

Lattice in flatland: Investigations of IR conformality and topological defects

Relativistic Fermions in Flatland: theory and application

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Features of flatland: fixed points ...

- * Can we produce a map showing the conformal and mass-gapped IR phases of fermionic gauge theories?
- * How to understand the conformality in non-compact QED3 at small number of flavors?

... and topology

- * How to formulate topological CS theories on lattice? Are there non-trivial examples doable by Monte-Carlo?
- * How to quantify the relevance of topological defects (monopoles, vortices, etc) at IR fixed points?
- * Can we study consequences of particle-vortex dualities by Monte-Carlo simulations?

Parity-invariant QED and QCD in 2+1 dimensions

Field content: $N = 2n$ flavors of two-component Dirac fermions coupled to gauge-field through Dirac operator in a ℓ^3 box

$$S = \sum_{i=1}^n \bar{\psi}_i^+ \not{D} \psi_i^+ - \sum_{i=1}^n \bar{\psi}_i^- \not{D}^\dagger \psi_i^- + \frac{1}{g^2} S_g$$

Continuum limit at Gaussian FP: Keep $g^2 \ell$ fixed and take $g^2 a \rightarrow 0$

Finite-size scaling/thermodynamic limit by changing $g^2 \ell$

IR limit: $|x| \rightarrow \infty$ for correlators $\langle \mathcal{O}(0)\mathcal{O}(x) \rangle$

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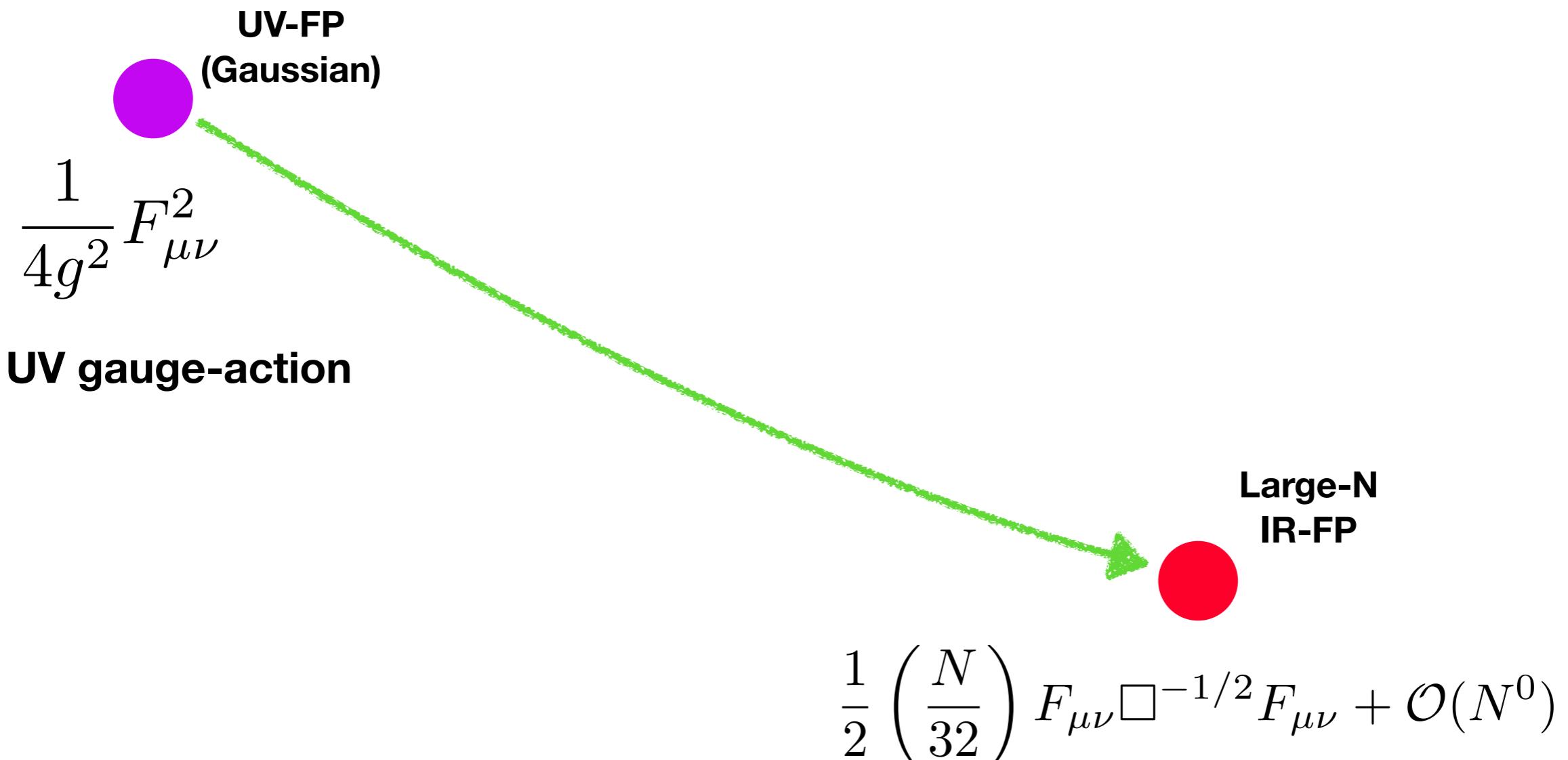
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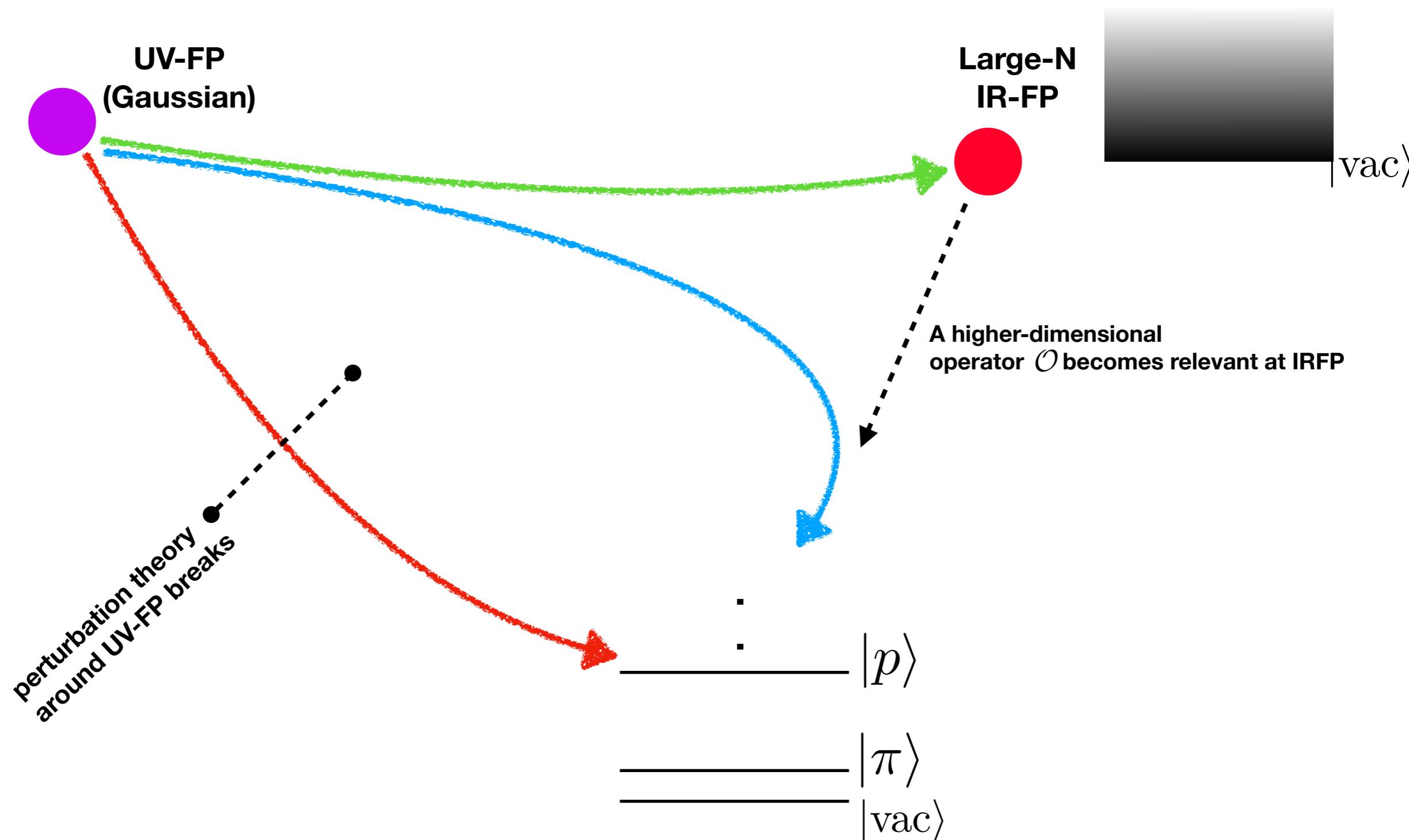
Why large-N infrared fixed point exists

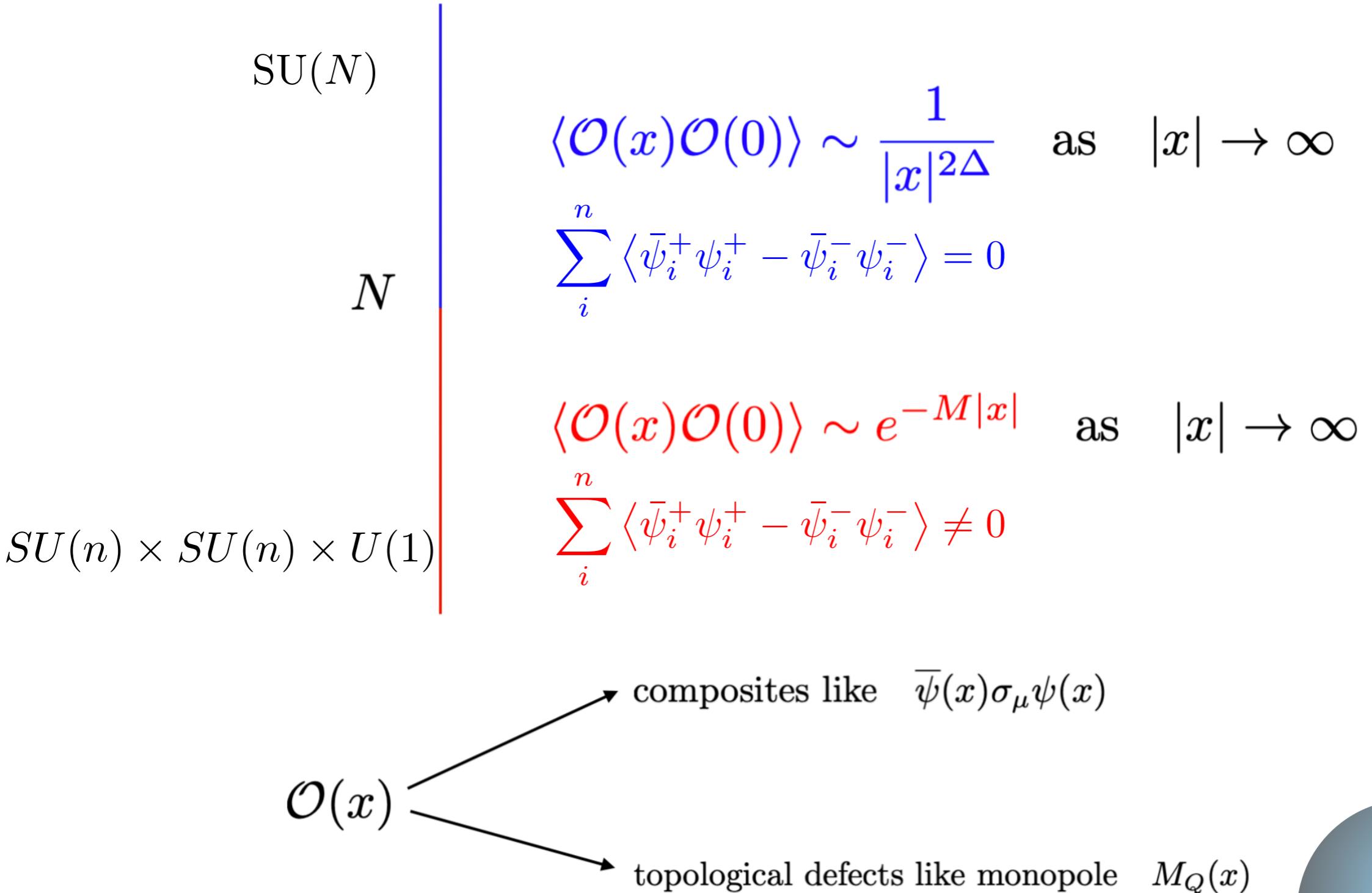


Fermion-induced conformal gauge-action gets dominant

(Appelquist et al, 85)

