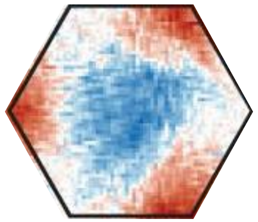
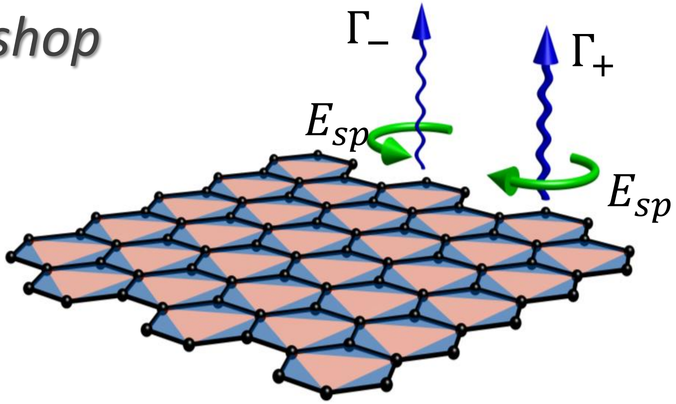
Topology with ultracold atoms in hexagonal optical lattices

*Trento ECT**

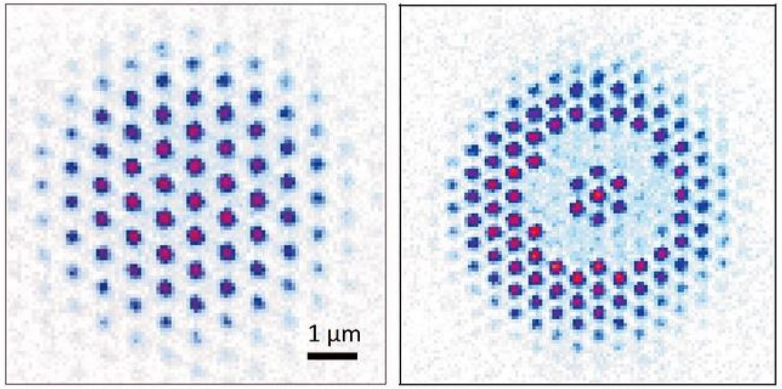
Nuclear Physics meets condensed Matter Workshop

19.7.2021





Luca Asteria



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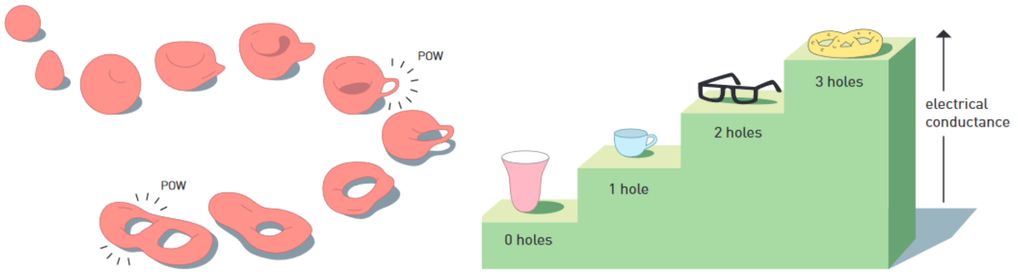
Motivation: Geometry and Topology

Topological quantum matter
(Nobel prize 2016)



nobelprize.org

Topology of closed surfaces

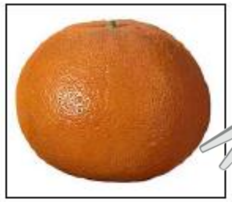
- **Local geometry:**
  Gaussian curvature *K*

$$K = \frac{1}{r_1} \cdot \frac{1}{r_2}$$

- **Topological invariant:**
  Number of holes *N*

$$2(1 - N) = \chi = \frac{1}{2\pi} \int_M K dA$$

(Gauß-Bonnet Theorem)



sphere $\chi = 2$ torus $\chi = 0$

Topology of Bloch bands

- **Local geometry:** (in k space)
  Berry curvature *Ω*

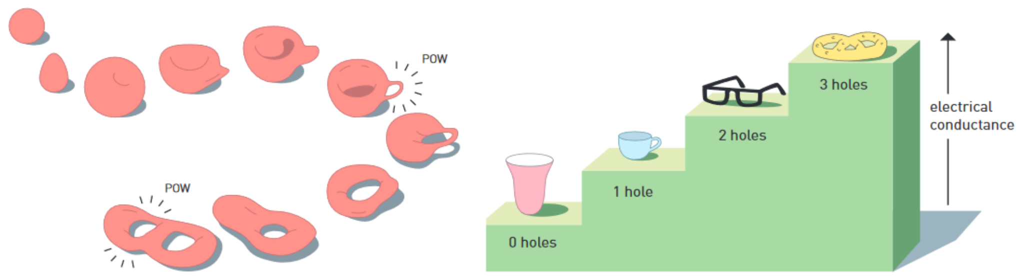
$$\mathbf{\Omega}_n(\mathbf{k}) = \nabla_{\mathbf{k}} \times i \left\langle u_{n,\mathbf{k}} \middle| \frac{\partial}{\partial \mathbf{k}} \middle| u_{n,\mathbf{k}} \right\rangle$$

- **Topological invariant:**
  Chern number *C*

$$\mathcal{C}_n = \frac{1}{2\pi} \int_{\mathbf{BZ}} d\mathbf{S} \cdot \mathbf{\Omega}_n(\mathbf{k})$$

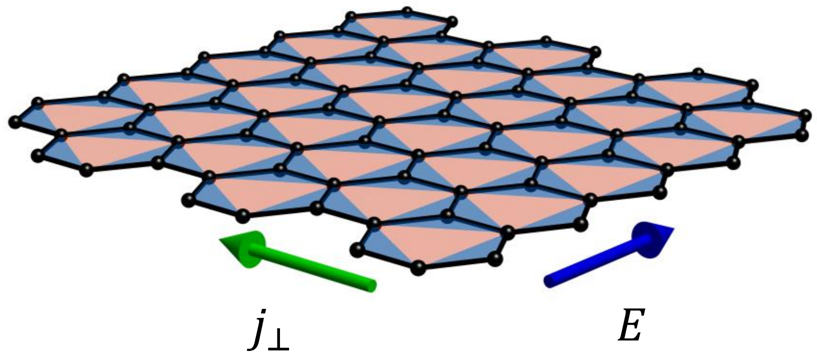
The Chern Number must be integer quantized

Motivation: Geometry and Topology



nobelprize.org

Quantized transport $j_{\perp} = (e^2/\hbar)C \cdot E$



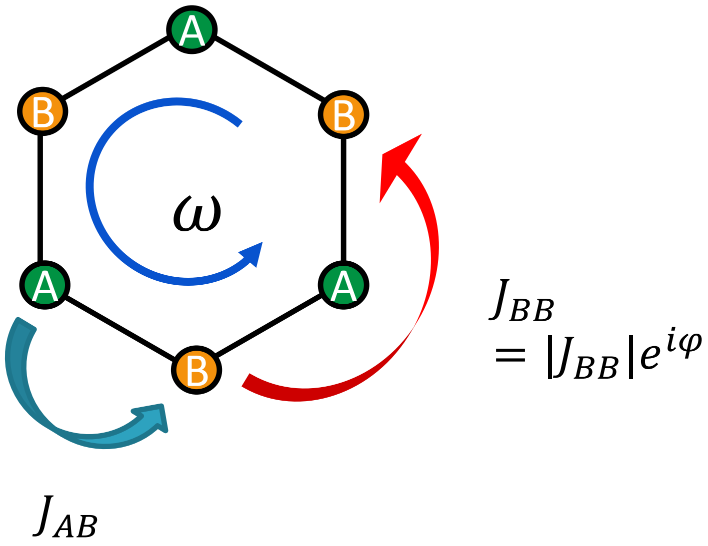
- The Chern number is related to conduction properties of real materials.
- Topological properties are robust, lot of practical applications

-> Quantum Simulation of (new) topological materials/properties with ultracold atoms!

