

QED₃ in condensed matter: From quantum criticality to instanton zero modes

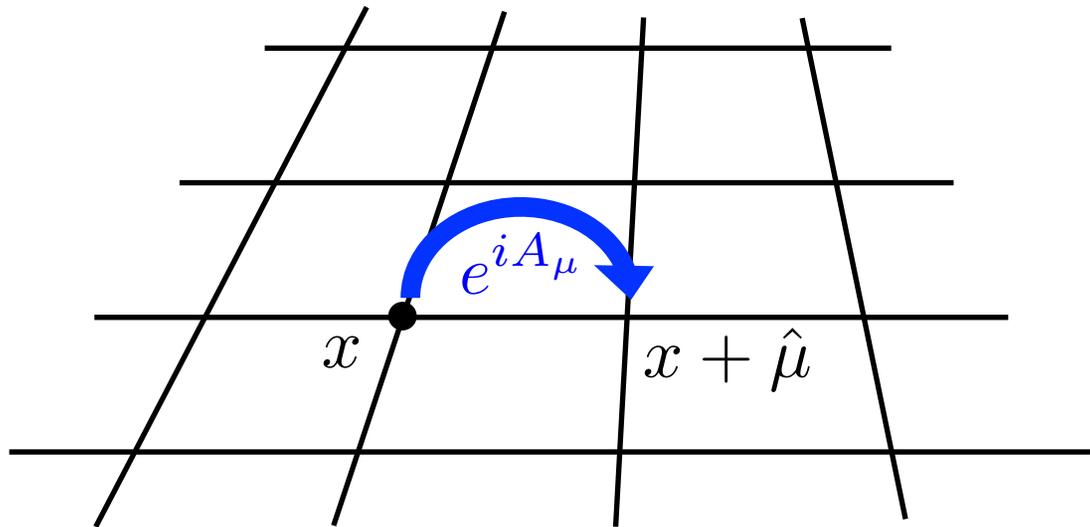
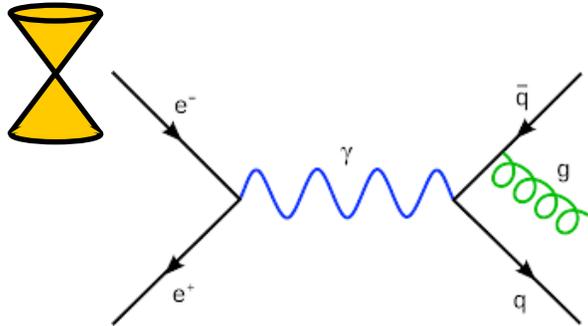
**Joseph Maciejko
University of Alberta**

Relativistic Fermions in Flatland: Theory & Application (ECT*)

July 6, 2021



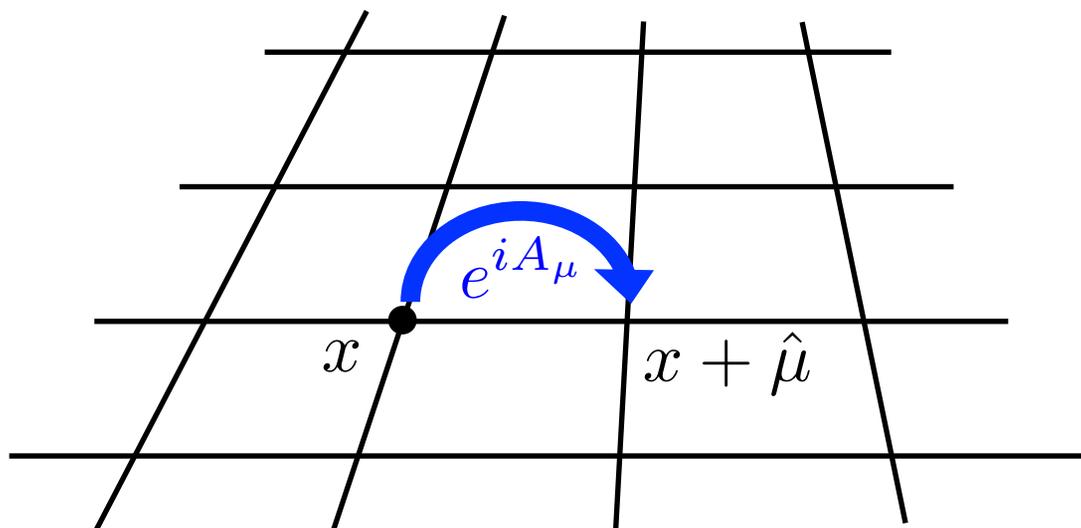
Lattice QED / QCD



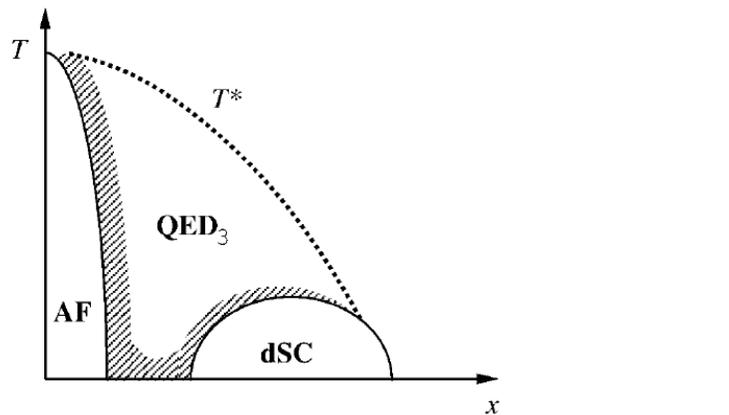
Wilson, Smit, Polyakov, Creutz, ...

Lattice QED₃ in CMP

?

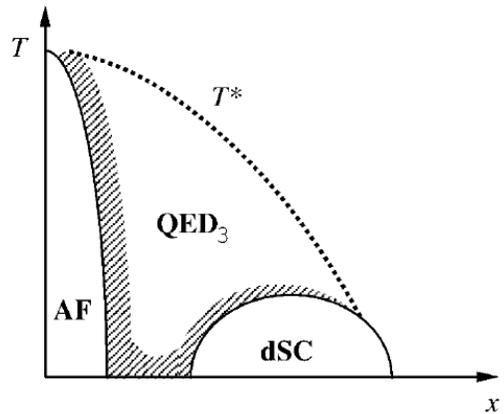


Lattice QED₃ in CMP

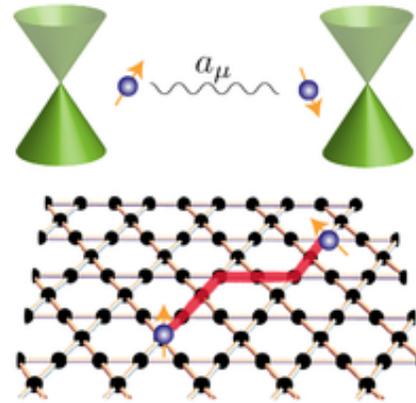


high- T_c superconductors
(Franz, Tešanović, Vafeek, Herbut, ...)

