Simulating both parity sectors of the Hubbard Model with Tensor Networks

arXiv:2106.13583 [physics.comp-ph]

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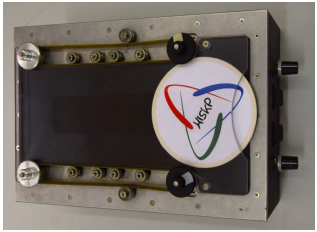
Helmholtz-Institut für Strahlen- und Kernphysik
Bonn University



July 7, 2021

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Hybrid Monte Carlo simulations of the Hubbard model [Ostmeyer *et al.* 2020, 2021]

$$H = -\sum_{\langle x,y\rangle,s} c^\dagger_{x,s} c_{y,s} + \frac{1}{2} U \sum_x q_x^2$$

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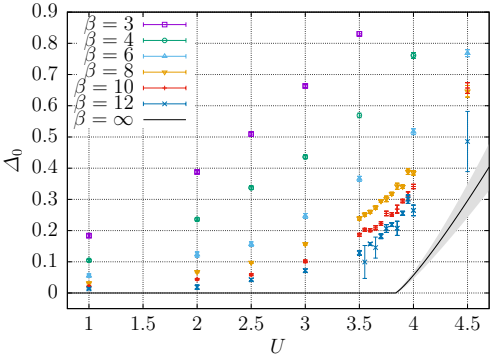

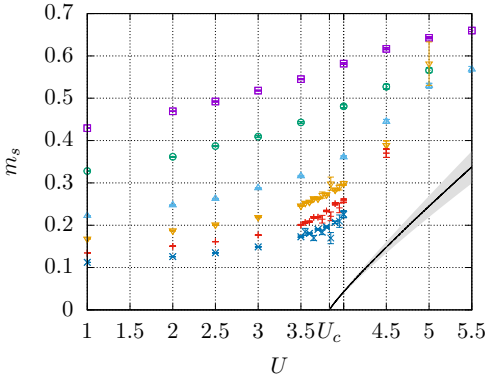

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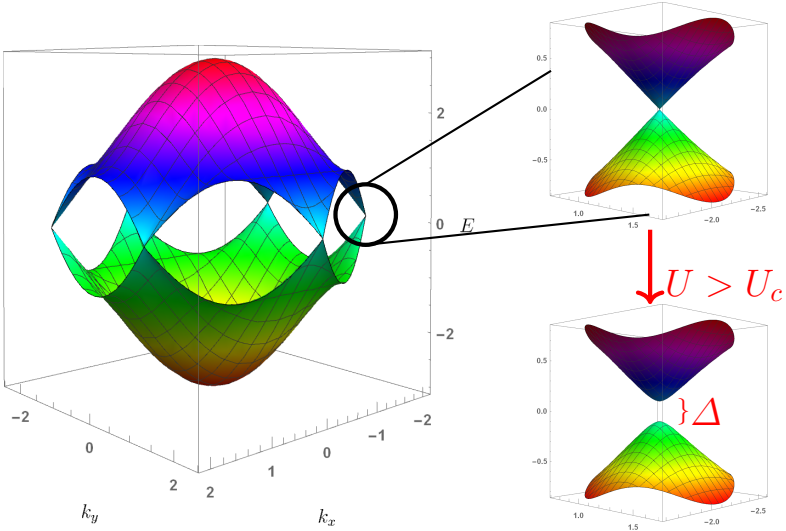

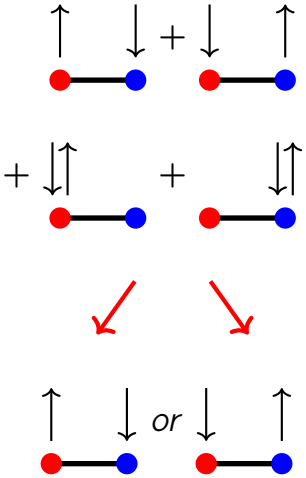

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Sign problem!