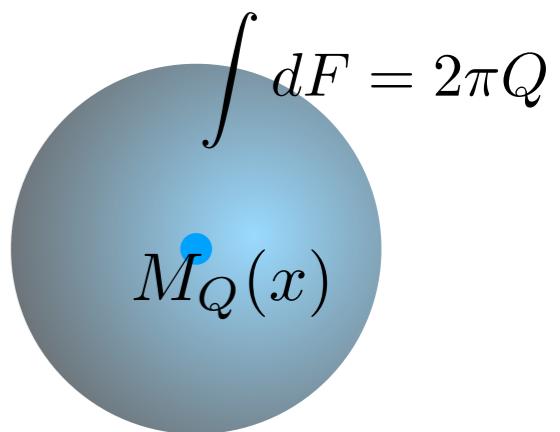
Characterizing the UV-to-IR flow



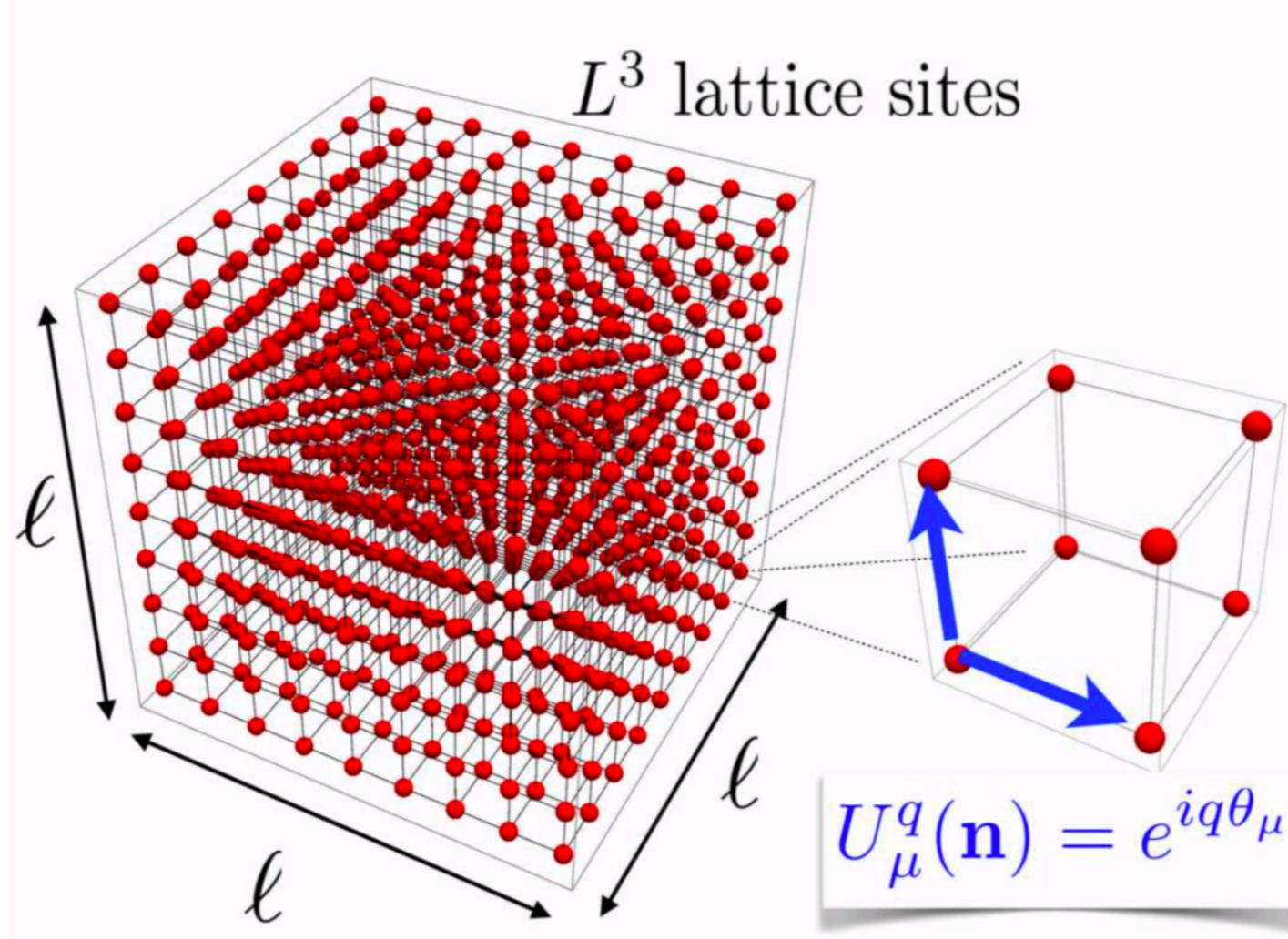


QED_3

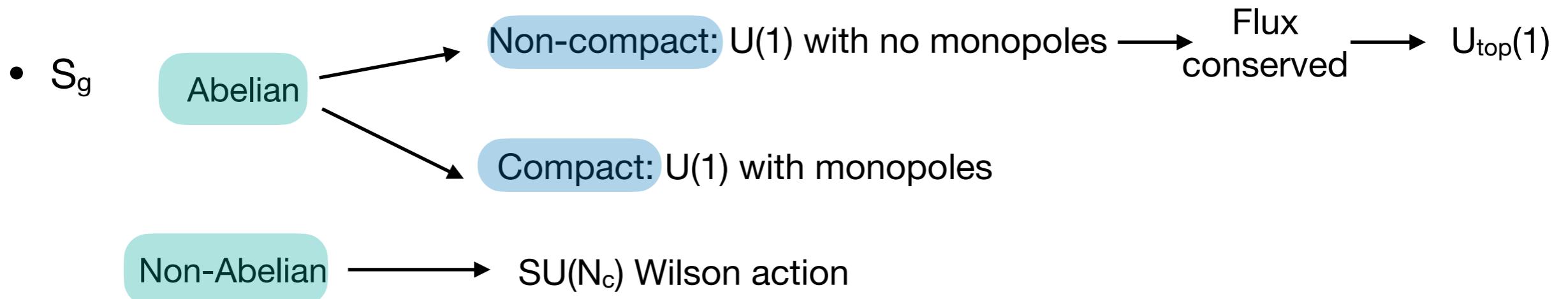
Previous lattice $N>2$, Hands et al, Raviv et al
large-N scalar dimension (Pufu): $N < 2$



Lattice Regularization using massless overlap fermions



- Regulate the theory on physical ℓ^3 torus using L^3 lattice



Lattice Regularization using massless overlap fermions

Effective fermion action

U(2) symmetry at finite lattice spacing

$$Z(\eta^+, \eta^-) = \det \left[\frac{1+V}{2} \frac{1+V^\dagger}{2} \right] e^{-\eta^+ \mathcal{G} \eta^+ - \eta^- \mathcal{G} \eta^-}$$

$$V = \left(\not{D}_w(m_w) \not{D}_w^\dagger(m_w) \right)^{-1/2} \not{D}_w; \quad VV^\dagger = 1$$

$2L^3 \times 2L^3$ unitary matrix

Details of construction in 1606.04109

Distinguishing conformal and scale-broken theories

Look at the microscopic eigenvalues of the Dirac operator in a finite-volume

$$\mathcal{G}^{-1}v_j = i\Lambda_i v_j$$

At fixed finite volume, take the continuum limit

$$\lambda_j \ell = \lim_{L \rightarrow \infty} \Lambda_j L$$

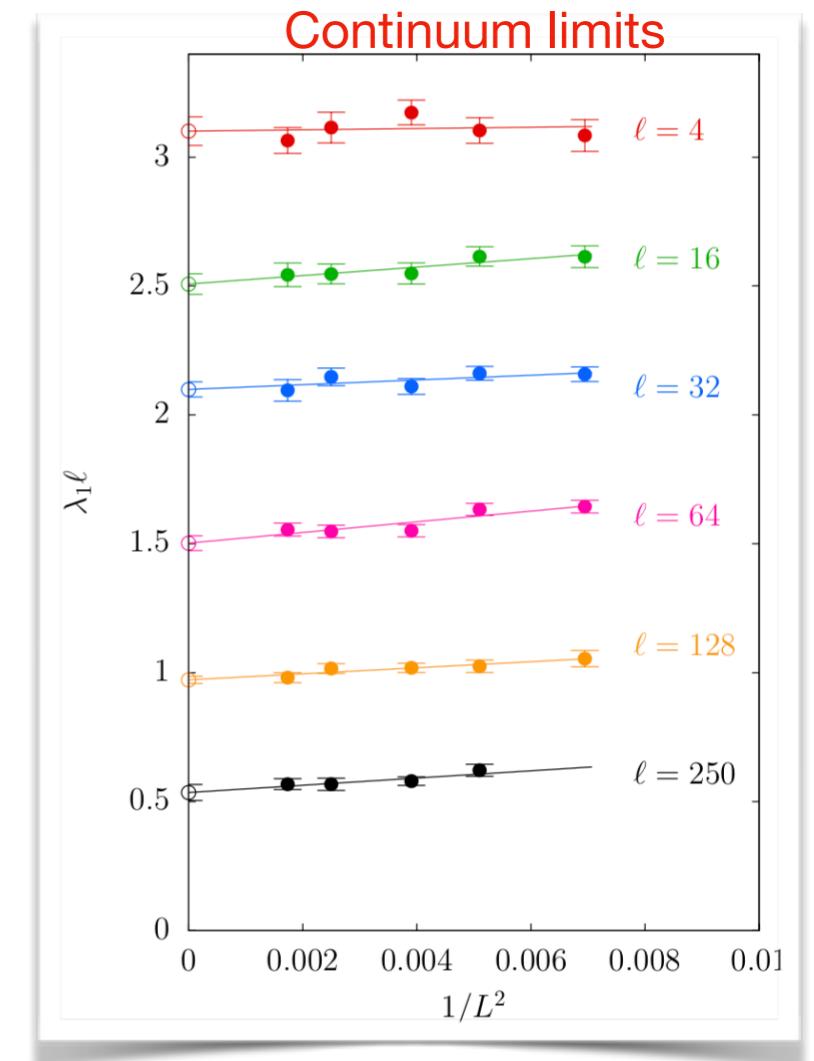
Look at the finite-size scaling of eigenvalues:

$$\lim_{\ell \rightarrow \infty} \lambda_j \propto \ell^{-1-\gamma_S}$$

SU(N)

$$\lim_{\ell \rightarrow \infty} \lambda_j \propto \ell^{-3}$$

SU(n) \times SU(n)



Distinguishing conformal and scale-broken theories

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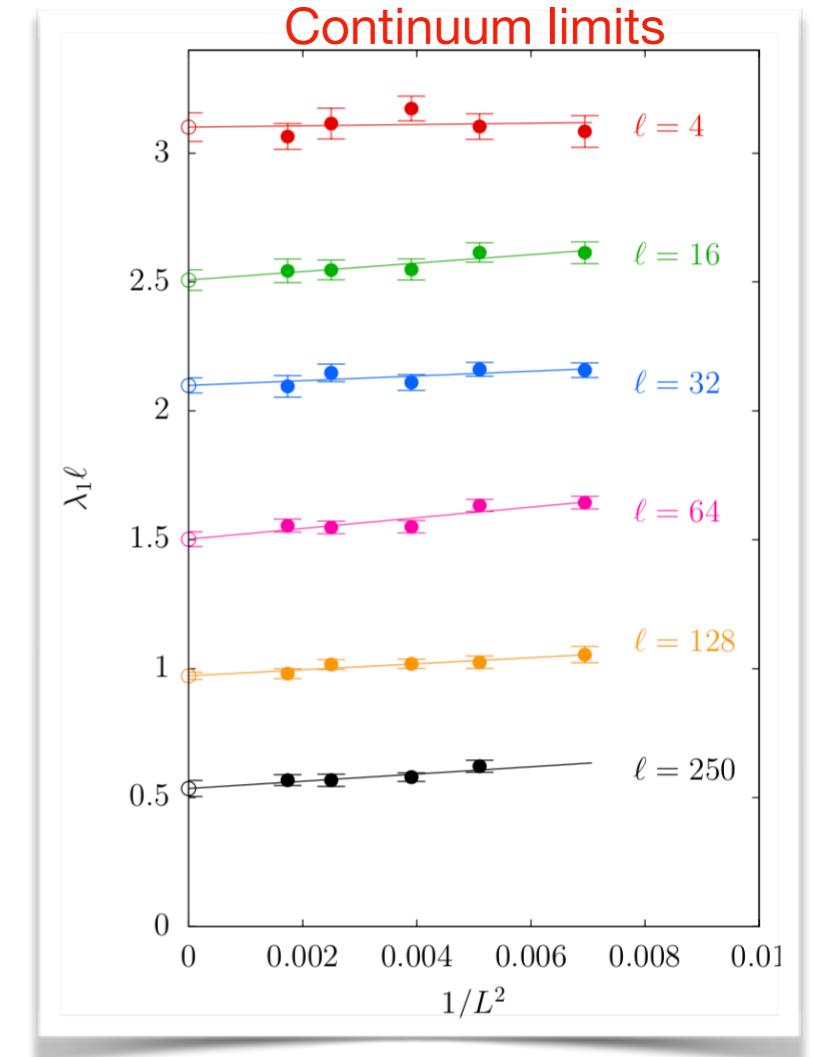
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$$\begin{aligned} & \text{SU}(N) \quad \left| \begin{array}{l} \lim_{\ell \rightarrow \infty} \lambda_j \propto \ell^{-1-\gamma_S} \end{array} \right. \\ & \text{SU}(n) \times \text{SU}(n) \quad \left| \begin{array}{l} \lim_{\ell \rightarrow \infty} \lambda_j \propto \ell^{-3} \end{array} \right. \end{aligned}$$



$$\Sigma = \lim_{\ell \rightarrow \infty} \left\{ \Sigma_i(\ell) = \frac{z_i}{\lambda_i \ell^3} \right\}$$



