

Fakher F. Assaad Relativistic Fermions in Flatland: theory and application. 5-9 July 2021 ECT* - Trento (Online)

Negative sign free realistic and toy model quantum Monte Carlo calculations of
correlated Dirac systems



SFB1170
ToCoTronics



Leibniz-Rechenzentrum
der Bayerischen Akademie der Wissenschaften



Gauss Centre for Supercomputing



Très Grand Centre de calcul du CEA



Center of excellence – complexity and
topology in quantum matter



Fakher F. Assaad Relativistic Fermions in Flatland: theory and application. 5-9 July 2021 ECT* - Trento (Online)

Outline.

Fermion Monte Carlo

Fermi velocity renormalization in graphene

Skyrmion superconductivity

Phases of (2+1) dimension SO(5) non-linear sigma model with WZW term

Conclusions



SFB1170
ToCoTronics

lrz
Leibniz-Rechenzentrum
der Bayerischen Akademie der Wissenschaften

GCS
Gauss Centre for Supercomputing



TGCC
Très Grand Centre de calcul du CEA



Center of excellence – complexity and
topology in quantum matter



Many thanks to



M. Raczkowski



Z. Wang
→ MPIPKS



T. Sato



J. S.E. Portela



A. Götz



G. Rein



M. Ulybyshev



F. Parisen Toldin



J. Schwab



B. Danu



Z. Liu

$$Z = \text{Tr} e^{-\beta \hat{H}} = \int D\{\Phi(i, \tau)\} e^{-S\{\Phi(i, \tau)\}}$$

$\Phi(i, \tau)$: Hubbard-Stratonovich
(or arbitrary field with
predefined dynamics)

Multidimensional integral
→ Monte Carlo

One body problem in external
field → Polynomial complexity

R. Blankenbecler, D. J. Scalapino, and R. L. Sugar, Phys. Rev. D 24 (1981), 2278

J. E. Hirsch, Phys. Rev. B 31 (1985), 4403

White, D. Scalapino, R. Sugar, E. Loh, J. Gubernatis, and R. Scalettar, Phys. Rev. B 40 (1989), 506

S. Duane, A. D. Kennedy, B. J. Pendleton, and D. Roweth, Phys. Lett. B195 (1987), 216–222.

.....

$$\hat{H} = \hat{H}_0 + \sum_{i,j} (\hat{n}_i - 1) V_{i,j} (\hat{n}_j - 1)$$

$$L_\tau \Delta\tau = \beta$$

$$e^{-S\{\Phi(i,\tau)\}} = e^{-4 \sum_{i,j,\tau} \Delta\tau \Phi(i,\tau) V_{i,j}^{-1} \Phi(j,\tau)} \text{Tr} \left[\prod_{\tau=1}^{L_\tau} e^{-\Delta\tau \hat{H}_0} e^{-i\Delta\tau \sum_i \Phi(i,\tau)(\hat{n}_i - 1)} \right]$$

$$= e^{-S_0\{\Phi(i,\tau)\}} [\det M(\Phi)]^2 \quad V \text{ has to be positive definite !}$$

The action is real (particle-hole / time reversal symmetry)

R. Brower, C. Rebbi and D. Schaich PoS(Lattice 2011)056

M. V. Ulybyshev, P. V. Buividovich, M. I. Katsnelson, and M. I. Polikarpov. Phys. Rev. Lett., 111, 056801, (2013).

M. Hohenadler, F. Parisen Toldin, I. Herbut and F. F. Assaad, 90, 085146 (2014)

.....

$$\hat{H} = \hat{H}_0 + \sum_{i,j} (\hat{n}_i - 1) V_{i,j} (\hat{n}_j - 1)$$

$$L_\tau \Delta\tau = \beta$$

$$e^{-S\{\Phi(i,\tau)\}} = e^{-4 \sum_{i,j,\tau} \Delta\tau \Phi(i,\tau) V_{i,j}^{-1} \Phi(j,\tau)} \text{Tr} \left[\prod_{\tau=1}^{L_\tau} e^{-\Delta\tau \hat{H}_0} e^{-i\Delta\tau \sum_i \Phi(i,\tau) (\hat{n}_i - 1)} \right]$$

$$= e^{-S_0\{\Phi(i,\tau)\}} [\det M(\Phi)]^2 \quad V \text{ has to be positive definite !}$$

The action is real (particle-hole / time reversal symmetry)

BSS: Work with the Fermion determinant. \rightarrow Memory intensive. CPU time $L^{3d} L_\tau$ Flexible and robust.

HMC: Get rid of fermion determinant \rightarrow Not memory intensive CPU time $(L^d L_\tau)^{1.5}$ provided that M is well conditioned.
(i.e. *far* from GN critical point)

$$\det(M)^2 = \det(MM^\dagger) \propto \int D\{\eta, \eta^\dagger\} e^{-\eta^\dagger (MM^\dagger)^{-1} \eta}$$

Kinetic

$$\hat{H} = \sum_{k=1}^{M_T} \sum_{\sigma=1}^{N_{\text{col}}} \sum_{s=1}^{N_{\text{fl}}} \sum_{x,y} \hat{c}_{x\sigma s}^\dagger T_{xy}^{(ks)} \hat{c}_{y\sigma s} + \sum_{k=1}^{M_V} U_k \left\{ \sum_{\sigma=1}^{N_{\text{col}}} \sum_{s=1}^{N_{\text{fl}}} \left[\left(\sum_{x,y} \hat{c}_{x\sigma s}^\dagger V_{xy}^{(ks)} \hat{c}_{y\sigma s} \right) + \alpha_{ks} \right] \right\}^2$$

Potential (sum of perfect squares)

Coupling of fermions to bosonic fields with predefined dynamics

$$+ \sum_{k=1}^{M_I} \hat{Z}_k \left(\sum_{\sigma=1}^{N_{\text{col}}} \sum_{s=1}^{N_{\text{fl}}} \sum_{x,y} \hat{c}_{x\sigma s}^\dagger I_{xy}^{(ks)} \hat{c}_{y\sigma s} \right) + \hat{H}_{\text{Ising}}$$

- Block diagonal in flavors, N_{fl}
- $SU(N_{\text{col}})$ symmetric in colors N_{col}
- Arbitrary Bravais lattice for $d=1,2$
- Model can be specified at minimal programming cost
- Fortran 2003 standard
- MPI implementation
- Global and local moves, Parallel tempering, Langevin
- Projective and finite T approaches
- pyALF: easy access python interface
- Predefined models



F. Goth



M. Bercx



J. Hoffmann



J. S.E. Portela



J. Schwab



Z. Liu



E. Huffman



A. Götz



F. Parisen Toldin



Wissenschaftliche
Literaturversorgungs
und Informationssysteme (LIS)



Fakher F. Assaad Relativistic Fermions in Flatland: theory and application. 5-9 July 2021 ECT* - Trento (Online)

Fermi velocity renormalization in graphene : Bridging the gap between experiment and theory.

M. Ulybyshev, S. Zafeiropoulos , C. Winterowd, FFA arXiv:2104.09655



SFB1170
ToCoTronics



Leibniz-Rechenzentrum
der Bayerischen Akademie der Wissenschaften



Gauss Centre for Supercomputing



Center of excellence – complexity and
topology in quantum matter
KONWIHR

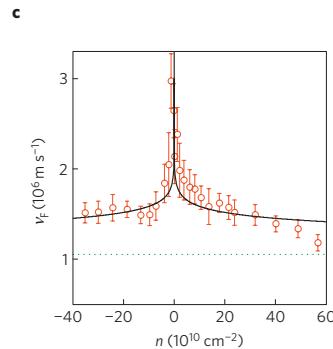
Fermi velocity renormalization: Bridging the gap between experiment and theory

Experiment



Dirac cones reshaped by interaction effects in suspended graphene

D. C. Elias¹, R. V. Gorbachev¹, A. S. Mayorov¹, S. V. Morozov², A. A. Zhukov³, P. Blake³, L. A. Ponomarenko¹, I. V. Grigorieva¹, K. S. Novoselov¹, F. Guinea^{4*} and A. K. Geim^{1,3}



Theory

PRL 106, 236805 (2011)

PHYSICAL REVIEW LETTERS

week ending
10 JUNE 2011

Strength of Effective Coulomb Interactions in Graphene and Graphite

T. O. Wehling,¹ E. Şaşioğlu,² C. Friedrich,² A. I. Lichtenstein,¹ M. I. Katsnelson,³ and S. Blügel²

	Graphene		Graphite	
	Bare	cRPA	Bare	cRPA
$U_{00}^{A \text{ or } B}$ (eV)	17.0	9.3	17.5, 17.7	8.0, 8.1
U_{01} (eV)	8.5	5.5	8.6	3.9
$U_{02}^{A \text{ or } B}$ (eV)	5.4	4.1	5.4, 5.4	2.4, 2.4
U_{03} (eV)	4.7	3.6	4.7	1.9

Add tail that interpolates between cRPA and $\epsilon = 1$

Comparison Experiments involve finite doping.

Numerical calculations are done at the particle-hole symmetric point. One then carries out a ridged band shift to compare with experiments.

Fermi velocity renormalization: Bridging the gap between experiment and theory

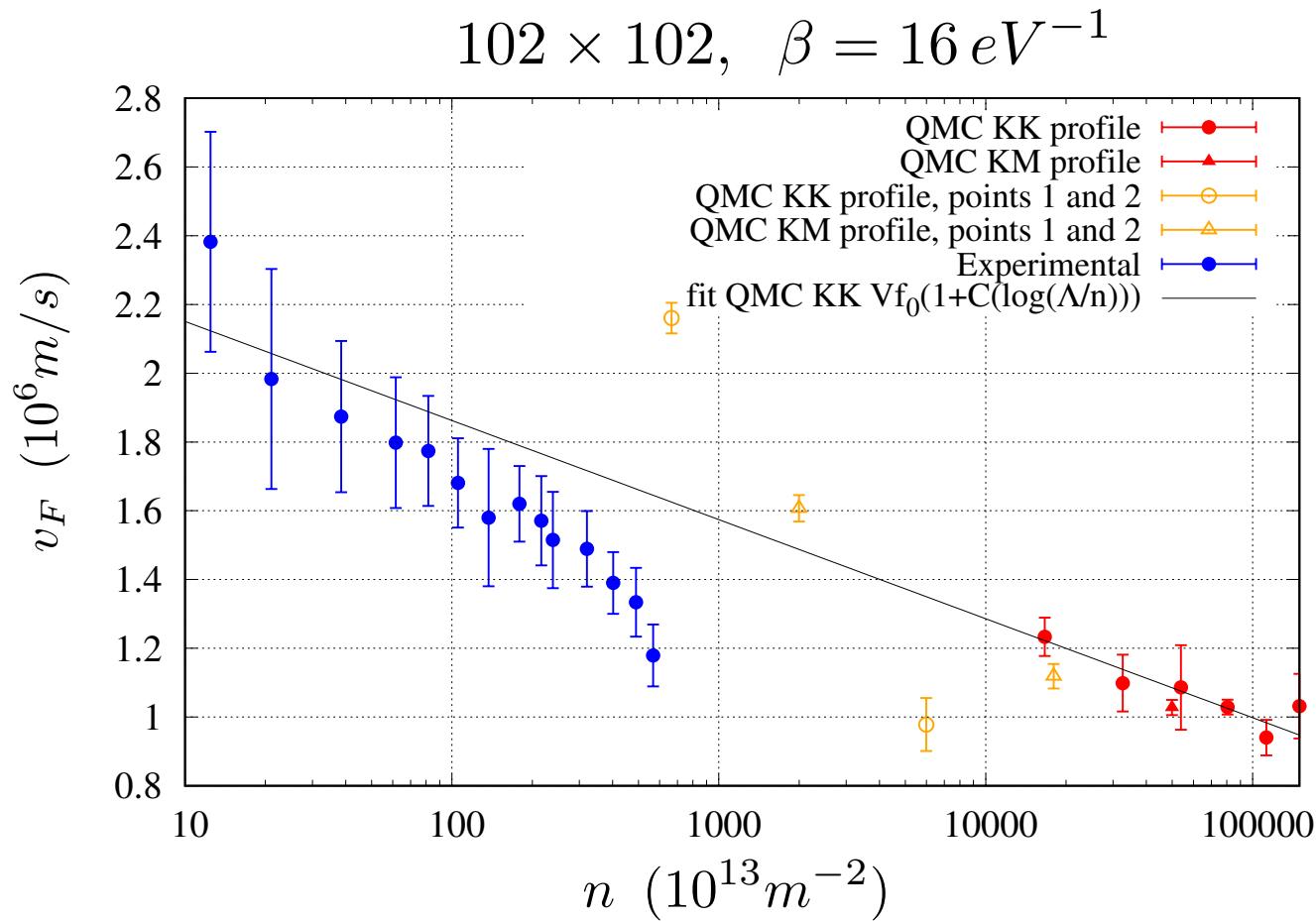
M. Ulybyshev,
S. Zafeiropoulos,
C. Winterowd,
FFA

arXiv:2104.09655

Experiment:

Screening kicks
in at finite doping

→ Deviation from
logarithmic form



→ Maksim's talk this afternoon.

Theory

Deviations from
logarithm.

Temperature effects?

Size effects ?

→ 204 X 204 Lattices
are achievable
and would bridge
the gap between
experiment and
theory.