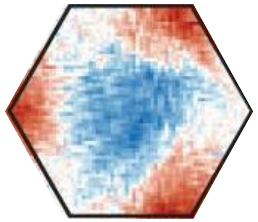
Feynman, *Int. J Theor. Phys.* **21**, 467 (1982)

Outline

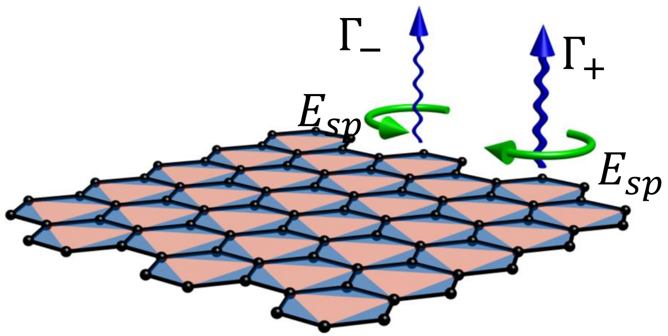
- Engineering Topological Systems



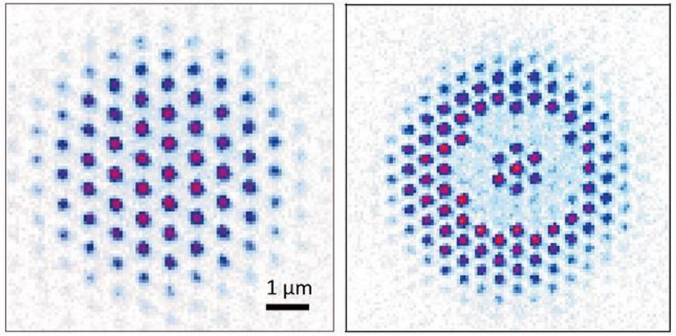
- Measuring Geometry



- New Topological effects

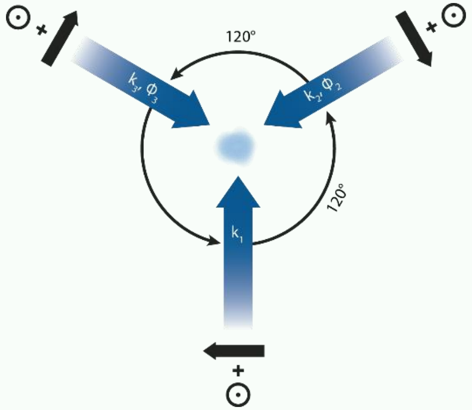


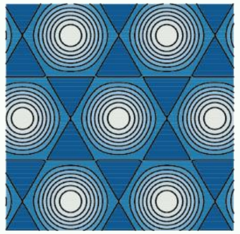
- New microscopy method (towards topology in real space?)



Optical lattice

- Optical lattices (to mimic a real crystal)





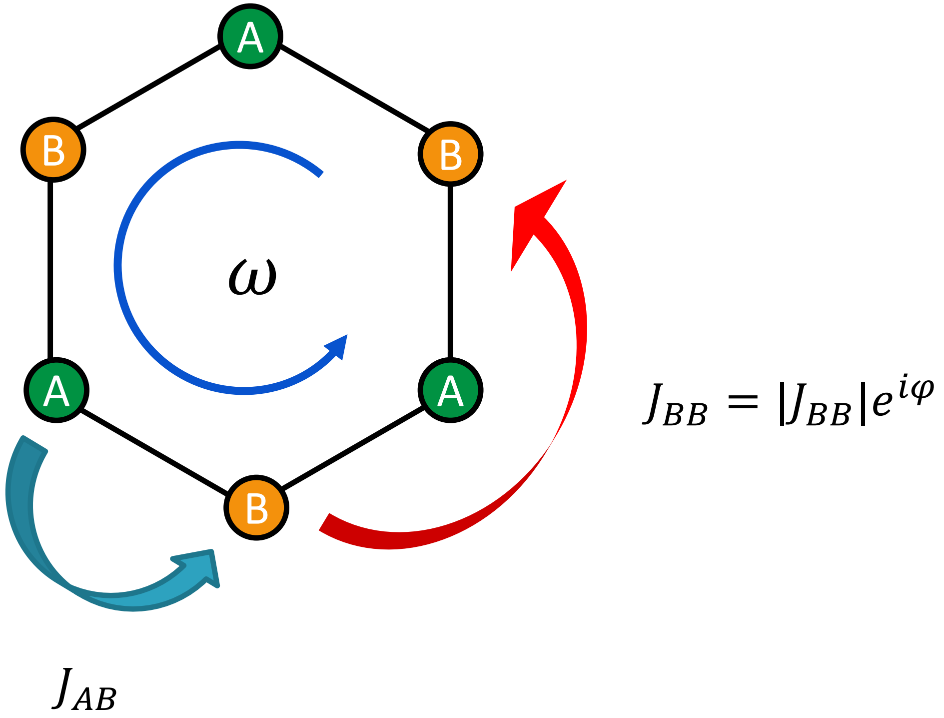
-> Honeycomb lattice
„artificial graphene“ with a tunable AB_Offset , or triangular lattice

Becker et al. New J. Phys. 12, 065025 (2010).
Soltan-Panahi et al., Nat. Phys. 7, 434 (2011)

- Fermions (40K) and/or bosons (87Rb) cooled to quantum degeneracy and loaded into the lattice, play the role of the electrons in a solid

Floquet Engineering

- $H(t+T) = H(t)$ The Hamiltonian is periodic in time with Floquet period T
  The dynamics can be described by an effective, time-independent,
  Floquet Hamiltonian H_F such that $U(T) = e^{iH_F T}$



- Floquet Engineering of a Hamiltonian which e.g. breaks time-reversal symmetry,
  with complex tunneling elements; e.g. the Haldane model

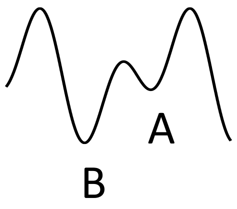
Haldane 1988:

While the particular model presented here is unlikely to be directly physically realizable, it indicates that, at least in principle, the QHE can be placed in the wider context of phenomena associated with broken time-reversal invariance, and does not necessarily require external magnetic fields, but could occur as a consequence of magnetic ordering in a quasi-two-dimensional system.

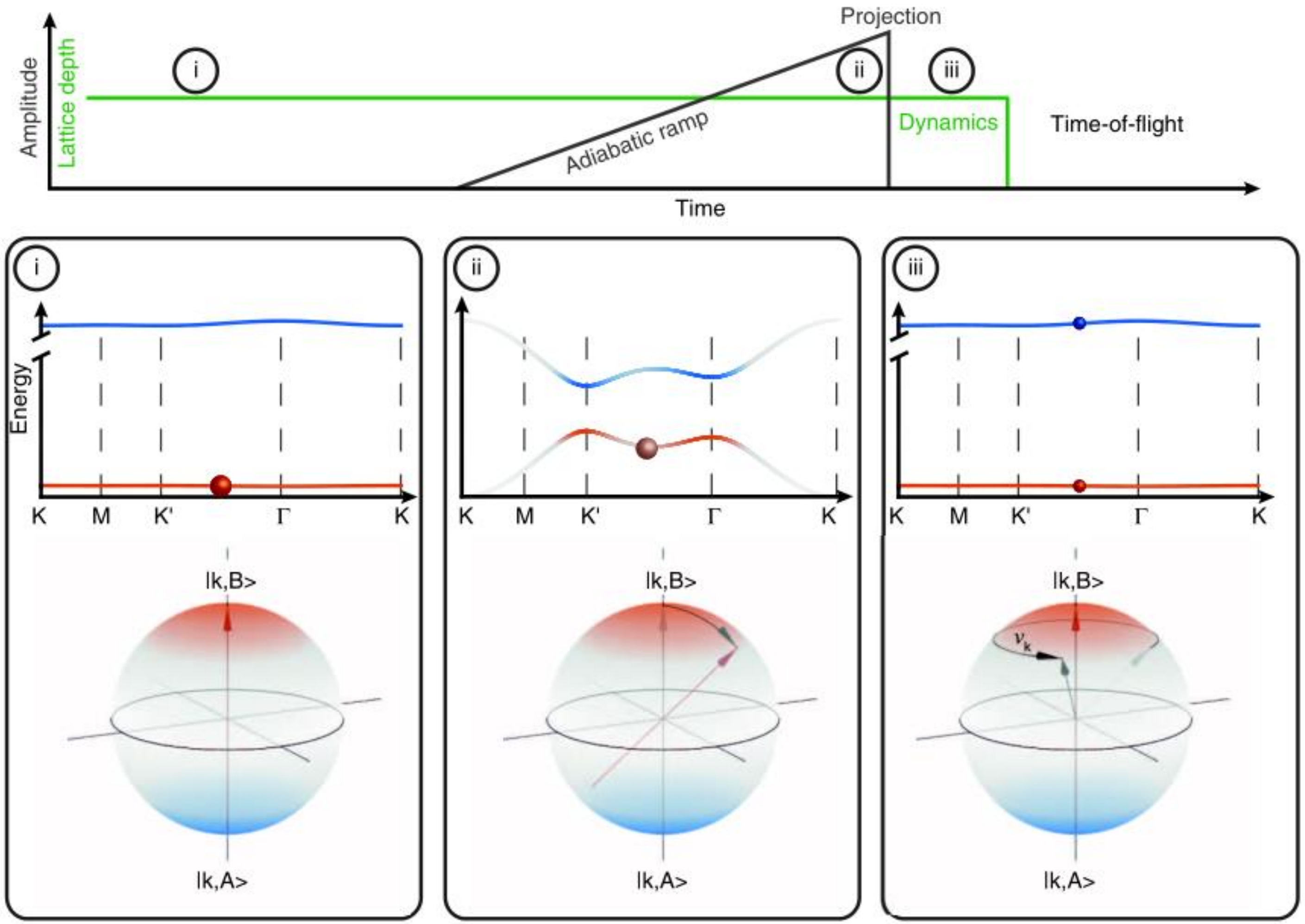
Haldane, F. D. M. , *Phys. Rev. Lett.* **61**, 2015 (1988)
Aka & Aoki, *Phys. Rev. B* **79**, 081406 (2009)
Jotzu et al, *Nature* ***515***, 237–240 (2014)

Other Schemes for generating Chern numbers:
Aidelsburger et al. *Nat. Phys.* **11,** 162–166 (2015)

