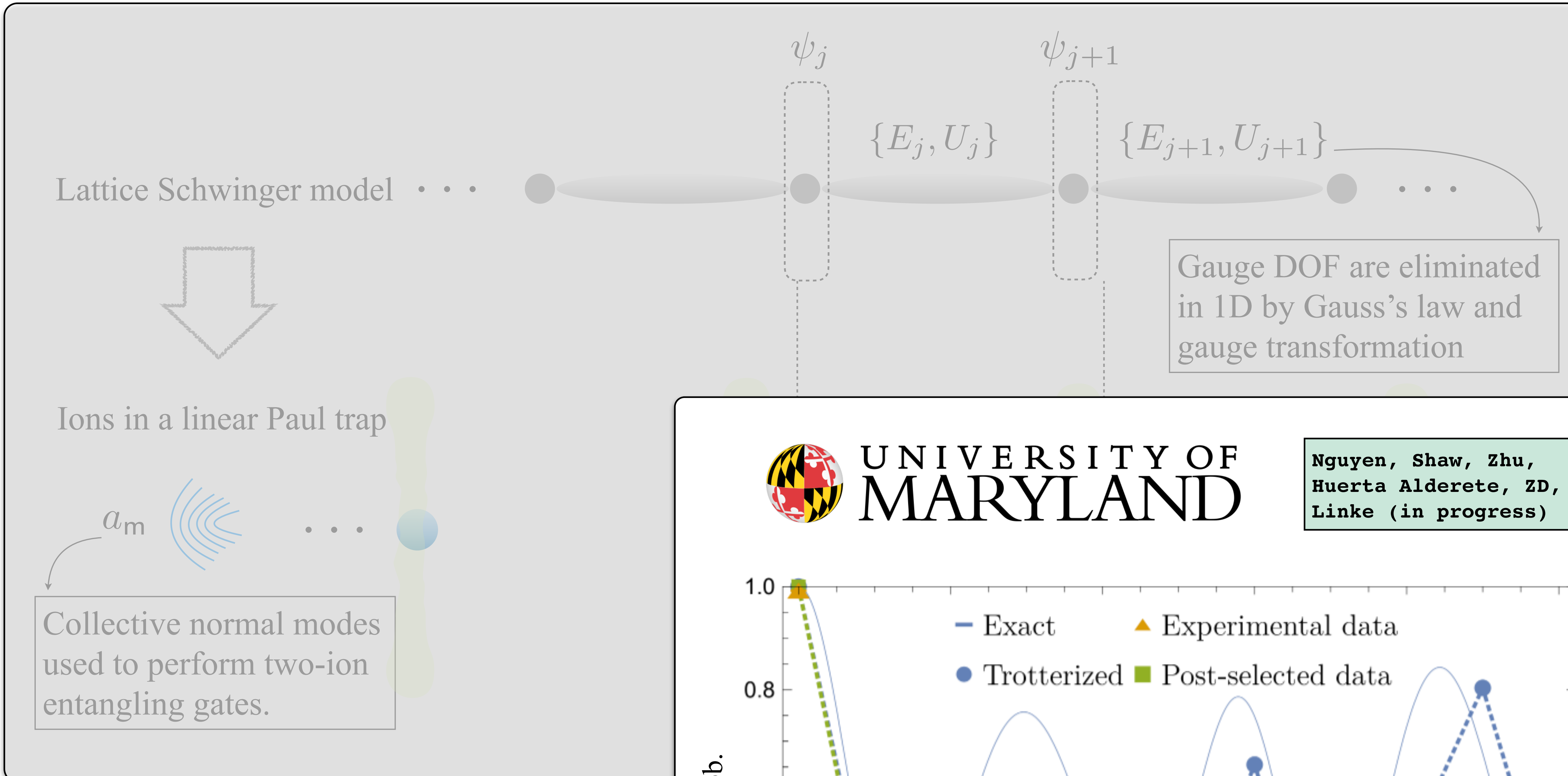


See also: Klco, Savage, et al, Phys. Rev. A 98, 032331 (2018) for an implementation using superconducting qubits.



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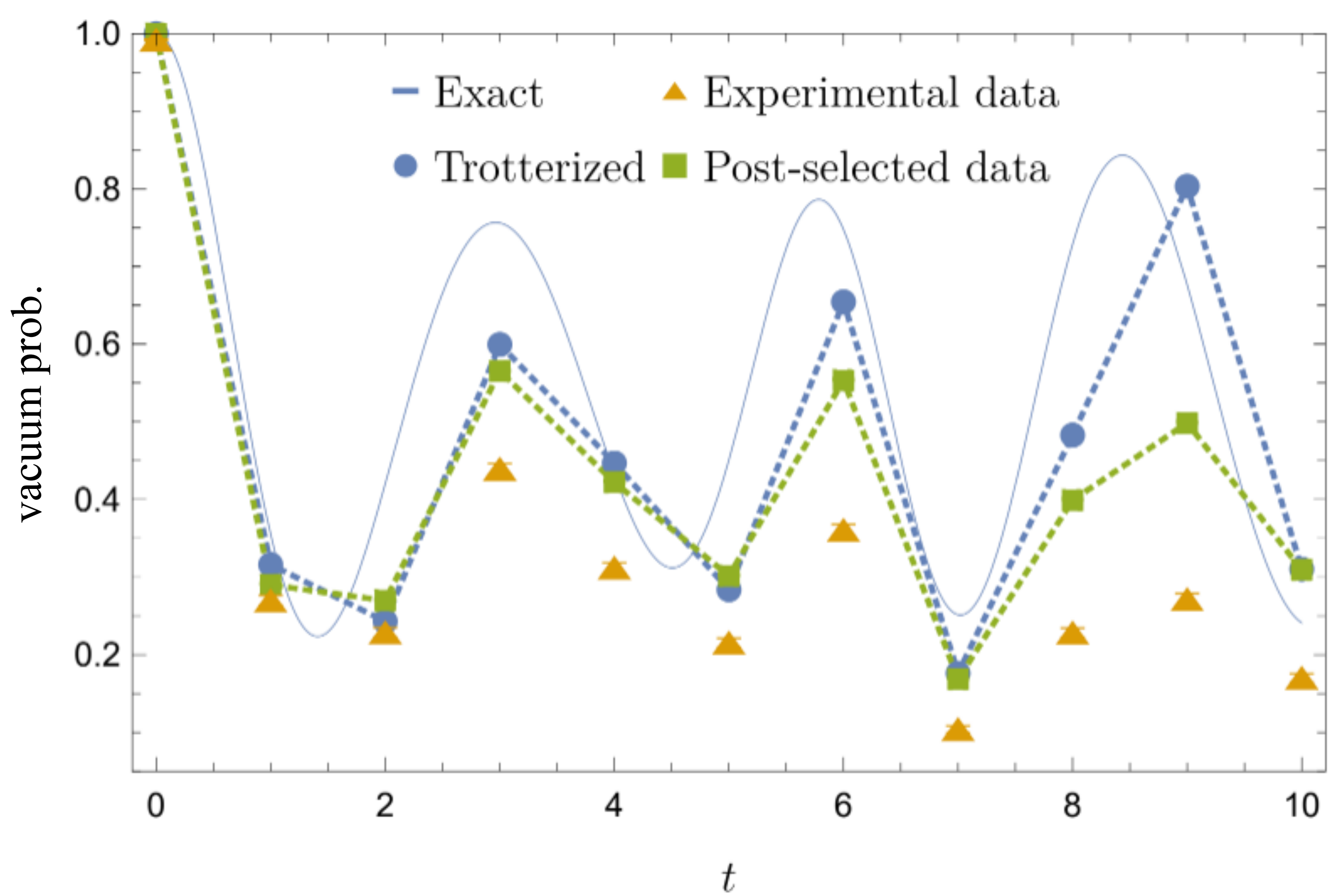
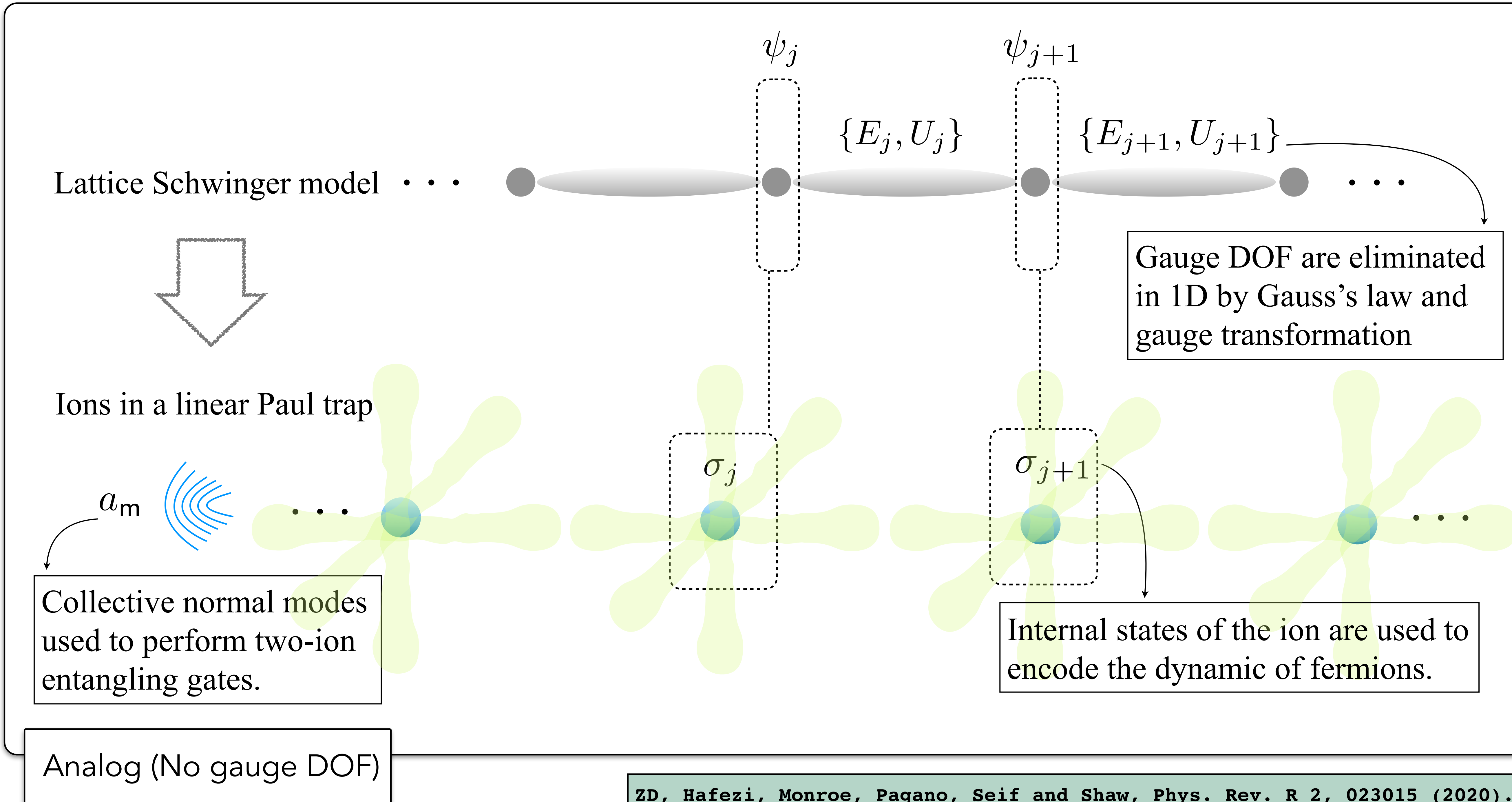
Nguyen, Shaw, Zhu, Huerta Alderete, ZD, Linke (in progress)

