Emergent supersymmetry at the critical point of Yukawa systems

Omar Zanusso

Università di Pisa

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Trento

Just to show you what we are missing, I believe that ECT* is somewhere on the frame

Disclaimers

Based on

- my small contribution to the topic which dates back to 2017 Hellwig et al.
- some unpublished material from 2018
- new material from 2020-21 thanks to O. Gelber see Gelber's poster

Emegent SUSY has been mentioned several times already during this conference

- My recent focus is on field theory and RG with a bit of CFT
- My knowledge of the topic is rather incomplete, to say the least
- For much more complete sets of references see H. Yao and I. Affleck's talks

Additional informations

- Currently in the middle of a heat wave, must keep the window open
- Italy is the country of noisy Vespas, be patient

Introduction

- Coleman-Mandula theorem:
  "Space-time and internal symmetries cannot be combined in any but a trivial way"

$$ISO(3,1) \times G$$

- Several assumptions of CM theorem to break. Simplest:
  Non-bosonic additional generators, imagine square root of translations

$$\delta_\beta \Phi = i\overline{\beta}^a \mathcal{Q}_a \Phi \qquad\qquad \left\{\mathcal{Q}_a, \overline{\mathcal{Q}}^b\right\} \sim 2\left(\gamma^\mu\right)_a{}^b \mathcal{P}_\mu$$

- Haag-ŁopuszańskiSohnius theorem: uniqueness and N_{susy}

$$\left\{\mathcal{Q}_{I\,a}, \overline{\mathcal{Q}}_J{}^b\right\} \sim 2\left(\gamma^\mu\right)_a{}^b \mathcal{P}_\mu \delta_{IJ} + \cdots$$

- Fermionic generators, schematically

$$\mathcal{Q}\,|\mathrm{boson}\rangle = |\mathrm{fermion}\rangle \qquad \mathcal{Q}\,|\mathrm{fermion}\rangle = |\mathrm{boson}\rangle$$

- Superfield and superspace

$$\Phi(x,\theta,\overline{\theta}) = \phi(x) + \overline{\theta}^a \psi_a(x) + \frac{1}{2}\overline{\theta}^a \theta_a F(x)$$

- From potential to superpotential

$$\int d^d x\, U(\phi) \longrightarrow \int d^d x d\theta d\overline{\theta}\, W(\Phi)$$

Why SUSY?

Upsides

- Development driven by a perid of mathematical success in QFT for particle physics
- Originally proposed as solution to hierarchy problem at Λ_{EW} ☹
- Plays well in GUT, string theory, etc.

Downsides

- Doubling of DOFs (but we do need particles for DM/cosmology, right? ☺)
- No evidence ☺
- Even less evidence of strings (strings need SUSY is an understatement)

A snapshot from what feels like a long time ago

Day 2: Thursday, 29 September 2016
Chairs: C. Muñoz, L. Ibáñez

Schedule	Speaker	Title	Video	Slides
10:00 - 10:30	A. Strumia	No	Video	PDF
10:30 - 11:00	L. Hall	Saxion cosmology and dark matter	Video	PDF
11:00 - 11:30	Coffee Break	Coffee Break		
11:30 - 12:00	J. Lykken	Emergent Supersymmetry	Video	PDF
12:00 - 12:30	J. March-Russell	Journeying beyond the realm of the MSSM	Video	PDF
12:30 - 13:00	D. Shih	Natural SUSY vs. the LHC	Video	PDF
13:00 - 15:00	Lunch Break	Lunch Break		
15:00 - 15:30	G. G. Ross	Is (low energy) SUSY alive?	Video	PDF
15:30 - 16:00	M. Cicoli	Soft terms in string models	Video	PDF
16:00 - 16:30	Coffee Break	Coffee Break		
16:30 - 17:30	Discussion	Convener: X. Tata	Video	Survey
21:00	Conference Dinner	Conference Dinner		

Day 3: Friday, 30 September 2016
Chair: A. Casas

Schedule	Speaker	Title	Video	Slides
10:00 - 10:30	J. Ellis	Keep the Sfaith	Video	PDF

So, seriously, why SUSY?