Lattice QED₃ in CMP

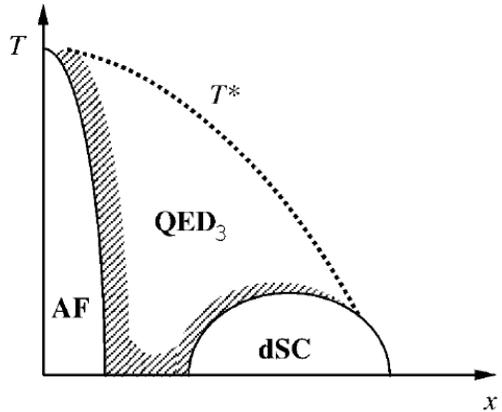


high- T_c superconductors
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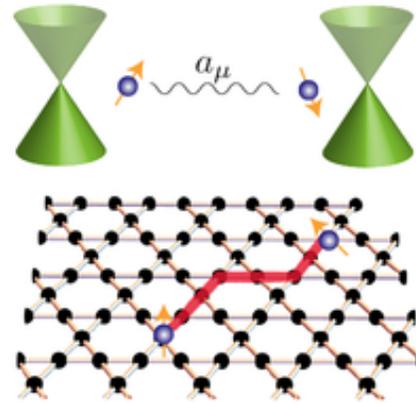


frustrated magnets
(Hastings, Lee, Wen, Hermele, Ran, ...)

Lattice QED₃ in CMP



high- T_c superconductors
(Franz, Tešanović, Vafeek, Herbut, ...)



frustrated magnets
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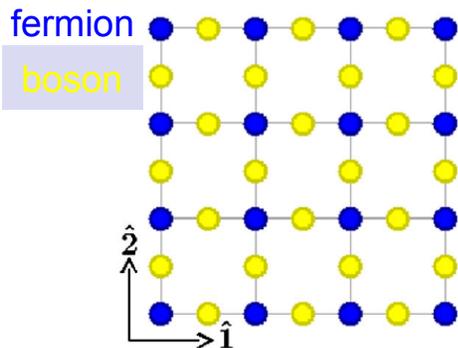
PRL 110, 055302 (2013)

PHYSICAL REVIEW LETTERS

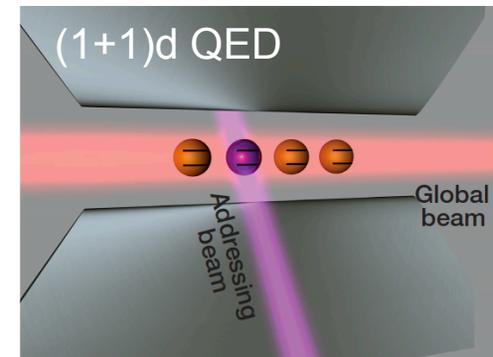
week ending
1 FEBRUARY 2013

Simulating (2 + 1)-Dimensional Lattice QED with Dynamical Matter Using Ultracold Atoms

Erez Zohar,¹ J. Ignacio Cirac,² and Benni Reznik¹



ultracold atoms

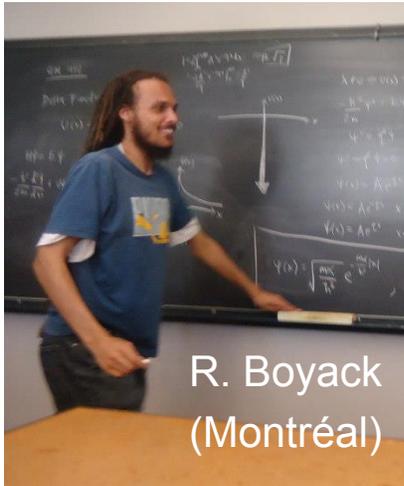


Martinez et al., Nature 534, 516 (2016)

Outline

- From lattice QED_3 to conformal QED_3 : the U(1) Dirac spin liquid
- What is the universality class of zero-temperature transitions out of the Dirac spin liquid?
- Confinement dynamics: massive QED_3 , instanton zero modes, and symmetry breaking

Collaborators (part I)



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C.-H. Lin
(UMN)



P. Marquard
(DESY)



A. Rayyan
(Toronto)



N. Zerf
(HU Berlin)

R. Boyack, C.-H. Lin, N. Zerf, A. Rayyan, & JM, PRB 98, 035137 (2018)

N. Zerf, P. Marquard, R. Boyack, & JM, PRB 98, 165125 (2018)

R. Boyack, A. Rayyan, & JM, PRB 99, 195135 (2019)

N. Zerf, R. Boyack, P. Marquard, J. A. Gracey, & JM, PRB 100, 235130 (2019)

N. Zerf, R. Boyack, P. Marquard, J. A. Gracey, & JM, PRD 101, 094505 (2020)

R. Boyack & JM, arXiv:1911.09768 (QTS-XI proceedings, 2021)



Review

Quantum phase transitions in Dirac fermion systems

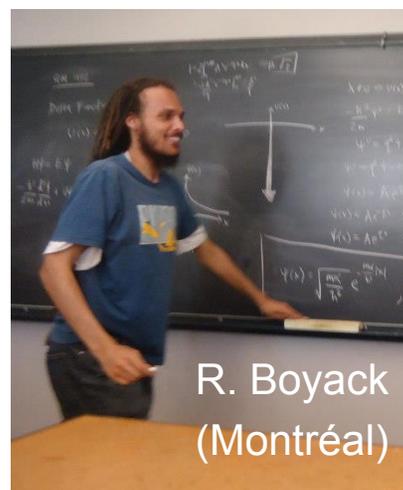
Rufus Boyack^{1,2}, Hennadii Yerzhakov¹, and Joseph Maciejko^{1,2,a}

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² Theoretical Physics Institute, University of Alberta, Edmonton, Alberta T6G 2E1, Canada

Received 20 April 2020 / Accepted 5 January 2021 / Published online 25 May 2021

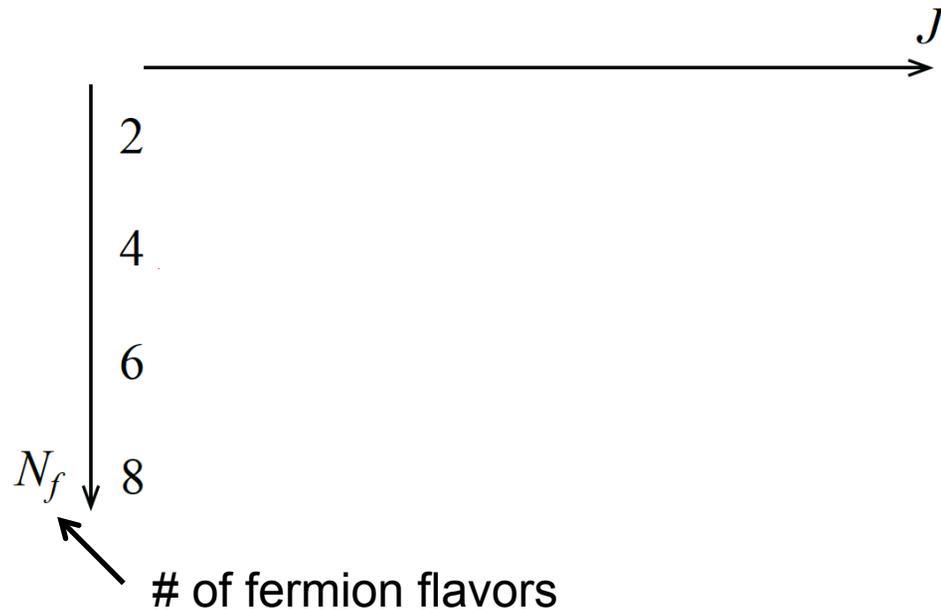
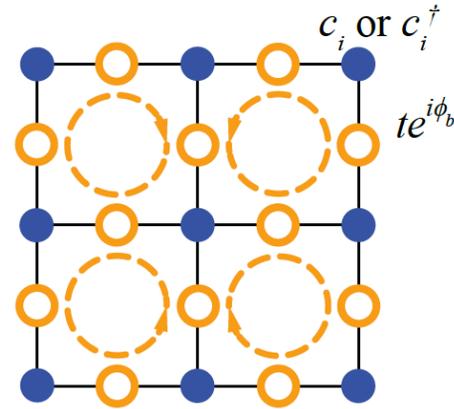
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H. Yerzhakov
(Bar-Ilan)

