Interacting Massless Dirac Fermions with Spin-Charge Flip Symmetry

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Outline

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Hanqing Liu's Poster

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Emilie Huffman's Poster

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Motivation

Relativistic fermions in flatland have a rich class of fixed points.

Difficult to access them without Monte Carlo calculations

Monte Carlo calculations are difficult due to fermion sign problems

When sign problems can be solved, Monte Carlo methods scale poorly with system size!

We have been exploring a new method: “Fermion Bag” approach.

Meron-clusters are also fermion bags!

What can we learn from them?

Lattice Models for Fermion Bags

Unfortunately at the moment, fermion bag ideas are rather restrictive in the type of lattice Hamiltonians we can solve.

A simple class of lattice models we can solve using them are given by

$$H = -\sum_{\langle ij \rangle} \underbrace{\omega \; e^{2\alpha \; \eta_{ij} \sum_{a=1}^{N_f} (c_i^\dagger c_j + c_j^\dagger c_i)}}_{H_{ij} \text{ "bond" operator}}$$

η_{ij} (pi-flux)

The model has one tunable parameter

In terms of the familar scales t, V

$$\omega = (t^2/V)(1-(V/2t)^2) \qquad \sinh(2\alpha) = (V/t)/(1-(V/2t)^2)$$

When $V = 2t$ fermion bags become meron clusters!

Why does this structure help for the “fermion bag” approach?

Let us write $H = \sum_b H_b \;=\; H_0 + H_{\text{int}}$ where

$$H_0 \;=\; 0, \quad H_{\text{int}} = \sum_b H_b \qquad H_b \;=\; -H_{ij}$$

We can write the partition function as

$$Z \;=\; \int_0^\beta dt_1 \int_0^{t_1} dt_2 \ldots \int_0^{t_{k-1}} dt_k \, \text{Tr}\Big(\text{e}^{-H_0(\beta - t_1)} \, (-H_{\text{int}}) \, \text{e}^{-H_0(t_1 - t_2)} \, (-H_{\text{int}}) \ldots \text{e}^{-H_0 t_k} \Big)$$

CT INT method, Rubtsov, Lichtenstein,…

$$= \int [dt] \sum_{[b]} \text{Tr}\Big((-H_{b_1})(-H_{b_2}) \ldots (-H_{b_k}) \Big)$$

exponential of a free nearest neighbor hopping term

We can now use the BSS formula

$$Z = \int [dt] \sum_{[b]} \underbrace{\mathrm{Tr}\Big((-H_{b_1})(-H_{b_2})...(-H_{b_k})\Big)}_{\omega^k \mathrm{Det}(1 \; + \; B_1 B_2 .... B_k)}$$

BSS formula

Positive!

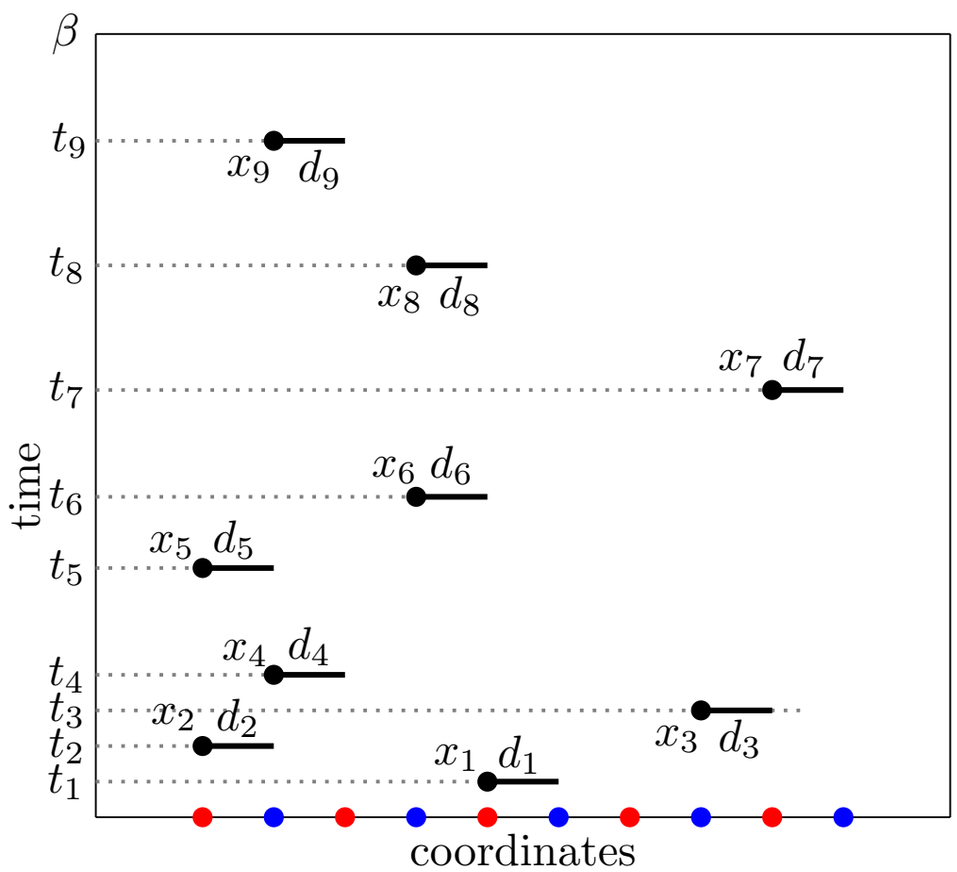
Every "B" matrix is associated with a bond and is a N_s x N_s unit matrix except for a 2 x 2 block that correspond to two nearest neighbor sites of a bond.

$$B = \begin{pmatrix} 1 & 0 & ... & 0 & ... & 0 & ... & 0 \\ 0 & 1 & ... & 0 & ... & 0 & ... & 0 \\ . & . & ... & . & ... & . & ... & . \\ . & . & ... & . & ... & . & ... & . \\ . & . & ... & . & ... & . & ... & . \\ 0 & 0 & ... & \cosh(2\alpha) & ... & \sinh(2\alpha) & ... & 0 \\ . & . & ... & . & ... & . & ... & . \\ . & . & ... & . & ... & . & ... & . \\ . & . & ... & . & ... & . & ... & . \\ 0 & 0 & ... & \sinh(2\alpha) & ... & \cosh(2\alpha) & ... & 0 \\ . & . & ... & . & ... & . & ... & . \\ . & . & ... & . & ... & . & ... & . \\ . & . & ... & . & ... & . & ... & . \\ 0 & 0 & ... & 0 & ... & 0 & ... & 1 \end{pmatrix}$$

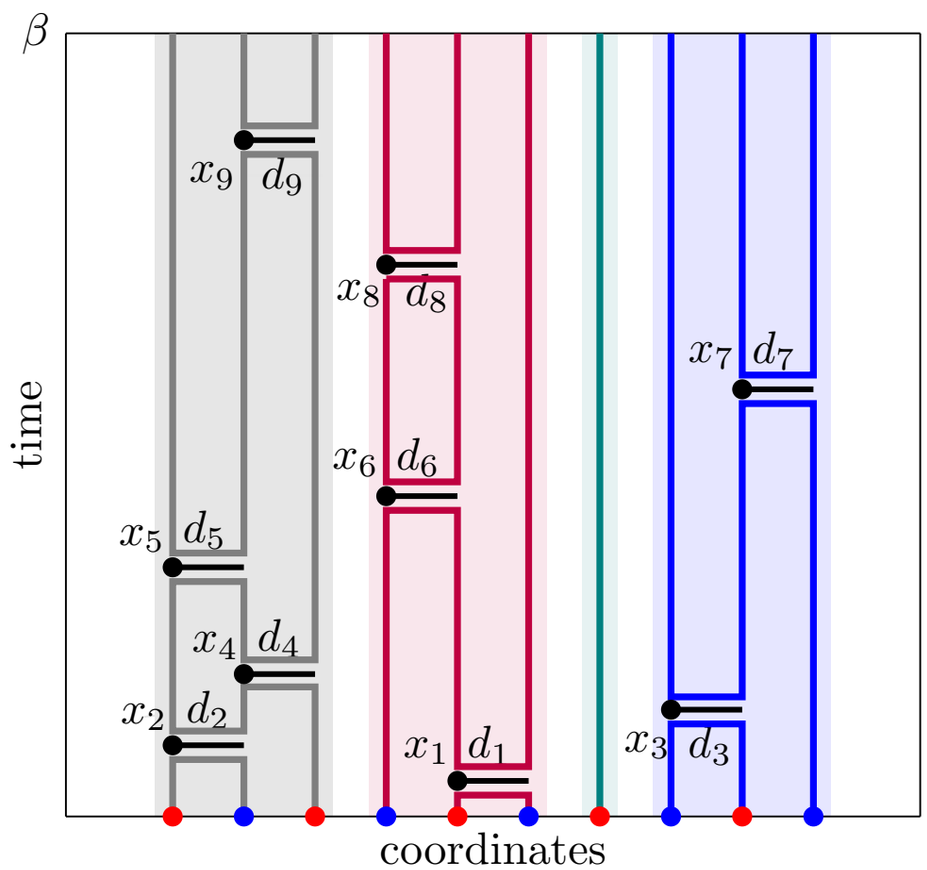
In traditional "auxiliary field" Monte Carlo approach B is more non-local. This slows the calculations.

