# Structure and Response in Nuclear and Cold Atom Physics

J. Carlson, LANL

L. Andreoli, A. Baroni, J. Barrow, S. Gandolfi, A. Gezerlis, A. Li, A. Lovato, M. Piarulli, N. Rocco, S. Pastore, A. Roggero, R. Schiavilla, R. B. Wiringa

## Outline

- EOS/pairing in Unitary Fermi Gas, neutron matter: bulk and finite systems
- Electron / Neutrino Scattering from Nuclei
- Spin and Density Response in Cold Fermions
- Explicit Final States: Quantum vs. Classical
- Outlook

# Well known similarity between cold fermions at unitarity and low-density neutron matter

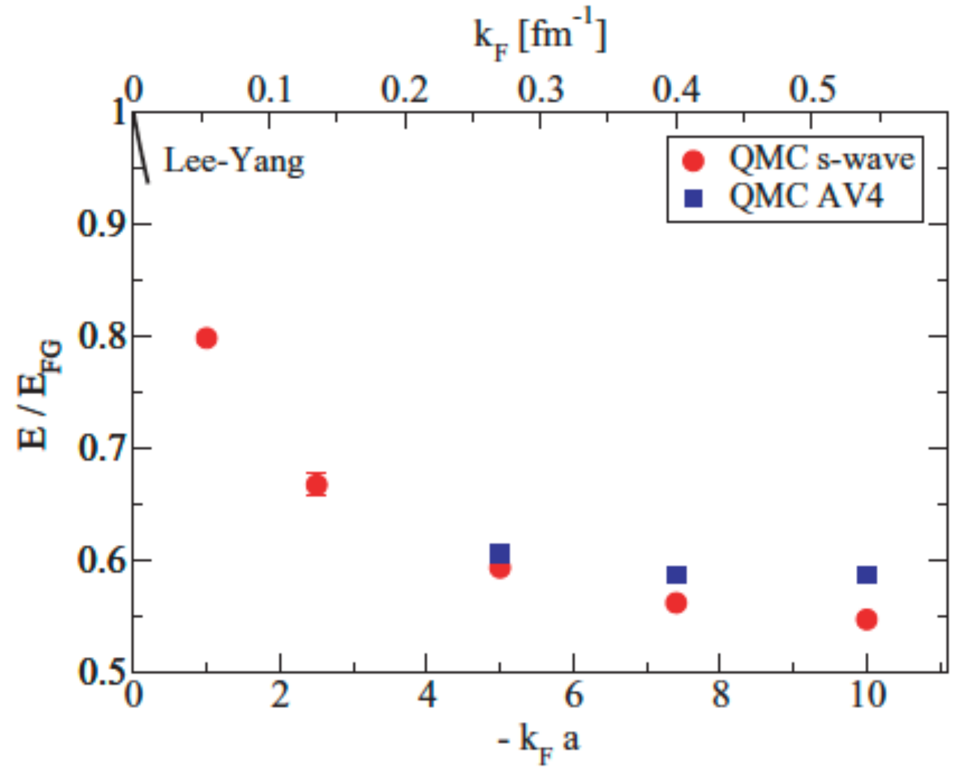
**Scattering length nn, cold atom**

$a = -18 \text{ fm}, \infty$

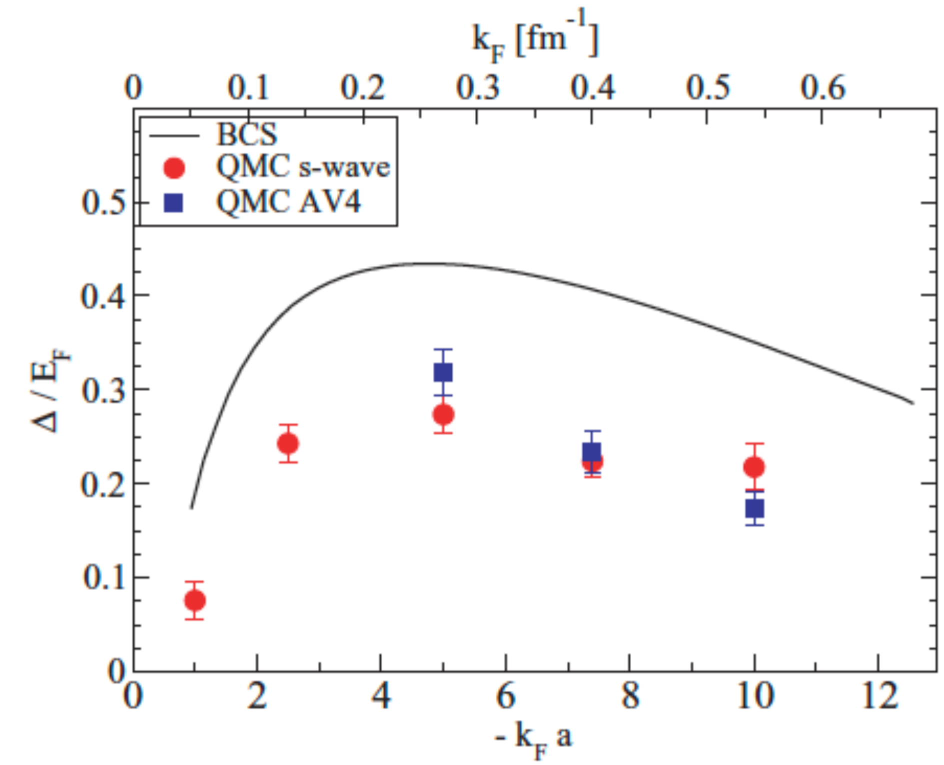
**Effective range nn, cold atom**

$r_e = 2.5 \text{ fm}, 0$

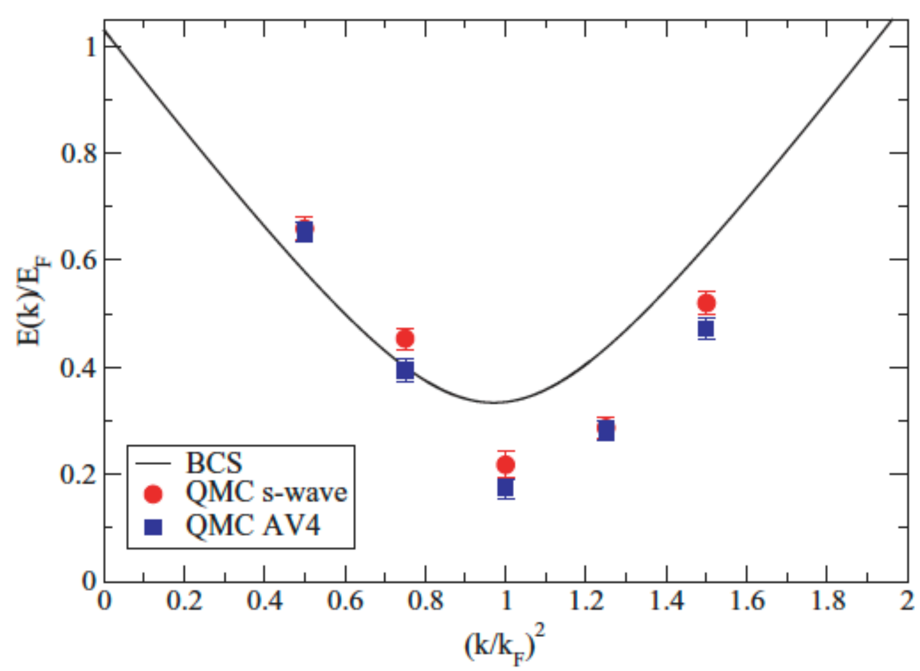
## T=0 EOS: Neutron Matter



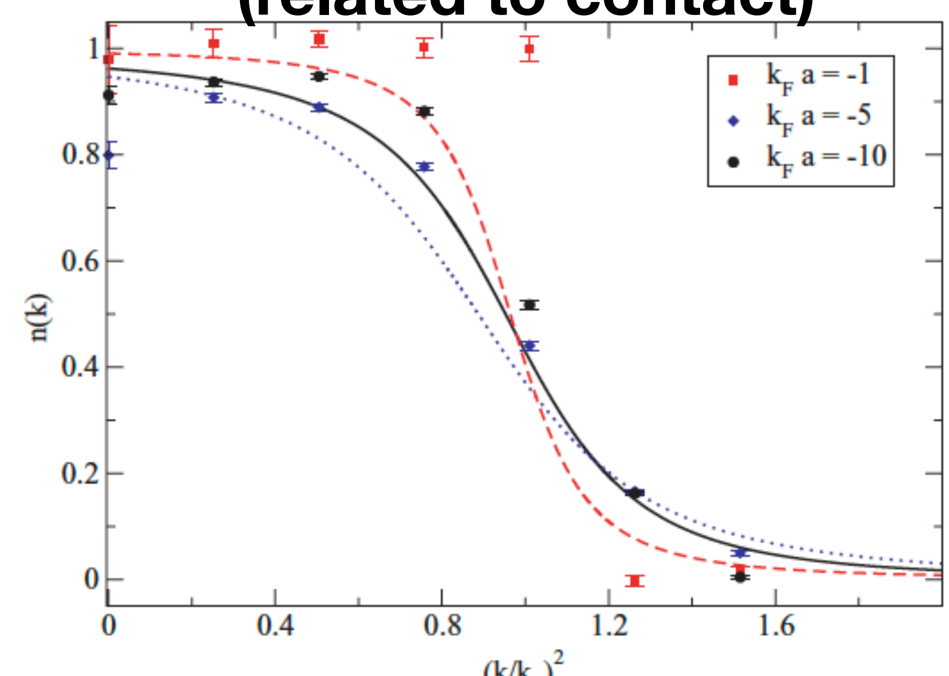

## Superfluid Pairing Gap




## Quasi-particle spectrum (NM)



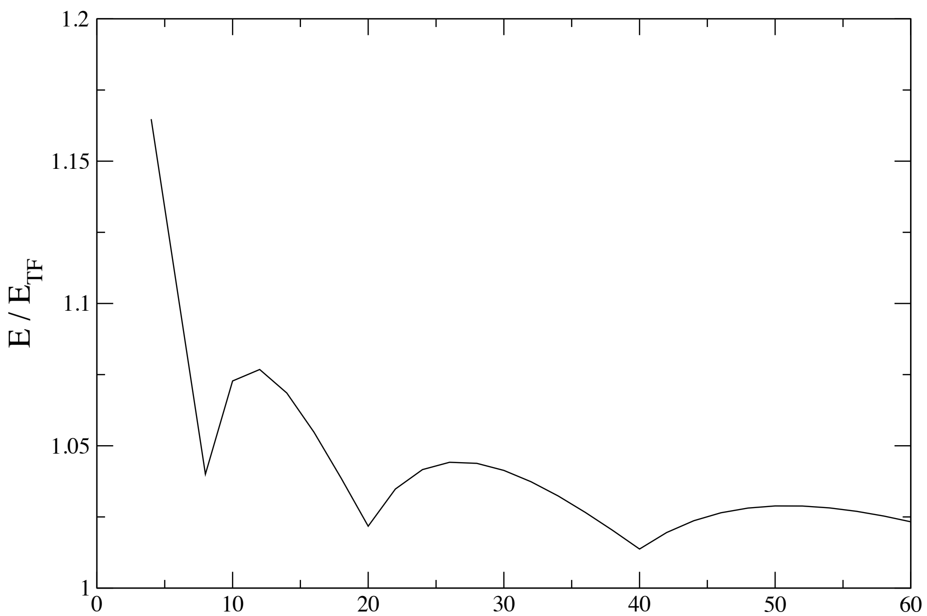

## Momentum Distributions NM (related to contact)




Gezerlis, JC; PRC (2008, 2010); Gandolfi, Gezerlis, JC; ARNP (2015)

# Finite Systems:
# Can we make connections to nuclei and nuclear matter more generally?

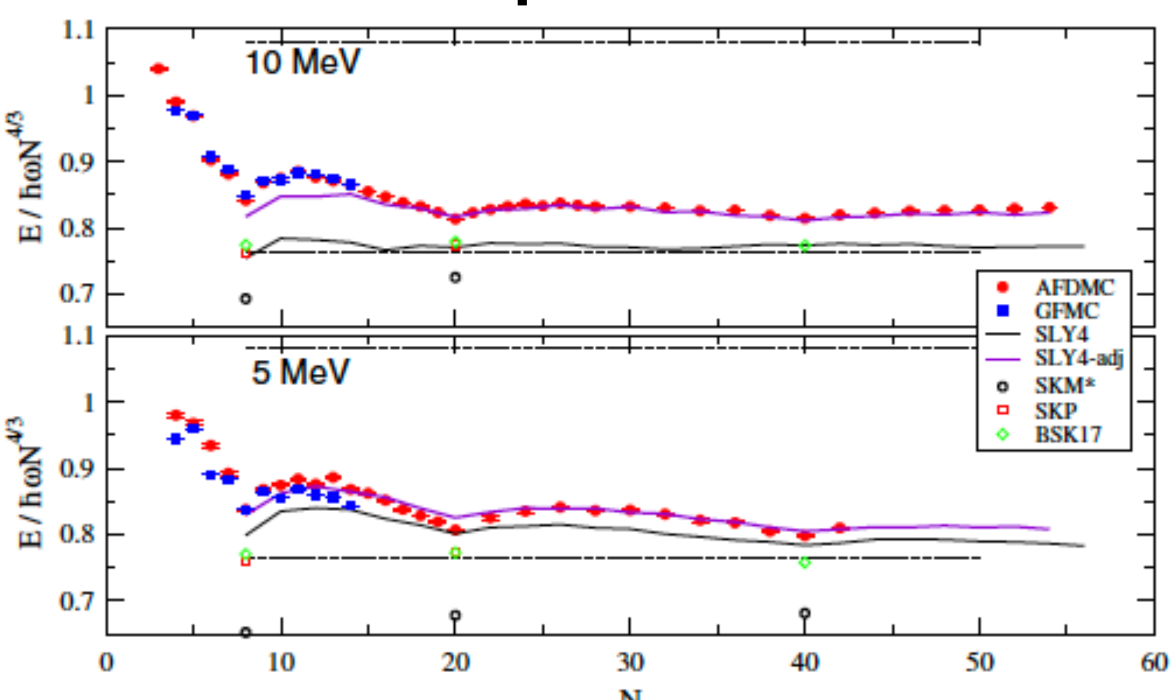
**Free Fermions shell structure**




Cold atom experimental capabilities:

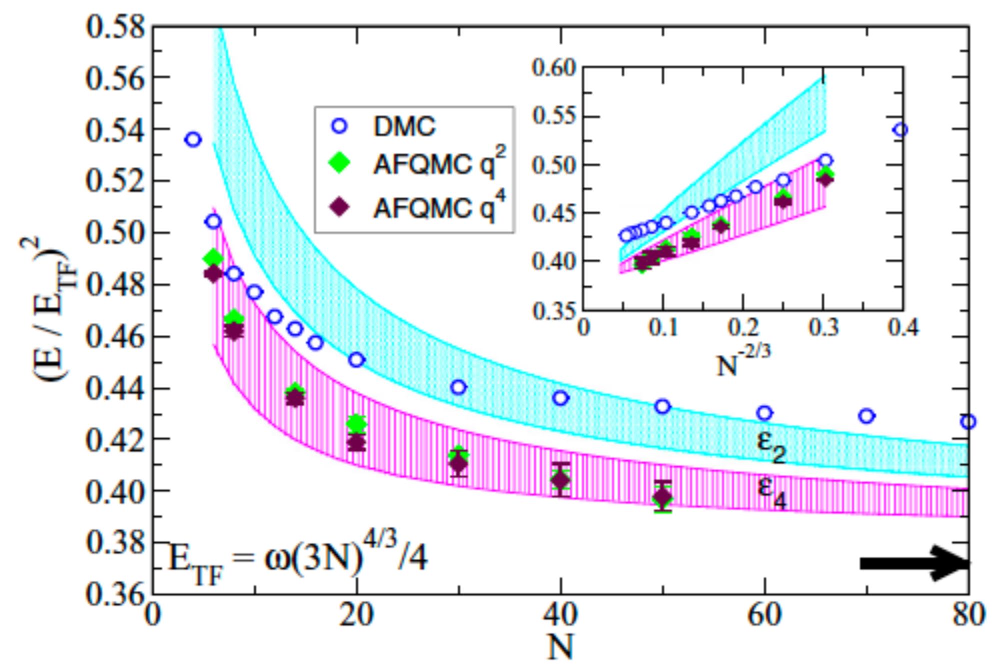
interaction strength
spin balance
external potentials (flat, oscillating),
spin, density response functions
+many more

**Neutrons / HO potential**



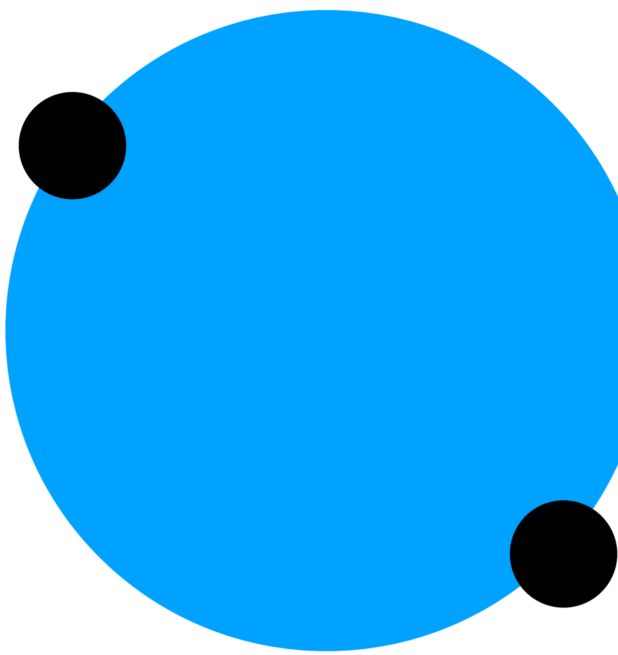

Gandolfi, JC, Pieper, 2011

**UFG / HO potential**

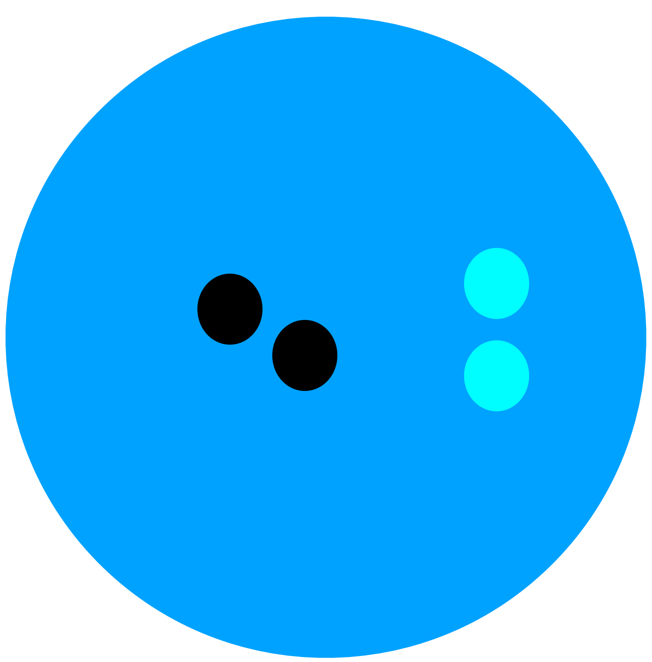



JC, Gandolfi, PRA, 2014

# What else could be addressed in structure?
# for example pairing and coherence length



Coherence length of pairing in nuclei
typically assumed to extend over entire nucleus
(pairing on the Fermi Surface)



Pairing in the UFG / HO is essentially local
extends over a few inter particle distances

short in the center, larger at the surface

Can we probe this transition by changing the coupling, changing the trapping potential
(also imbalance can impact this)?

***What about dynamics?***
***Linear Response***

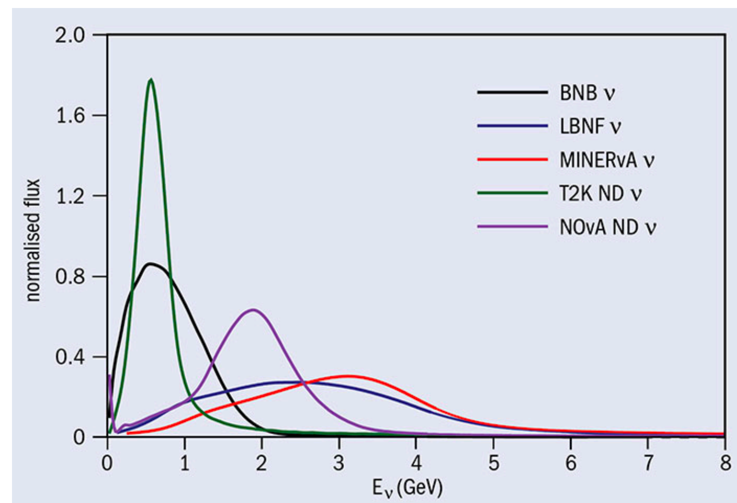
# Why are we interested? Accelerator Electron and Neutrino Experiments

wide range of neutrino energies

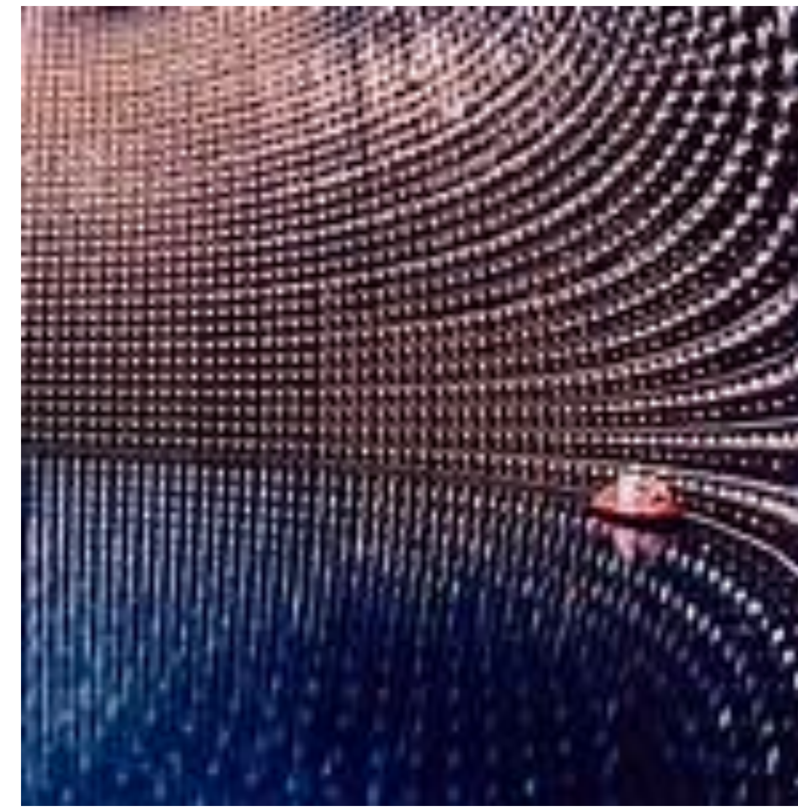
importance of oscillations/cross sections for energies ~1-3 GeV

need inclusive cross section for different flavors to extract neutrino parameters

