Phases and critical flavor number of 3d Thirring model

Andreas Wipf

Theoretisch-Physikalisches Institut
Friedrich-Schiller-University Jena

seit 1558

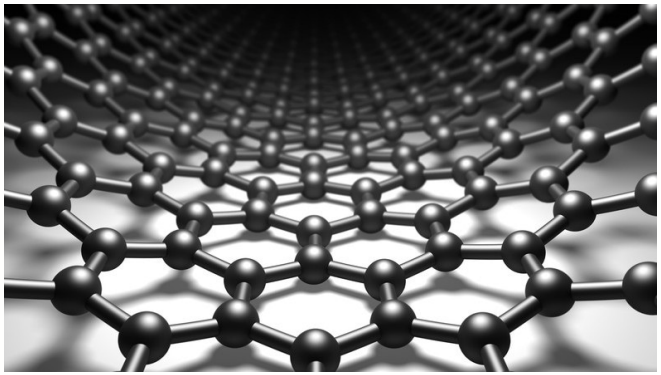
Relativistic Fermions in Flatland

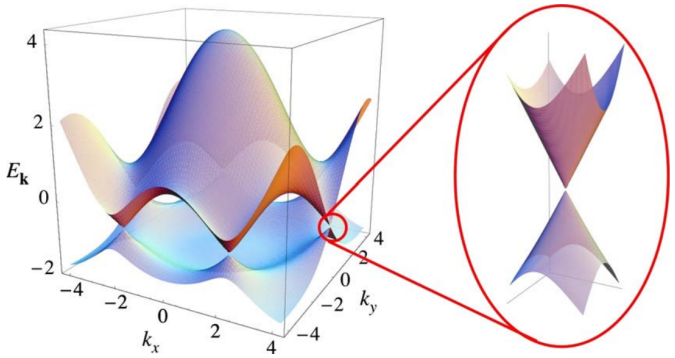
Trento, 8. July 2021

in collaboration with:
Björn Wellegehausen, Julian Lenz, Michael Mandl und Daniel Schmidt

condensed matter

- low energy description of tight binding model with NN hopping
- graphen's honeycomb lattice (GN)
- increase coupling $\Rightarrow$ PT semi-metal $\rightarrow$ (Mott) insulator
- long-range order $\Rightarrow$ AF, CDW, ...
- Dirac materials: two Dirac points $\Rightarrow$ reducible ψ with 4 components





particle physics

- non-renormalizable Fermi theory of weak interaction
- effective models for chiral phase transition in real-life QCD

- 2 spacetime dimensions:
  - integrable systems — Coleman, ...
  - massless $\mathrm{N_f}$-flavor ThM: soluble CFT — Thirring, Klaiber
    $\Rightarrow$ general massless 1-flavor system: soluble CFT
  - same in curved space and with finite density — Sachs+AW, ...
- 3 spacetime dimensions:
  - not renormalizable in PT
  - renormalizable in large-$\mathrm{N_f}$ expansion — Gawedzki, Kupiainen; Park, Rosenstein, Warr
  - interacting UV fixed points $\rightarrow$ asymptotically safe — de Veiga; da Calen; Gies, Janssen
  - parity breaking at low T?
- lattice approach:
  - critical behavior, mass spectrum, finite density, inhomogeneous condensates ...
  - sign problem for odd $\mathrm{N_f}$ — partial solution: Schmidt, Wellegehausen, Lenz, AW
  - chiral fermions on lattice? — S. Hands et al. and Jena group

- Euclidean 4×4 gamma-matrices (reducible)

$$\{\Gamma_\mu, \Gamma_\nu\} = 2\delta_{\mu\nu}\mathbb{1}_4, \quad \mu = 1,2,3, \qquad \mathcal{L}_0 = \bar{\Psi}\mathrm{i}\partial\!\!\!/\Psi, \quad \partial\!\!\!/ = \Gamma^\mu\partial_\mu$$

$\Gamma_4, \Gamma_5 \Rightarrow$ two notions of „chirality“, $\Gamma_{45} = \mathrm{i}\Gamma_4\Gamma_5$

- $\mathrm{N_f}$-flavors $\Psi = (\psi_1, \ldots, \psi_{\mathrm{N_f}})^T$, non-interacting system:

global $\mathrm{U}(2\mathrm{N_f})$: generated by $T_\mathrm{f} \otimes \Gamma$, $\quad \Gamma \in \{\mathbb{1}, \Gamma_4, \Gamma_5, \Gamma_{45}\}$

$\mathbb{Z}_2$ parity : $\quad \Psi(x) \to \mathrm{i}\,\Gamma_1\,\Gamma_4\,\Psi(x'), \quad x' = (x_0, -x_1, x_2)$

- $\mathbb{Z}_2 \times \mathrm{U}(2\mathrm{N_f})$ invariant Thirring interaction

$$\mathcal{L} = \mathcal{L}_0 - \frac{g^2}{2\mathrm{N_f}}(\bar{\Psi}\Gamma^\mu\Psi)^2$$

- $\Sigma = \bar{\Psi}\Psi$ scalar: $\quad U(2\mathrm{N_f}) \otimes \mathbb{Z}_2 \xrightarrow{\Sigma} U(\mathrm{N_f}) \otimes U(\mathrm{N_f}) \otimes \mathbb{Z}_2$
- $\Sigma_{45} = \bar{\Psi}\Gamma_{45}\Psi$ pseudo-scalar: $U(2\mathrm{N_f}) \otimes \mathbb{Z}_2 \xrightarrow{\Sigma_{45}} U(2\mathrm{N_f})$

- irreducible ψ with 2 components (two inequivalent reps)

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}\mathbb{1}_2, \quad \mu = 1,2,3, \qquad \mathcal{L}_0 = \bar{\Psi}\mathrm{i}\partial\!\!\!/\Psi, \quad \partial\!\!\!/ = \gamma^\mu\partial_\mu$$

- no notion of chirality ($d = 3$)
- $\mathrm{N_f^{ir}}$ flavours of Dirac spinors: $\Psi = (\psi_1, \ldots, \psi_{\mathrm{N_f^{ir}}})^T$

$$\mathcal{L} = \mathcal{L}_0 - \frac{g^2}{2\mathrm{N_f^{ir}}}(\bar{\Psi}\gamma^\mu\Psi)^2$$

- global $\mathrm{U}(\mathrm{N_f^{ir}})$ symmetry: $\Psi \to U\Psi$
- $\mathbb{Z}_2$ parity:

$$\psi(x) \to \mathrm{i}\gamma_3\psi(\tilde{x}), \quad \bar{\psi}(x) \to \mathrm{i}\bar{\psi}(\tilde{x})\gamma_3, \qquad \tilde{x} = (x_1, x_2, -x_3)$$

- $\Sigma^{\mathrm{ir}} = \bar{\Psi}\Psi$ pseudo-scalar: $U(\mathrm{N_f^{ir}}) \otimes \mathbb{Z}_2 \xrightarrow{\Sigma^{\mathrm{ir}}} U(\mathrm{N_f^{ir}})$

- Γ^μ necessarily contains both irred. reps., e.g. $\Gamma_\mu = \sigma_3 \otimes \gamma_\mu$
- two irred. spinors $\equiv$ one reducible spinor

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \Longrightarrow \bar\psi \Gamma^\mu \psi = \bar\psi_1 \gamma^\mu \psi_1 - \bar\psi_2 \gamma^\mu \psi_2$$

- transform to standard kinetic term $\psi \to \Gamma_{45}\psi, \quad \bar\psi \to \bar\psi$
- transformation of order parameters: $\mathrm{N_f} = 2\mathrm{N_f^{ir}}$ flavors

$$\Sigma \to \sum_1^{\mathrm{N_f^{ir}}} (-1)^{a+1} \bar\psi_a \psi_a \quad \text{and} \quad \Sigma_{45} \to \sum_1^{\mathrm{N_f^{ir}}} \bar\psi_a \psi_a$$

- Hubbard-Stratonovich $\Rightarrow$

$$\mathcal{L}_{\mathrm{Th}} = \bar\Psi \mathrm{i} \not{D} \Psi + \lambda v_\mu v^\mu, \quad \not{D} = \not\partial - \mathrm{i}\not{v}, \quad \lambda = \frac{\mathrm{N_f^{ir}}}{2g^2}$$

- fermionic integration

$$Z_{\mathrm{Th}} = \int \mathcal{D}v_\mu \, \mathrm{e}^{-\mathrm{N_f^{ir}} S_{\mathrm{eff}}}, \quad S_{\mathrm{eff}} = \frac{1}{2g^2} \int \mathrm{d}^3x \, v_\mu v^\mu - \log\det(\mathrm{i}\not{D})$$

