

Quantum critical behaviour of the Gross-Neveu SO(3) universality class from field theory beyond leading order



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Collaborators:

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Spin-orbital liquids and quantum criticality

$$H = -K \sum_{\langle ij \rangle_\gamma} \vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \tau_i^\gamma \tau_j^\gamma$$

$K > 0$

σ^α (τ^γ) — Pauli matrices in spin (orbital) space

- Soluble spin-orbital generalization of Kitaev model on honeycomb lattice

⇒ fractionalization, fermionic excitations, ...

Chulliparambil, Seifert, Vojta, Janssen & Tu,
Phys. Rev. B(R) '20

Spin-orbital liquids and quantum criticality

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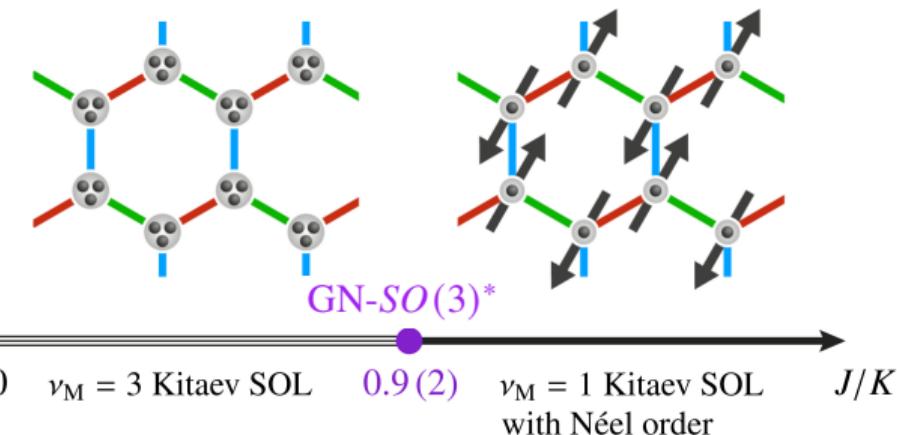
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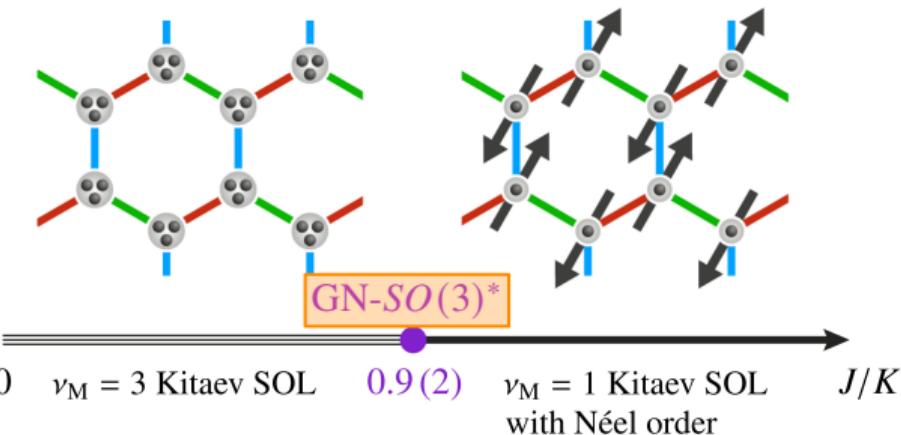
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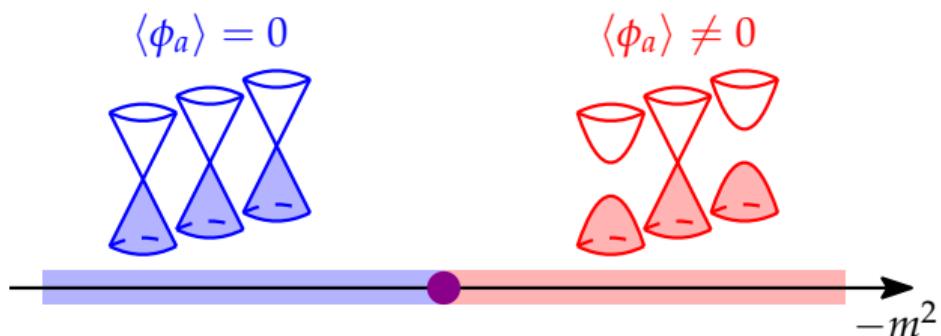
Universality class

Effective field theory Lagrangian [Seifert *et al.*, Phys. Rev. Lett. '20]

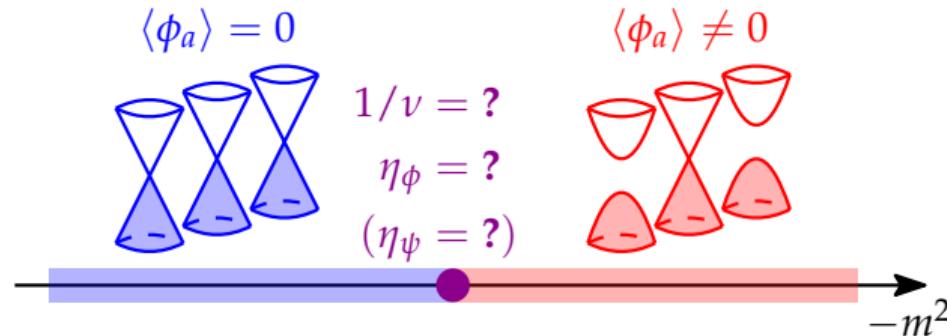
$$\mathcal{L} = \bar{\psi}_i \not{\partial} \psi_i + \frac{1}{2} (\partial_\mu \phi_a)^2 - g \phi_a \bar{\psi}_i T_{ij}^a \psi_j + \frac{1}{2} m^2 \phi^2 + \frac{1}{4!} \lambda \phi^4$$

$\mu = 0, 1, 2$: (2+1)D spacetime; T^a : $SO(3)$ generators ($a = 1, 2, 3$)

- 3 fermion flavours with $SO(3)$ symmetry
- one flavour remains gapless after SSB
contrast with chiral Ising or chiral Heisenberg!
- Novel universality class, state of the art: first-order ϵ and $1/N$ expansion
ibid.