My personal motivations

- Still theoretical interesting symmetry evading no-go theorem
- Low-dimensional **tunable** condensed matter with correct DOFs can display it as **emergent** symmetry
- Realistic possibility of observing SUSY in lab, just not as we originally thought

From the Yukawa model to supersymmetry

The Gross-Neveu-Yukawa model

Take φ real scalar and ψ 2-component spinor (Majorana)

$$S_Y = \int d^d x \Big\{ \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \frac{i}{2}\overline{\psi}\partial\!\!\!/\psi + U(\varphi) + \frac{1}{2}H(\varphi)\overline{\psi}\psi \Big\}$$

- Perturbatively renormalizable in $d = 4 - \epsilon$ for

$$U(\varphi) \sim \frac{1}{4!}\lambda\varphi^4 \qquad\qquad H(\varphi) \sim y\varphi$$

- Invariant under $x \to \tilde{x} = (x_1, -x_2, x_3)$ in $d = 3$ for

$$\varphi(x) \to -\varphi(\tilde{x}) \qquad\qquad \psi(x) \to i\gamma^2\psi(\tilde{x})$$

Enhanced symmetry

Suppose the functions satisfy

$$U(\varphi) = \frac{1}{2} W'(\varphi)^2 \qquad\qquad H(\varphi) = W''(\varphi)$$

for some $W(\varphi)$

$$S_{\text{susy}} = \frac{1}{2} \int d^d x \Big\{ \partial_\mu \varphi \partial^\mu \varphi + i\overline{\psi} \partial\!\!\!/ \psi + W'(\varphi)^2 + W''(\varphi)\overline{\psi}\psi \Big\}$$

- Invariant under a supersymmetric transformation (on-shell)

$$\delta\varphi = \overline{\theta}\psi \qquad\qquad \delta\psi = (i\partial\!\!\!/ \varphi + W'(\varphi))\theta$$

- Perturbatively renormalizable in $d = 4 - \epsilon$ ($\lambda = 6y^2$) for

$$W(\varphi) \sim \frac{1}{3!} y \varphi^3$$

On-shell vs off-shell SUSY

The transformation is non-linear

$$\delta\psi = (i\partial\!\!\!/\varphi + W'(\varphi))\theta \sim (i\partial\!\!\!/\varphi + {}^{1}\!/\!{}_{2}\,y\varphi^2)\theta$$

Introduce an auxiliary field F

$$S_{\text{susy}} = \int d^d x \Big\{ \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \frac{i}{2}\overline{\psi}\partial\!\!\!/\psi - \frac{1}{2}F^2 + FW'(\varphi) + \frac{1}{2}W''(\varphi)\overline{\psi}\psi \Big\}$$

Off-shell transformations (wrt F)

$$\delta\varphi = \overline{\theta}\psi \qquad \delta\psi = (i\partial\!\!\!/\varphi + F)\theta \qquad \delta F = i\overline{\theta}\partial\!\!\!/\psi$$

Consider $\Phi(x,\theta,\overline{\theta}) = \varphi(x) + \overline{\theta}\psi(x) + \frac{1}{2}\overline{\theta}\theta F$

$$S_{\text{susy}} = \int d^d x\, d\theta\, d\overline{\theta} \Big\{ \frac{1}{2}\mathcal{D}\Phi\overline{\mathcal{D}}\Phi + W(\Phi) \Big\}$$

Supercovariant derivatives, $\mathcal{D} = \frac{\partial}{\partial\overline{\theta}} + i\gamma^\mu\theta\partial_\mu$

$$\Big\{ \mathcal{D}_a, \overline{\mathcal{D}}^b \Big\} = -2\not{\partial}_a{}^b$$

They offer an explicit differential realization of $\Big\{ \mathcal{Q}_a, \overline{\mathcal{Q}}^b \Big\}$.

General idea for emergece of SUSY

- Yukawa models have a critical point as a function of N_{scalars} and N_{fermions},
- but SUSY model is also a Yukawa model (ex: $N_{\mathrm{scalars}} = N_{\mathrm{fermions}}$),
- so critical point has enhanced SUSY for certain N_{scalars} and N_{fermions}!

The crucial points are how many parameters must be tuned
and how Weyl fermions are constructed to interplay with scalar order parameter...

Similar considerations can be made for Nambu-Jona-Lasinio-Yukawa model with charged scalar (twice as many generators), we concentrate on $N_{\mathrm{susy}} = 1$.

Let's review some proposals before coming back to field theory.

Some proposed realizations

Qiu original proposal

In $d = 2$:

- Tower of conformal CFTs $c = 1 - \frac{6}{m(m+1)}$, potential: $\varphi^{2(m-1)}$ Zamolodchikov
- Tower of superconformal CFTs $c = \frac{3}{2}\left(1 - \frac{8}{\hat{m}(\hat{m}+2)}\right)$ Friedan et al.