Fakher F. Assaad Relativistic Fermions in Flatland: theory and application. 5-9 July 2021 ECT* - Trento (Online)

Skyrmion superconductivity

Y. Liu, Z. Wang, T. Sato, M. Hohenadler, C. Wang, W. Guo and FFA
Z. Wang, Y. Liu, T. Sato, M. Hohenadler, C. Wang, W. Guo and FFA
Y. Liu, Z. Wang, T. Sato, W. Guo and FFA arXiv:2103.08434

Nature Communications 10 (2019), 2658
Phys. Rev. Lett. 126, 205701 (2021)



SFB1170
ToCoTronics



Leibniz-Rechenzentrum
der Bayerischen Akademie der Wissenschaften

GCS
Gauss Centre for Supercomputing



TGCC
Très Grand Centre de calcul du CEA

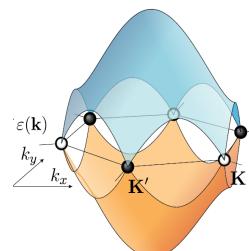


Center of excellence – complexity and
topology in quantum matter



Why would it be interesting to dynamically generate a QSH state?

$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + \text{H.c.}) + \lambda \sum_{\bigcirc=x} N(x) \cdot \sum_{\langle\langle i,j \rangle\rangle \in \bigcirc} (i\nu_{ij} \hat{c}_i^\dagger \boldsymbol{\sigma} \hat{c}_j + \text{H.c.})$$



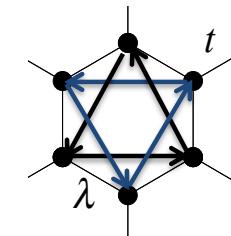
$$L_0 = \sum_{\sigma} \bar{\psi}_{\sigma}(\mathbf{x}, \tau) \partial_{\mu} \gamma_{\mu} \psi_{\sigma}(\mathbf{x}, \tau)$$

$$\{\gamma_{\mu}, \gamma_{\nu}\} = 2\delta_{\mu,\nu}, \quad \bar{\psi}_{\sigma} = \psi_{\sigma}^{\dagger} \gamma_0$$

$$N(x) \in S^2$$

$$\hat{c}_i^\dagger = (\hat{c}_{i,\uparrow}^\dagger, \hat{c}_{i,\downarrow}^\dagger)$$

$$j = i + \delta_1 + \delta_2 \rightarrow \nu_{i,j} = \text{sign}[e_z \cdot (\delta_1 \times \delta_2)]$$

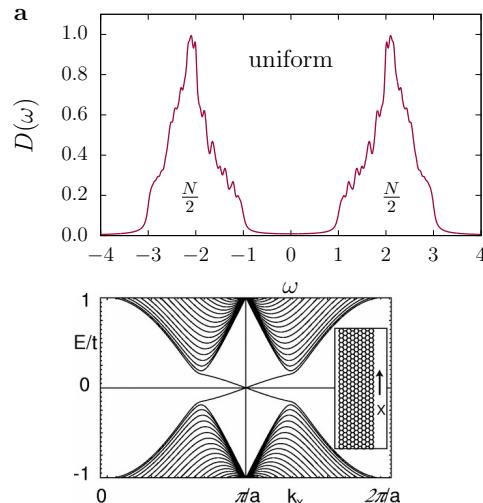


Why would it be interesting to dynamically generate a QSH state?

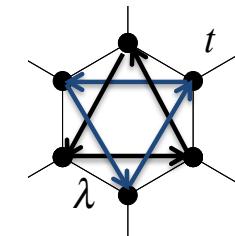
$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + \text{H.c.}) + \lambda \sum_{\bigcirc=x} \mathbf{N}(x) \cdot \sum_{\langle\langle i,j \rangle\rangle \in \bigcirc} (i\nu_{ij} \hat{c}_i^\dagger \boldsymbol{\sigma} \hat{c}_j + \text{H.c.})$$

Uniform $\mathbf{N}(x) = \mathbf{e}_z$

Quantum spin Hall insulator



C. L. Kane and E. J. Mele, PRL, 2005



$$\mathbf{N}(x) \in S^2$$

$$\hat{c}_i^\dagger = (\hat{c}_{i,\uparrow}^\dagger, \hat{c}_{i,\downarrow}^\dagger)$$

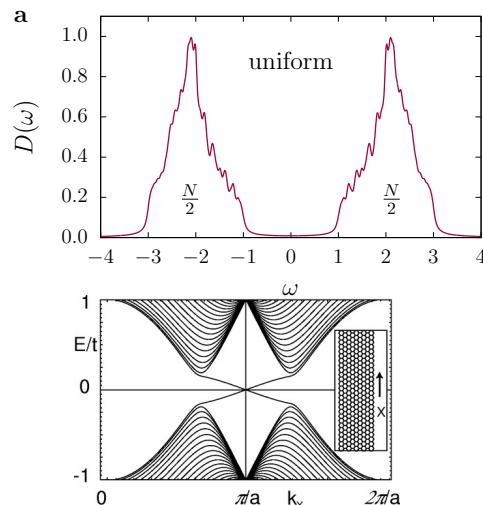
$$j = i + \delta_1 + \delta_2 \rightarrow \nu_{i,j} = \text{sign} [e_z \cdot (\delta_1 \times \delta_2)]$$

Why would it be interesting to dynamically generate a QSH state?

$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + \text{H.c.}) + \lambda \sum_{\bigcirc=x} \mathbf{N}(x) \cdot \sum_{\langle\langle i,j \rangle\rangle \in \bigcirc} (i\nu_{ij} \hat{c}_i^\dagger \boldsymbol{\sigma} \hat{c}_j + \text{H.c.})$$

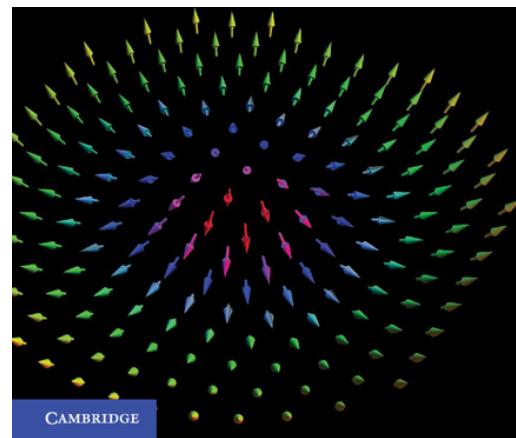
Uniform $\mathbf{N}(x) = \mathbf{e}_z$

Quantum spin Hall insulator



C. L. Kane and E. J. Mele, PRL, 2005

One Skyrmiон

$$Q = \frac{1}{4\pi} \int dx dy \mathbf{N}(x) \cdot (\partial_x \mathbf{N}(x) \times \partial_y \mathbf{N}(x)) = 1$$


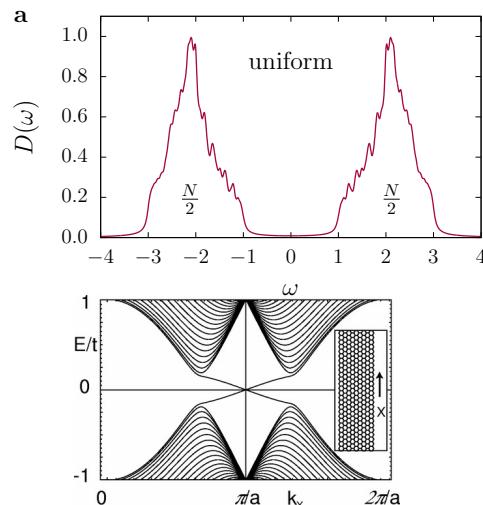
Field theories of condensed matter physics. E. Fradkin

Why would it be interesting to dynamically generate a QSH state?

$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + \text{H.c.}) + \lambda \sum_{\bigcirc=x} \mathbf{N}(x) \cdot \sum_{\langle\langle i,j \rangle\rangle \in \bigcirc} (i\nu_{ij} \hat{c}_i^\dagger \boldsymbol{\sigma} \hat{c}_j + \text{H.c.})$$

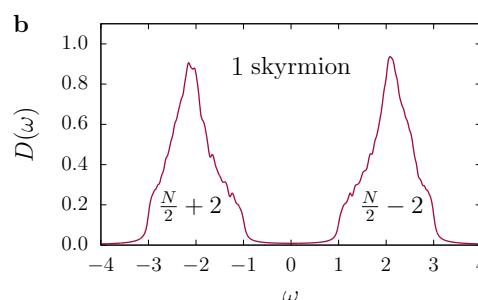
Uniform $\mathbf{N}(x) = \mathbf{e}_z$

Quantum spin Hall insulator



C. L. Kane and E. J. Mele, PRL, 2005

One Skyrmiон

$$Q = \frac{1}{4\pi} \int dx dy \mathbf{N}(x) \cdot (\partial_x \mathbf{N}(x) \times \partial_y \mathbf{N}(x)) = 1$$


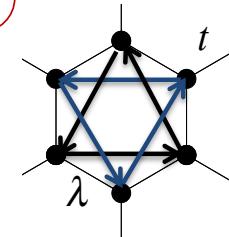
Skyrmiон carries charge 2e → proliferation of
skyrmiions destroys QSH and could lead to SC.

T. Grover and T. Senthil, PRL, 2008.

$$\hat{H} = -t \sum_{\langle ij \rangle} (\hat{c}_i^\dagger \hat{c}_j + \hat{c}_j^\dagger \hat{c}_i) - \lambda \sum_{\bigcirc} \left(\sum_{\langle\langle i,j \rangle\rangle \in \bigcirc} \nu_{i,j} i \left(\hat{c}_i^\dagger \boldsymbol{\sigma} \hat{c}_j - \hat{c}_j^\dagger \boldsymbol{\sigma} \hat{c}_i \right) \right)^2$$

$$\hat{c}_i^\dagger = (\hat{c}_{i,\uparrow}^\dagger, \hat{c}_{i,\downarrow}^\dagger)$$

$$j = i + \delta_1 + \delta_2 \rightarrow \nu_{i,j} = \text{sign} [e_z \cdot (\delta_1 \times \delta_2)]$$



Order parameters

$$\mathbf{J}_{\bigcirc} = \sum_{\langle\langle i,j \rangle\rangle \in \bigcirc} \nu_{i,j} \left(i \hat{c}_i^\dagger \boldsymbol{\sigma} \hat{c}_j - i \hat{c}_j^\dagger \boldsymbol{\sigma} \hat{c}_i \right)$$

Quantum spin Hall [SO(3)]

$$\hat{\Delta}_i^\dagger = \hat{c}_{i,\uparrow}^\dagger \hat{c}_{i,\downarrow}^\dagger$$

s-wave superconductor [U(1)]

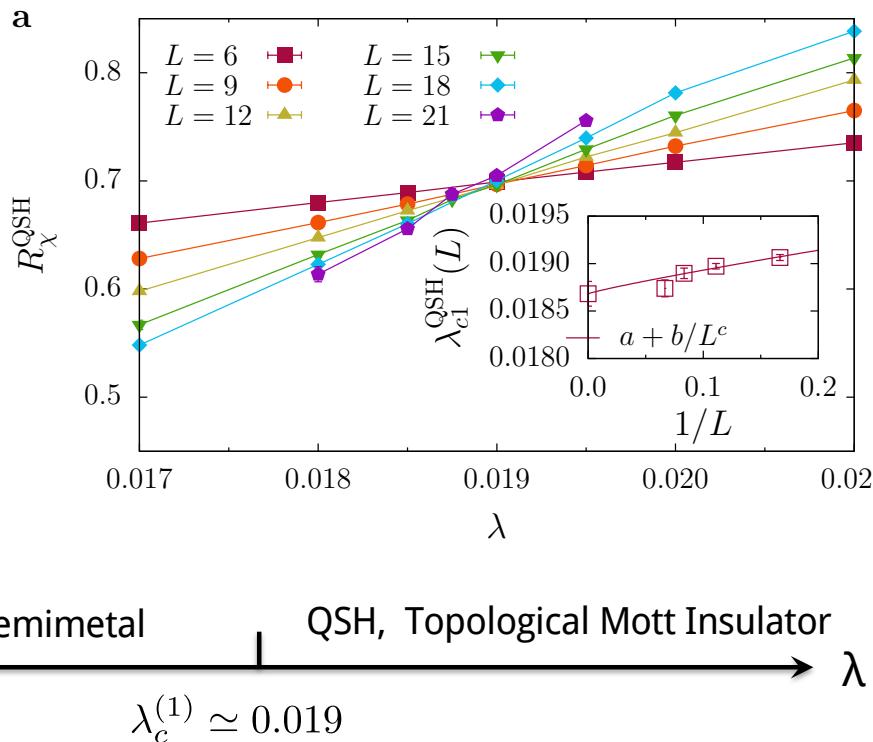
Skyrmion superconductivity

Y. Liu, Z. Wang, T. Sato, M. Hohenadler, C. Wang, W. Guo and FFA
Nature Communications 10 (2019), 2658
Phys. Rev. Lett. 126, 205701 (2021)

$$\chi_{QSH}(q) = \frac{1}{N} \int_0^\beta d\tau \sum_{i,j} \langle \vec{J}_{\bigcirc_i}(\tau) \vec{J}_{\bigcirc_j} \rangle e^{iq(i-j)}$$