Our work / this talk ...



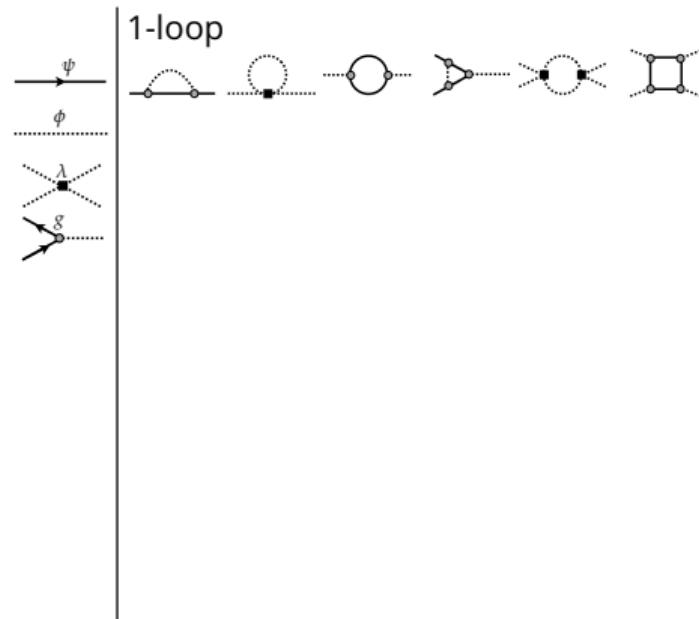
... series expansion parameters $\epsilon, 1/N$ in reality ~ 1

This talk:

- Series expansions to high(-er) orders
- Complementary non-perturbative calculation (FRG)
- Combined estimates with high-confidence error bars

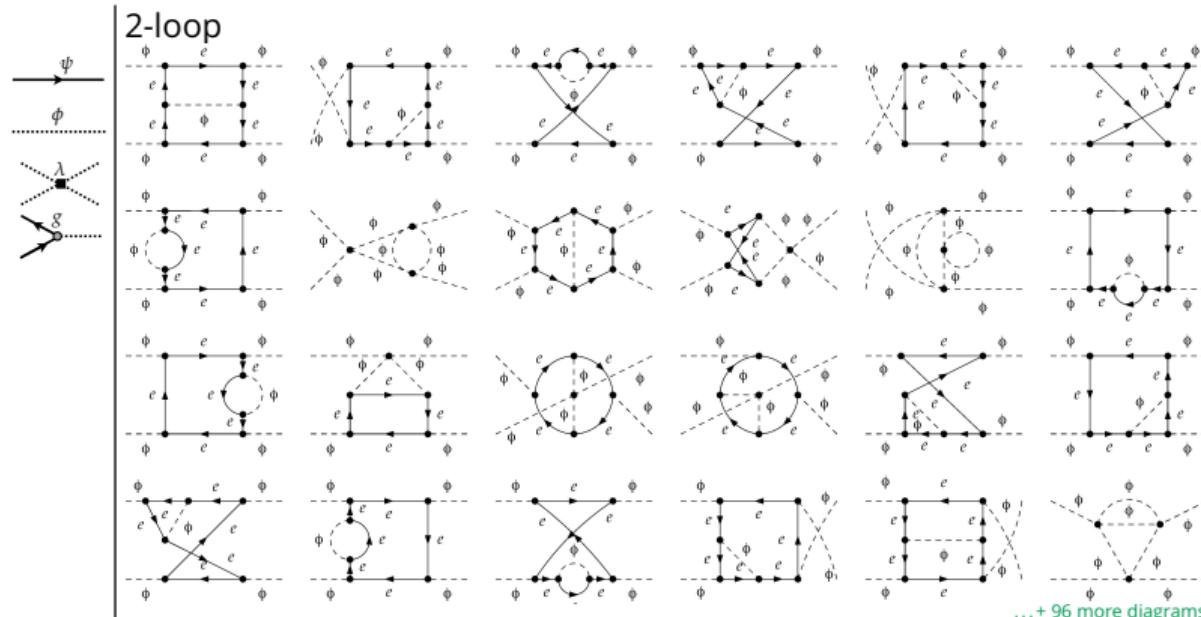
ϵ expansion

- Basic idea: loop expansion; need small interaction strengths
- Couplings at criticality of order ϵ in $D = 4 - \epsilon$ spacetime dimensions