$c = {}^7\!/_{10}$ shared by $m = 4$ and $\hat{m} = 3 \implies$ tricritical Ising model has SUSY?

Ising model with vacancies has first vs second order behavior in this universality

$$H = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j - h \sum_i \sigma_i + \sum_i \Delta \sigma_i^2$$

ψ comes from "disorder" operator (nonlocal) and magnetization with ad hoc BCs
Realization: mixture of ^{4}He and ^{3}He at critical concentration
Unfortunately, no conclusive observation. Also, works only in $d = 2$

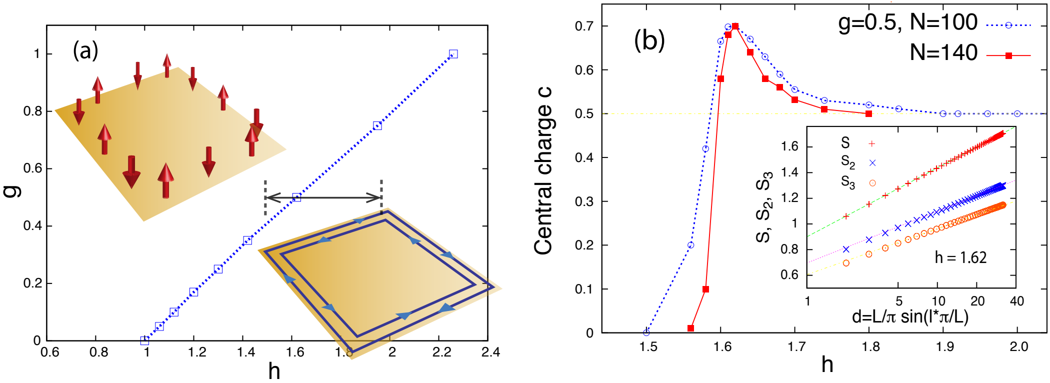
Grover-Sheng-Vishwanath proposal

$d = 1 + 1$ boundary of $2 + 1$ topological superconductor, g ∼Yukawa, h =magnetic

$$H = -i \sum \left\{ \left(1 - g\mu^z_{j+1/2}\right)\chi_j\chi_{j+1} + \mu^z_{j-1/2}\mu^z_{j+1/2} - h\mu^z_{j+1/2} \right\}$$

Weyl spinors ψ at boundary (nonlocal)

DMRG simulation → central charge; $c = 0$ (gapped), $c = 1/2$ Ising, $c = 7/10$ SUSY



Rahmani-Zhu-Franz-Affleck proposal

- Unpaired Majoranas, localized to topological defects (vortices, domain walls) $\Longrightarrow$ topological superconductor
- Difference from Grover et al. $\Longrightarrow$ purely self-interacting Majoranas $\Longrightarrow$ local interactions (vs long-range boson-mediated)

$$H = it \sum_j \gamma_j \gamma_{j+1} + g \sum_j \gamma_j \gamma_{j+2} \gamma_{j+3} \gamma_{j+4}$$

Evidence for tricritical Ising universality $c = {}^7\!/_{10}$ by tuning one parameter.
See Affleck's talk at this conference

Back to field theory in $d \leq 4$

RG flow of the Yukawa system with one Majorana

Define dimensionless renormalized quantities ($v = \frac{\lambda}{4!}\varphi^4$ and $h = y\varphi$)

$$u(\varphi) = k^{-d} U(\varphi\, k^{d/2-1} Z_\varphi^{-1/2}) \qquad h(\varphi) = k^{-1} Z_\psi^{-1} H(\varphi\, k^{d/2-1} Z_\varphi^{-1/2})$$

Leading perturbative RG flow in $d = 4 - \epsilon$

$$\beta_u = -4u + \varphi u' + \epsilon\Big(u - \frac{1}{2}\varphi u'\Big) + \frac{\eta_\varphi}{2}\varphi u' + \frac{1}{2(4\pi)^2}(u'')^2 - \frac{1}{2(4\pi)^2} h^4$$

$$\beta_h = -h + \varphi h' - \epsilon\frac{1}{2}\varphi h' + \eta_\psi h + \frac{\eta_\varphi}{2}\varphi h' + \frac{2}{(4\pi)^2} h(h')^2$$

Hint of SUSY

$$\eta_\psi = \eta_\varphi = \frac{1}{(4\pi)^2} h'(0)^2$$

which is necessary for them to combine in the same supermultiplet.