$$R_{QSH} = 1 - \frac{\chi_{QSH}(q_0 + \delta q)}{\chi_{QSH}(q_0)}$$

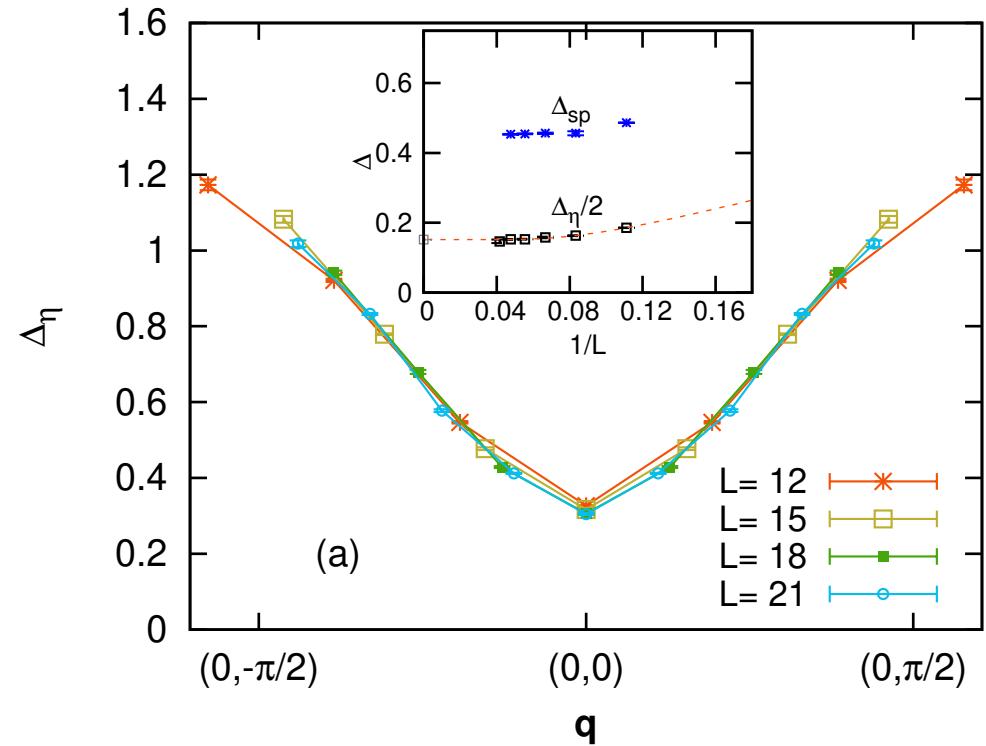
$$R_{QSH}(L, \lambda) = F(L^z/\beta, (\lambda - \lambda_c)L^{1/\nu}, L^{-\omega})$$



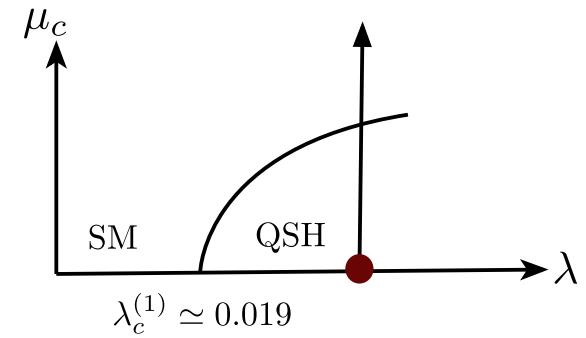
	$1/\nu$	η_ϕ	η_ψ
This study	1.11(4)	0.80(9)	0.29(2)
Ref. ⁴ (AFQMC)	0.95(5)	0.75(4)	0.23(4)
Ref. ⁴⁶ (HMC)	0.861	0.872(22)	—
Ref. ²⁵ (AFQMC)	1.14(9)	0.79(5)	—
Ref. ⁶ (AFQMC)	0.98(1)	0.49(2)	0.20(2)
Ref. ⁷ (AFQMC)	1.19(6)	0.70(15)	—
Ref. ⁴⁷ ($4 - \epsilon$), ϵ^4 , Padé [2/2]	0.6426	0.9985	0.1833
Ref. ⁴⁷ ($4 - \epsilon$), ϵ^4 , Padé [3/1]	0.6447	0.9563	0.1560
Ref. ⁴⁸ FRG	0.795	1.032	0.071
Ref. ⁴⁹ FRG	0.76	1.01	0.08

Y. Liu, Z. Wang, T. Sato, W. Guo and FFA arXiv:2103.08434

Preformed pairs ?



Two holes pair since: $2\Delta_{sp} - \Delta_\eta > 0$

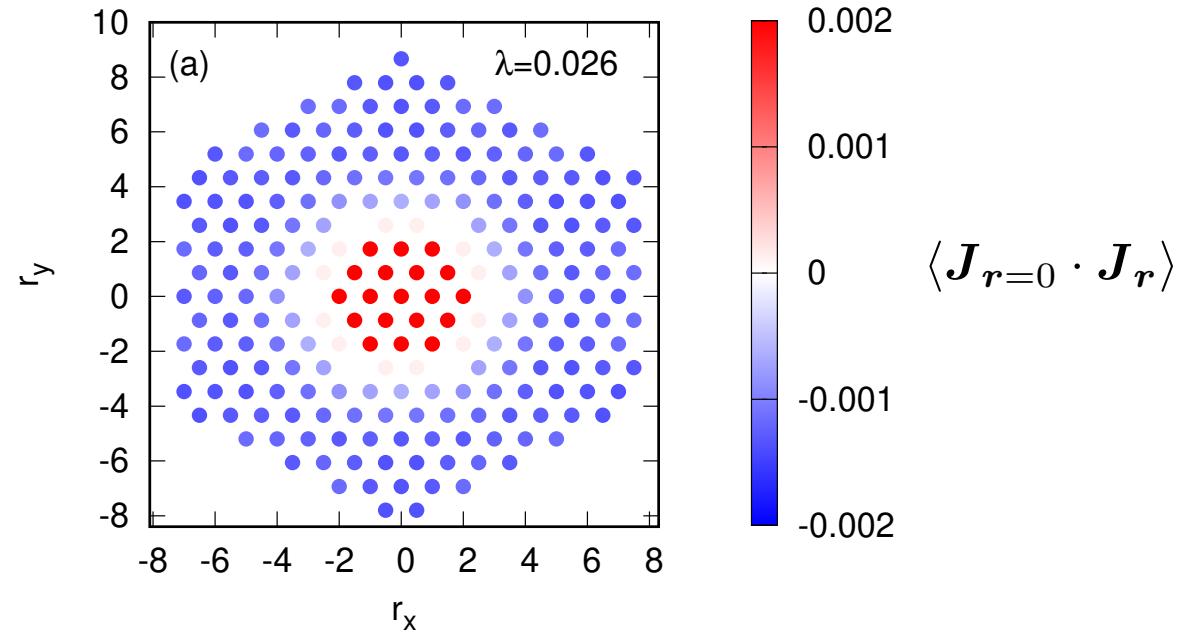
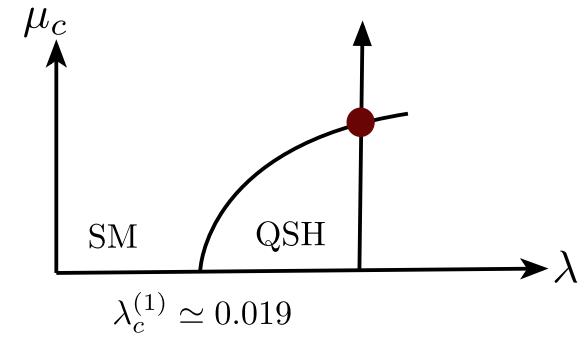


Do the hole pairs correspond to Skyrmions?

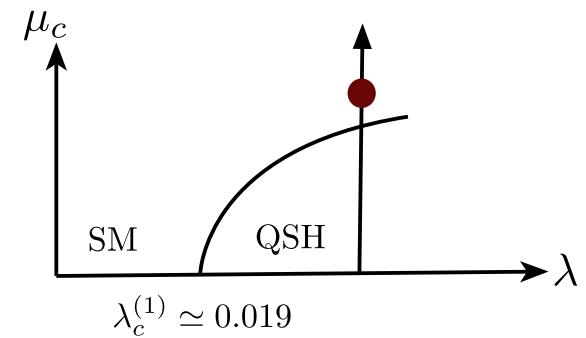
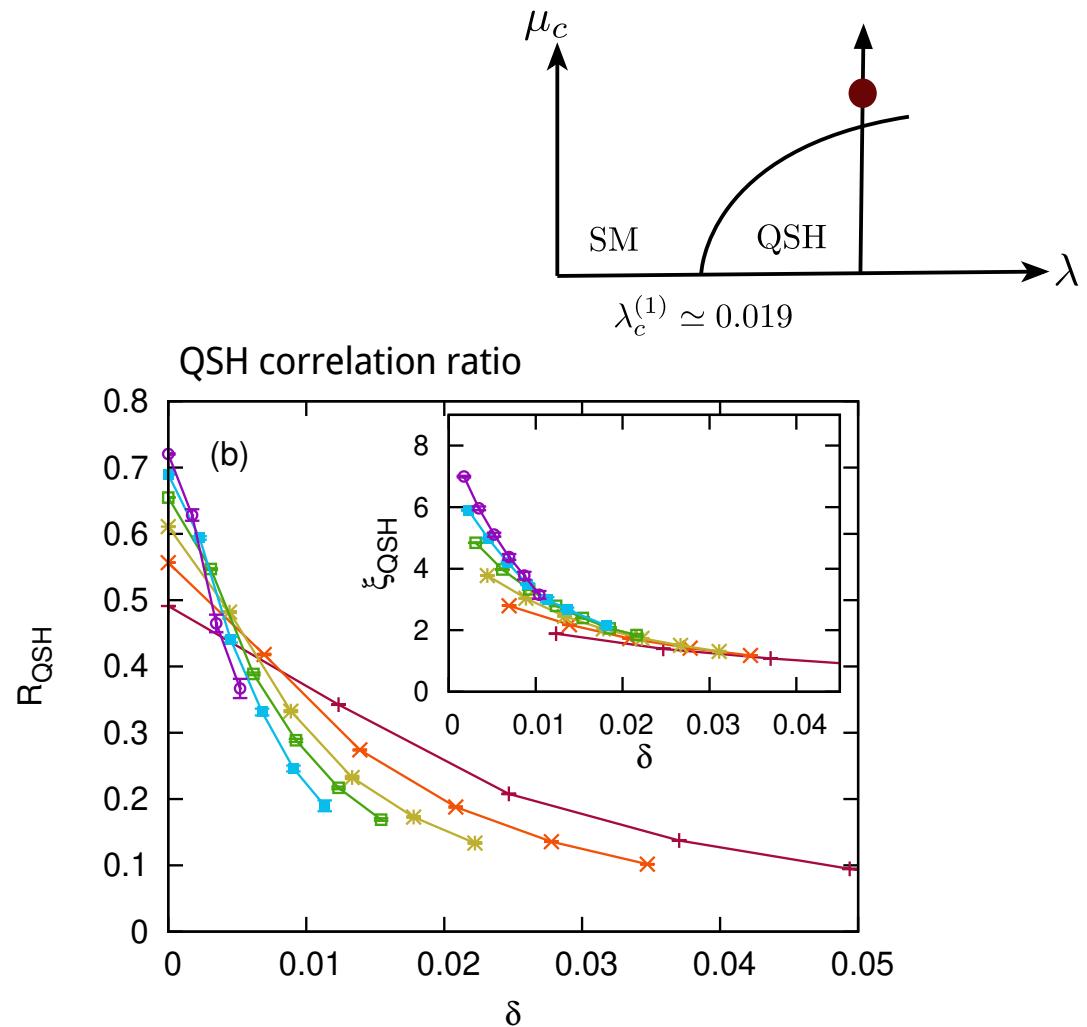
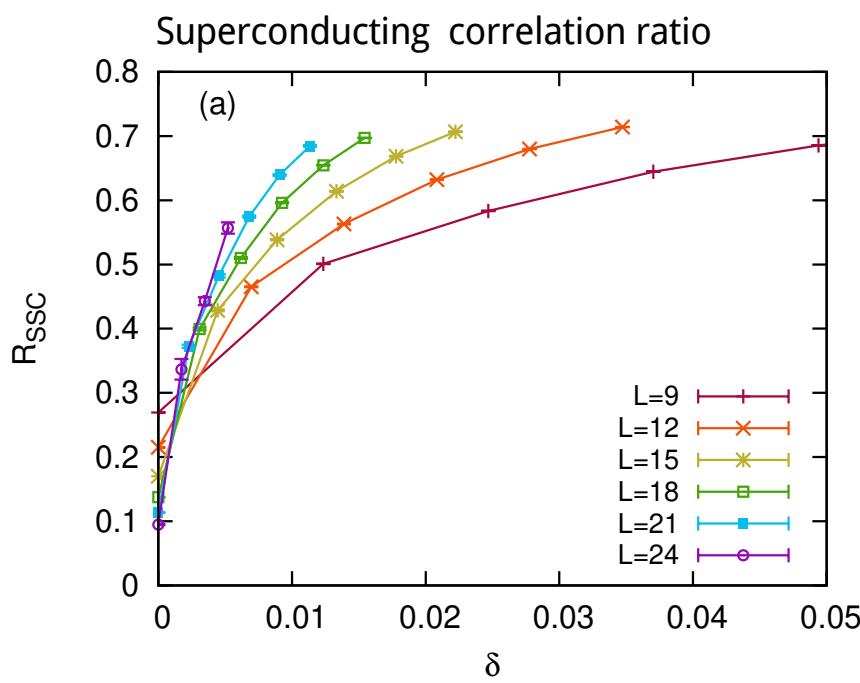
Imaging the Skyrmion:

$$\rho(\mathbf{r}) = \frac{2e}{4\pi} \mathbf{N}(\mathbf{r}) \cdot \left(\frac{\partial}{\partial r_x} \mathbf{N}(\mathbf{r}) \times \frac{\partial}{\partial r_y} \mathbf{N}(\mathbf{r}) \right)$$

