Quantum simulation of gauge theories: from non-Abelian to Abelian via the *encoding route*

Virtual workshop@ECT*

19/7/21

Marcello Dalmonte

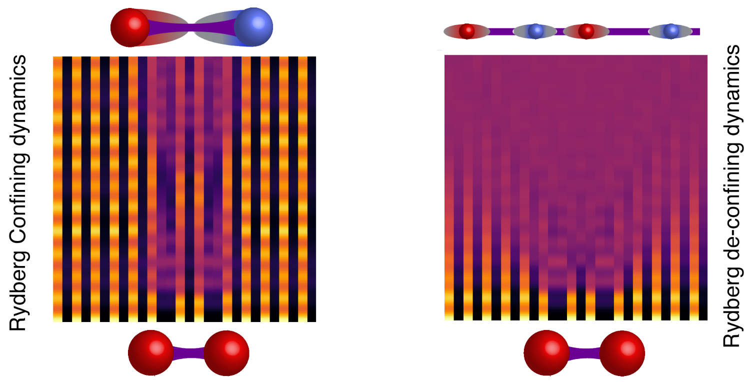
ICTP&SISSA, Trieste

Joint work with **E. Rico** (Bilbao), P. Zoller (Innsbruck), U.-J. Wiese, P. Stebler, **D. Banerjee** and M. Bögli (Bern) and
F. Surace, P. Mazza, G. Giudici, A. Lerose, A. Gambassi (Trieste)

Based on: Annals of Phys. 393 (2018) 466 and PRX 10, 021041 (2020)

Brief outline

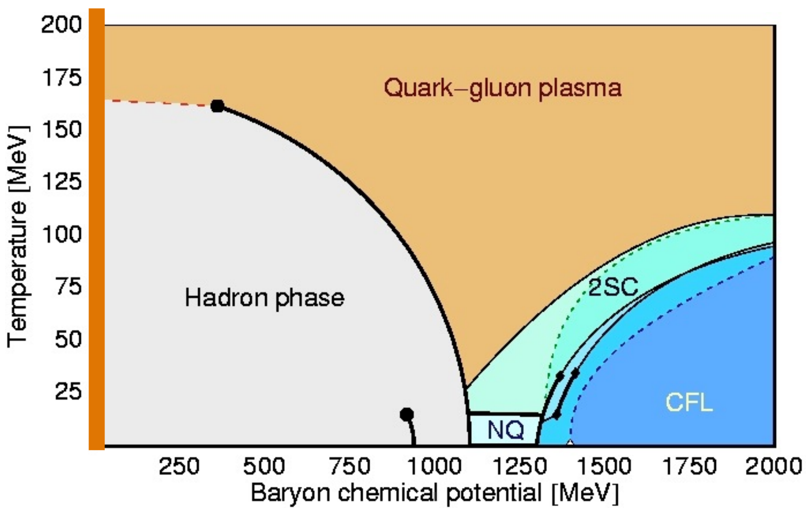
- A brief panoramic on analog quantum simulators for gauge theories
- How to deal with gauge invariance? encoding strategies
- “Nuclear” physics with SO(3) models in cold atoms



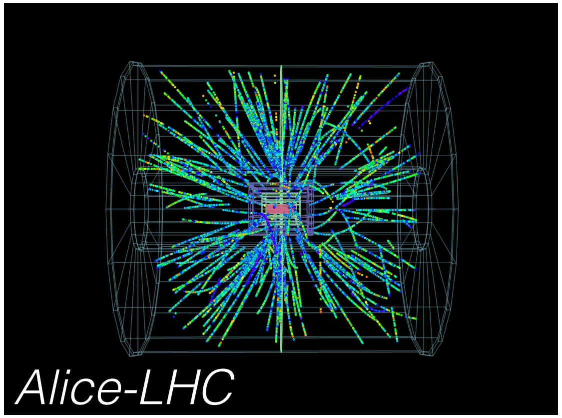
- (Large scale) quantum simulations of U(1) theories

Challenges in gauge field theories

Tackling gauge theories is of pivotal importance for quantum simulation of HEP **- real time, sign problems**



UniFrankfurt website



Clear challenges:

- **real time dynamics**
- **'finite-density'**

Montvay and Münster, *Quantum fields on a Lattice*

Panorama of ‘quantum simulations’ for NP/HEP

Models

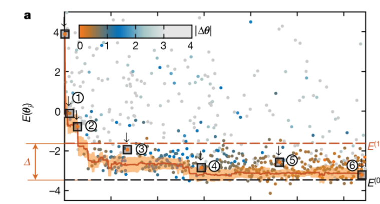
ab initio

Gauge theories

effective

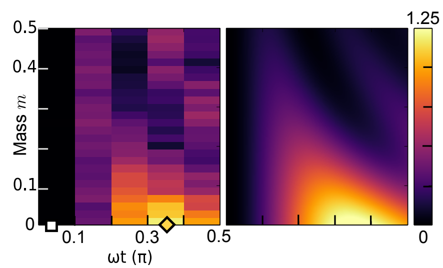
Unitary Fermi gas

Hybrid



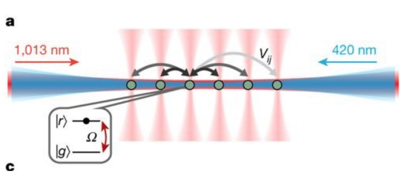
Kokail et al., Nature 2019

Digital



Martinez et al., Nature 2016

Analog



Bernier et al., Nature 2017

Outputs

New perspectives / interfaces

Qualitative insight

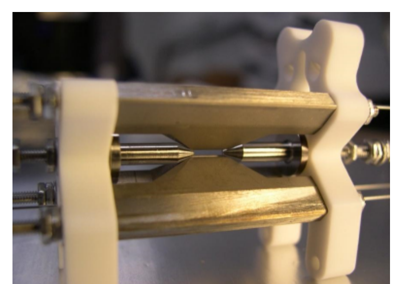
Quantitative predictions

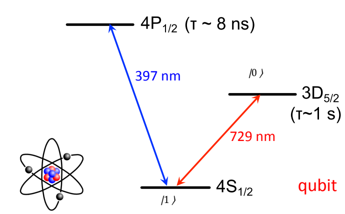
Techniques

Analog quantum simulators in a nutshel

$$H = H_1 + H_2 + \ldots$$

a) (exact) mapping to microscopic degrees of freedom
b) probing tools, protocols (e.g., state preparation)
c) ‘understanding’ of errors





Analog simulation: challenges

Typical challenges for quantum simulators:

- initial state preparation
- probing
- engineer the desired dynamics
- validate / control
- probing

- same as SM quantum simulators
- novel HEP challenges!

Main challenge: engineer gauge invariance

—> shift of paradigm: from *interaction engineering*, to *symmetry engineering*

Full Hilbert space: state of the art

Theory proposals:

- early 2000's: first quenched proposals
- 2012: first proposals including dynamical matter
- 2012/3: first (and almost last) non-Abelian
- more following 2013:
    - Abelian: >100 theory proposals.
    - Non-Abelian: <5 works.

Ions - see Zohreh's talk later today!

Incomplete (!) list of contributors:
Banerjee, Rico, Wiese, Zoller, Marcos, Hafezi, Hauke, Stebler, Cirac, Zohar, Reznik, Meurice, Celi, Tagliacozzo, Lewenstein, Glaetzle, Moessner, Nath, Kapit, Müller, Davoudi, Pagano, Savage, Monroe, Barbiero, Kaplan, Syrker, Farace, Lukin, Pichler, Solano, …

HEP Reviews: U. J. Wiese, Ann. Phys. 525, 777 (2013); Preskill, arXiv.1811.10085 (2018).
"Pedagogical": MD and S. Montangero, Cont. Rev. Phys. 2016 / 1602.03776.
More advanced ones: Rep. Prog. Phys. 79, 014401 (2016); 1910.00257; 1911.00003.

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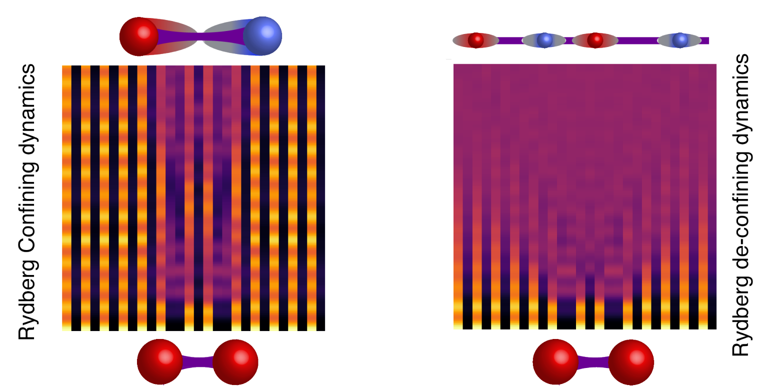


- (Large scale) quantum simulations of U(1) theories

Gauge theories with Heisenberg models?

Q: can we formulate a model, that

A) shows interesting features connected with nuclear physics and QCD (and possibly more)

Chiral condensation and symmetry breaking

Bound states

B) can be encoded onto a simple dynamics, such as the one described by super-exchange in mixtures?

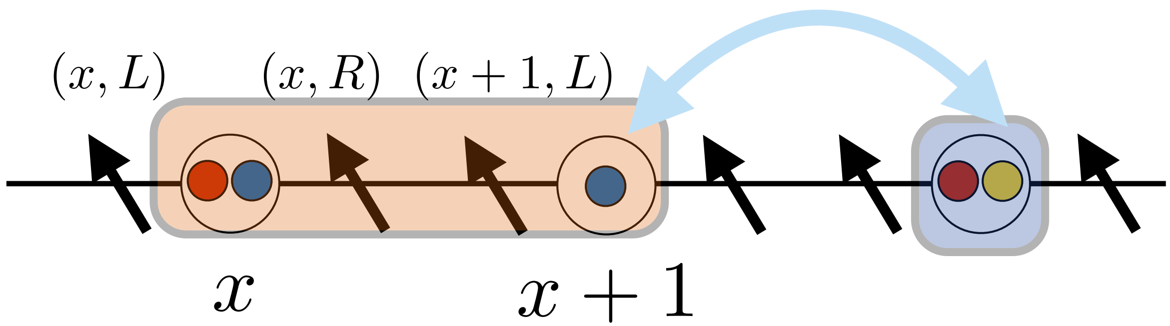
$$H_{\text{enc.}} \simeq \sum_{i,j;\alpha} J_\alpha S_j^\alpha S_i^\alpha$$