Lattice QED₃: sign-problem-free QMC

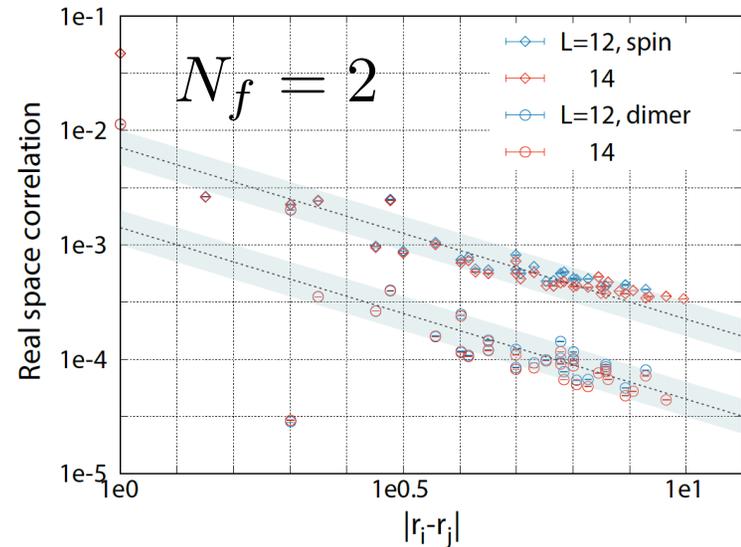
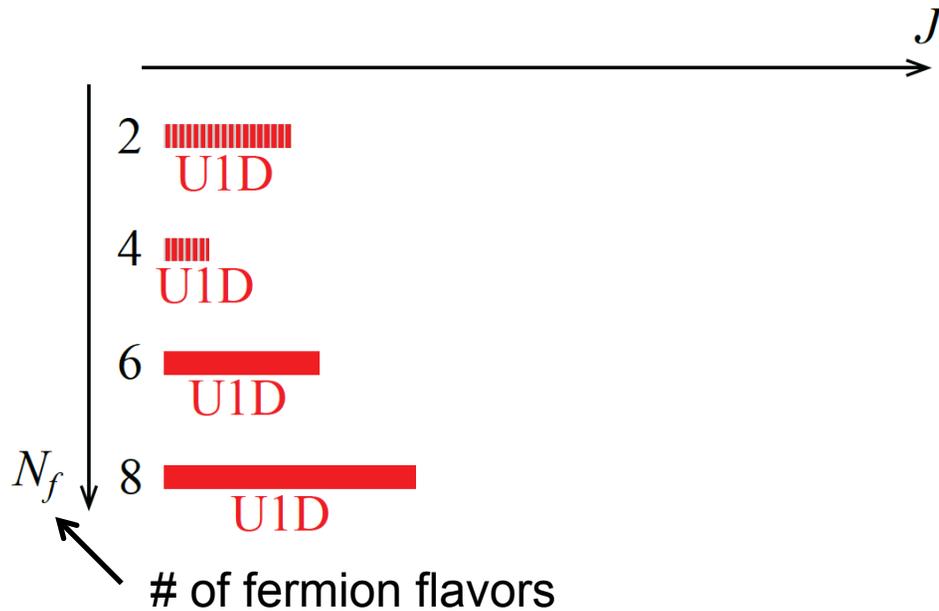
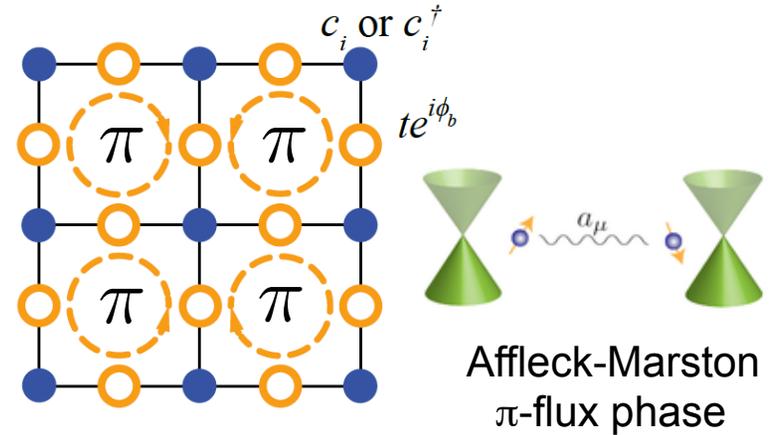
$$H = \frac{1}{2} J N_f \sum_{\langle i,j \rangle} \frac{1}{4} \hat{L}_{ij}^2 - t \sum_{\langle i,j \rangle \alpha} \left(\hat{c}_{i\alpha}^\dagger e^{i\hat{\theta}_{ij}} \hat{c}_{j\alpha} + \text{h.c.} \right) + \frac{1}{2} K N_f \sum_{\square} \cos(\text{curl} \hat{\theta})$$



Xu, Qi, Zhang, Assaad, Xu, & Meng,
PRX 9, 021022 (2019)

U(1) deconfined phase: a Dirac spin liquid

$$H = \frac{1}{2} J N_f \sum_{\langle i,j \rangle} \frac{1}{4} \hat{L}_{ij}^2 - t \sum_{\langle i,j \rangle \alpha} \left(\hat{c}_{i\alpha}^\dagger e^{i\hat{\theta}_{ij}} \hat{c}_{j\alpha} + \text{h.c.} \right) + \frac{1}{2} K N_f \sum_{\square} \cos(\text{curl} \hat{\theta})$$

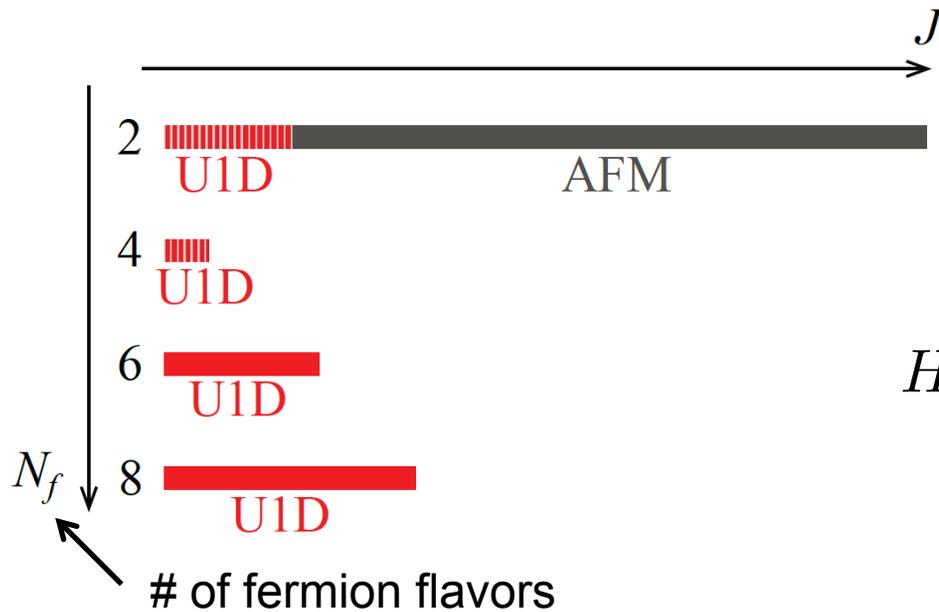
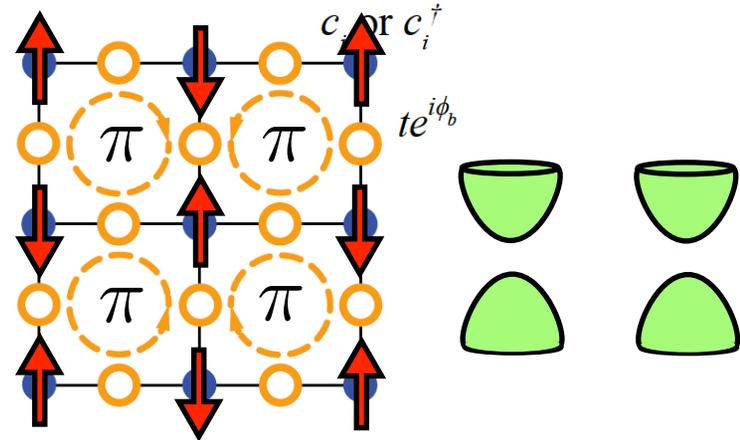