- no sign problem for reducible Thirring (and GN) models

$$\not{D}_{\rm red} = \sigma_3 \otimes \not{D}_{\rm ir}, \quad {\rm i}\not{D}_{\rm ir} \text{ hermitean}$$

- $m = 0$: reducible $\mathrm{N_f}$-model $\equiv$ irreducible $\mathrm{N_f^{ir}} = 2\mathrm{N_f}-$model
- large $\mathrm{N_f^{ir}}$ limit of massive model

$$Z = \int \mathcal{D}v_\mu \, {\rm e}^{-\mathrm{N_f^{ir}} S_{\rm eff}[v_\mu]} \xrightarrow{\mathrm{N_f^{ir}} \to \infty} {\rm e}^{-\mathrm{N_f^{ir}} \min_v S_{\rm eff}[v_\mu]}$$

- homogeneity $\Rightarrow$ v_μ constant $\Rightarrow$ $S_{\rm eff}[v_\mu] = \text{volume} \times U_{\rm eff}(v_\mu)$

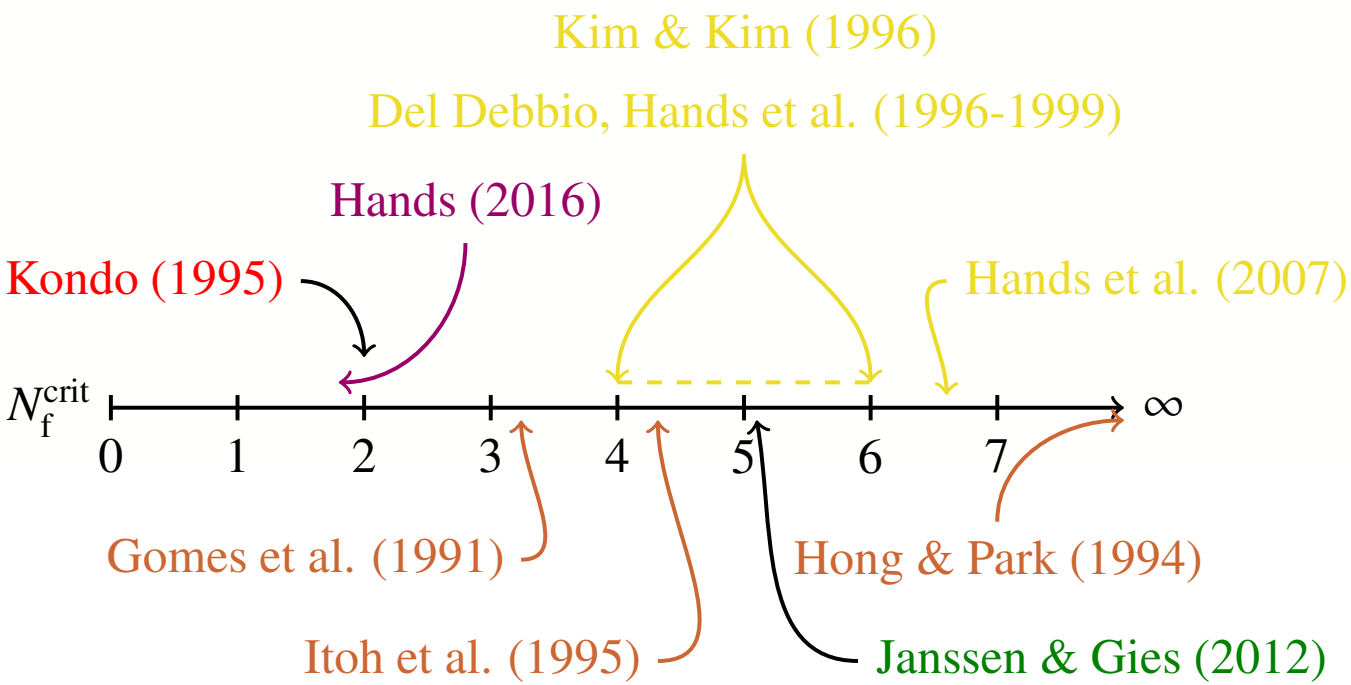
$$U_{\rm eff} = \frac{1}{2g_{\rm ren}^2} v_\mu v^\mu + U_{\rm free}(T, m^2), \quad g^2 = \frac{4\pi g_{\rm ren}^2}{4\pi + \Lambda g_{\rm ren}^2}$$

- condensate

$$N_f^{ir} \to \infty : \quad \langle \bar{\Psi}\Psi \rangle = \frac{1}{V}\frac{\partial}{\partial m} \log Z \overset{m\to 0}{\longrightarrow} 0 \quad v_\mu\text{-independent}$$

$$N_f^{ir} = 1 : \quad \langle \bar{\Psi}\Psi \rangle \neq 0 \quad \text{since equivalent to Gross-Neveu}$$

- critical flavor number N_f^{crit}: phase transitions for $N_f \leq N_f^{crit}$
  no phase transition for $N_f > N_f^{crit}$



- SD equations
- $1/N_f$-expansion
- FRG
- lattice, staggered
- lattice, domain wall
- lattice, SLAC

- irreducible and reducible chiral SLAC fermions

  N_f^{ir} even $\Rightarrow$ no breaking — B. Wellegehausen, D. Schmidt, AW, PRD 96 (2017)

  simulations for $0.5 \leq N_f \leq 1$ reducible flavors

  $\Rightarrow N_f^{crit} = 0.80(4)$ — J. Lenz, B. Wellegehausen, AW, PRD 100 (2019)

  N_f^{ir} odd $\Rightarrow$ breaking for $N_f^{ir} \leq N_f^{ir,crit} = 9$ — B. Wellegehausen, D. Schmidt, AW, PRD 96 (2017)
- reducible „domain wall“ fermions (DWF)

  condensate, spectrum of Goldstone-bosons

  surface formulation: $N_f^{crit} \lessapprox 1$ — S. Hands, JHEP 11 (2016), PRD 99 (2019)

  bulk formulation: $1 < N_f^{crit} < 2$ — S. Hands, M. Mesiti, J. Worthy, PRD 102 (2020)
- FRG

  momentum-dependent couplings $\Rightarrow N_f^{crit}$ decreases

  new results could be compatible with $N_f^{crit} < 1$ — L. Dabelow, H. Gies, B. Knorr, PRD 99, 2019

- what is it — Drell, Weinstein, Yankielowicz
  - continuum: Fourier transform $\partial_\mu f(x) \to \mathrm{i} p_\mu \tilde{f}(p)$
  - on lattice: define derivative in p-space by $\mathrm{i} p_\mu \tilde{f}(p)$, p_μ discrete
  - transform back to x-space (FFT) $\to (\partial_\mu^{\mathrm{slac}} f)(x)$
  - periodic BC: odd number of sites; antiperiodic BC: even number of sites
  - code optimization since 2008 — Kästner, Bergner, Wozar, Körner, Wellegehausen, Schmidt, Lenz, Mandl
- successfully applied to
  - scalar field theories, non-linear sigma-models — Körner, Wozar, AW
  - supersymmetric Yukawa models — Wozar, Wellegehausen, Bergner, Kästner, AW
  - critical exponents of GN model (poster T. Lang) — Lang, Läuchli; Schmidt
- advantages
  - exact $\mathrm{U}(\mathrm{N}_\mathrm{f}^\mathrm{ir}) \times \mathbb{Z}_2$ symmetry on hypercubic lattice
  - lattice derivative ∂_μ real, antisymmetric $\Rightarrow$ $\mathrm{i}\partial\!\!\!/$ hermitean
  - auxiliary v_μ non-compact site variable (not gauge field)
  - no doublers $\Rightarrow$ odd $\mathrm{N}_\mathrm{f}^\mathrm{ir}$ no rooting
  - cheap compared to Ginsparg-Wilson fermions
- disadvantage
  - non-local fermions $\to$ useless in gauge theories — Smit
  - no such problems without gauge fields! — Bergner

- $\mathrm{N_f^{ir}} \geq 2$: no sign problem
- $\mathrm{N_f^{ir}} = 1$: dual formulation $\rightarrow$ fermion bag approach S. Chandrasekharan, SLAC: B. Wellegehausen
- $N \times (N-1)^2$ lattices, $N = 8, 12, 16, 24$
- rational HMC with

$$\left(\det(D^\dagger D)^{\mathrm{N_f^{ir}}/2N_{\mathrm{PF}}} \right)^{N_{\mathrm{PF}}}, \quad N_{\mathrm{PF}} \approx 2\mathrm{N_f^{ir}}$$