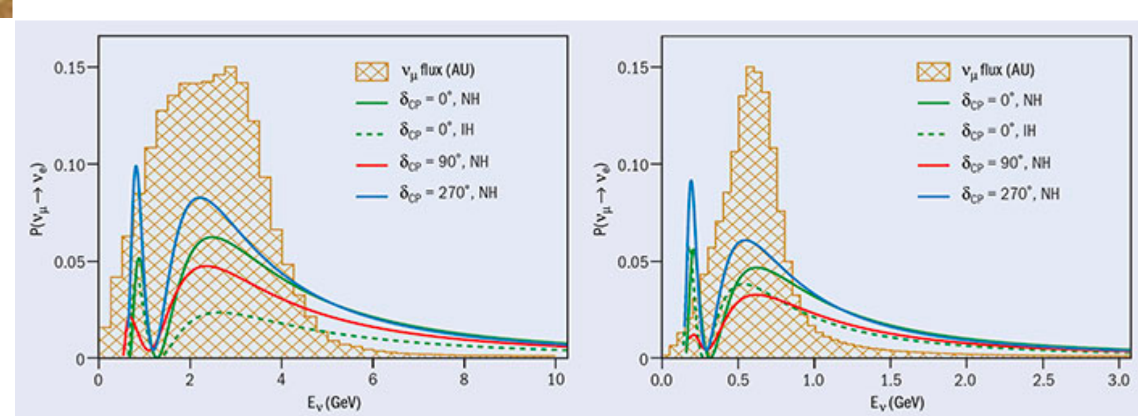

DUNE




CP violating phase in neutrino sector



T2K



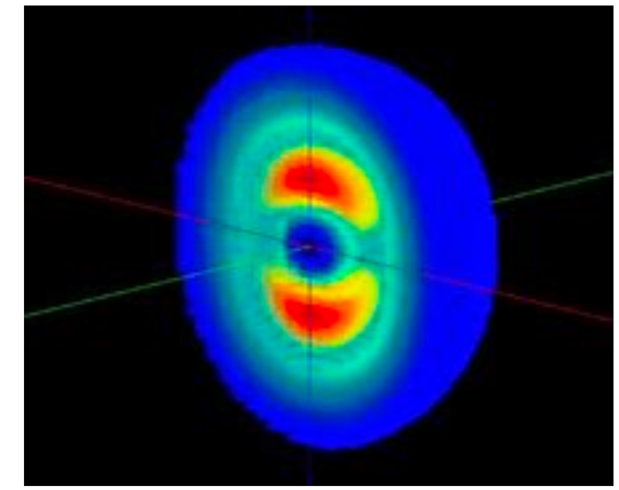

DUNE T2K

# Why study electron scattering?

*not to determine properties of electron or photon*

## Quasi-elastic scattering: higher q, E

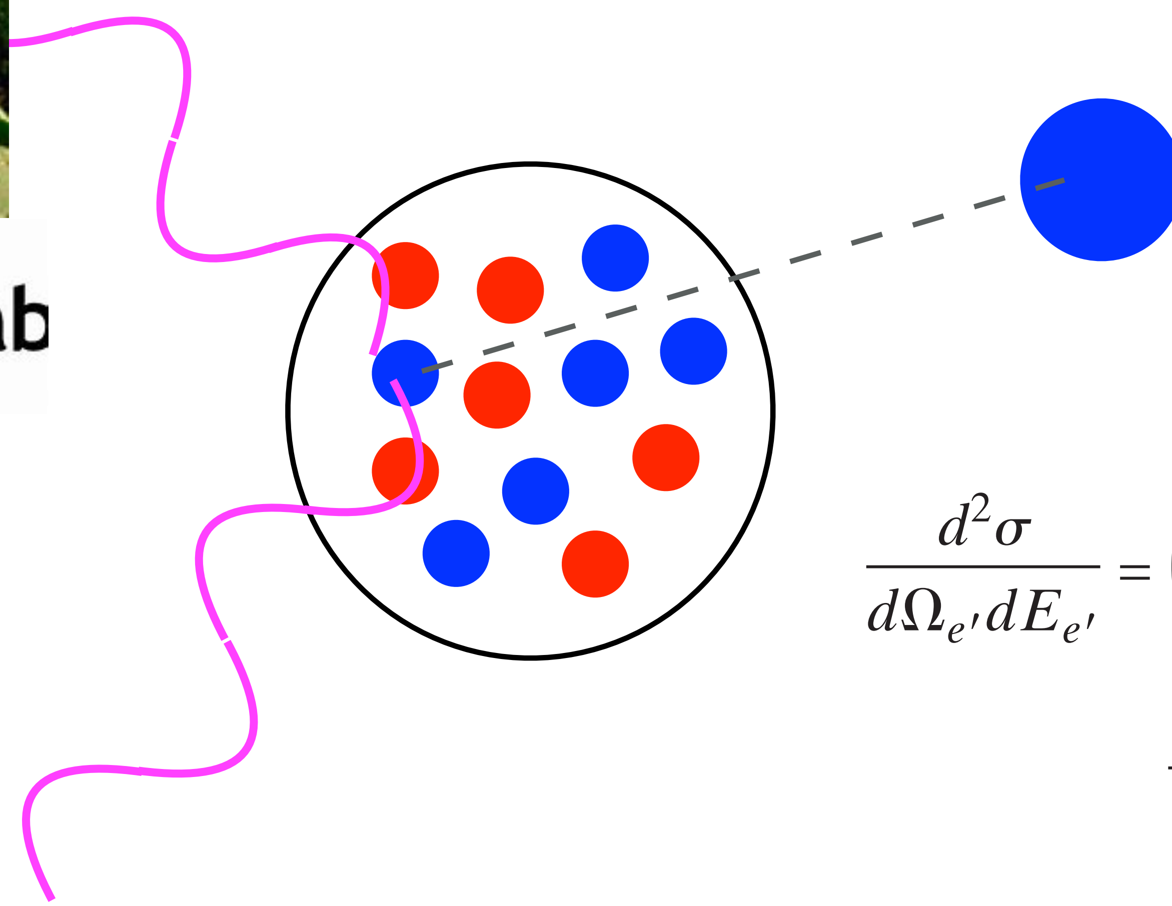
$q \sim k_F \quad E \sim E_F$

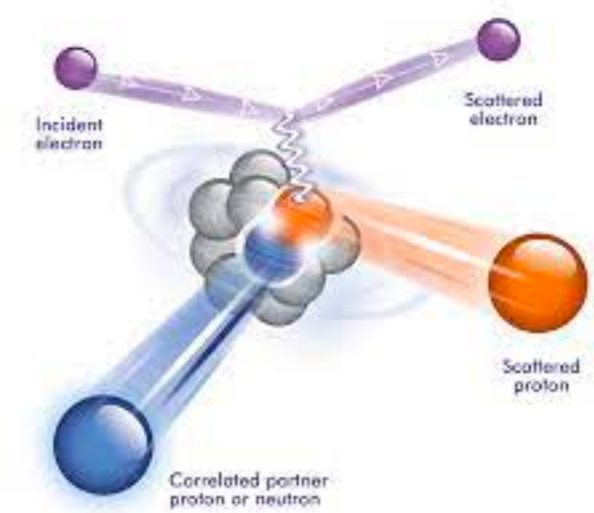


Jefferson Lab

Electron Scattering: 2 response functions

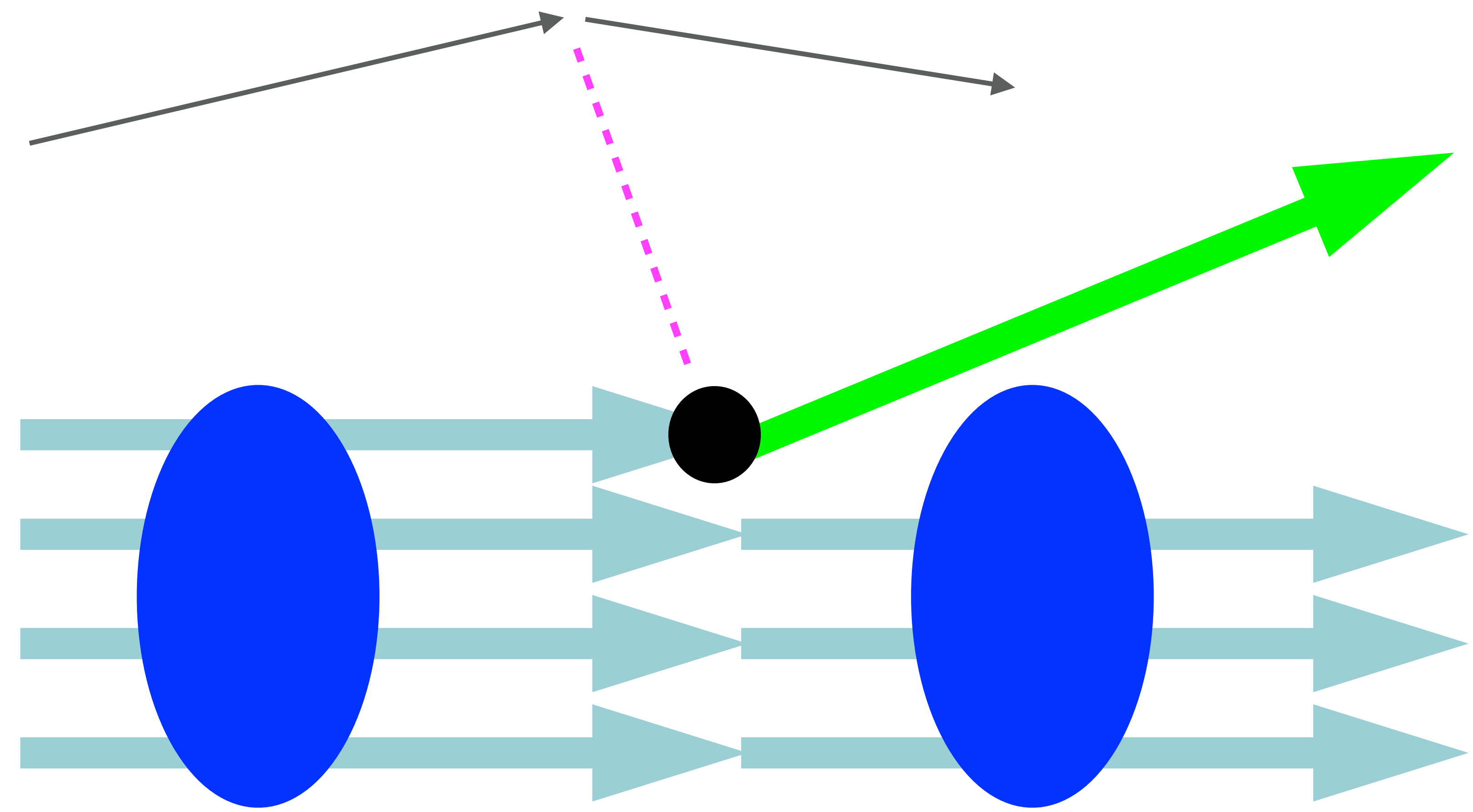
Neutrino/Antineutrinos: 5 response functions






$$\frac{d^2\sigma}{d\Omega_{e'}dE_{e'}} = \left(\frac{d\sigma}{d\Omega_{e'}}\right)_M \left[\frac{Q^4}{|\mathbf{q}|^4} R_L(|\mathbf{q}|,\omega) + \left(\frac{1}{2}\frac{Q^2}{|\mathbf{q}|^2} + \tan^2\frac{\theta}{2}\right) R_T(|\mathbf{q}|,\omega)\right]$$

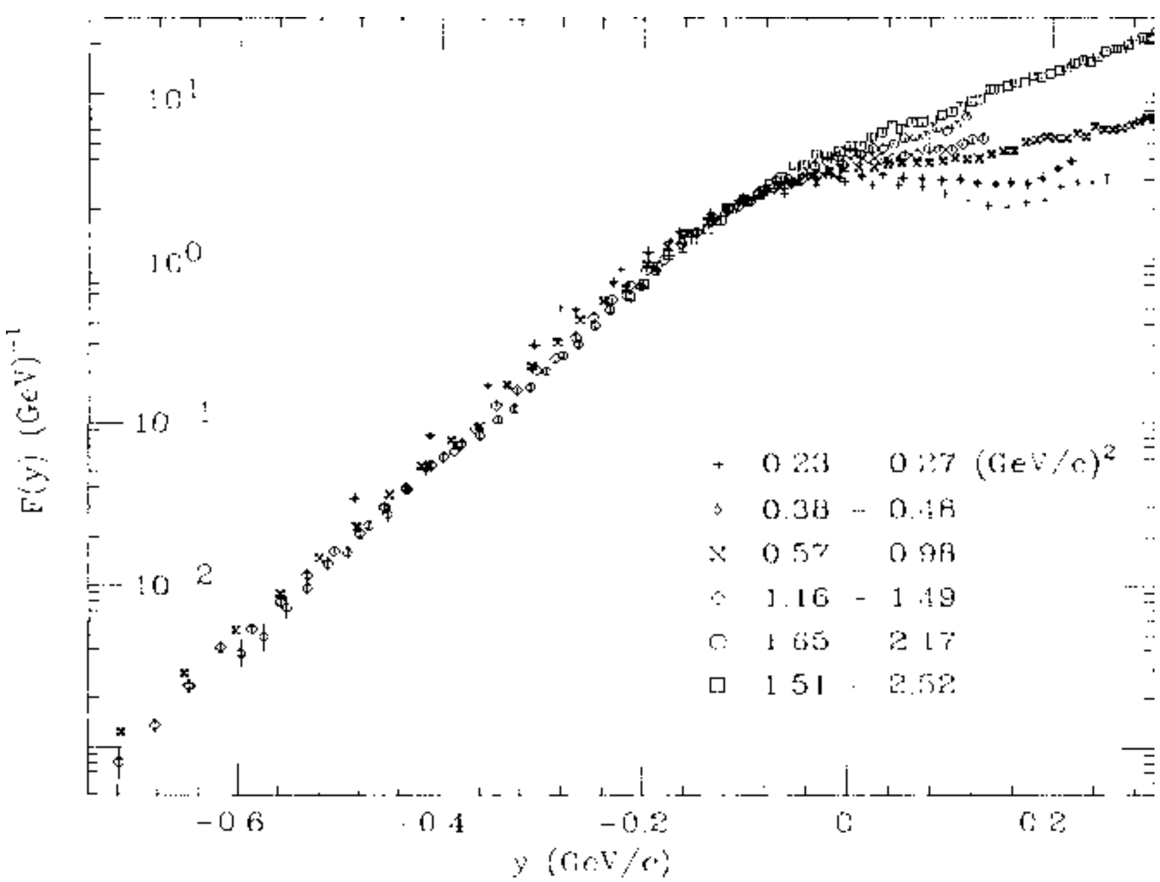
# Quasi-Elastic Scattering and Plane Wave Impulse Approximation



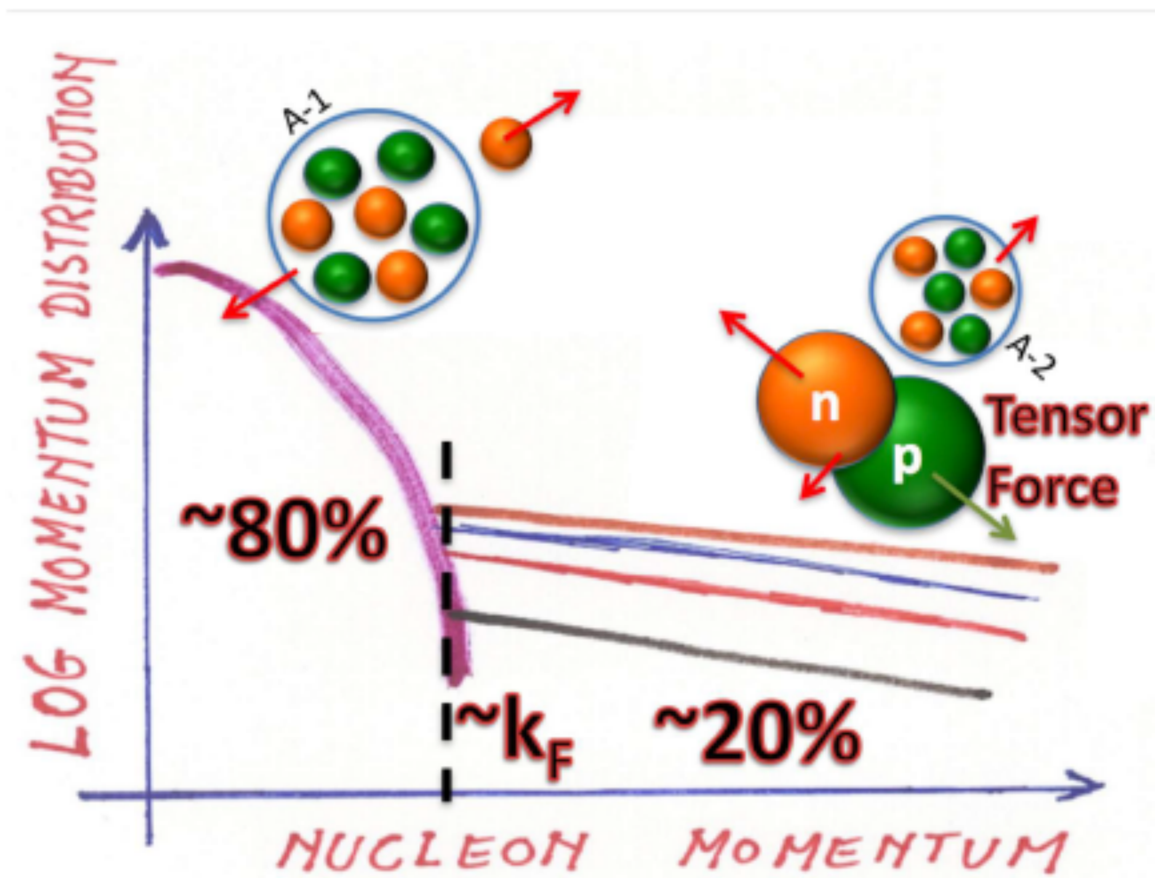
Incorporates incoherent scattering of single nucleons:
n(k) or spectral function S(k,w)
and single-nucleon form factors

# Basic Observations from Electron Scattering

Scaling with momentum transfer: ***y-scaling***
incoherent sum over scattering from single nucleons
- scaling of 1st kind-