RG flow of the superpotential

Dimensionless renormalized superpotential

$$w(\varphi) = k^{-d+1} W(\varphi\, k^{d/2-1} Z^{-1/2})$$

Leading perturbative RG flow of the superpotential

$$\beta_w = -3w(\varphi) + \varphi w'(\varphi) + \epsilon\Big(w(\varphi) - \frac{1}{2}\varphi w'(\varphi)\Big) + \frac{\eta}{2}\varphi w'(\varphi) + \frac{1}{3(4\pi)^2} w''(\varphi)^3$$

Anomalous dimension

$$\eta = \frac{1}{(4\pi)^2} w'''(0)^2$$

SUSY is a solution of the Yukawa RG

Take $u = (w')^2/2$ and $h = w''$, it is easy to show

$$\begin{aligned} \beta_u &= w'\Big\{-2w' + \frac{1}{2}(2+\eta)\varphi w'' + \frac{1}{2}\epsilon\left(w' - \varphi w''\right) + \frac{1}{16\pi^2}w'''(w'')^2 + \frac{1}{32\pi^2}w'(w''')^2\Big\} \\ &\simeq w'\left(k\partial_k w'\right) = k\partial_k \frac{(w')^2}{2} = \beta_{(w')^2/2} \end{aligned}$$

and

$$\begin{aligned} \beta_h &= (-1+\eta)w'' - \frac{1}{2}\epsilon\varphi w''' + \frac{1}{2}\varphi(2+\eta)w''' + \frac{1}{8\pi^2}w''(w''')^2 \\ &\simeq k\partial_k w'' = \beta_{w''} \end{aligned}$$

RG flows consistent for properly ranked polynomials (LO and NLO perturbation theory) or up to irrelevant operators (in a functional interpretation).

The SUSY fixed point

Take $w(\varphi) = \frac{(4\pi)^2}{3!}\lambda\varphi^3$, at NLO

$$\beta_\lambda = -\frac{\epsilon}{2}\lambda + \frac{7}{2}\lambda^3 - \frac{21}{2}\lambda^5 \qquad\qquad \eta = \lambda^2 - \lambda^4$$

The fixed point

$$(\lambda^*)^2 = \frac{\epsilon}{7} + \frac{3\epsilon^2}{49} \quad\Longrightarrow\quad \eta = \frac{1}{7}\epsilon + \frac{2}{49}\epsilon^2$$

Super-superscaling relation determines ν for one superfield

$$\Delta_\Phi = \frac{d-2+\eta}{2} \quad\Longrightarrow\quad \Delta_{\Phi^2} = 2 + \Delta_\Phi$$

Operator Φ^2 turns on the same mass for φ and $\psi \Longrightarrow \frac{1}{\nu} = \frac{d-\eta}{2} = 2 - \frac{4}{7}\epsilon - \frac{1}{49}\epsilon^2$

Idea: reinterpret Yukawa as a broken SUSY RG

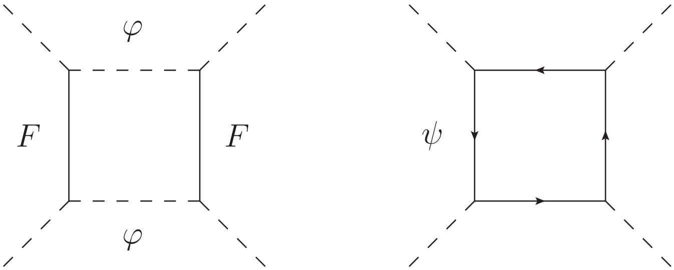
Neglect zero point energy and set $H(\varphi) = W''(\varphi) + Y(\varphi)$

$$S_{\text{Yukawa}} = \int d^d x \left\{ \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \frac{i}{2}\bar{\psi}\not{\partial}\psi - \frac{1}{2}F^2 + FW'(\varphi) + \frac{1}{2}W''(\varphi)\bar{\psi}\psi + \frac{1}{2}Y(\varphi)\bar{\psi}\psi \right\}$$