The fermion bag idea

$$Z = \int [dt] \sum_{[b]} \omega^k \mathrm{Det}(1 + B_1 B_2 .... B_k)$$



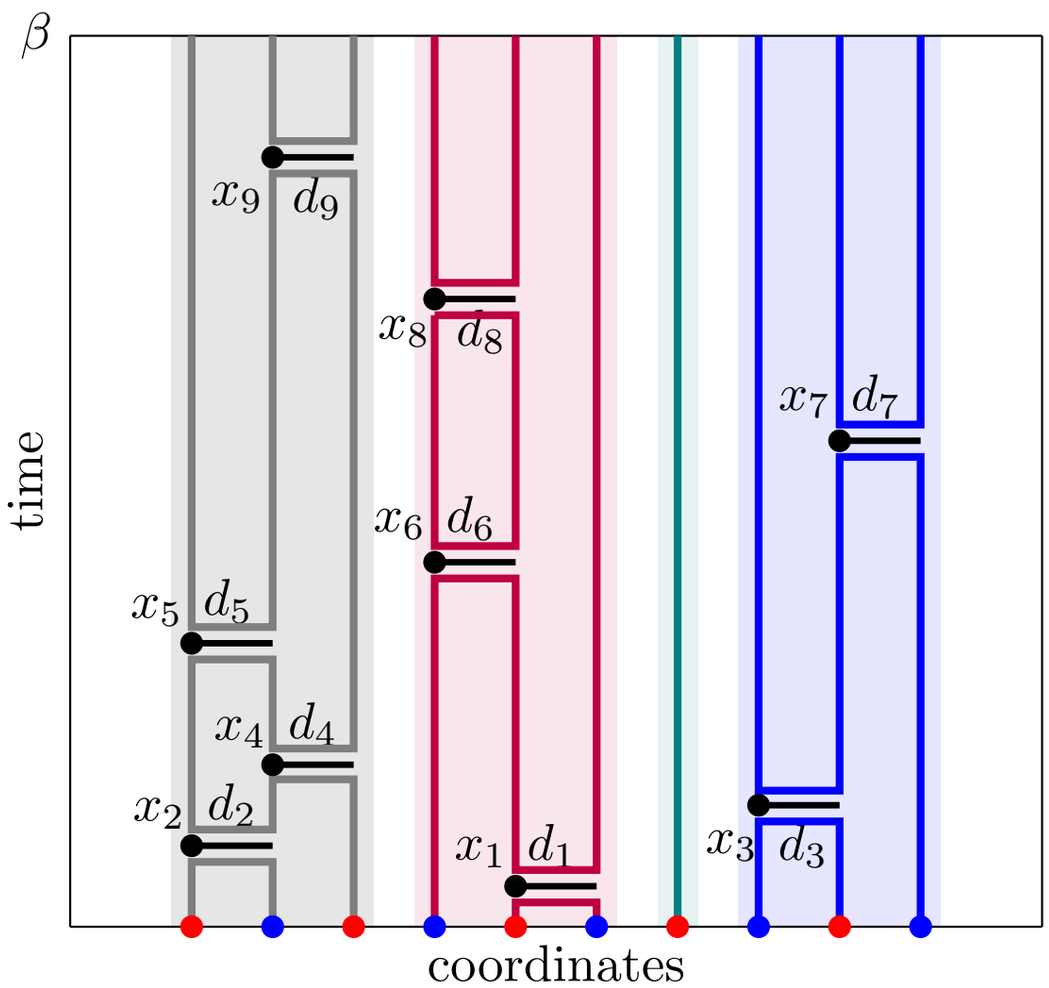
[b] configuration

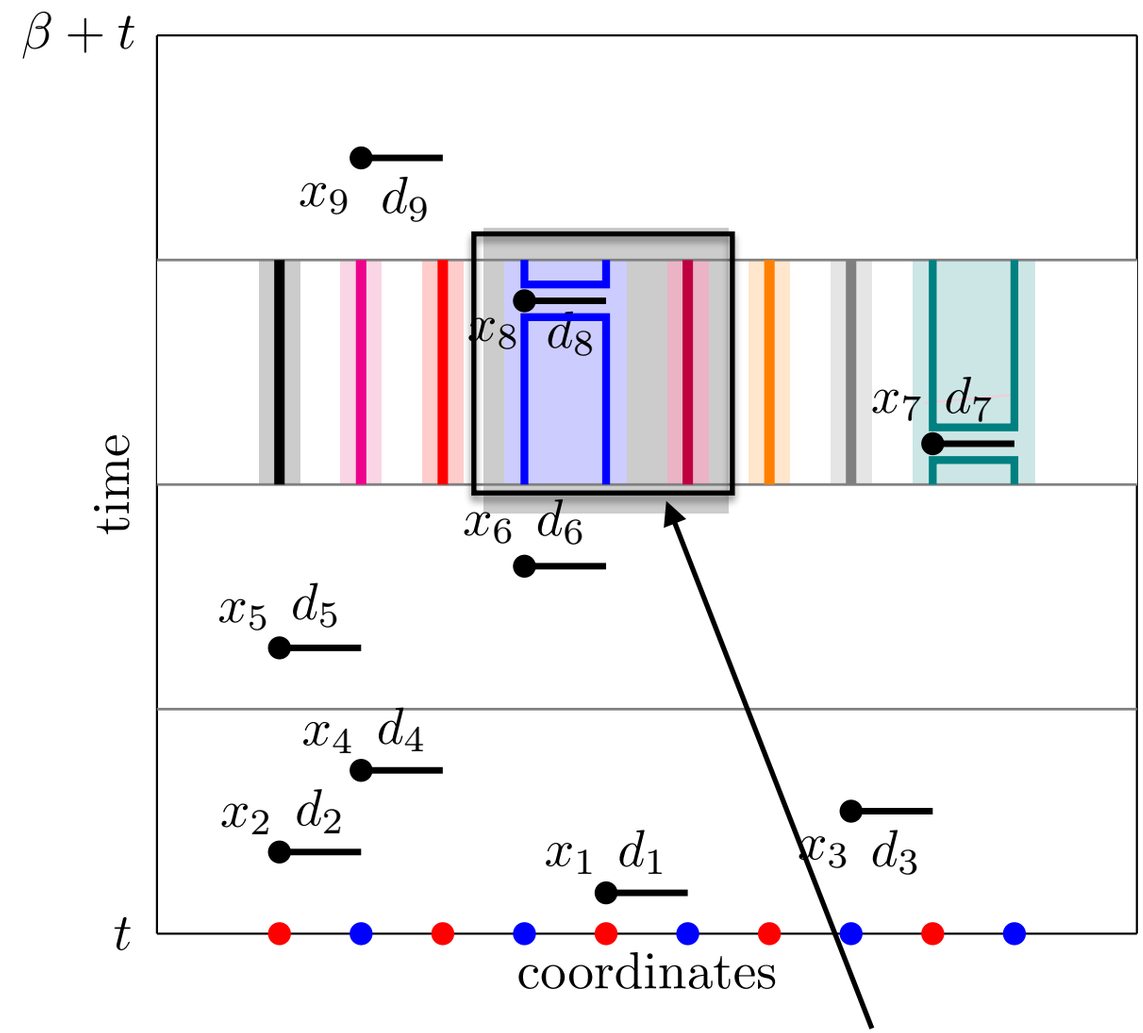


Fermion Bags

The determinant is block diagonal

At small temperatures fermion bags merge!
but at high temperatures fermion bags naturally split