State Tomography Protocol



offset: Δ

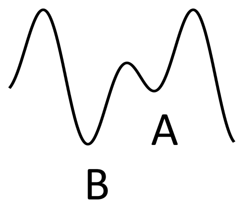


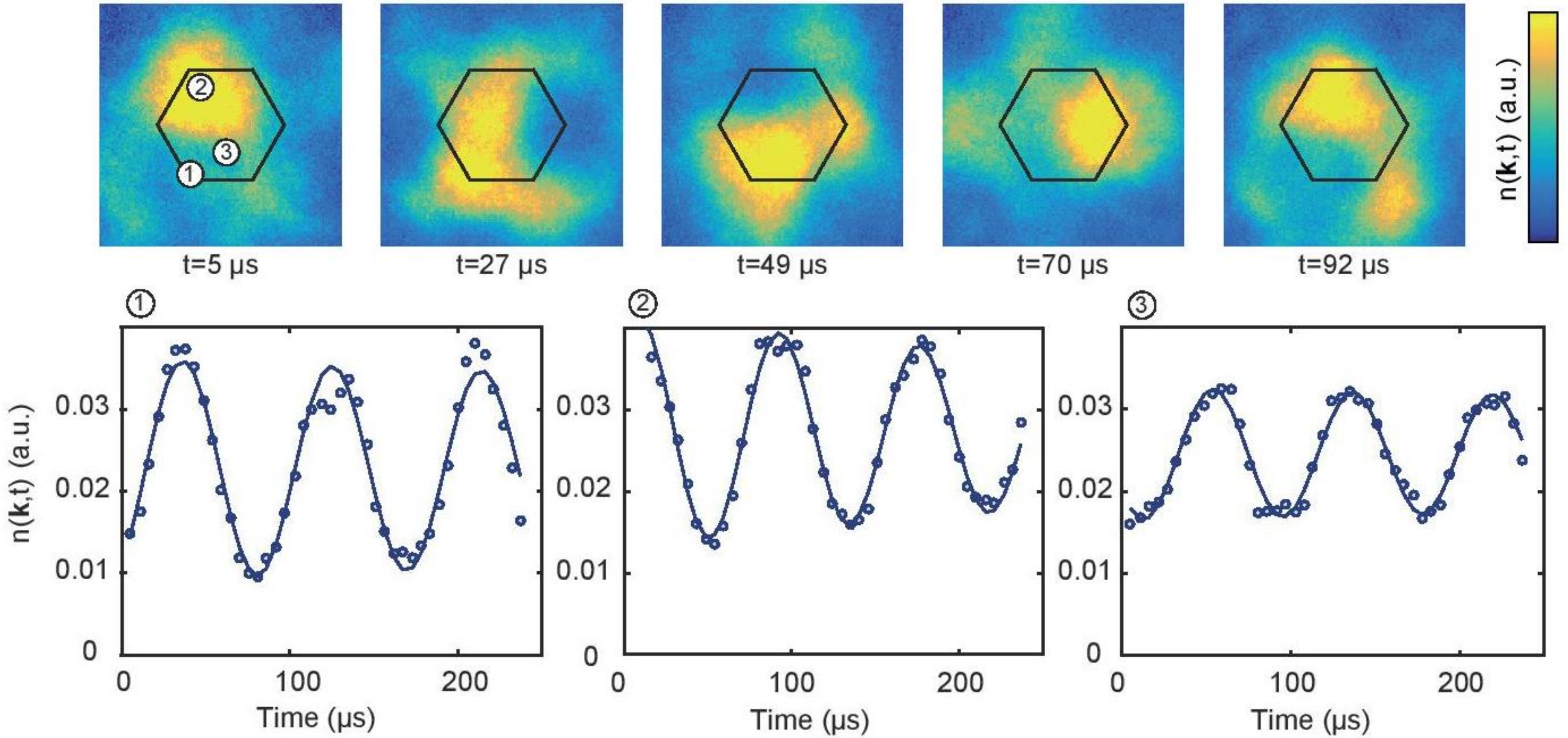
Idea: Hauke et al. *PRL* **113**, 045303 (2014),
Exp: Fläschner et al, *Science* **352,** 1091 (2016)

Adiabatically loading into the Floquet System, new eigenstates

$$|\mathbf{k}\rangle = \sin(\theta_k/2)|\mathbf{k},A\rangle - \cos(\theta_k/2)\exp(i\phi_k)|\mathbf{k},B\rangle \qquad n(\boldsymbol{k},t) = |c|^2\left(1-\sin\theta\cos(t\,\Delta/\hbar+\phi)\right)$$

Density Oscillations in Time of Flight





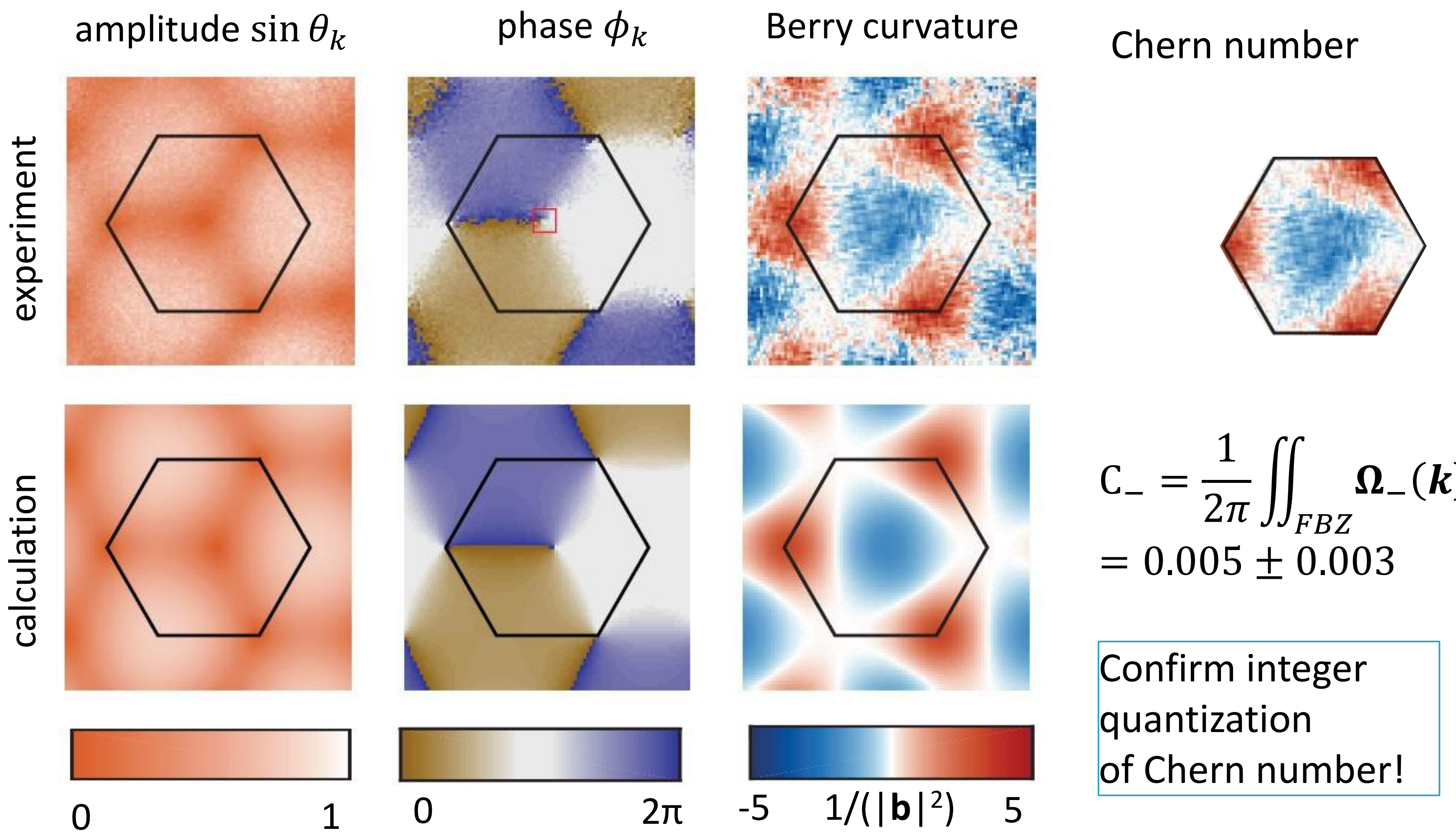
$$n(\boldsymbol{k},t) = |c|^2 \left(1 - \sin\theta \cos(t\,\Delta/\hbar + \phi)\right)$$

$$|\mathbf{k}\rangle = \sin(\theta_k/2)|\mathbf{k},A\rangle - \cos(\theta_k/2)\exp(i\phi_k)|\mathbf{k},B\rangle$$

Full State Tomography!

Fläschner et al., *Science* **352,** 1091 (2016)

Berry Curvature



$$C_- = \frac{1}{2\pi}\iint_{FBZ} \boldsymbol{\Omega}_-(\boldsymbol{k})\cdot d\boldsymbol{k}$$
$$= 0.005 \pm 0.003$$

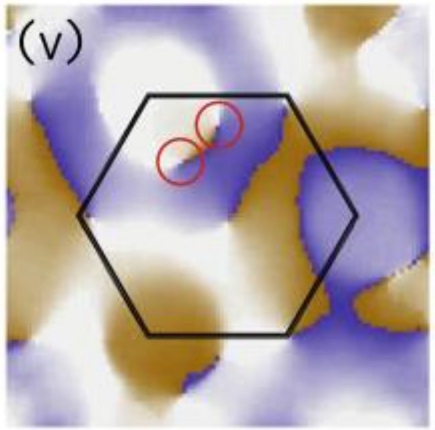
Confirm integer quantization of Chern number!