1. Dope two holes away from half-filling and pin them at the origin by modulating the chemical potential
2. Measure real space spin-spin correlations around the hole pair

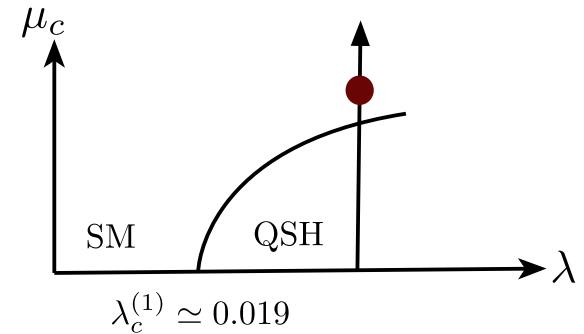
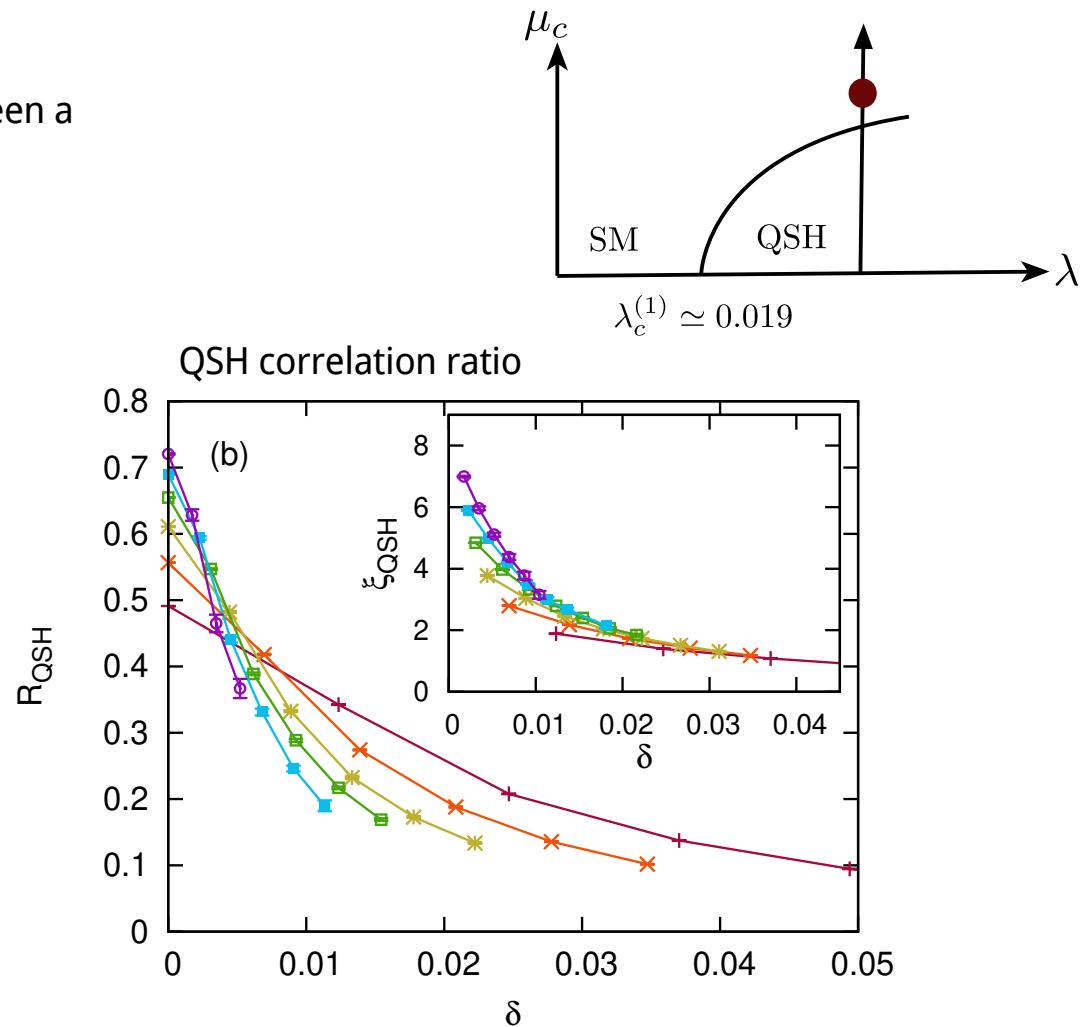
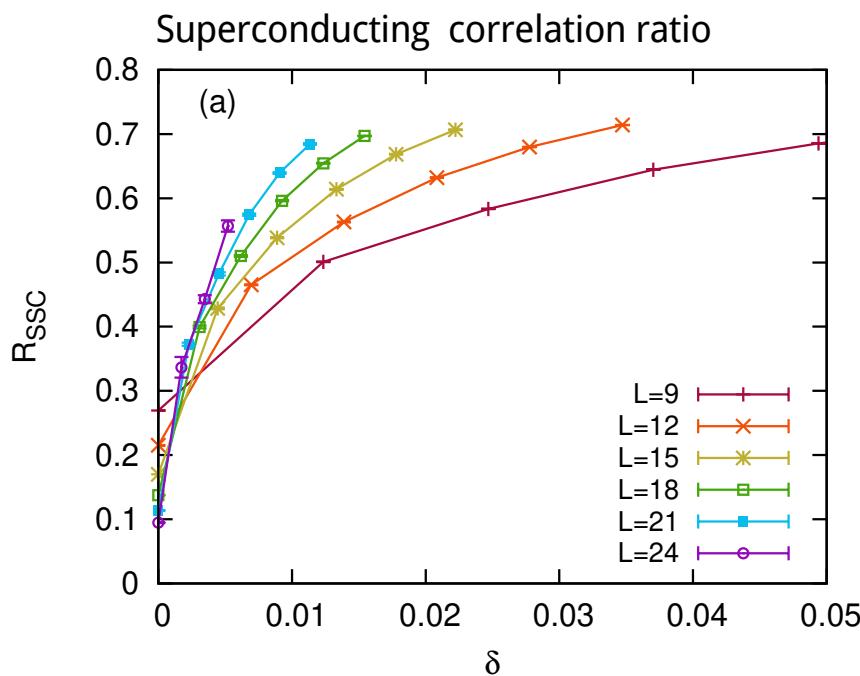


Nature of doping induced transition?



Nature of doping induced transition ?

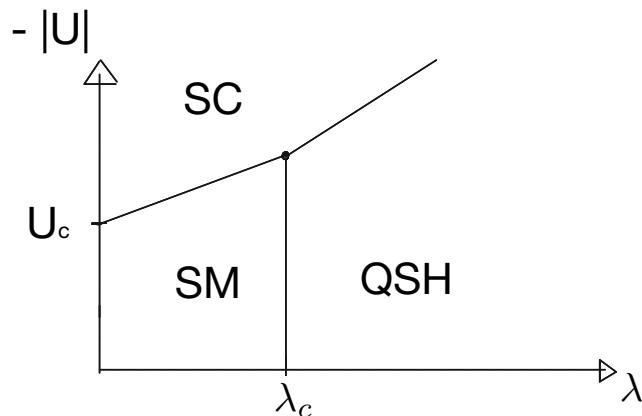
- Direct and continuous doping induced transition between a QSH insulator and s-wave superconductor
- Resolution $\delta = 0.0017$



Bandwidth induced transition.

Add an attractive Hubbard U-term

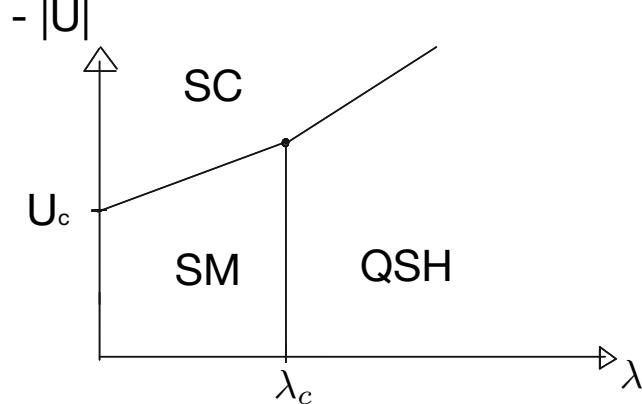
(Is technically possible and does not trigger a negative sign problem)



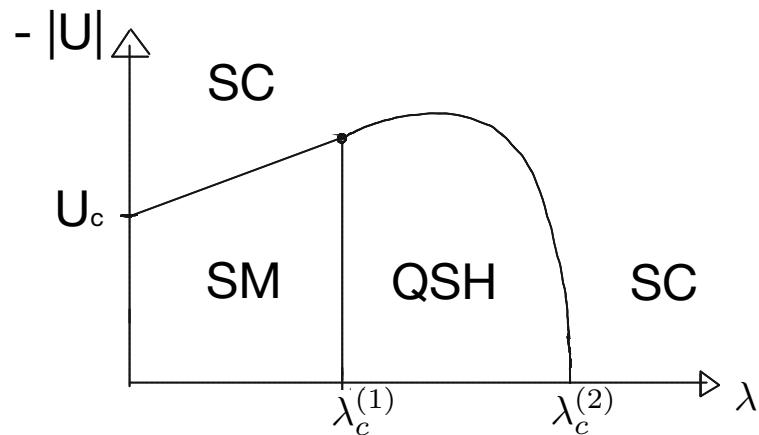
Bandwidth induced transition.

Add an attractive Hubbard U-term

(Is technically possible and does not trigger a negative sign problem)



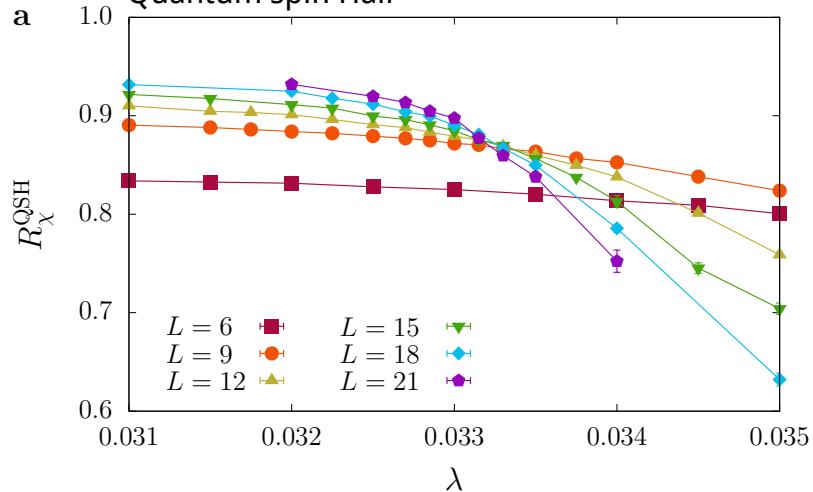
Adding a Hubbard U-term is not necessary!



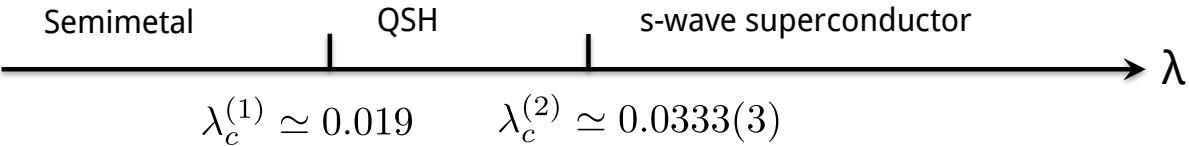
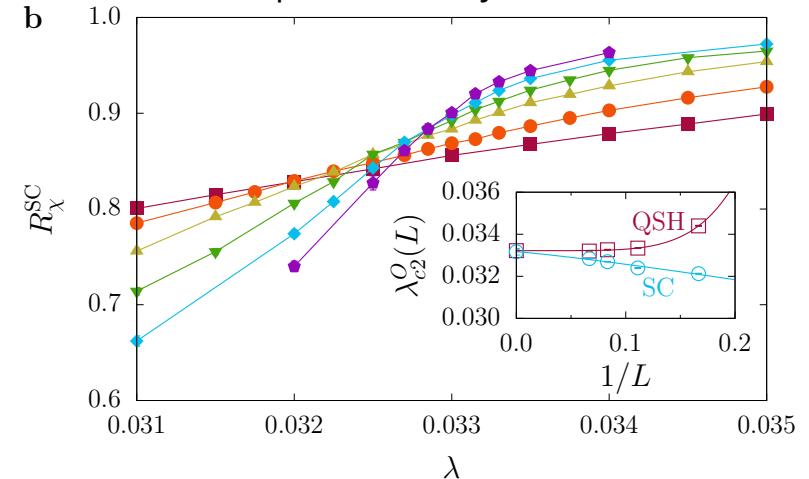
Skyrmiон superconductivity