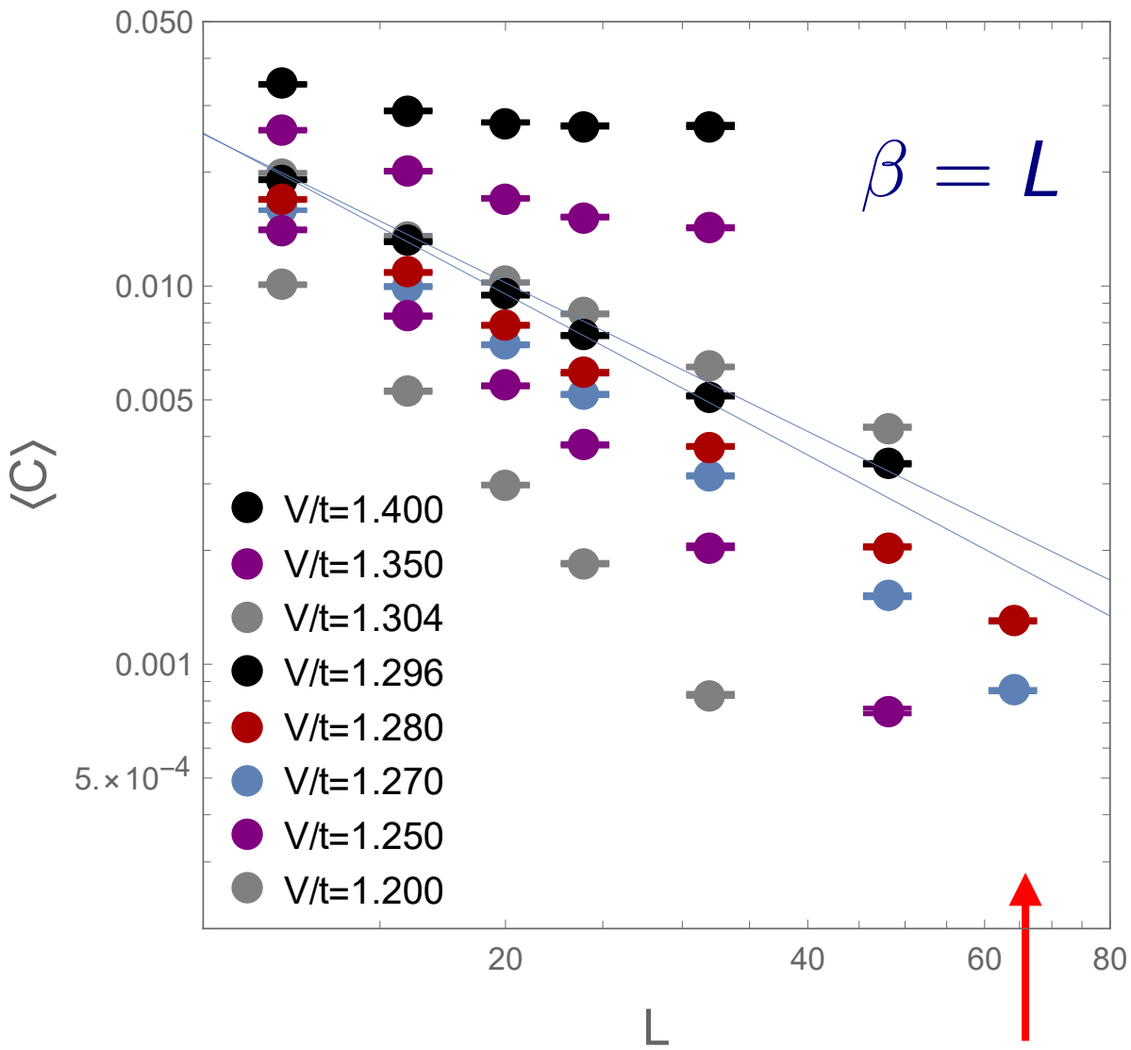


space-time regions
to update

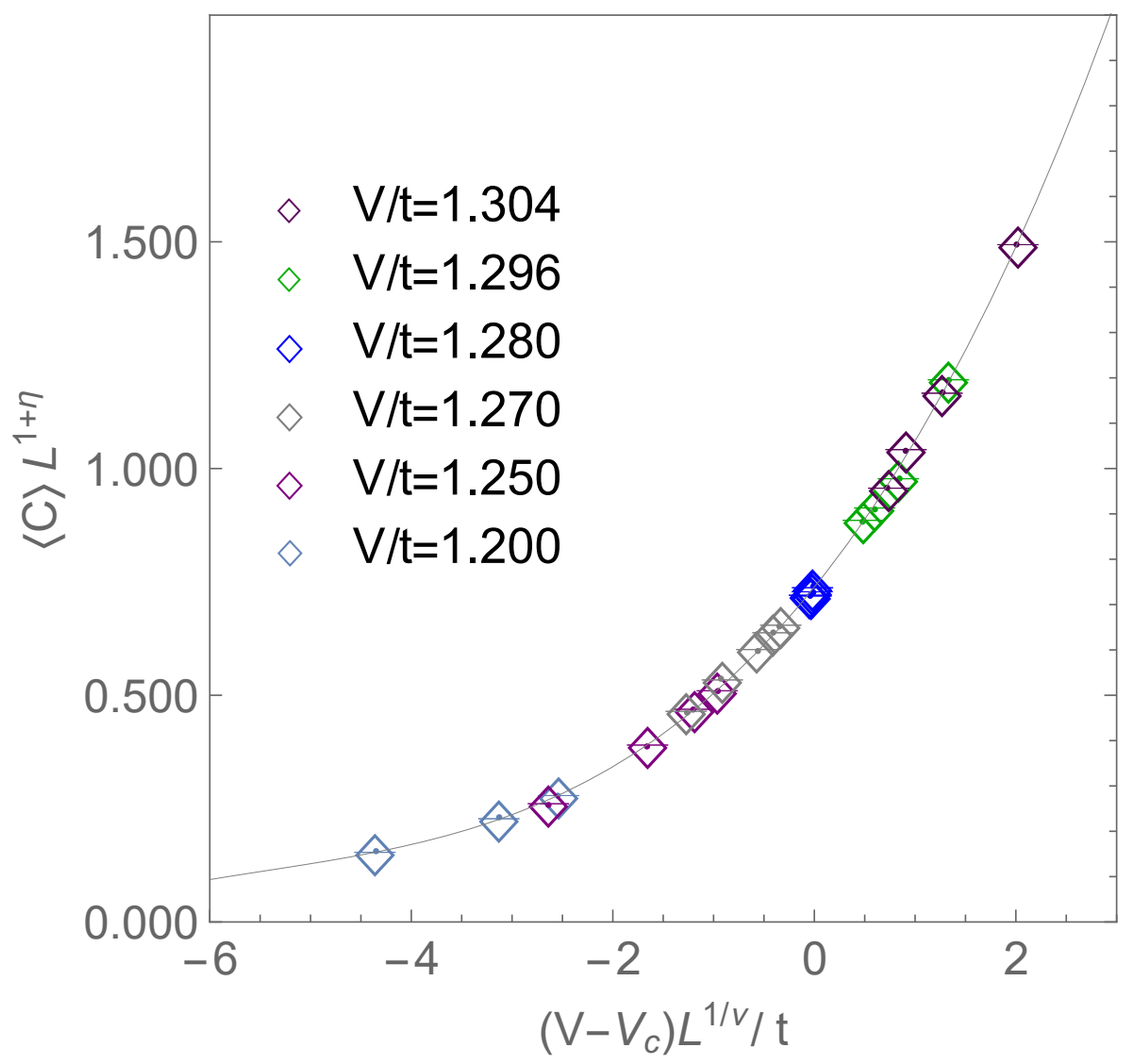
Results with fermion bag method:

Huffman, SC Phys.Rev.D 101 (2020) 7, 074501

$N_f = 1$ model, in the continuous-time formulation

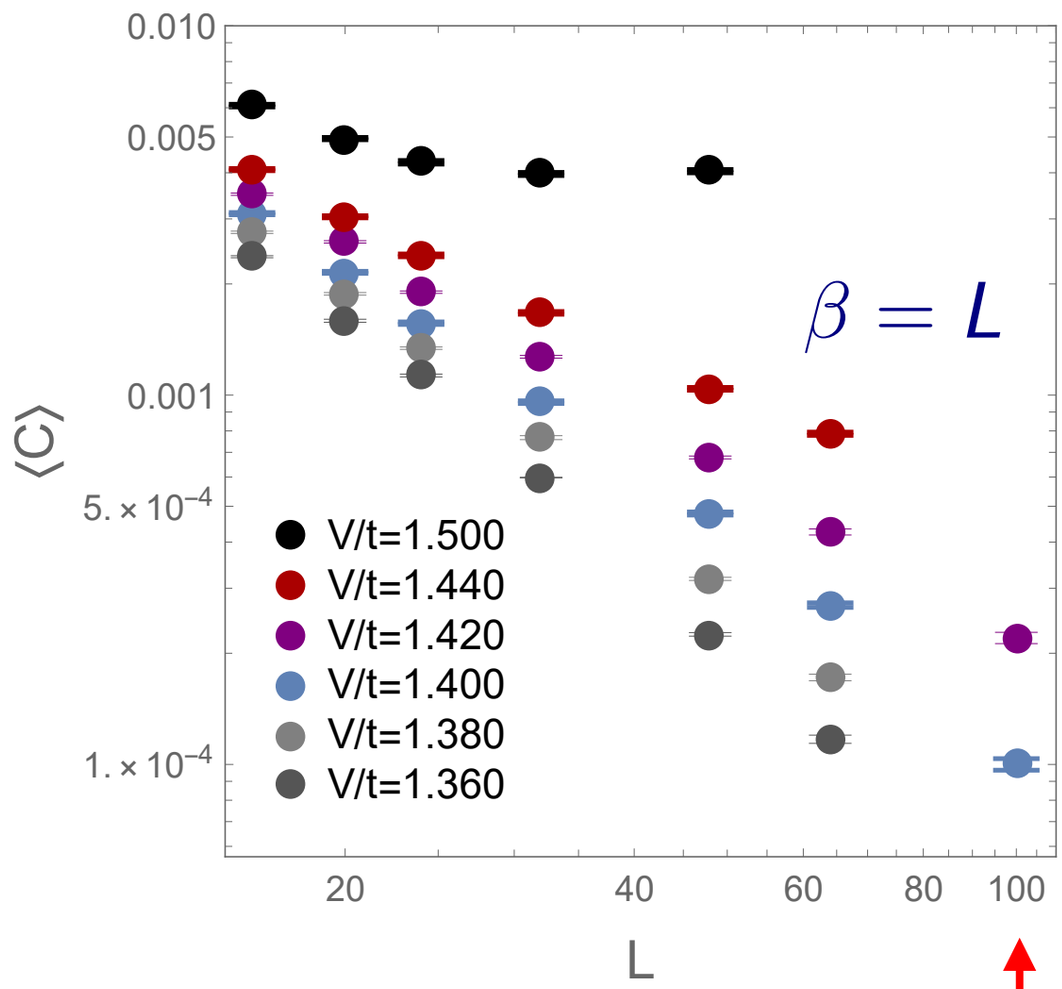


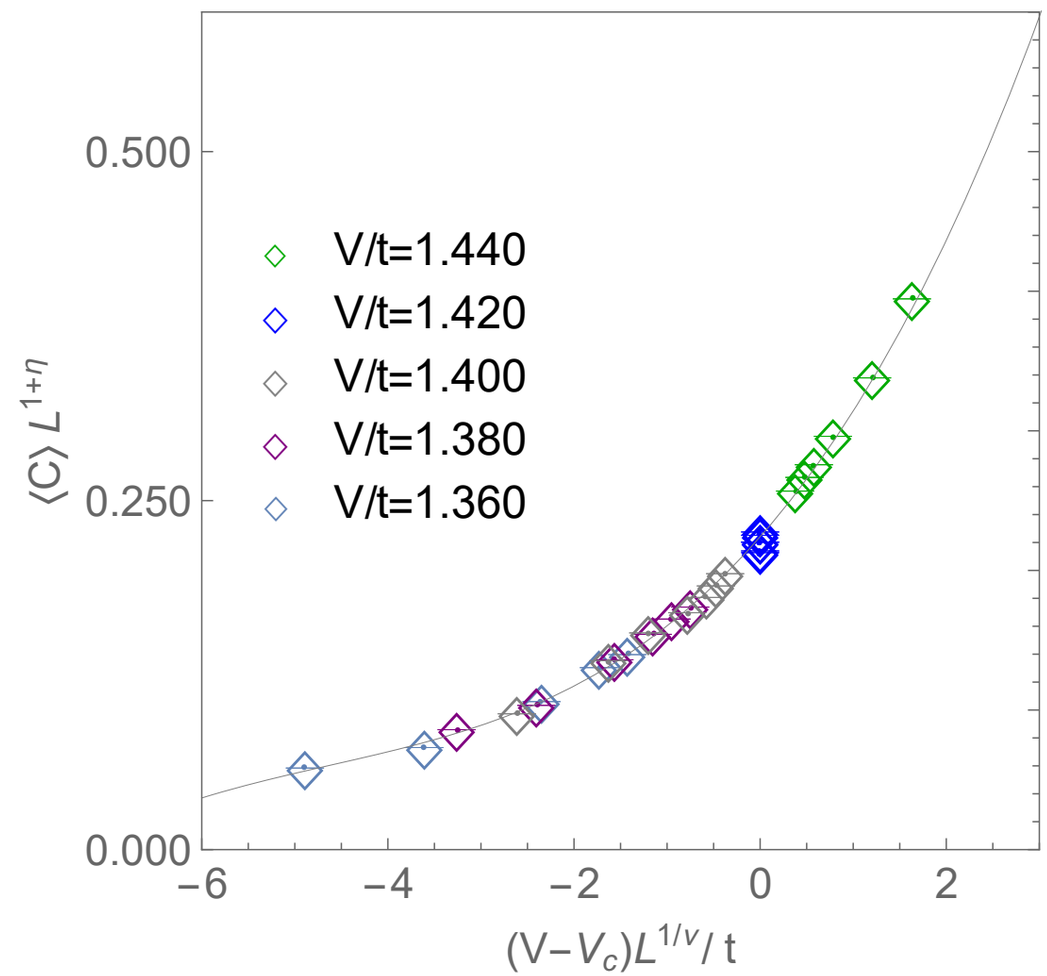
4096 sites



$V_c = 1.281(2)$ $\eta = 0.51(3)$ $\nu = 0.89(1)$

$N_f = 1$ model, in the discrete-time formulation



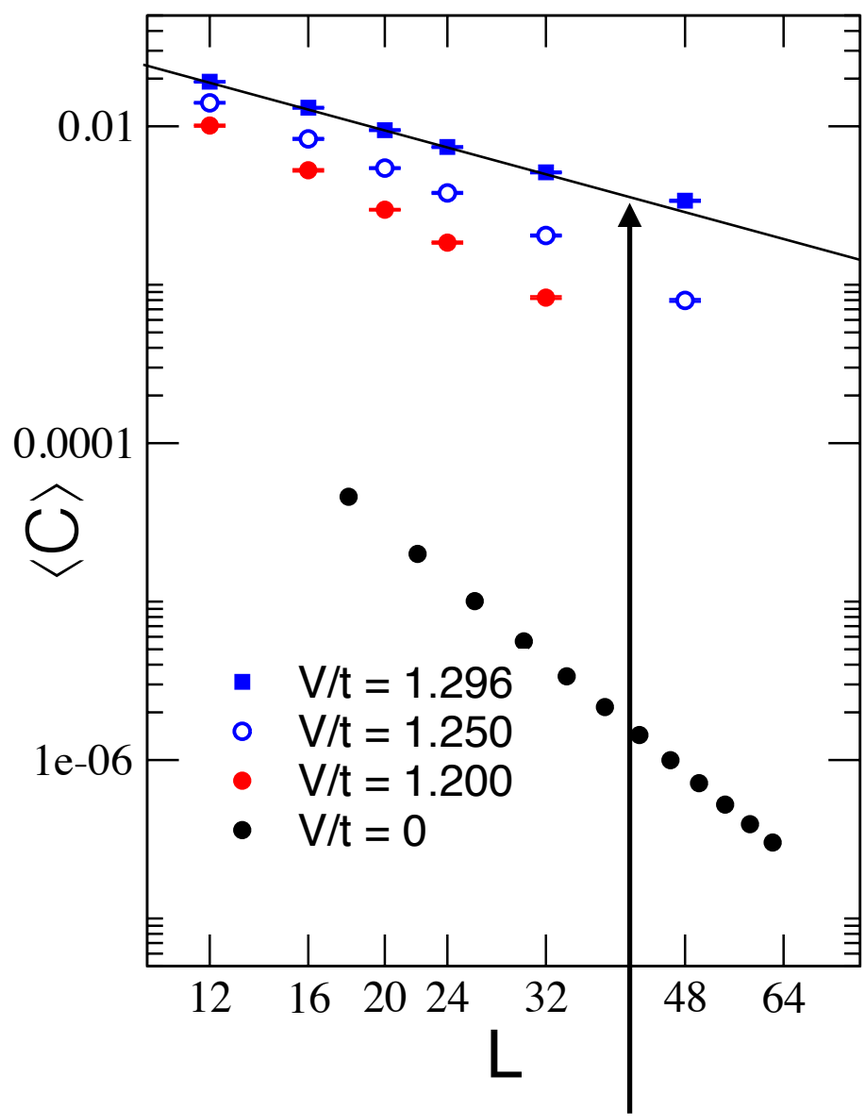


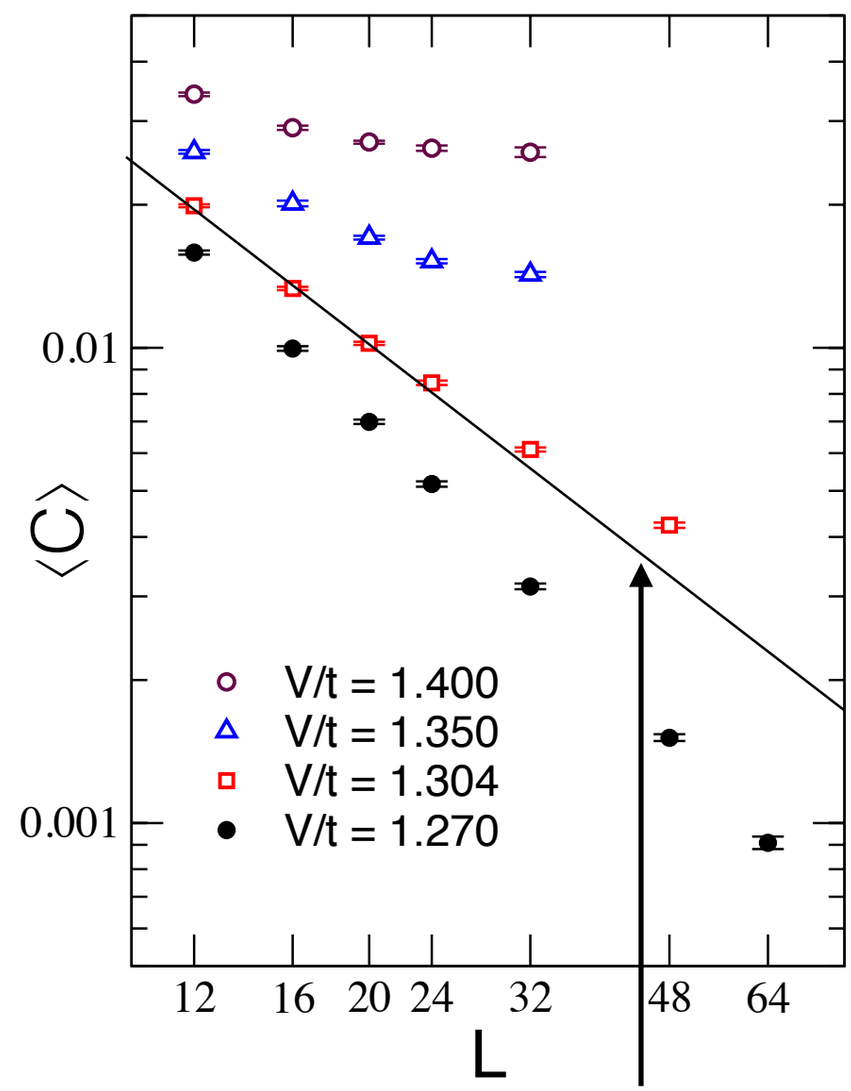
10,000 sites

$V_c = 1.420(2)$

$\eta = 0.49(4) \quad \nu = 0.94(3)$

Importance of large lattice sizes in calculations





Our values

$\nu = 0.89(1)$

$\eta = 0.51(3)$

$V_c = 1.281(2)$

Li, Jiang and Yao, 2014
(968 sites)

$\nu = 0.77(3), \eta = 0.45(2)$

$V_c = 1.296$

Wang, Carboz and Troyer, 2014
(225 sites)

$\nu = 0.80(2), \eta = 0.30(1)$

$V_c = 1.304$

Gross-Neveu (N_f = 1, Z_2 chiral universality)

Method	ν	η	N_s
$4-\epsilon$ [22]	0.898(30)	0.487(12)	–
FRG [24]	0.93(1)	0.55	–
Large-N [7, 84]	0.938	0.509	–
bootstrap [85]	1.32	0.544	–
LCT-INT QMC [37]	0.80(3)	0.30(2)	2×18^2
LCT-INT QMC [86]	0.74(4)	0.275(25)	2×21^2
MQMC [87]	0.77(3)	0.45(2)	2×24^2
SLAC QMC [88]	0.912(34)	–	32^2
CT-FB QMC	0.89(1)	0.51(3)	64^2
DT-FB QMC	0.94(3)	0.49(4)	100^2