- stochastic estimators for fermionic propagator $N_{\mathrm{est}} = 200 \times \mathrm{N_f^{ir}}$ ≈ 5000 configurations
- finite size effects under control
- parity-even extensions to non-integer $\mathrm{N_f} \in [0.5, 1.1]$

- Fierz-rearrangement

$$(\bar{\Psi}\gamma^\mu\Psi)(\bar{\Psi}\gamma_\mu\Psi) = -2\sum(\bar{\psi}^a\psi^b)(\bar{\psi}^b\psi^a) - (\bar{\Psi}\Psi)^2$$

- Hubbard-Statonovich $\Rightarrow$ equivalent tensor-scalar model

$$\mathcal{L} = \bar{\Psi}(\mathrm{i}\partial\!\!\!/ + \mathrm{i}T + \mathrm{i}\phi)\Psi + \frac{\mathrm{N_f^{ir}}}{4g^2}\,\mathrm{tr}\,T^2 + \frac{\mathrm{N_f^{ir}}}{2g^2}\phi^2, \quad T^\dagger = T,\ \phi \in \mathbb{R}$$

- Dyson-Schwinger-equations $\Rightarrow \langle T_{ab}\rangle \propto \langle\bar{\psi}_a\psi_b\rangle$
- $T \to UTU^\dagger \Rightarrow \langle T\rangle$ order parameter for SSB
- hard sign problem, but useful for probing various condensates ☺
- tool: dual formulation as sum over spin-configuration $k_{xi}^{ab} \in \{0,1\}$
- lattice filling factor

$$k = \frac{1}{2V\mathrm{N_f^{ir}}}\sum_{i=1}^{2}\sum_{a,b=1}^{\mathrm{N_f^{ir}}}\sum_{x=1}^{V} k_{xi}^{ab}$$

- effective potential for Cartan part T^c of T

$$V(T^c) = -\frac{1}{V}\log\sum_{n=0}^{2\mathrm{N}_\mathrm{f}^\mathrm{ir}}\sum_{i=1}^{\mathrm{N}_\mathrm{f}^\mathrm{ir}} a_{n,i}\,(t_i)^n\,, \qquad T^c = t_i H^i$$

- dual formulation $\Rightarrow a_{n,i} = \langle \mathcal{O}_{n,i}\rangle$, $\mathcal{O}_{n,i}$ powers of fermion bilinears
- $\langle \mathcal{O}_{n,i}\rangle$ calculated with simulations in vector field formulation
- minima of V at

$$T^c_{\mathrm{min}} = \frac{2x}{\mathrm{N}_\mathrm{f}^\mathrm{ir}}\,\mathrm{diag}\big(\underbrace{1,\ldots,1}_{n_+},\underbrace{-1,\ldots,-1}_{n_-}\big), \qquad n_+ \geq n_-$$

- permutations of entries $\rightarrow$ equivalent minima (Weyl-orbit)
- physically distinct minima characterized by x and

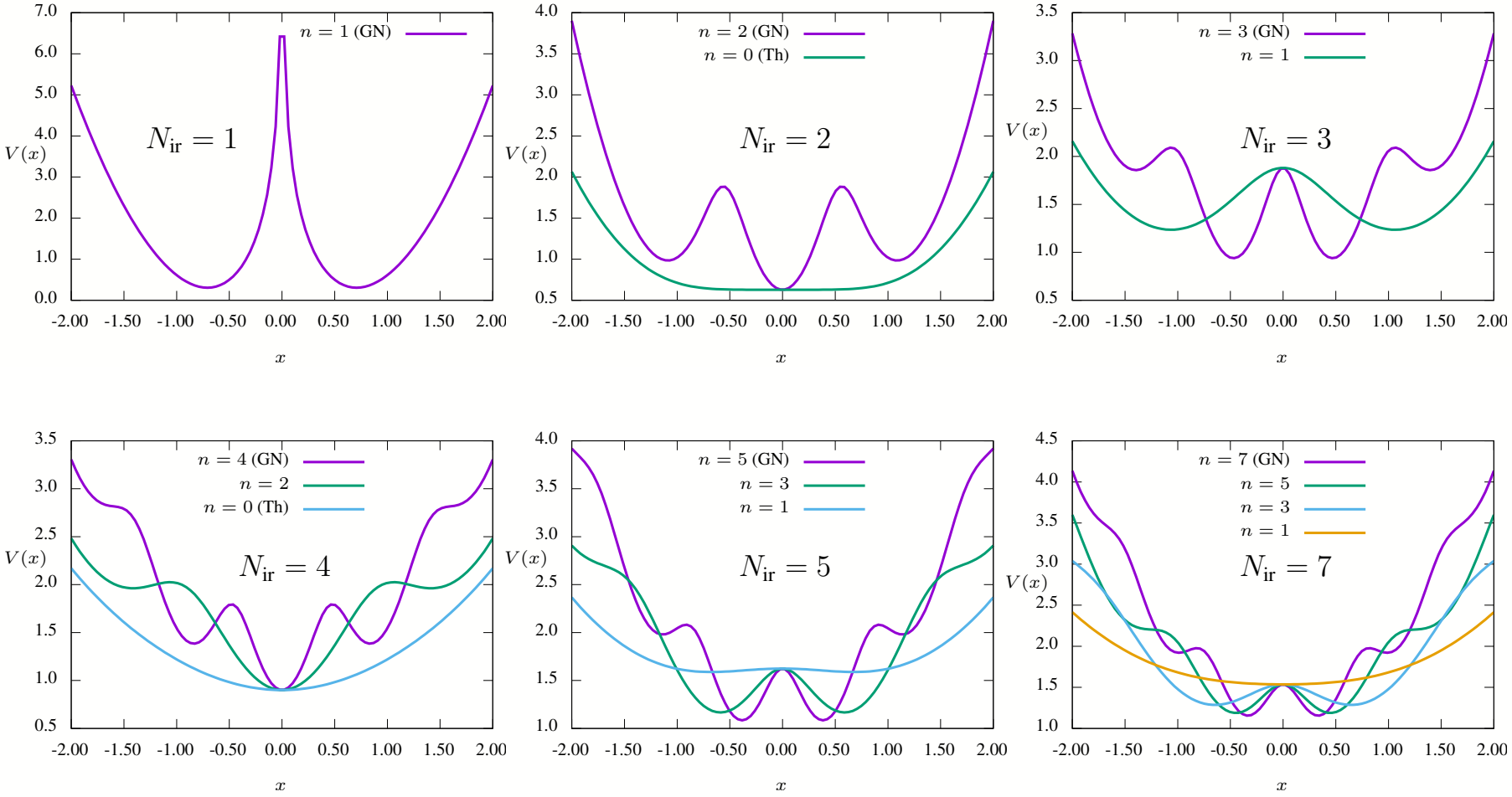
$$n = n_+ + n_- = \begin{cases} 0,2,4,\ldots,\mathrm{N}_\mathrm{f}^\mathrm{ir} & \mathrm{N}_\mathrm{f}^\mathrm{ir} \quad \text{even} \\ 1,3,5,\ldots,\mathrm{N}_\mathrm{f}^\mathrm{ir} & \mathrm{N}_\mathrm{f}^\mathrm{ir} \quad \text{odd} \end{cases}$$

- breaking pattern:

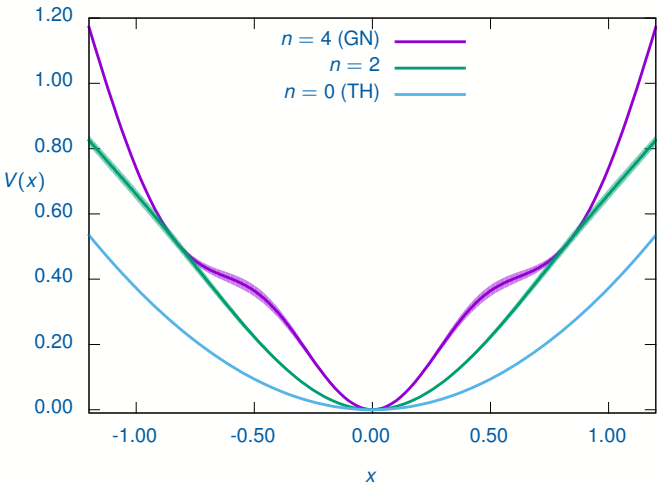
$$U(\mathrm{N}_\mathrm{f}^\mathrm{ir}) \otimes \mathbb{Z}_2 \to U(n_+) \otimes U(n_-)$$

$\mathrm{N}_\mathrm{f}^\mathrm{ir} = 7$	$n =$	1	3	5	7	$\mathrm{N}_\mathrm{f}^\mathrm{ir} = 8$	$n =$	0	2	4	6	8
	$n_+ =$	4	5	6	7		$n_+ =$	4	5	6	7	8
	$n_- =$	3	2	1	0		$n_- =$	4	3	2	1	0

