Quantum criticality between spin liquids and long-range order

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Würzburg-Dresden Cluster of Excellence



Shouryya Ray (Dresden) \rightarrow Wed 17:00 John Gracey (Liverpool) Bernhard Ihrig (Cologne) Daniel Kruti (Cologne) Michael Scherer (Cologne)

Complexity and Topology in Quantum Matter





Outline

Introduction: Spin fractionalization Ι.

Example #1: Confinement transition П. in flatland U(1) gauge theory

III. Example #2: Spinon-metal—insulator transition in flatland \mathbb{Z}_2 gauge theory

IV. Conclusions: Spin-liquid criticality







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Introduction: Quantum numbers of composites

Free spins-1/2:



+



Introduction: Quantum numbers of composites

Free spins-1/2:



Interacting spins-1/2:



+

 $s_2 = \frac{1}{2}$ S = 0 S = 1M=0 $M=0, \pm 1$

> Magnons S = 1

Introduction: Quantum numbers of composites

Free spins-1/2:



Interacting spins-1/2:



Fractionalization:



Quantum numbers of collective excitations $\neq n \cdot quantum numbers of constituents$

Fractionalization:

Parton decomposition:





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U(1) gauge redundancy:

$$\Psi_i \mapsto \mathrm{e}^{\mathrm{i}\lambda_i}\Psi_i$$

... full gauge redundancy: SU(2)[Affleck et al., PRB '88]

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Quantum numbers of collective excitations $\neq n \cdot quantum$ numbers of constituents

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U(1) gauge redundancy:

$$\Psi_i \mapsto \mathrm{e}^{\mathrm{i}\lambda_i}\Psi_i$$

... full gauge redundancy: SU(2)[Affleck et al., PRB '88]

Models featuring "deconfined" spinons?

Introduction: Kitaev honeycomb model

Spins-1/2 on honeycomb lattice:

Hamiltonian:







Introduction: Kitaev honeycomb model

Spins-1/2 on honeycomb lattice:

Hamiltonian:

Parton decomposition:



Introduction: Spin-liquid criticality



[Wolter, Corredor, LJ, et al., PRB '17] [Gass, ..., LJ, et al., PRB '21]





[Wolter, Corredor, LJ, et al., PRB '17] [Gass, ..., LJ, et al., PRB '21]

[Kasahara *et al.*, Nature '18] [Czajka et al., Nat. Phys. '21]



[Gass, ..., LJ, et al., PRB '21]

This talk:

How do fractionalized excitations affect quantum criticality?

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Flatland U(1) gauge theory: Lattice compact QED₃ Action (square lattice):

$$S = \int_{0}^{eta} d au \left[\sum_{\langle ij
angle, lpha} \psi^{\dagger}_{ilpha} (\partial_{ au} \delta_{ij} - t e^{i arphi_{ij}}) \psi_{j} + rac{4}{JN_{
m f}} \sum_{\langle ij
angle} rac{1 - \cos[arphi_{ij}(au+1) - arphi_{ij}]}{\Delta au^2}
ight]$$





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Phase diagram (K > 0):



U(1) Dirac spin liquid







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Phase diagram (K > 0):





Field theory: QED₃-Gross-Neveu-XY

Action (continuum):

 $S = \int d\tau \int d^{2}\vec{x} \left[\bar{\psi}\gamma_{\mu}(\partial_{\mu} - iA_{\mu})\psi + \phi^{a}\bar{\psi}\mu^{a}\psi + \frac{r}{2}\phi^{a}\phi^{a} + \frac{1}{2e^{2}}(\epsilon_{\mu\nu\rho}\partial_{\nu}A_{\rho})^{2} \right]$ $= \int d\tau \int d^{2}\vec{x} \left[\bar{\psi}\gamma_{\mu}(\partial_{\mu} - iA_{\mu})\psi + \phi^{a}\bar{\psi}\mu^{a}\psi + \frac{r}{2}\phi^{a}\phi^{a} + \frac{1}{2e^{2}}(\epsilon_{\mu\nu\rho}\partial_{\nu}A_{\rho})^{2} \right]$ $= \int d\tau \int d^{2}\vec{x} \left[\bar{\psi}\gamma_{\mu}(\partial_{\mu} - iA_{\mu})\psi + \phi^{a}\bar{\psi}\mu^{a}\psi + \frac{r}{2}\phi^{a}\phi^{a} + \frac{1}{2e^{2}}(\epsilon_{\mu\nu\rho}\partial_{\nu}A_{\rho})^{2} \right]$





Field theory: QED₃-Gross-Neveu-XY

Action (continuum):

RG flow:





 \rightarrow Talk Joseph Maciejko Tue 17:45



a = x, y



Evidence for QED₃-Gross-Neveu-XY criticality Scenario 1: Conventional paramagnet XY: $u \simeq 0.672$ $\eta_{oldsymbol{\phi}}\simeq 0.039$

Scenario 2: Dirac fermions Gross-Neveu-XY: $u \simeq 1.07$ $\eta_{oldsymbol{\phi}}\simeq 0.97$... for $N_{\rm f} = 8$ → Talk Sandro Sorella Mon 14:15