Distinguishing conformal and scale-broken theories

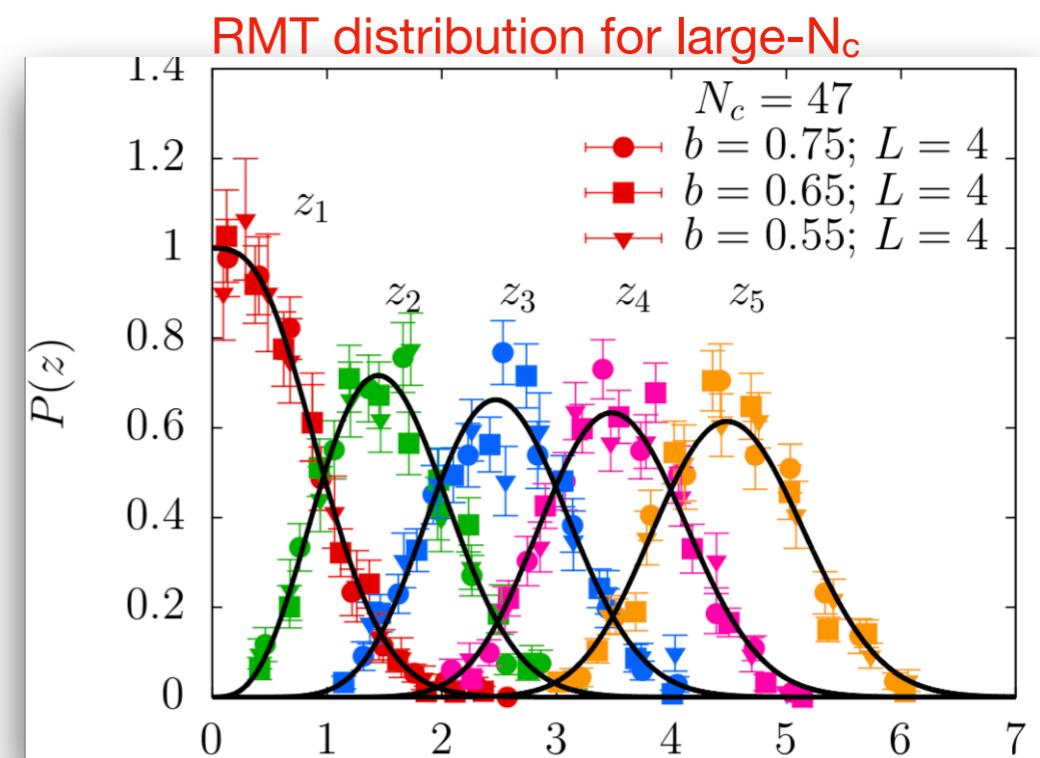
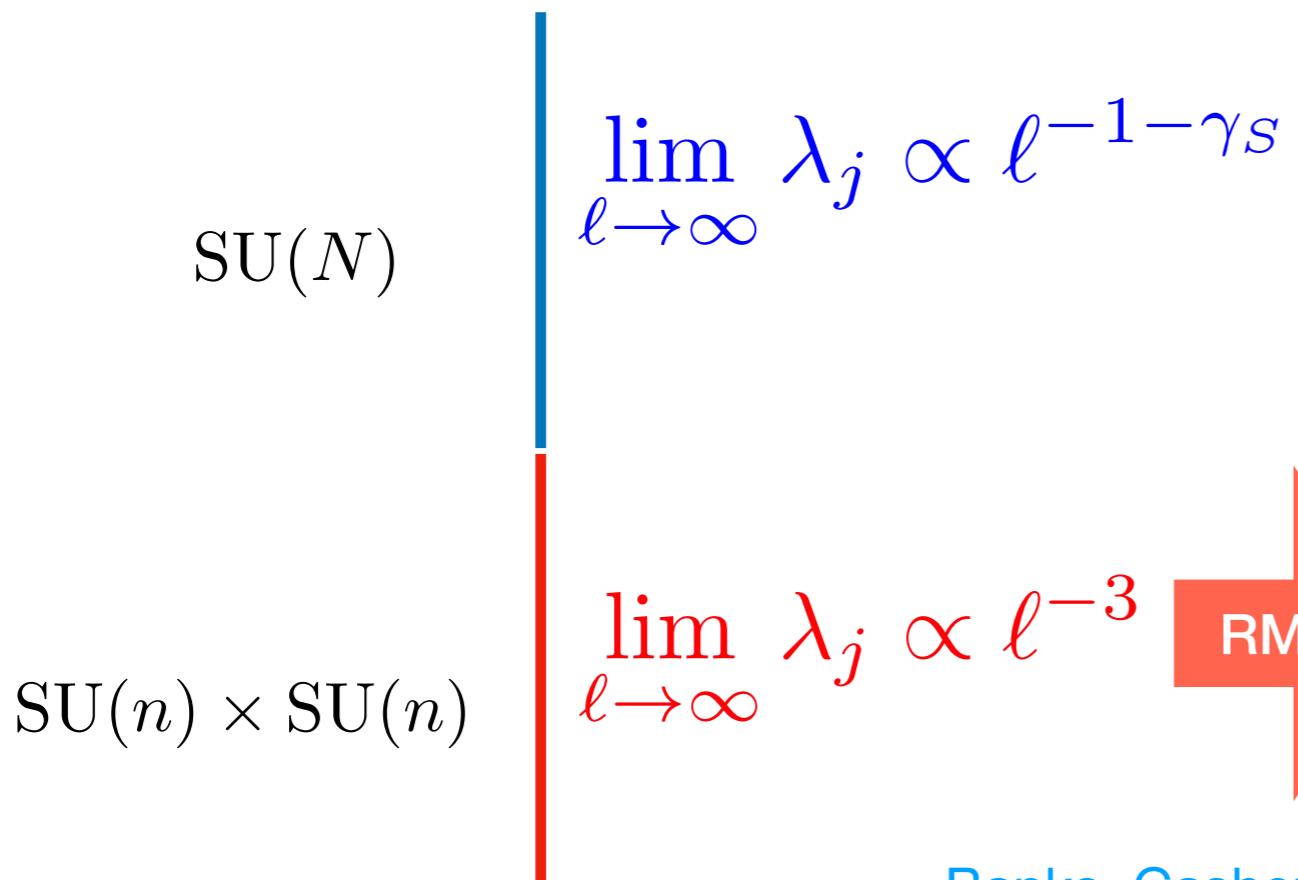
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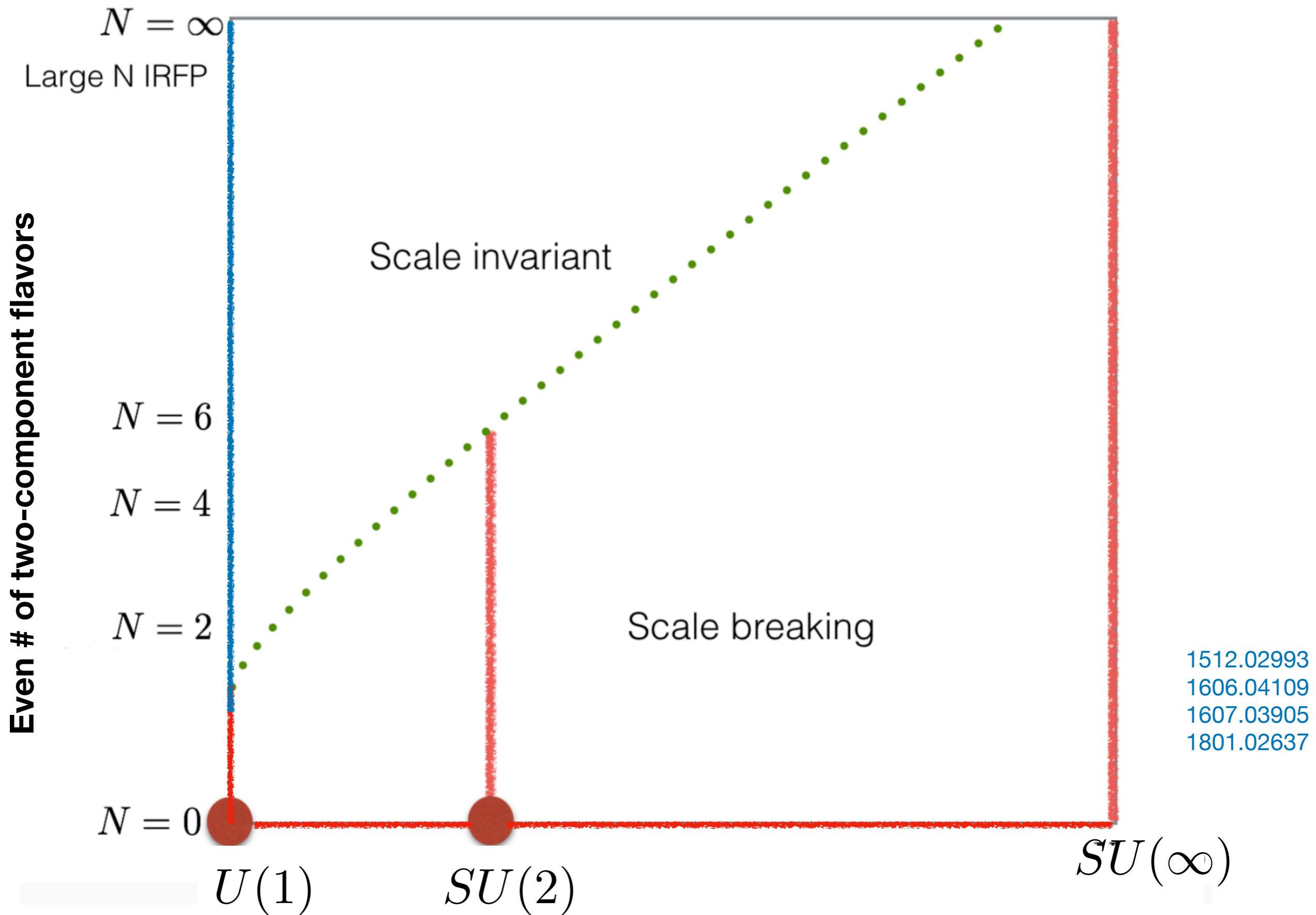
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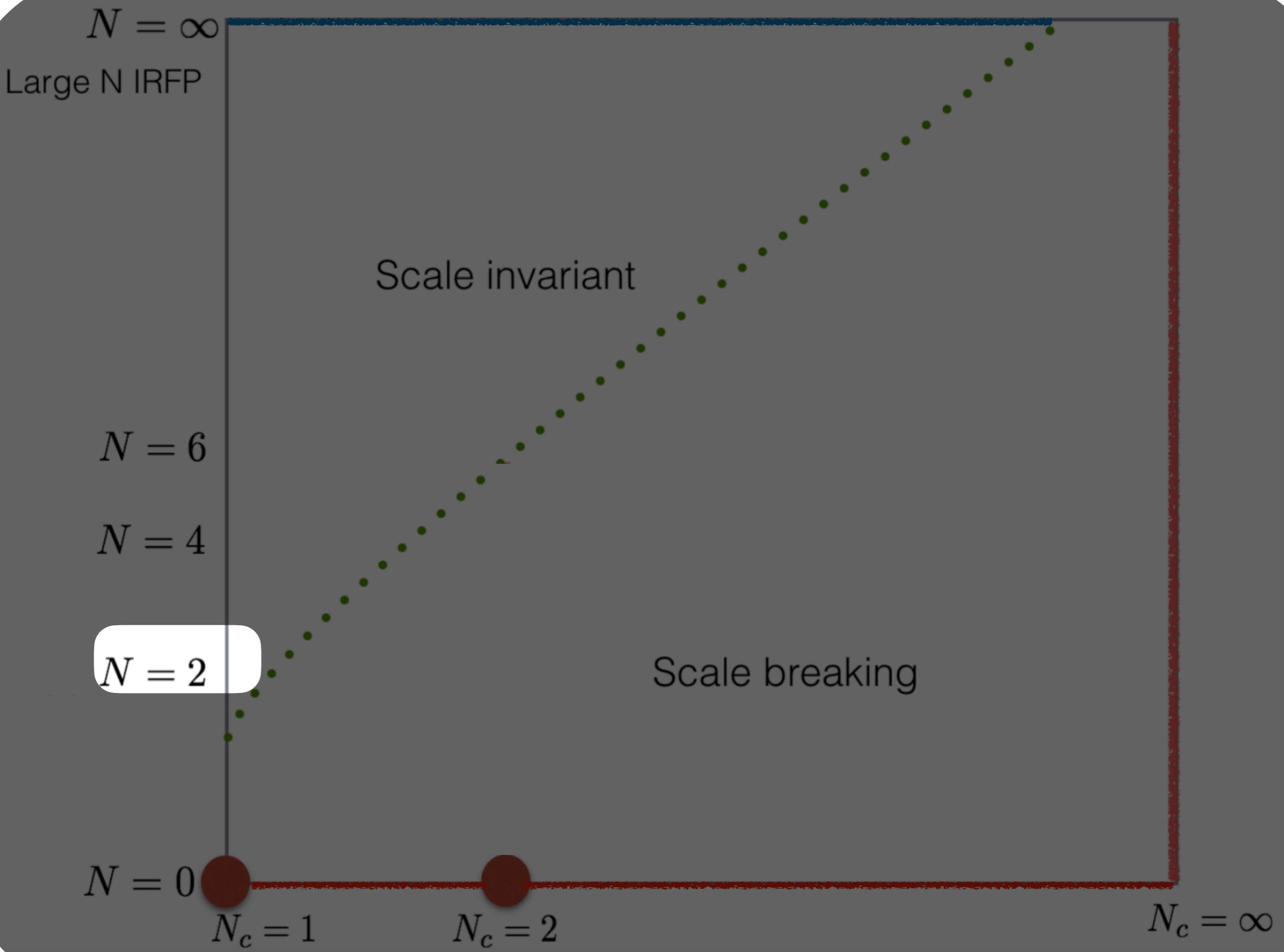


$$\Sigma = \lim_{\ell \rightarrow \infty} \left\{ \Sigma_i(\ell) = \frac{z_i}{\lambda_i \ell^3} \right\}$$

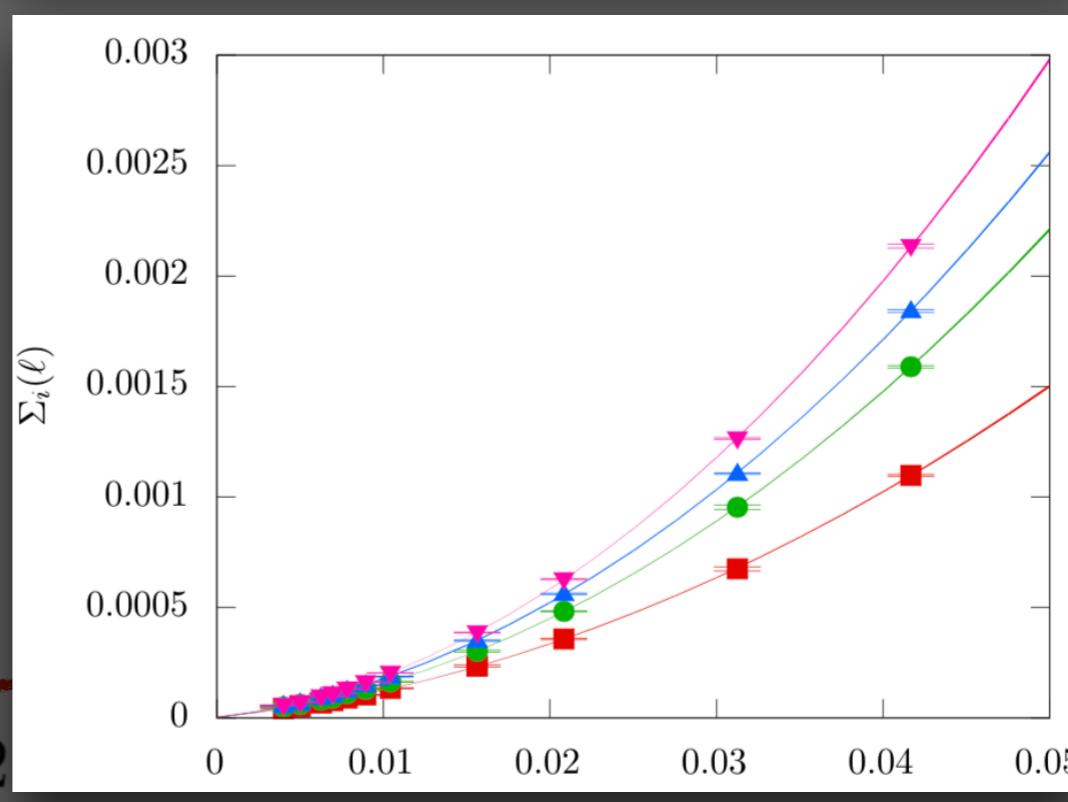
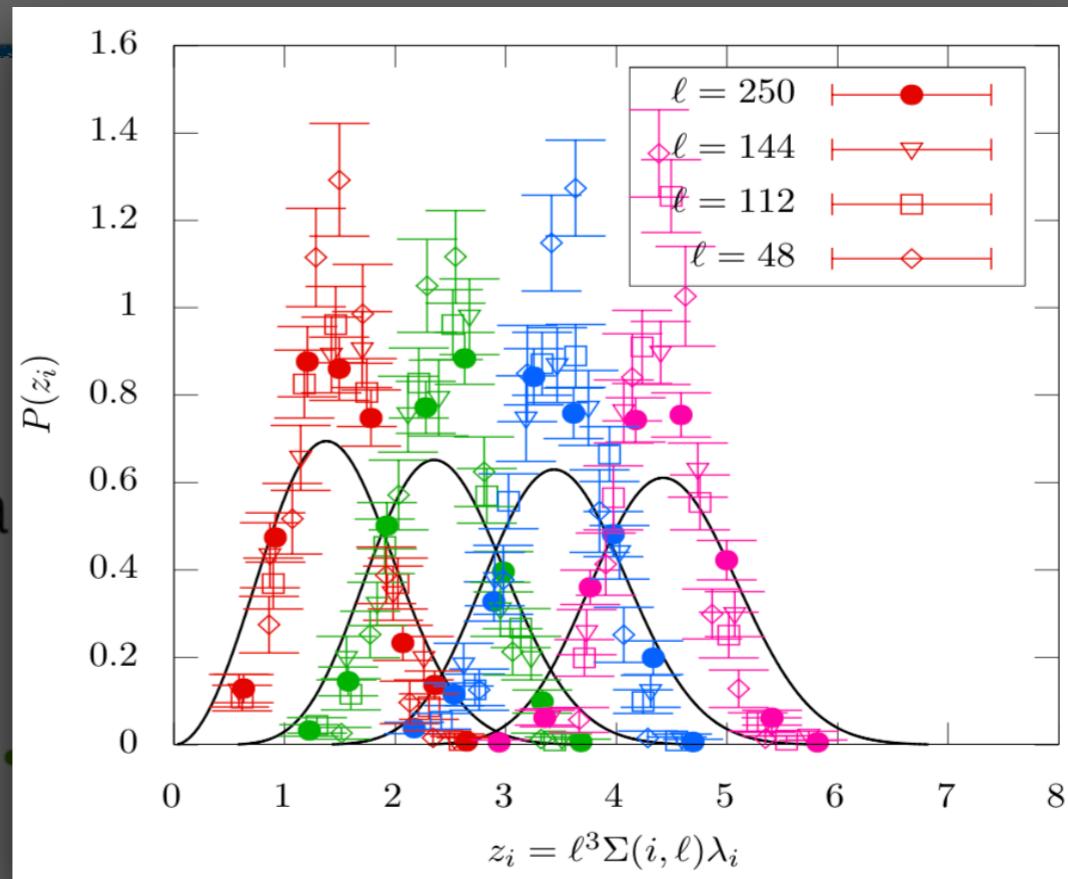
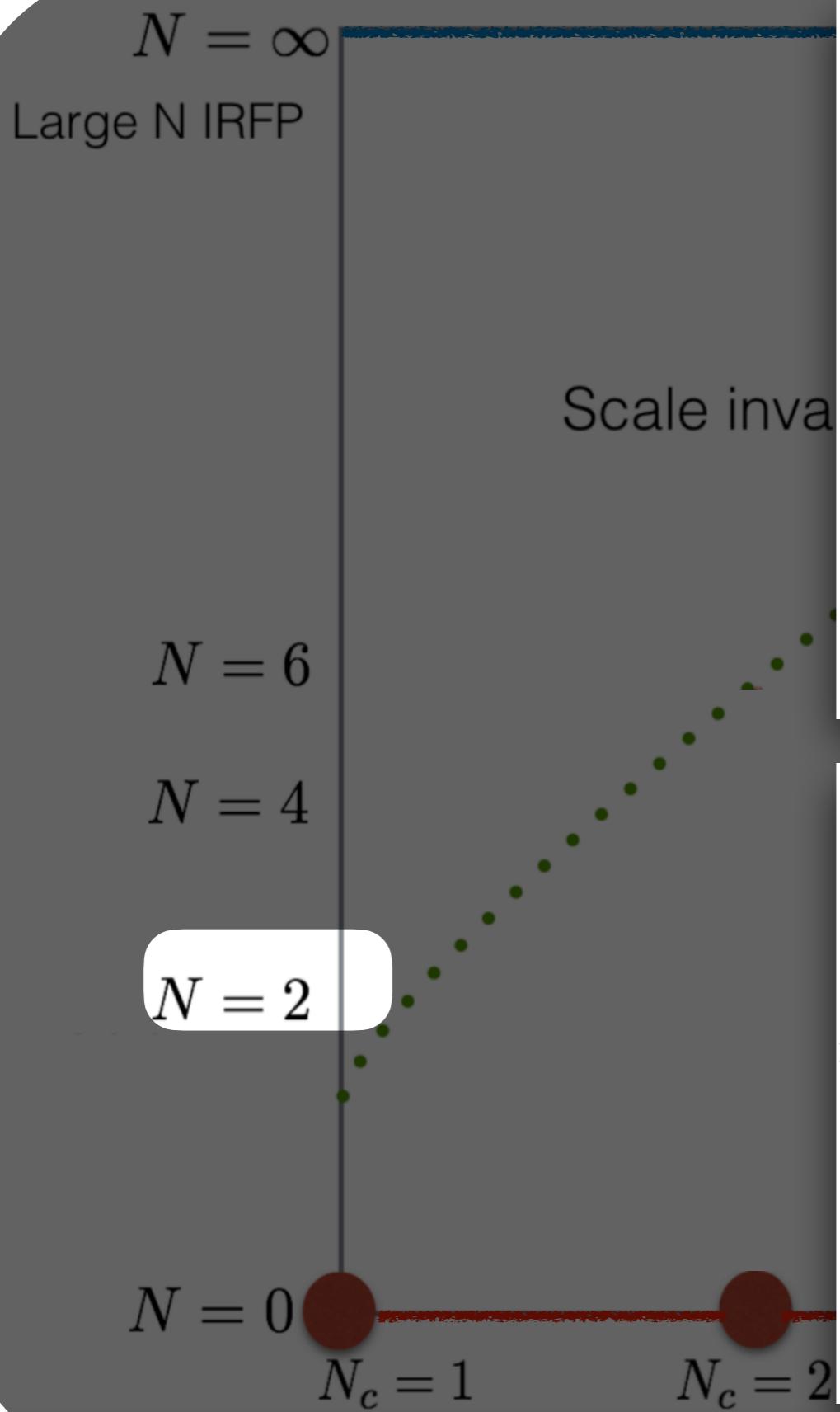
Mapping the IR phase diagram of 2+1d massless QCD