Obtain Berry curvature from derivatives of the data:

$$\boldsymbol{\Omega}_-(\boldsymbol{k}) = \boldsymbol{\nabla}_{\mathbf{k}} \times i\hbar \langle u_k^- | \frac{\partial}{\partial \boldsymbol{k}} | u_k^- \rangle = -\frac{1}{2}\sin\theta(\partial_{k_x}\theta\partial_{k_y}\phi - \partial_{k_y}\theta\partial_{k_x}\phi)\,\hat{e}_z$$

Fläschner et al., *Science* **352,** 1091 (2016)

Follow up Measurements



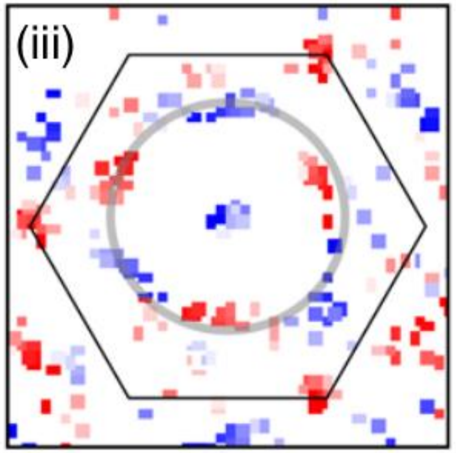
Experiment:
- Nick Fläschner
- Dominik Vogel
- Benno Rem
- Matthias Tarnowski
- Christoph Becker
- (LA)

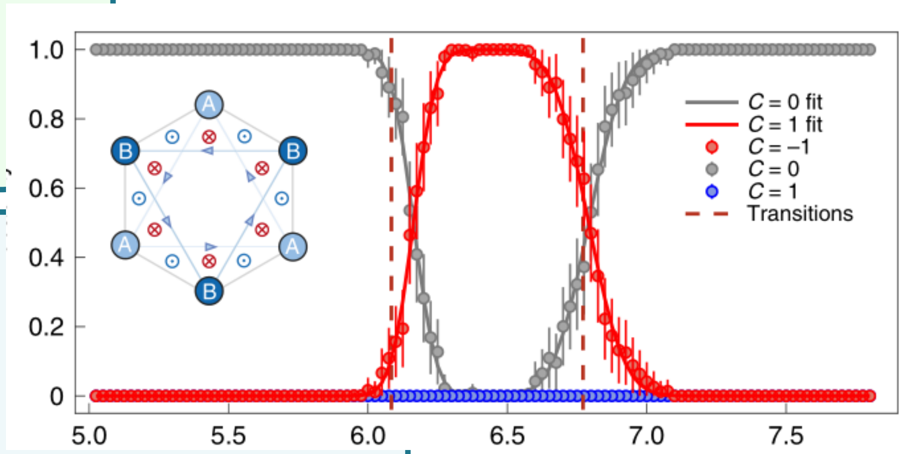
Machine Learning:
- Niklas Käming

Theory Hamburg:
- Ludwig Mathey
- Marlon Nuske
- Lukas Freystatzky
- Dirk-Sören Lühmann

Theory Dresden:
- André Eckardt
- Nur Ünal

Theory Innsbruck:
- Jan Carl Budich
- Markus Heyl





-Observation of dynamical vortices after quenches in a system with topology,
Fläschner et al, Nat. Phys. **14**, 265 (2018)

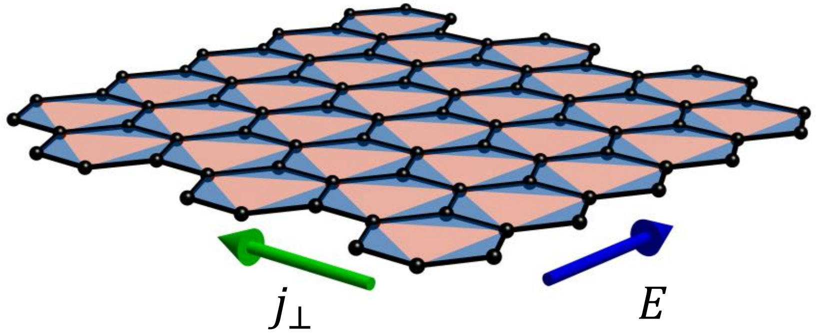
-Observation of topological Bloch-state defects and their merging transition,
Tarnowski et al., PRL **118**, 240403 (2017)

-Measuring topology from dynamics by obtaining the Chern number from a linking number,
Tarnowski et al, Nat. Comm. **10**, 1728 (2019)

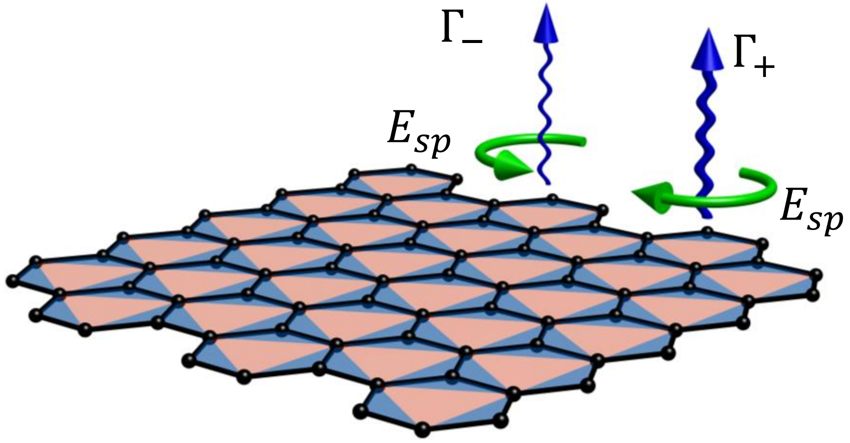
-Identifying Quantum Phase Transitions using Artificial Neural Networks on Experimental Data,
Rem et al,Nat. Phys. **15**, 917 (2019)

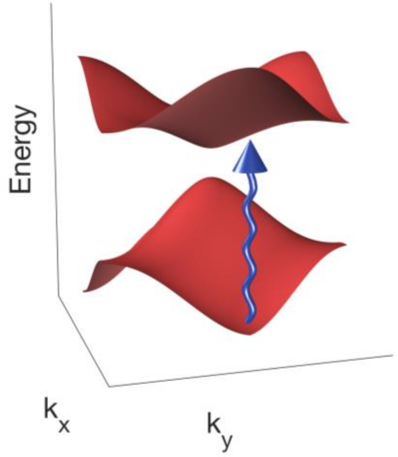
Quantized circular dichroism

Quantized transport $j_\perp = (e^2/\hbar) C \cdot E$



Quantized depletion $\Delta\Gamma_{\pm}^{int}/A_{cell} = \left(\frac{1}{\hbar^2}\right) C \cdot E_{sp}^2$





Idea: D. T. Tran, et. al, *Science Advances* **3**, e1701207 (2017)
Exp: Asteria et al, *Nat. Phys.* **15**,449-454 (2019)

Quantized circular dichroism in Chern insulators

$$\hat{H}_{\pm}(t) = \hat{H}_0 + 2E\{\cos(\omega t)\,\hat{x} \pm \sin(\omega t)\,\hat{y}\}$$

Topological Hamiltonian (e.g. Haldane model)

Chiral spectroscopy

- Transformation into the accelerated frame… $H'_{\pm} = R^{-1}\, H_{\pm}\, R$
- Leads to $H'_{\pm}(t) \approx \hat{H}_0 + \frac{2E}{\hbar\omega}\left\{\sin(\omega t)\frac{\partial \hat{H}_0}{\partial k_x} \mp \cos(\omega t)\frac{\partial \hat{H}_0}{\partial k_y}\right\}$

- ´Fermi‘s golden rule: $\Gamma_{\pm}(\omega) = \frac{2\pi}{\hbar}\left(\frac{E}{\hbar\omega}\right)^2 \sum_k \sum_{n>0} \left|\left\langle n \left| \frac{1}{i}\frac{\partial \hat{H}_0}{\partial k_x} \mp \frac{\partial \hat{H}_0}{\partial k_y} \right| 0 \right\rangle\right|^2 \delta(\varepsilon_n(k) - \varepsilon_0(k) - \omega)$
- **Differential integrated rate:**

$$\Delta\Gamma_{\pm}^{int} \equiv \int \frac{d\omega[\Gamma_+(\omega) - \Gamma_-(\omega)]}{2} = \left(\frac{E}{\hbar}\right)^2 \underbrace{4\pi\,\mathrm{Im} \sum_k \sum_{n>0} \left\langle 0 \left| \frac{\partial H_0}{\partial k_x} \right| n \right\rangle \left\langle n \left| \frac{\partial H_0}{\partial k_y} \right| 0 \right\rangle / (\varepsilon_0 - \varepsilon_n)^2}_{= C \cdot A_{\mathrm{cell}}}$$