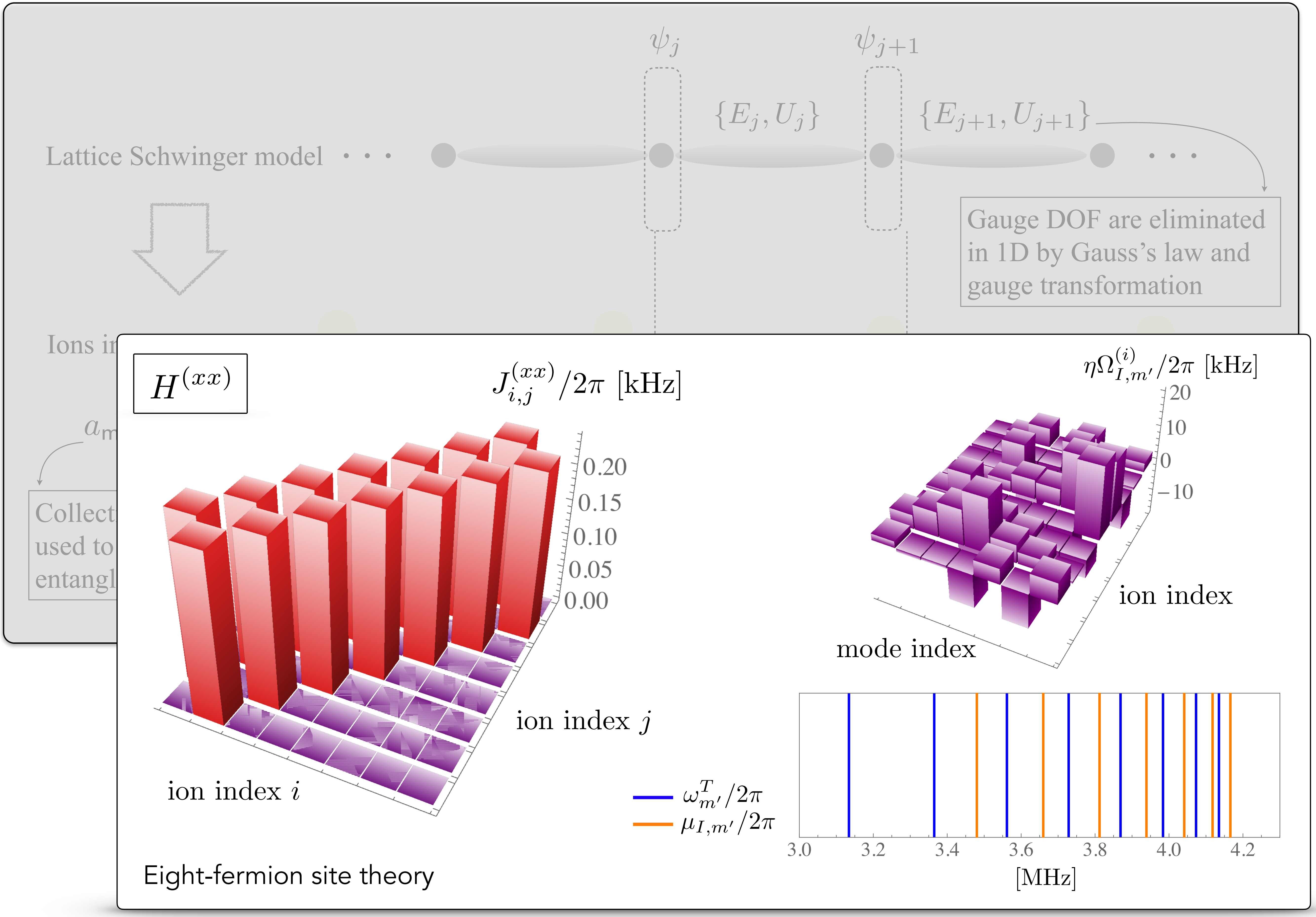




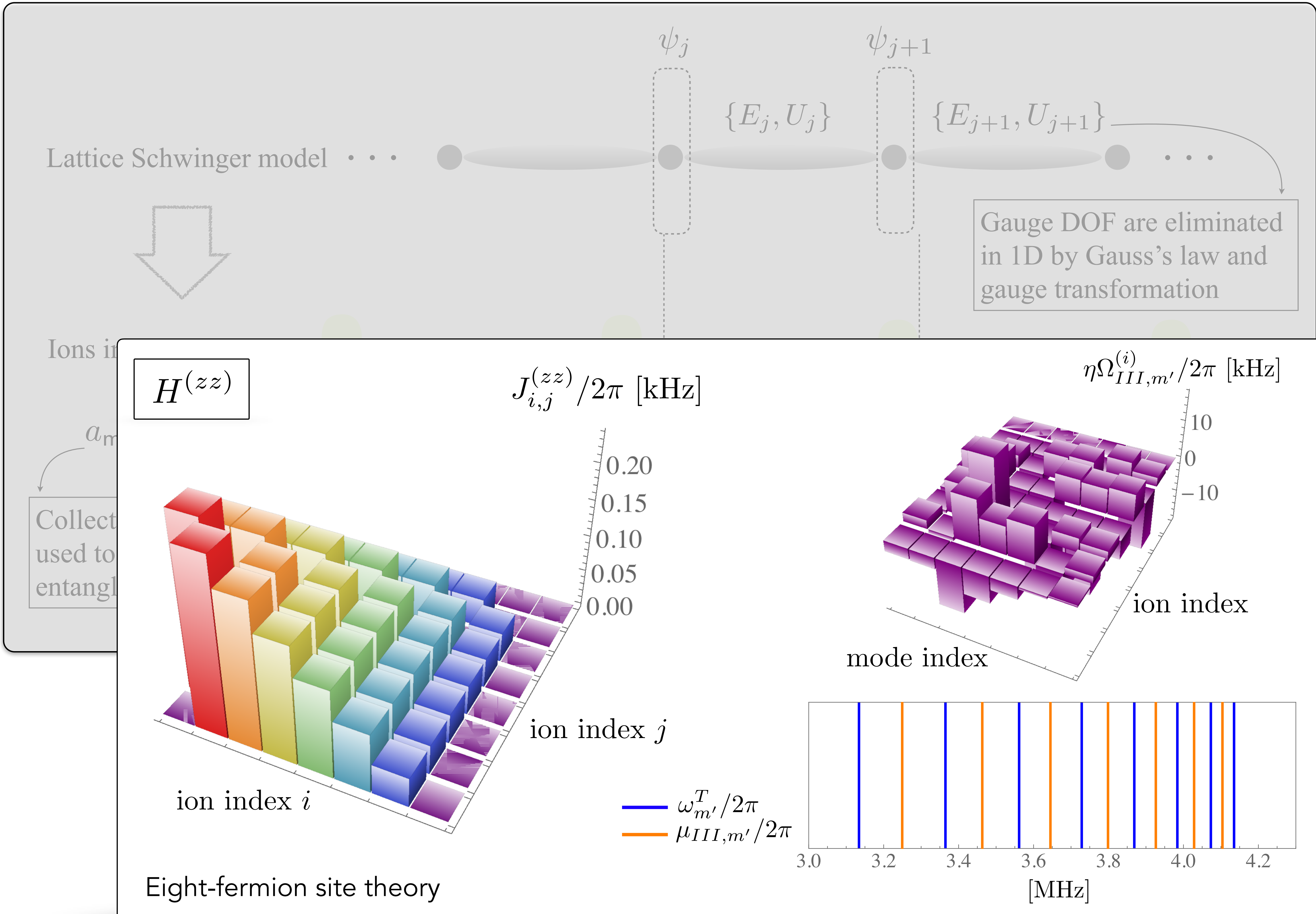
ZD, Hafezi, Monroe, Pagano, Seif and Shaw, Phys. Rev. R 2, 023015 (2020).