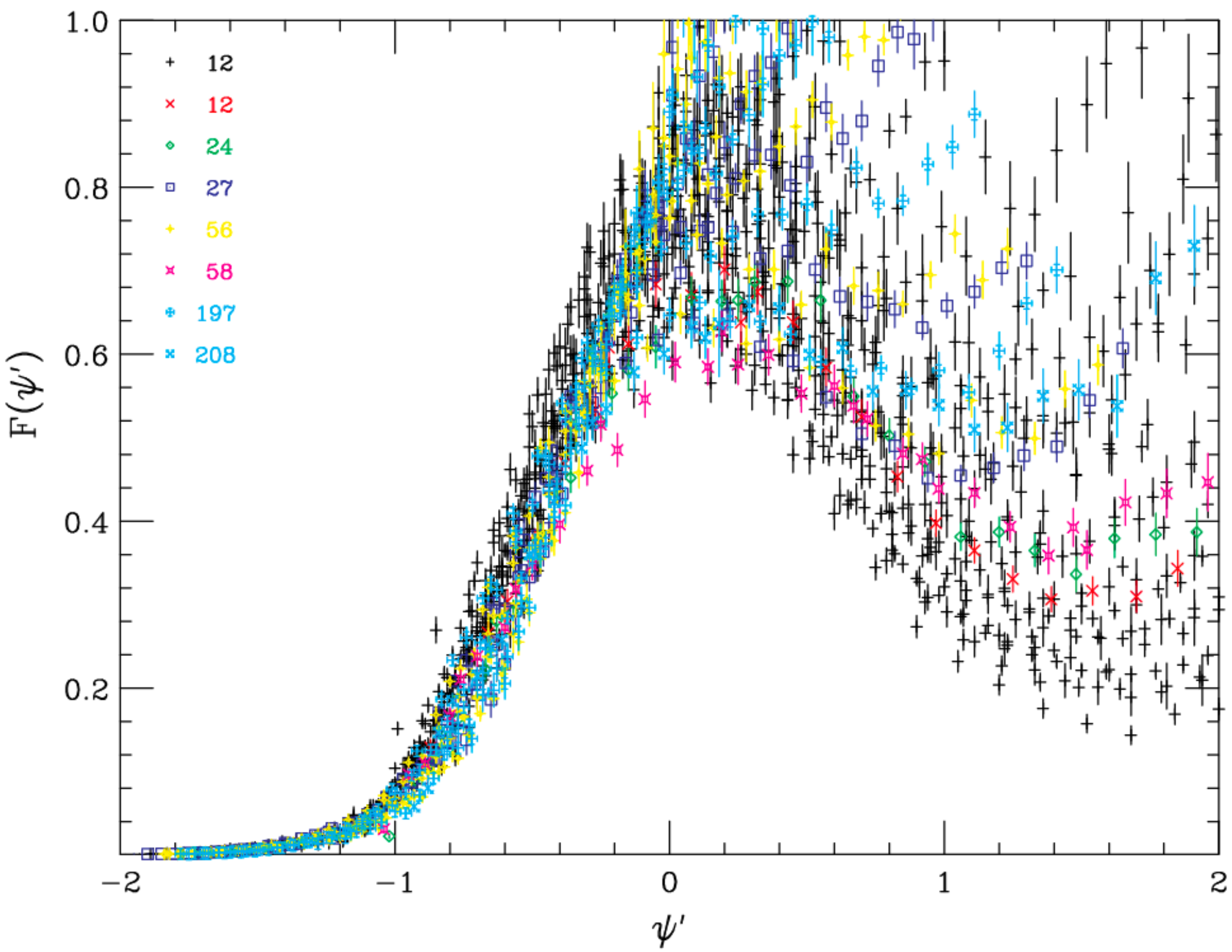



y-scaling in NP
Day, McCarthy, PRL 1987



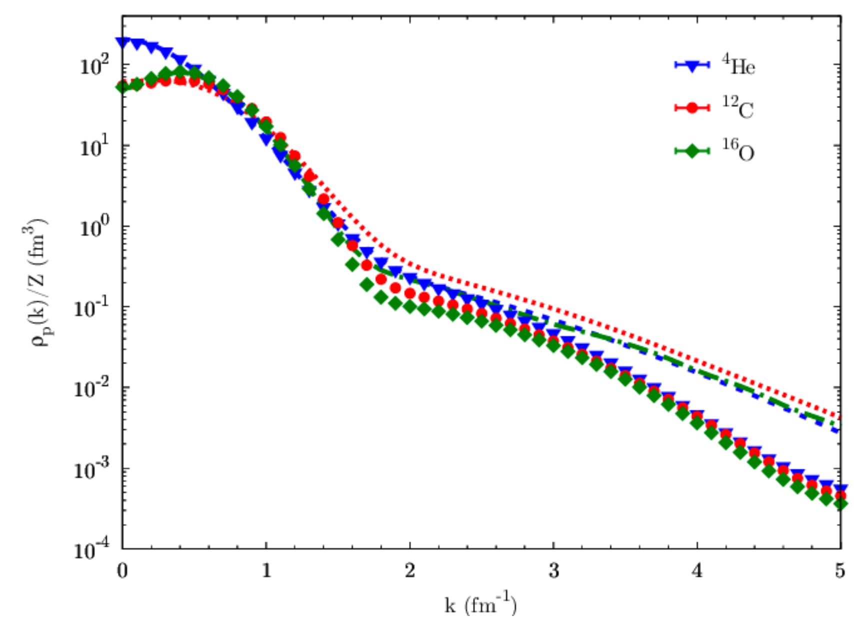
# Some basic Observations from Electron Scattering

***Superscaling:*** for the same kinematics, response looks similar for different nuclei ($q > k_F$ )

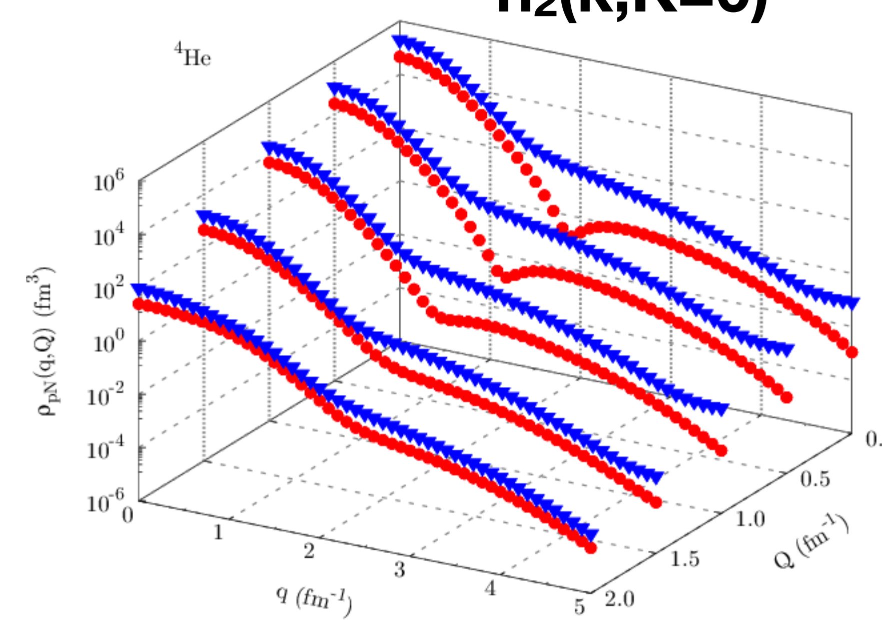



Superscaling in inclusive e-nucleus scattering
Different nuclei at the same kinematics
Same kinematics: same ratio of L/T response
Donnelly, Sick PRL (1999)

**n(k)**

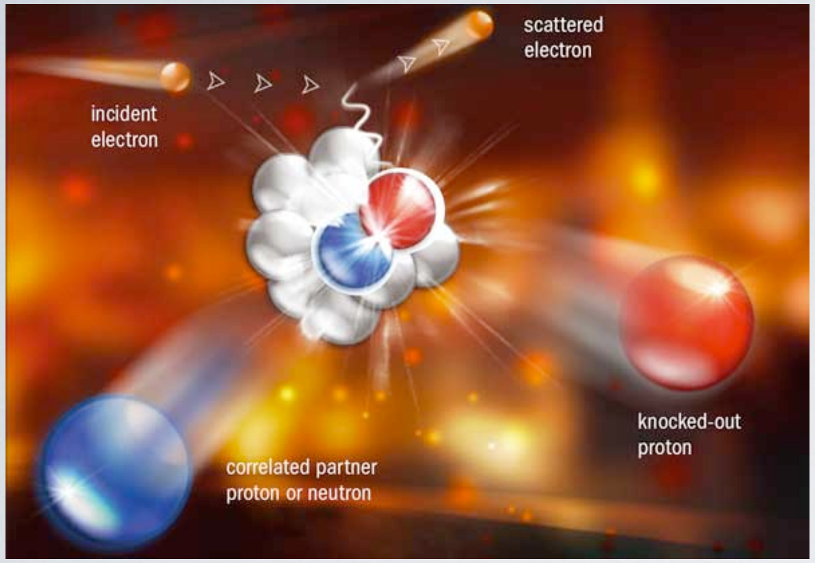



**$n_2$(k,K=0)**



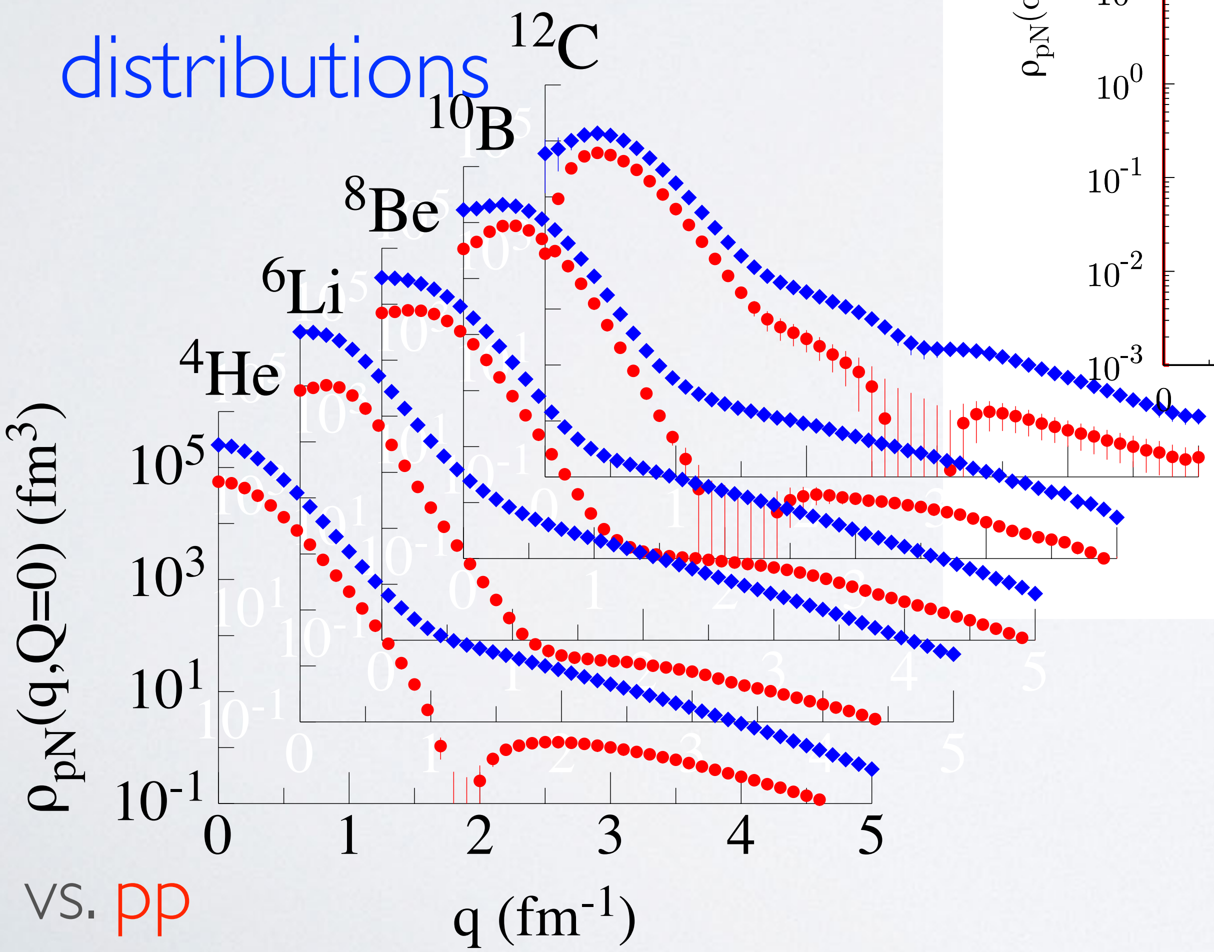

Lonardoni, Gandolfi, Wang, Carlson (2018)

# Back to Back Nucleons (total Q~0)
# np pairs dominate over nn and pp



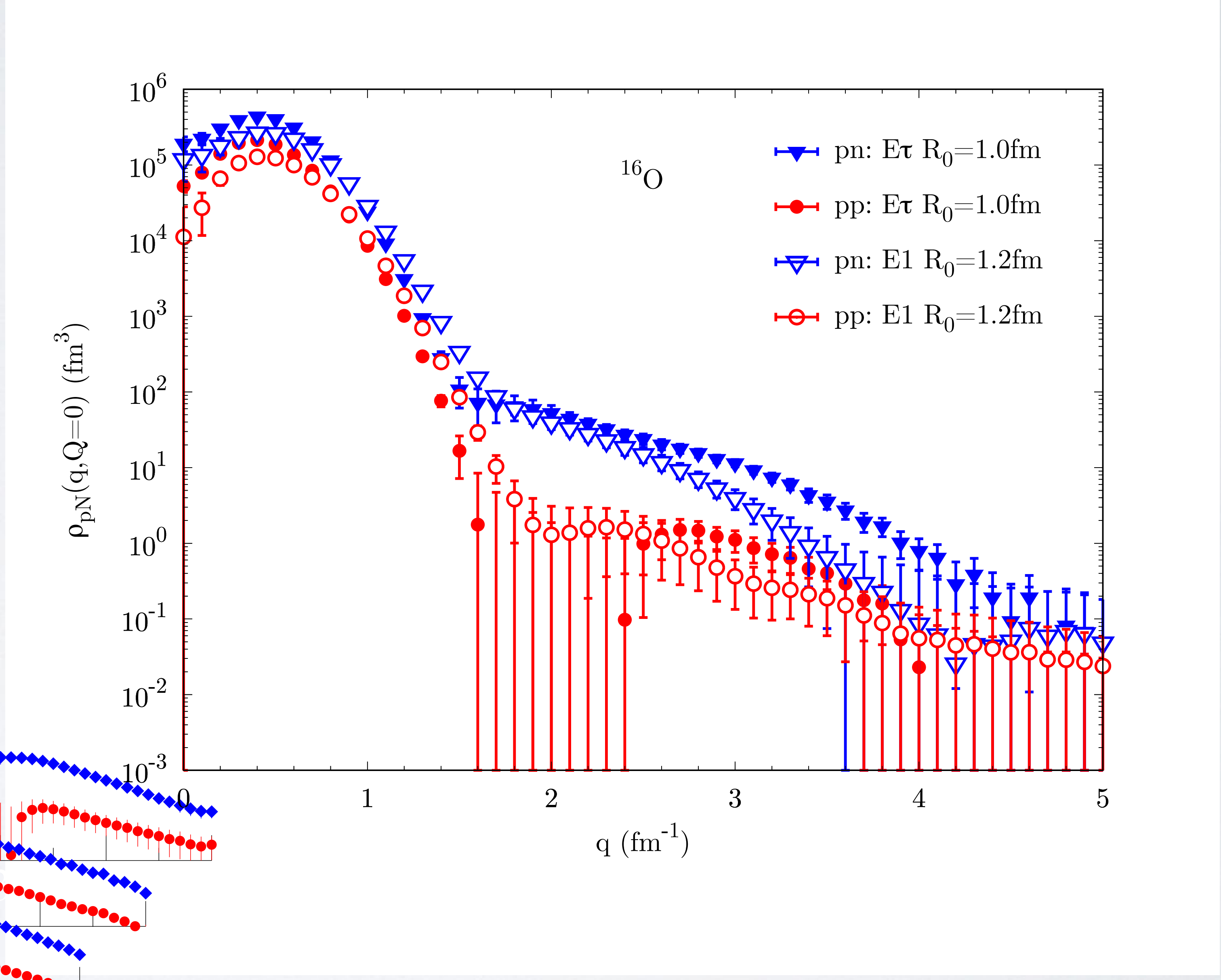

E Piasetzky *et al.* 2006 **Phys. Rev. Lett. 97** 162504.
M Sargsian *et al.* 2005 **Phys. Rev. C 71** 044615.
R Schiavilla *et al.* 2007 **Phys. Rev. Lett. 98** 132501.
R Subedi *et al.* 2008 **Science 320** 1475.