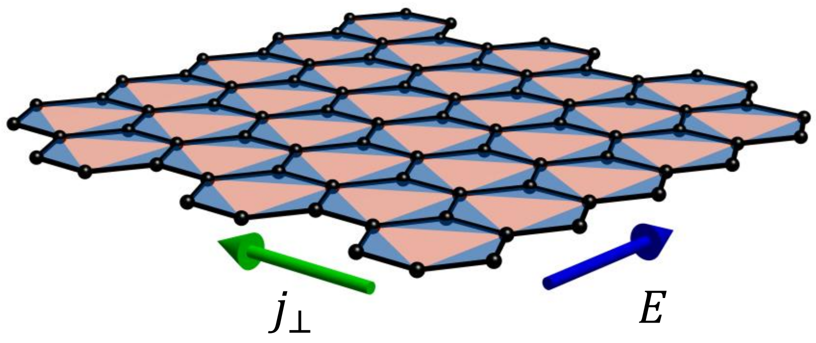
$= C \cdot A_{\mathrm{cell}}$ Chern number

$$\Delta\Gamma_{\pm}^{int} / A_{cell} = (1/\hbar^2)\, C \cdot E^2$$

D. T. Tran, et. al, *Science Advances* **3**, e1701207 (2017)

Quantum Hall effect and Quantized circular dichroism

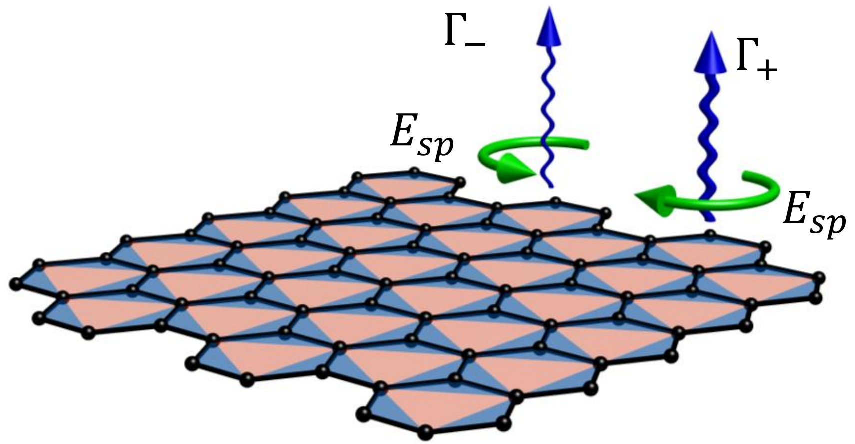
Quantized transport $j_{\perp} = (e^2/\hbar)C \cdot E$



„reactive“ response

Kramers-Kronig
Relations

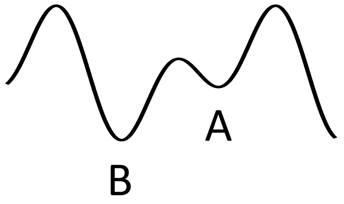
Quantized depletion $\Delta\Gamma_{\pm}^{int}/A_{cell} = \left(\frac{1}{\hbar^2}\right) C \cdot E_{sp}^2$