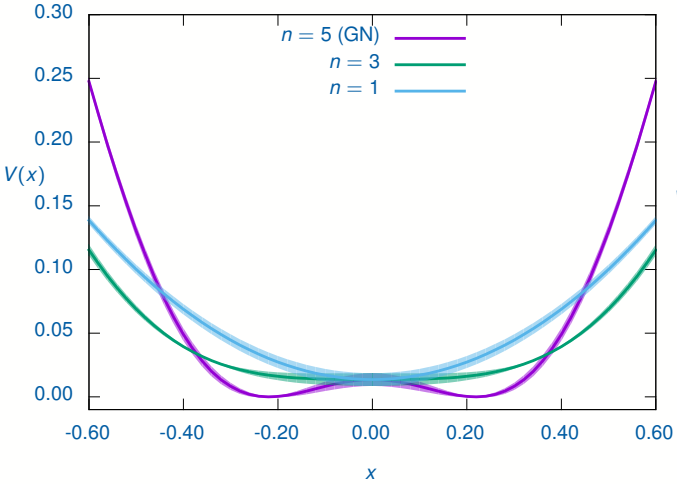
- $\mathrm{N}_\mathrm{f}^\mathrm{ir} = 2\mathrm{N}_\mathrm{f}$ even, $n_+ = n_- \Rightarrow$ condensation of staggered $\sum(-1)^a \bar{\psi}_a \psi_a$
  $\Rightarrow$ Thirring breaking $U(\mathrm{N}_\mathrm{f}^\mathrm{ir}) \to U(\mathrm{N}_\mathrm{f}) \times U(\mathrm{N}_\mathrm{f})$
- $n = \mathrm{N}_\mathrm{f}^\mathrm{ir} \Rightarrow$ condensation of invariant $\sum \bar{\psi}_a \psi_a$
  $\Rightarrow$ GN-breaking: $U(\mathrm{N}_\mathrm{f}^\mathrm{ir}) \times \mathbb{Z}_2 \to U(\mathrm{N}_\mathrm{f}^\mathrm{ir})$
- strong coupling limit: $\lambda \to 0 \equiv$ complete filling in dual formulation

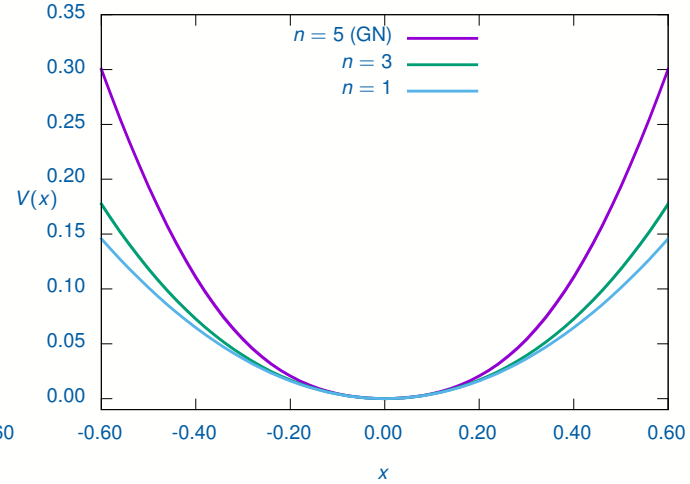


- N_f^{ir} odd: chiral symmetry always broken in strong coupling
- N_f^{ir} even: chiral symmetry always unbroken in strong coupling



- $\mathrm{N}_\mathrm{f}^\mathrm{ir} = 4$
  $\lambda = 0.118$
  $L = 16$



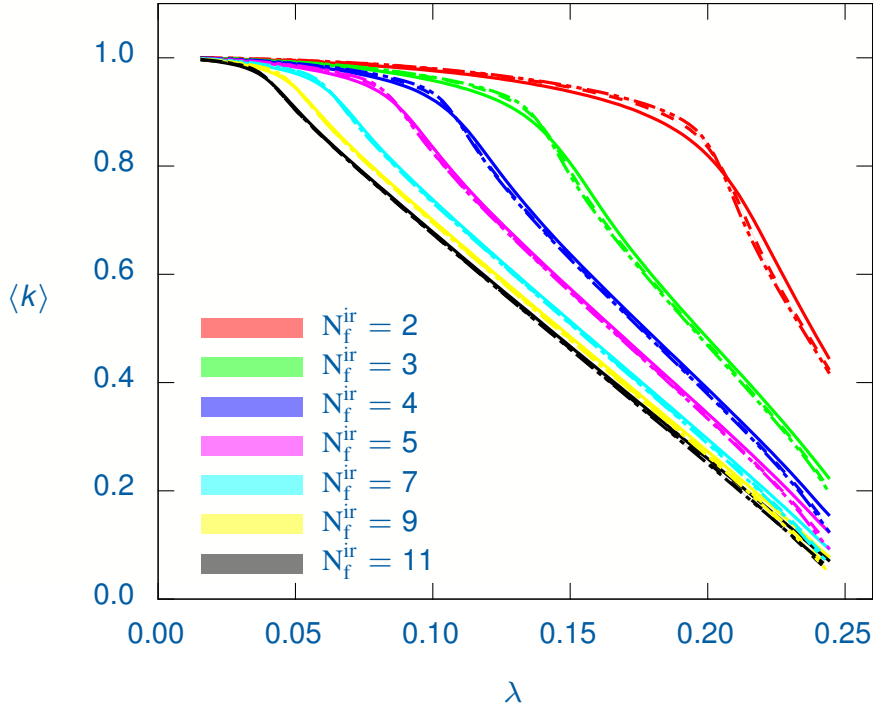


- $\mathrm{N}_\mathrm{f}^\mathrm{ir} = 5$
  $L = 16$
  left: $\lambda = 0.102$
  right: $\lambda = 0.118$

- no SSB for $\mathrm{N}_\mathrm{f}^\mathrm{ir} = 4$ or $\mathrm{N}_\mathrm{f} = 2$
- SSB for $\mathrm{N}_\mathrm{f}^\mathrm{ir} = 5$ in Gross-Neveu channel (SSB of parity)

- dual formulation with $k_{xi}^{ab} \in \{0,1\}$, mean filling density $\langle k \rangle \in [0,1]$

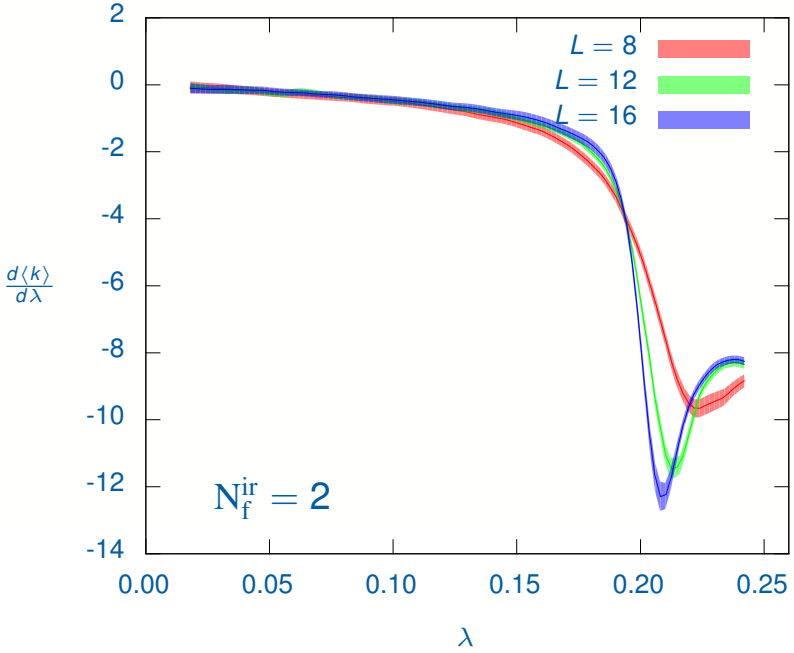
$$\langle k \rangle = -\frac{1}{2\mathrm{N}_\mathrm{f}^\mathrm{ir}} \frac{\lambda}{V} \frac{\dot{Z}(\lambda)}{Z(\lambda)} + c = \frac{1}{4\mathrm{N}_\mathrm{f}\lambda} \langle j^\mu j_\mu \rangle + c$$