Xu, Qi, Zhang, Assaad, Xu, & Meng, PRX 9, 021022 (2019)

Universal power-law correlations (Rantner, Wen, Hermele, Senthil, Fisher, ...)

Néel phase

$$H = \frac{1}{2} J N_f \sum_{\langle i,j \rangle} \frac{1}{4} \hat{L}_{ij}^2 - t \sum_{\langle i,j \rangle \alpha} \left(\hat{c}_{i\alpha}^\dagger e^{i\hat{\theta}_{ij}} \hat{c}_{j\alpha} + \text{h.c.} \right) + \frac{1}{2} K N_f \sum_{\square} \cos(\text{curl} \hat{\theta})$$

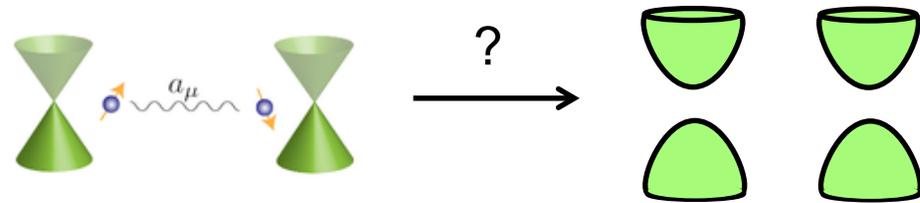
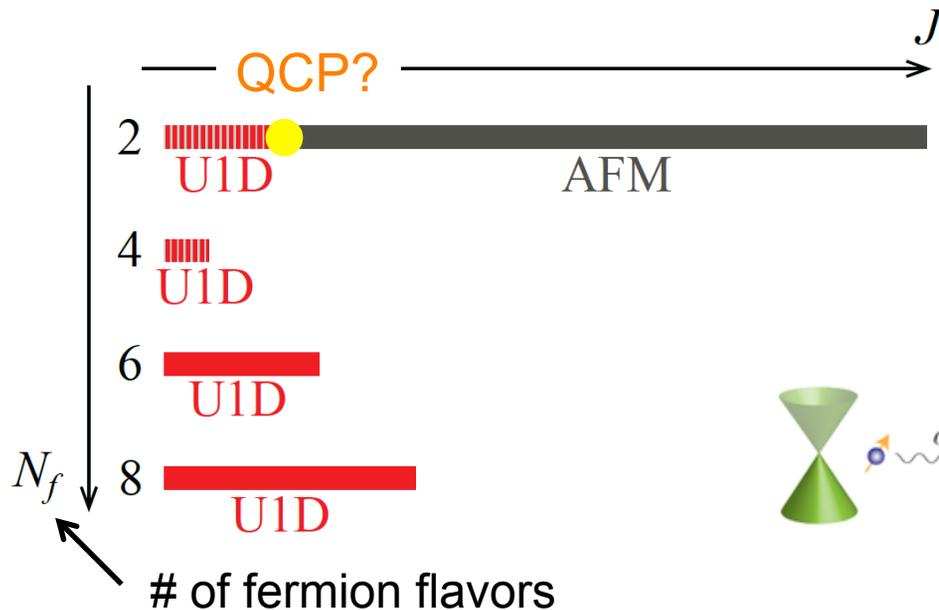
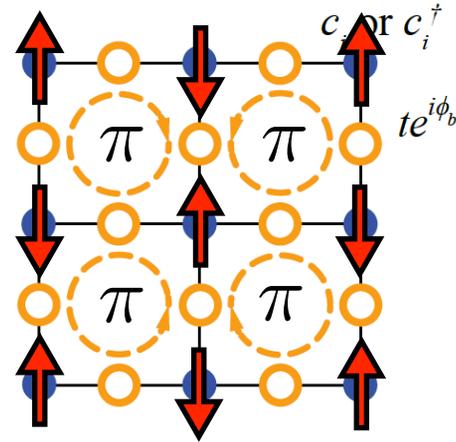


$$H_{\text{eff}}(J \rightarrow \infty) \sim \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

Xu, Qi, Zhang, Assaad, Xu, & Meng,
PRX 9, 021022 (2019)

U(1) Dirac to Néel transition

$$H = \frac{1}{2} J N_f \sum_{\langle i,j \rangle} \frac{1}{4} \hat{L}_{ij}^2 - t \sum_{\langle i,j \rangle \alpha} \left(\hat{c}_{i\alpha}^\dagger e^{i\hat{\theta}_{ij}} \hat{c}_{j\alpha} + \text{h.c.} \right) + \frac{1}{2} K N_f \sum_{\square} \cos(\text{curl} \hat{\theta})$$



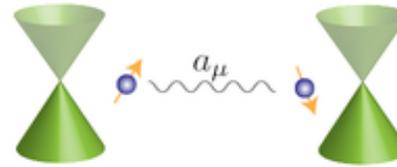
Xu, Qi, Zhang, Assaad, Xu, & Meng,
PRX 9, 021022 (2019)

U(1) Dirac

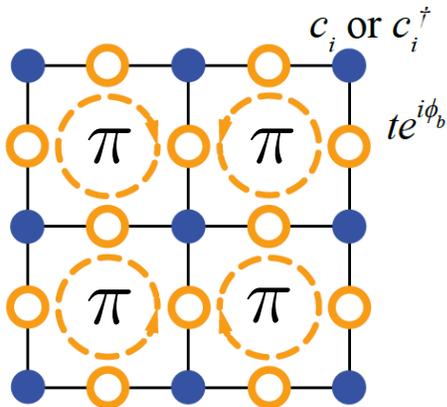
- Continuum effective theory:

$$\mathcal{L} = \sum_{i=1}^{N_f} \bar{\Psi}_i \not{D} \Psi_i + \frac{1}{4} F_{\mu\nu}^2$$