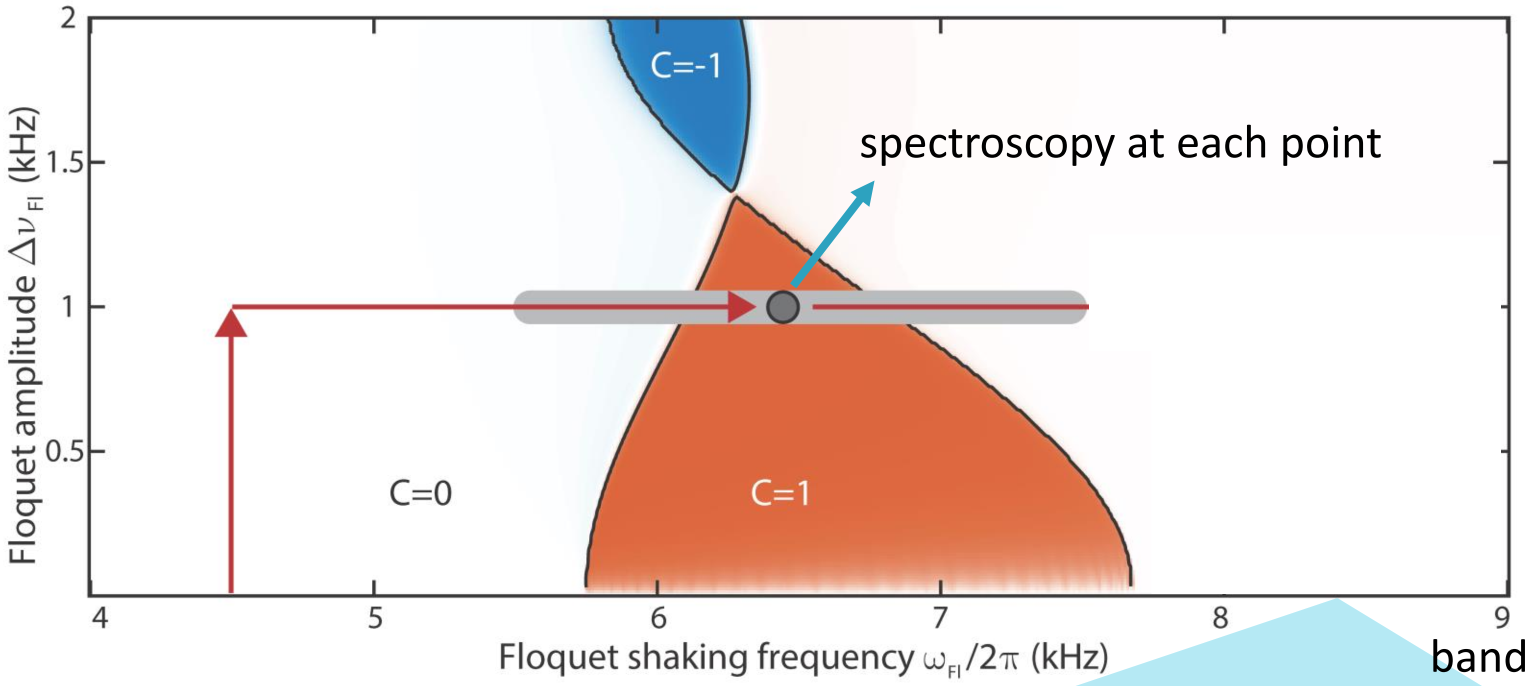
„dissipative“ response

D. T. Tran, et. al, *Science Advances* **3**, e1701207 (2017)

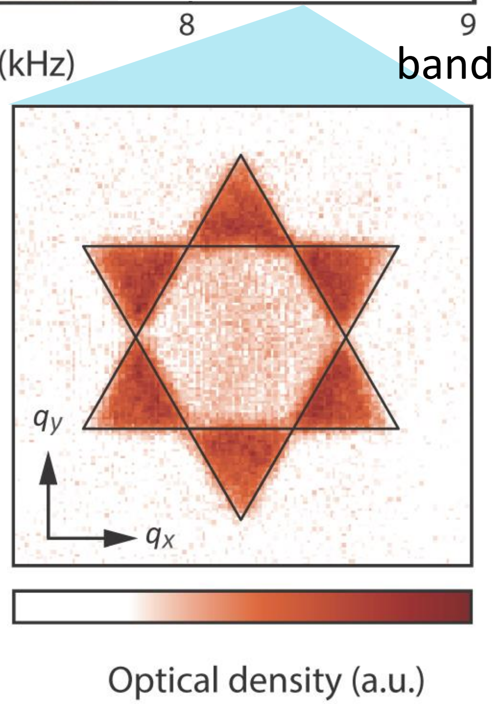
Experimental procedure



offset:
$\Delta \sim 6$ kHz



band mapping

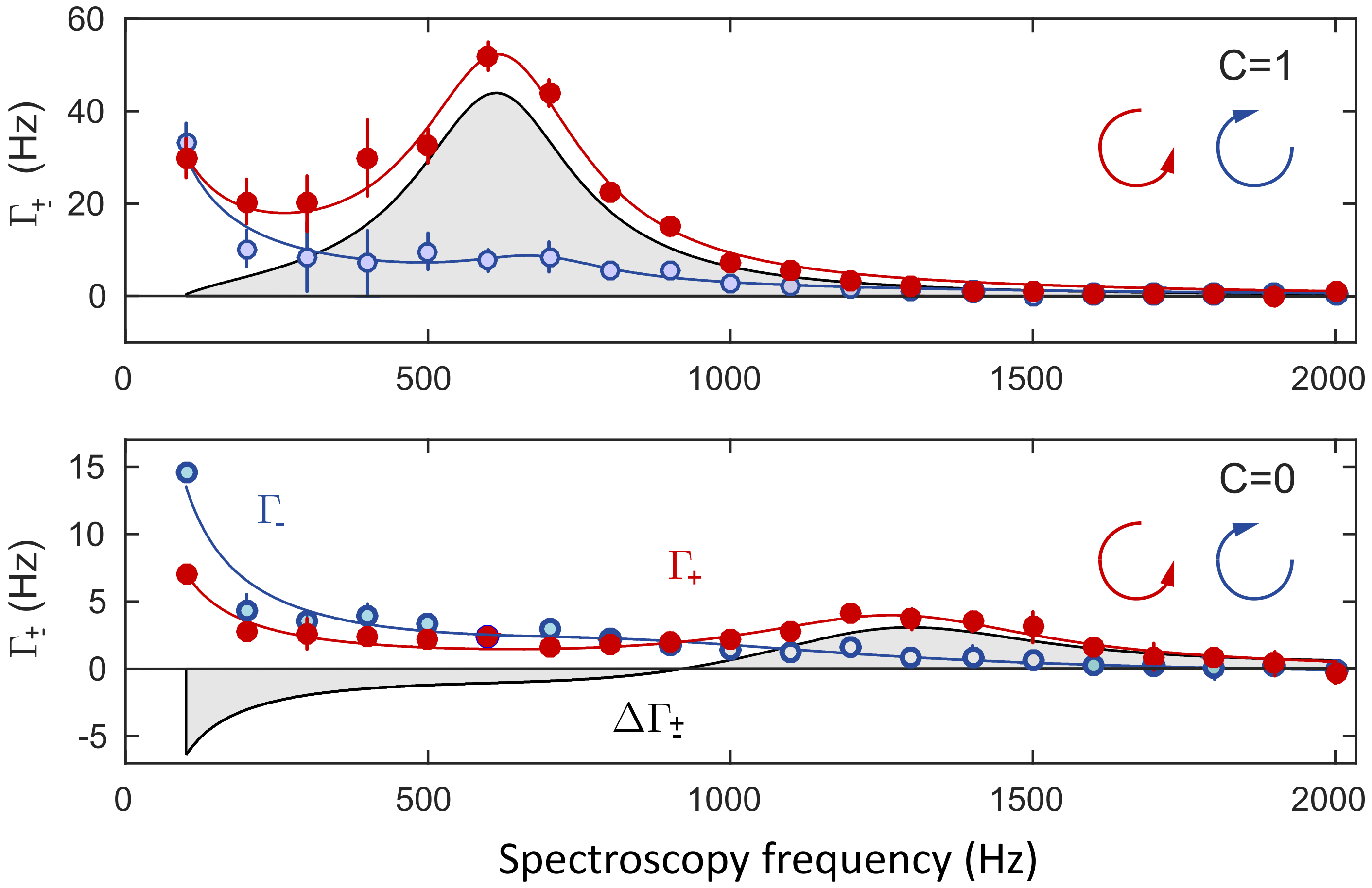


- Spectroscopy via (additional) circular lattice shaking
- Spectroscopy at smaller frequencies than Floquet engineering frequencies

-> Separation of timescales (~1kHz < ~ 6kHz)

Asteria et al, *Nat. Phys.* **15**,449-454 (2019)

Chiral spectra



$C_{\text{exp}} = 0.92(12)$

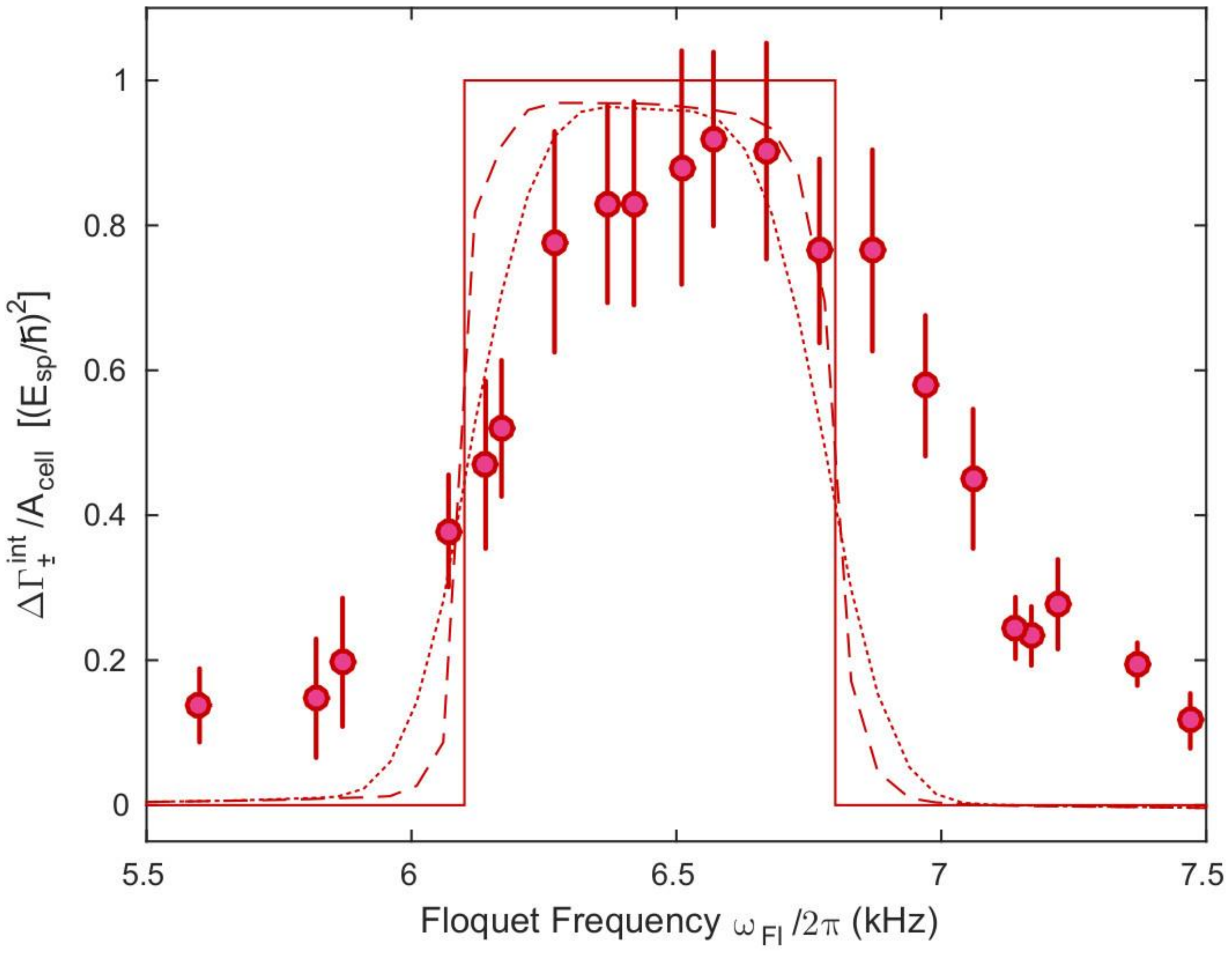
$C_{\text{exp}} = 0.12(4)$

Grey area $\Delta\Gamma_\pm^{int} = \int d\omega[\Gamma_+(\omega) - \Gamma_-(\omega)]/2$

Dichroic signal $C_{\text{exp}} = \Delta\Gamma^{\text{int}}/A_{\text{cell}} \cdot (\hbar/E_{\text{sp}})^2$

Exp. confirmation of quantized circular dichroism!

Asteria et al, *Nat. Phys.* **15**,449-454 (2019)



Circular Dichroism:

- New Topological effect
- Useful for detecting topology, also for fractional states

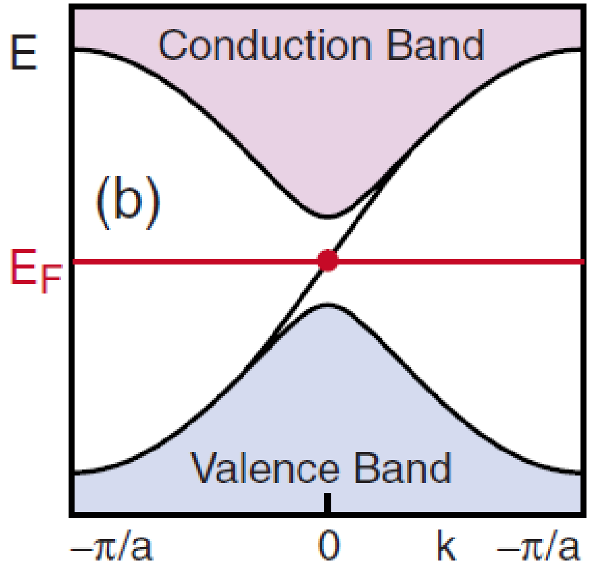
Repellin & Goldman, **PRL 122, 166801 (2019)**

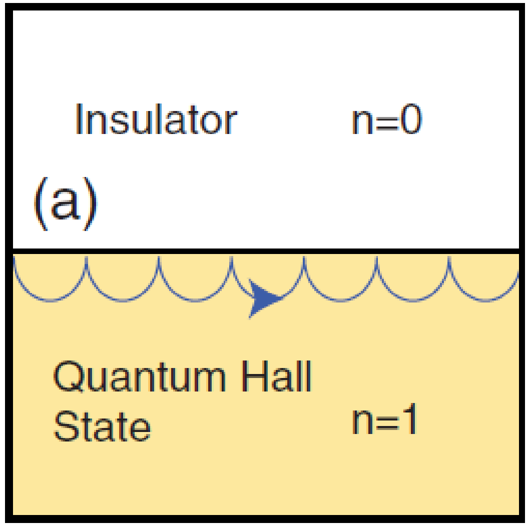
Measuring quantized circular dichroism in ultracold topological matter

Asteria et al, *Nat. Phys.* **15**,449-454 (2019)