The relation $U(\varphi) = \frac{1}{2}W'(\varphi)^2$ can be integrated (use h over y for convenience)

$$\begin{aligned}
\beta_w = & -3w(\varphi) + \varphi w'(\varphi) + \epsilon\Big(w(\varphi) - \frac{1}{2}\varphi w'(\varphi)\Big) + \frac{\eta_\varphi}{2}\varphi w'(\varphi) + \frac{1}{3(4\pi)^2}w''(\varphi)^3 \\
& + \int_0^\varphi dx\, \frac{w''(x)^4 - h(x)^4}{2(4\pi)^2 w'(x)} \\
\beta_h = & -h(\varphi) + \varphi h'(\varphi) - \frac{\epsilon}{2}\varphi h'(\varphi) - \eta_\psi h(\varphi) + \frac{\eta_\varphi}{2}\varphi h'(\varphi) + \frac{2}{(4\pi)^2}h(\varphi)h'(\varphi)^2
\end{aligned}$$

We have broken SUSY in the Yukawa sector with $Y(\varphi)$,
but RG generates $C(\varphi) \propto W''(\varphi)^4 - H(\varphi)^4 \sim \varphi^4$ outside the potential



These cancel only if $W''(\varphi) - H(\varphi) = Y(\varphi) = 0$

General form of the RG steps

$\overline{\mathrm{MS}}$ preserves on/off-shell susy at 2-loops

$$S_{\rm susy} \to S_{\rm susy} - \frac{\delta\mu}{\mu}\int d^dx \Big\{ \beta'_W F + \frac{1}{2}\beta''_W \bar\psi\psi \Big\}$$

In general (beyond NLO or nonperturbatively), for some RG scale k

$$S \to S - \frac{\delta k}{k}\int d^dx \Big\{ A(\varphi) F + \frac{1}{2} B(\varphi)\bar\psi\psi + C(\varphi) \Big\}$$

Three independent functions to attribute to the RG of $W(\varphi)$ and $Y(\varphi)$

F-field redefinition

Redefine F along the flow: $\beta_F \equiv k\partial_k F = D(\varphi)$

$$S \to S - \frac{\delta k}{k}\int d^dx\Big\{A(\varphi)F + \frac{1}{2}B(\varphi)\bar{\psi}\psi + C(\varphi) - \frac{\delta S}{\delta F}D(\varphi)\Big\}$$

Choose

$$\beta_F = D(\varphi) = \frac{C(\varphi)}{W'(\varphi)}$$

to get

$$\begin{aligned} S &\to S - \frac{\delta k}{k}\int d^dx\Big\{\Big(A(\varphi) - \frac{C(\varphi)}{W'(\varphi)}\Big)F + \frac{1}{2}B(\varphi)\bar{\psi}\psi\Big\} \\ &\equiv S - \frac{\delta k}{k}\int d^dx\Big\{\beta_{W'}F + (\beta_{W''} + \beta_Y)\bar{\psi}\psi\Big\} \end{aligned}$$

Now we can use nonperturbative methods such as FRG for a broken-SUSY flow

Some numbers in $d = 3$

Big advantage of broken SUSY RG and F-redefinition: $\eta = \eta_\varphi = \eta_\psi$.
Decent numbers even for LPA truncations, which is surprising, in need of improvement.

	FRG_1	FRG_2	[2/2]	[3/1]	CB	FRG_{old}
η	0.174	0.167	0.171	0.170	0.164	0.185*
ν^{-1}	1.385	1.395	1.415	1.415	1.418*	1.29

- Hellwig et al. 1705.08312
- Zerf et al. 1709.05057
- Iliesu et al. 1508.00012
- Vacca-Zambelli 1503.09136

Flavors of SUSY

Generalize to N_f superfields

Coupling generalizes to a "tensor"

$$W(\varphi) = \frac{1}{3!}\lambda_{ijk}\varphi^i\varphi^j\varphi^k$$

Structural similarity with $U(\varphi) = \frac{1}{3!}g_{ijk}\varphi^i\varphi^j\varphi^k$ in $d = 6 - \epsilon$ (Lee-Yang, Landau-Potts)!