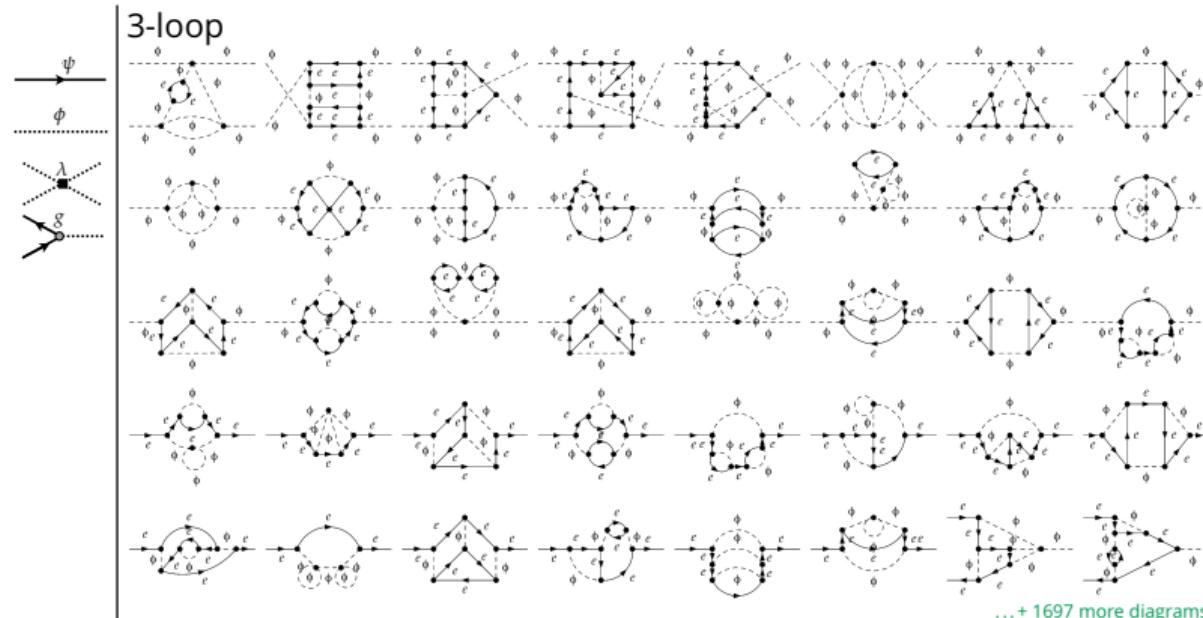
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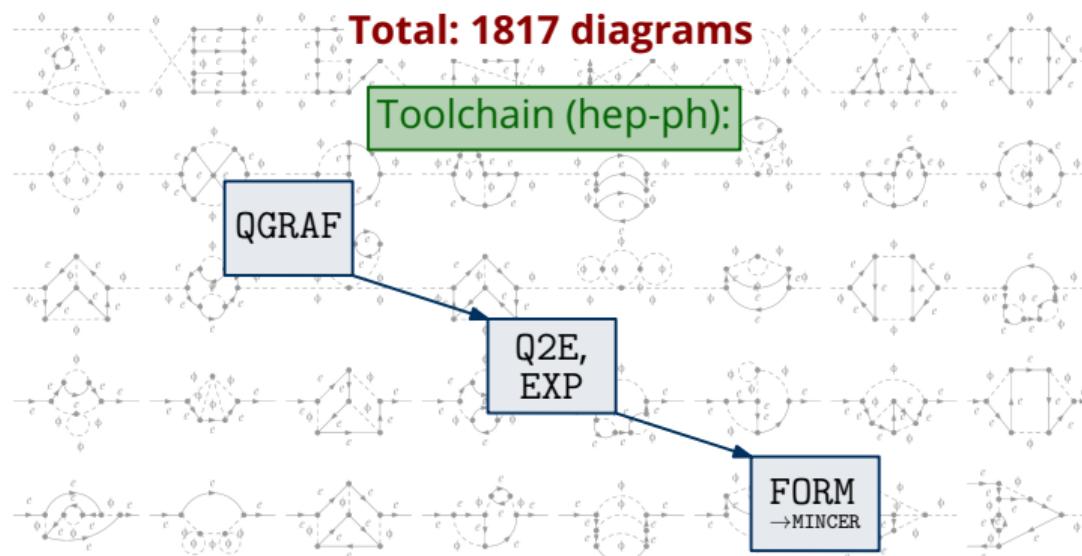
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cf.: Nogueira J Comput. Phys. '93; Nogueira Nucl. Instrum. Methods Phys. Res. '06; Harlander *et al.* Phys. Lett. B '98; Seidensticker et al. arXiv:hep-ph/9905298; Vermaseren arXiv:math-ph/0010025; Kuipers *et al.* Comput. Phys. Commun. '13; Ruijl *et al.* arXiv:1707.06453; Czakon Nucl. Phys. B '05; Gorishnii *et al.* Comput. Phys. Commun. '89; Larin *et al.* NIKHEF-H-91-18 '91

$1/N$ expansion

- Basic idea: flavour number $N \rightarrow \infty$ with gN fixed \simeq mean-field theory
- Systematic corrections in $1/N$: resummation of (infinitely many!) loop diagrams (perturbation theory reorganized according to 'closed fermion loops per Yukawa vertex')
⇒ self-consistent Dyson-Schwinger equations

$$0 = (\overline{\psi} \psi)^{-1} + \overline{\psi} \overset{\phi}{\circlearrowleft} \psi + \overline{\psi} \overset{\phi}{\circlearrowleft} \psi \overset{\phi}{\circlearrowleft} \psi$$

$$0 = (\overline{\phi} \phi)^{-1} + \dots \overset{\bar{\psi}}{\circlearrowright} \phi \dots + \dots \overset{\bar{\psi}}{\circlearrowright} \phi \overset{\bar{\psi}}{\circlearrowleft} \phi \dots$$

(+ 6 more diagrams at NNLO)

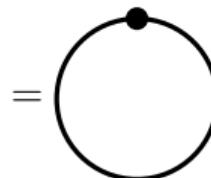
cf.: Vasil'ev *et al.* Theor. Math. Phys. '81a,b; '93;
Gracey Int. J. Mod. Phys. A '94; '18 and refs. therein