Scenario 3: Dirac fermions + U(1) gauge field QED₃-Gross-Neveu-XY: $\nu \simeq 1.51$ $\eta_{oldsymbol{\phi}}\simeq 1.24$... for $N_{\rm f} = 8$



Evidence for QED₃-Gross-Neveu-XY criticality Scenario 1: Conventional paramagnet SBV 0.2 -XY: $u \simeq 0.672$ $\eta_{oldsymbol{\phi}}\simeq 0.039$

Scenario 2: Dirac fermions $\nu \simeq 1.07$ • $\eta_{\phi} \approx 0.97$ •••• Gross-Neveu-XY: ... for $N_{\rm f} = 8$ → Talk Sandro Sorella Mon 14:15





[LJ, Wang, Scherer, Meng, Xu, PRB '20]





Scenario 2: Dirac fermions

Gross-Neveu-XY:

$$\nu \simeq 1.07$$

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... for $N_{\rm f} = 8$

→ Talk Sandro Sorella Mon 14:15

Scenario 3: Dirac fermions + U(1) gauge field QED₃-Gross-Neveu-XY: $\nu \simeq 1.51$ η_{ϕ} 2222 η_{ϕ} 0 0 00 ... for $N_{\rm f} = 8$



[LJ, Wang, Scherer, Meng, Xu, PRB '20]





Scenario 2: Dirac fermions

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Scenario 3: Dirac fermions + U(1) gauge field η_{ϕ} QED₃-Gross-Neveu-XY: $\nu \simeq 1.51$ 0 0 00 ... for $N_{\rm f} = 8$







[LJ, Wang, Scherer, Meng, Xu, PRB '20]



 η_{ϕ} QED₃-Gross-Neveu-XY: $\nu \simeq 1.51$ 0 0 00

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[LJ, Wang, Scherer, Meng, Xu, PRB '20]



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Flatland \mathbb{Z}_2 gauge theory: Spin-orbital liquid

Spin-orbital generalization of Kitaev model:

 $H = -K \sum \left(\sigma_i^{\times} \sigma_j^{\times} + \sigma_i^{y} \sigma_j^{y} \right) \otimes \tau_i^{\gamma} \tau_j^{\gamma} + J^z \sum \sigma_i^{z} \sigma_j^{z} \otimes \mathbb{1}_i \mathbb{1}_j$ $\langle ij
angle_{\gamma}$ XY spin





[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, PRL '20]

Kitaev orbital

... recover known j = 3/2 model $J^{z} = 0$: [Yao, Zhang, Kivelson, PRL '09] [Nakai, Ryu, Furusaki, PRB '12]





Flatland \mathbb{Z}_2 gauge theory: Spin-orbital liquid

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Phase diagram:



"Kitaev" spin-orbital liquid



 $\langle ij \rangle$

[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, PRL '20]

 $J^{z} = 0$:

Kitaev orbital

... recover known j = 3/2 model [Yao, Zhang, Kivelson, PRL '09] [Nakai, Ryu, Furusaki, PRB '12]







Parton decomposition:

$$\sigma^{y} \otimes \tau^{x} = ib^{1}c^{x}$$
 $\sigma^{y} \otimes \tau^{y} = ib^{2}c^{x}$
 $\sigma^{y} \otimes \tau^{z} = ib^{3}c^{x}$



 $\sigma^{x} \otimes \mathbb{1} = ib^{4}c^{x}$ $\sigma^z \otimes \mathbb{1} = ic^y c^x$

Parton decomposition: $\sigma^{y} \otimes \tau^{x} = ib^{1}c^{x}$ $\sigma^y \otimes \tau^y = ib^2 c^x$ $\sigma^{y} \otimes \tau^{z} = ib^{3}c^{x}$ hopping parameter t = 2Kuij = ibibj π -flux model: $\langle ij \rangle$



 $\sigma^{x} \otimes \mathbb{1} = ib^{4}c^{x}$ $\sigma^z \otimes \mathbb{1} = ic^y c^x$

nearest-neighbor repulsion $V = 4 J^z$ $H \mapsto \sum_{i=1}^{\infty} \left[2Ku_{ij}(f_i^{\dagger}f_j + f_j^{\dagger}f_i) + 4J^z(n_i - \frac{1}{2})(n_j - \frac{1}{2}) \right]$ electron density ft f

Ground-state: π flux

 $f = \frac{1}{2}(c^{*})$







 $\sigma^{x} \otimes \mathbb{1} = ib^{4}c^{x}$ $\sigma^z \otimes \mathbb{1} = ic^y c^x$

nearest-neighbor repulsion $V = 4 J^z$ $\sum \left[2Ku_{ij}(f_i^{\dagger}f_j + f_j^{\dagger}f_i) + 4J^z(n_i - \frac{1}{2})(n_j - \frac{1}{2}) \right]$ electron density ft f