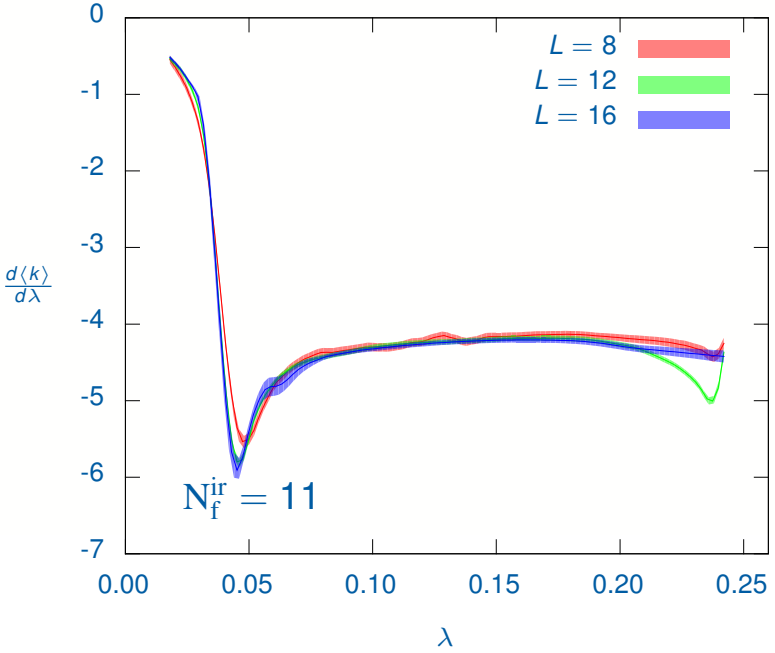


- artifact phase $\lambda < \lambda^*$
  - $\langle k \rangle$ near 1
  - $\langle k \rangle$ weakly N-dependent
- physical phase $\lambda > \lambda^*$
  - $\langle k \rangle \to 0$ for $\lambda \gg \lambda^*$
  - $\langle k \rangle$ for different V intersect at λ^*
- increasing $\mathrm{N}_\mathrm{f}^\mathrm{ir} \to$ decreasing λ^*
- small finite size effects

- peak of susceptibility $\partial_\lambda \langle k \rangle$ at λ^*
- λ^* decreases when N increases

N	N_f^{ir} =2	3	4	5	7	9	11
8	λ^* =0.223(6)	0.158(4)	0.122(4)	0.098(2)	0.073(2)	0.058(2)	0.048(2)
12	λ^* =0.214(4)	0.149(4)	0.114(3)	0.094(3)	0.068(2)	0.054(2)	0.046(2)
16	λ^* =0.208(4)	0.146(4)	0.112(3)	0.091(2)	0.068(2)	0.054(2)	0.046(2)

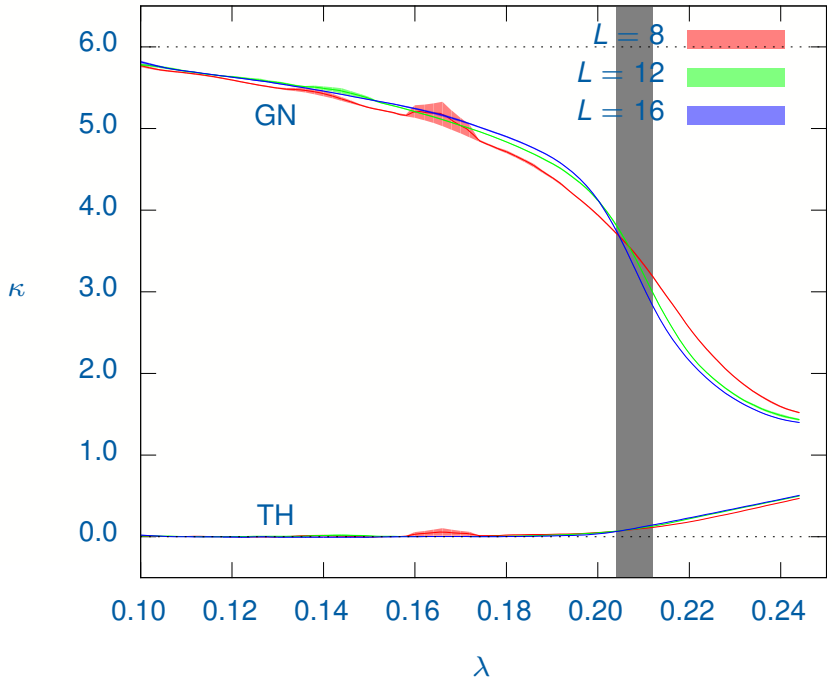


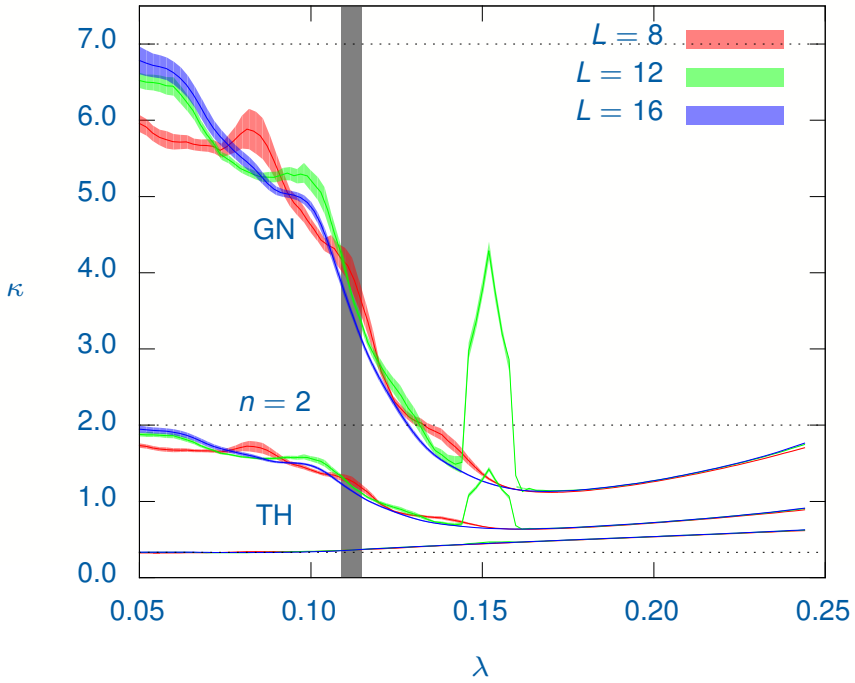


- SSB if

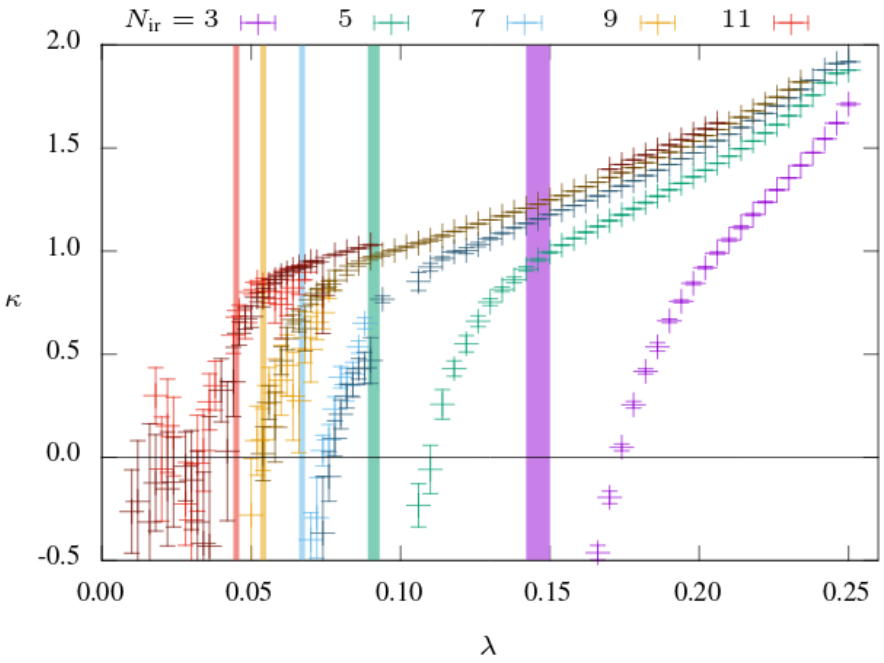
$$\kappa(n) = V''_{\mathrm{eff}}(x,n)\Big|_{x=0} < 0 \quad \text{for one} \quad n$$