Functional Renormalization Group (FRG)

- Basic idea: solve Wetterich equation self-consistently

Wetterich, Phys. Lett. B '93

$$\frac{\partial \Gamma_k[\Phi]}{\partial \ln k} = \text{STr} \left[\left(\frac{\delta^2 \Gamma_k[\Phi]}{\delta \Phi \delta \Phi^\top} + R_k[\Phi] \right)^{-1} \frac{\partial R_k[\Phi]}{\partial \ln k} \right] =$$

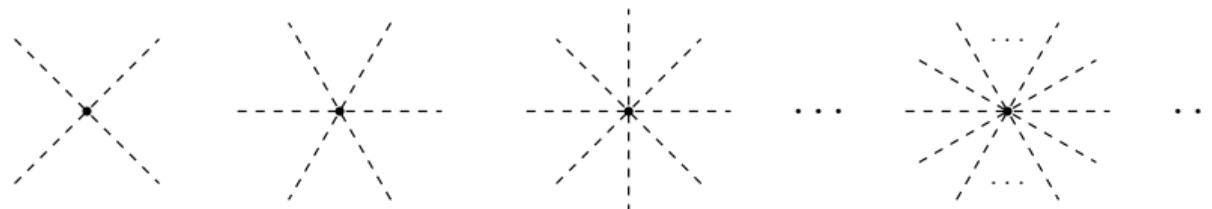


$\Phi = (\phi_a, \psi, \bar{\psi})^\top$; k — RG scale; R — regulator

cf., e.g.: Berges *et al.* Phys. Rep. '02; Metzner *et al.* Rev. Mod. Phys. '12; Dupuis *et al.* Phys. Rept. '21; and refs. therein

- truncation: improved local potential approximation (LPA')

- * standard wavefct. and Yukawa vertex renorm.: $\psi, \phi, g \rightarrow \sqrt{Z_\psi}\psi, \sqrt{Z_\phi}\phi, Z_g g$
- * arbitrary renormalized boson effective potential: $m^2\phi^2/2 + \lambda\phi^4/4! \rightarrow V(\phi)$
⇒ self-couplings of arbitrary order



Post processing

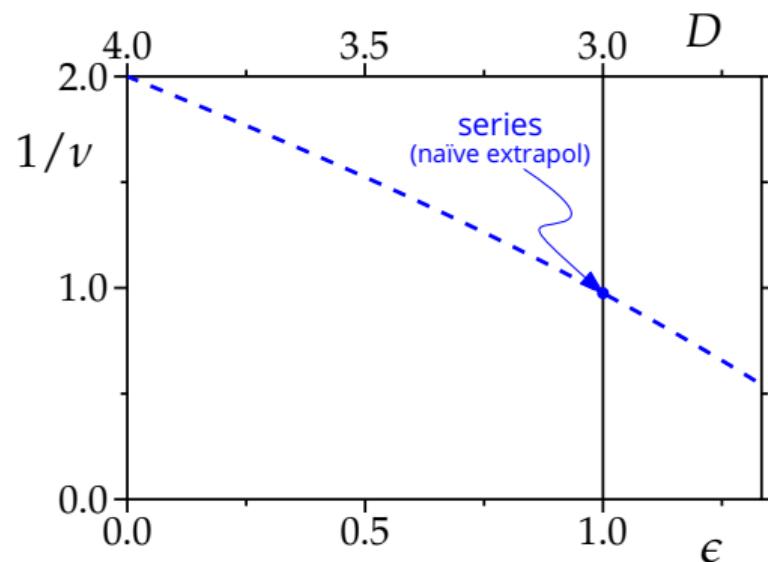
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- Physical case: $\epsilon = 1 \longrightarrow$ **extrapolation**; here: **Padé approximants**

$$[m/n] := \frac{a_0 + a_1\epsilon + \dots + a_m\epsilon^m}{1 + b_1\epsilon + \dots + b_n\epsilon^n} = c_0 + c_1\epsilon + \dots + c_{m+n}\epsilon^{m+n} + \mathcal{O}(\epsilon^{m+n+1})$$