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  [Corboz 2016; Schneider *et al.* 2021]

Projected Entangled Pair States (PEPS) [Orús 2014; Verstraete & Cirac 2004]

$$|\psi\rangle = \sum_{s_1} \sum_{s_2} \cdots \sum_{s_N} A_{s_1,s_2,\ldots,s_N} |s_1\rangle \otimes |s_2\rangle \otimes \cdots \otimes |s_N\rangle$$

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Truncate $\alpha_i \leq D\ \forall i$

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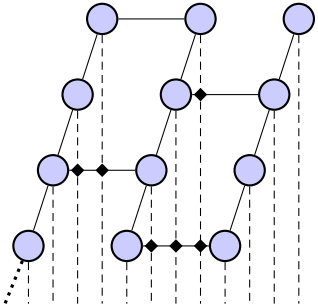

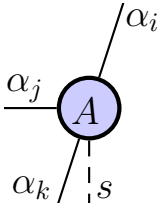

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Fermionic PEPS [Corboz *et al.* 2010]

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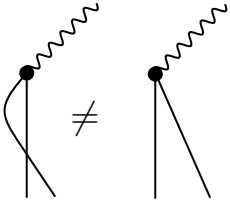

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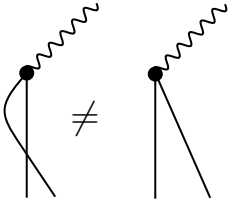

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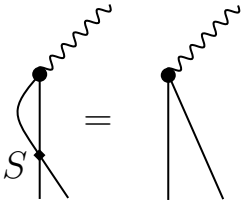
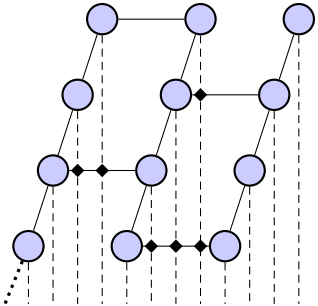
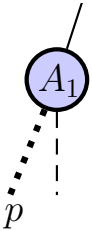


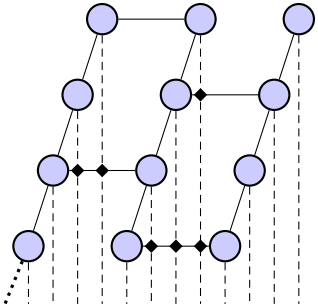
$$S = \begin{array}{c} \overbrace{\qquad\qquad}^{\text{even}} \quad \overbrace{\qquad\qquad}^{\text{odd}} \\ \begin{pmatrix} 1 & \dots & 1 & 1 & \dots & 1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 1 & \dots & 1 & 1 & \dots & 1 \\ 1 & \dots & 1 & -1 & \dots & -1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 1 & \dots & 1 & -1 & \dots & -1 \end{pmatrix} \end{array}$$

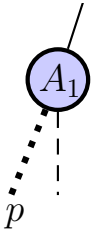
Parity link





Parity link





$p = \pm 1$

$\Rightarrow$ even- and odd-parity subspaces without communication

Ground state search

- Fix bond dimension D

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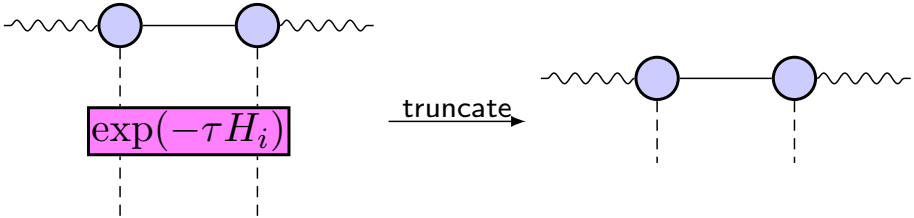
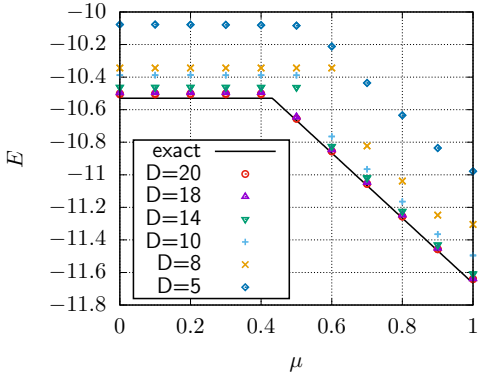
- Local updates
- Contract network to calculate expectation values