Y. Liu, Z. Wang, T. Sato, M. Hohenadler, C. Wang, W. Guo and FFA
Nature Communications 10 (2019), 2658
Phys. Rev. Lett. 126, 205701 (2021)

a Quantum spin Hall



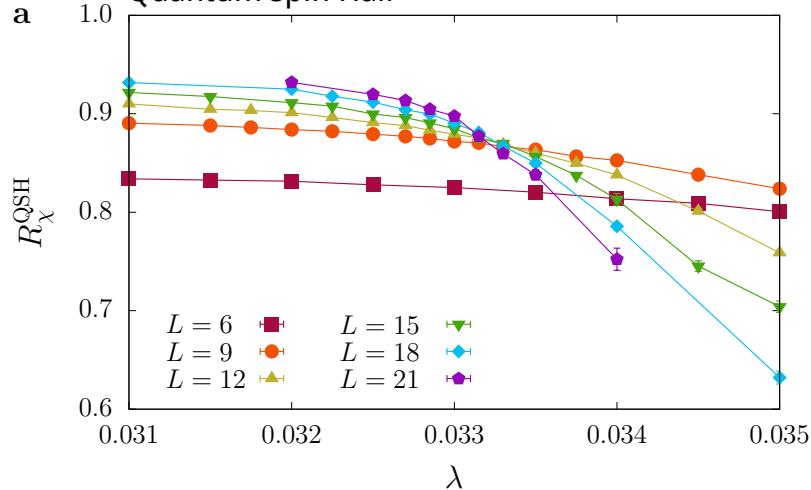
b s-wave superconductivity



Skyrmiон superconductivity

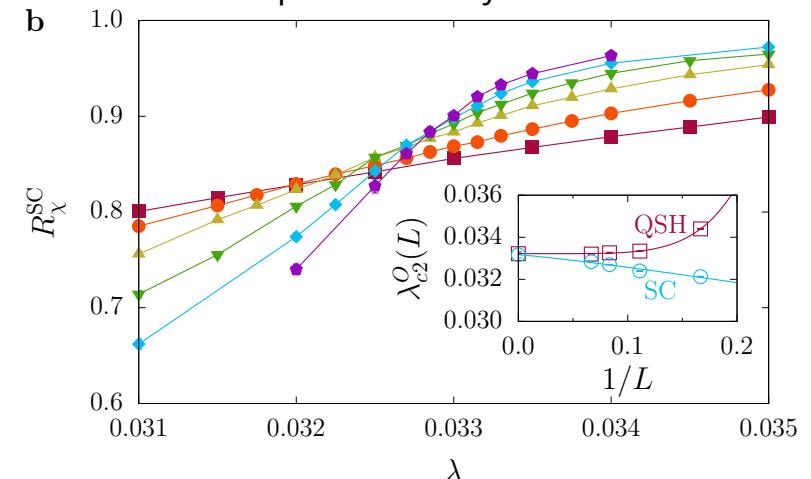
Y. Liu, Z. Wang, T. Sato, M. Hohenadler, C. Wang, W. Guo and FFA
Nature Communications 10 (2019), 2658
Phys. Rev. Lett. 126, 205701 (2021)

a Quantum spin Hall

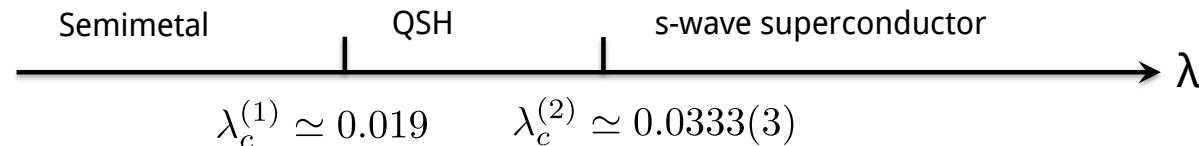
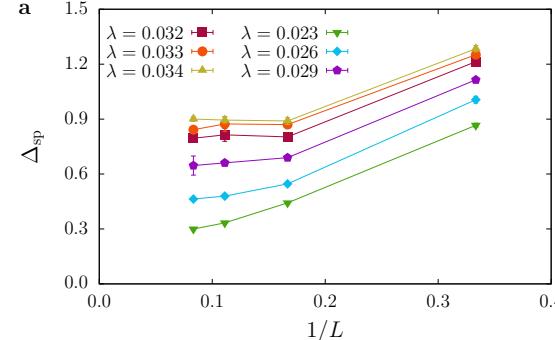


Single particle gap remains open over the transition

b s-wave superconductivity

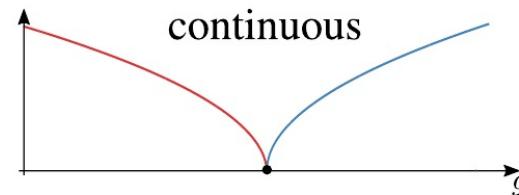
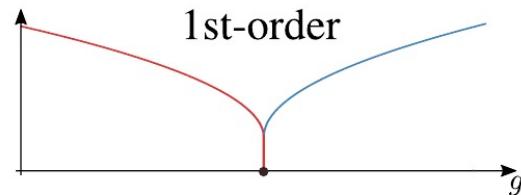
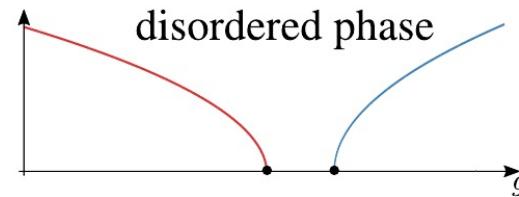
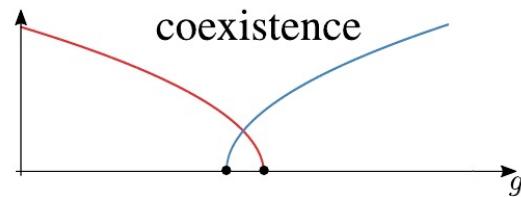


a



Direct and continuous transition between two broken symmetry phases.

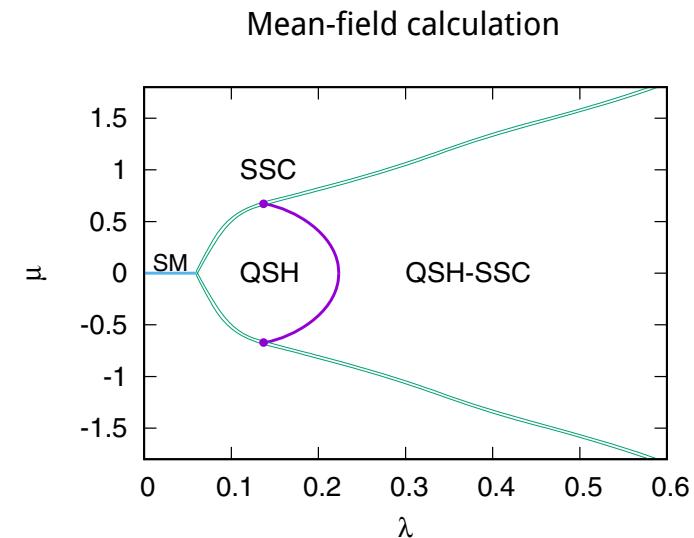
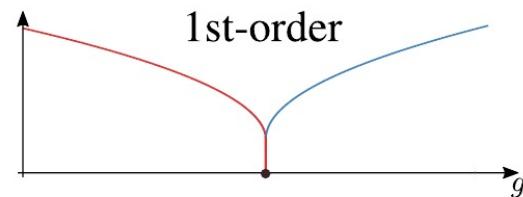
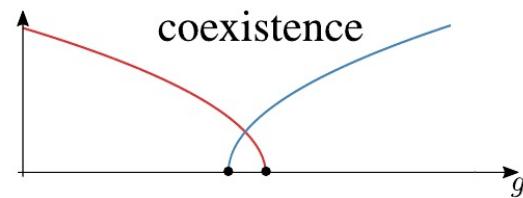
Can we understand this in the realm of Ginzburg-Landau order parameter theory?



Fine tuning \rightarrow not generic

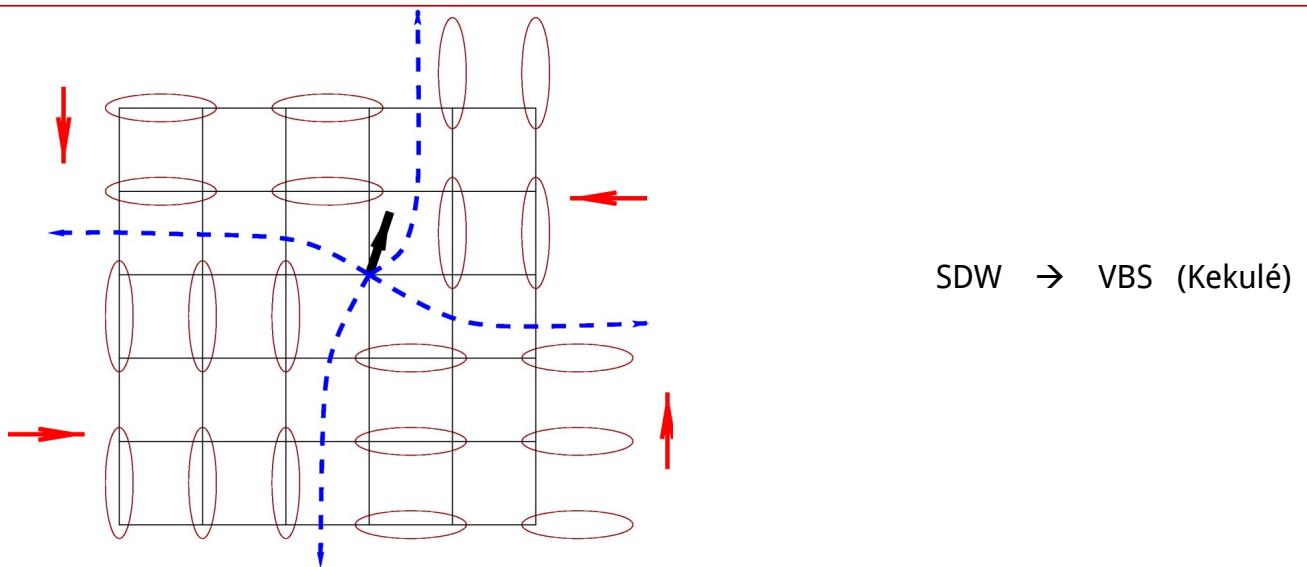
Direct and continuous transition between two broken symmetry phases.

Can we understand this in the realm of Ginzburg-Landau order parameter theory?



Deconfined quantum criticality T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M. P. A. Fisher, Science 303 (2004), 1490–1494.

Topological defects of one phase carry the charge of the other. When they condense they simultaneously destroy one phase and create the other.



C₄ VBS vortex carries a spinon. Proliferation of vortices destroys the VBS and generates SDW order.

M. Levin and T. Senthil, Phys. Rev. B 70 220403 (2004).

Why is the quantum spin Hall state intertwined with s-wave superconductivity?

The three quantum spin Hall mass terms are part of the quintuplet of mutually **anti-commuting** QSH and s-wave superconductivity mass terms.

$$M = \{ \underbrace{\tau_z \sigma_x i \gamma_0 \gamma_3 \gamma_5, \tau_0 \sigma_y i \gamma_0 \gamma_3 \gamma_5, \tau_z \sigma_z i \gamma_0 \gamma_3 \gamma_5}_{M_{QSH}}, \underbrace{\tau_y \sigma_y i \gamma_0 \gamma_2 \gamma_3, \tau_x \sigma_y i \gamma_0 \gamma_2 \gamma_3}_{M_{SC}} \}$$

Bogoliubov index

s-wave superconductivity

A. Tanaka and X. Hu, PRL 95 (2005), 036402.

Def: Mass term

$$H(k) = \overbrace{\sum_{i=1}^2 i k_i \gamma_0 \gamma_i}^{H_0(k)} + m M$$

M is a mass term if $\{M, H_0(k)\} = 0$ and $M^2 = 1 \rightarrow E(k) = \pm \sqrt{k^2 + m^2}$

Skyrmion superconductivity

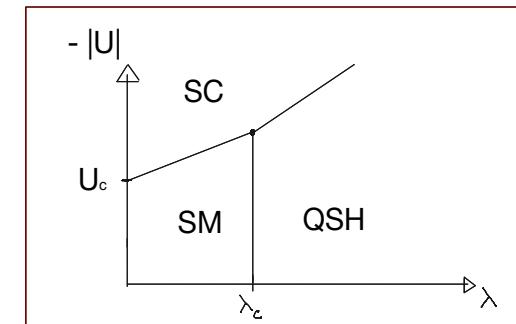
Y. Liu, Z. Wang, T. Sato, M. Hohenadler, C. Wang, W. Guo and FFA
 Nature Communications 10 (2019), 2658
 Phys. Rev. Lett. 126, 205701 (2021)

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_{SO(3) \times U(1)}(\varphi(\mathbf{x}, \tau))$$

$$\mathcal{L}_F = \mathcal{L}_{\text{Dirac}} + g\Psi^\dagger(\mathbf{x}, \tau) \overbrace{\begin{bmatrix} \Phi(\mathbf{x}, \tau) \\ \chi(\mathbf{x}, \tau) \end{bmatrix}}^{\equiv \varphi(\mathbf{x}, \tau)} \cdot \begin{bmatrix} \mathbf{M}_{QSH} \\ \mathbf{M}_{SC} \end{bmatrix} \Psi(\mathbf{x}, \tau)$$