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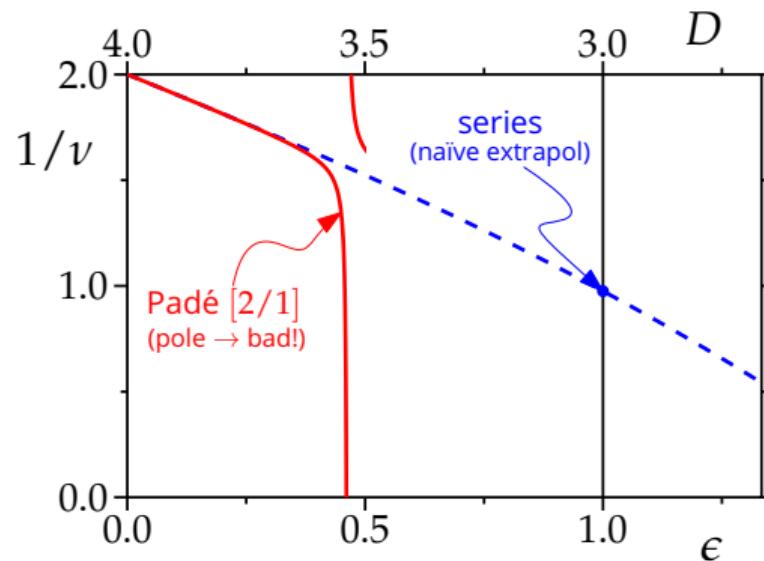
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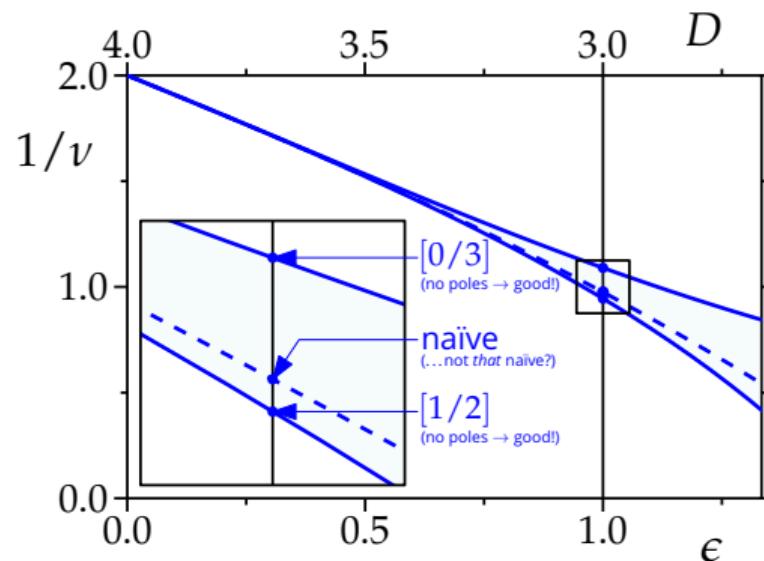
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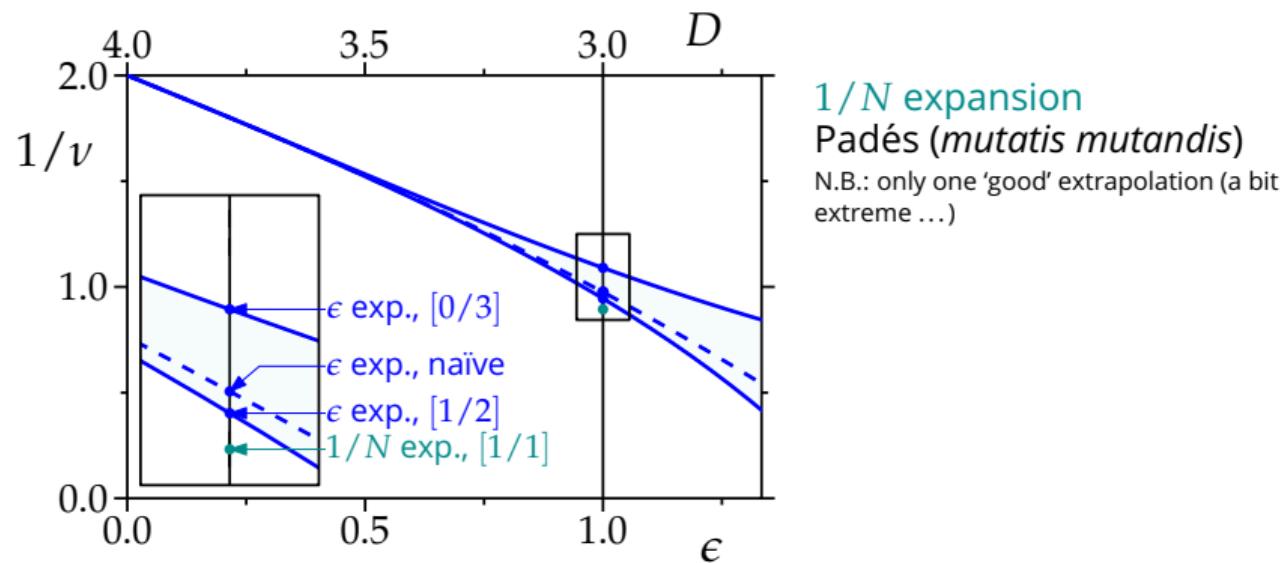
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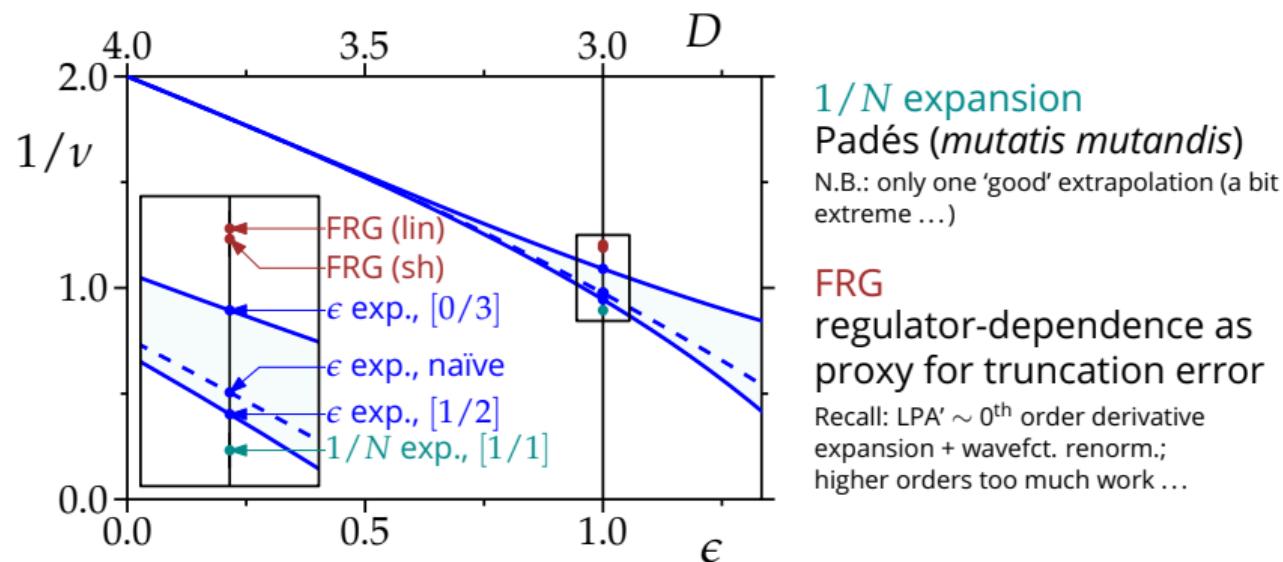
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1/ N expansion