Simple Update

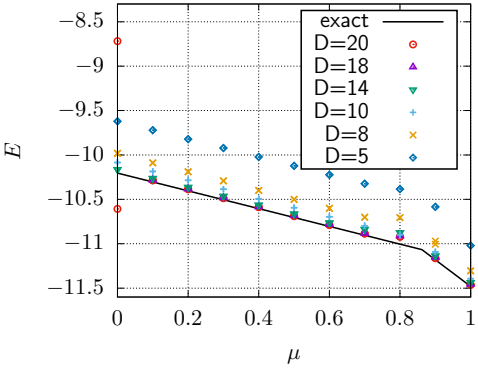


Simulations with chemical potential (3×4 hex. lattice, $U = 2$)

even parity



odd parity



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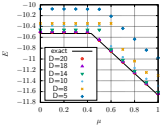
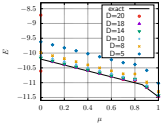
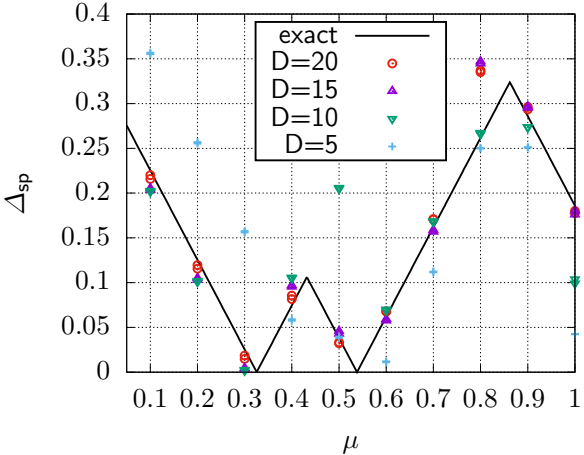
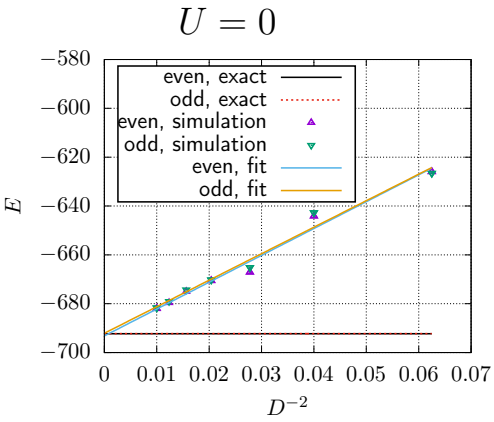
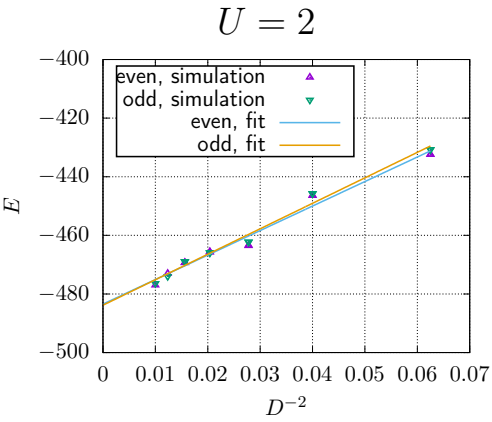




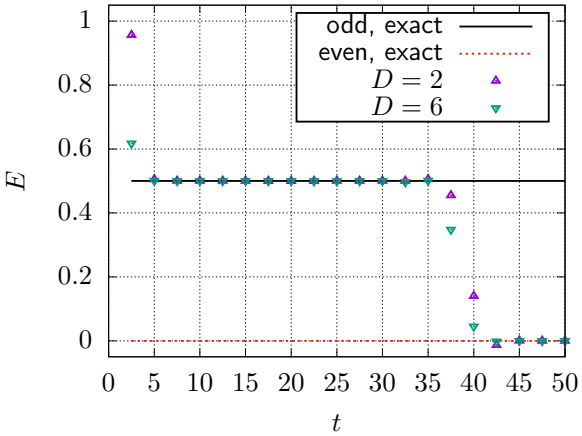


Simulations with chemical potential (30×15 hex. lattice, $\mu = 0.5$)