See also Yang et al, Physical Review A 94, 052321 (2016) for a phonon-ion based analog proposal of lattice Schwinger Model.

$$H = x\sum_{n=1}^{N-1}\left[\sigma_+^{(n)}\sigma_-^{(n+1)} + \sigma_+^{(n+1)}\sigma_-^{(n)}\right] + \sum_{n=1}^{N-1}\left[\epsilon_0 + \frac{1}{2}\sum_{m=1}^{n}\left(\sigma_z^{(m)} + (-1)^m\right)\right]^2 + \frac{\mu}{2}\sum_{n=1}^{N}(-1)^n\sigma_Z^{(n)}.$$



ZD, Hafezi, Monroe, Pagano, Seif and Shaw, Phys. Rev. R 2, 023015 (2020).



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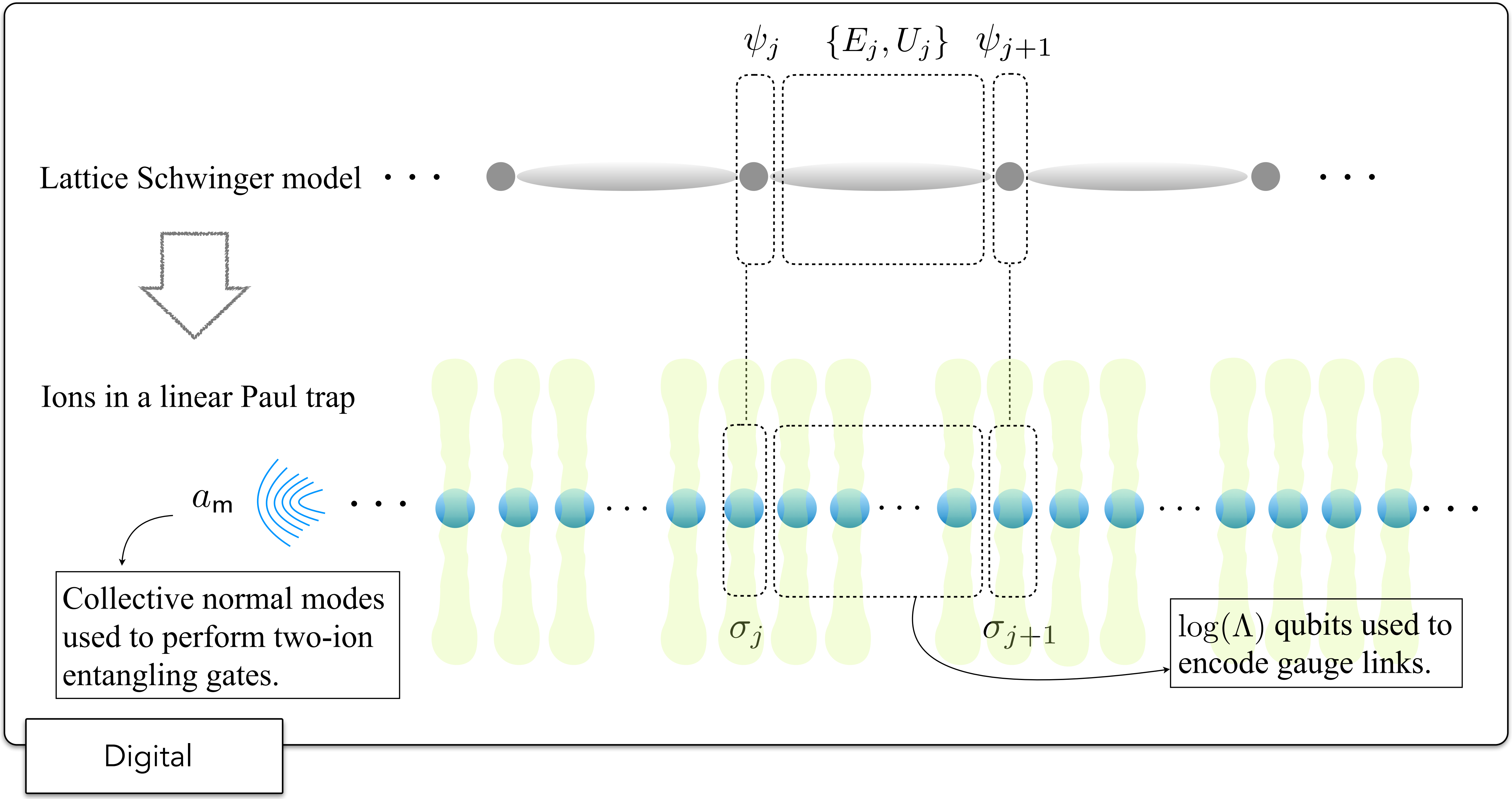

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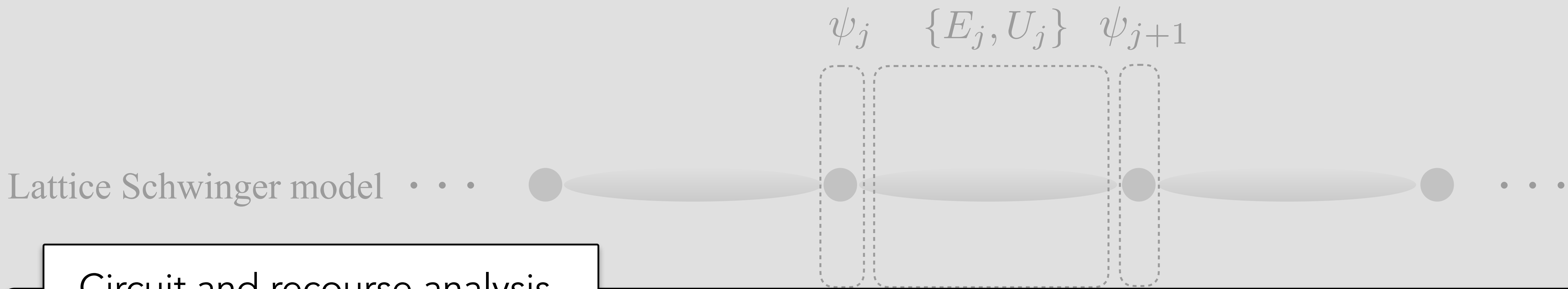
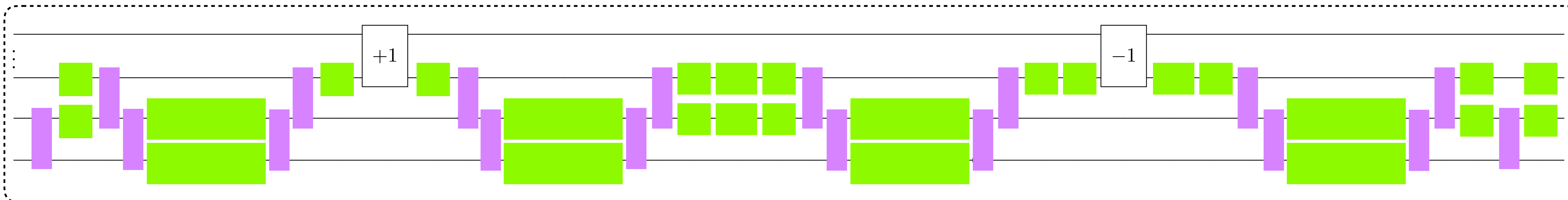