Padés (*mutatis mutandis*)

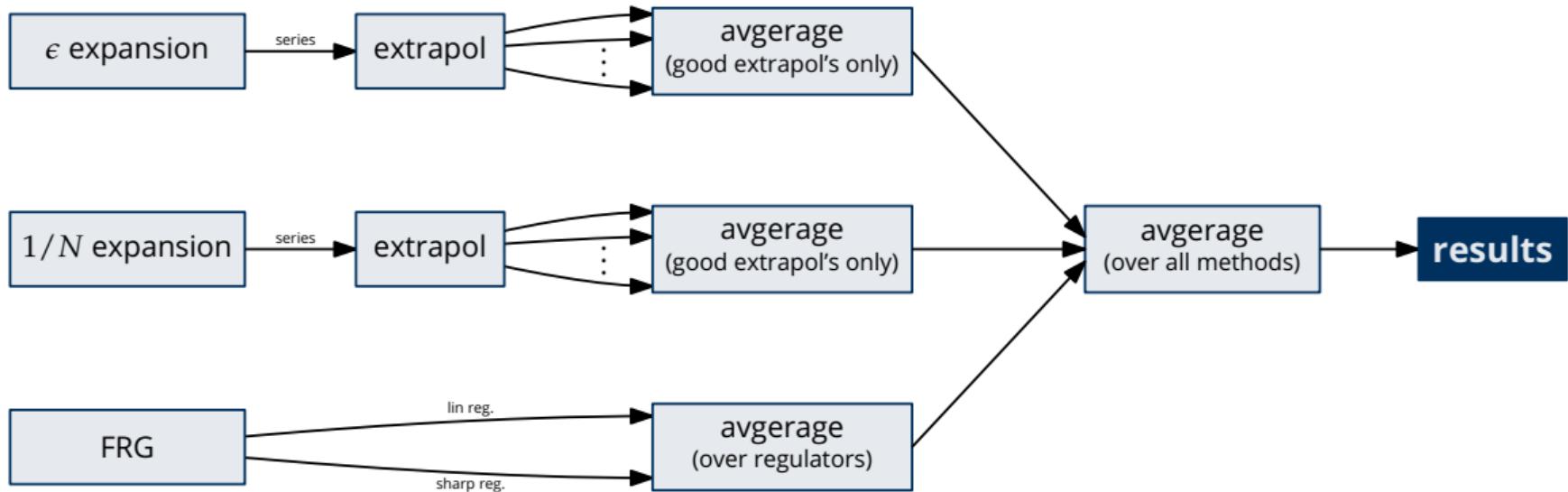
N.B.: only one 'good' extrapolation (a bit extreme ...)

FRG

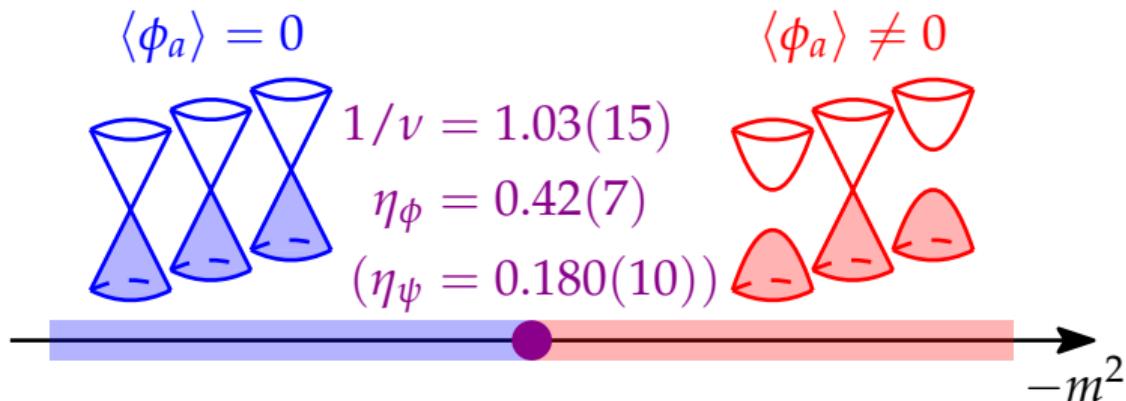
regulator-dependence as proxy for truncation error

Recall: LPA' $\sim 0^{\text{th}}$ order derivative expansion + wavefct. renorm.; higher orders too much work ...