SO(3) gauge theory

$$H = -t \sum_x [(\psi_x^a)^\dagger \sigma^a_{x,R} \sigma^b_{x+1,L} \psi^b_{x+1} + \text{h.c.}] +$$
$$+ \sum_x [V n_x n_{x+1} + G n_x^2]$$

$a = 1, 2, 3$

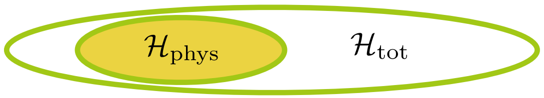
Color index

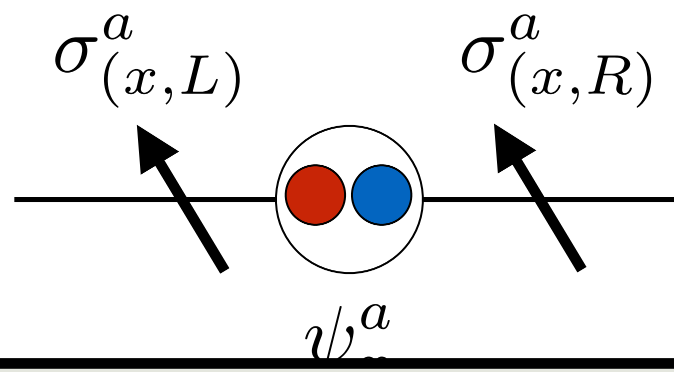


Formalism: D-theories / quantum link models

Horn 1981, Orland & Rohrlich 1990,
Chandrasekharand & Wiese, 1997

Gauge invariant Hilbert space





$a = 1, 2, 3$

Gau

The local gauge invariant Hilbert space **looks really like a Spin 3/2**! But before encoding, let us look at the **phase diagram**

(9.32)

$]|0\rangle_x$

$|b_x = 3\rangle_x = \frac{i}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)\, \psi^{1\dagger}\psi^{2\dagger}\psi^{3\dagger}|0\rangle_x$

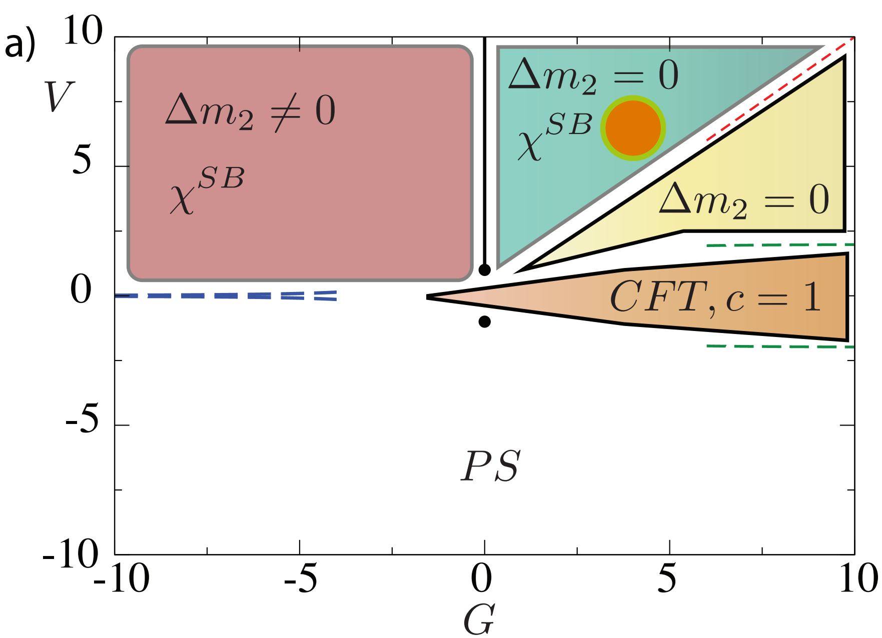
$\dim[\mathcal{H}_{\text{phys}}] = 4$

Chiral symmetry breaking

$$H = -t\sum_x [(\psi_x^a)^\dagger \sigma^a_{x,R}\sigma^b_{x+1,L}\psi^b_{x+1} + \text{h.c.}] + \\ + \sum_x [V n_x n_{x+1} + G n_x^2]$$

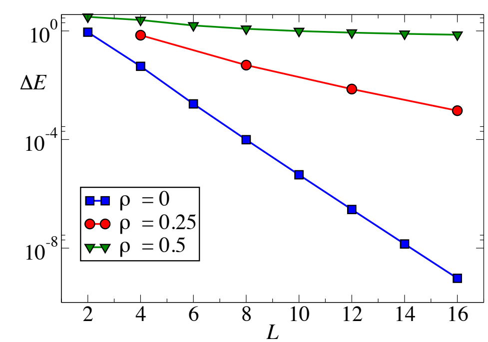
Chiral Symmetry: translation by one lattice spacing

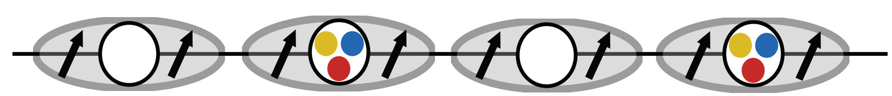
$$\chi \psi_x^a = (-1)^x \psi^a_{x+1}$$
$$\chi \sigma^a_{x,\beta} = (-1)^x \sigma^a_{x+1,\beta}$$