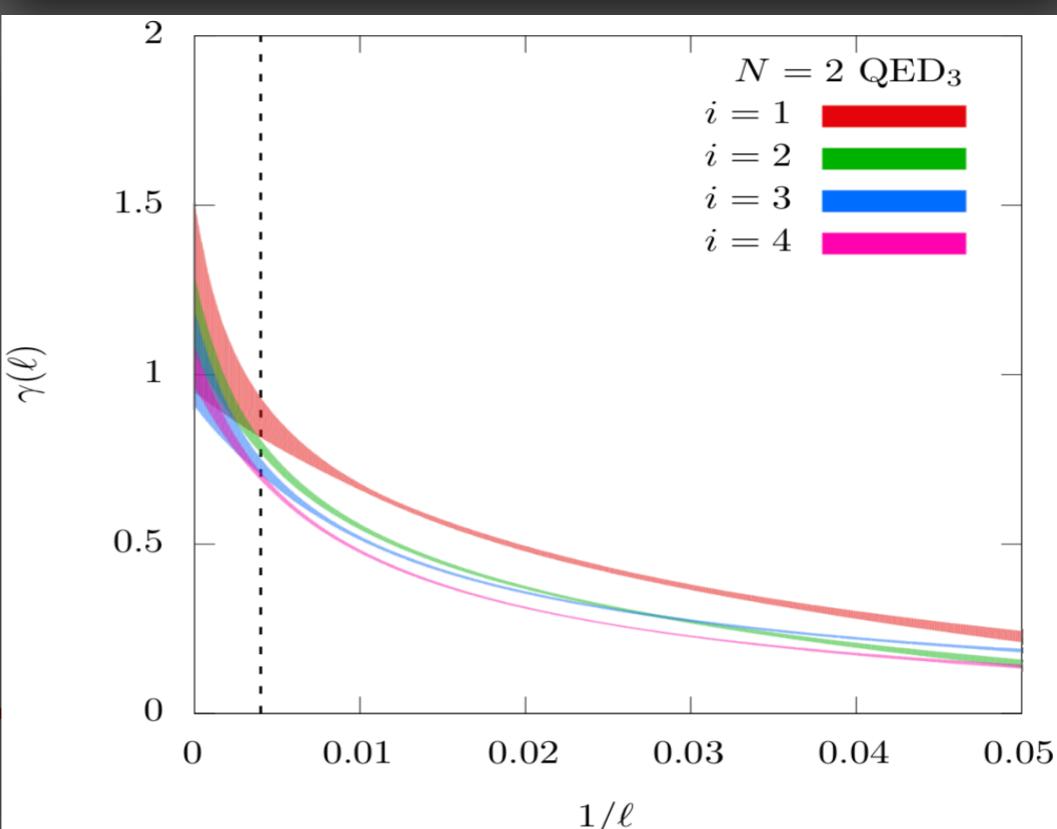
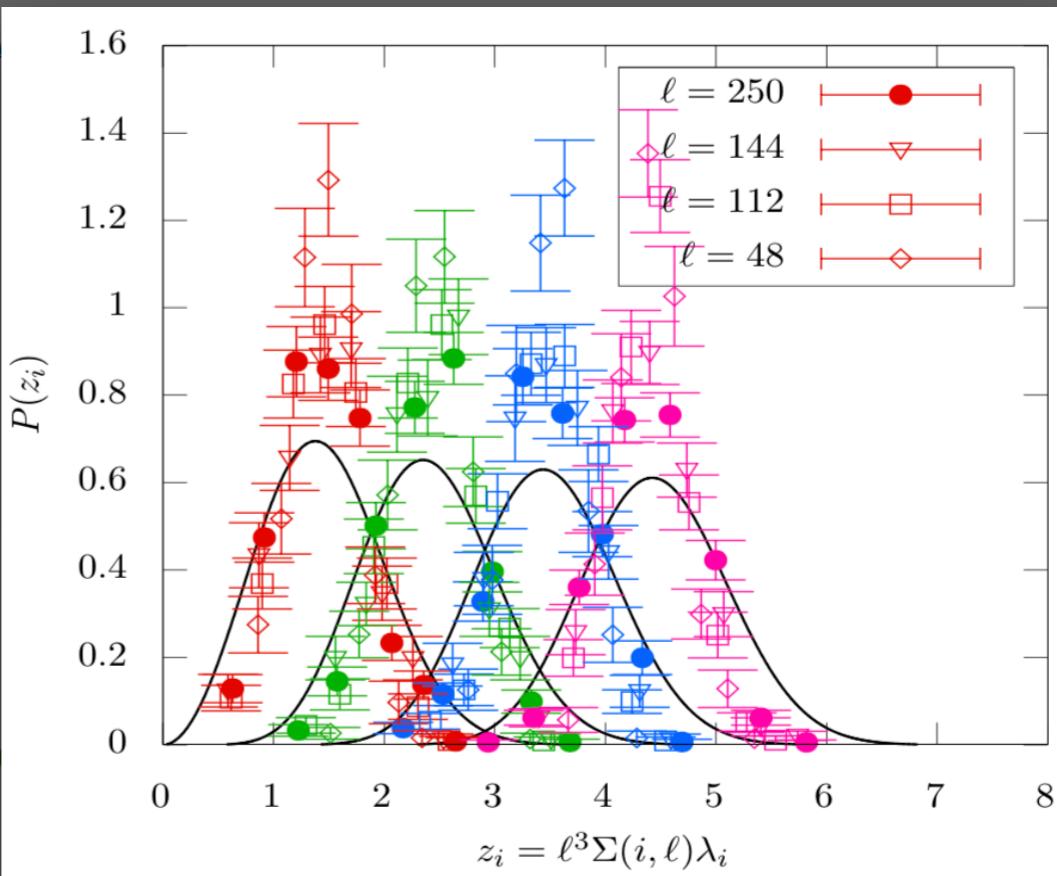
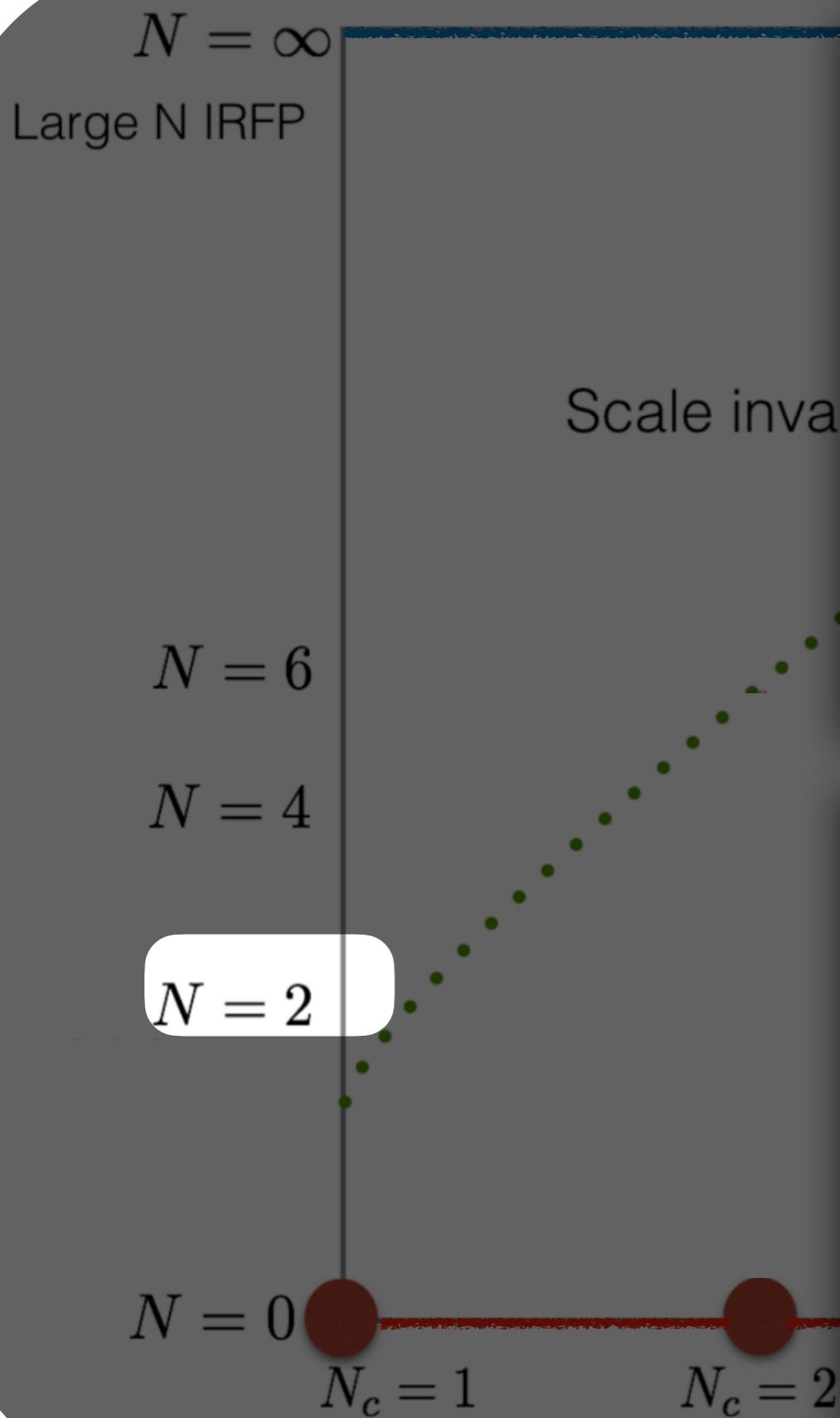
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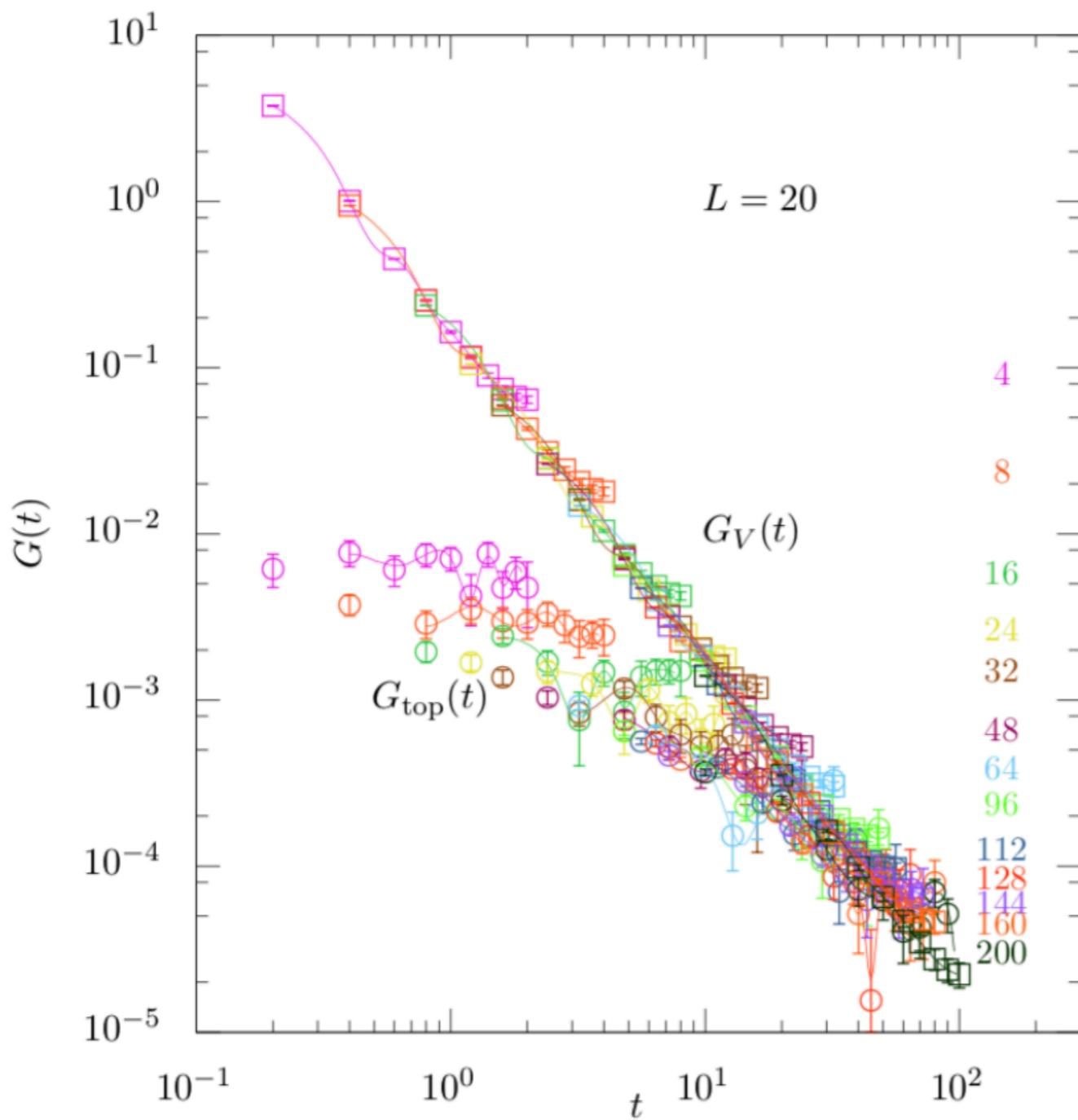
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Mapping the IR phase diagram of 2+1d massless QCD



Self-duality of QED₃: a prediction from particle-vortex duality



1707.11143

$$\bar{\psi}^+ \not{D}(a + B)\psi^+ - \bar{\psi}^- \not{D}^\dagger(a - B)\psi^-$$

↑ self-dual? ↓

$$\bar{\chi}^+ \not{D}(a)\chi^+ - \bar{\chi}^- \not{D}^\dagger(a)\chi^- + B_\mu \left(\frac{1}{4\pi} \epsilon_{\mu\nu\rho} F^{\nu\rho} \right)$$

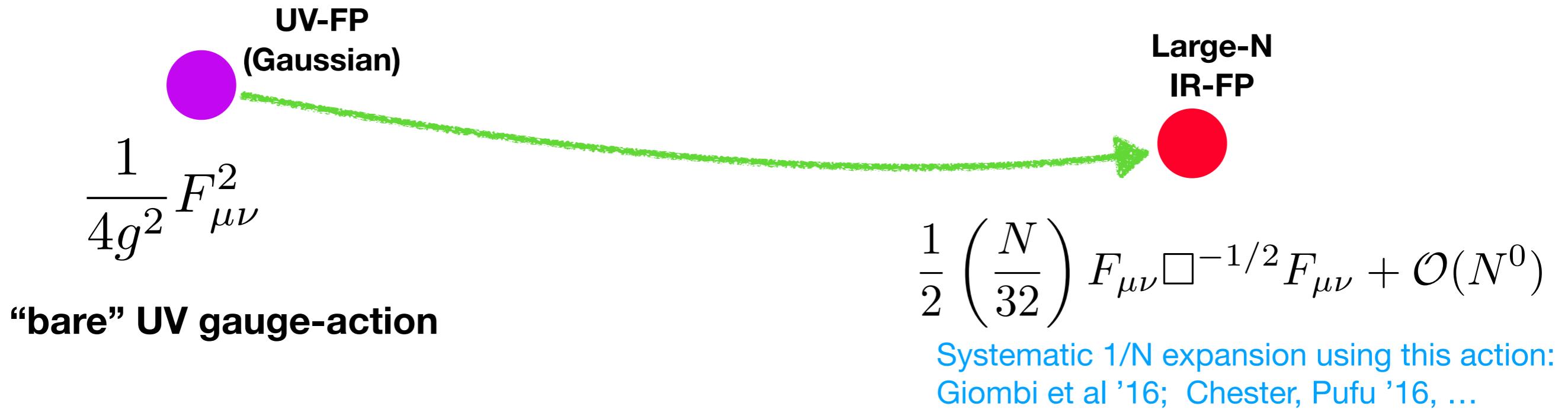
$$\langle V^\mu(0)V_\mu(x) \rangle = C_V |x|^{-4}$$