\mathcal{L}_F has SO(5) symmetry

$$\mathcal{L}_{\text{Dirac}} + g\Psi^\dagger(\mathbf{x}, \tau) \left(R^{SO(5)} \begin{bmatrix} \Phi(\mathbf{x}, \tau) \\ \chi(\mathbf{x}, \tau) \end{bmatrix} \right) \cdot \begin{bmatrix} \mathbf{M}_{QSH} \\ \mathbf{M}_{SC} \end{bmatrix} \Psi(\mathbf{x}, \tau) = \mathcal{L}'_{\text{Dirac}} + g\Psi'^\dagger(\mathbf{x}, \tau) \begin{bmatrix} \Phi(\mathbf{x}, \tau) \\ \chi(\mathbf{x}, \tau) \end{bmatrix} \cdot \begin{bmatrix} \mathbf{M}_{QSH} \\ \mathbf{M}_{SC} \end{bmatrix} \Psi'(\mathbf{x}, \tau)$$



Thereby the single particle gap is given by: $\Delta_{sp} = g|\varphi|$

Assume that the single particle gap remains finite across the transition, then one can omit

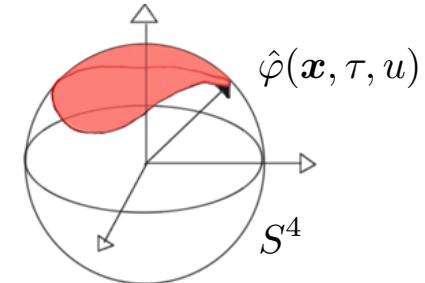
amplitude fluctuations of the field $\varphi(\mathbf{x}, \tau)$ and retain only phase fluctuations $\hat{\varphi}(\mathbf{x}, \tau)$, $|\hat{\varphi}(\mathbf{x}, \tau)| = 1$

A. Tanaka and X. Hu, PRL 95 (2005), 036402.

Integrating out the fermions gives an effective field theory for the field $\hat{\varphi}(\mathbf{x}, \tau)$

$$S = \int d^2\mathbf{x} d\tau \frac{1}{G} (\partial_\mu \hat{\varphi}(\mathbf{x}, \tau))^2 + 2\pi i \Gamma [\hat{\varphi}] + S_{SO(3) \times U(1)}$$

$$\Gamma [\hat{\varphi}] = \frac{\epsilon_{abcde}}{\text{Area}(S^4)} \int_0^1 du \int d^2\mathbf{x} d\tau \hat{\varphi}_a \partial_x \hat{\varphi}_b \partial_y \hat{\varphi}_c \partial_\tau \hat{\varphi}_d \partial_u \hat{\varphi}_e$$



$$\begin{aligned}\hat{\varphi}(\mathbf{x}, \tau, u=0) &= \mathbf{e}_z \\ \hat{\varphi}(\mathbf{x}, \tau, u=1) &= \hat{\varphi}(\mathbf{x}, \tau)\end{aligned}$$

$$S_{SO(3) \times U(1)} = \int d^2\mathbf{x} d\tau \alpha_{SO(3) \times U(1)} (\varphi_1^2 + \varphi_2^2 + \varphi_3^2 - \varphi_4^2 - \varphi_5^2) + \dots$$

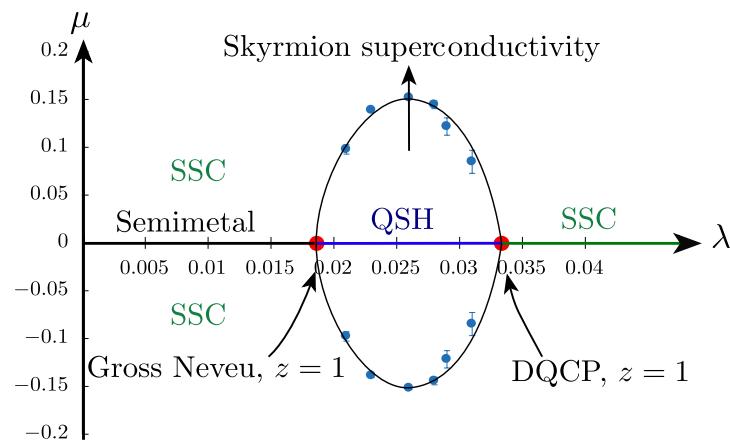
A. Tanaka and X. Hu, PRL 95 (2005), 036402.

T. Grover and T. Senthil, PRL. 2008.

Model	Symmetries	Anomalous Dimension	Correlation length
JQ ⁽¹⁾ /Loop ⁽²⁾	SO(3)xZ ₄	$\eta^{\text{AFM}} = 0.259(6)$, $\eta^{\text{VBS}} = 0.25(3)$	$1/\nu = 2.24(4)$
QSH-SC	SO(3)xU(1)	$\eta^{\text{QSH}} = 0.21(5)$, $\eta^{\text{SC}} = 0.22(6)$	$1/\nu^{\text{QSH}} = 1.7(4)$, $1/\nu^{\text{SC}} = 1.8(2)$

(1) H. Shao, W. Guo, and A. W. Sandvik,
 Science 352 (2016), no. 6282, 213–216.

(2) A. Nahum et. al. Phys. Rev. X 5 (2015), 041048.



Fakher F. Assaad Relativistic Fermions in Flatland: theory and application. 5-9 July 2021 ECT* - Trento (Online)

Phases of (2+1) dimension SO(5) non-linear sigma model with WZW term

$$S = \int d^2x d\tau \frac{1}{G} (\partial_\mu \hat{\varphi}(x, \tau))^2 + 2\pi i \Gamma [\hat{\varphi}]$$

$$\Gamma [\hat{\varphi}] = \frac{\epsilon_{abcde}}{\text{Area}(S^4)} \int_0^1 du \int d^2x d\tau \hat{\varphi}_a \partial_x \hat{\varphi}_b \partial_y \hat{\varphi}_c \partial_\tau \hat{\varphi}_d \partial_u \hat{\varphi}_e$$

Matteo Ippoliti, Roger S. K. Mong, Fakher F. Assaad, and Michael P. Zaletel, Phys. Rev. B 98 (2018), 235108.

Zhenjiu Wang, Michael P. Zaletel, Roger S. K. Mong, and Fakher F. Assaad, Phys. Rev. Lett. 126 (2021), 045701.



SFB1170
ToCoTronics

lrz
Leibniz-Rechenzentrum
der Bayerischen Akademie der Wissenschaften

GCS
Gauss Centre for Supercomputing



TGCC
Très Grand Centre de calcul du CEA

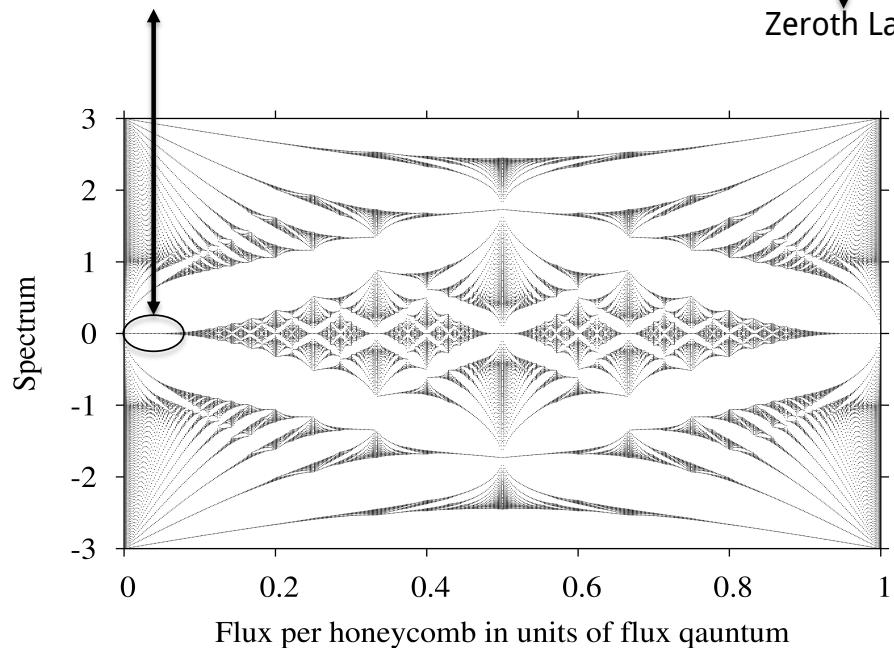


KONWIHR

Single particle Hilbert space
has dimension $4N_\phi$

$$\hat{\psi}_\alpha^\dagger(\mathbf{x}) = \sum_{k_y=1}^{N_\phi} \langle k_y, n=0 | \mathbf{x} \rangle \hat{c}_{k_y, n=0, \alpha}^\dagger, \quad \alpha = 1 \dots 4$$

Zeroth Landau level

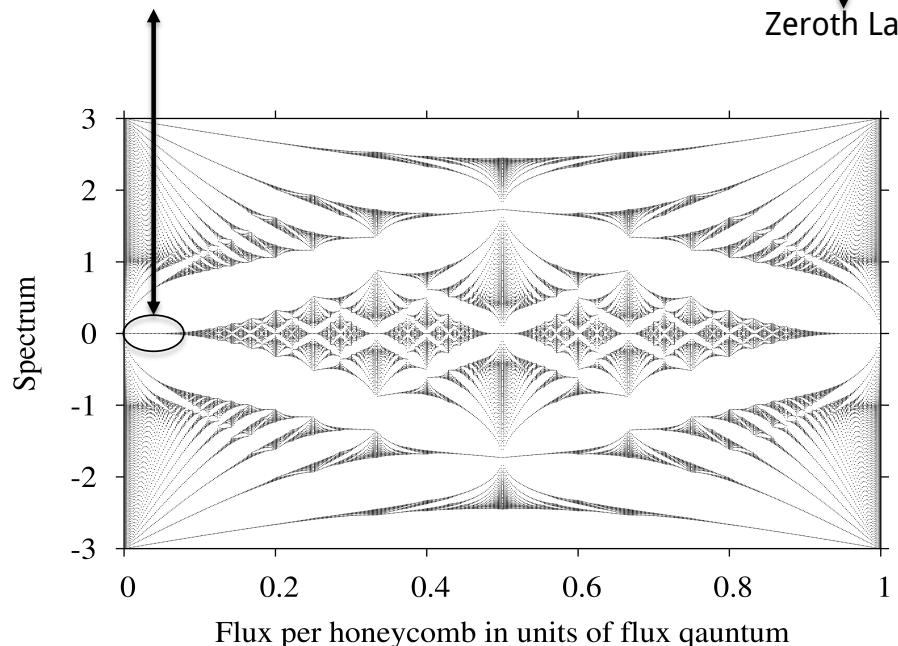


$$\hat{H} = \int d^2x \left[U_0 \left(\hat{\psi}^\dagger(x) \hat{\psi}(x) \right)^2 - U_M \left(\hat{\psi}^\dagger(x) \tau_z \boldsymbol{\sigma} \hat{\psi}(x) \right)^2 - U_K \left(\hat{\psi}^\dagger(x) \tau_x \hat{\psi}(x) \right)^2 - U_K \left(\hat{\psi}^\dagger(x) \tau_y \hat{\psi}(x) \right)^2 \right]$$

Single particle Hilbert space
has dimension $4N_\phi$

$$\hat{\psi}_\alpha^\dagger(x) = \sum_{k_y=1}^{N_\phi} \langle k_y, n=0 | x \rangle \hat{c}_{k_y, n=0, \alpha}^\dagger, \quad \alpha = 1 \dots 4$$

↓
Zeroth Landau level



Quintuplet of anti-commuting mass terms

$$\mathbf{M} = (\tau_z \boldsymbol{\sigma}, \tau_x, \tau_y)$$

$$M_{a,b} \equiv -\frac{i}{2} [M_a, M_b], \quad a > b = 1 \dots 5$$

10 generators of SO(5)

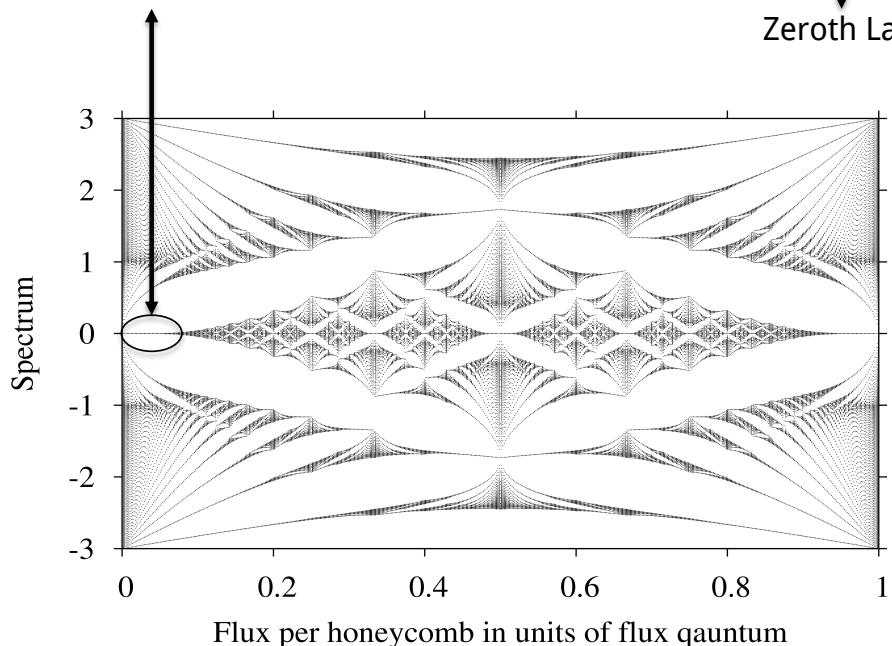
$$@ \quad U_K = U_M \quad [\hat{H}, \hat{M}_{a,b}] = 0$$

$$\hat{H} = \int d^2x \left[U_0 \left(\hat{\psi}^\dagger(x) \hat{\psi}(x) \right)^2 - U_M \left(\hat{\psi}^\dagger(x) \tau_z \boldsymbol{\sigma} \hat{\psi}(x) \right)^2 - U_K \left(\hat{\psi}^\dagger(x) \tau_x \hat{\psi}(x) \right)^2 - U_K \left(\hat{\psi}^\dagger(x) \tau_y \hat{\psi}(x) \right)^2 \right]$$

Single particle Hilbert space has dimension $4N_\phi$

$$\hat{\psi}_\alpha^\dagger(x) = \sum_{k_y=1}^{N_\phi} \langle k_y, n=0 | x \rangle \hat{c}_{k_y, n=0, \alpha}^\dagger, \quad \alpha = 1 \dots 4$$

↓
Zeroth Landau level



PRL 114, 226801 (2015)

PHYSICAL REVIEW LETTERS

 Wess-Zumino-Witten Terms in Graphene Landau Levels

Junhyun Lee¹ and Subir Sachdev^{1,2}

$$U = U_K = U_M$$

$$S = \int d^2x d\tau \frac{1}{G} (\partial_\mu \hat{\varphi}(x, \tau))^2 + 2\pi i \Gamma[\hat{\varphi}]$$

$$\Gamma[\hat{\varphi}] = \frac{\epsilon_{abcde}}{\text{Area}(S^4)} \int_0^1 du \int d^2x d\tau \hat{\varphi}_a \partial_x \hat{\varphi}_b \partial_y \hat{\varphi}_c \partial_\tau \hat{\varphi}_d \partial_u \hat{\varphi}_e$$

Candidate theory for critical point of DQCP.