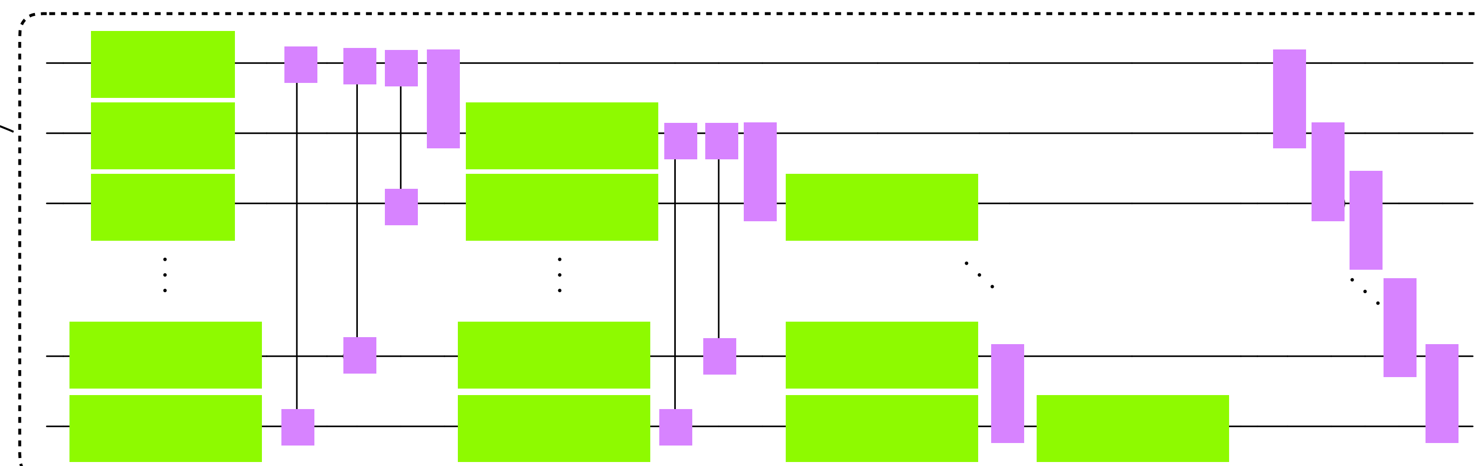
Circuit and recourse analysis

Shaw, Lougovski, Stryker, Wiebe, Quantum 4, 306 (2020)



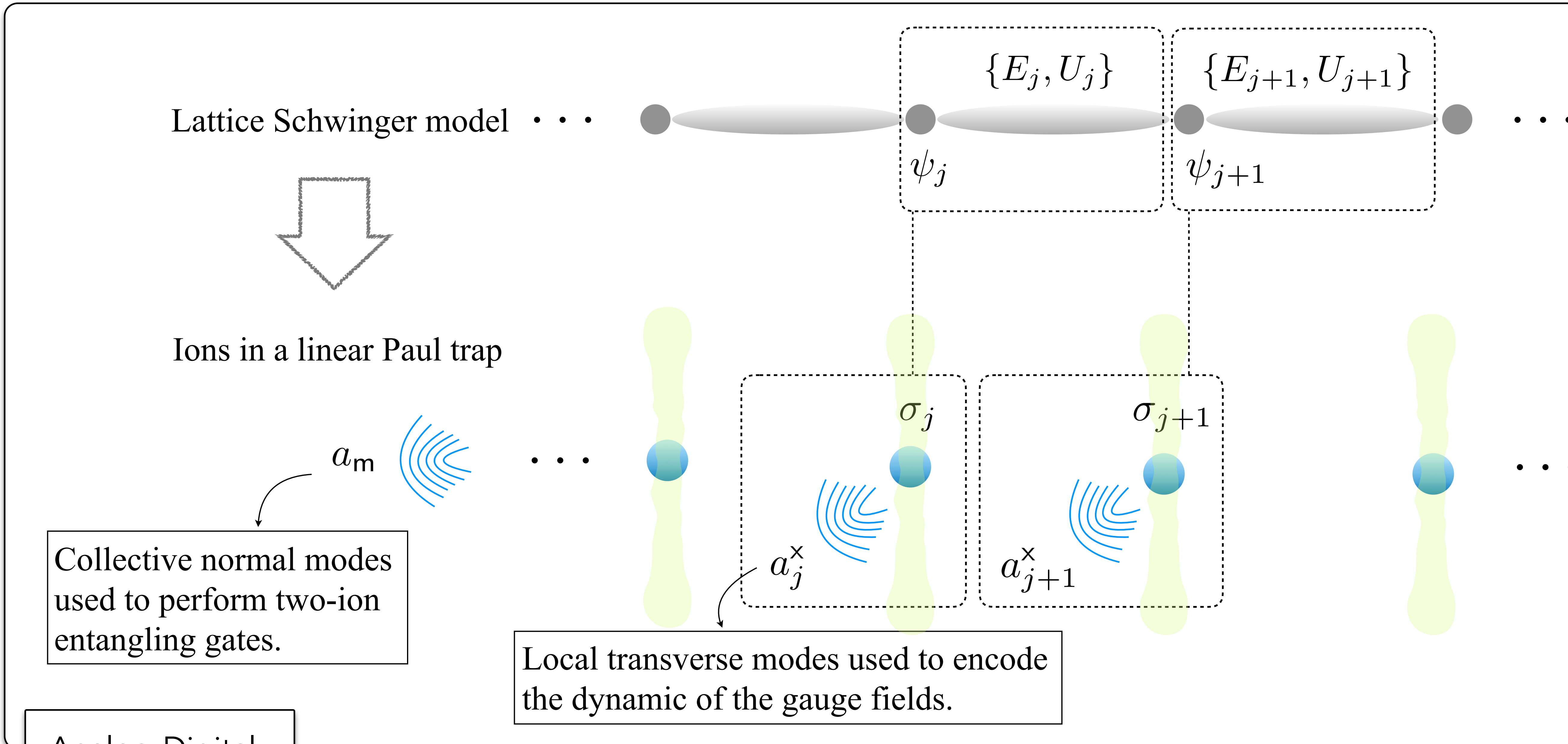
Sample gauge-fermion interaction block

Part of electric field interactions acting on gauge DOF registers



Near term cost

	$\delta_g = 10^{-3}$		$\delta_g = 10^{-4}$		$\delta_g = 10^{-5}$		$\delta_g = 10^{-6}$		$\delta_g = 10^{-7}$	
	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT
$x = 10^{-2}$	—	7.3e4	—	1.6e5	—	3.4e5	—	7.3e5	5.6e-2	1.6e6
$x = 10^{-1}$	—	1.6e4	—	3.5e4	—	7.5e4	5.9e-2	1.6e5	2.7e-3	3.5e5
$x = 1$	—	4.6e3	—	9.9e3	1.0e-1	2.1e4	4.7e-3	4.6e4	2.2e-4	9.9e4
$x = 10^2$	—	2.8e3	8.3e-1	6.1e3	3.8e-2	1.3e4	1.8e-3	2.8e4	8.2e-5	6.0e4