$\mathbb{Z}_2$ Chiral Symmetry breaking

$$\Delta E \simeq e^{-\alpha L}$$

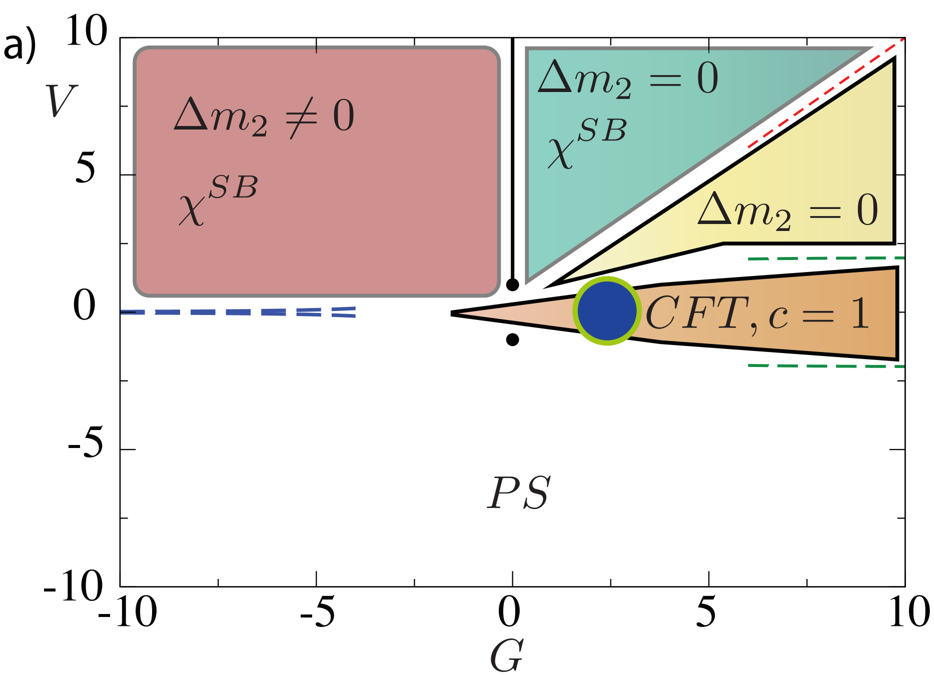




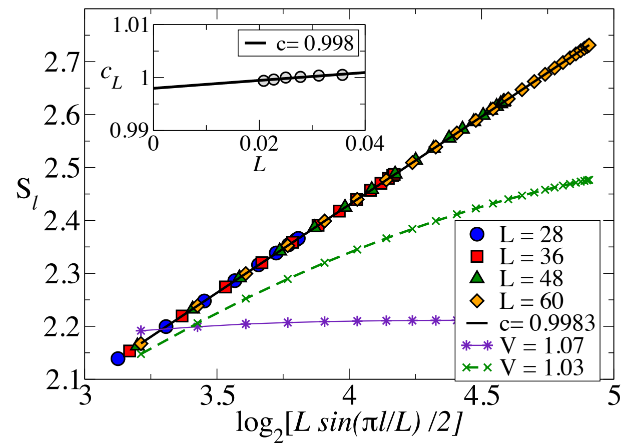
Results: ED (L up to 16), DMRG/PBC (L up to 72)

Conformal window

$$H = -t\sum_x [(\psi_x^a)^\dagger \sigma^a_{x,R}\sigma^b_{x+1,L}\psi^b_{x+1} + \text{h.c.}] +$$
$$+\sum_x [V n_x n_{x+1} + G n_x^2]$$



Conformal Window



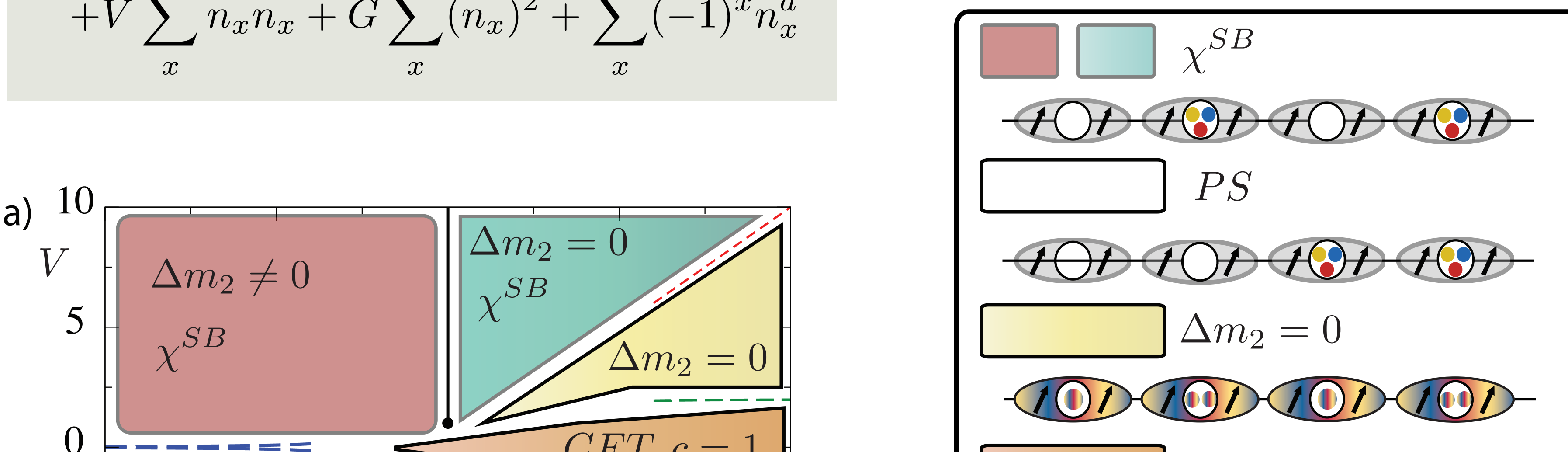
$$S_\ell = \frac{c}{3}\ln[\frac{L}{\pi}\sin(\pi\ell/L)] + ...$$