↑ degenerate? ↓

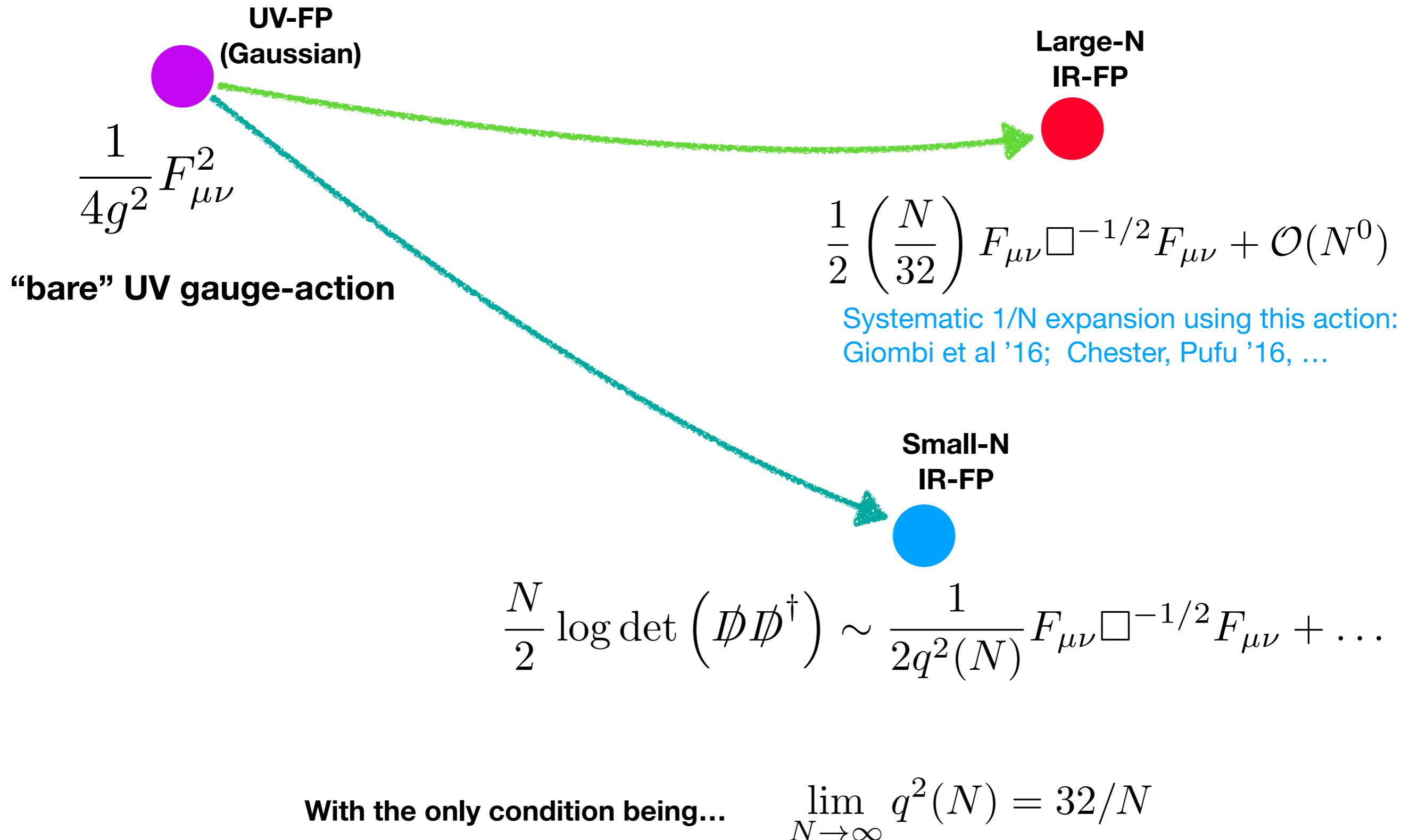
$$\langle V_\mu^{\text{top}}(0)V_\mu^{\text{top}}(x) \rangle = C_V^{\text{top}} |x|^{-4}$$

Xu, You '15; Seiberg, Hsin '16; Wang et al, '16

Capturing IR of noncompact QED_3 by a quadratic conformal induced gauge action



Capturing IR of noncompact QED_3 by a quadratic conformal induced gauge action



Study a conformal lattice gauge theory coupled to fermion sources

Charge q is the dimensionless parameter

$$Z = \int [da] e^{-\eta^+ g \eta^+ - \eta^- g \eta^-} e^{-\frac{1}{2q^2} F \square^{-1/2} F}$$

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Charge q is the dimensionless parameter

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Proposal:

N-flavor QED3:

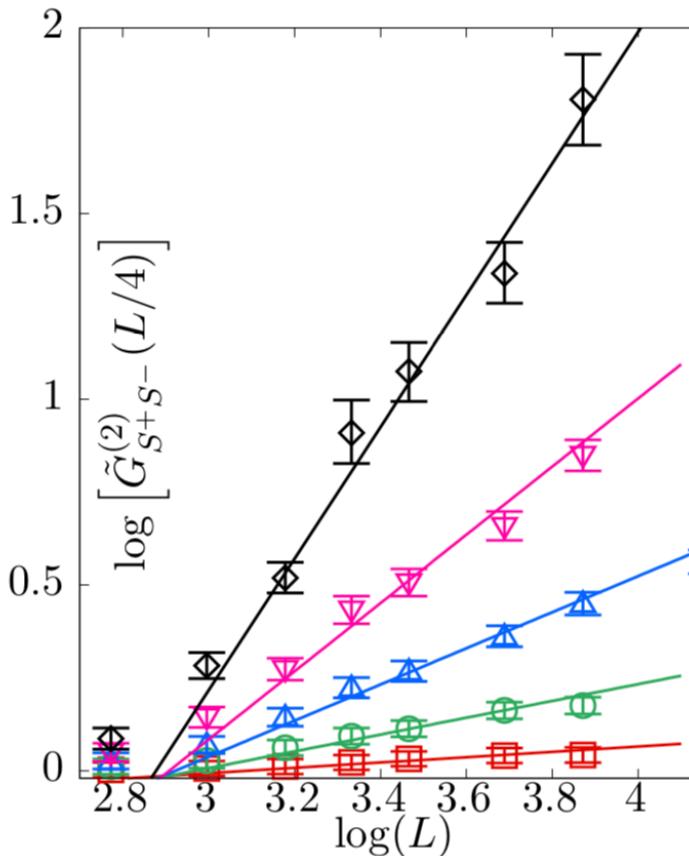
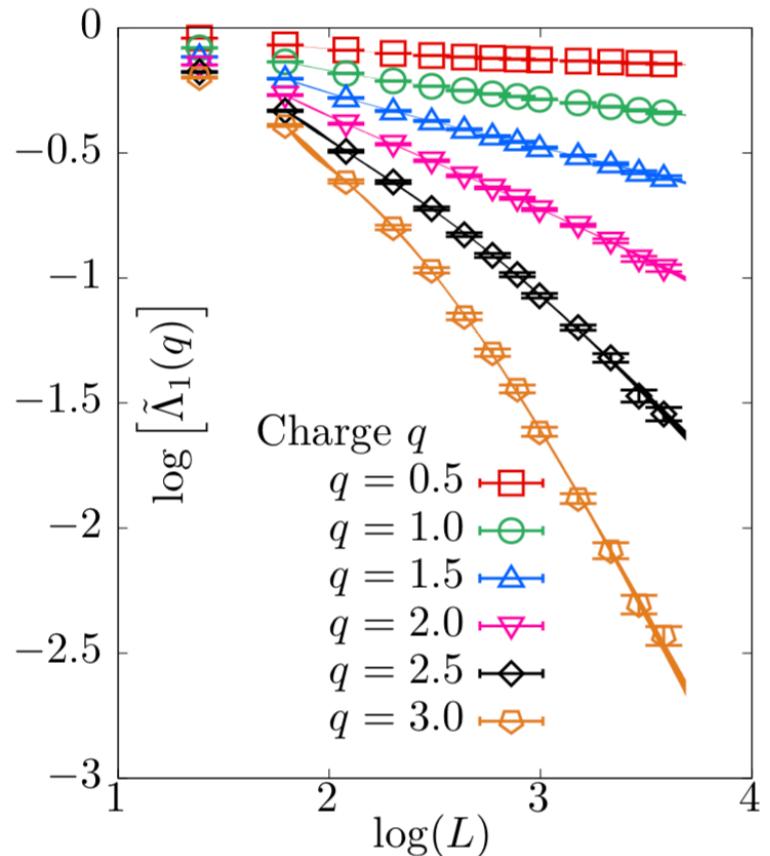
determine infrared CFT data
nonperturbatively

Charge- q lattice model:

determine CFT data in fermionic
observables

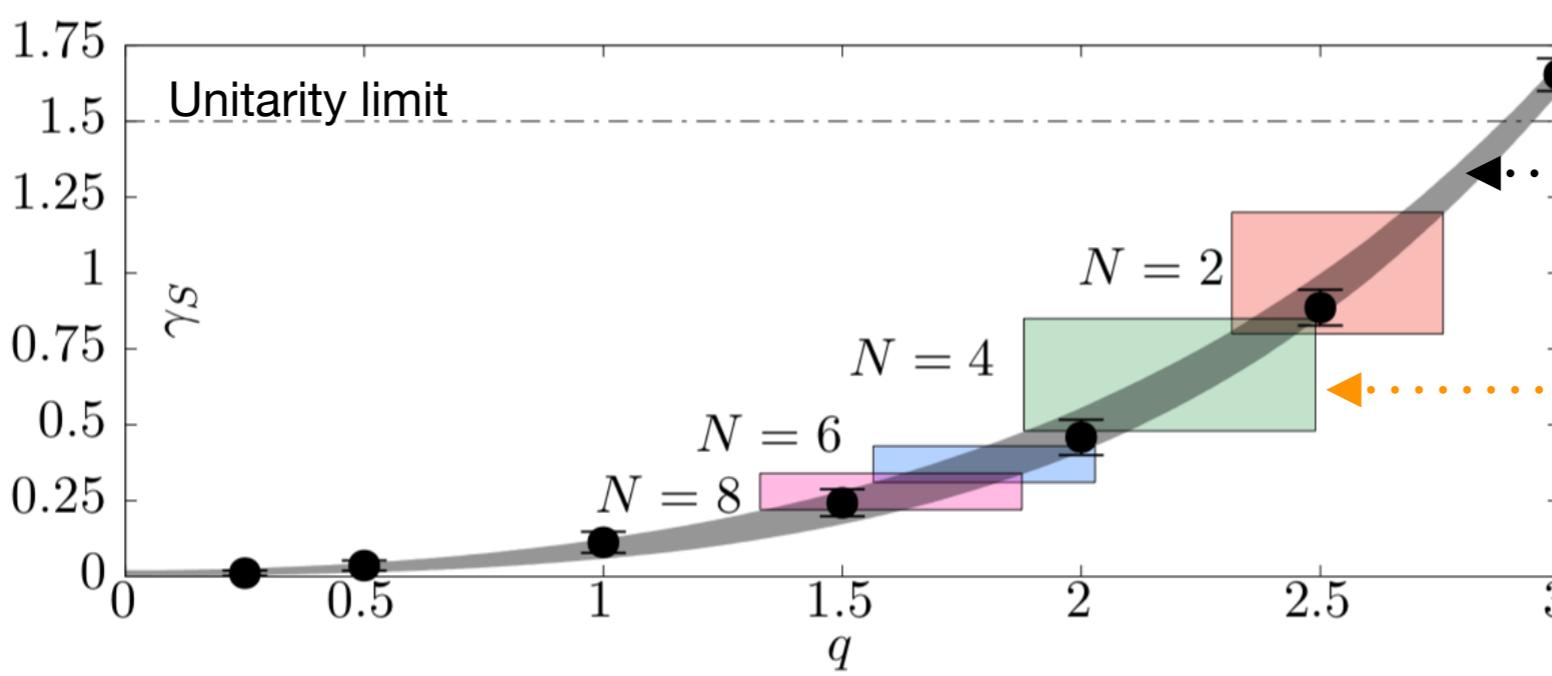
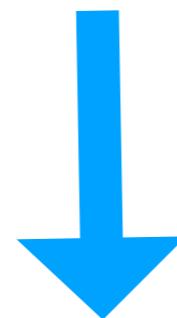
Approximate **q -N map** by
matching scaling dimensions

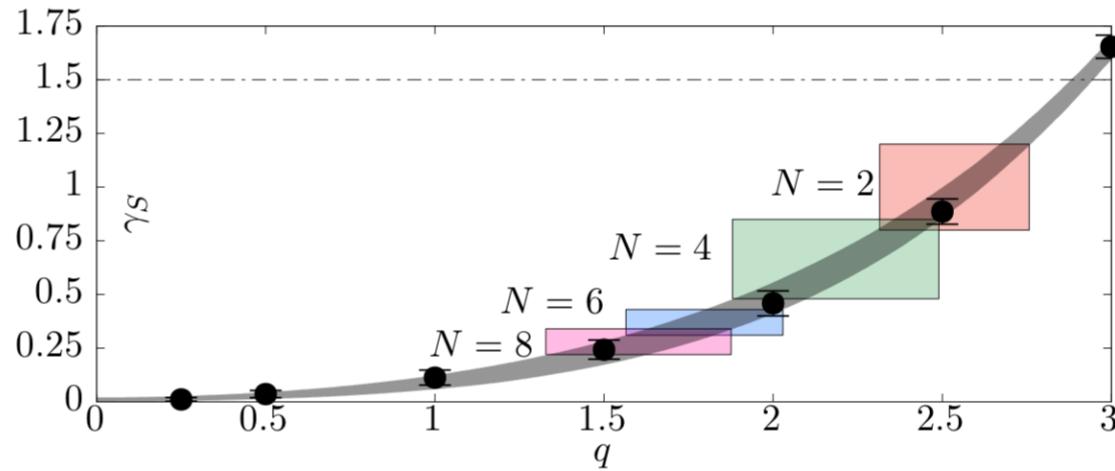
Test for other agreements
between QED3 and model at
matched q