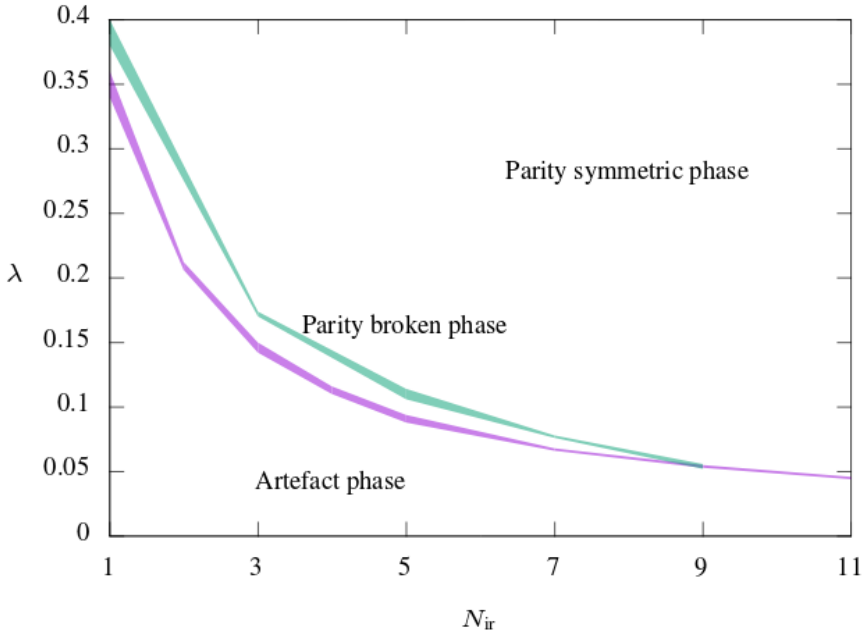
- even $\mathrm{N_f^{ir}}$: all $\kappa > 0$ in physical domain $\rightarrow$ no SSB in reducible models
- plots: curvatures for $\mathrm{N_f^{ir}} = 2$ and $\mathrm{N_f^{ir}} = 4$ (dotted: strong coupling)
- dark bar: estimate for λ^*





- odd N_f^{ir} : SSB for small N_f^{ir}





odd $N_f^{ir} \Longrightarrow N_{f,irr}^{crit} \approx 9$ for odd N_f^{ir} ($N_f^{crit} \approx 4.5$)

Phys. Rev. D96 (2017) 094504

- staggered fermions „fail“: wrong universality class
- domain wall fermions favor $1 < \mathrm{N_f^{crit}} < 2$
- chiral SLAC fermions favor $\mathrm{N_f^{crit}} < 1$

  $\Rightarrow$ zoom to neighborhood of $\mathrm{N_f} = 1$
  continue parity invariant systems
- zooming in with
  - chiral condensate $\Sigma_{L,\mathrm{N_f}}$
  - mean spectral density $\bar{\varrho}_{L,\mathrm{N_f}}(E)$
  - masses of light mesons $\mathrm{N_f} \in \{0.8, 1.0\}$
    symmetry of meson spectrum
  - susceptibilities
- problem: condensate vanishes on finite lattice for any v_μ
- simulations with „small“ fermion masses $m = m_0/L,\ m_0 \ll 1$

- fermion operator $D_{\rm red} = \sigma_0 \otimes \not{D}_{\rm ir} + m\sigma_3 \otimes \sigma_0 \Rightarrow$

$$S_{\rm eff} = \lambda \int {\rm d}^3x\, v^\mu v_\mu - {\rm N_f}\, \ln\det\left(m^2 - \not{D}^2\right)$$

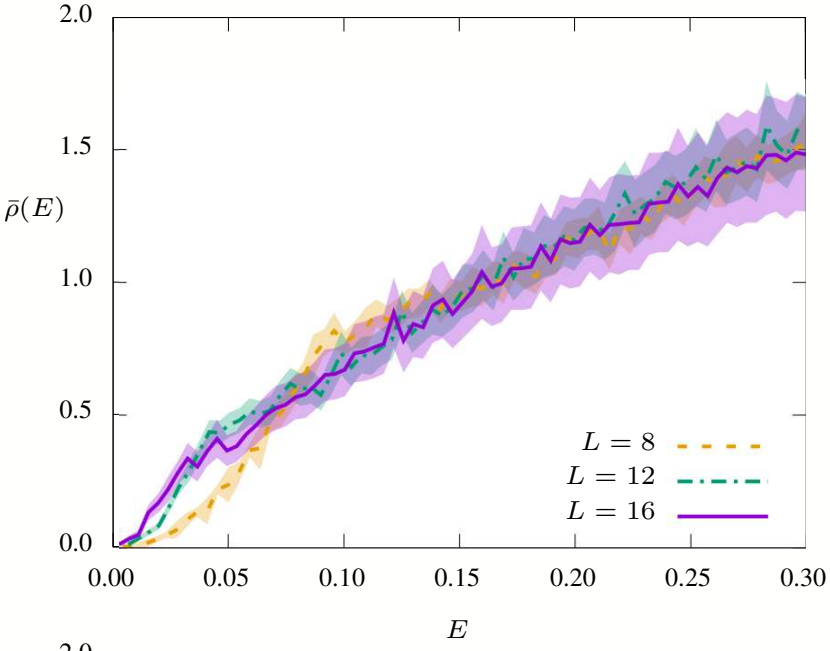
- order parameter for $\mathrm{U}(2\mathrm{N_f}) \to \mathrm{U}(\mathrm{N_f}) \times \mathrm{U}(\mathrm{N_f})$ breaking

$$\Sigma = \frac{\rm i}{2{\rm N_f}} \sum_a \langle \bar\psi_a \Gamma_{45} \psi_a \rangle = \frac{1}{V}\frac{1}{Z} \int \mathcal{D}v^\mu\, {\rm tr}\left(\frac{m}{m^2 - \not{D}^2}\right) e^{-S_{\rm eff}(v)}$$
$$= \frac{2m}{V} \int_0^\infty \frac{{\rm d}E}{E^2 + m^2}\, \bar\rho(E)$$

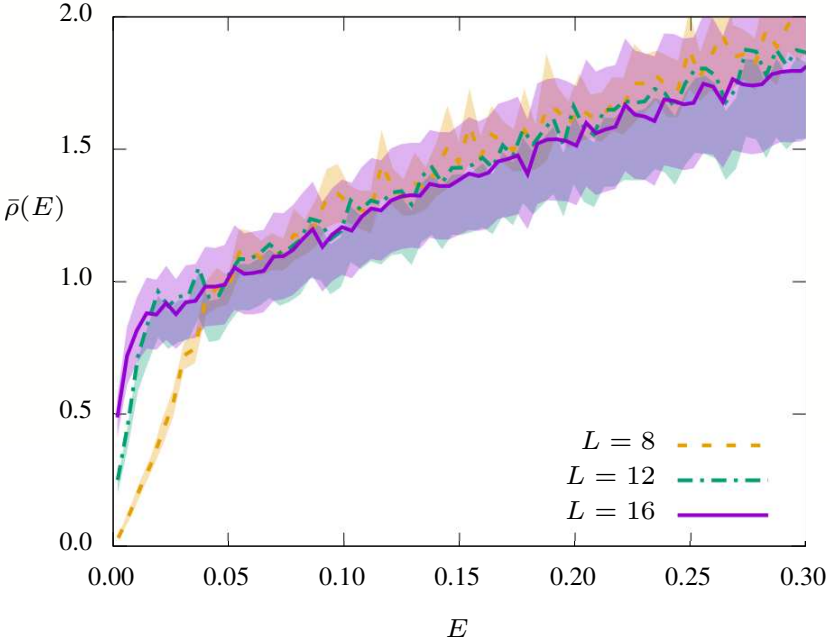
Banks-Casher relation

- average spectral density $\bar\rho(E) \overset{cc}{=} \bar\rho(-E)$:

$${\rm tr}\, f({\rm i}\not{D}) = \int_{-\infty}^{\infty} {\rm d}E\, f(E) \rho_v(E), \quad \bar\rho(E) = \frac{1}{Z} \int \mathcal{D}v^\mu\, e^{-S_{\rm eff}(v)} \rho_v(E)$$



symmetric phase
$\mathrm{N_f} = 1.0,\ L = 8, 12, 16$
density stays small near $E = 0$