Holzhey et al, 1994; Calabrese and Cardy 2004

Connected to beyond-Higgs physics - slowly walking technicolor?

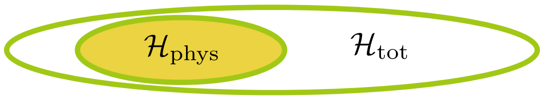
SO(3) gauge theory: phase diagram

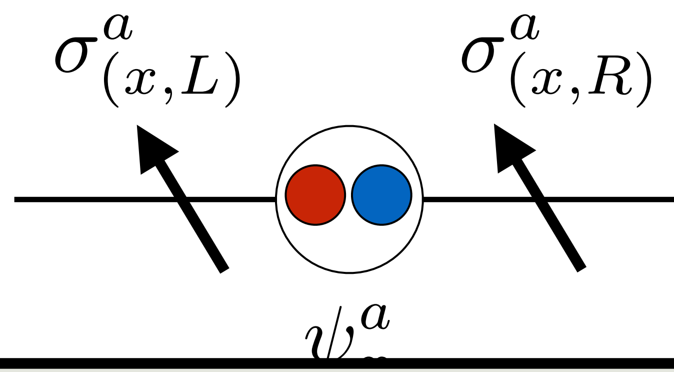
$$H_{\text{enc.}} = -t\sum_x [(\psi_x^a)^\dagger \sigma^a_{x,R}\sigma^b_{x+1,L}\psi^b_{x+1} + \text{h.c.}]$$
$$+V\sum_x n_x n_x + G\sum_x (n_x)^2 + \sum_x (-1)^x n_x^a$$



Take-home message: the toy model displays:
1) **chiral symmetry breaking**
2) non-trivial **Baryons physics**
3) stable **conformal window**

Gauge invariant Hilbert space





$a = 1, 2, 3$

Gau

The local gauge invariant Hilbert space **is equivalent to a spin 3/2**!

(9.32)

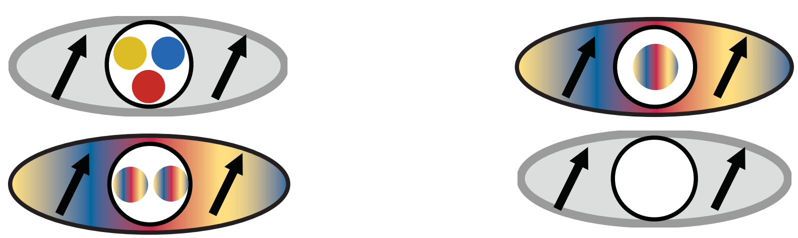
$]\,|0\rangle_x$

$$\dim[\mathcal{H}_{\rm phys}] = 4$$

$$|b_x = 3\rangle_x = \frac{i}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)\psi^{1\dagger}\psi^{2\dagger}\psi^{3\dagger}|0\rangle_x$$

$|M = 3\rangle$

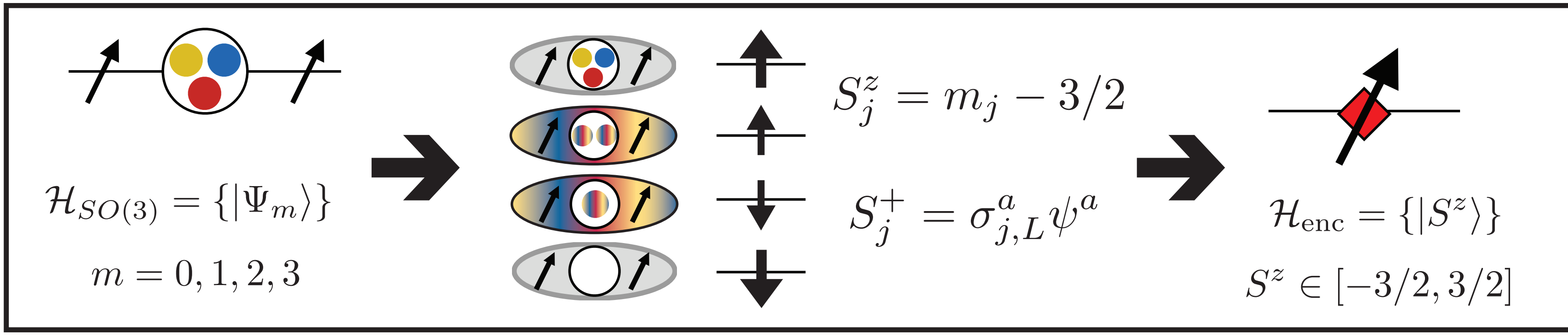
$|M = 2\rangle$



$|M = 1\rangle$

$|M = 0\rangle$

SO(3) gauge theory and spin chains: encoding



What happens to the operators?