Microscopy of topological Systems

Transport properties, topological edge states are real space observables



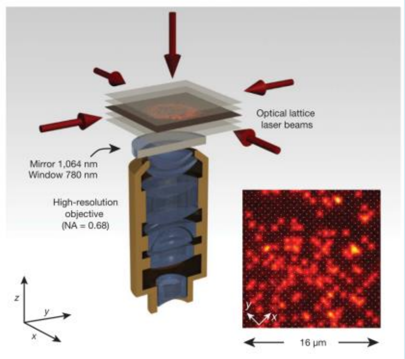


-> Need to look at the system in real space

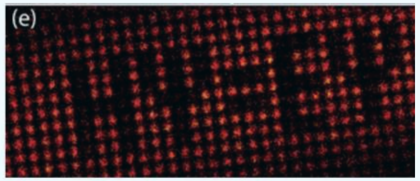
Microscopy of the interacting Harper–Hofstadter model in the two-body limit
Tai et al., *Nature* **546**, 519–523 (2017)

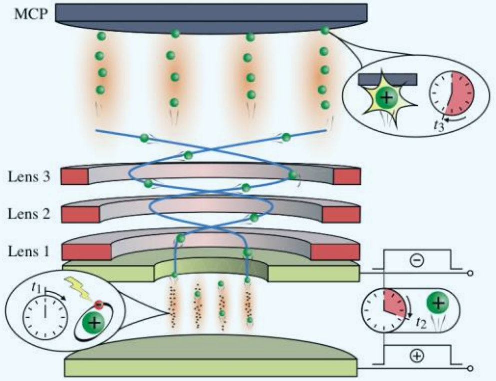
Microscopy of ultracold Atoms

Quantum Gas Microscopes



W. S. Bakr et al., Science 329, 547 (2010)
J. F. Sherson et al., Nature 467, 68 (2010),
and many more…





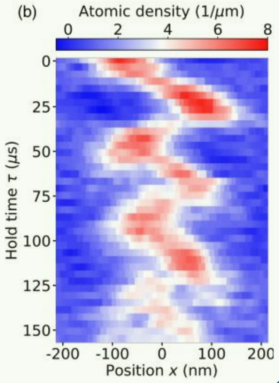
Electron Beam

Ion Microscope

P. Würtz et al., PRL 103, 080404 (2009)
C. Veit et al., Phys. Rev. X 11, 011036 (2021).

Superresolution
Microscopes

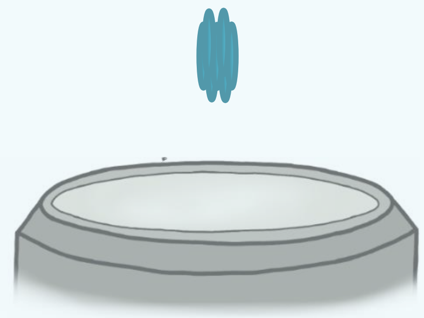
M. McDonald et al., Phys. Rev. X 9, 21001 (2019).
S. Subhankar et al., Phys. Rev. X 9, 21002 (2019).



Alternative approach:
Quantum Gas Magnifier

New approach: magnify the quantum System

High resolution imaging

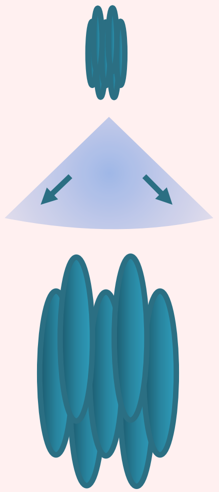


Optical lens

Getting closer to the quantum system….

…or bring it closer to us!

Quantum Gas Magnifier



Matter wave lens

Get a single-shot, sub-lattice resolved image of the 3D system!

Asteria et al., arXiv:2104.10089 (2021)

The Quantum Gas Magnifier

Array of „tubes“, triangular lattice, $\sim 10^5$ ^{87}Rb atoms

Initial position x_0

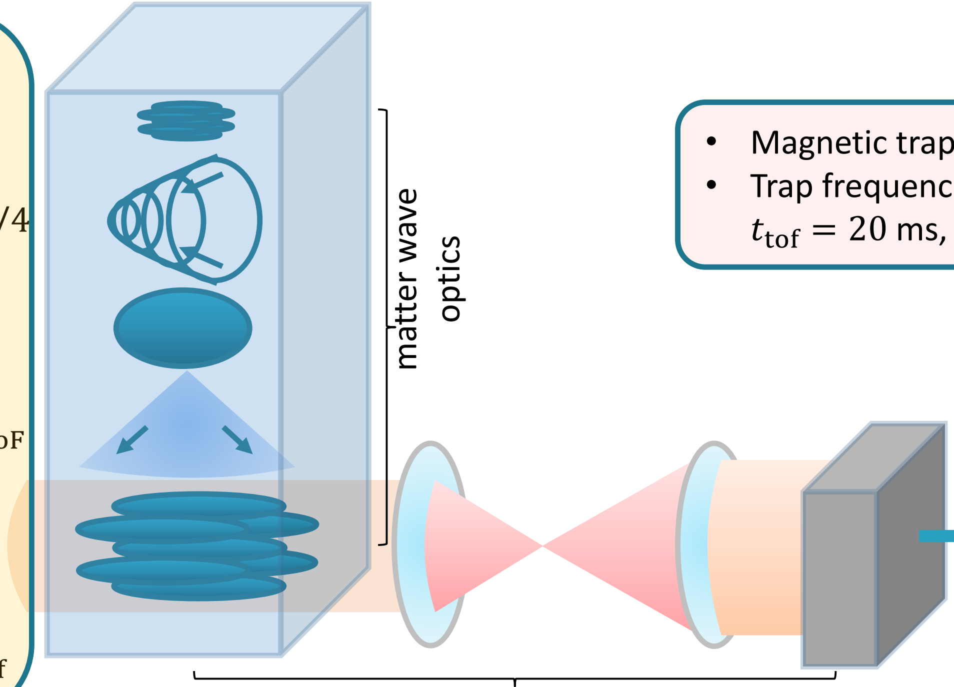
Harmonic trap ω for $T/4$

Momentum $p = m\omega x_0$

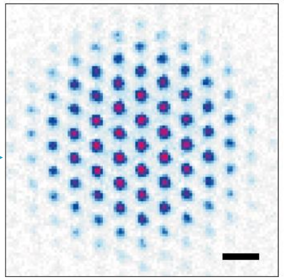
Free expansion for t_{ToF}

Final position
$x = p/m\, t_{\mathrm{tof}} = \omega t_{\mathrm{tof}} x_0$

Magnification $M = \omega t_{\mathrm{tof}}$



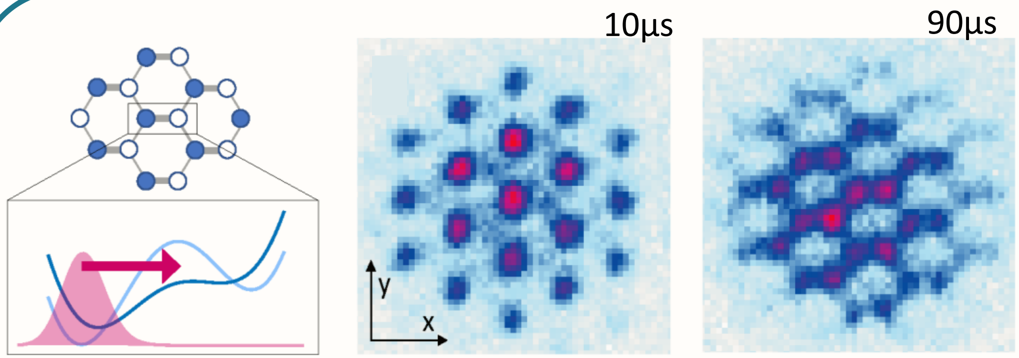
- Magnetic trap is smooth and isotropic
- Trap frequencies up to $\omega/2\pi$ =700 Hz, $t_{\mathrm{tof}} = 20$ ms, up to 93-fold magnification



- Optical imaging ($a_{\mathrm{lat}} \triangleq 60$ µm)

Fantastic Resolution, single-shot image!

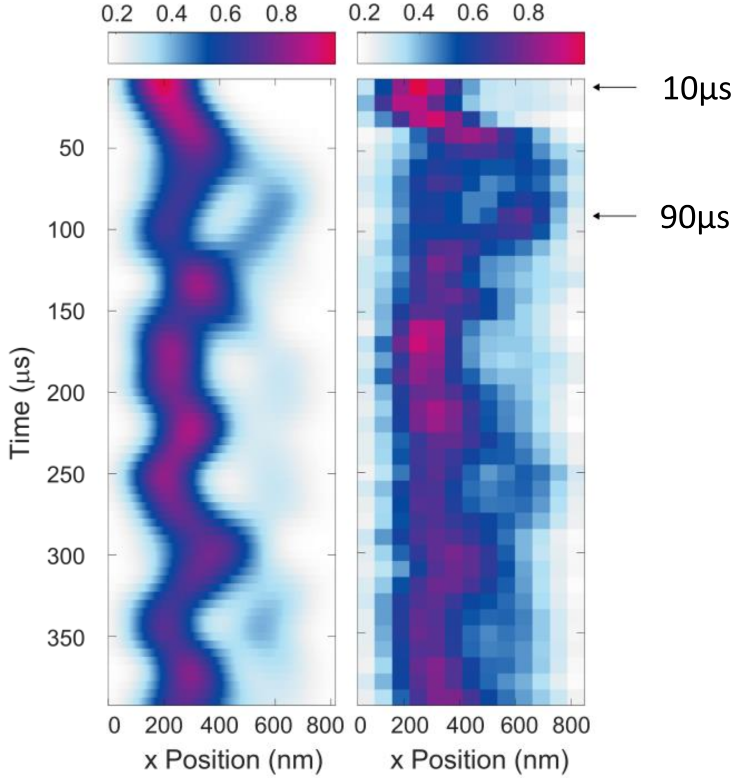
Asteria et al., arXiv:2104.10089 (2021)



- Quench to uncoupled dimers potential
- Induce both an oscillation and a tunneling dynamics

- Observing dynamics at the sub-lattice-spacing level
- Towards real-space study of multi-band systems (?)