Huffman, SC Phys.Rev.D 101 (2020) 7, 074501

We would like to extend this success to N_f = 2

Symmetries: Lattice vs. Continuum

Symmetries of the lattice Hamiltonian are the same as those of the free lattice Hamiltonian

$$H = -\sum_{\langle ij \rangle} \omega \; e^{\,2\alpha \;\eta_{ij} \sum_{a=1}^{N_f} \; (c_i^\dagger c_j + c_j^\dagger c_i)}$$

Lattice Symmetries independent of N_f

Lattice Translations T^1 , T^2

Lattice Rotations R

Parity P

Time Reversal Θ

Charge Conjugation C

Symmetries for N_f=2

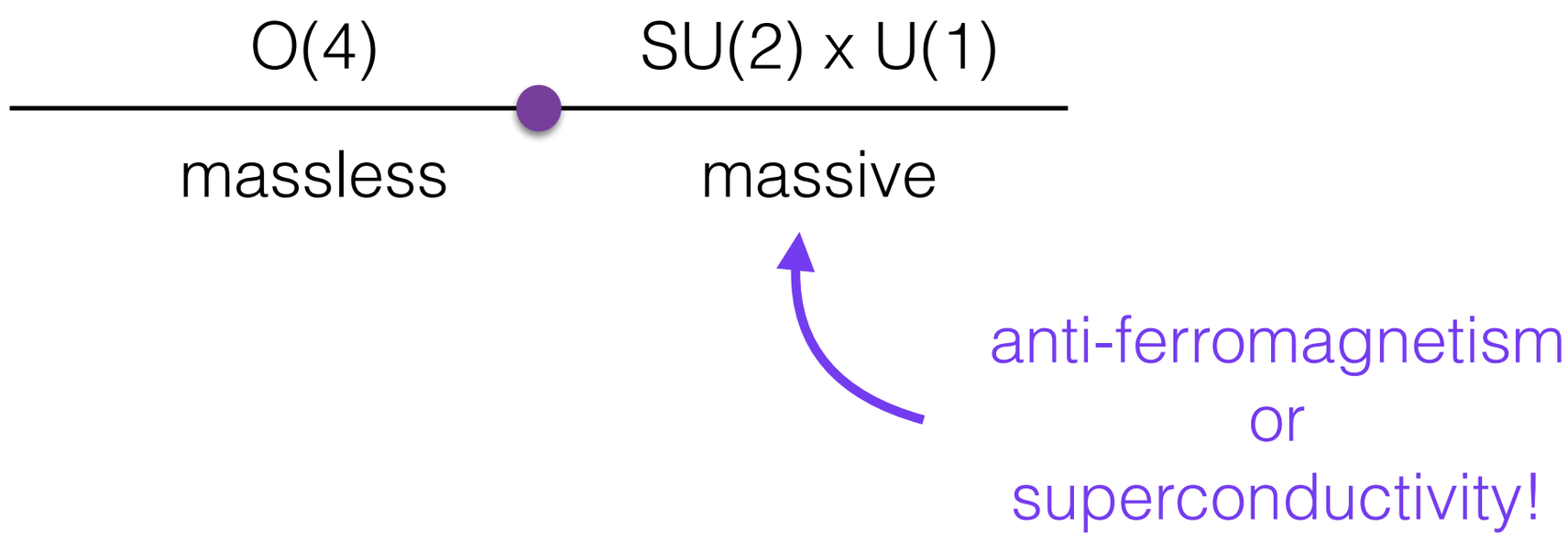
Spin Rotations $\vec{S}$

Charge Rotation $\vec{C}$

Spin-Charge Flip F

Internal Symmetry: $$\frac{SU(2) \times SU(2)}{\mathbb{Z}_2} \times \mathbb{Z}_2 \equiv O(4)$$

phase diagram (numerical evidence):



Comparing with the Hubbard Model

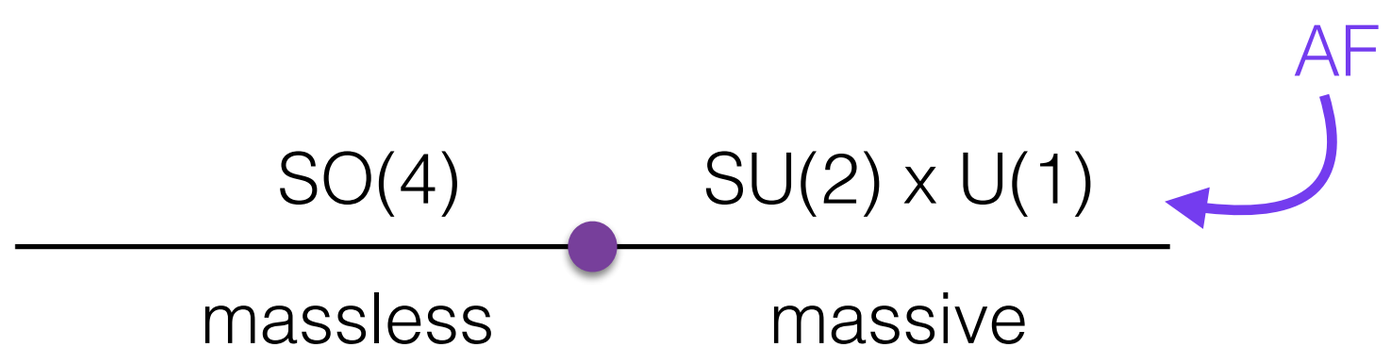
The Hubbard model does not have the spin-charge flip symmetry

Internal Symmetry: $$\frac{SU(2) \times SU(2)}{\mathbb{Z}_2} \times \cancel{\mathbb{Z}_2} \equiv SO(4)$$

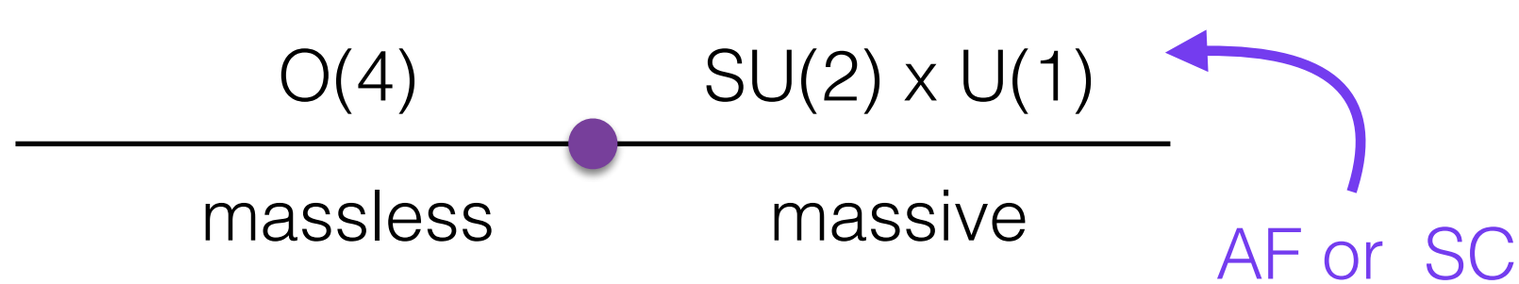
The Hubbard interaction breaks the spin-charge flip symmetry!

Phase Diagrams

Hubbard model:



Our model:



Symmetries and their breaking patterns determine the critical behavior

Does the spin-charge flip symmetry change the critical behavior?

Embedding lattice symmetries in the Continuum

The free fermion lattice theory in the continuum limit is described by the Euclidean action

$$S = \int d^3x \quad \overline{\psi}_a(x)\gamma_\mu\partial_\mu\psi_a(x) \qquad a = 1, 2$$

Here we will choose $\mu = 1, 2, 3$ where 1, 2 are spatial and 3 is temporal (Euclidean), and γ_μ are 4 x 4 Dirac matrices.

From the lattice theory we can choose γ_1, γ_2 to be purely imaginary.

The remaining three gamma matrices $\gamma_3, \gamma_4, \gamma_5$ will then be real.

The free theory has an O(8) internal symmetry not U(4)!

To see the O(8) symmetry we have to go to the Majorana representation:

$$S = \int d^3x \sum_{i=1}^{4} \xi_i^T(x)\gamma_3\gamma_\mu\partial_\mu\xi_i(x) \qquad i = 1, 2, 3, 4$$

$$\Gamma_\mu \otimes \mathbb{I}$$

We can now see the internal O(8) symmetry explicitly.

Lattice Symmetries are embedded in this internal O(8) symmetry and the usual space-time symmetries of the continuum.

In particular the lattice O(4) symmetry mixes the four Majorana fields

$$\xi_i \to V_{ij}\, \xi_j\,, \quad V \in O(4)$$

The other discrete lattice symmetries are embedded as follows:

$$T_1 : \ \psi(x) \ \rightarrow \ i\gamma_4\psi(x)$$

$$T_2 : \ \psi(x) \ \rightarrow \ i\gamma_5\psi(x)$$

$$R : \ \psi(x) \ \rightarrow \ e^{i\pi/4(i\gamma_1\gamma_2 \pm i\gamma_4\gamma_5)}\psi(x)$$

$$P : i\gamma^5\gamma^1\psi(Px)$$

These involve mixing in the Dirac space.

There are 36 mass terms and 28 current terms that are allowed in the continuum based on the lattice symmetries and possible symmetry breaking patterns.

Poster by Hanqing Liu (this workshop)
Ryu, Mudry, Hou and Chamon, PRB 80, 205319

There is a large “four-fermion” coupling constant space within which our lattice model lies.

Similar to Bitan Roy’s talk (this workshop)

To make some progress we focus on Gross-Neveu models.