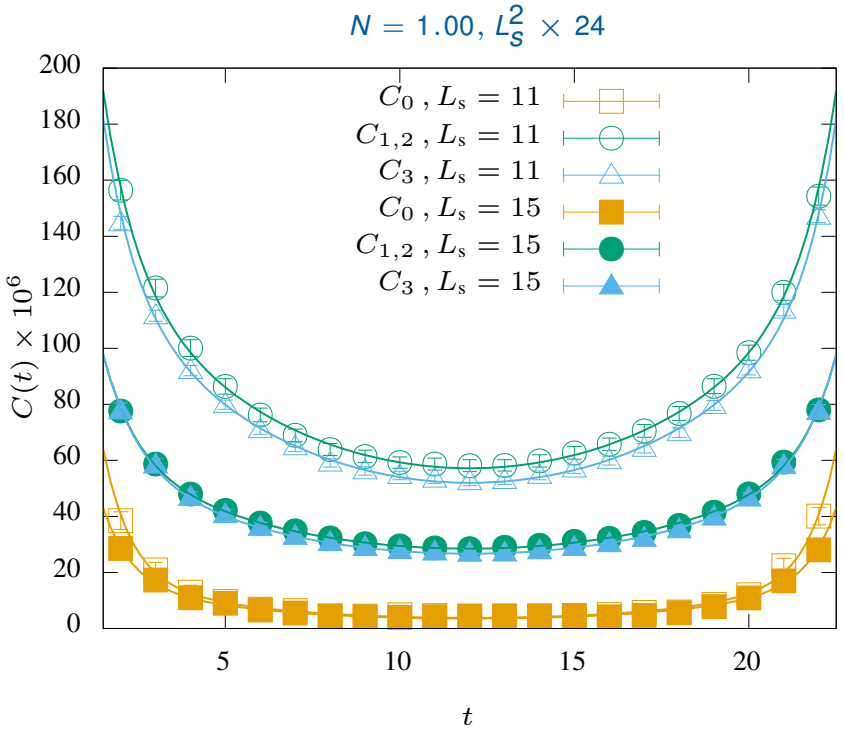


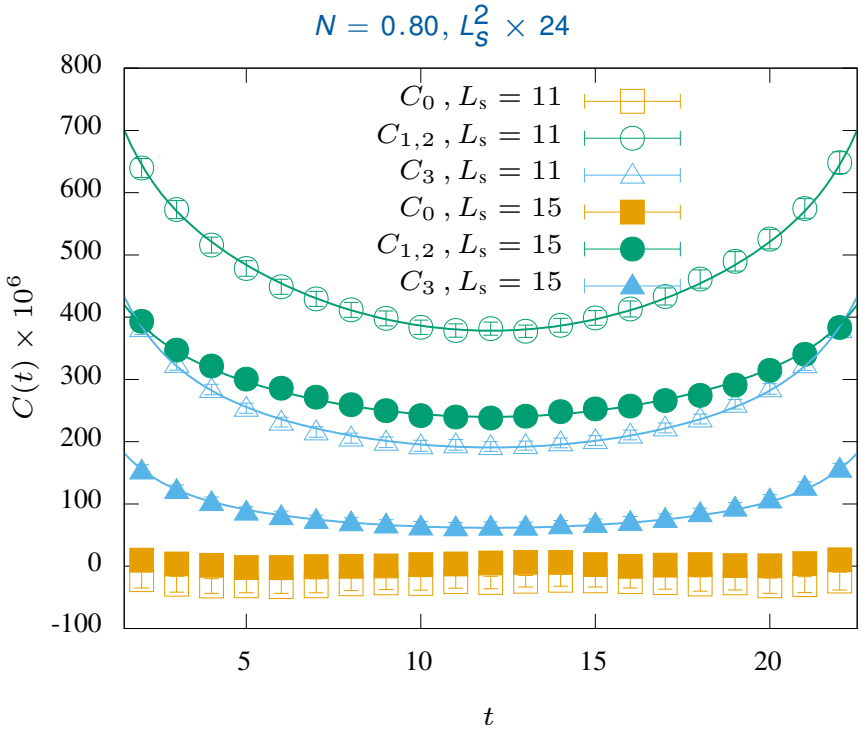
broken phase
$\mathrm{N_f^{crit}} = 0.8,\ L = 8, 12, 16$
density builds up near $E = 0$

- Goldstone spectrum with DWF $\Rightarrow \mathrm{N_f^{crit}} > 1$ S. Hands
- mesons: two scalars, two pseudo scalars, ψ reducible $\rightarrow$

$$\mathcal{O}_a(x) = \bar{\psi}(x)(\sigma_a \otimes \sigma_0)\psi(x), \quad \mathcal{O}_a(t) = \sum_{\boldsymbol{x}} \mathcal{O}_a(t, \boldsymbol{x}), \quad 0 \leq a \leq 3$$

- $\langle \mathcal{O}_3 \rangle \propto$ chiral condensate, $\langle \mathcal{O}_0 \rangle \propto$ would-be parity condensate ($= 0$)
- correlation matrix $C_{ab}(t) = \langle \mathcal{O}_a(t) \mathcal{O}_b(t) \rangle_c = C_a(t) \delta_{ab}$
- U(2) unbroken: expect singlet and triplett
  triplet $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3 \Rightarrow C_1 = C_2 = C_3, \quad m_1 = m_2 = m_3$
- SSB U(2) $\rightarrow$ U(1)$\otimes$ U(1): two Goldstone modes
- interpolating operaters $\mathcal{O}_1, \mathcal{O}_2, \Rightarrow m_1 = m_2$

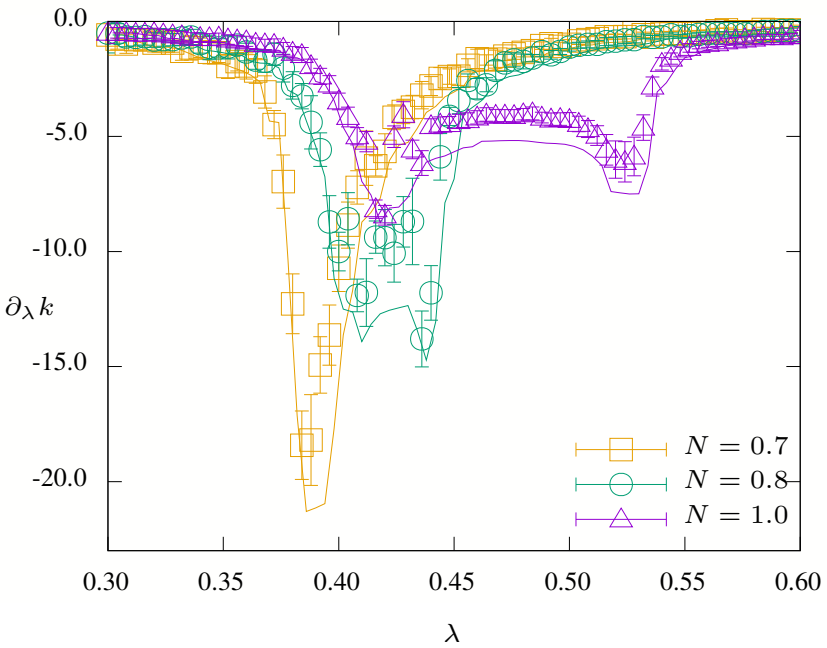




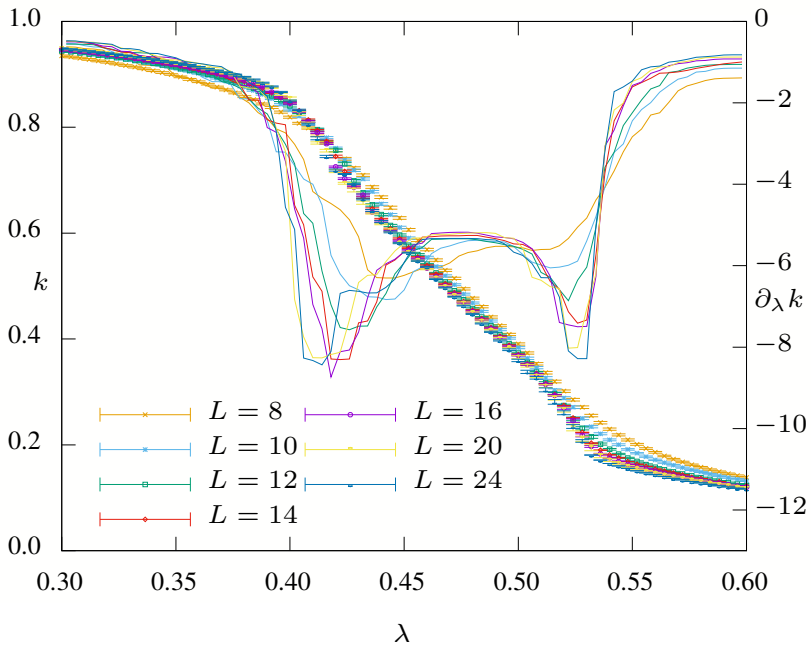
C	$m(11)$	$m(15)$	$m^*(11)$	$m^*(15)$	N_f	Symm.
C_0	0.21(2)	0.21(2)	1.27(6)	1.22(7)	1.0	U(2)
$C_{1,2}$	0.134(3)	0.128(2)	1.03(5)	1.02(3)	1.0	
C_3	0.138(2)	0.131(2)	1.08(4)	0.98(3)	1.0	
$C_{1,2}$	0.103(2)	0.095(3)	1.04(12)	0.93(17)	0.8	U(1)$\times$ U(1)
C_3	0.109(4)	0.127(7)	0.81(7)	0.81(10)	0.8	