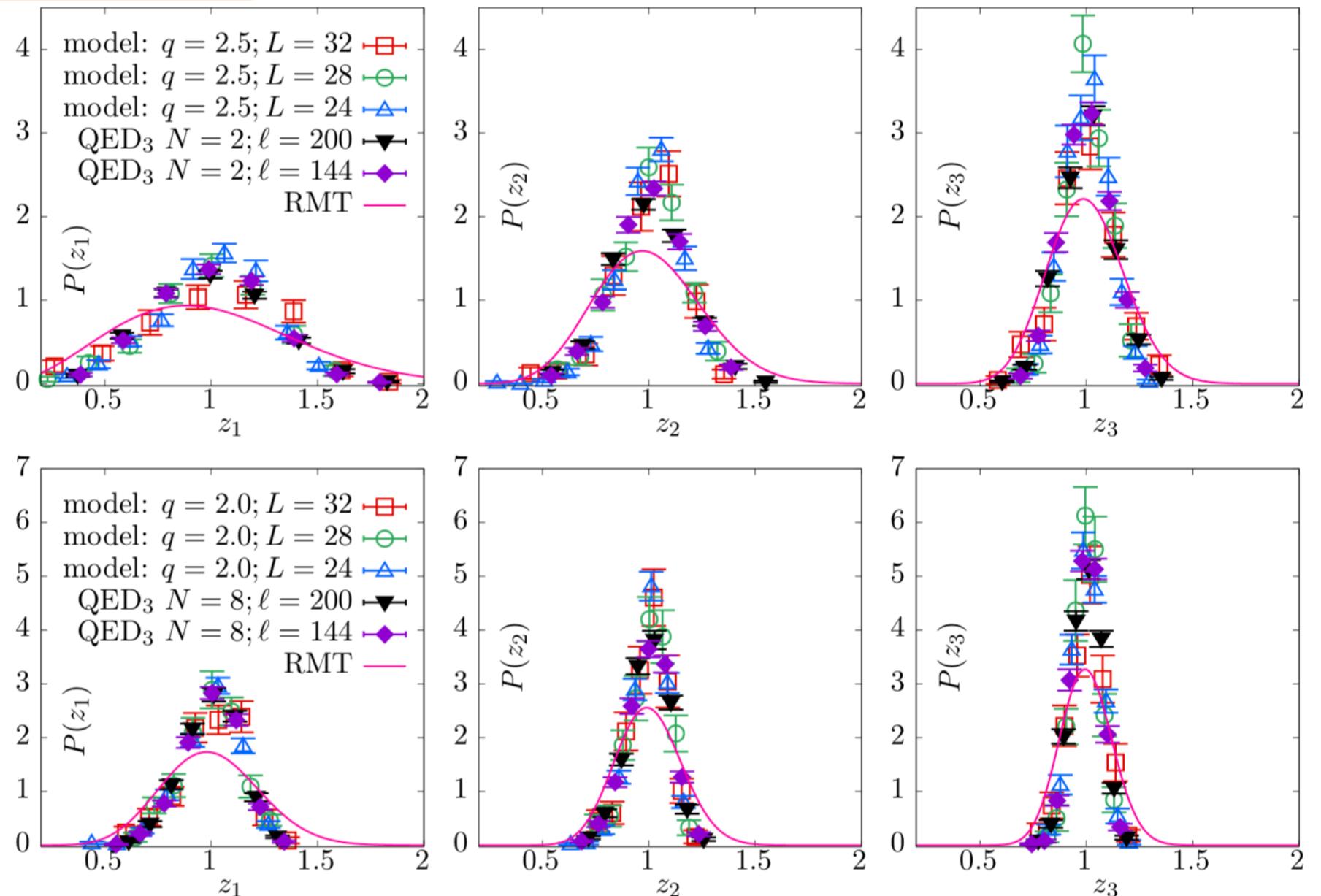
Determine scalar anomalous dimension γ_S in the model

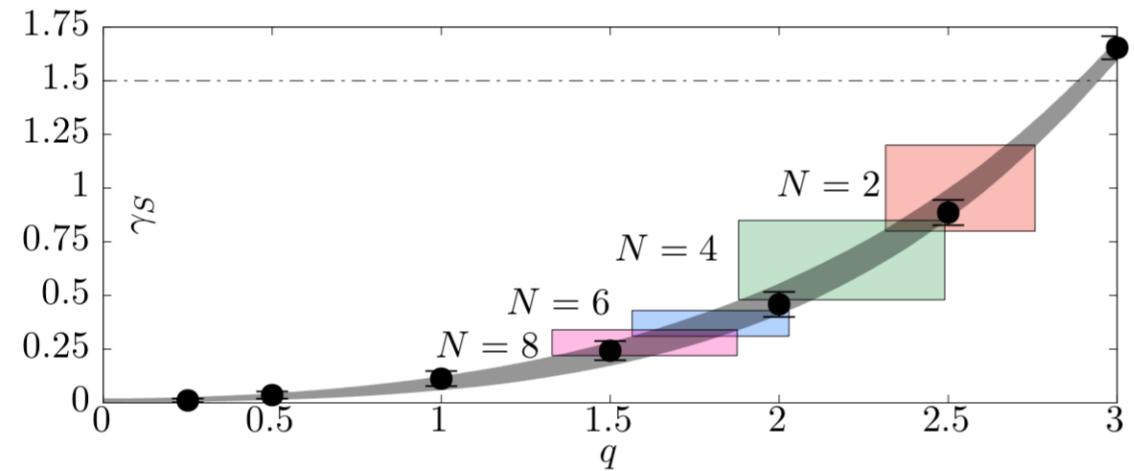
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QED₃ Dirac eigenvalue distribution
matches better with a CFT than RMT

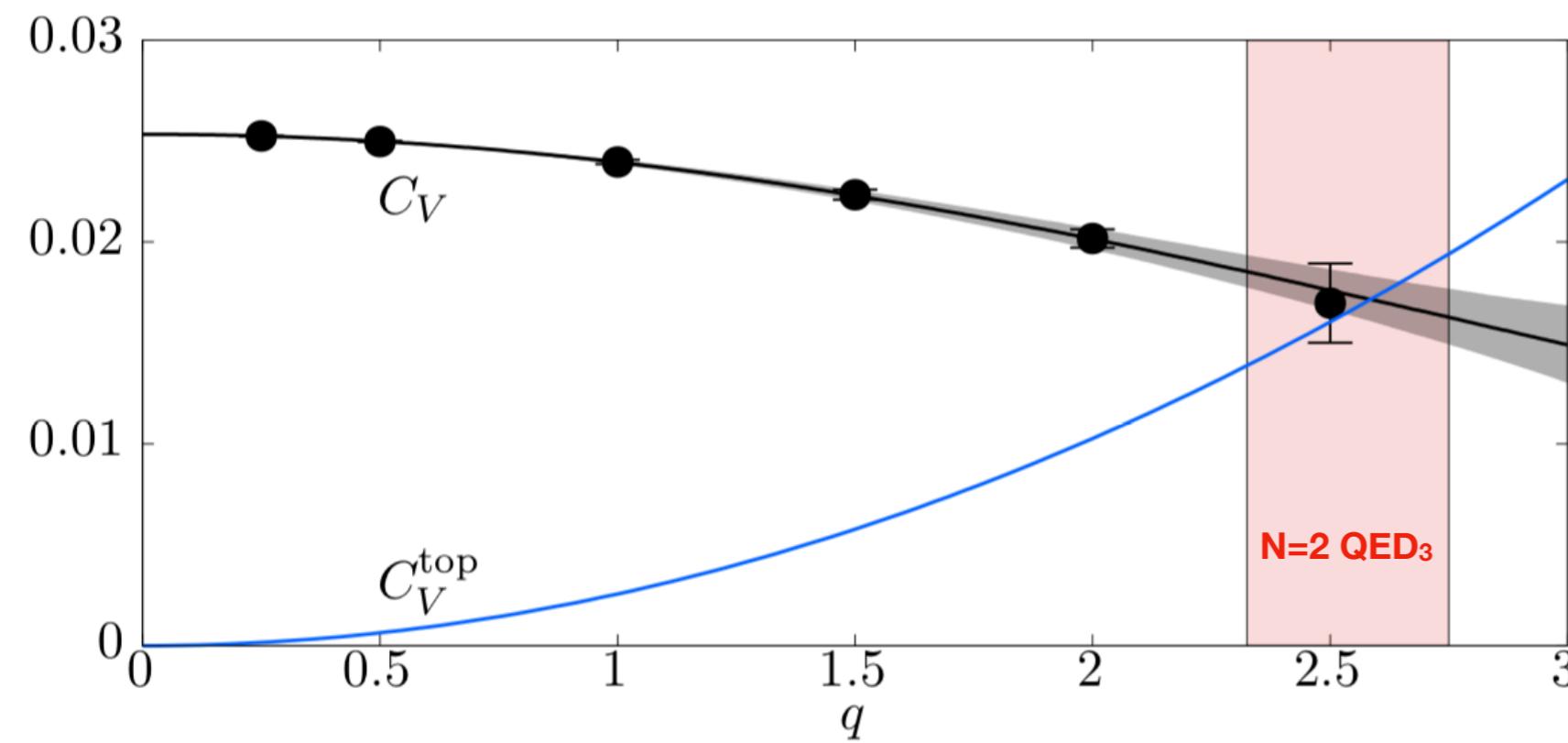




$$\bar{\psi}^+ \not{D}(a+B)\psi^+ - \bar{\psi}^- \not{D}^\dagger(a-B)\psi^-$$

self-dual?

$$\bar{\chi}^+ \not{D}(a)\chi^+ - \bar{\chi}^- \not{D}^\dagger(a)\chi^- + B_\mu \left(\frac{1}{4\pi} \epsilon_{\mu\nu\rho} F^{\nu\rho} \right)$$



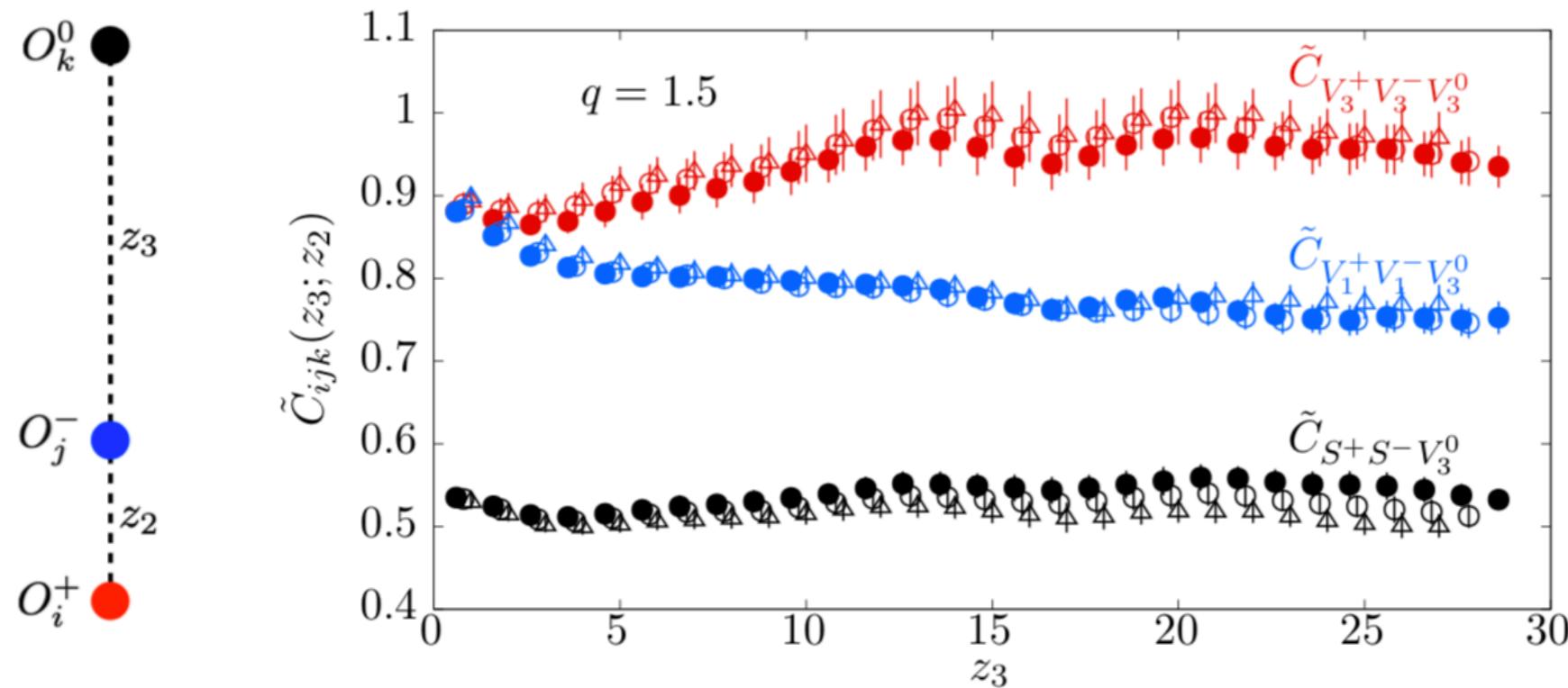
$$\langle V^\mu(0)V_\mu(x) \rangle = C_V |x|^{-4}$$

degenerate?

$$\langle V_\mu^{\text{top}}(0)V_\mu^{\text{top}}(x) \rangle = C_V^{\text{top}} |x|^{-4}$$

1) A nice system to use lattice to obtain CFT data in 3-pt functions

$$\langle O_i(z_1)O_j(z_2)O_k(z_3) \rangle = \frac{C_{ijk}}{|z_2 - z_1|^{-\Delta_1 - \Delta_2 + \Delta_3} |z_3 - z_2|^{-\Delta_2 - \Delta_3 + \Delta_1} |z_1 - z_3|^{-\Delta_3 - \Delta_1 + \Delta_2}}$$



2) What are the scaling dimensions of flavor-singlet parity-even 4-fermi operators as function of q ? (fermion line disconnected pieces)

3) Extension of non-compact CFT model to include monopoles?

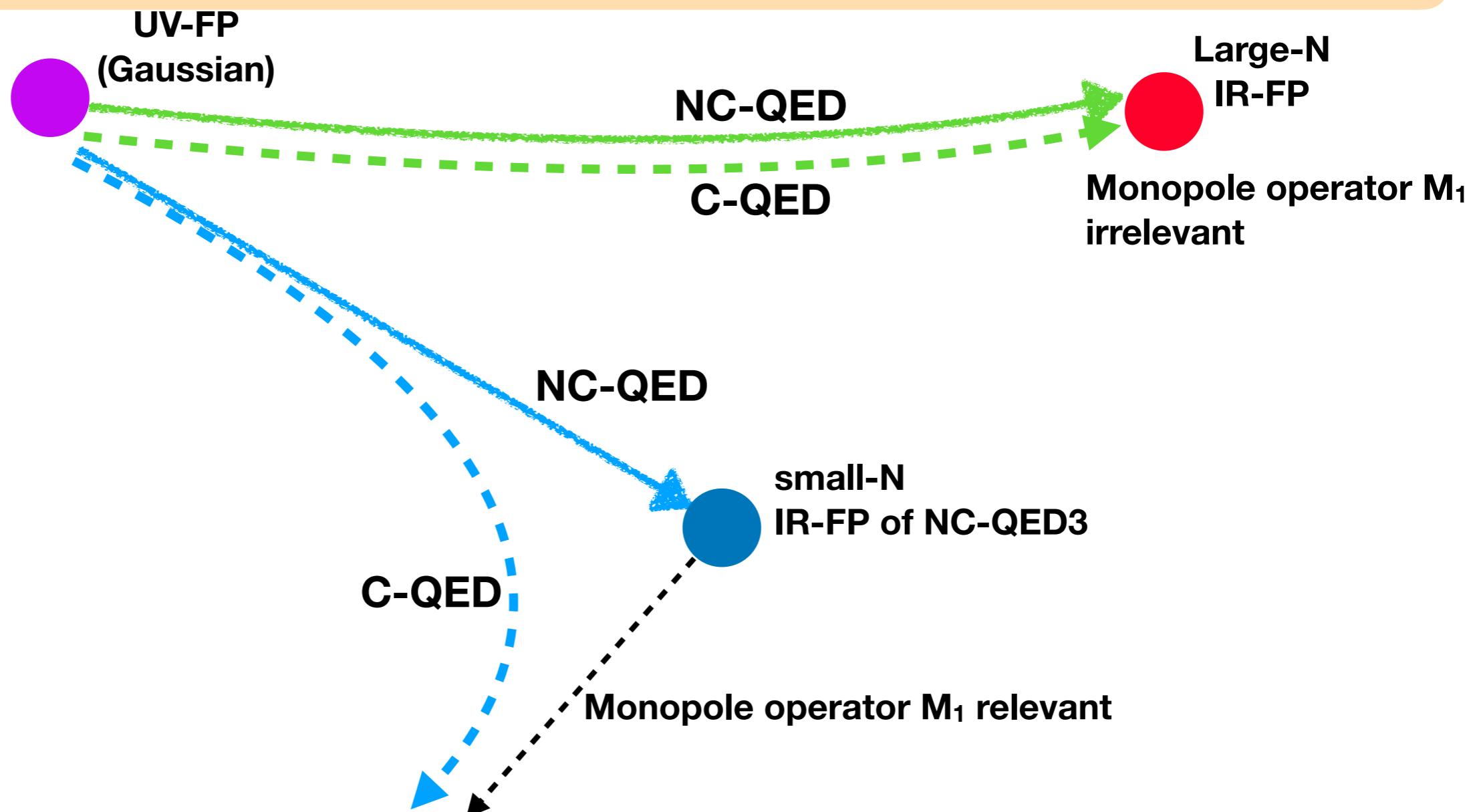
Scaling dimension of topological defects