QED₃



UID



U(1) Dirac to Néel transition

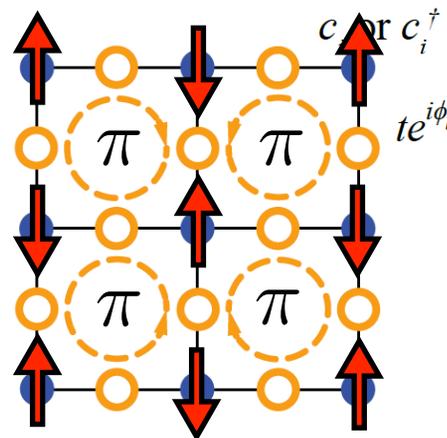
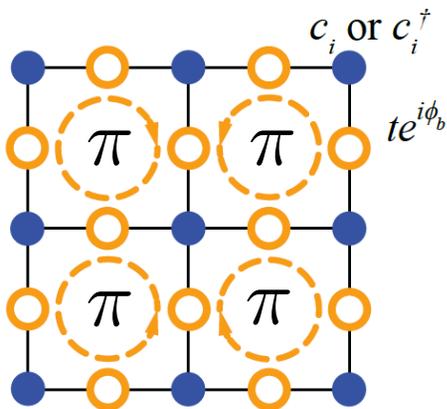
- Continuum effective theory: Heisenberg QED₃-Gross-Neveu-Yukawa model

$$\mathcal{L} = \sum_{i=1}^{N_f} \bar{\Psi}_i \not{D} \Psi_i + \frac{1}{4} F_{\mu\nu}^2$$


$$+ \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 + \lambda^2 (\phi^2)^2 + g \phi \cdot \sum_{i=1}^{N_f} \bar{\Psi}_i \frac{\sigma}{2} \Psi_i$$

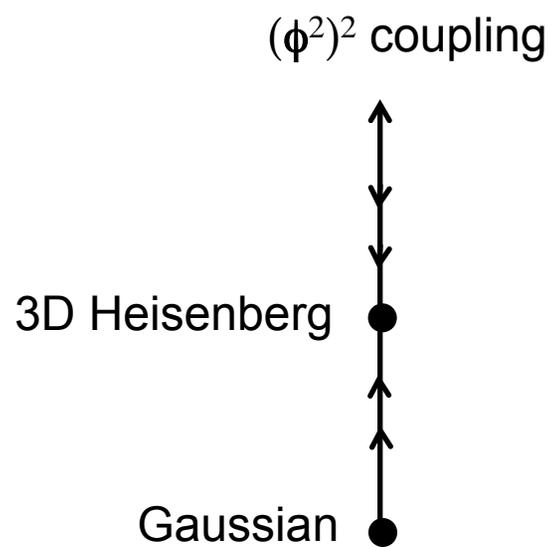


$$m^2 \propto J_c - J$$

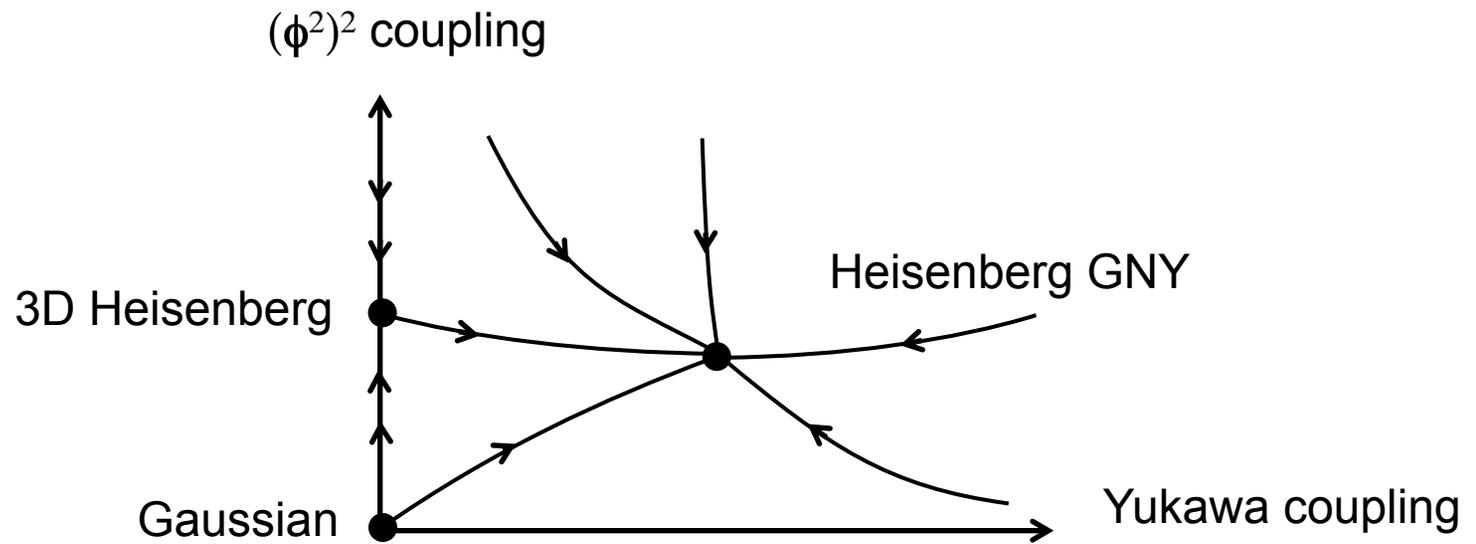


Ghaemi & Senthil,
PRB 73, 054415 (2006)