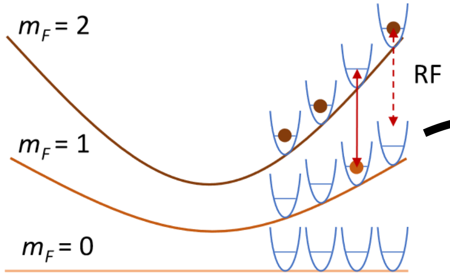
Compare with:
McDonald/Chin et al., *PRX* **9**, 21001 (2019).
Subhankar/Porto et al., *PRX* **9**, 21002 (2019).



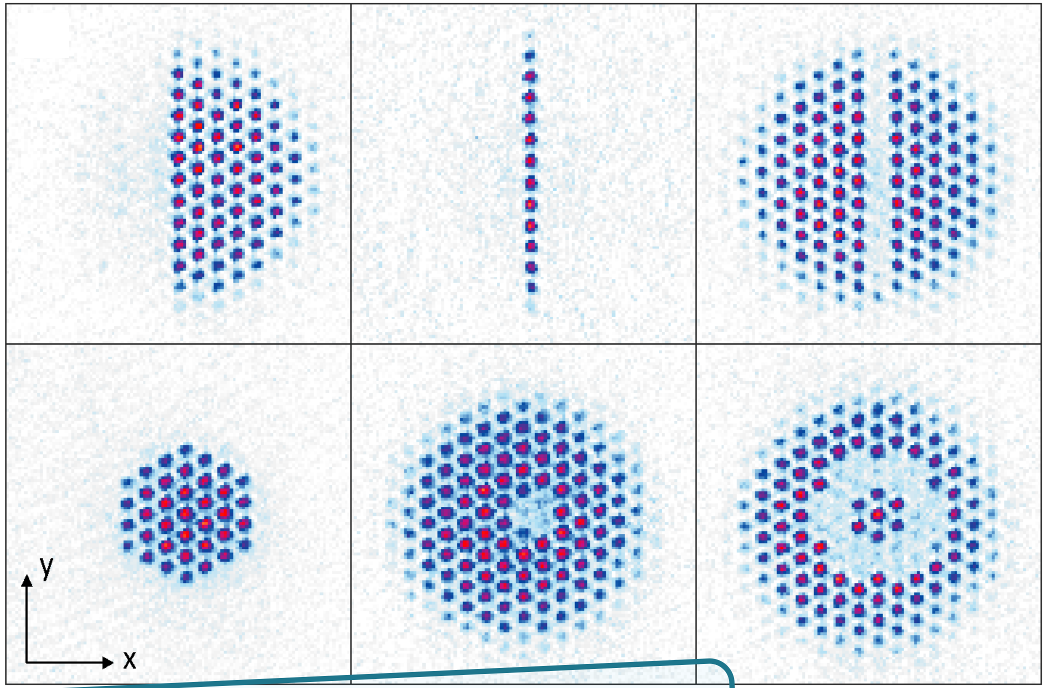
Simulation
~80nm Gauss filter

Experiment

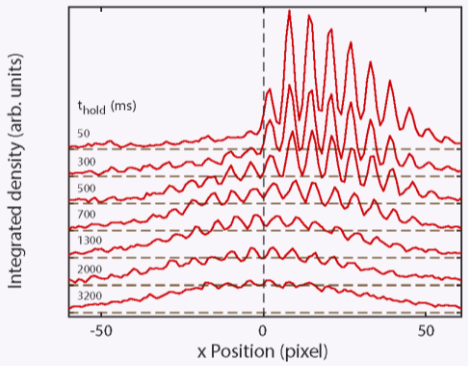
Site-Resolved RF Addressing



Position Dependent Resonances to a state with losses in magnetic field gradient $(\Delta E \sim 10^2 - 10^3 \text{Hz})$



Shape the density distribution at the single site level !



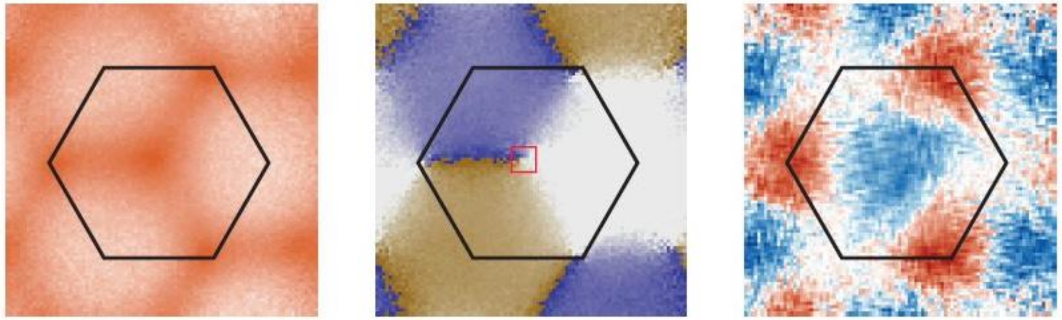
- Thermalization as a function of time

Choi/Bloch et al., *Science* **352**, 1547 (2016).

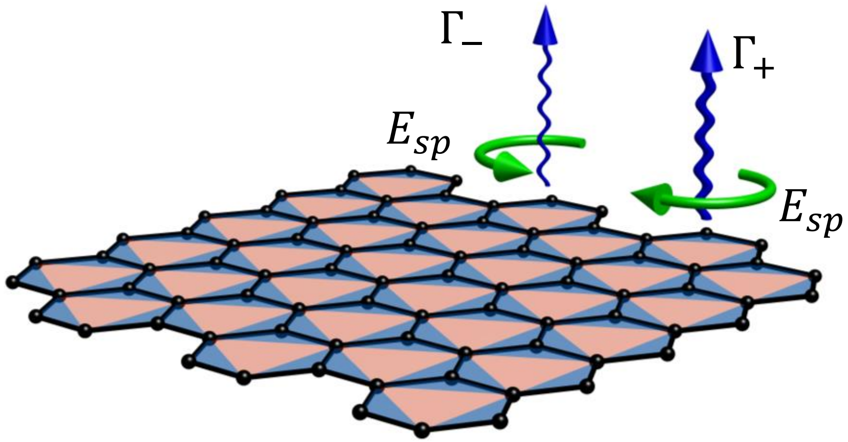
- Systems out of equilibrium
- Preparing wavepackets with high overlap with topological edge states

Summary

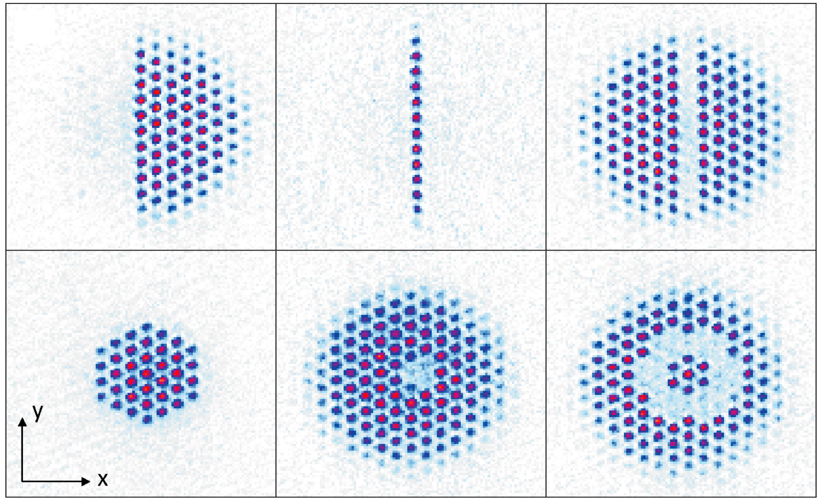
- Ultracold atoms are very *cool*
- Measurement of the Berry Curvature
- Quantization of Circular Dichroism
- Towards topology in real space?



Experimental reconstruction of the Berry curvature in a Floquet Bloch band
Fläschner et al., *Science* **352,** 1091 (2016)



Measuring quantized circular dichroism in ultracold topological matter
Asteria et al, *Nat. Phys.* **15**,449-454 (2019)



Quantum gas magnifier for sub-lattice-resolved imaging of three-dimensional quantum systems
Asteria et al., arXiv:2104.10089 (2021)

The "Topology" of the Collaborations

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Circular Dichroism

Quantum Gas Magnifier

Thank You for your Attention!