Anti-ferromagnetism and superconductivity form due to mass terms that break $SU_s(2)$ or $SU_c(2)$

Anti-ferromagnetic mass terms: $\vec{M}_s(x) \;=\; \overline{\psi}(x)\;\vec{\sigma}\;\psi(x)$

Superconducting mass terms:

$$M_c^1(x) \;=\; \psi_1^T(x)\gamma_3\psi_2(x) \;+\; \overline{\psi}_2(x)\gamma_3\overline{\psi}_1^T(x)$$

$$M_c^2(x) \;=\; i\Big(\psi_1^T(x)\gamma_3\psi_2(x) \;-\; \overline{\psi}_2(x)\gamma_3\overline{\psi}_1^T(x)\Big)$$

$$M_c^3(x) \;=\; \Big(\overline{\psi}_1(x)\psi_1(x) \;+\; \overline{\psi}_2(x)\psi_1(x)\Big)$$

Continuum Analysis: V_{eff} + RG + new FP

How does the free fermion behave, when perturbed by the two order parameter fluctuations through appropriate four-fermion couplings.

We begin with the Euclidean four-fermion action:

$$S = S_0 - \frac{G_s}{2}\int d^3x\ \vec{M}_s(x)\cdot\vec{M}_s(x) - \frac{G_c}{2}\int d^3x\ \vec{M}_c(x)\cdot\vec{M}_c(x)$$

Our lattice model has spin-charge flip symmetry: $G_s = G_c$

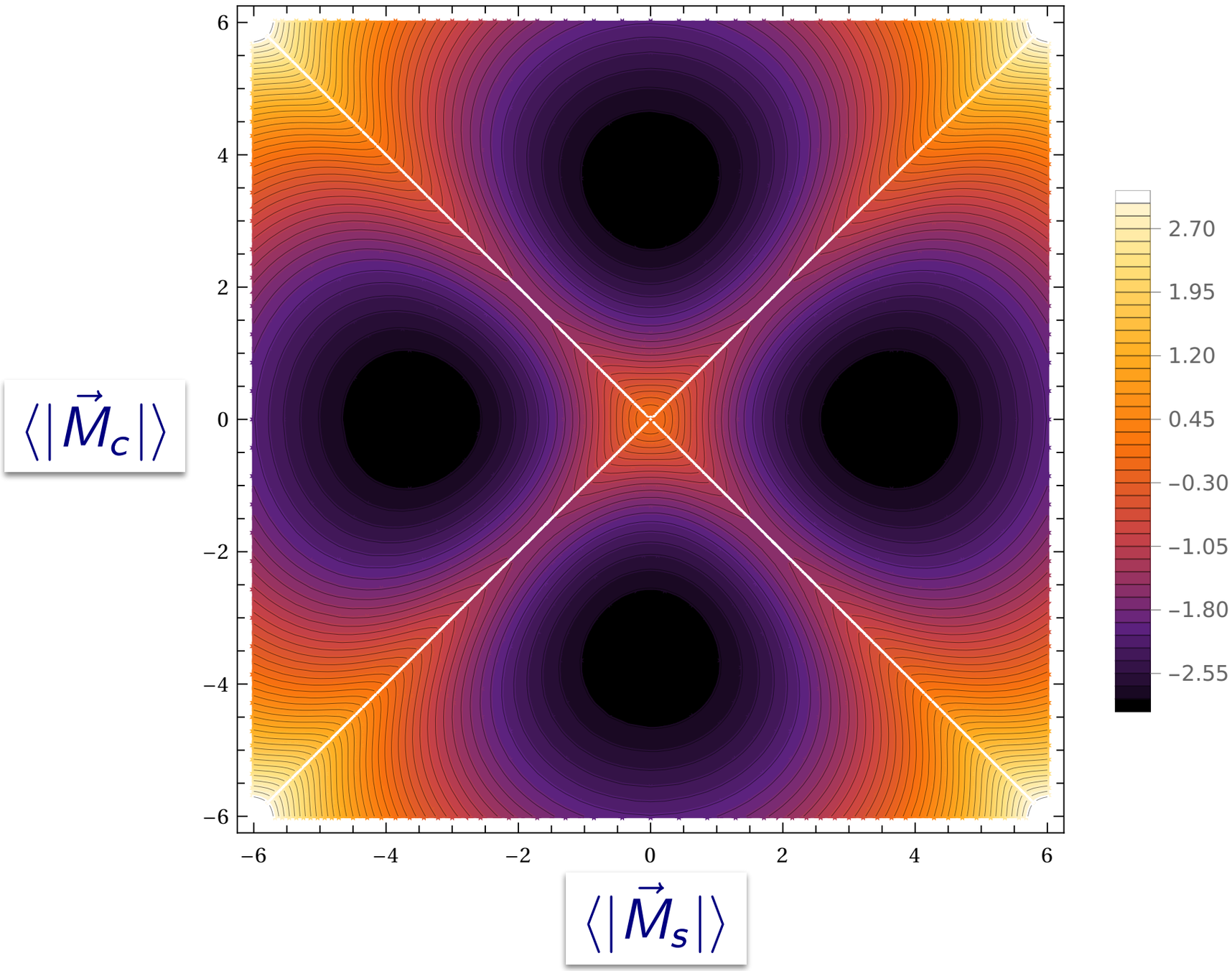
We can perform a mean field analysis and compute the effective potential that arises due to the presence of the two couplings.

We find two phases:

massless fermions for small couplings

broken phase at large couplings

Effective Potential in the broken phase



Renormalization Group Analysis

We have performed a $2+\varepsilon$ and $4-\varepsilon$ calculations, and find that when $G_s = G_c$ there is a new fixed point.

The Hubbard model, where we expect $G_s \neq G_c$, is described by a different fixed point.

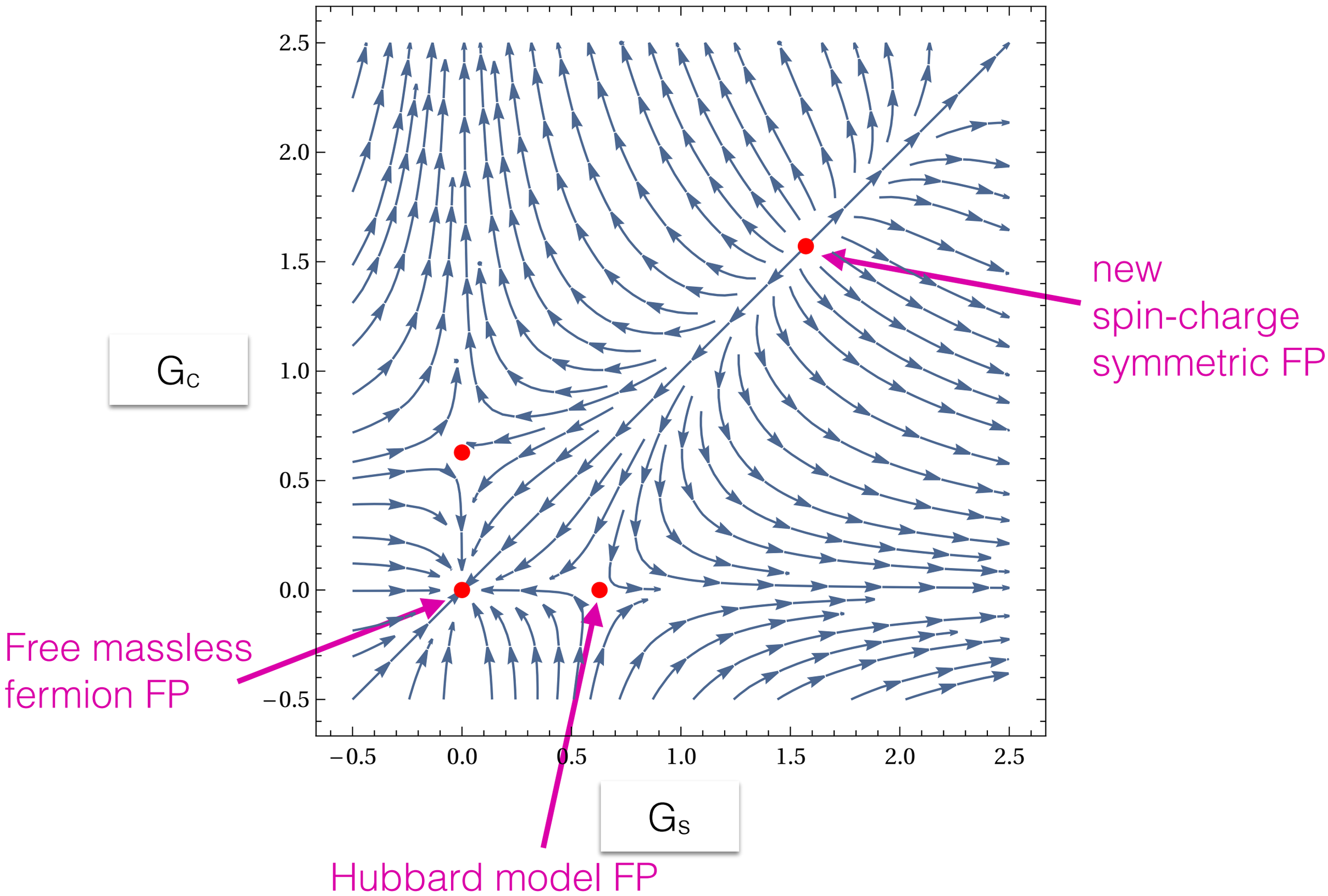
One-loop beta function in $2+\varepsilon$ expansion

$$\beta(G_s) = \varepsilon G_s - \frac{1}{\pi}(5G_s^2 - 3G_sG_c)$$

$$\beta(G_c) = \varepsilon G_c - \frac{1}{\pi}(5G_c^2 - 3G_cG_s)$$

Four-fixed points

RG flow diagram in $2+\varepsilon$



Critical Exponents to leading order

	η	ν^{-1}
$2+\varepsilon$	2	ε
$4-\varepsilon$	$2\varepsilon/7$	$2-6\varepsilon/7$

These are different from those we get in the Hubbard model.