4- ϵ expansion

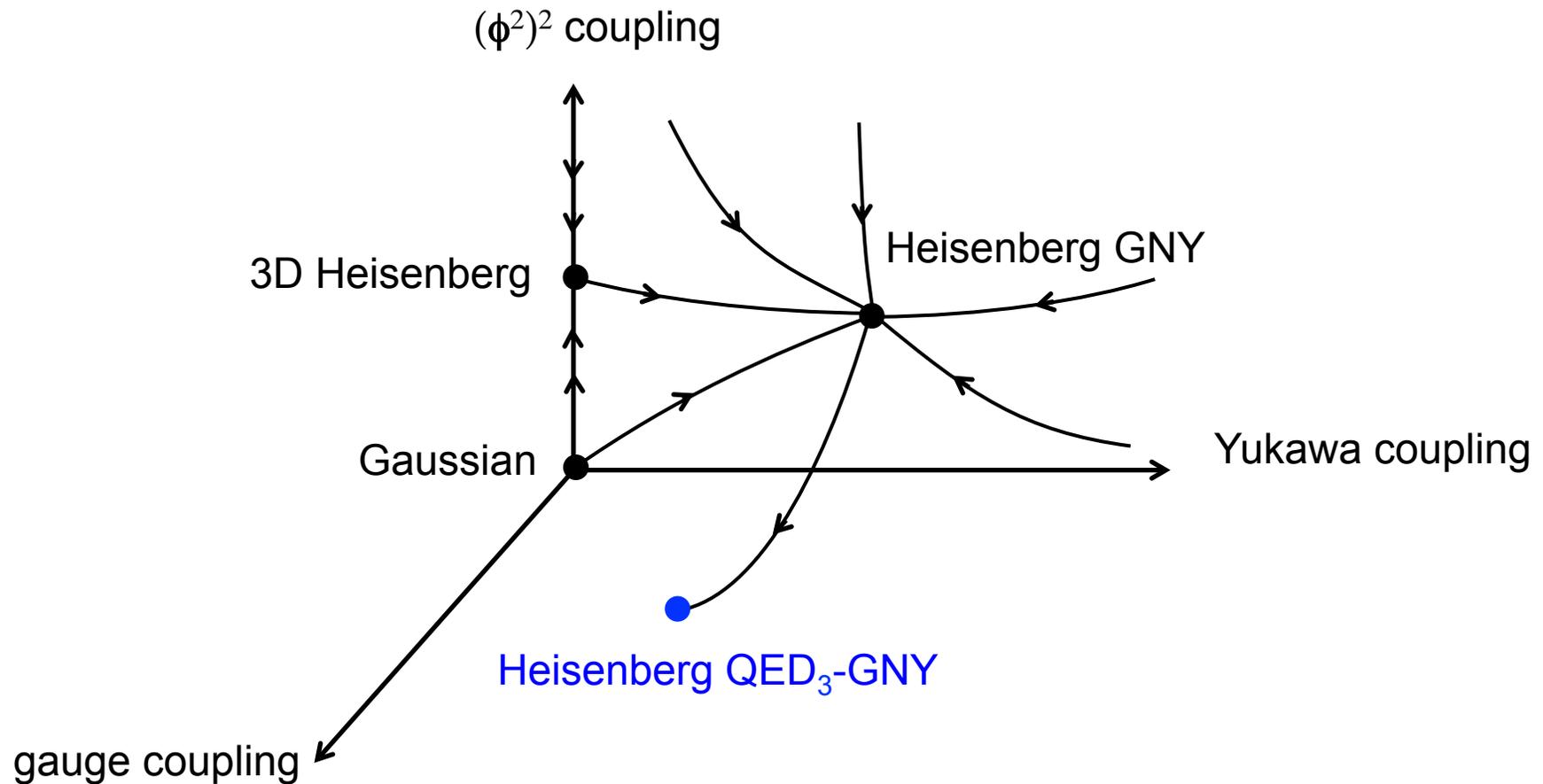


4- ϵ expansion



4- ϵ expansion

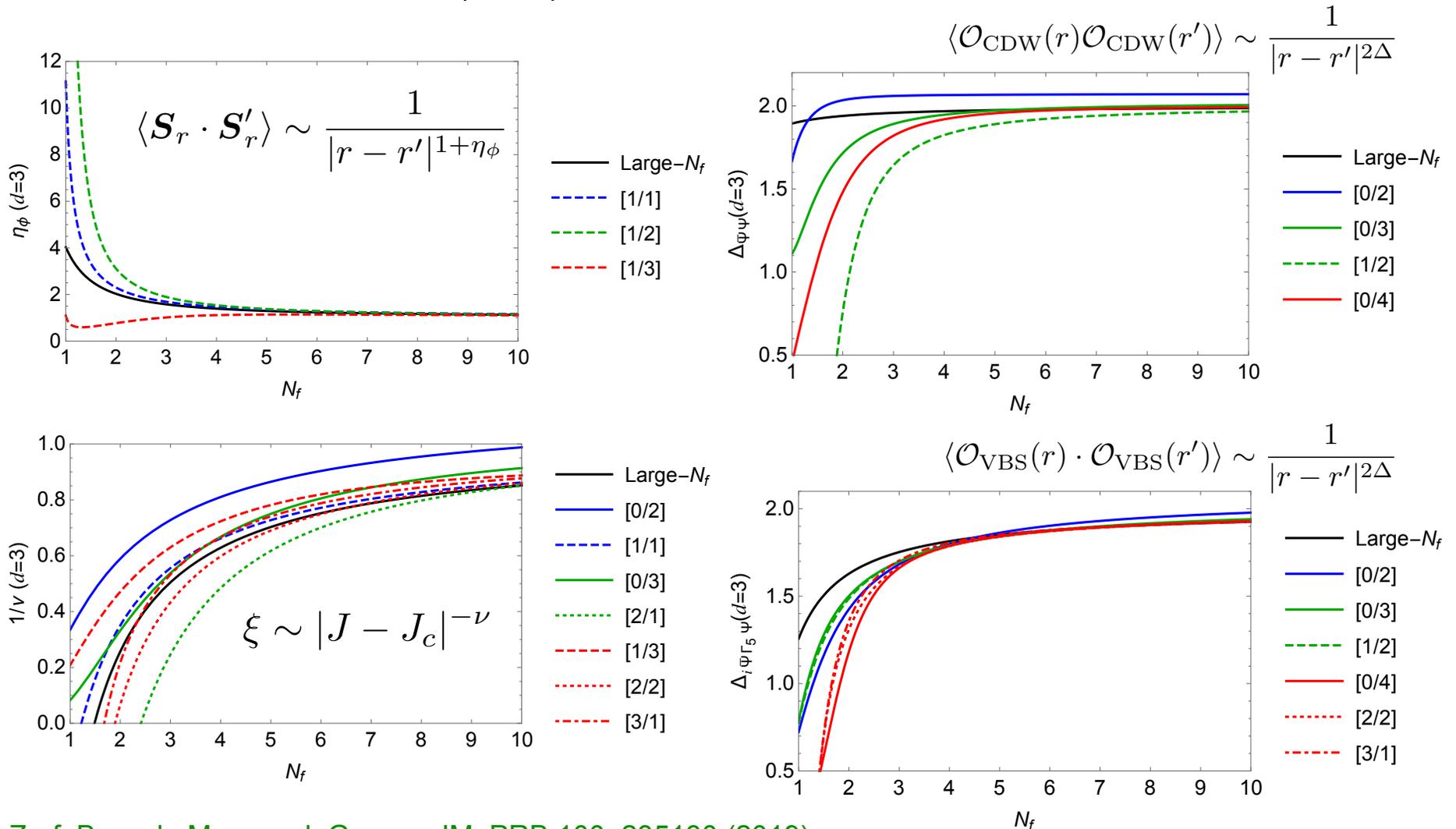
- Stable fixed point for all N_f : continuous transition, agrees with QMC. New $O(3)$ universality class!



Dupuis, Paranjape, Witczak-Krempa, PRB 100, 094443 (2019)
Zerf, Boyack, Marquard, Gracey, JM, PRB 100, 235130 (2019)

Critical exponents

- ϵ (4-loop) & large- N_f ($1/N_f^2$) expansions, eventually compare to QMC

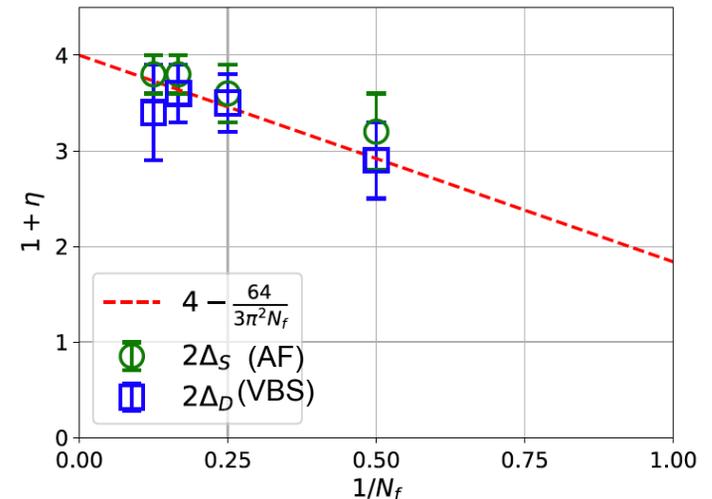
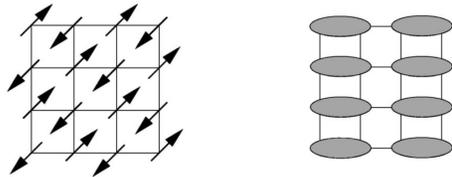


AF vs VBS correlations

- U1D fixed point: emergent SU(4) flavor symmetry implies AF and VBS correlations decay with the same exponent (Hermele, Senthil, Fisher, PRB 72, 104404 (2005))

$$\langle \mathcal{O}(r)\mathcal{O}(r') \rangle \sim |r - r'|^{-\lambda}$$

$$\lambda_{\text{AF}}(\text{U1D}) = \lambda_{\text{VBS}}(\text{U1D}) = 4 - \frac{64}{3\pi^2 N_f} + \mathcal{O}(1/N_f^2)$$