Stability issues with odd parity



3×4 hex. lattice, $U = 1$, no hopping

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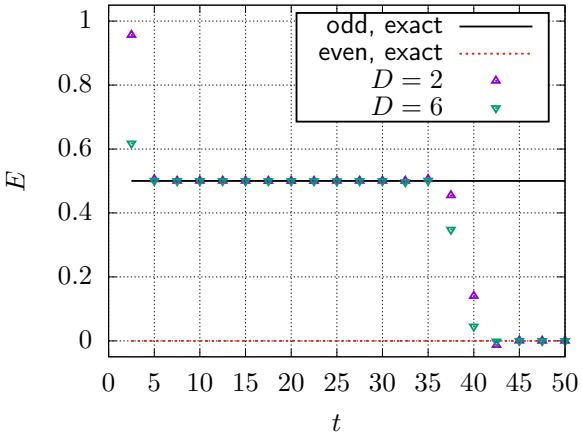

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Large gap (strong coupling) $\Rightarrow$ jump to even parity ground state

Possible solutions

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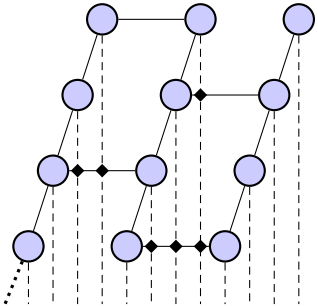
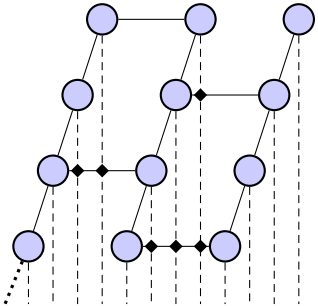
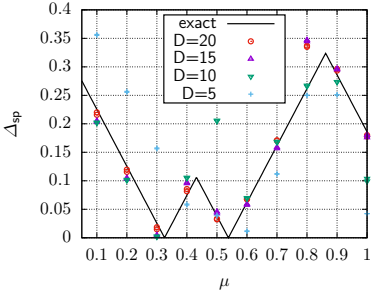
- Open for suggestions!

Summary



Summary

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D=20
D=15
D=10
D=5
Δ_{sp}
μ
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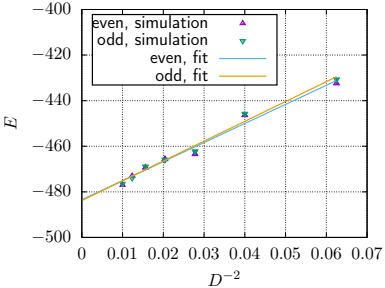
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1


even, simulation
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E
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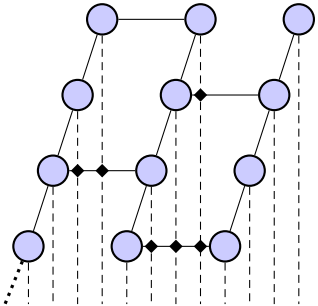
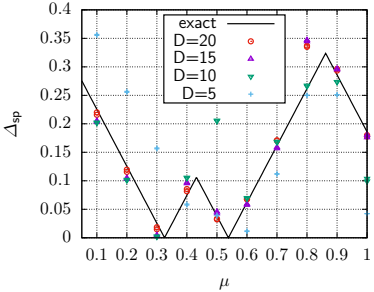
Summary

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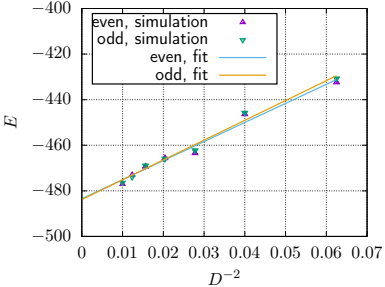
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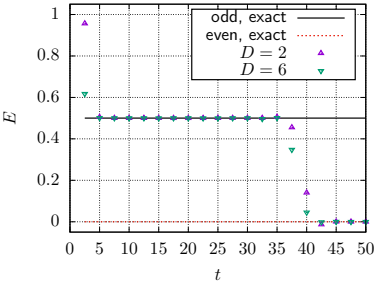
odd, exact
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Bibliography I

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