We have not seen this new FP discussed in the literature,
but if we have missed the discussion please let us know.

Monte Carlo Results

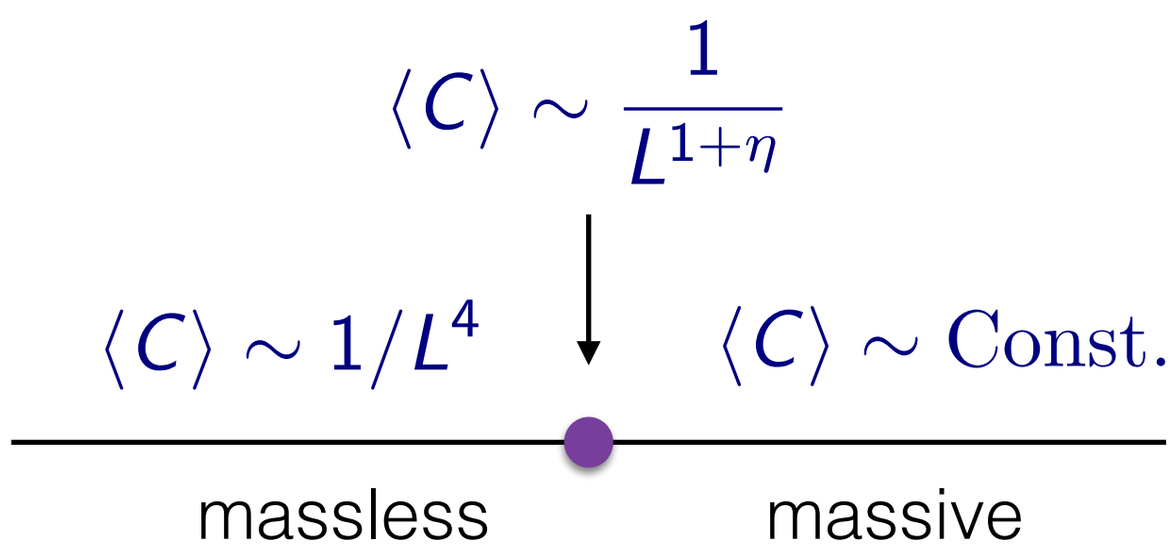
Observable: $$\langle C \rangle \;=\; \frac{1}{Z}\mathrm{Tr}\Big(e^{-\beta H}\,\mathcal{O}_{L/2}\,\mathcal{O}_0\Big)$$

Here $\mathcal{O}_j \;=\; (-1)^j\,(n_{j,\uparrow} - n_{j,\downarrow})$ is the z-component of the

Anti-ferromagnetic order parameter.

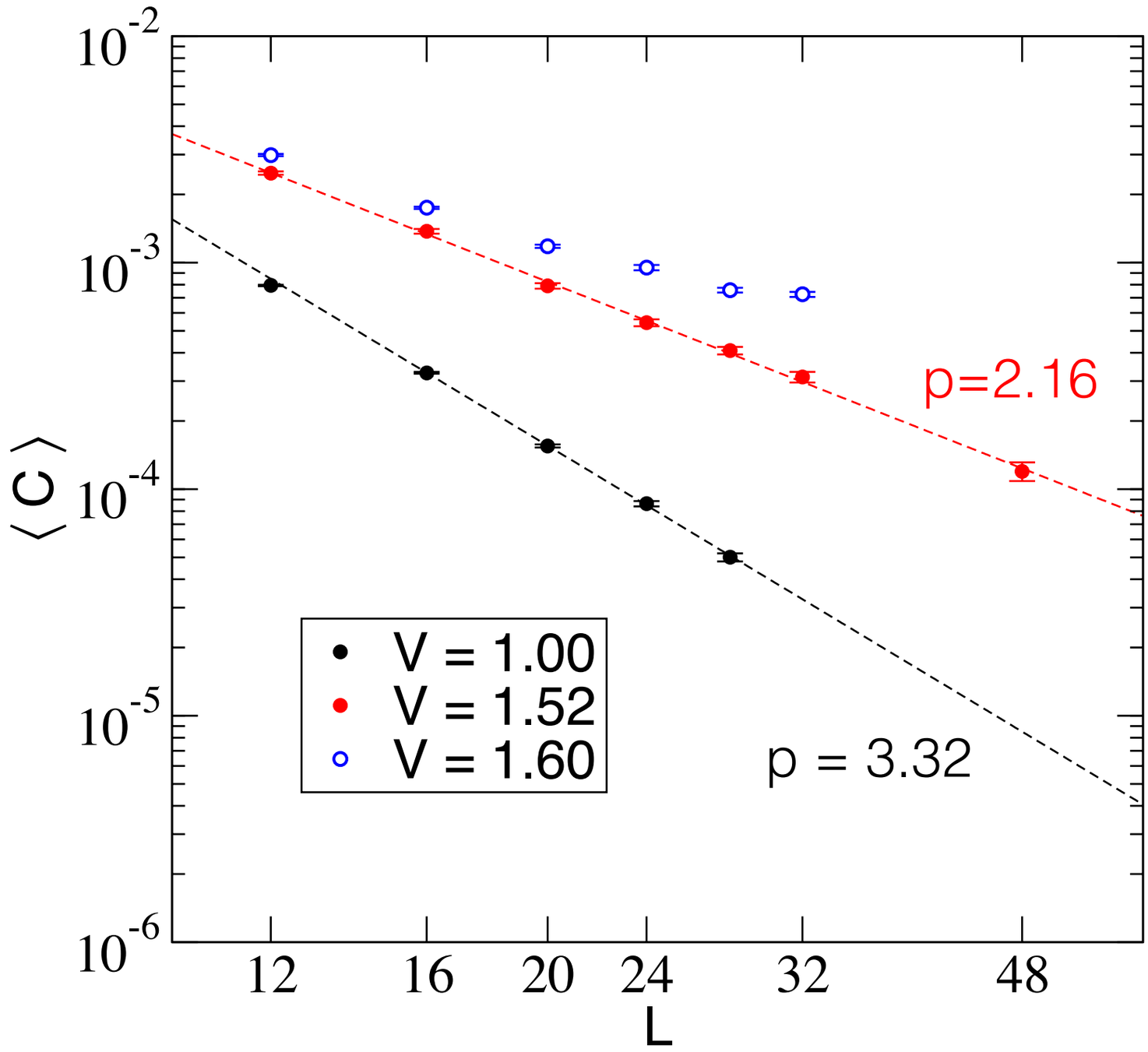
Expected Behavior

in the $\lim\limits_{L\to\infty}$



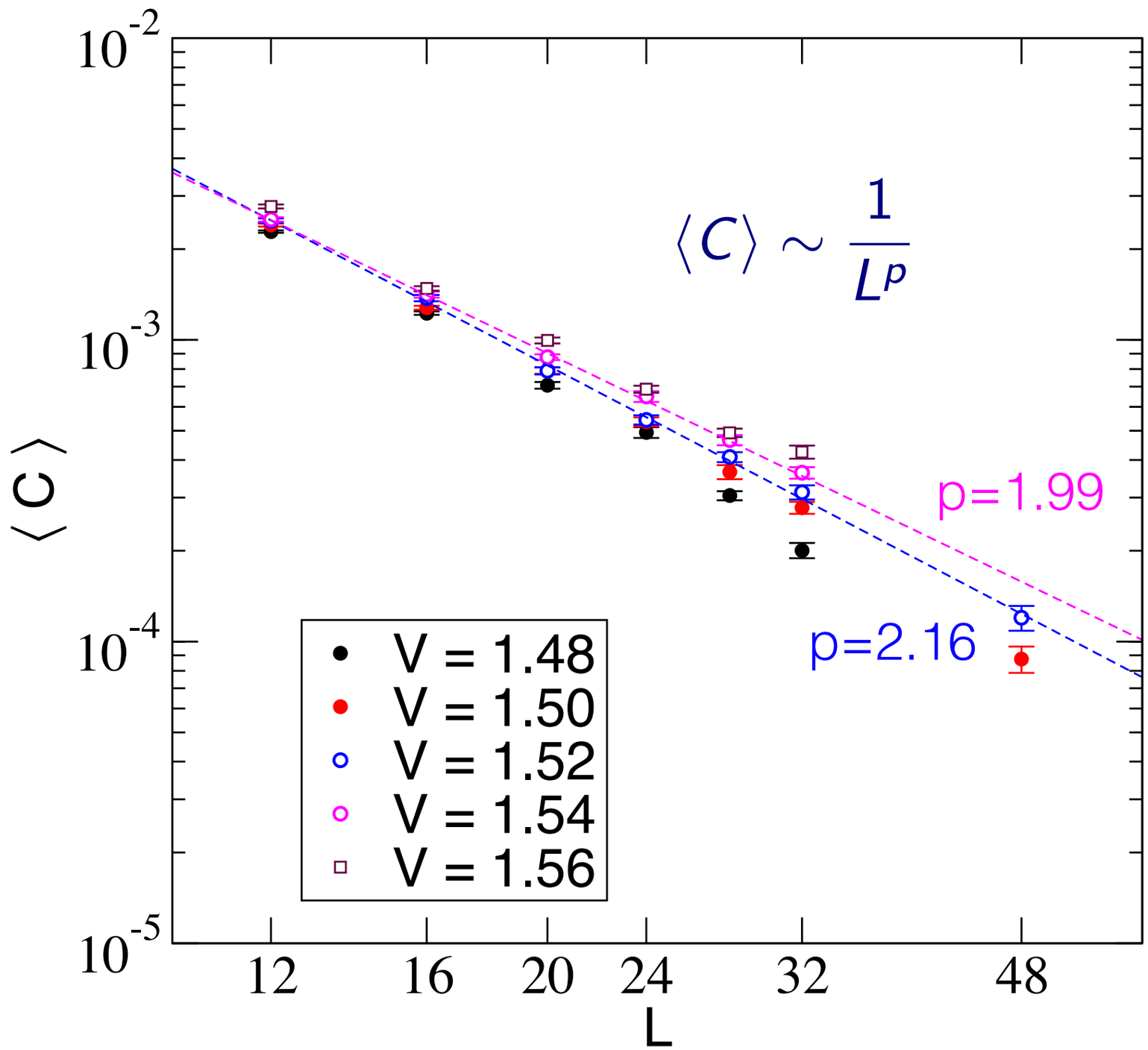
Evidence for two phases and a transition