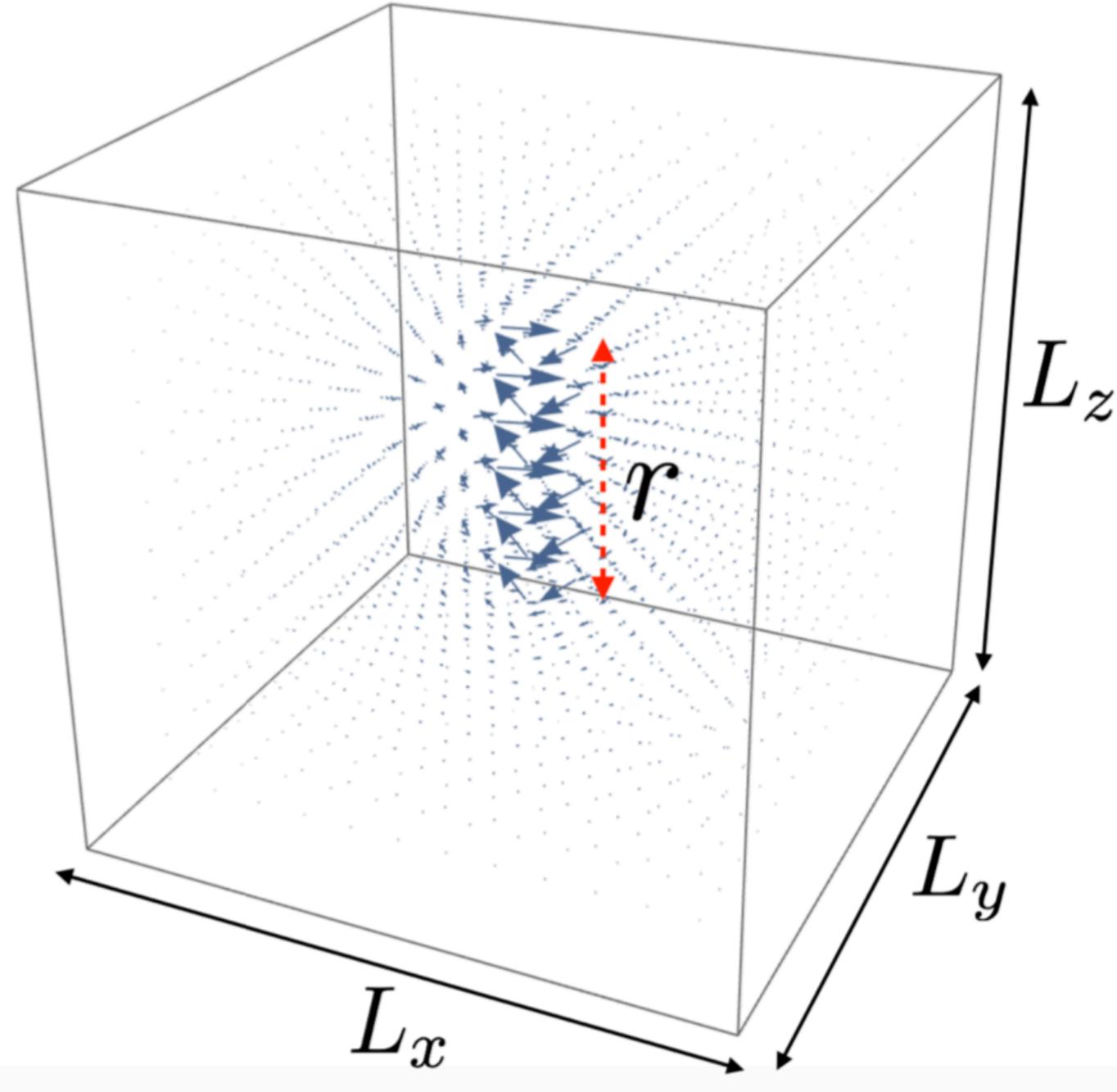
Compact QED₃ → Monopoles occur in the U(1) path-integral

Pure compact lattice theory: confining but no lines of constant physics

Problem: how to define compact QED₃ as a QFT with a good UV completion?
Perhaps as a limit of SU(2) Georgi-Glashow model

Indirect approach: find RG relevance of monopole operators M_Q at non-compact QED₃
IRFP (Pufu)



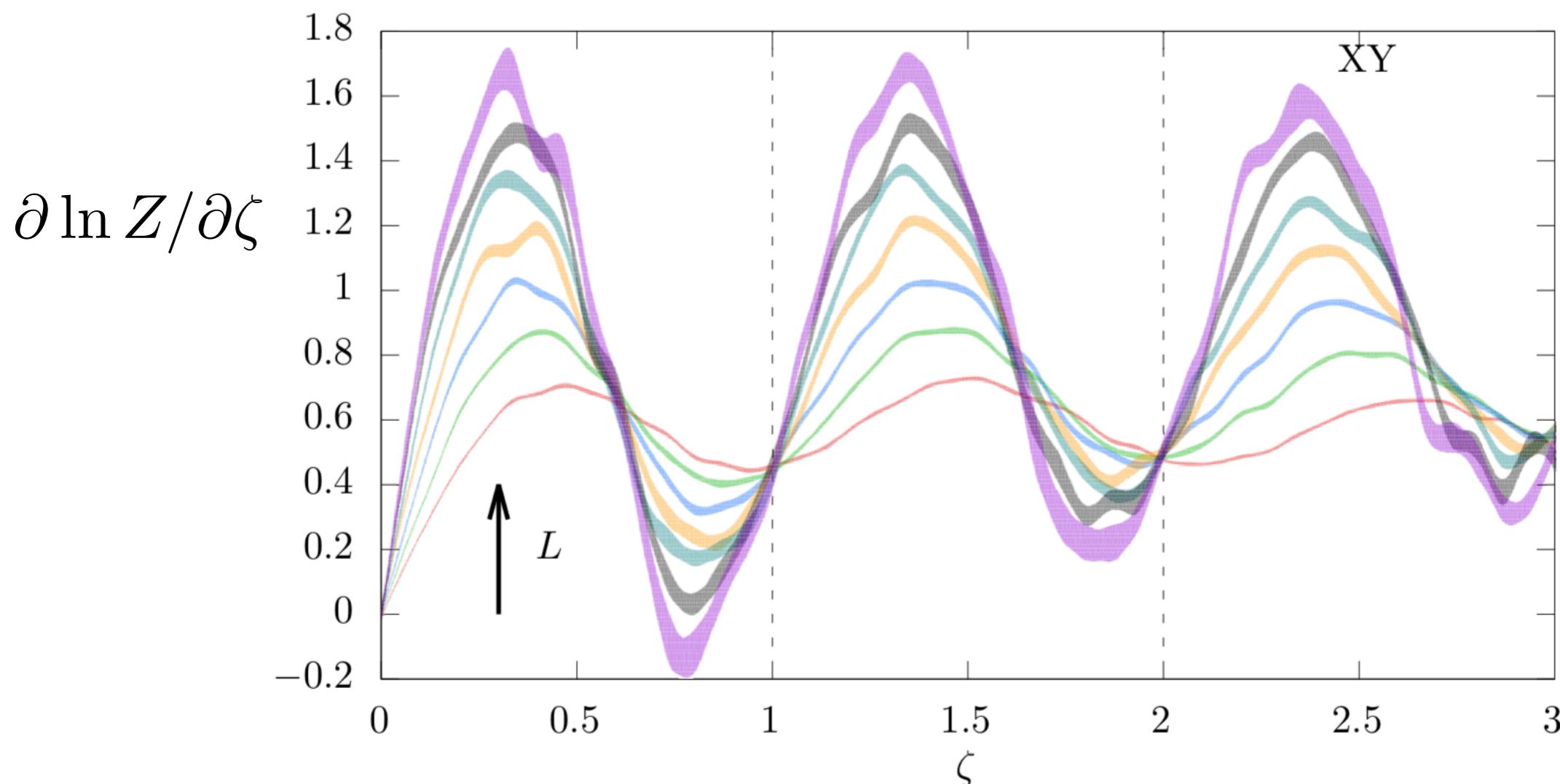


$$\langle M_Q(0)M_{-Q}(0)\rangle \equiv \frac{Z_{\text{NC-QED}}(A_{QQ}(r))}{Z_{\text{NC-QED}}(0)} \sim \frac{1}{r^{2\Delta_Q}}$$

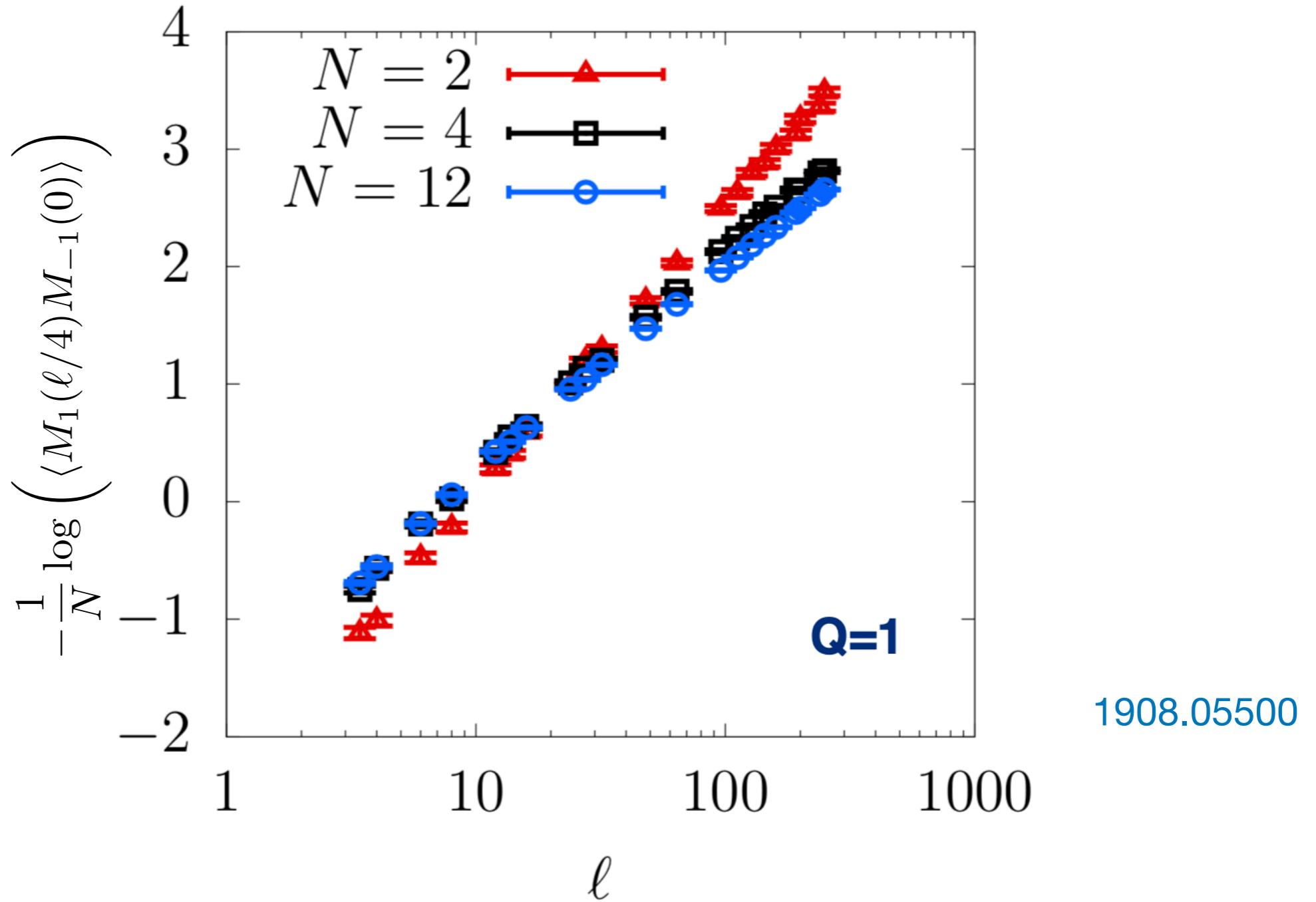
$$\langle M_Q(0)M_{-Q}(0) \rangle \equiv \frac{Z_{\text{NC-QED}}(A_{QQ}(r))}{Z_{\text{NC-QED}}(0)} \sim \frac{1}{r^{2\Delta_Q}}$$

$$= \exp \left(\int_0^Q d\zeta \frac{\partial}{\partial \zeta} \ln Z(\zeta A_{Q\bar{Q}}) \right)$$

Evaluate by M.C. for each ζ



Results for $N = 2, 4, 12$ QED₃: $Q\bar{Q}$ pair separated by $\ell/4$ as box-size ℓ is increased.



$$\Delta_1(N = 12) = 3.2(2) \Rightarrow \text{marginal}$$

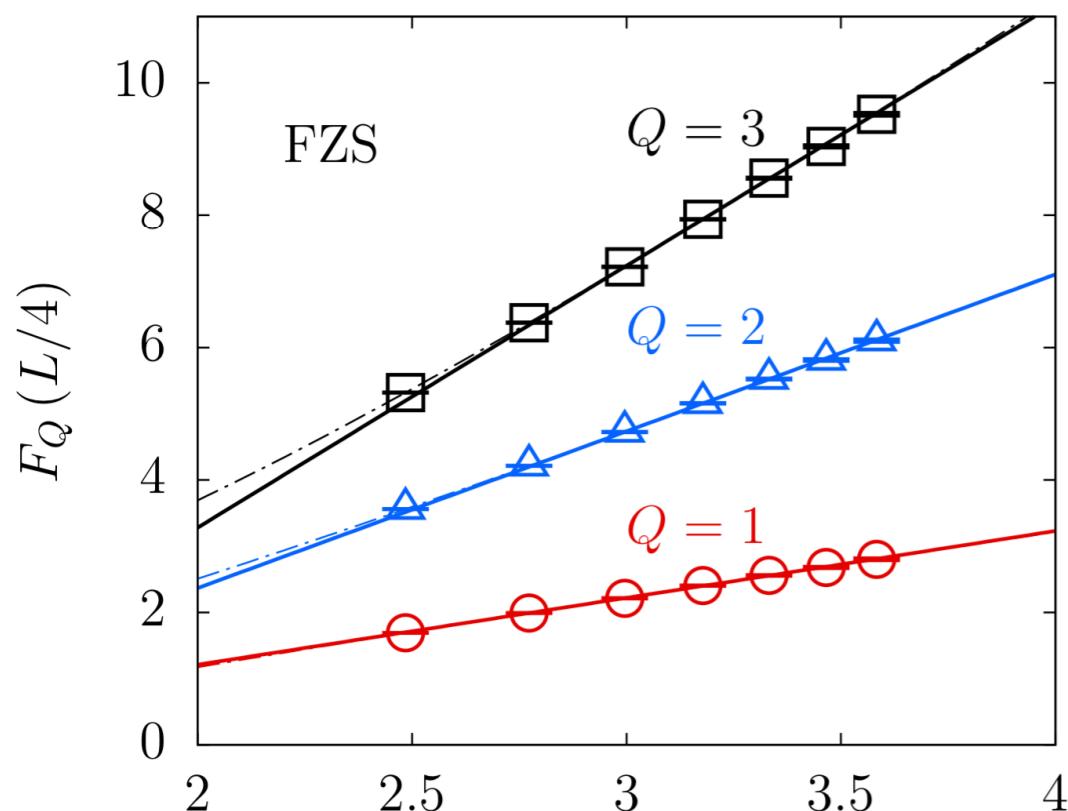
$$\Delta_1(N = 2) = 0.82(2) > 0.5 \text{ (unitarity min)}$$