$$n_x = (\psi^a_x)^\dagger \psi^a_x = S^z_x + 3/2$$

$$\psi^a_x \sigma^a_{x,R} = S^+_x$$

Requires a Jordan-Wigner-like transformation to be made rigorous:

$$\tilde{\psi}^\alpha_x = \psi^\alpha_x e^{i\pi[\sum_{\ell<x} M_\ell + \sum_{\beta<\alpha} n_{\beta,x}]}$$

Encoded Hamiltonian

$$H = -t \sum_x [(\psi_x^a)^\dagger \sigma_{x,R}^a \sigma_{x+1,L}^b \psi_{x+1}^b + \text{h.c.}] + \sum_x [V n_x n_{x+1} + G n_x^2]$$

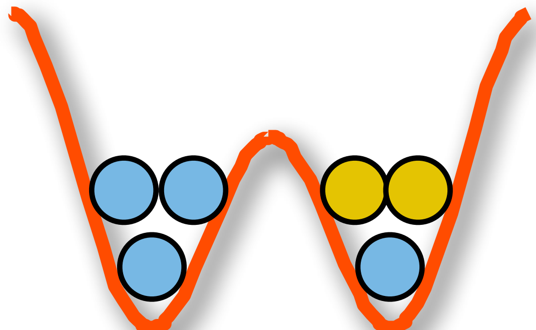
$$n_x = (\psi_x^a)^\dagger \psi_x^a = S_x^z + 3/2$$

$$\psi_x^a \sigma_{x,R}^a = S_x^+$$

$$H_{\text{enc}} = -t \sum_x (S_x^+ S_{x+1}^- + \text{h.c.}) + \sum_x [V S_x^z S_{x+1}^z + G (S_x^z)^2]$$

Spin-S Heisenberg with cold atoms

Bose Mixtures in optical lattices

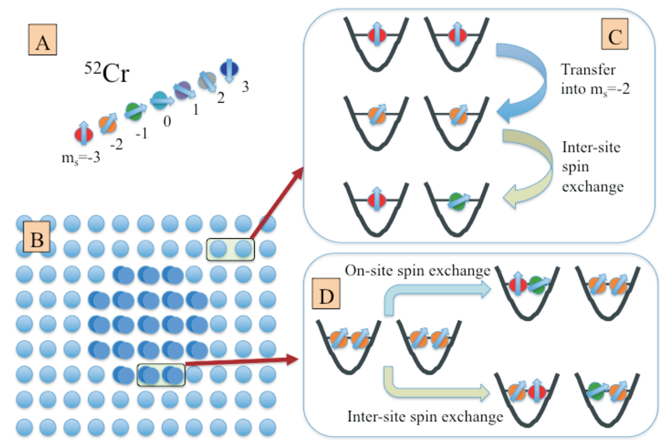


$$S_x^z = \frac{n_x^I - n_x^{II}}{2}$$

Exp. double well: Munich, JQI

NB: three-body losses may limit timescales

(Fermionic) Magnetic atoms (Dy, Er)

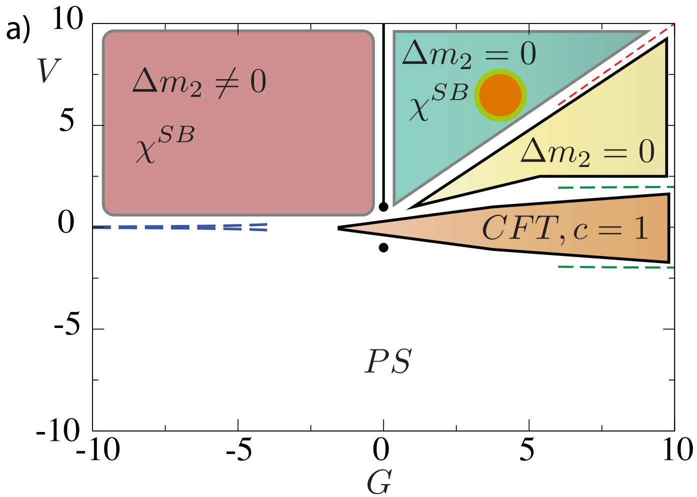


Paris, Stuttgart, Stanford, Innsbruck,…

Other dipolar systems, e.g., polar molecules dressed with MWs

Micheli, Brennen, Zoller, Nat.Phys. 2006. For S=3/2, see also Gorshkov et al. 1301.5636.

Observables: an example



How to detect chiral symmetry breaking?