## 2-nucleon momentum distributions




np vs. pp

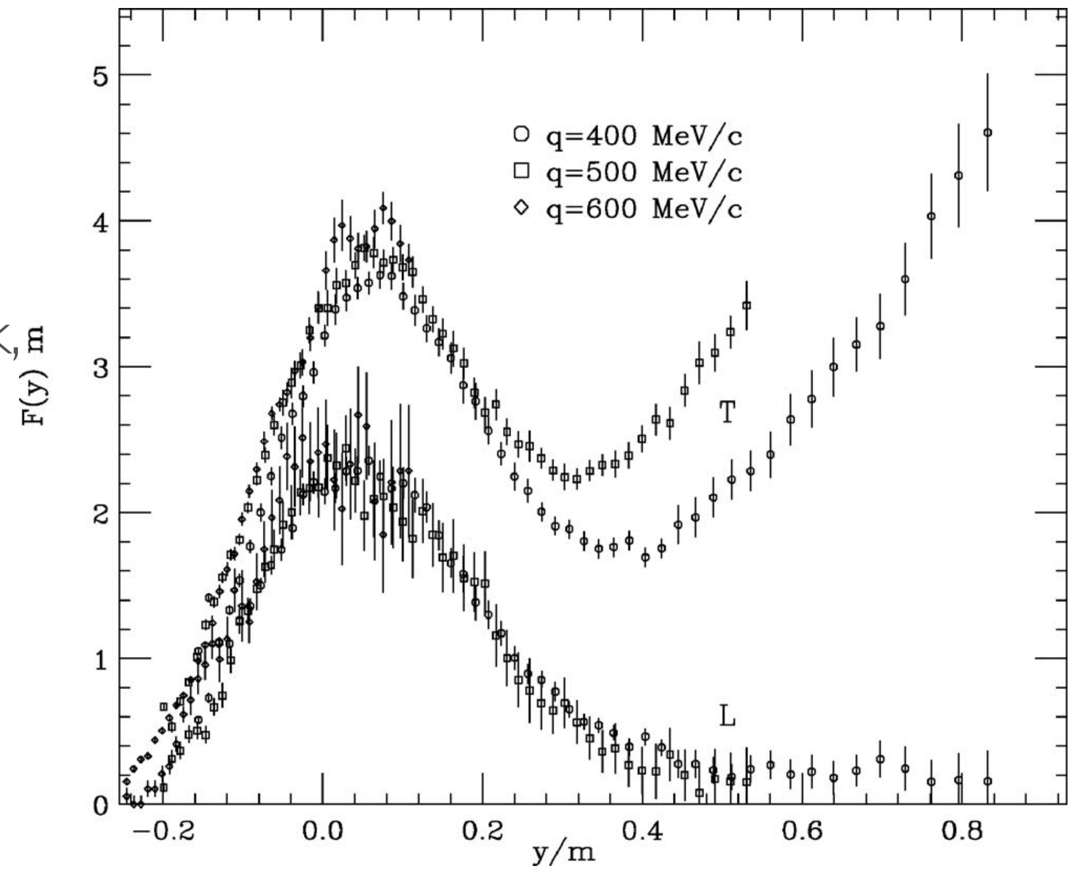
Wiringa et al.; Carlson, et al, RMP 2015

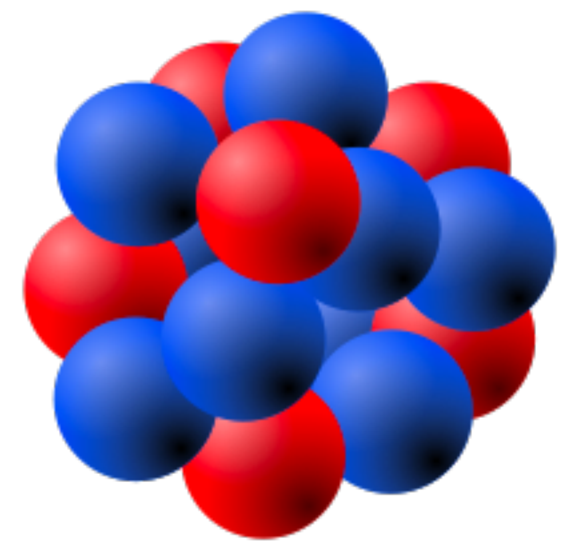



see work of Bob Wiringa and Diego Lonardoni

# Electron Scattering: Longitudinal vs. Transverse
# Single Nucleon form factors (squared) divided out

from Benhar, Day, Sick,
RMP 2008
data Finn, et al 1984



Scaled longitudinal vs.
transverse scattering from $^{12}$C

## Distances probed at various q

| q | $r \sim \pi/q$ |
|---|---|
| 0.3 GeV/c | 2.1 fm |
| 0.5 GeV/c | 1.2 fm |
| 1 GeV/c | 0.6 fm |

Nearest neighbor nucleons at

$\rho = 0.16\ \mathrm{fm}^{-1} = 1 / (4/3\ \pi\ r^3)$

$r = 1.14$ fm

$d = 2.28$ fm

# Electron Scattering: Longitudinal and Transverse Response

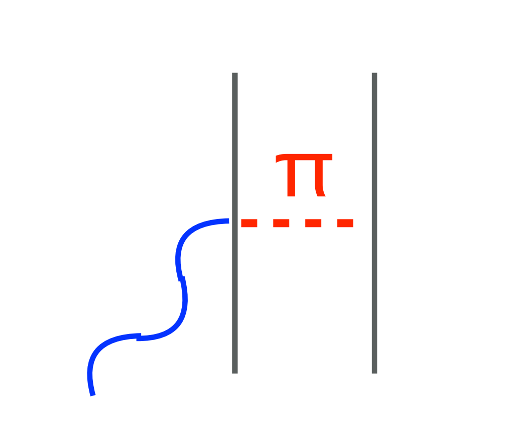
Transverse (current) response:

$$R_T(q,\omega) \; = \; \sum_f \langle 0|\; \mathbf{j}^\dagger(q)\; |f\rangle\langle f|\; \mathbf{j}(q)\; |0\rangle\; \delta(w-(E_f-E_0))$$

Longitudinal (charge) response:

$$R_L(q,\omega) \; = \; \sum_f \langle 0|\; \rho^\dagger(q)\; |f\rangle\langle f|\; \rho(q)\; |0\rangle\; \delta(w-(E_f-E_0))$$

$$\mathbf{j} \; = \sum_i \mathbf{j}_i \; + \sum_{i<j} \mathbf{j}_{ij} + \ldots$$




Two-nucleon currents required by current conservation

Response depends upon all the excited states of the nucleus

# Euclidean Response

*Want to calculate*

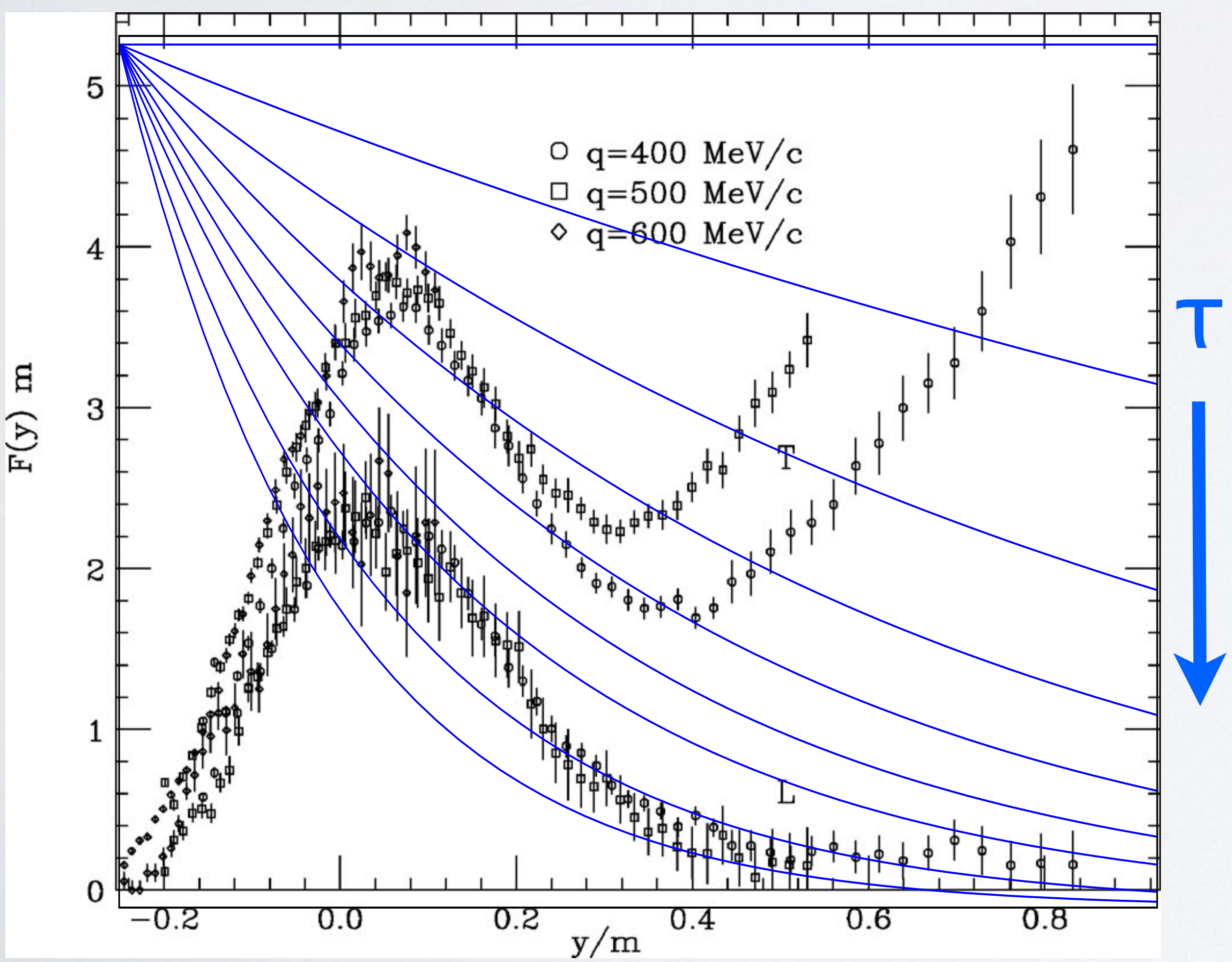
$$R(q,\omega) \;=\; \int dt \;\langle 0|\; \mathbf{j}^\dagger \exp[i(H-\omega)t]\; \mathbf{j}\; |0\rangle$$

*Can calculate*

$$\tilde{R}(q,\tau) \;=\; \langle 0|\; \mathbf{j}^\dagger \;\exp[-(\mathbf{H}-\mathbf{E_0}-\mathbf{q^2}/(\mathbf{2m}))\tau]\;\; \mathbf{j}\;|\mathbf{0}\rangle >$$

- Exact given a model of interactions, currents
- `Thermal' statistical average
- *Full final-state interactions*
- *All contributions included - elastic, low-lying states, quasi elastic, …*