$$\langle C \rangle \sim \frac{1}{L^p}$$



$V_c \approx 1.52?$

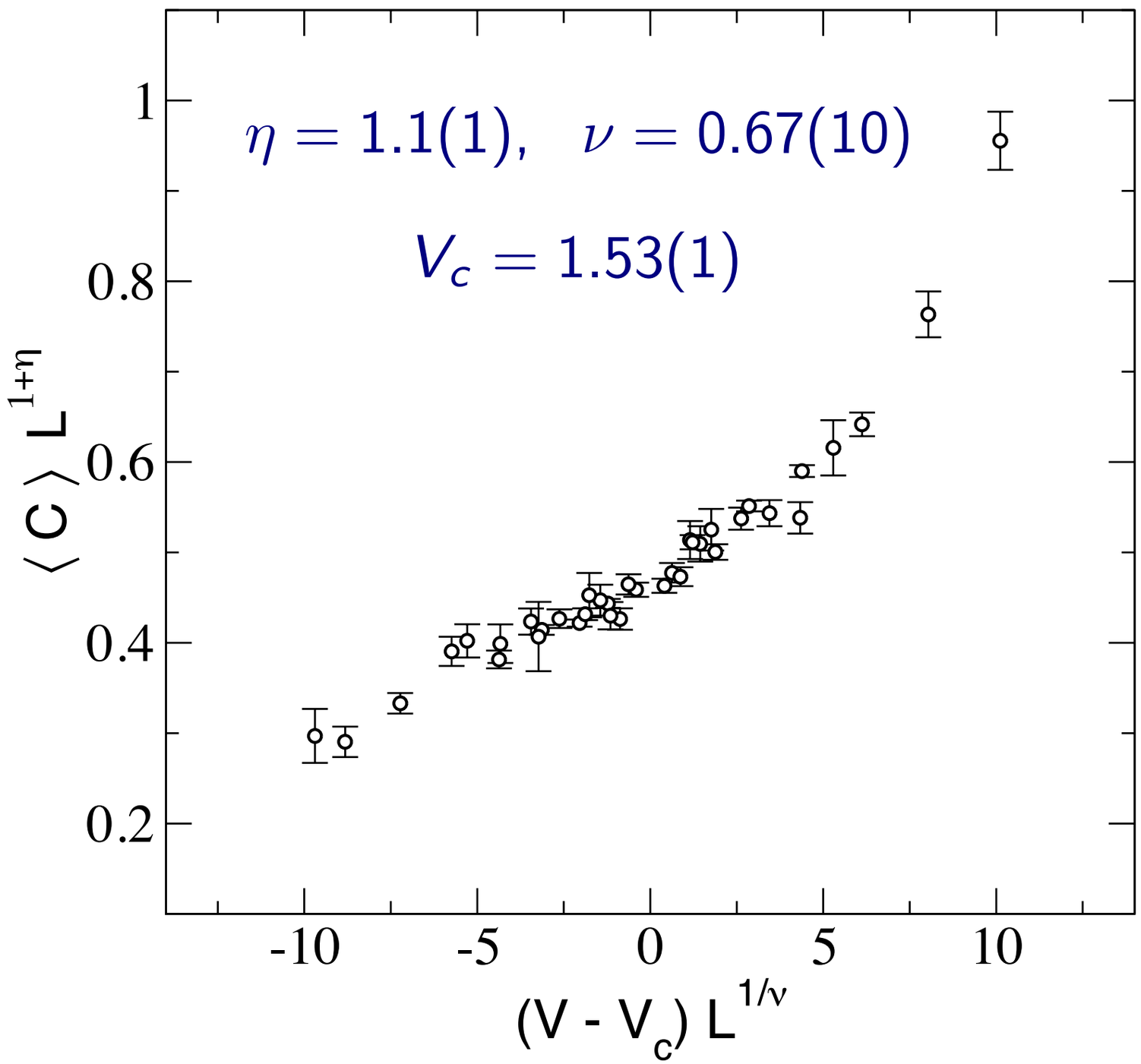
Critical region



$$p = 1 + \eta$$

$$\eta = 0.99 - 1.16$$

Critical Scaling



Computer time used: 400,000 CPU core hours

Evidence for SSB of spin-charge flip symmetry

From our effective potential analysis we expect the spin-charge flip symmetry F to be broken spontaneously.

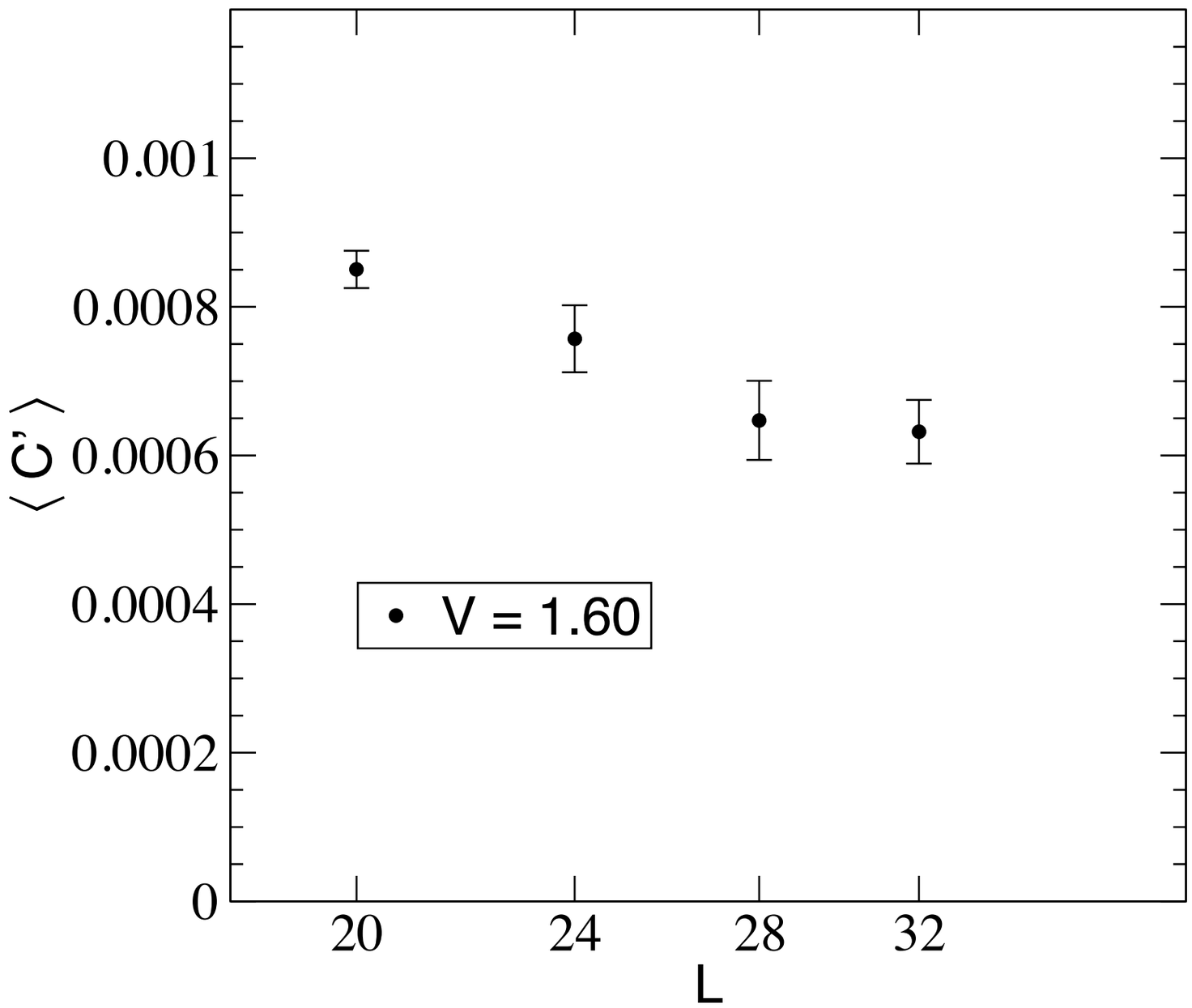
A new observable is necessary to study this

$$\langle C' \rangle \;=\; \frac{1}{Z}\mathrm{Tr}\Big(e^{-\beta H}\; \mathcal{O}'_{L/2}\; \mathcal{O}'_0\Big)$$

Here $\mathcal{O}'_j = (n_{j,\uparrow} - 1/2)(n_{j,\downarrow} - 1/2)$ is odd under spin-charge symmetry, but invariant under spin and charge transformations.

In the broken phase we expect $\lim_{L\to\infty} \langle C' \rangle \;\sim \mathrm{Const.}$

Scaling with system size



Conclusions

Lattice Model that is suitable for fermion bag calculations naturally contains a spin-charge flip symmetry.

This symmetry can lead to new fixed points different from that usually observed with the Hubbard interaction.

Our model shows an interesting quantum critical point, governed by this new FP.

The phase transition is between a semi-metal and an anti-ferromagnet (or a superconductor) which is accompanied by the breaking of the spin-charge flip symmetry.

An analysis up to L = 48 lattices suggests $\eta = 1.1(1), \ \nu = 0.67(10)$