- strong coupling $\langle k \rangle \to 1$, weak coupling $\langle k \rangle \to 0$
- susceptibility

$$\partial_\lambda \langle k \rangle = -\frac{1}{16 \mathrm{N_f} \lambda^3} \sum_x \left\langle (j^\mu j_\mu)(x)(j^\mu j_\mu)(0) \right\rangle_c + \frac{c - \langle k \rangle}{\lambda}$$

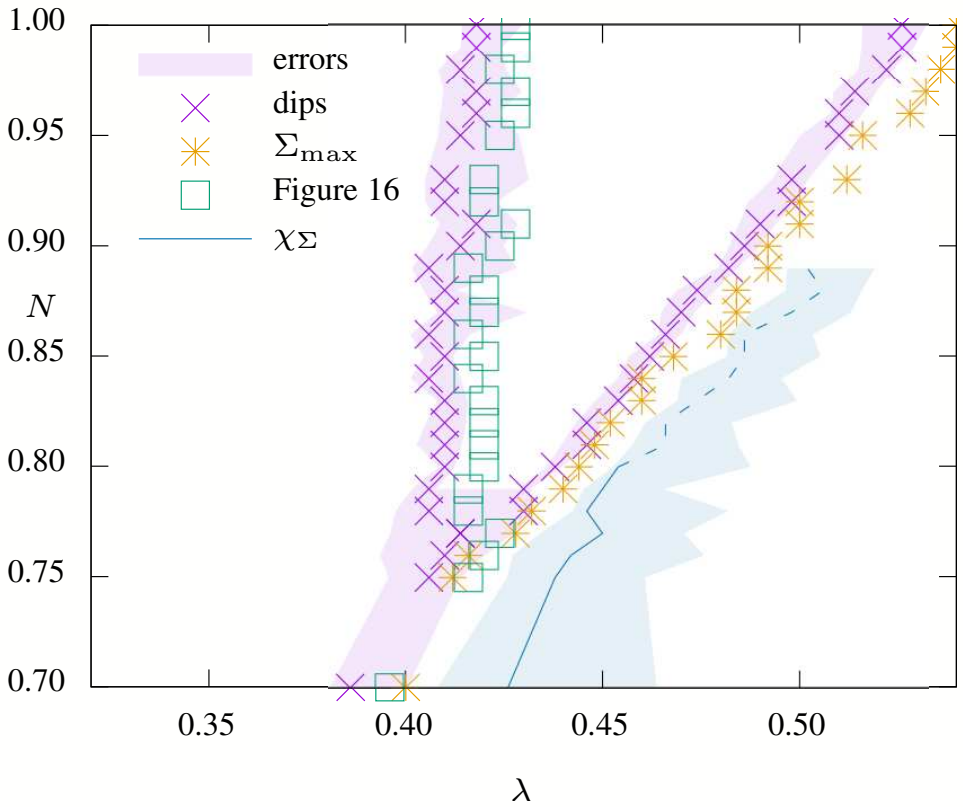


susceptibility for $\mathrm{N_f} = 0.7, 0.8, 1.0$



$\langle k \rangle$ and susceptibility for $\mathrm{N_f} = 1$, various L

Phase diagram in $(\lambda, \mathrm{N_f})$-plane



- $\partial_\lambda \langle k \rangle$ two dips for $\mathrm{N_f} \gtrapprox 0.76$
- artifact transitions for small λ
- $\mathrm{N_f} \lessapprox 0.80$ only one CPT
- $\mathrm{N_f} \gtrapprox 0.80$: second PT without order parameter
- nature of this PT?

- simulations of 3*d* four-Fermi theories with chiral SLAC fermions
  - relativistic four-Fermi theories (Th, GN, cGN) with correct symmetries
  - no sign-problem with reducible systems (including μ)
  - sign-problem for odd $\mathrm{N_f^{ir}}$ – under control
  - useful dual formulation
- no SSB for all all reducible models with $\mathrm{N_f} = 1, 2, 3, \ldots$

$$\mathrm{N_f^{crit}} = 0.80(4)$$

- odd $\mathrm{N_f^{ir}}$: parity breaking PT for $\mathrm{N_f^{ir}} \lessapprox 9$, corresponds to $\mathrm{N_f^{crit}} \lessapprox 4.5$
- choice of fermion species relevant: staggered, Kähler, DWF, SLAC, ...
- parity breaking PT with order parameter $\mathrm{N_f} \leq 0.8$

  new PT without order parameter $\mathrm{N_f} \geq 0.8$ ($\rightarrow$ needs clarification)
- symmetries of lattice fermion action essential for strongly coupled system
- still discrepancy SLAC $\leftrightarrow$ DWF

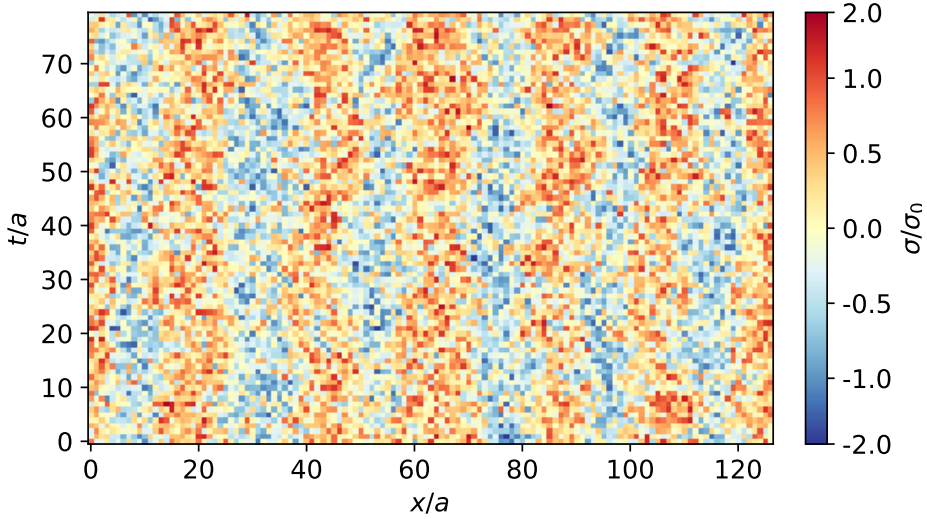
- inhomogeneous condensates at finite density
  - known to exist $1+1$d GN and cGN in large $\mathrm{N_f}$ limit
  - do they exist for finite $\mathrm{N_f}$?
  - inhomogeneous condensates in $d > 2$?

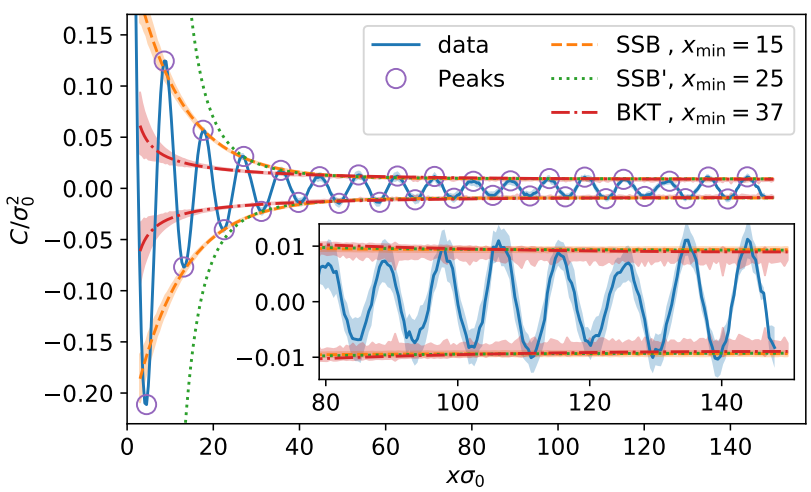
J.Lenz, M. Mandl, L. Pannullo, M. Wagner, M. Winstel, AW

- interacting fermions in external fields

J. Lenz, M. Mandl, AW, in progress

- condensates in rotating Fermi liquids





Lenz, Pannullo, Wagner, Wellegehausen, AW; Phys. Rev. D101 (2020) 094512 and Phys. Rev. D102 (2020) 114501