Excellent agreement w/ EM (L & T) response in A=4,12
Lovato, 2015, PRL 2016




Sum rule → elastic FF² w/ increasing

Note: for q > $k_F$
Basically a nearly local operator

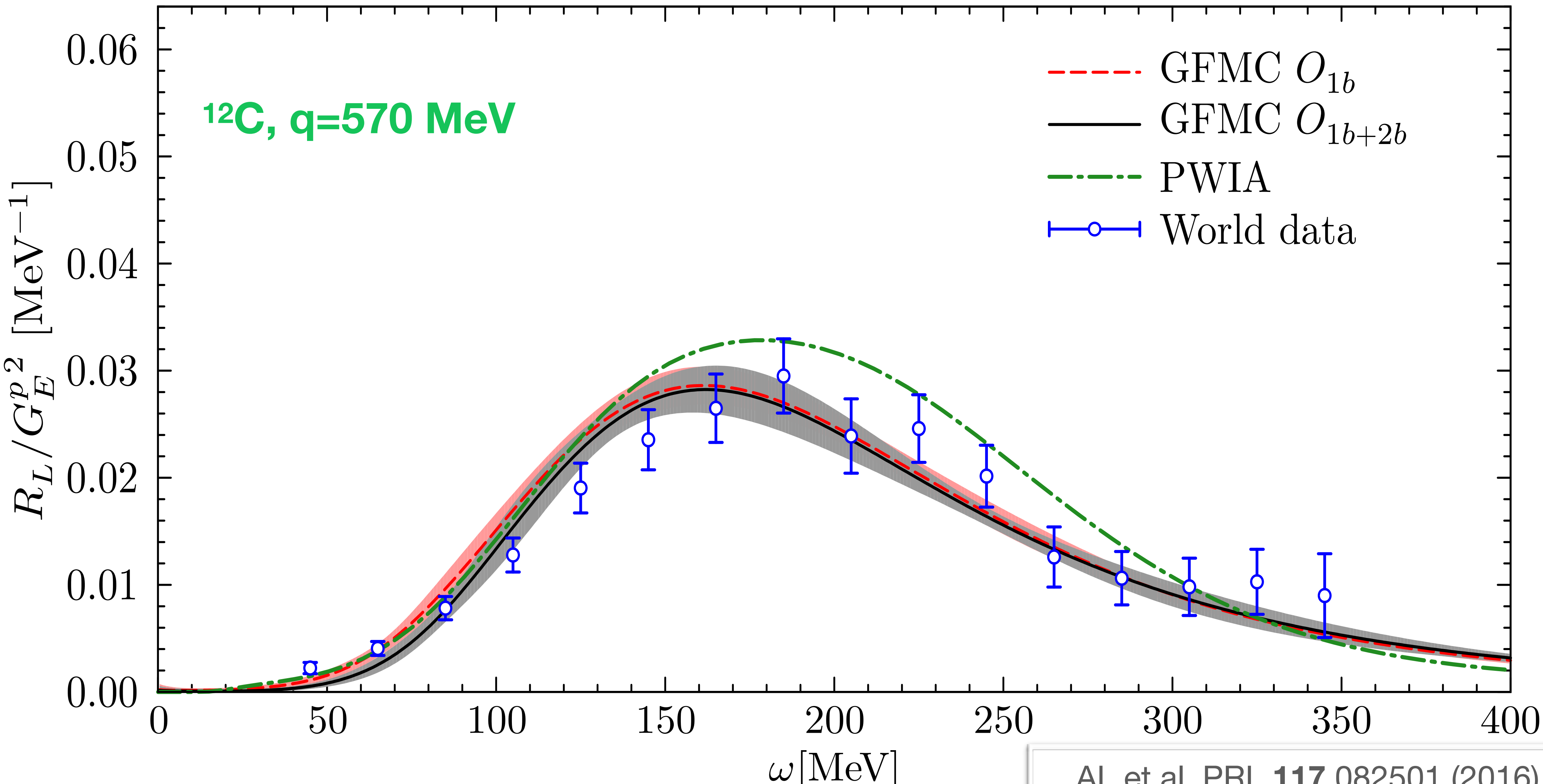
$$E\tau \;\approx\; \frac{q^2}{2m}\tau = 1$$

All nuclei have same density
approximately same ratio of n/p

→ Superscaling
Does not necessarily imply incoherent scattering

# Electron Scattering from $^{12}C$: Longitudinal (Charge) Response

- We inverted the electromagnetic Euclidean response of $^{12}C$
- Good agreement with data without in-medium modifications of the nucleon form factors
- Small contribution from two-body currents.

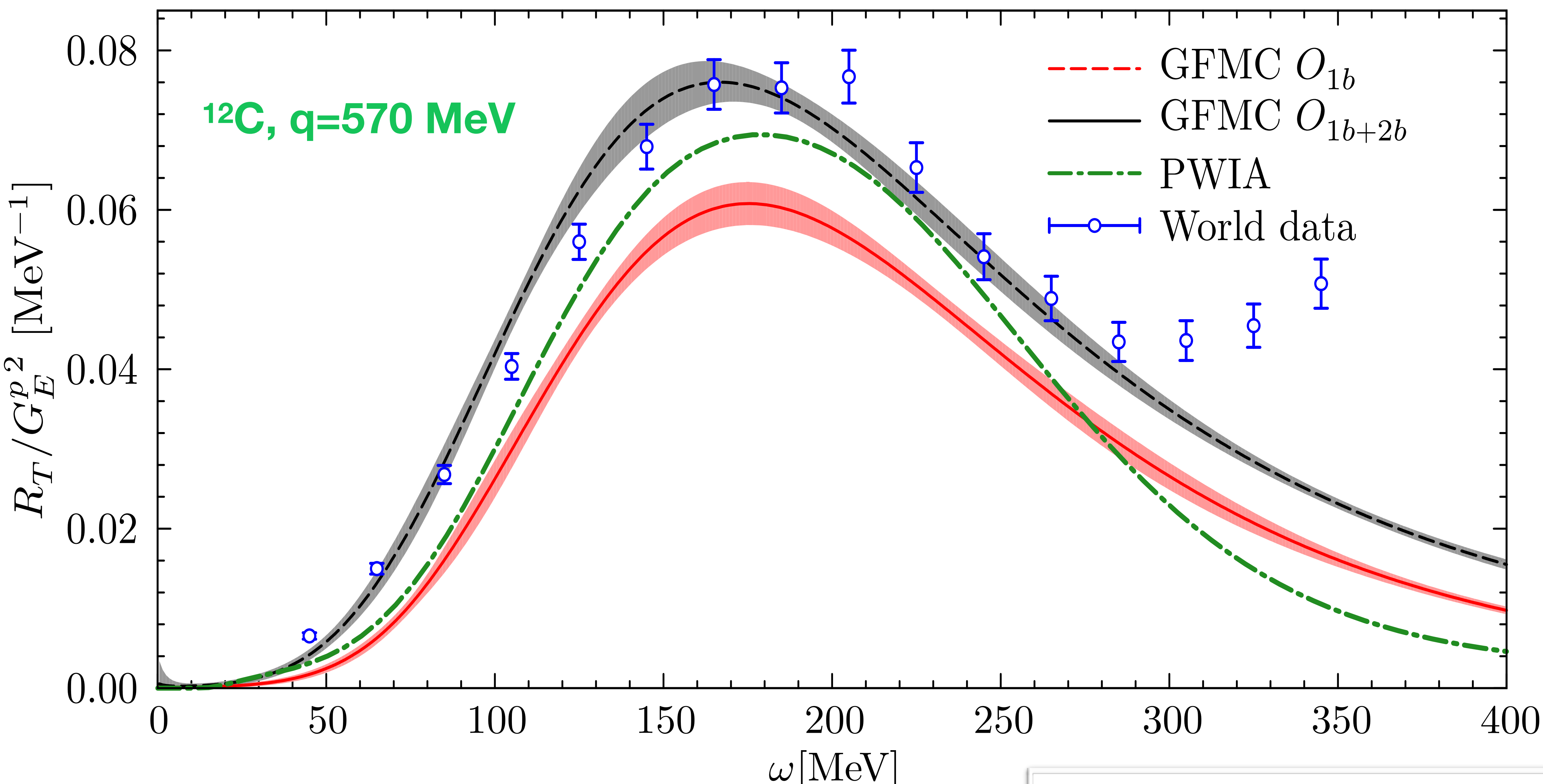



AL et al. PRL **117** 082501 (2016)

see also recent work in CC theory: 40Ar

# Electron Scattering from $^{12}C$: Transverse (Spin) Response

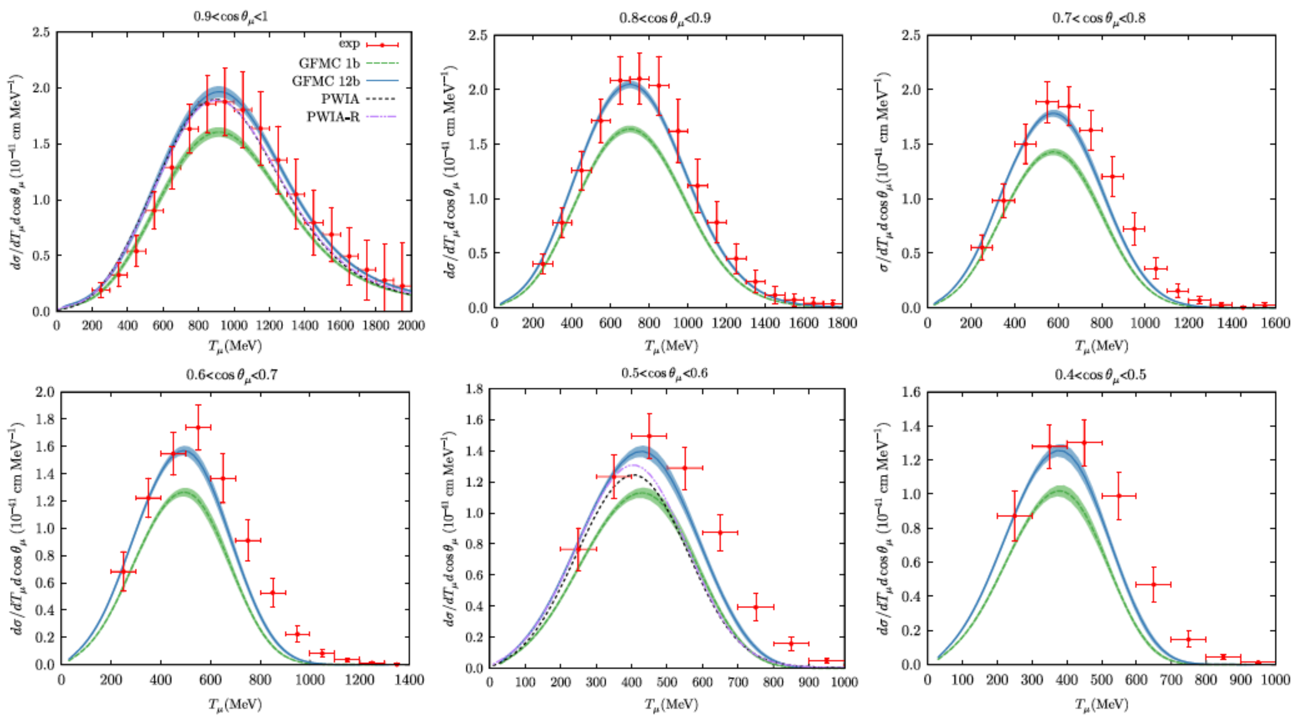
- We inverted the electromagnetic Euclidean response of $^{12}C$
- Good agreement with the experimental data once two-body currents are accounted for
- Need to include relativistic corrections in the kinematics




AL et al. PRL **117** 082501 (2016)

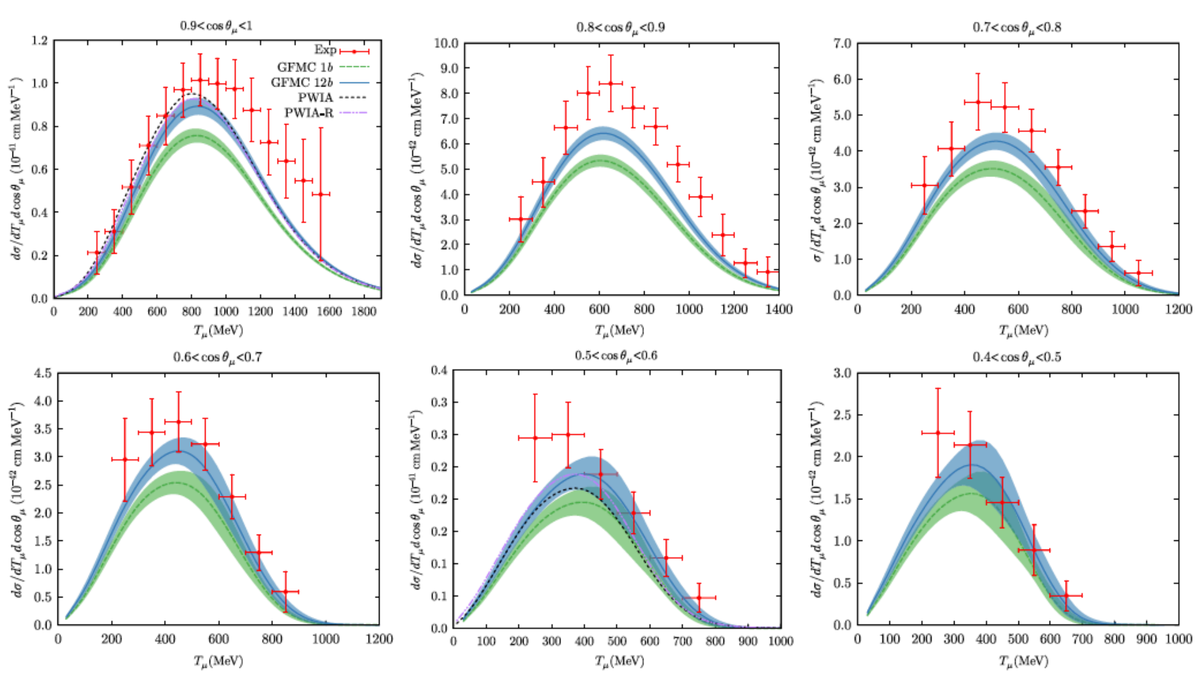
# Neutrino Scattering from $^{12}C$: MiniBoone and T2K

## MiniBoone: Neutrino




Neutrinos and Antineutrinos
Vector Axial-vector response
and V-A interference
Enhancement in two-body currents