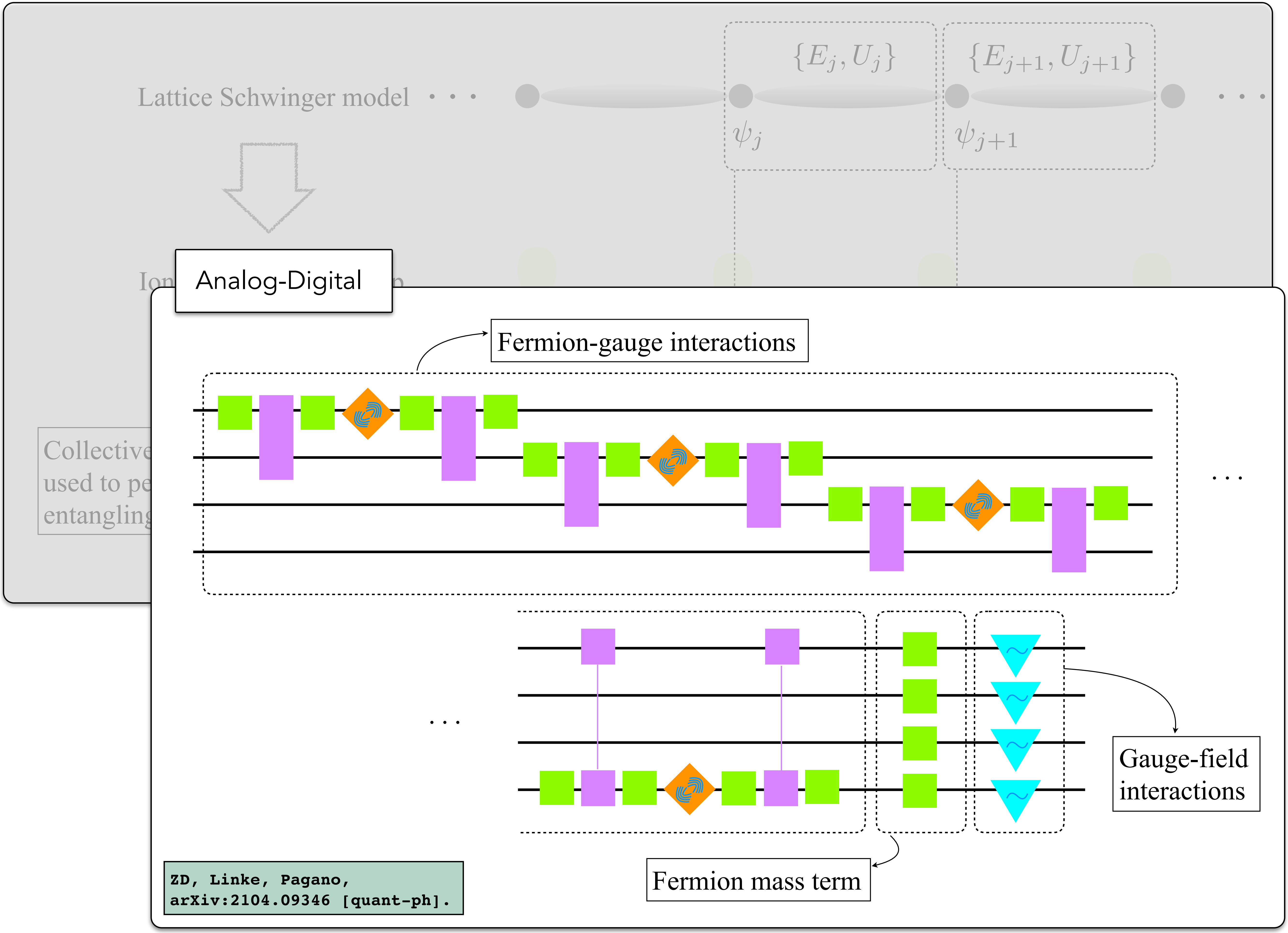
Analog-Digital

ZD, Linke, Pagano, arXiv:2104.09346 [quant-ph].

See Yang et al, Phys. Rev. A 94, 052321 (2016) for the highly-occupied bosonic model of the Schwinger model.

See also Casanova et al, Phys. Rev. Lett. 108, 190502 (2012), Lamata et al, EPJ Quant. Technol. 1, 9 (2014), and Mezzacapo et al, Phys. Rev. lett. 109, 200501 (2012) for analog-digital approaches to other interacting fermion-boson theories.

$$H = -ix \sum_{n=1}^{N-1} \left[\psi_n^\dagger U_n \psi_{n+1} - \text{h.c.}\right] + \sum_{n=1}^{N-1} E_n^2 + \mu \sum_{n=1}^{N} (-1)^n \psi_n^\dagger \psi_n$$



Lattice Schwinger model
$\{E_j, U_j\}$
$\{E_{j+1}, U_{j+1}\}$
ψ_j
ψ_{j+1}
Analog-Digital
Fermion-gauge interactions
Gauge-field interactions
Fermion mass term
ZD, Linke, Pagano, arXiv:2104.09346 [quant-ph].

Let us compare the circuit structure of digital and analog-digital cases when gauge DOF are present:

Schwinger model			
	Fermion-gauge interaction	Fermion mass	Electric-field term
Analog-digital	$\mathcal{O}(N)$	$\mathcal{O}(1)$	$\mathcal{O}(N)$
Digital	$\mathcal{O}\left(N^2 (\log \Lambda)^2\right)$	$\mathcal{O}(1)$	$\mathcal{O}\left(N (\log \Lambda)^2\right)$

ZD, Linke, Pagano, arXiv:2104.09346 [quant-ph].

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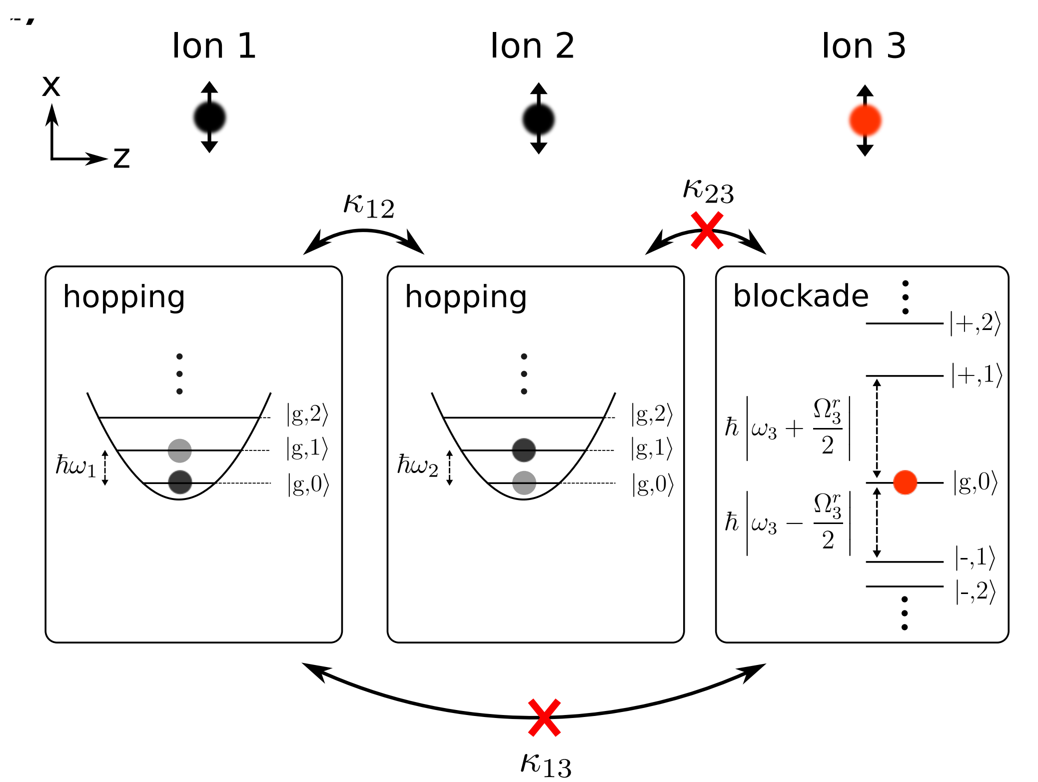
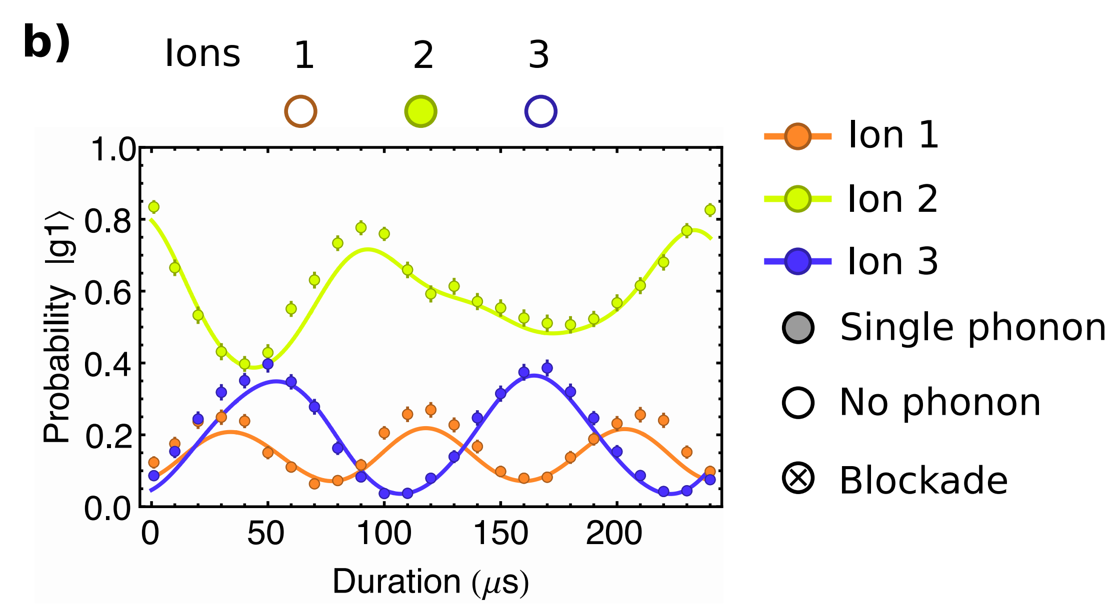
ZD, Linke, Pagano, arXiv:2104.09346 [quant-ph].

Is phonon control experimentally feasible? Yes…at least for small systems so far!



Debnath et al, Phys. Rev. Lett. 120, 073001 (2018).