Averaging — workflow

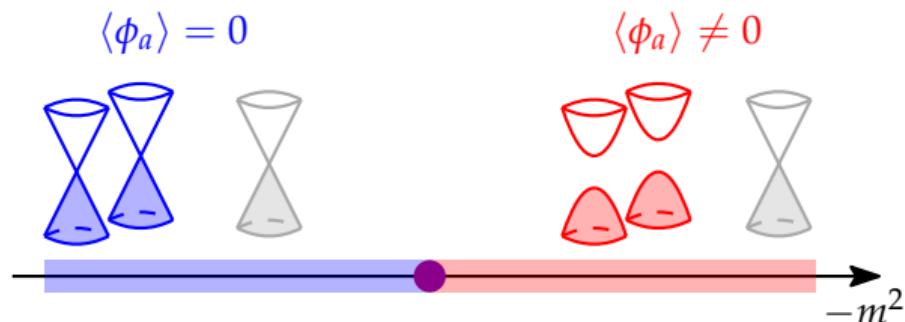
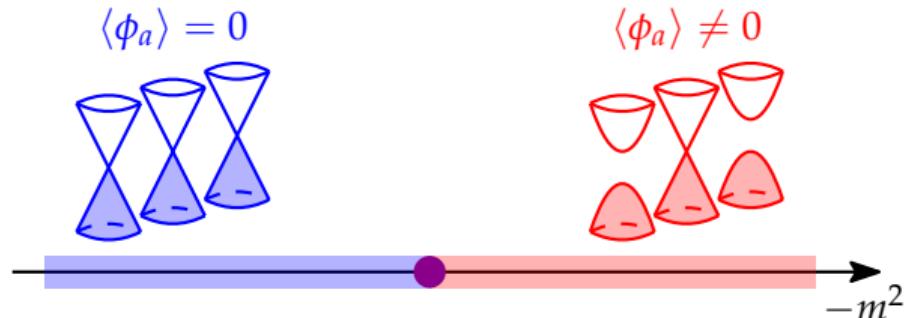


Numbers



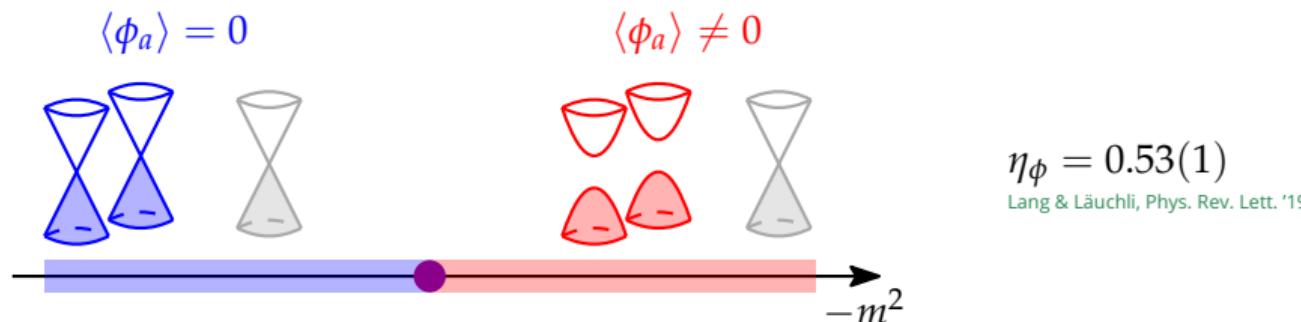
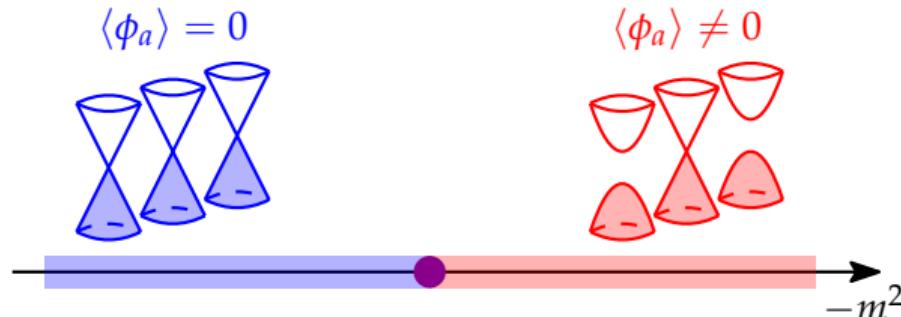
Quantum criticality as diagnostic?