Numerical results.

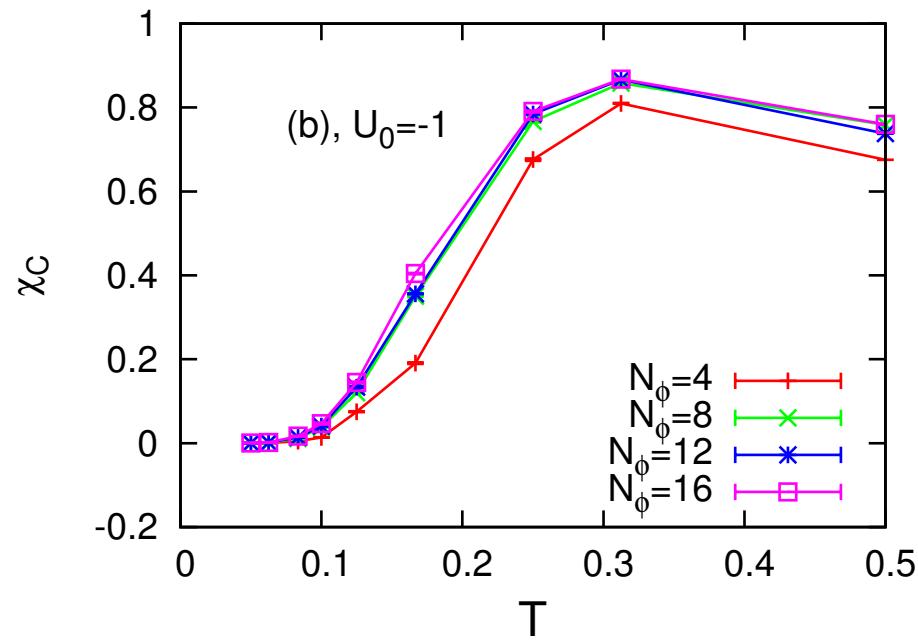
$U_K = U_M = U = 1$ Sign free formulation possible for $U_0 > -1$

$CPU \propto N_\phi^5 \beta \propto V^5 \beta$

The half-filled Landau level is insulating.

$$\chi_C = \frac{\beta}{V} [\langle \hat{n}_{q=0} \hat{n}_{q=0} \rangle - \langle \hat{n}_{q=0} \rangle \langle \hat{n}_{q=0} \rangle]$$

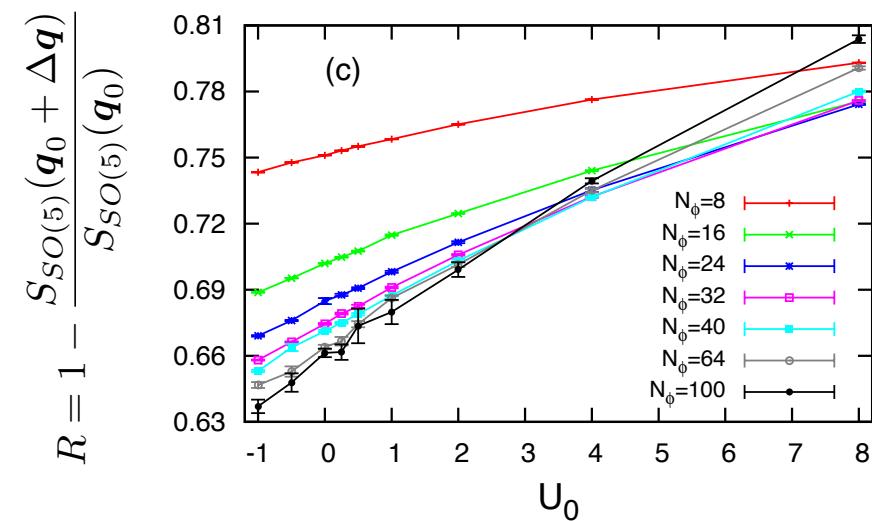
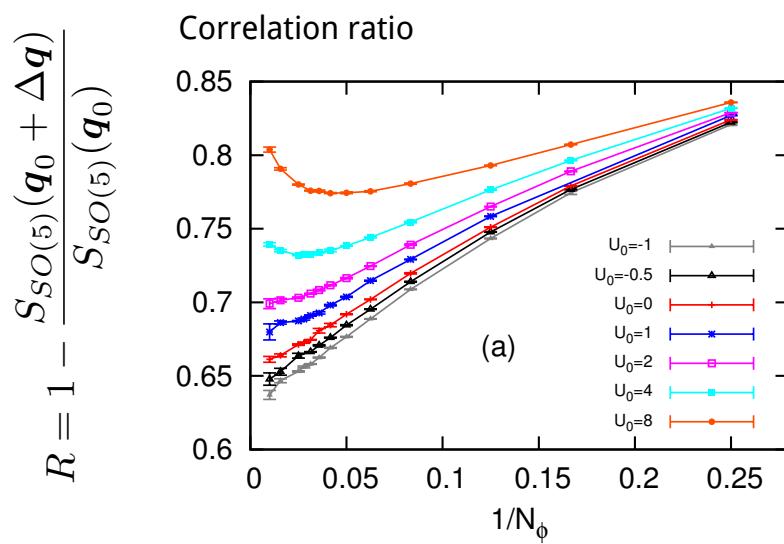
$$\hat{n}(\mathbf{q}) = \frac{1}{\sqrt{V}} \int_V d^2x e^{i\mathbf{q} \cdot \mathbf{x}} \hat{\Psi}^\dagger(\mathbf{x}) \hat{\Psi}(\mathbf{x})$$



Numerical results.

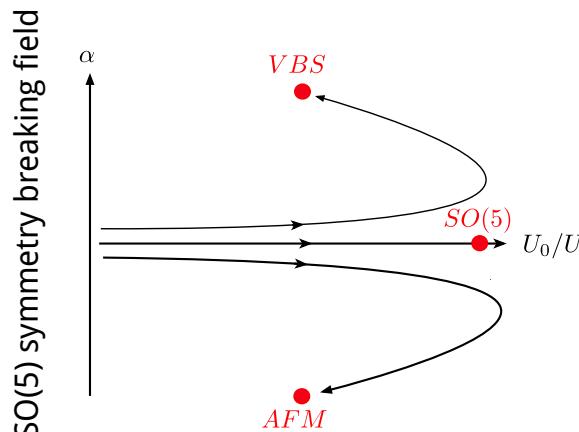
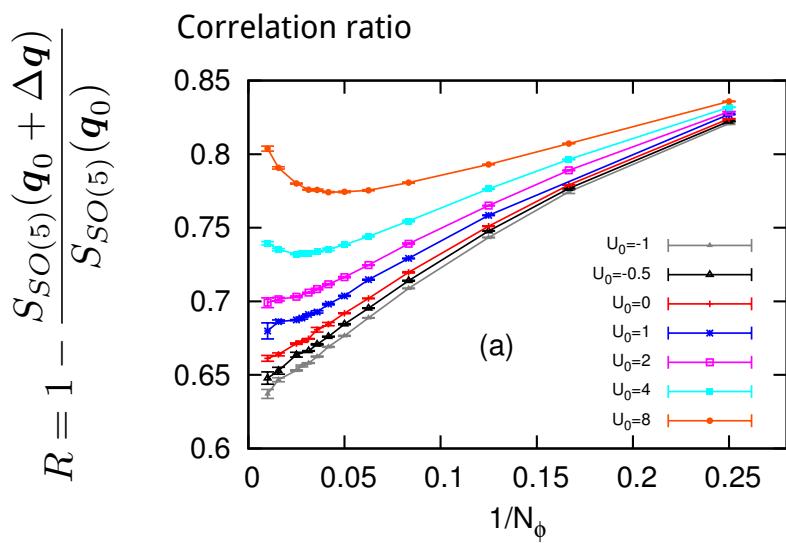
$$S_{SO(5)}(\mathbf{q}) = \langle \hat{\mathbf{M}}(\mathbf{q}) \cdot \hat{\mathbf{M}}(-\mathbf{q}) \rangle$$

$$\hat{\mathbf{M}}(\mathbf{q}) = \frac{1}{\sqrt{V}} \int_V d^2x e^{i\mathbf{q} \cdot \mathbf{x}} \hat{\Psi}(x)^\dagger \mathbf{M} \hat{\Psi}(x)$$



1) Ordered phase

2) At small U_0 the correlation-length exceeds our system size.

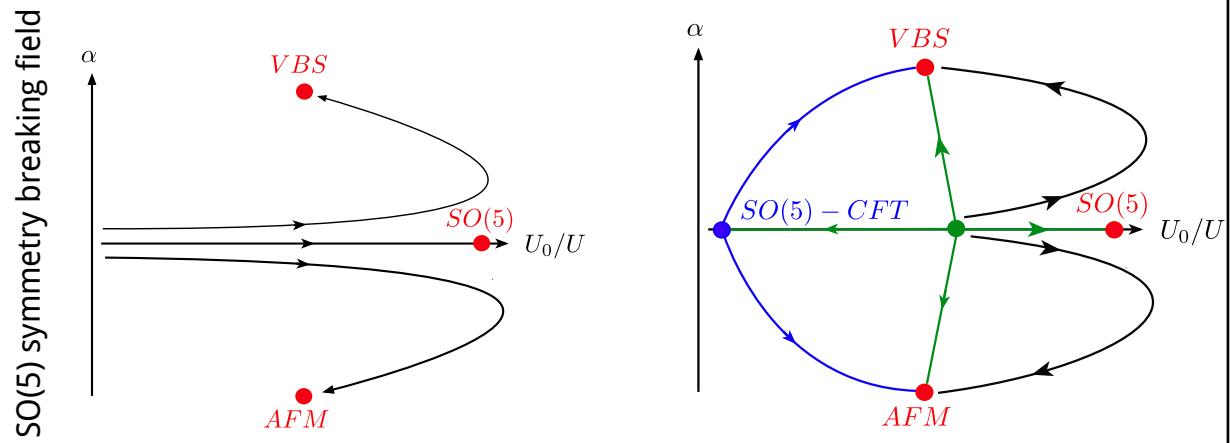
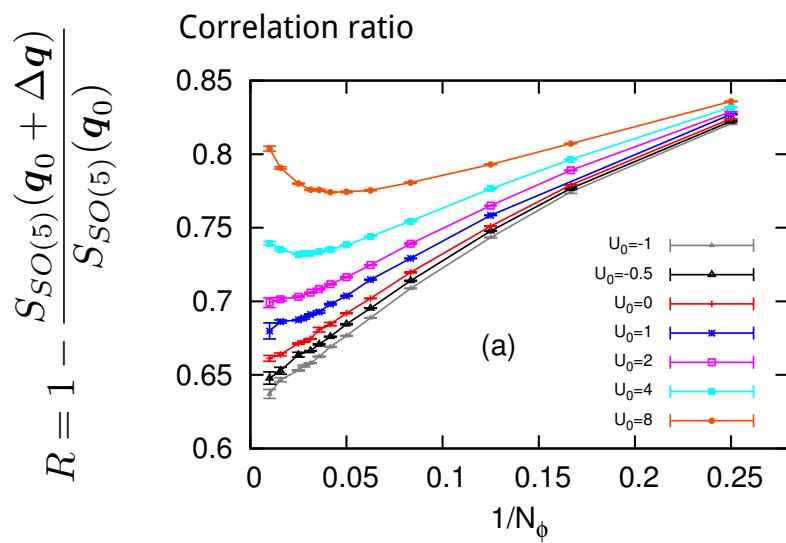


Color coding.

Symmetry broken phase

1) Ordered phase

2) At small U_0 the correlation-length exceeds our system size.