Measure a finite **chiral condensate fraction**

$$\chi = \langle \sum_x (-1)^x (\psi_x^a)^\dagger \psi_x^a \rangle = \langle \sum_x (-1)^x S_x^z \rangle$$

After encoding, this translates onto a **staggered magnetisation** (band mapping, microscope):

$$\chi = \sum_{jodd} n_j^I - \sum_{jeven} n_j^I$$

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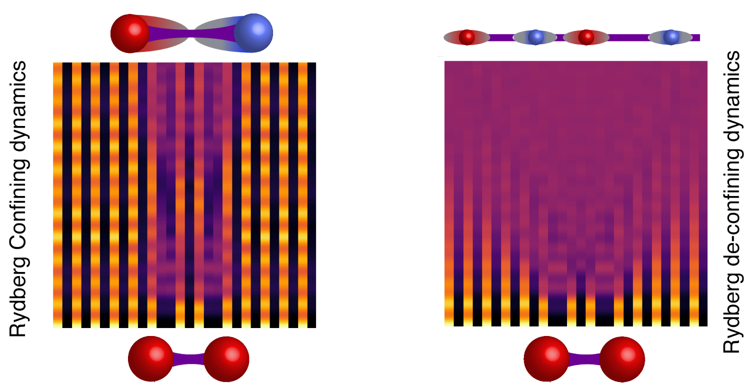
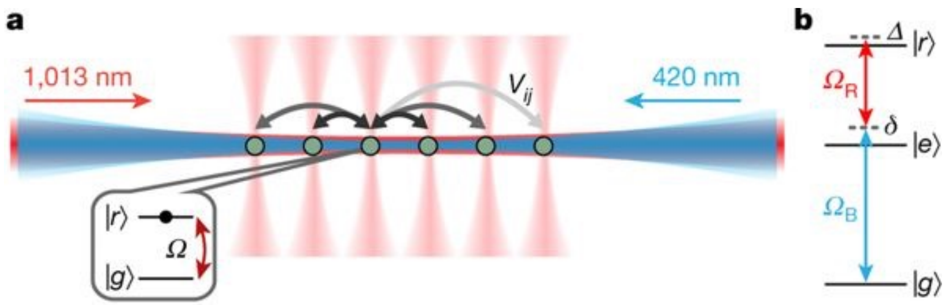
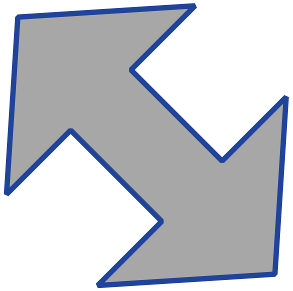
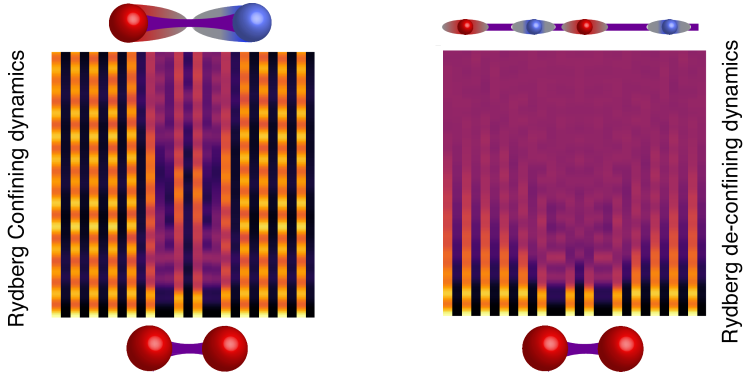


Encoding strategies for Abelian theories







Why slow dynamics? Gauge theory interpretation

Spin model maps onto:

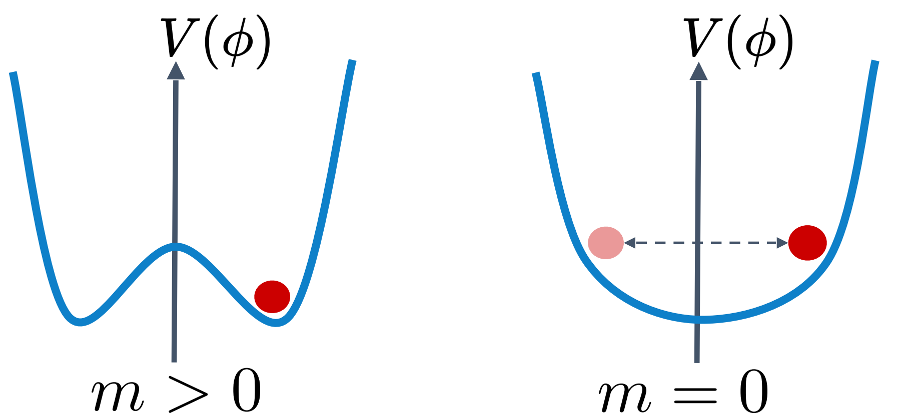
$$\hat{H}_{\mathrm{B}} = \int \mathrm{d}x \left[\frac{1}{2}\hat{\Pi}^2 + \frac{1}{2}(\partial_x\hat{\phi})^2 + \frac{1}{2}\frac{e^2}{\pi}\hat{\phi}^2\right] - cm\omega_0 \cos(2\sqrt{\pi}\hat{\phi} - \theta)$$

S. Coleman, Phys. Rev. D 11, 2088 (1975)

Integrable in the vanishing mass limit!