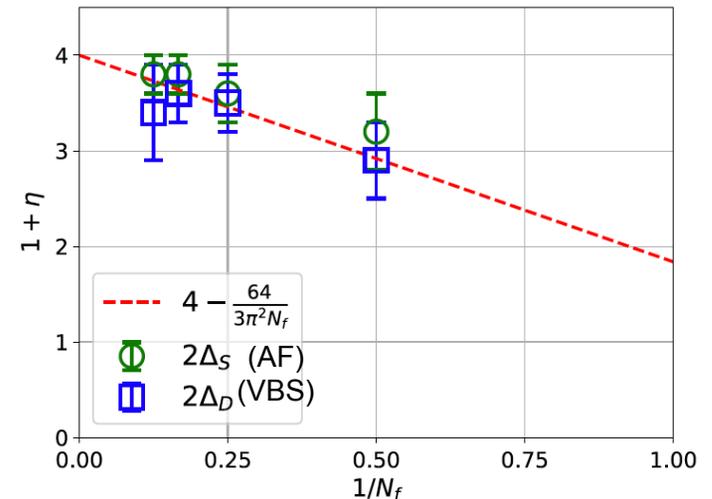
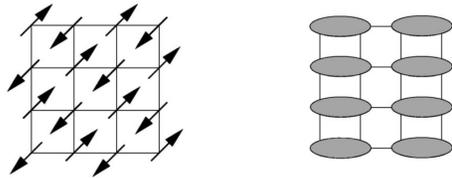
Xu, Qi, Zhang, Assaad, Xu, & Meng, PRX 9, 021022 (2019)

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Xu, Qi, Zhang, Assaad, Xu, & Meng, PRX 9, 021022 (2019)

- U1D-Néel QCP: Yukawa coupling breaks flavor (= spin + valley) degeneracy

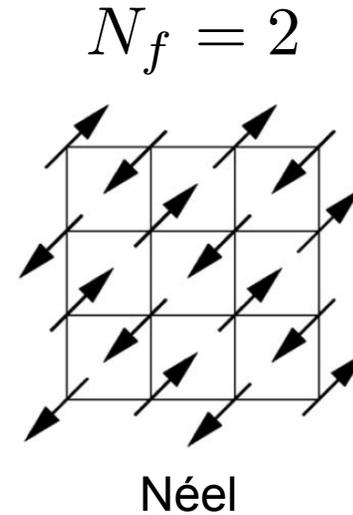
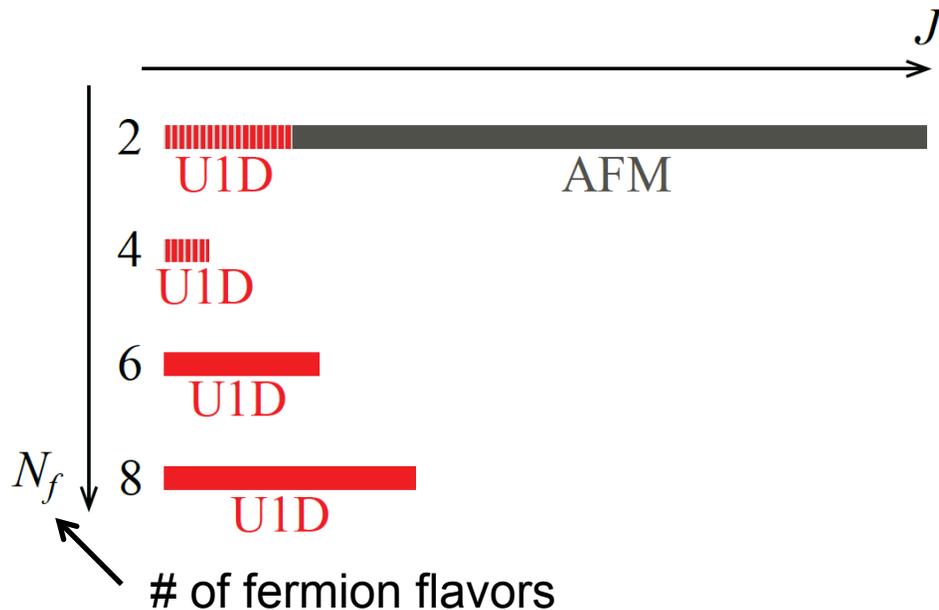
$$\lambda_{\text{AF}}(\text{U1D-Néel}) = 2 + \frac{64}{3\pi^2 N_f} + \mathcal{O}(1/N_f^2) \quad (\text{divergent susceptibility})$$

$$\lambda_{\text{VBS}}(\text{U1D-Néel}) = 4 - \frac{88}{3\pi^2 N_f} + \mathcal{O}(1/N_f^2) \quad (\text{OP fluctuations enhance VBS correlations})$$

Zerf, Boyack, Marquard, Gracey, JM, PRB 100, 235130 (2019)

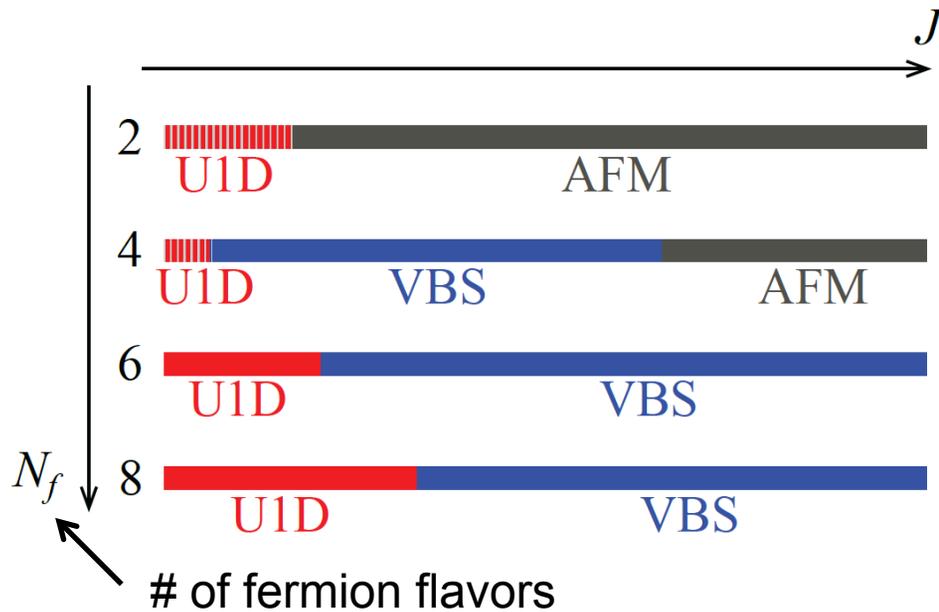
Lattice QED₃: results from QMC

$$H = \frac{1}{2} J N_f \sum_{\langle i,j \rangle} \frac{1}{4} \hat{L}_{ij}^2 - t \sum_{\langle i,j \rangle \alpha} \left(\hat{c}_{i\alpha}^\dagger e^{i\hat{\theta}_{ij}} \hat{c}_{j\alpha} + \text{h.c.} \right) + \frac{1}{2} K N_f \sum_{\square} \cos(\text{curl} \hat{\theta})$$