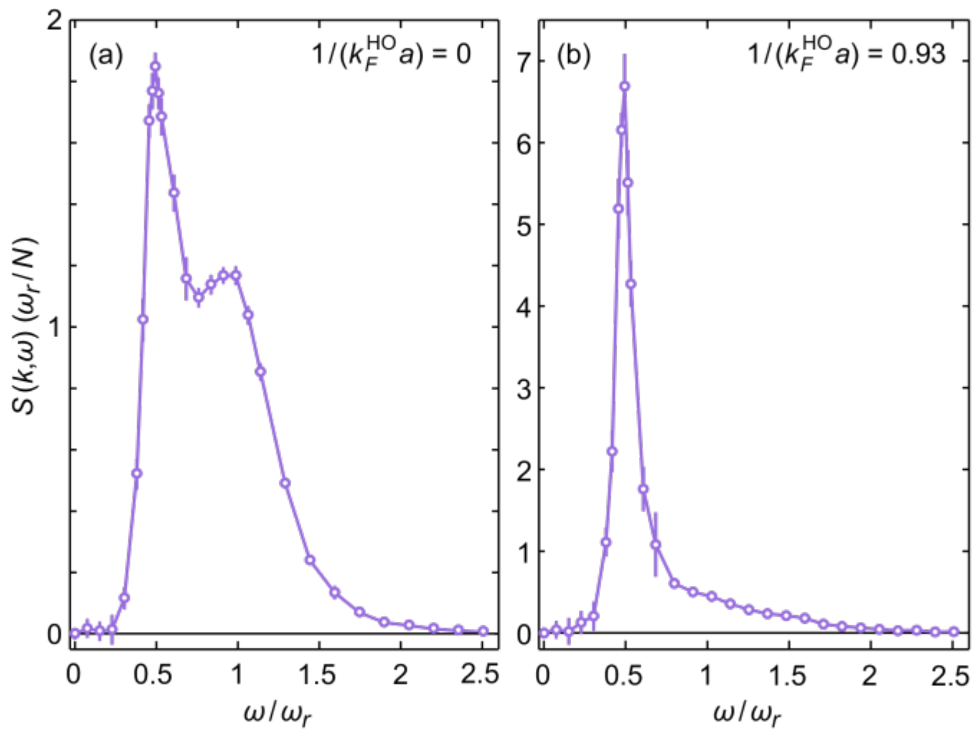
## MiniBoone: Anti-Neutrino




Also: T2K comparison

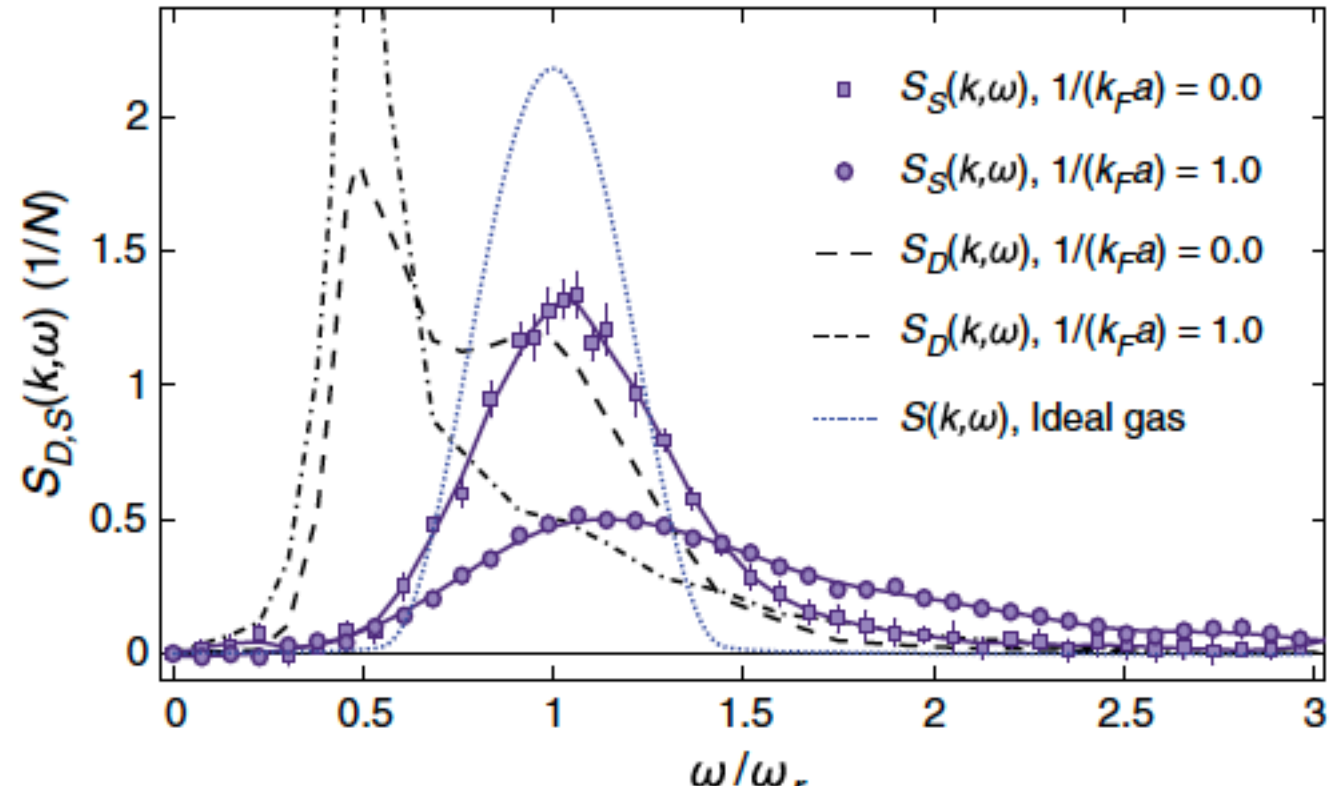
A Lovato et al. PRX (2020)

# *Spin and Density Response in Cold Atoms*



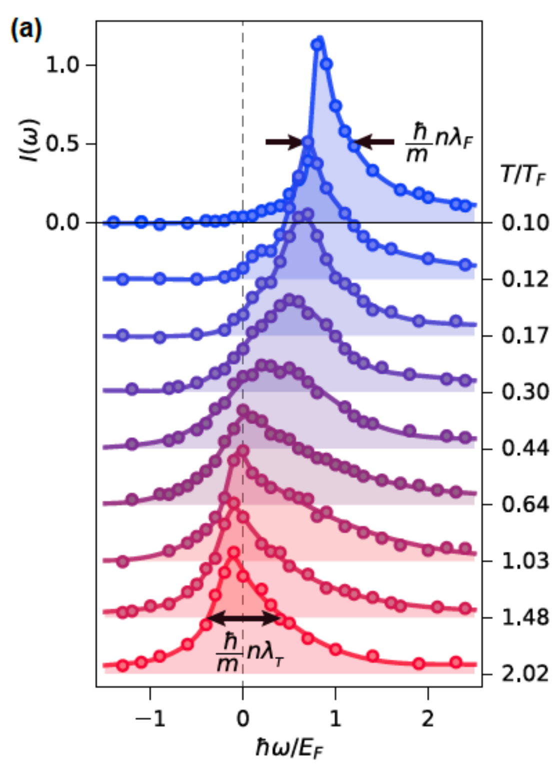

**Density-Density Response at k=4.3 $k_F^{HO}$**

Vale, Drut, Gandolfi, et al (2021)



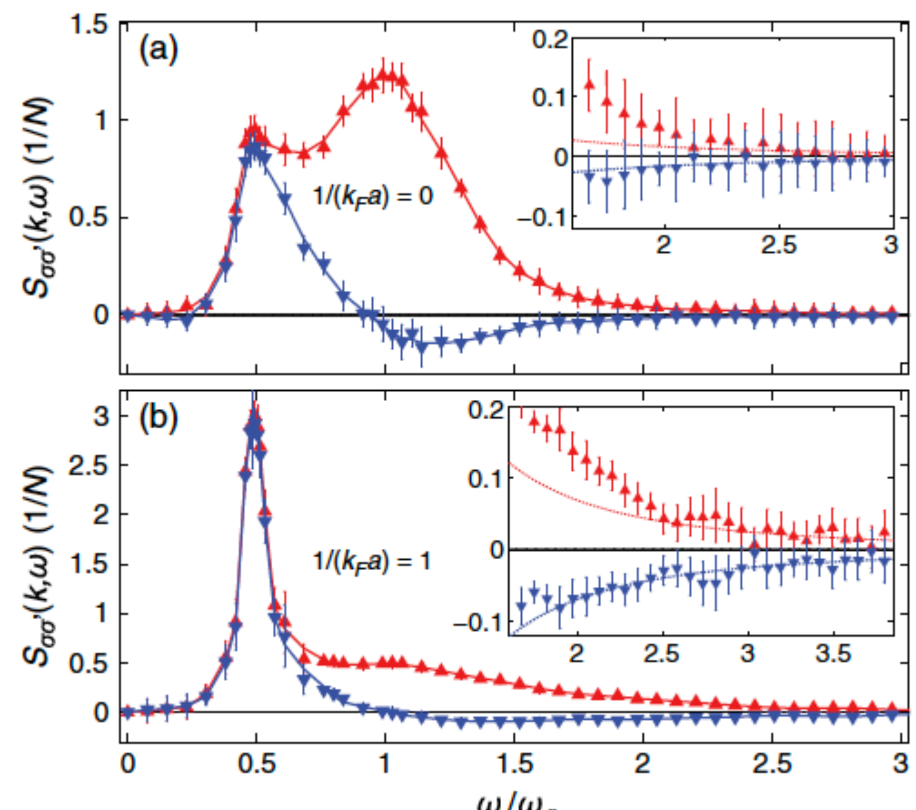

**Dynamic Spin Response of a Strongly-Interacting Fermi Gas**

Hoinka, et al, PRL (2012)




**Spectral Response / Contact**

Mukherjee, et.a all PRX (2020)




Spin parallel and anti-parallel components

## *Can we mimic or test models of nuclear and cold atom response?*

- y-scaling (different momenta),
- superscaling (different systems)
- QMC / short-time approximation
- Importance of short-range correlations
- Microscopic understanding of contacts

Extend to low-energy regime: superfluidity, etc.

Valuable Experimental Aspects in Cold Atom Systems

- Flexibility in Interaction Strength (and range?)
- Flexibility in External Potential
- Spin Imbalance (np vs pp, ↑↑ vs ↑↓)
- Others ?

# Towards real-time dynamics: Short-time approximation

Saori Pastore, et al, 2019

$$R^O(q,\omega) = \frac{\int d\Omega_q}{4\pi} \sum_f \langle \Psi_0 | \mathcal{O}^\dagger(\mathbf{q}) | \Psi_f \rangle \langle \Psi_f | \mathcal{O}(\mathbf{q}) | \Psi_0 \rangle \delta(E_f - E_0 - \omega),$$

$$R^O(q,\omega) = \frac{\int d\Omega_q}{4\pi} \int \frac{dt}{2\pi} \exp[i\omega t] \langle \Psi_0 | \mathcal{O}^\dagger(\mathbf{q}, t') \exp[-iHt] \mathcal{O}(\mathbf{q}, t=0) \Psi_0 \rangle,$$

At short time evolution can be described as a product of NN propagators

$$\langle \mathbf{R}', \sigma', \tau' | \exp[-iHt] | \mathbf{R}, \sigma, \tau \rangle \approx \langle \mathbf{R}', \sigma', \tau' | \prod_i \exp[-iH_i^0 t] \frac{\mathcal{S} \prod_{i<j} \exp[-iH_{ij} t]}{\prod_{i<j} \exp[-iH_{ij}^0 t]} | \mathbf{R}, \sigma, \tau \rangle$$

Evaluate as a sum of matrix elements of NN states embedded in the Nucleus

Incoherent sum of single nucleon currents

$$\sum_{q,Q,J,L,S,T} \langle \Psi_0 | \, \mathbf{j_i}^\dagger \, | \psi_{NN}(q,Q) \rangle \, \langle \psi_{NN}(q,Q) | \, \mathbf{j_i} \, | \Psi_0 \rangle \, \delta(E_f - E_i - \omega)$$

Interference of 1- and 2-nucleon currents

$$\sum_{q,Q,J,L,S,T} \langle \Psi_0 | \, \mathbf{j_{ij}}^\dagger \, | \psi_{NN}(q,Q) \rangle \, \langle \psi_{NN}(q,Q) | \, \mathbf{j_i} \, | \Psi_0 \rangle \, \delta(E_f - E_i - \omega)$$

Diagonal 2-nucleon currents

$$\sum_{q,Q,J,L,S,T} \langle \Psi_0 | \, \mathbf{j_{ij}}^\dagger \, | \psi_{NN}(q,Q) \rangle \, \langle \psi_{NN}(q,Q) | \, \mathbf{j_{ij}} \, | \Psi_0 \rangle \, \delta(E_f - E_i - \omega)$$

# Properties of short-time approximation

- Includes incoherent scattering plus interference between single nucleon terms and interference between one- and two-nucleon currents

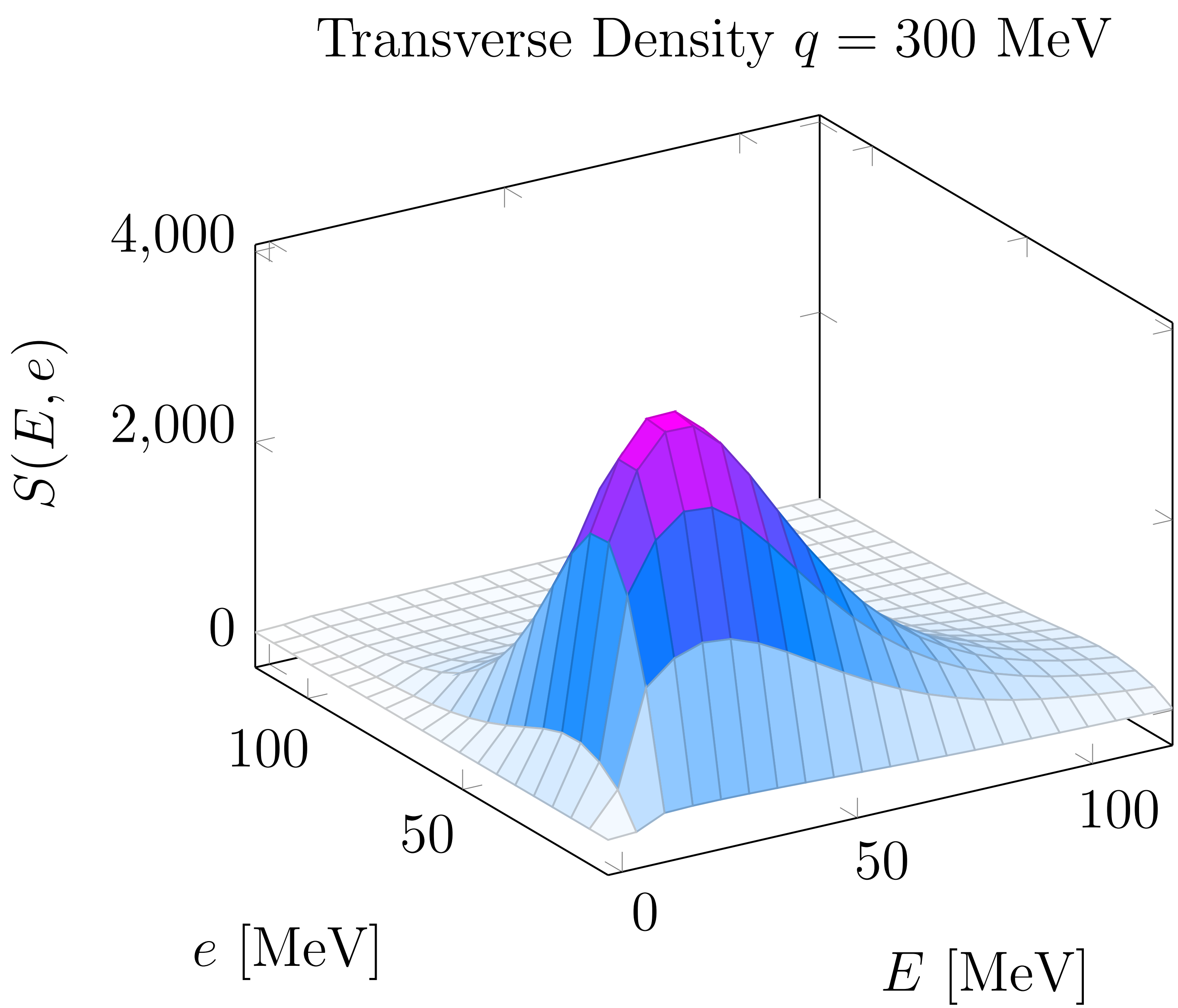
- Must be calculated at each q
  Fully incorporates Pauli exclusion principle
  Correct sum rule, good approximation to energy-weighted sum rule
  Includes charge propagation due to pion exchange