UNIVERSITY OF MARYLAND Monroe-Linke Group

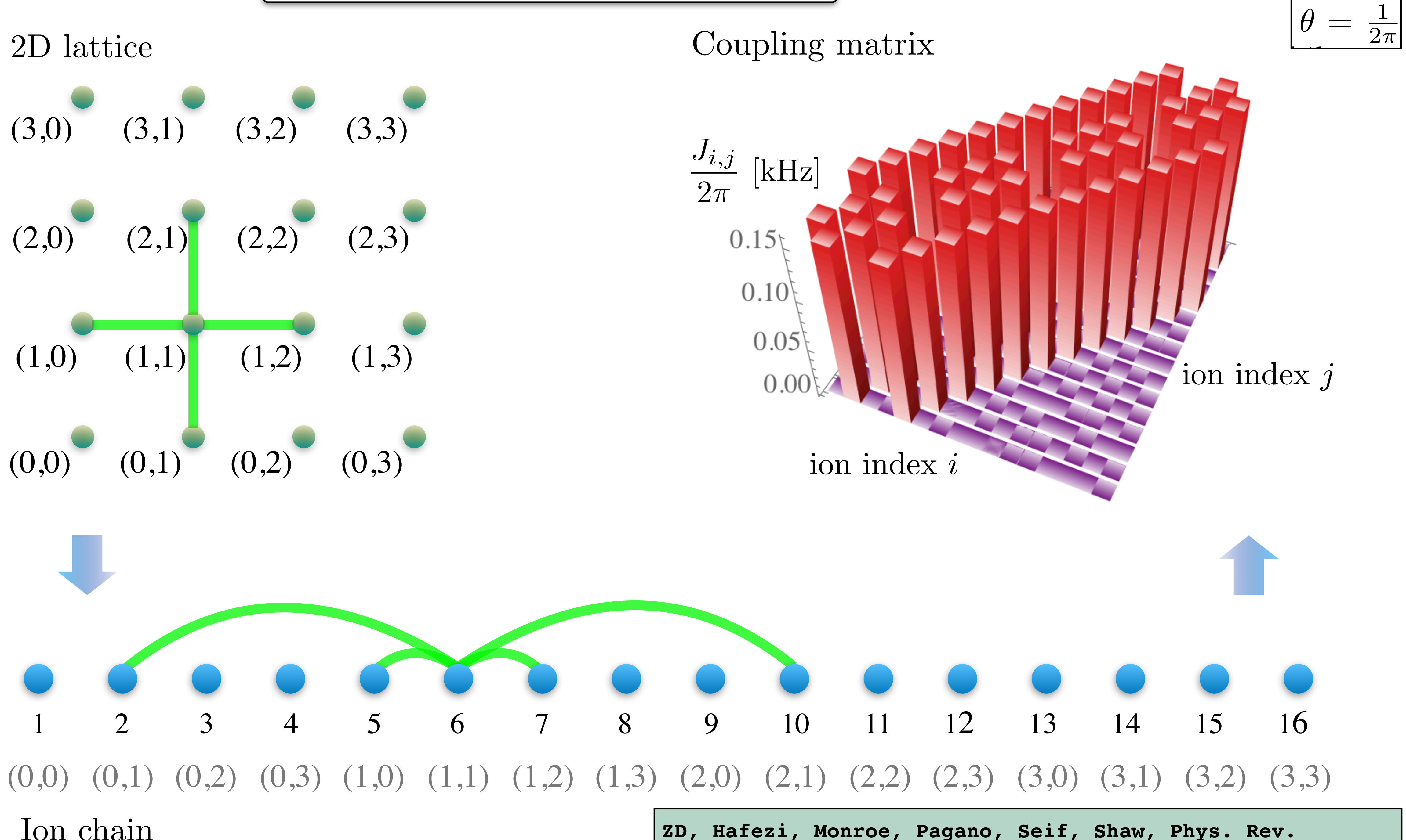


GAUGE THEORIES IN HIGHER DIMENSIONS WITH THE ANALOG MODE OF THE SIMULATOR

THE ANALOG SCHEME CAN BE APPLIED TO CHERN-SIMONS THEORY IN 2+1 D:

$$\mathcal{L}_{\mathrm{CS}} = a^\dagger(x) i D_0 a(x) - \sum_{j=1,2} \left[a^\dagger(x) e^{iA_j(x)} a(x+\hat{\boldsymbol{n}}_j) + \mathrm{h.c.} \right] - \frac{\theta}{4} \epsilon^{\mu\nu\lambda} A_\mu(x) F_{\nu\lambda}(x) \quad (24)$$

$$H_{\mathrm{CS}} = \sum_{\boldsymbol{n}} \sum_{j=1,2} \left[\sigma_+^{(\boldsymbol{n})} \sigma_-^{(\boldsymbol{n}+\hat{\boldsymbol{n}}_j)} + \mathrm{h.c.} \right]$$

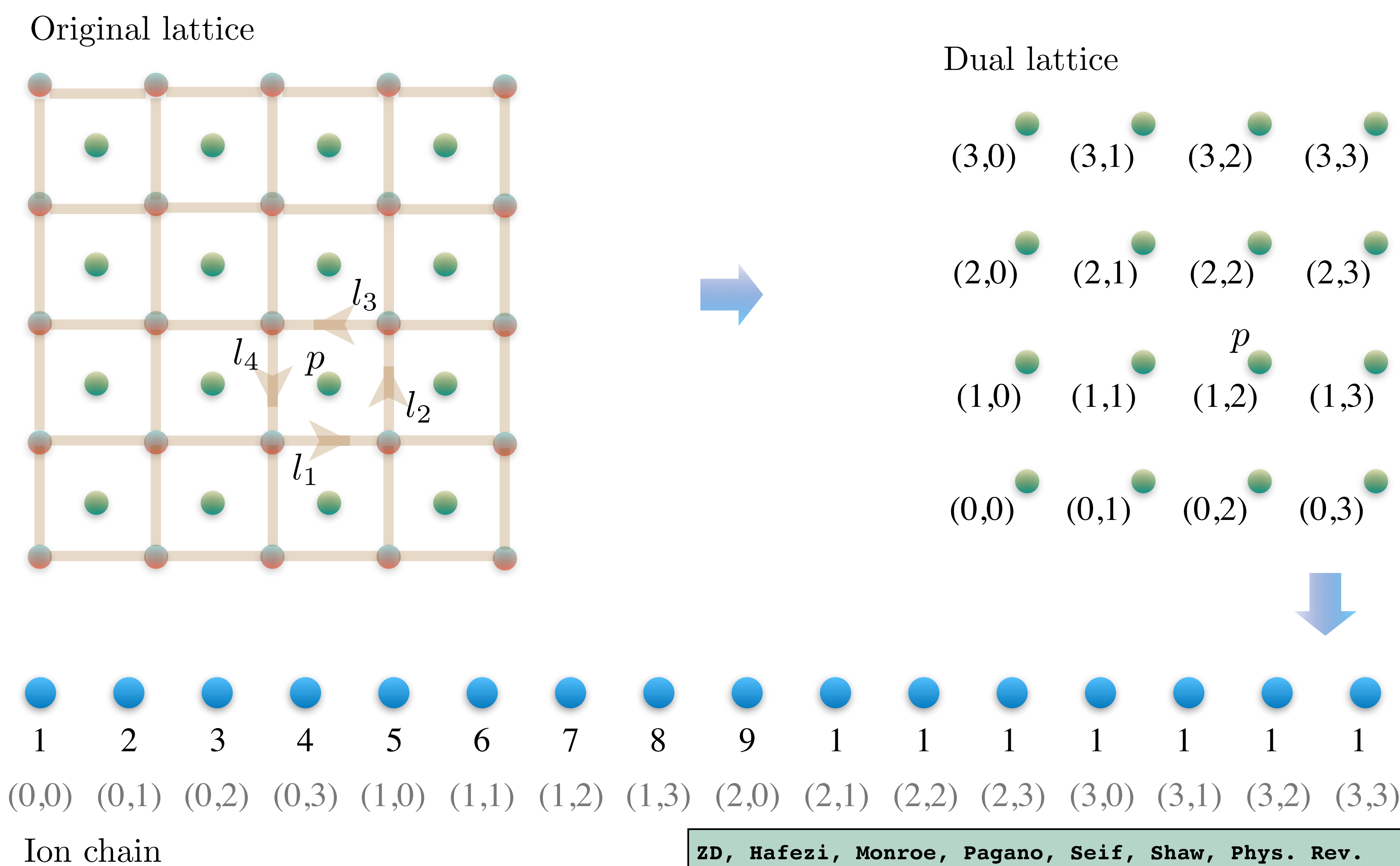


ZD, Hafezi, Monroe, Pagano, Seif, Shaw, Phys. Rev. Research, 2, 023015 (2020), arXiv: 1908.03210 [quant-ph].