- Two possible scenarios compatible with gapless DOF count



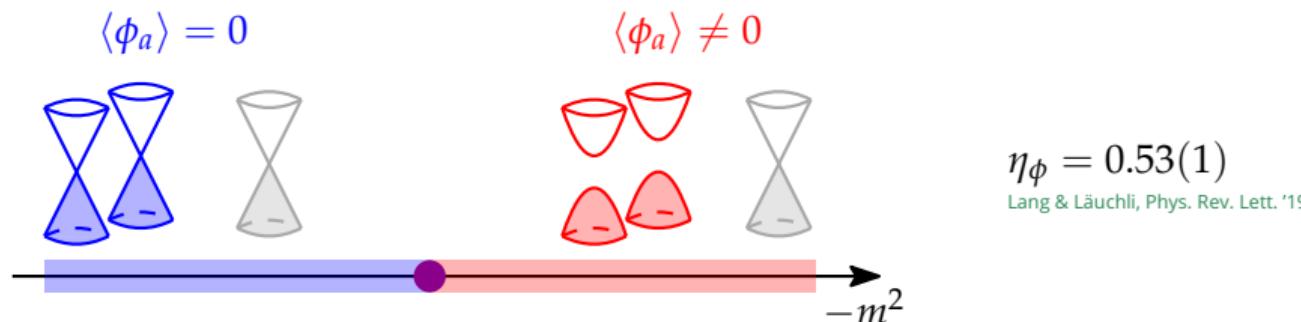
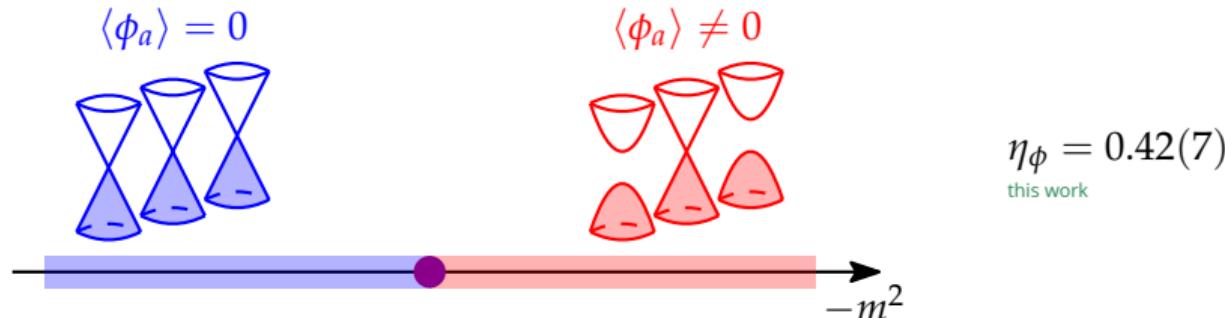
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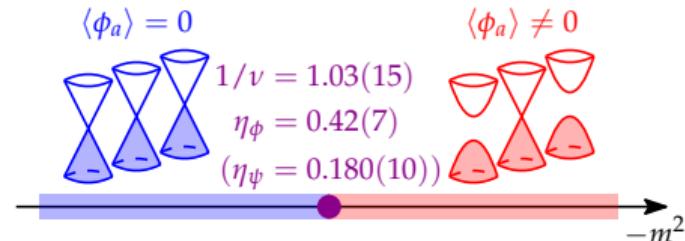


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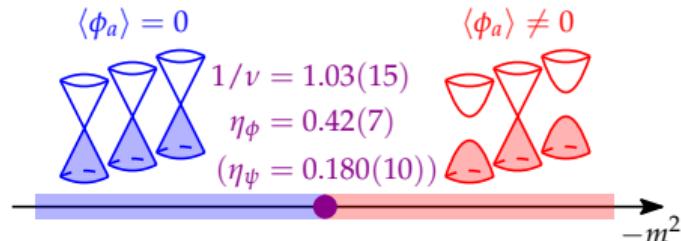


Closing remarks



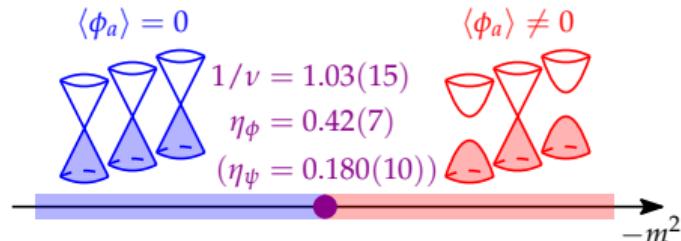
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Further details and results:

S.R., B. Ihrig, D. Kruti, J.A. Gracey, M.M. Scherer, and L. Janssen (2021):
Phys. Rev. B **103**, 155160

Acknowledgement



B. Ihrig



D. Kruti
(Köln)



M. M. Scherer



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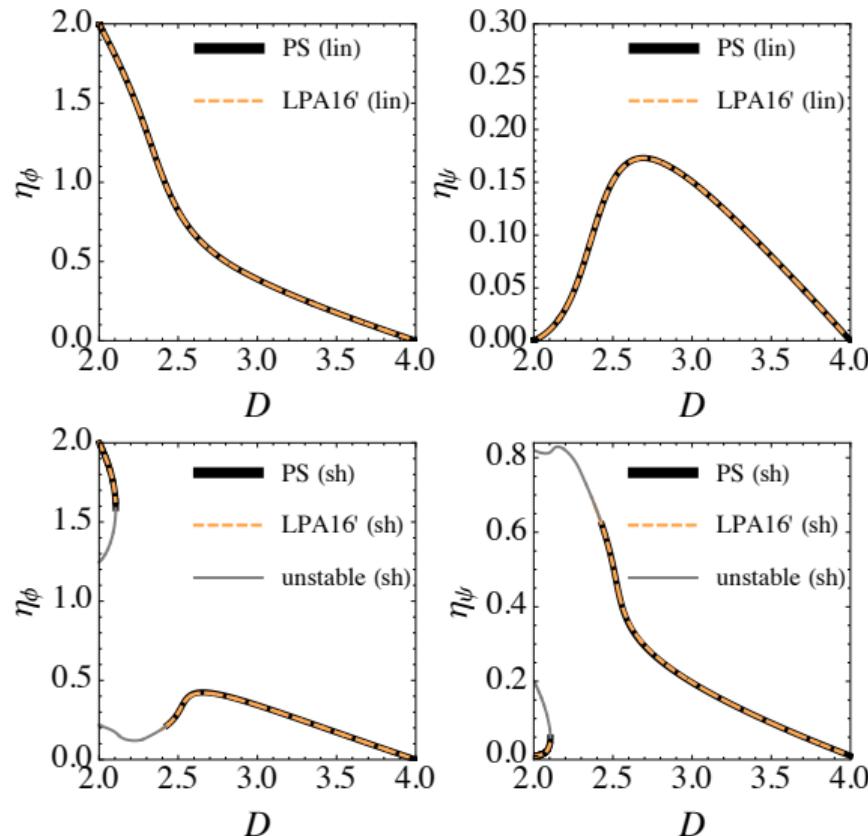
J. A. Gracey
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L. Janssen
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Thank you!

Cool non-perturbative stuff



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regularization (FRG) = momentum-dep. mass of fluctuation modes