- Gives two-nucleon information after the vertex
  Now: relative and CM energy (or momentum) of the pair
  separation into different kind of pairs
  Future: angular dependence of q.P’, q.p’
  (where p’,P’ are the momenta of the final state pair)

- Intermediate between fully quantum evolution and single-nucleon vertex
  Classical evolution of the pair interacting with other nucleons still required

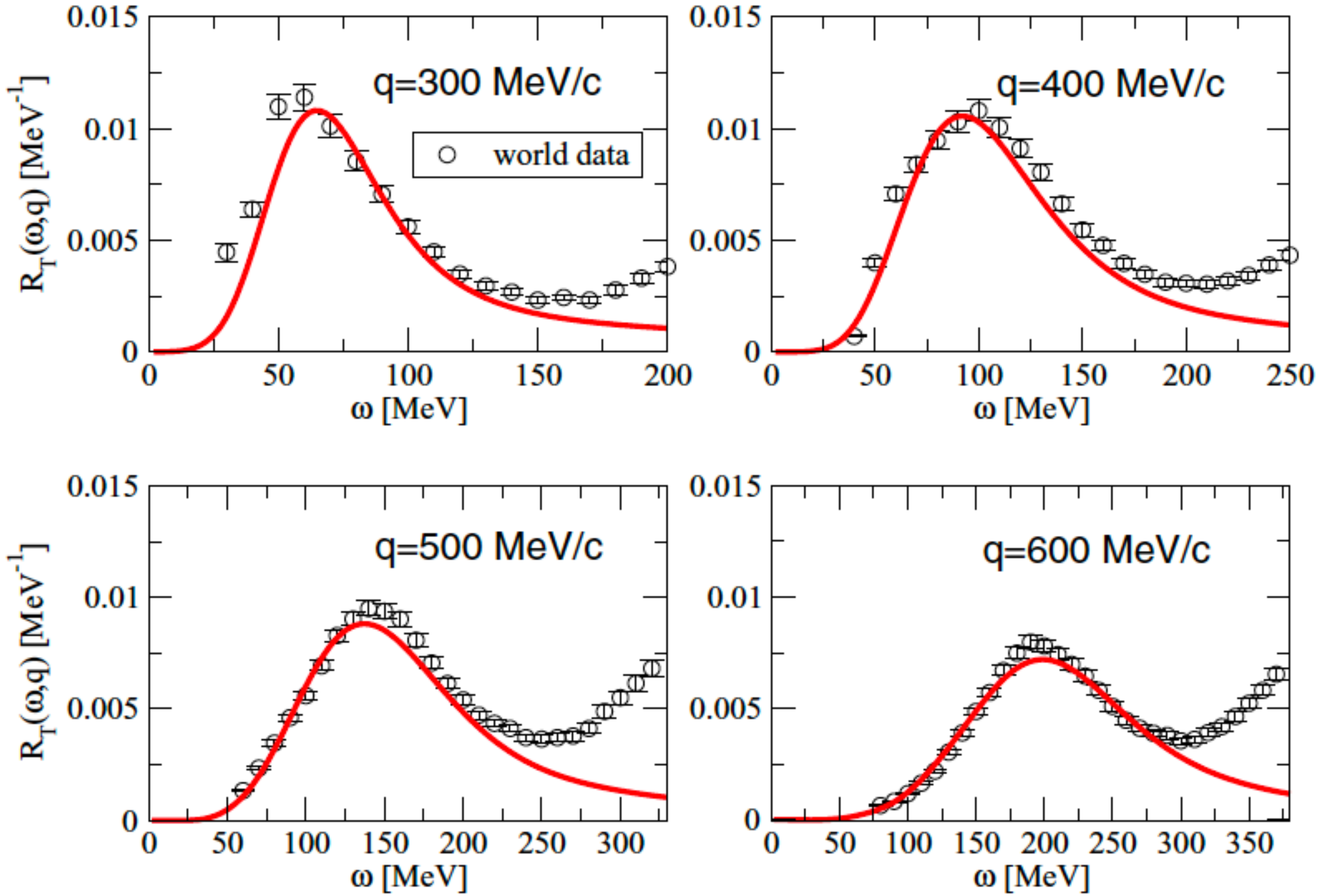
# Towards real-time dynamics: Short-Time Approx

- Integral over surfaces w/ constant e+E gives full response



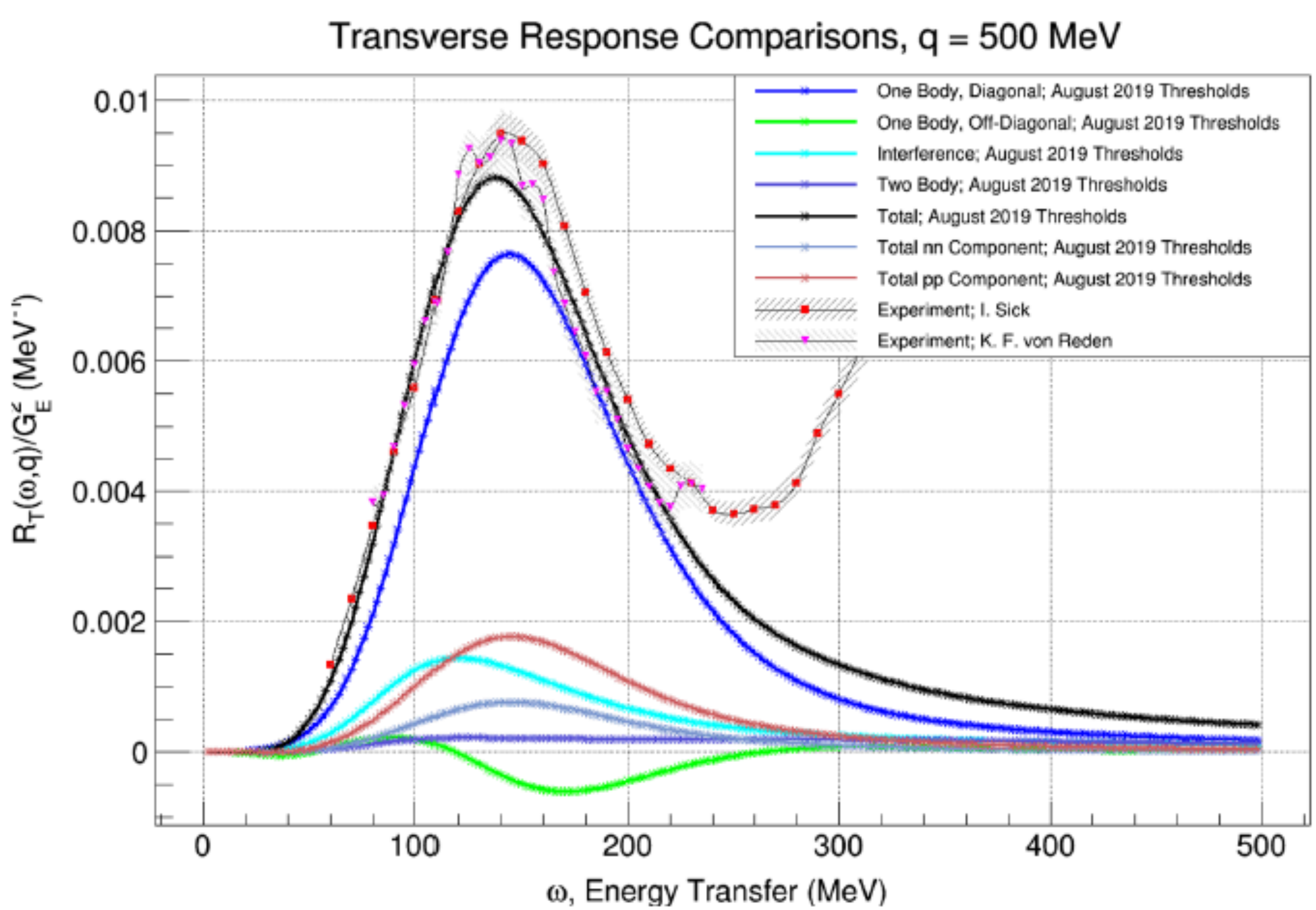

S. Pastore, et al, PRC (2020)

- STA Comparison to data/ GFMC : electron scattering on A=4



# Towards exclusive final states

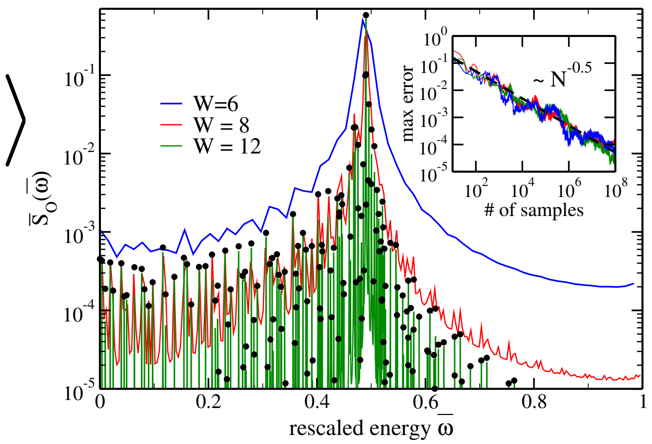
event generators take vertex (now moving to two nucleon vertex and perform semiclassical propagation through the nucleus




Barrow, et al, PRD 2021

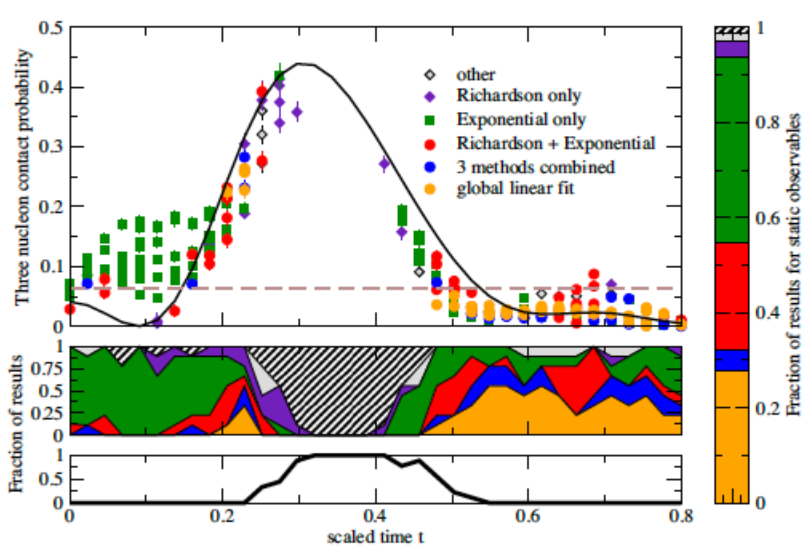
# Lower Energy Response/ Exclusive Final States Quantum Computing and Real-Time quantum dynamics

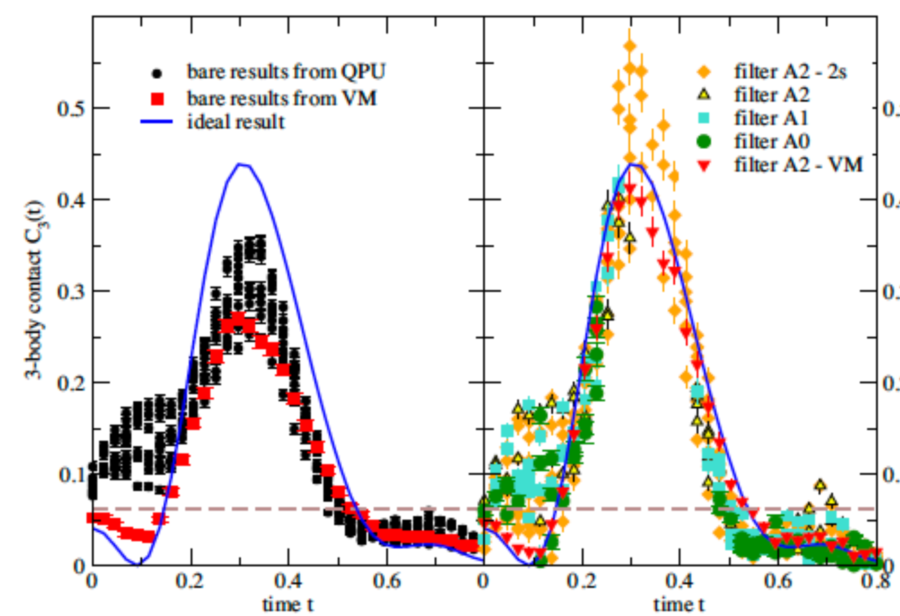
$$R(q,\omega) = \int dt \ \langle 0|\ \mathbf{j}^\dagger \exp[i(H-\omega)t]\ \mathbf{j}\ |0\rangle$$




- use quantum computer
- test ideas on simple problems
- gradually extend to more realistic cases

- Only fairly modest time-propagation is required (modest coherence time)
- Quantum vs. Classical dynamics
- Dynamics of entanglement
- Impact on specific observables




Alessandro Roggero, et al (2018)
Roggero, Lu, PRD (2020)
Roggero, Baroni, et al. (in prep)

# Future directions

- Compare theories of nuclear and cold atom response with data
- Can we merge quantum/classical approaches to get
  exclusive cross sections? What are the requirements
- Tests of this quantum/classical transition
  Nuclear physics
  Finite cold atom systems
- Other (Bulk) Responses (Spin susceptibility, viscosity, …)
- Beyond Linear Response …. Quench, more general scattering, …