OR THAT OF Z(2) GAUGE THEORY IN 2+1 D

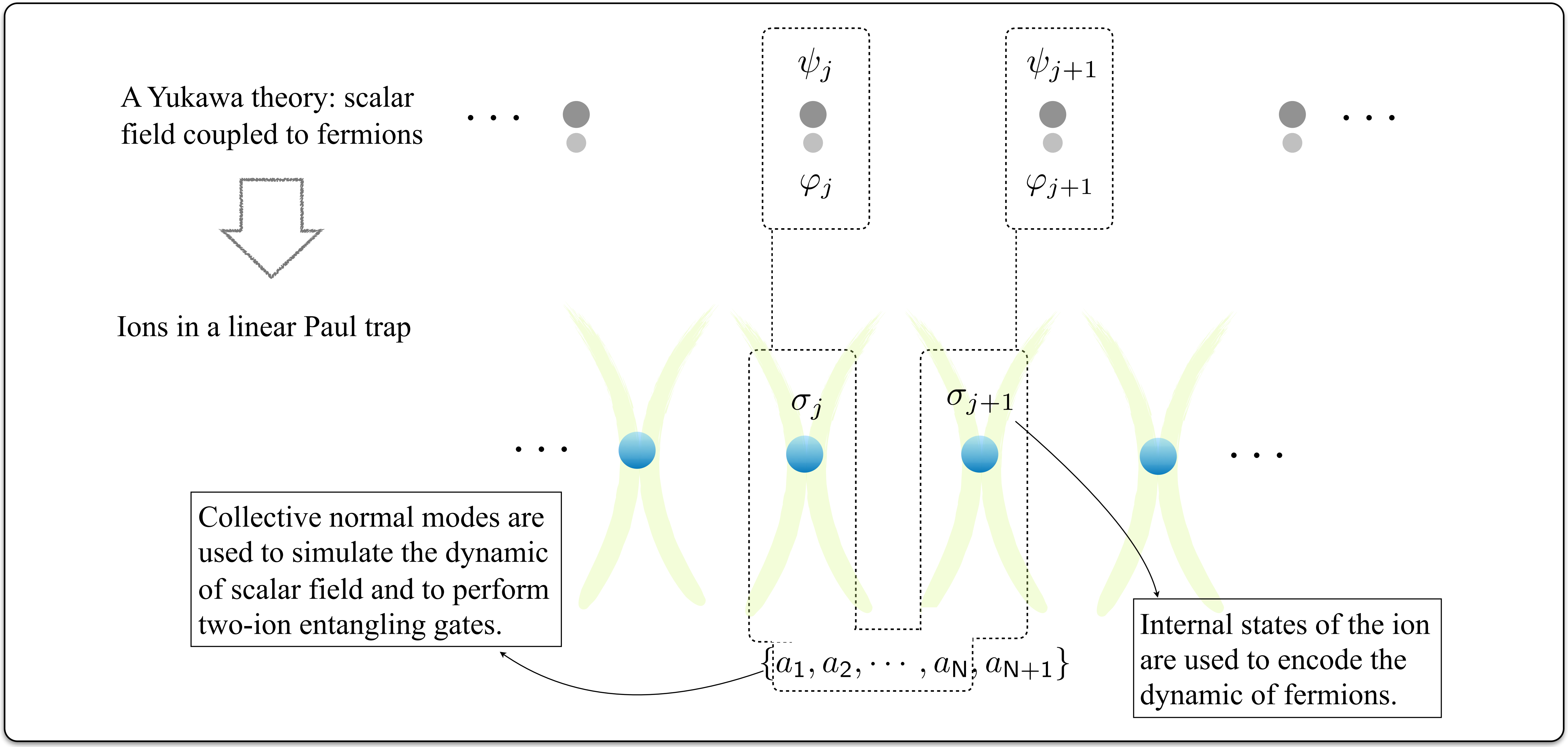
$$H_{2+1\text{D } Z_2} = -\sum_l \sigma_x(l) - \lambda \sum_p \sigma_z(l_1)\sigma_z(l_2)\sigma_z(l_3)\sigma_z(l_4)$$

$$H_{\text{2D Ising}} = -\lambda \sum_{\boldsymbol{n}} \sigma_x^{(\boldsymbol{n})} - \sum_{\boldsymbol{n}} \sum_{j=1,2} \sigma_z^{(\boldsymbol{n})} \sigma_z^{(\boldsymbol{n}+\hat{\boldsymbol{n}}_j)}$$

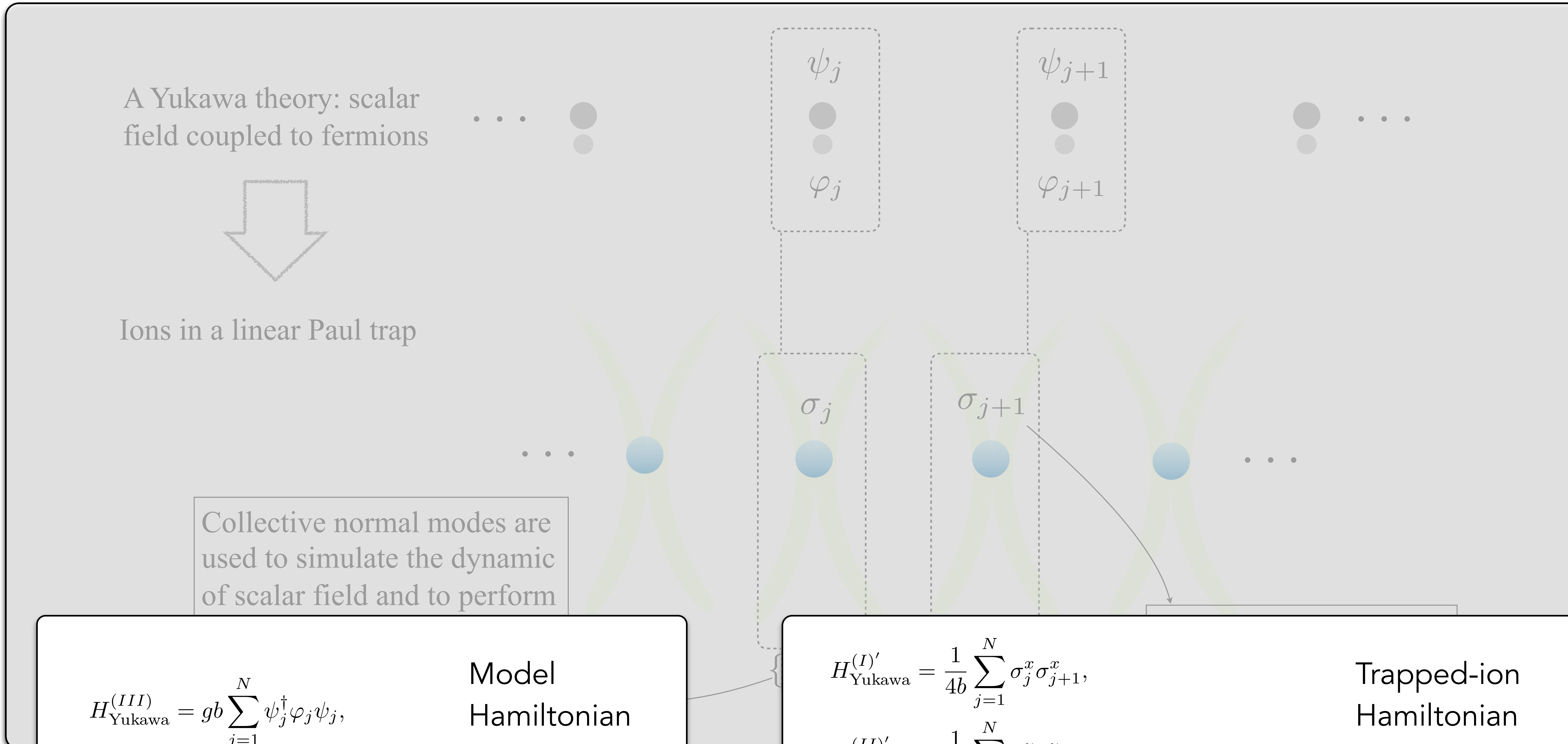


ZD, Hafezi, Monroe, Pagano, Seif, Shaw, Phys. Rev. Research, 2, 023015 (2020), arXiv: 1908.03210 [quant-ph].

A SCALAR FIELD THEORY COUPLED TO FERMIONS (YUKAWA THEORY) WITH THE HYBRID MODE OF THE SIMULATOR



ZD, Linke, Pagano, arXiv:2104.09346 [quant-ph].



Model Hamiltonian

$$H^{(III)}_{\text{Yukawa}} = gb\sum_{j=1}^{N}\psi_j^\dagger\varphi_j\psi_j,$$

$$H^{(II)}_{\text{Yukawa}} = b\sum_{j=1}^{N}\left[\frac{\Pi_j^2}{2} + \frac{(\nabla\varphi_j)^2}{2} + \frac{m_\varphi^2}{2}\varphi_j^2\right]$$

$$H^{(I)}_{\text{Yukawa}} = \sum_{j=1}^{N}\left[\frac{i}{2b}(\psi_j^\dagger\psi_{j+1} - \psi_{j+1}^\dagger\psi_j) + m_\psi(-1)^j\psi_j^\dagger\psi_j\right]$$