Corresponds to a Yukawa model with N_f scalars and N_f fermions

$$\mathcal{L}_{\rm int} = \lambda_{ijk}\varphi_i\overline{\psi}_j\psi_k + \frac{1}{4}\lambda_{k(ij}\lambda_{mn)k}\varphi_i\varphi_j\varphi_m\varphi_n$$

RG flow has a richer structure since there is no symmetry input. In $d = 4 - \epsilon$

$$\beta_{\lambda_{ijk}} = -\frac{\epsilon}{2}\lambda_{ijk} + \frac{2}{3}\lambda_{abc}\lambda_{ab(i}\lambda_{jk)c} + 2\lambda_{iab}\lambda_{jbc}\lambda_{kca}$$

Collective index $a = (i, j, k)$. Admits a gradient form (at NLO)

$$\beta_{\lambda_a} = G^{ab}\frac{\partial}{\partial\lambda_b}A(\lambda) \qquad\qquad G_{ab} = \delta_{ab} + \dots$$

Checked: SUSY solves RG flow of general Yukawa system at NLO like for $N_f = 1$

Maximal symmetry and irreps

$w(\varphi)$ has no symmetry...

if $w^*(\varphi)$ is solution then also $\tilde{w}^*(\varphi) = w^*(\mathcal{R}\cdot\varphi)$ is for $\mathcal{R} \in O(N_f)$

$O(N_f)$ is maximal symmetry and induces an action on λ_{ijk} as

$$\tilde{w}(\varphi) = w(\mathcal{R}\cdot\varphi)$$

$O(N_f)$ is always broken because irreps have no singlet

$$\lambda_{ijk} = \kappa_{(i}\delta_{jk)} + \sigma_{ijk} \qquad \text{with} \qquad \sigma_{iij} = 0$$

Example: for $N_f = 3$ then $\lambda \in \underline{10} = \underline{3} \oplus \underline{7}$

General solutions for $N_f = 1, 2, 3$

Use global $O(N_f)$ freedom to fix κ_i axially (also, full irreps decomposition) and then solve numerically:

N_f	Anomalous dim./ϵ	Symmetry	A_*/ϵ^2
1	$\left(\frac{1}{7}\right)$	-	-0.01786
2	$\left(\frac{1}{3}, \frac{1}{3}\right)$	S_3	-0.08333
	$\left(\frac{18}{109}, \frac{13}{109}\right)$	$\mathbb{Z}_2$	-0.03555
3	$\left(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right)$	S_4	-0.15
	$(0.464138, 0.292417, 0.292417)$	$O(2)$	-0.26224
	$\left(\frac{47}{285}, \frac{47}{285}, \frac{9}{95}\right)$	S_3	-0.10615
	$(0.337931, 0.312268, 0.0877323)$	$\mathbb{Z}_2 \times \mathbb{Z}_2$	-0.18449
	$(0.164912, 0.164912, 0.0947368)$	$\mathbb{Z}_2$	-0.10648
	$(0.343715, 0.32918, 0.127884)$	$\mathbb{Z}_2$	-0.2002

$N_f = 2$ solution with $\mathbb{Z}_2$, is it worth a Dom Pèrignon?

Rychkov-Stergiou (1810.10541) offered a bottle of Champagne for finding a perturbatively unitary scalar theory with $\mathbb{Z}_2$ symmetry in $d = 4 - \epsilon$ and $N_{\text{scalars}} > 1$

Take our second solution and integrate-out the fermions

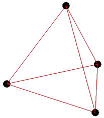
$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi^i \partial^\mu \varphi^i + \frac{1}{4}\lambda_{k(ij}\lambda_{mn)k}\varphi_i\varphi_j\varphi_m\varphi_n - \frac{1}{2}\mathrm{Tr}\log\Big(i\partial\!\!\!/ + \lambda_{i..}\varphi_i\Big)_{jk}$$

Hopefully this one qualifies... though it circumvents their original motivation.

Super-Potts solutions

S_q-invariance in $U \sim \varphi^3 \implies$ Landau-Potts field theory (universality of Potts model)

S_q-invariance with vertices e^α of $(q-1)$-symplex in $\mathbb{R}^{q-1}$



$\psi^\alpha = e_i^\alpha \varphi_i$ q-states order parameter, with potential $U = \frac{1}{3!} g \sum_\alpha (\psi^\alpha)^3$
which belongs to universality class of lattice Potts model ($q = 0$ are percolations)

What about a superpotential $W = \frac{1}{3!}\lambda \sum_\alpha e_i^\alpha e_j^\alpha e_k^\alpha \Phi_i \Phi_j \Phi_k$?

Family of solutions considered by Rong-Su in 1910.08578. Enhanced SUSY for $q = 3$
with $\Delta_\Phi = 1 - \frac{\epsilon}{3} \to 2/3$ for $d = 3$ (exact).

Conclusions

- Realistic possibility of observing SUSY in tabletop experiments
- Now regarded as more likely than observing it at LHC
- Potentially interesting families of Yukawa models in $d = 4 - \epsilon$ with enhanced sym

Thank you