C-QED

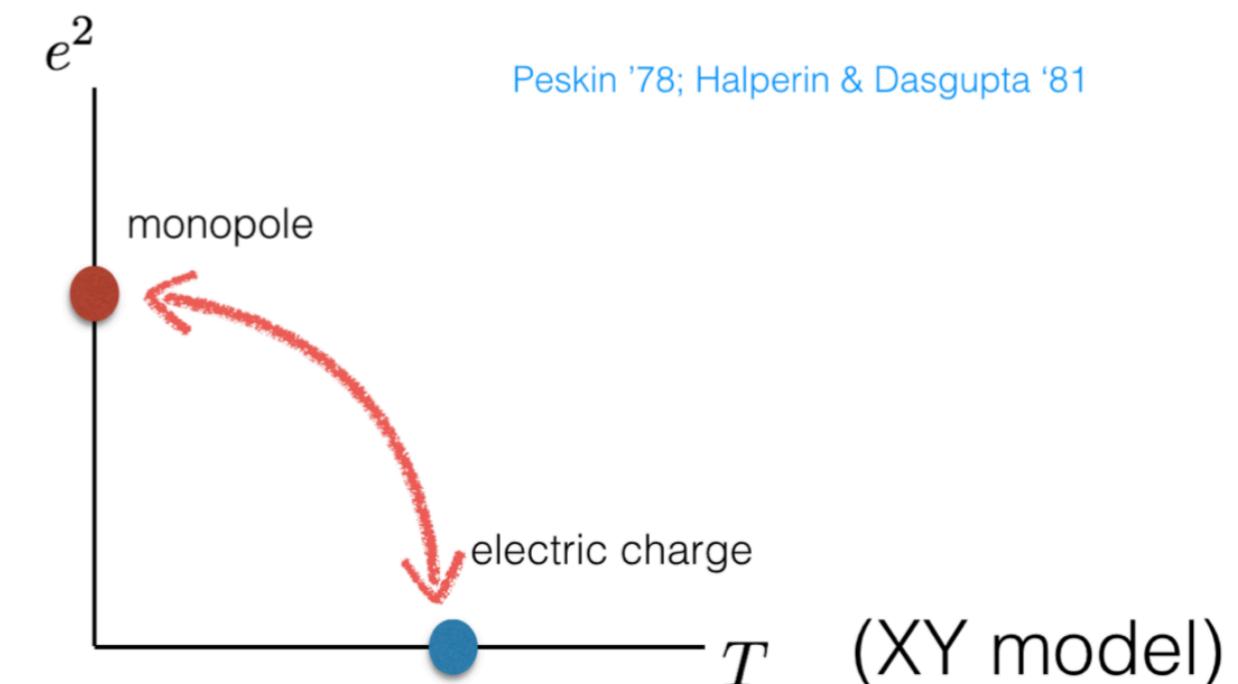
$N = 12 \pm 2$

NC-QED

Doing more with monopoles: particle-vortex dualities

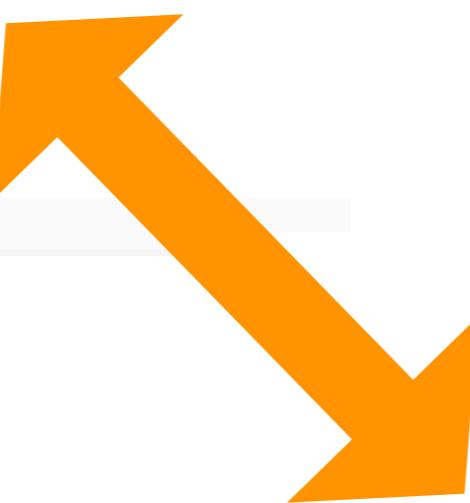


(FZS model)

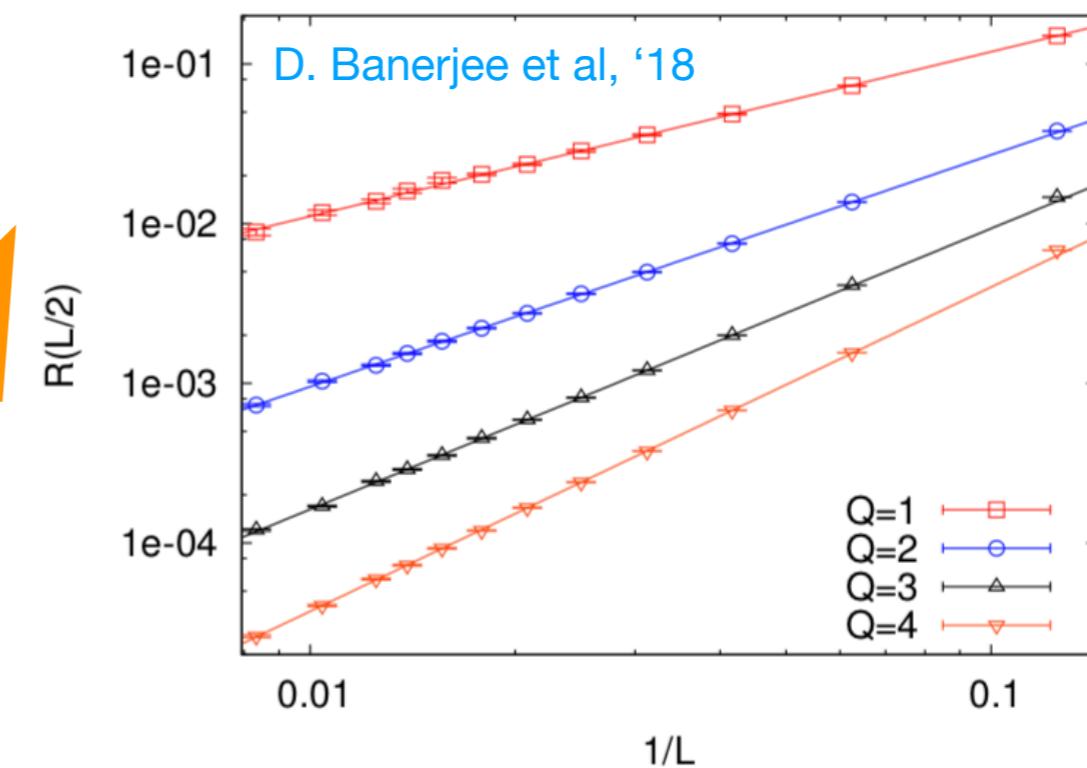


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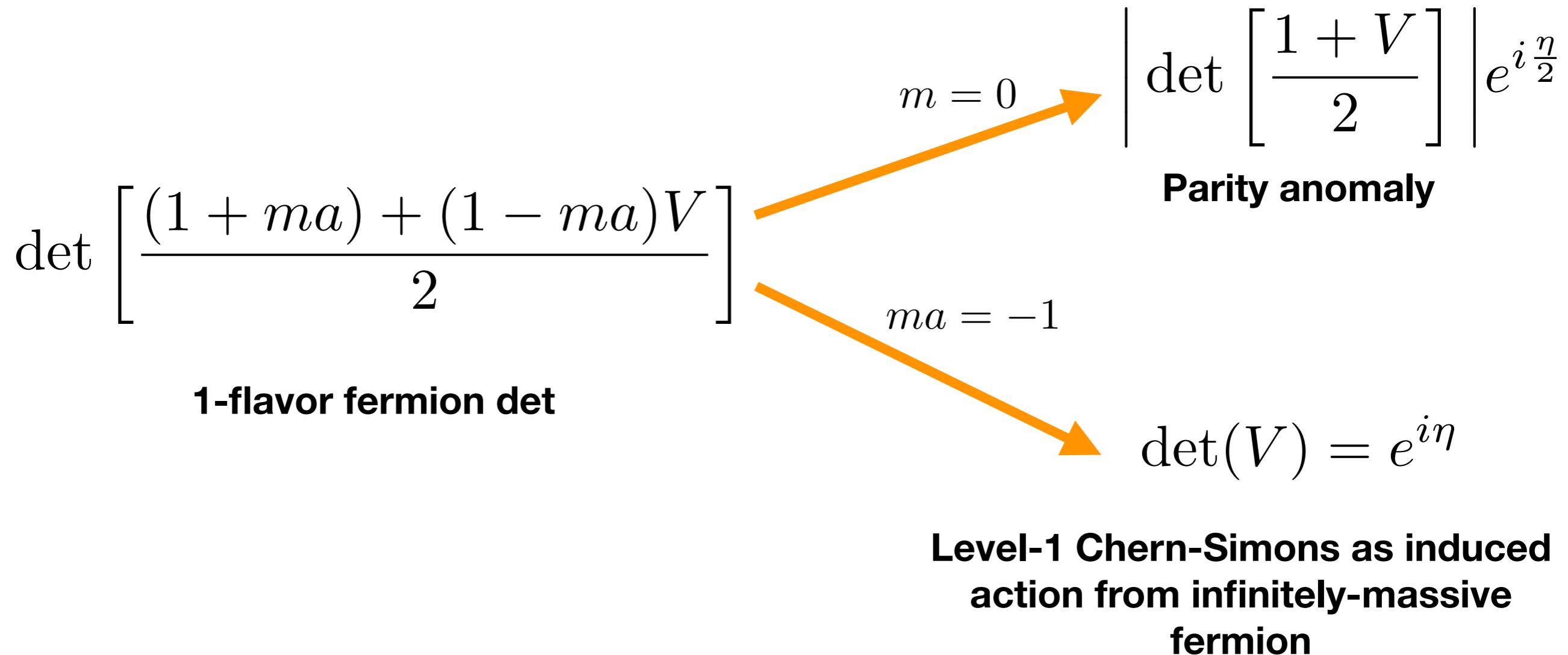
$\log(L)$



Extend similar method to
contemporary particle-vortex
dualities?



How to study gauge theories coupled to Chern-Simons term?



Lattice-continuum dictionary is very simple

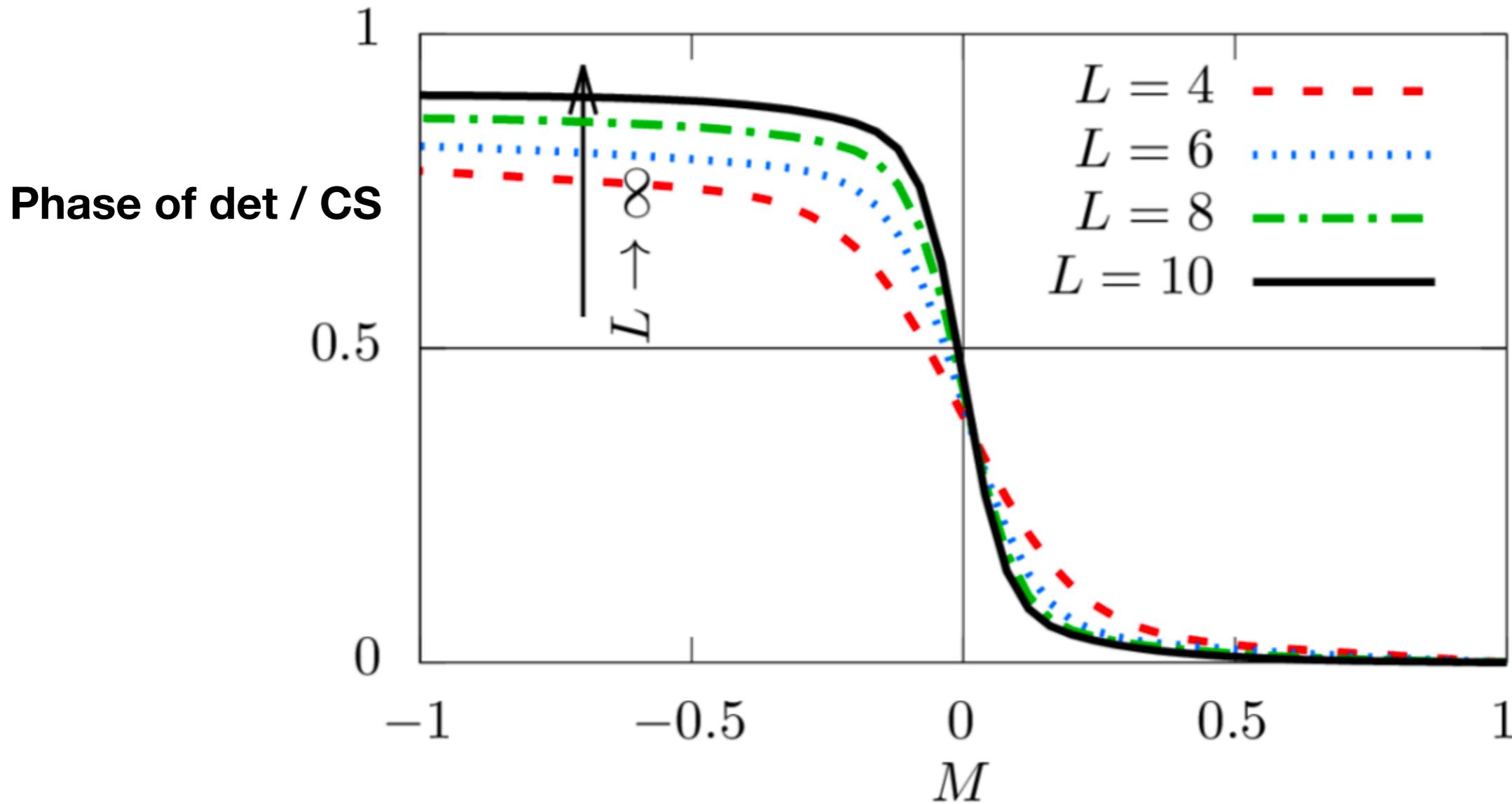
$$\frac{k}{4\pi} AdA \longrightarrow \det(V_A)^k$$

$$\frac{1}{8\pi}(AdB + BdA) \longrightarrow \det(V_{A+B}V_{A-B}^\dagger)$$

$$\text{fermion} + U(1)^{“k+1/2”} \longrightarrow \det\left(\frac{1 + V_A}{2}\right) \det(V_A)^k$$

Some examples of phases of overlap fermion determinant

$$a_1 = \frac{1}{\ell} \cos \left(\frac{2\pi x_3}{\ell} \right); a_2 = \frac{1}{\ell} \sin \left(\frac{2\pi x_3}{\ell} \right); a_3 = 0$$



Lattice CS only approaches continuum CS only in the continuum limit

Son's composite fermion theory for half-filled FQHE on lattice

also a toy-model for 4d chiral gauge theory on lattice!

$$\bar{\psi} \not{D}(2a) \psi - \frac{2i}{4\pi} \epsilon_{\mu\nu\rho} a_\mu \partial_\nu a_\rho$$

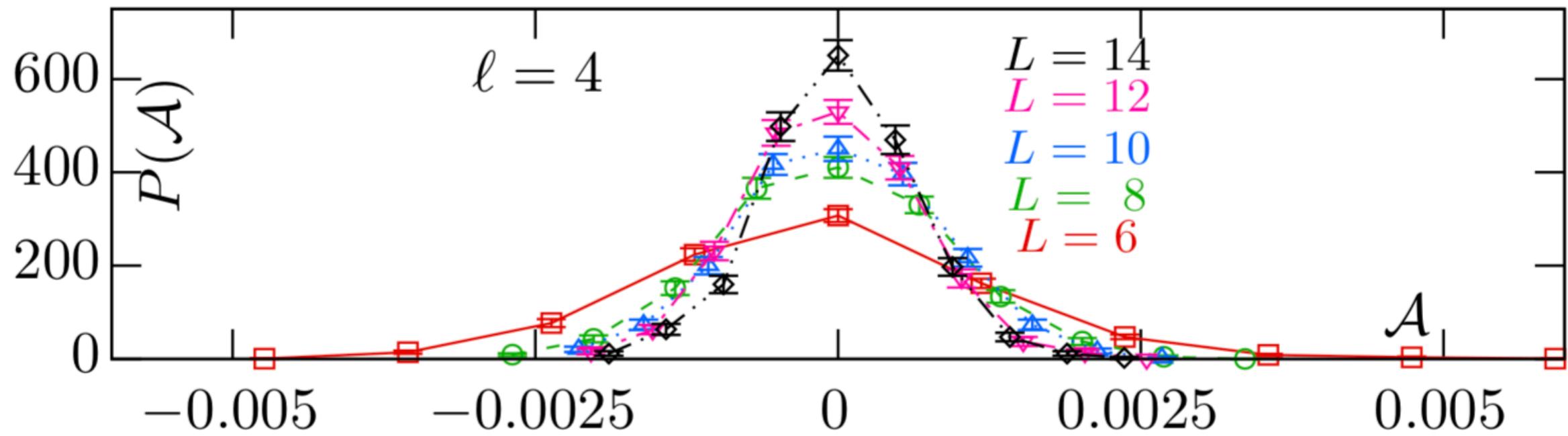
lattice

$$\det \left[\frac{1 + V(2a)}{2} \right] \det [V^\dagger(a)]^2$$

$$\pm |\det \not{D}|$$

$$\# e^{i\mathcal{A}}$$

M.C. Distribution of remnant phase



Summary

- * Can we produce a map showing the conformal and mass-gapped IR phases of fermionic gauge theories? If QED3 is conformal for all N, why so?

$N > 0$ parity-invariant QED3 appears to be conformal, and $N > 4$ for SU(2) QCD3. We can understand conformality in QED3 for any N from a simple quadratic induced gauge action.

- * How to quantify the relevance of topological defects (monopoles, vortices, etc) at IR fixed points?

FSS of monopole BF is feasible on lattice. It would be interesting to use it to test sign-problem free duality conjectures.

- * Can we study consequences of particle-vortex dualities by Monte-Carlo simulations?

Demonstrated some duality predictions in QED_3 . Approximate modeling of fermion-induced conformal gauge field dynamics could be a practical direction to study 2+1 d theories. For example: self-duality of QED3

- * How about TQFTs?
CS on lattice via fermion determinant. Sign-problem for an actual study.