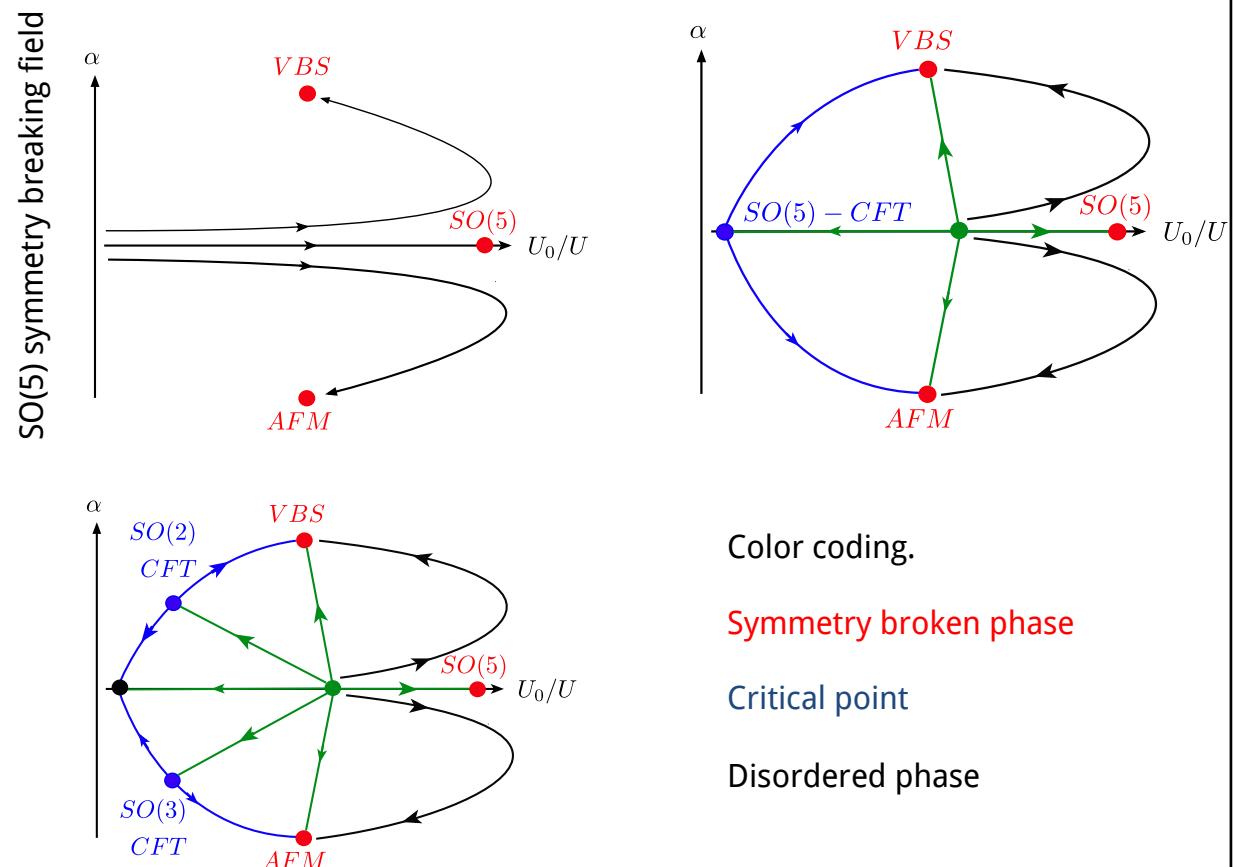
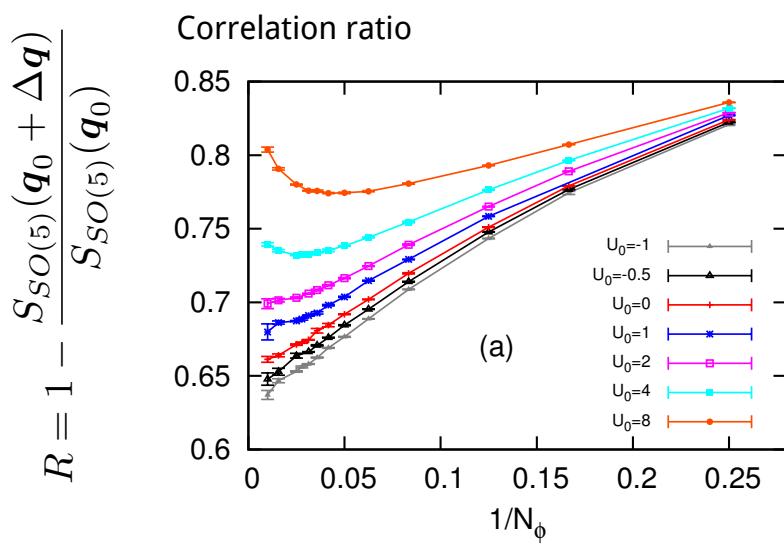
Color coding.

Symmetry broken phase

Critical point

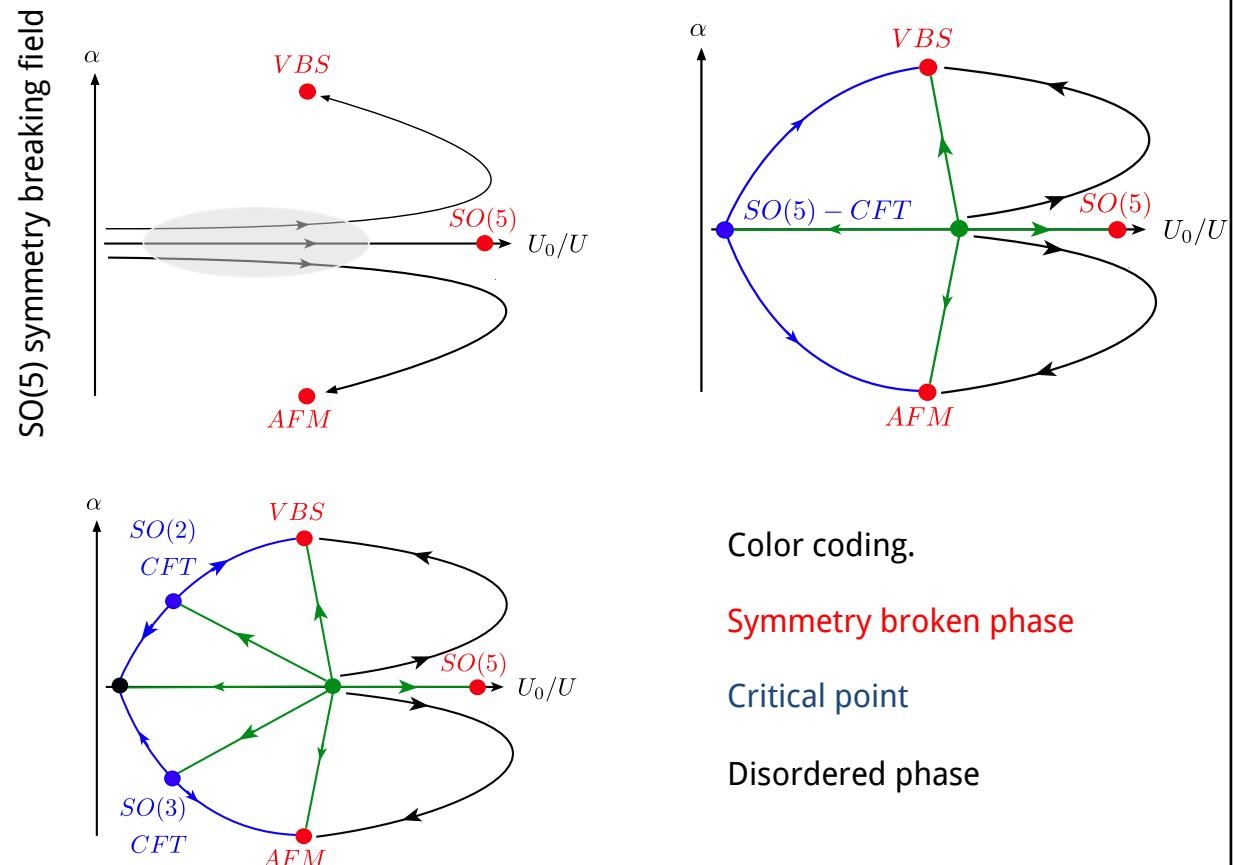
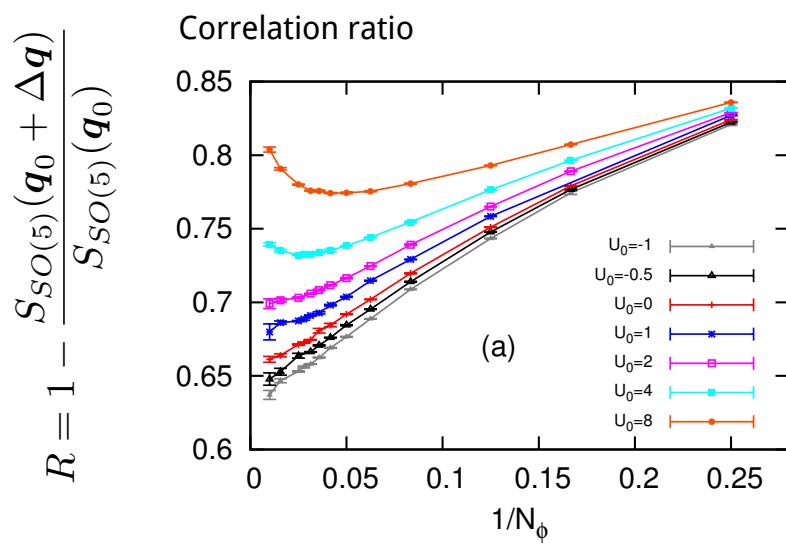
1) Ordered phase

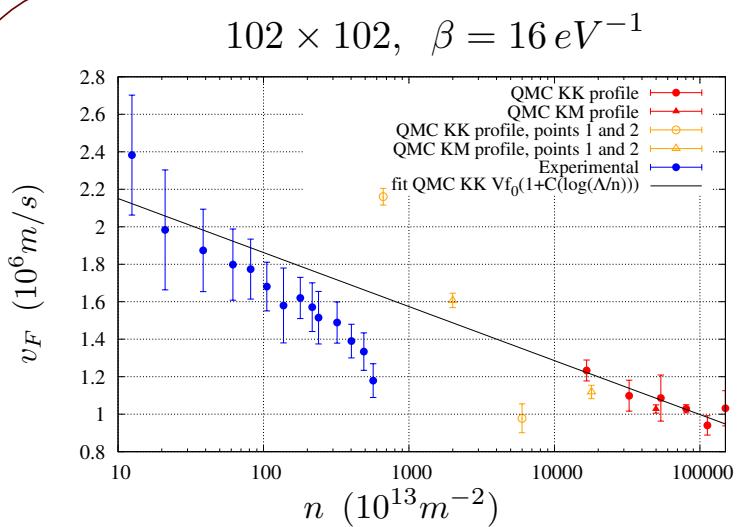
2) At small U_0 the correlation-length exceeds our system size.



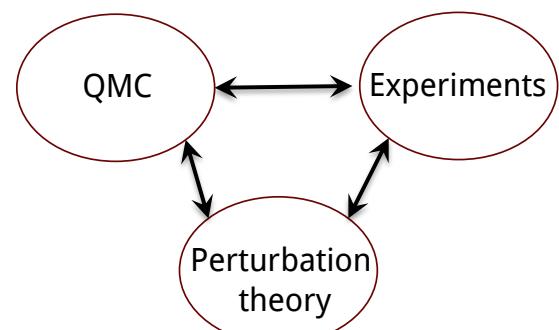
1) Ordered phase

2) At small U_0 the correlation-length exceeds our system size.

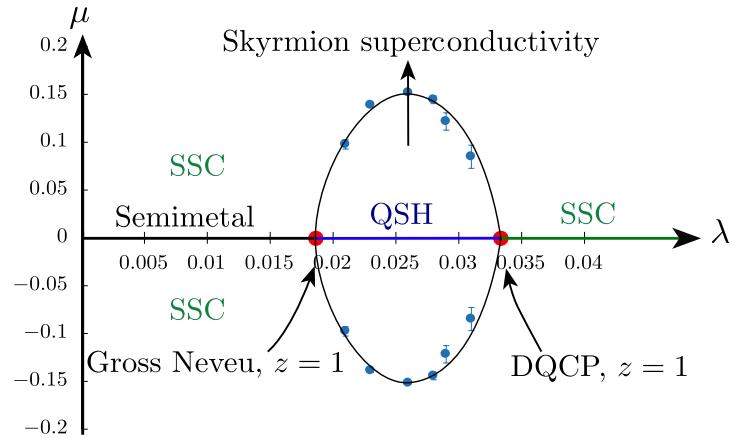




Can we reach $n = 100 \cdot 10^{13} \text{ m}^{-2}$?



M. Ulybyshev, S. Zafeiropoulos, C. Winterowd. FFA. arXiv:2104.09655



Y. Liu, Z. Wang, T. Sato, M. Hohenadler, C. Wang, W. Guo and FFA , Nature Communications 10 (2019), 2658.

Z. Wang, Y. Liu, T. Sato, M. Hohenadler, C. Wang, W. Guo, and FFA PRL 126 (2021), 205701

Landau level regularization scheme for.

$$S = \int d^2x d\tau \frac{1}{G} (\partial_\mu \hat{\varphi}(x, \tau))^2 + 2\pi i \Gamma [\hat{\varphi}]$$

Evidence for (proximity to) SO(5) CFT.

Z. Wang, M. P. Zaletel, R. S. K. Mong, and FFA Phys. Rev. Lett. 126 (2021), 045701