Ground-state: π flux





Kitaev spin-orbital model \rightarrow Interacting fermions on π -flux lattice



 $\sigma^{x} \otimes \mathbb{1} = ib^{4}c^{x}$ $\sigma^z \otimes \mathbb{1} = ic^y c^x$

nearest-neighbor repulsion $V = 4 J^z$ $\sum \left[2Ku_{ij}(f_i^{\dagger}f_j + f_j^{\dagger}f_i) + 4J^z(n_i - \frac{1}{2})(n_j - \frac{1}{2}) \right]$ electron density ft

Ground-state: π flux





Spinless fermions on π -flux lattice: QMC



Dirac semimetal

0







V/t



[Huffman & Chandrasekharan, PRD '17; PRD '20]

- [Wang, Corboz, Troyer, NJP '14]
 - [Li, Jiang, Yao, NJP '15]





Spinless fermions on π -flux lattice: QMC



1/
u = 1.12(1),Gross-Neveu- \mathbb{Z}_2 universality:



Charge density wave



[Huffman & Chandrasekharan, PRD '17; PRD '20]

[Wang, Corboz, Troyer, NJP '14]

[Hands, Kocic, Kogut, Ann. Phys. '93] [Gracey, IJMP '94] [Braun, Gies, D Scherer, PRD '11] [LJ & Herbut, PRB '14] [lliesiu et al., JHEP '18] [Ihrig, Mihaila, M Scherer, PRB '18]

\rightarrow Talk David Poland Tue 17:00

$$\eta_{oldsymbol{\phi}}=0.51(3)$$
, $\eta_{\psi}pprox 0.1$

V/t





Spinless fermions on π -flux lattice: QMC



 $1/
u = 1.12(1), \quad \eta_{m{\phi}} = 0.51(3),$ Gross-Neveu- \mathbb{Z}_2 universality:

Spin-orbital model:





Charge density wave

V/t

 $\eta_{oldsymbol{\psi}}pprox 0.1$



[Huffman & Chandrasekharan, PRD '17; PRD '20]

- [Wang, Corboz, Troyer, NJP '14]
 - [Li, Jiang, Yao, NJP '15]
- [Hands, Kocic, Kogut, Ann. Phys. '93] [Gracey, IJMP '94] [Braun, Gies, D Scherer, PRD '11] [LJ & Herbut, PRB '14] [lliesiu et al., JHEP '18] [Ihrig, Mihaila, M Scherer, PRB '18]

\rightarrow Talk David Poland Tue 17:00

Ising spin order J^z/K











Finite-size spectroscopy: Ising vs Ising*

Transverse-field Ising:

$$H = -J\sum_{\langle ij\rangle}\sigma_i^z\sigma_j^z - h\sum_i\sigma_i^x$$



Transverse-field toric code:

$$H = -J\sum_{s}\prod_{i\in s}\sigma_i^x - J\sum_{p}\prod_{i\in p}\sigma_i^z - h\sum_i$$

[Kitaev, Ann. Phys. '03] [Trebst et al., PRL '07]



Finite-size spectroscopy: Ising vs Ising*

Transverse-field Ising:

Ш





Transverse-field toric code:

Gross-Neveu vs Gross-Neveu*

Gross-Neveu- \mathbb{Z}_2



[Schuler, Hesselmann, Whitsitt, Lang, Wessel, Läuchli, PRB '21]

→ Poster Thomas Lang



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Spin-liquid criticality IV. Conclusions:





Conclusions: Spin-liquid criticality

Confinement transition:



[LJ, Wang, Scherer, Meng, Xu, PRB '20]

Spinon-metal—insulator transition:



[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, PRL '20]



Conclusions: Spin-liquid criticality

Confinement transition:



[LJ, Wang, Scherer, Meng, Xu, PRB '20]

Many further possibilities: \rightarrow Talk Shouryya Ray @ Wed 17:00

→ Poster Wilhelm Krüger

Spinon-metal—insulator transition:



[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, PRL '20]

[Ray, Ihrig, Kruti, Gracey, Scherer, LJ, PRB '21]

[Krüger, LJ, arXiv:2107.00661]





Thermal conductivity of α-RuCl₃: Thermal Hall effect

Transversal heat conductivity:





Smoking-gun signature of Majorana edge states?



 $\mu_0 H(\mathsf{T})$

[Czajka et al., Nat. Phys. '21]



Monopole proliferation



Monopole scaling dimension:

$$\Delta_{q=\frac{1}{2}} = \begin{cases} 0.265 \cdot 2N_{\rm f} - 0.\\ 0.195 \cdot 2N_{\rm f} + \mathcal{O} \end{cases}$$

Critical flavor number:

$$N_{\rm f,c} \simeq egin{cases} 5.7, & {\sf QED}_3 \ 7.7, & {\sf QED}_3-{\sf GN}-X \end{cases}$$

 $.0383 + O(1/N_{\rm f}),$ QED_3 $P(1/N_{\rm f}^0),$ QED₃-GN-XY Calculation originally done for QED₃-GN-Heisenberg:

[Dupuis, Paranjape, Witczak-Krempa, PRB '19]





VBS correlation ratio



Decay of critical correlators