Trapped-ion Hamiltonian

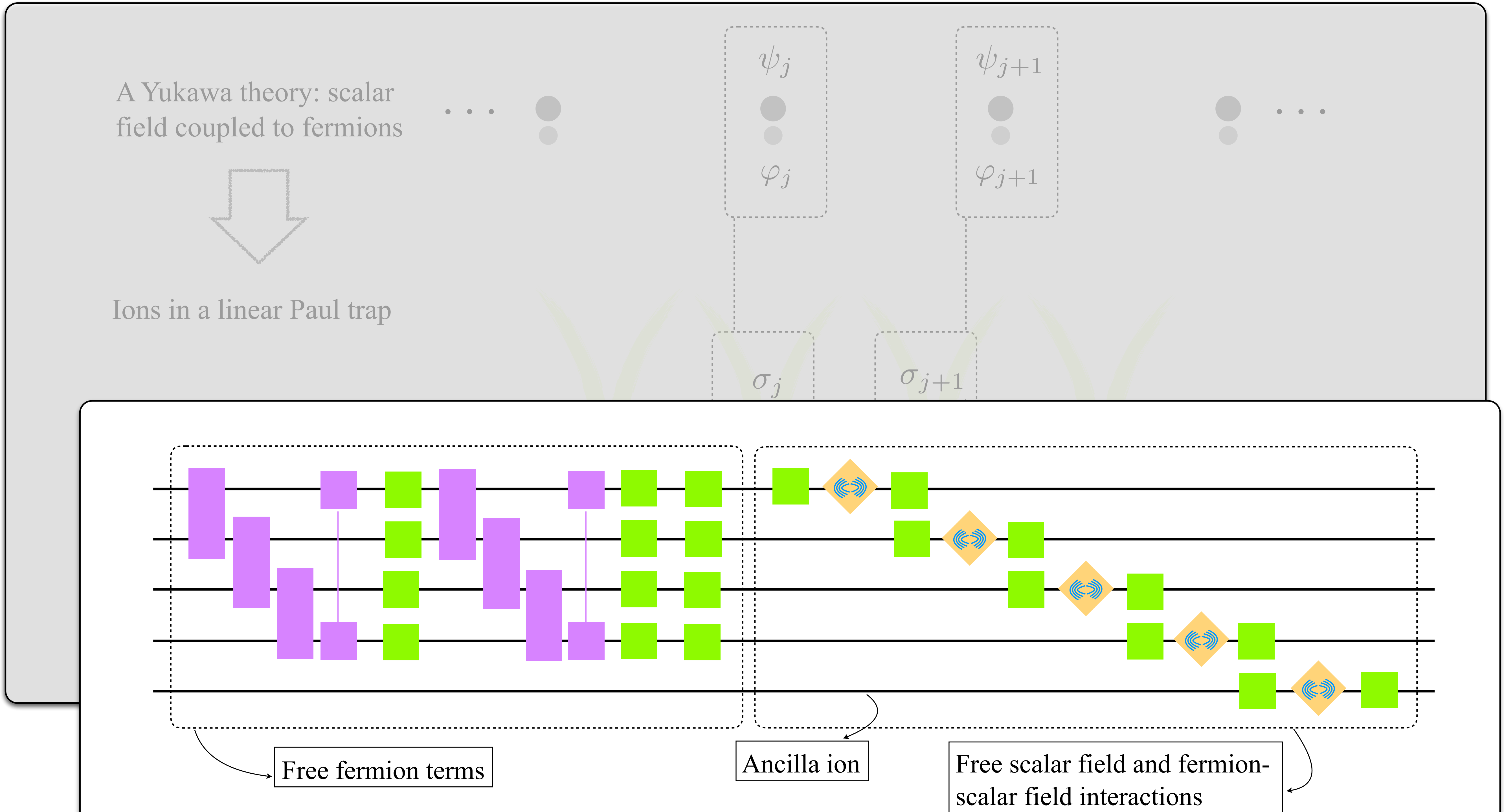
$$H^{(I)'}_{\text{Yukawa}} = \frac{1}{4b}\sum_{j=1}^{N}\sigma_j^x\sigma_{j+1}^x,$$

$$H^{(II)'}_{\text{Yukawa}} = \frac{1}{4b}\sum_{j=1}^{N}\sigma_j^y\sigma_{j+1}^y,$$

$$H^{(III)'}_{\text{Yukawa}} = \frac{m_\psi}{2}\sum_{j=1}^{N}(-1)^j\sigma_j^z + \text{const.},$$

$$H^{(IV)'}_{\text{Yukawa}} = \sqrt{\frac{g^2 b}{8N}}\sum_{j=1}^{N}(\mathbb{I}_j + \sigma_j^z)\sum_{\mathsf{m}=1}^{N}\frac{1}{\sqrt{\varepsilon_{\mathsf{m}}}}\times (a_{\mathsf{m}}^\dagger e^{-i\frac{2\pi j}{N}(\mathsf{m}-\frac{N}{2}-1)} + a_{\mathsf{m}}e^{i\frac{2\pi j}{N}(\mathsf{m}-\frac{N}{2}-1)}) + \sum_{\mathsf{m}=1}^{N}\varepsilon_{\mathsf{m}}(a_{\mathsf{m}}^\dagger a_{\mathsf{m}} + \frac{1}{2})$$

ZD, Linke, Pagano, arXiv:2104.09346 [quant-ph].



Yukawa theory				
	Fermion hopping	Fermion mass	Free scalar fields	Fermion scalar-field interaction
Analog-digital	$\mathcal{O}(N)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$
Digital	$\mathcal{O}(N)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}\left(N^2 (\log \Lambda)^2\right)$

ZD, Linke, Pagano, arXiv:2104.09346 [quant-ph].

SUMMARY

Trapped-ion simulators have become a successful platform for quantum simulation of many-body physics.

The first successful implementations of gauge-field theory dynamics on trapped-ion simulators/computers have emerged for small systems.

Phonons can be manipulated and measured in these systems, hence the possibility of using phonons as both virtual and dynamical degrees of freedom is realistic.

Simulating complex dynamics of quantum field theories may benefit from digital, analog, and hybrid implementations depending on the problem at hand.

OUTLOOK

As these systems scale and improve, we must come up with resource-efficient algorithms tailored to our problems, paying attention to hardware architecture.

Co-development will be crucial as special-purpose hardware may reduce the time to solution in near-term. Theory-experiment collaborations will be highly beneficial.

Theoretical development need to complement the program. Efficient Hamiltonian formulations for (non-Abelian) gauge theories along with best approaches to state preparation and measurement will continue to develop.

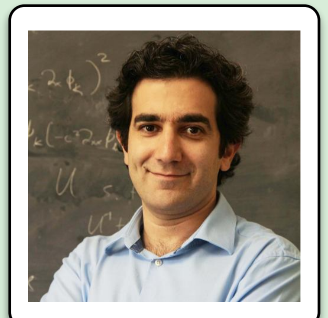
Effective field theory simulations can benefit from developments in gauge-theory simulations and vice versa. Analog and hybrid approaches for nuclear EFTs should be explored.

TOWARDS SIMULATING QUANTUM FIELD THEORIES WITH TRAPPED-ION SIMULATORS @MARYLAND+COLLABORATING INSTITUTIONS (RICE, DUKE, U CHICAGO, ICFO/BARCELONA)

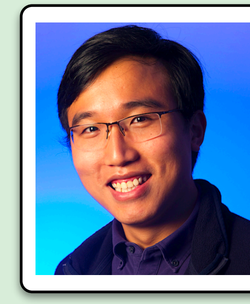
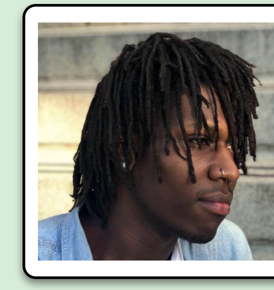



Toward analog quantum simulations of lattice gauge theories with trapped ions

ZD, Hafezi, Monroe, Pagano, Seif, Shaw, Phys. Rev. Research, 2, 023015 (2020), arXiv: 1908.03210 [quant-ph]

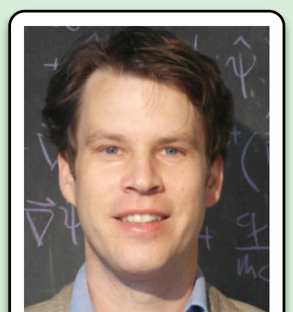
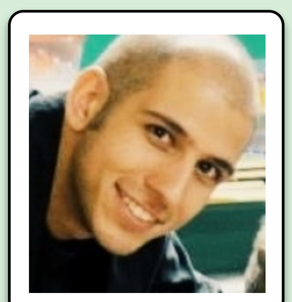




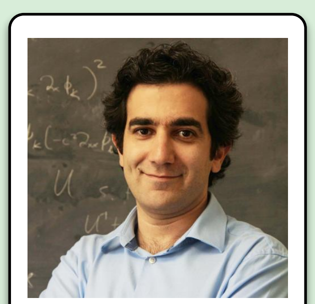
Lattice Schwinger model: Real-time dynamics and gauge-symmetry protection with a trapped-ion quantum computer

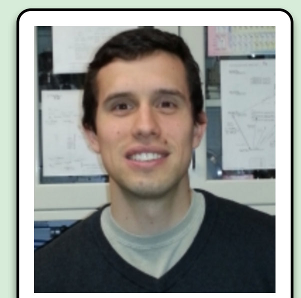
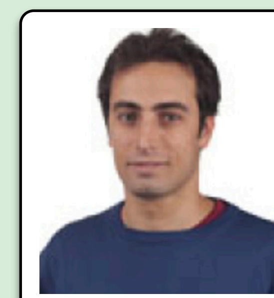
Nguyen, Tran, Shaw, Zhu, ZD, Linke, work in progress (2021).


Engineering an Effective Three-spin Hamiltonian for Applications in Quantum Simulation

Andrade, ZD, Grass, Hafezi, Pagano, Seif, arXiv: 2107.xxxx [quant-ph]



Toward simulating quantum field theories with controlled phonon-ion dynamics: A hybrid analog-digital approach

ZD, Linke, Pagano, arXiv:2104.09346 [quant-ph].

THANK YOU

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