Tricky aspect: continuum limit beyond RG

$$\theta = \pi$$

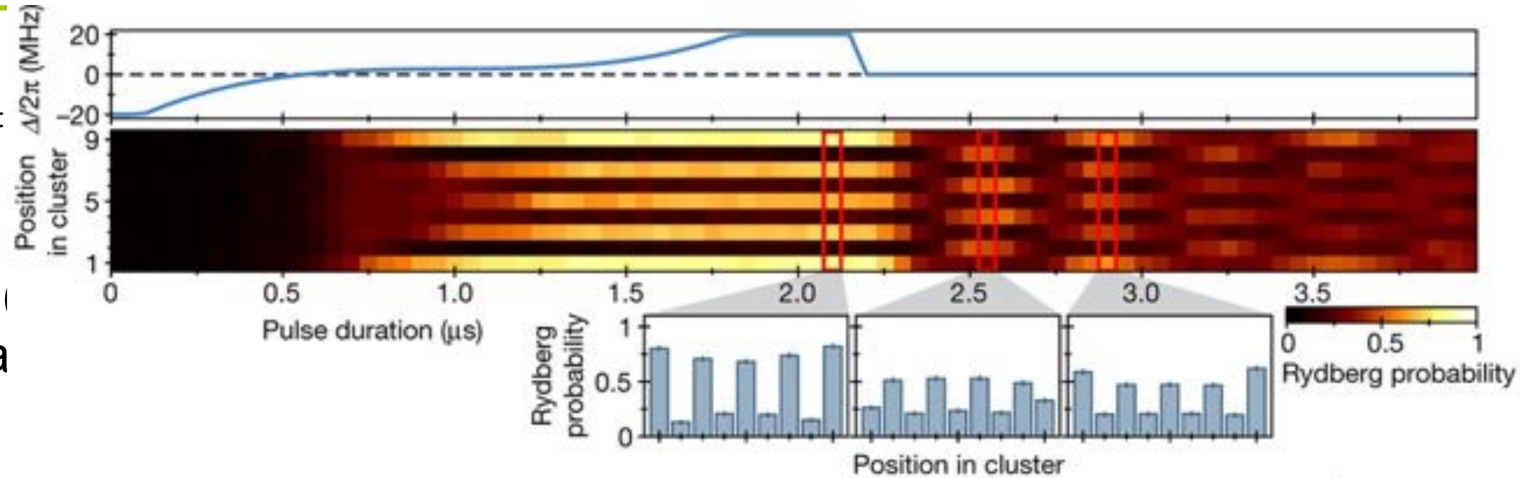


Why slow dynamics? Gauge theory interpretation

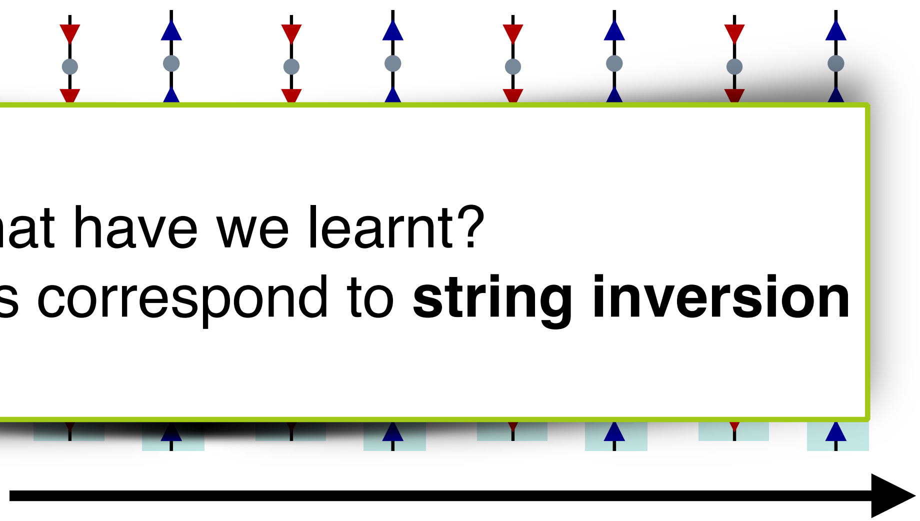
$t =$

String

sta



Oscillations between string (Neel) states



What have we learnt?
(2) slow dynamics correspond to **string inversion**

t

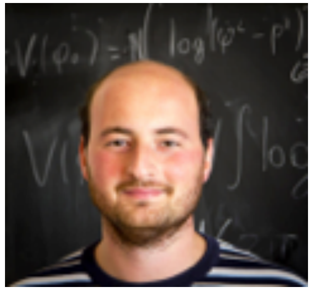
Recently observed in multi-Higgs U(1): Chanda et al., PRL 2020

Recap

- Quantum simulation for gauge theories: the ‘encoding route’
  - 1D SO(3) gauge theories:
    - simple toy models with basic, interesting features
    - proposal: cold atom mixtures
  - Schwinger model
    - mapping to constrained spin chains
    - already **experimentally realized**!
- open points:
  - scaling of errors in encoded versions non-trivial
  - 2D?
  - other non-Abelian groups?

ICTP and SISSA

Federica Surace



Paolo Mazza

Giuliano Giudici

Alessio Lerose

Andrea Gambassi

Bilbo

Enrique Rico

IQOQI / ITP Innsbruck

Peter Z

Einstein Institute / ITP Univ Bern

Uwe-Jens Wiese

Debasish Banerjee

Pascal Stebler

Michael Bögli

Thank you

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