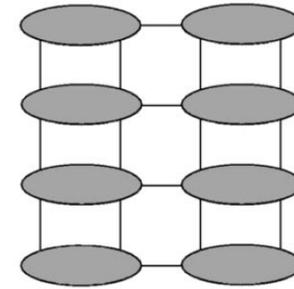


VBS phase

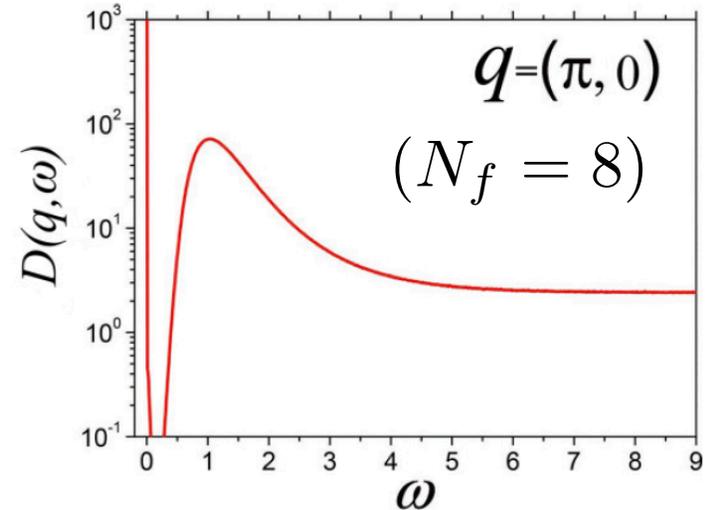
$$H = \frac{1}{2} J N_f \sum_{\langle i,j \rangle} \frac{1}{4} \hat{L}_{ij}^2 - t \sum_{\langle i,j \rangle \alpha} \left(\hat{c}_{i\alpha}^\dagger e^{i\hat{\theta}_{ij}} \hat{c}_{j\alpha} + \text{h.c.} \right) + \frac{1}{2} K N_f \sum_{\square} \cos(\text{curl} \hat{\theta})$$



$$N_f \geq 4$$



VBS



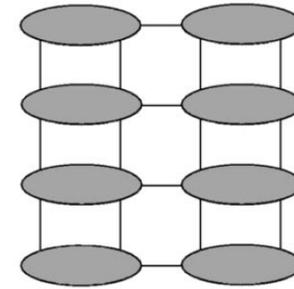
Xu, Qi, Zhang, Assaad, Xu, & Meng, PRX 9, 021022 (2019)

Wang, Lu, Xu, You, & Meng, PRB 100, 085123 (2019)

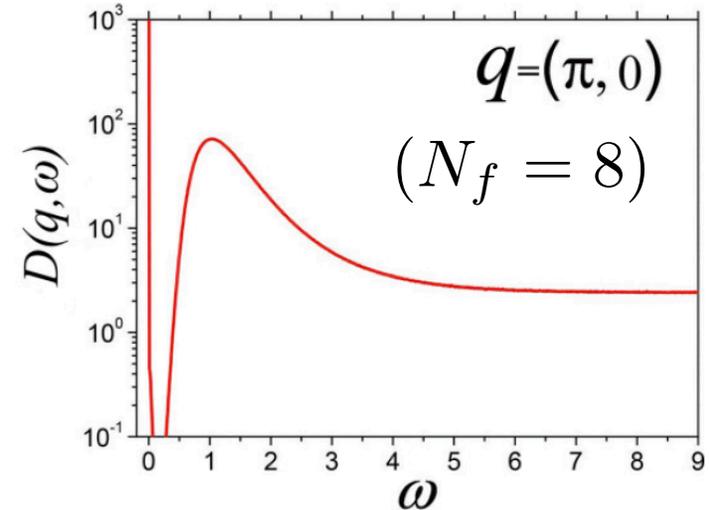
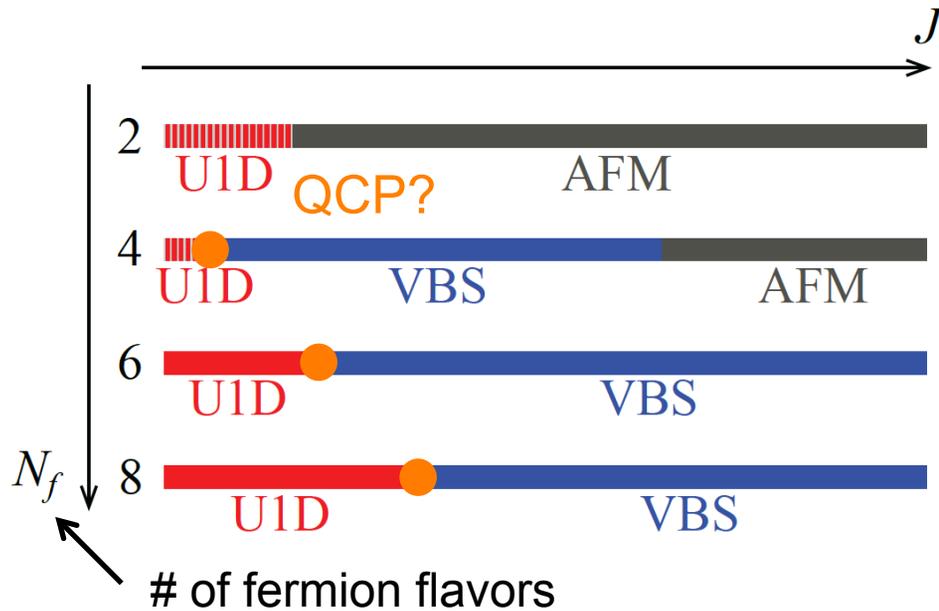
U(1) Dirac to VBS transition

$$H = \frac{1}{2} J N_f \sum_{\langle i,j \rangle} \frac{1}{4} \hat{L}_{ij}^2 - t \sum_{\langle i,j \rangle \alpha} \left(\hat{c}_{i\alpha}^\dagger e^{i\hat{\theta}_{ij}} \hat{c}_{j\alpha} + \text{h.c.} \right) + \frac{1}{2} K N_f \sum_{\square} \cos(\text{curl} \hat{\theta})$$

$$N_f \geq 4$$



VBS

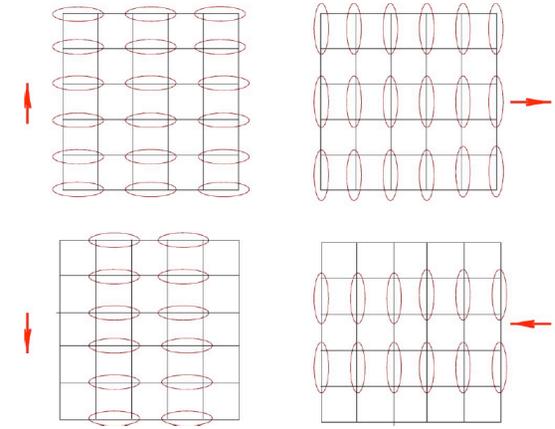


Xu, Qi, Zhang, Assaad, Xu, & Meng,
PRX 9, 021022 (2019)

Wang, Lu, Xu, You, & Meng,
PRB 100, 085123 (2019)

U(1) Dirac to VBS transition

- Continuum effective theory: XY QED₃-GNY model, equivalent to gauged Nambu–Jona-Lasinio (NJL) model



$$\mathcal{L}_{\text{NJL}} = \sum_{\alpha=1}^{N_f} \left[\bar{\psi}_\alpha \gamma_\mu \left(\partial_\mu + \frac{e}{\sqrt{N_f}} i A_\mu \right) \psi_\alpha + \frac{g}{\sqrt{N_f}} \bar{\psi}_\alpha (\phi_1 + i\phi_2 \gamma_5) \psi_\alpha \right] + \dots$$

- C₄ anisotropy terms $\propto (\phi_1 + i\phi_2)^4 + \text{c.c.}$ irrelevant at large N_f
- Nontrivial critical point in large-N_f expansion: new O(2) universality class!

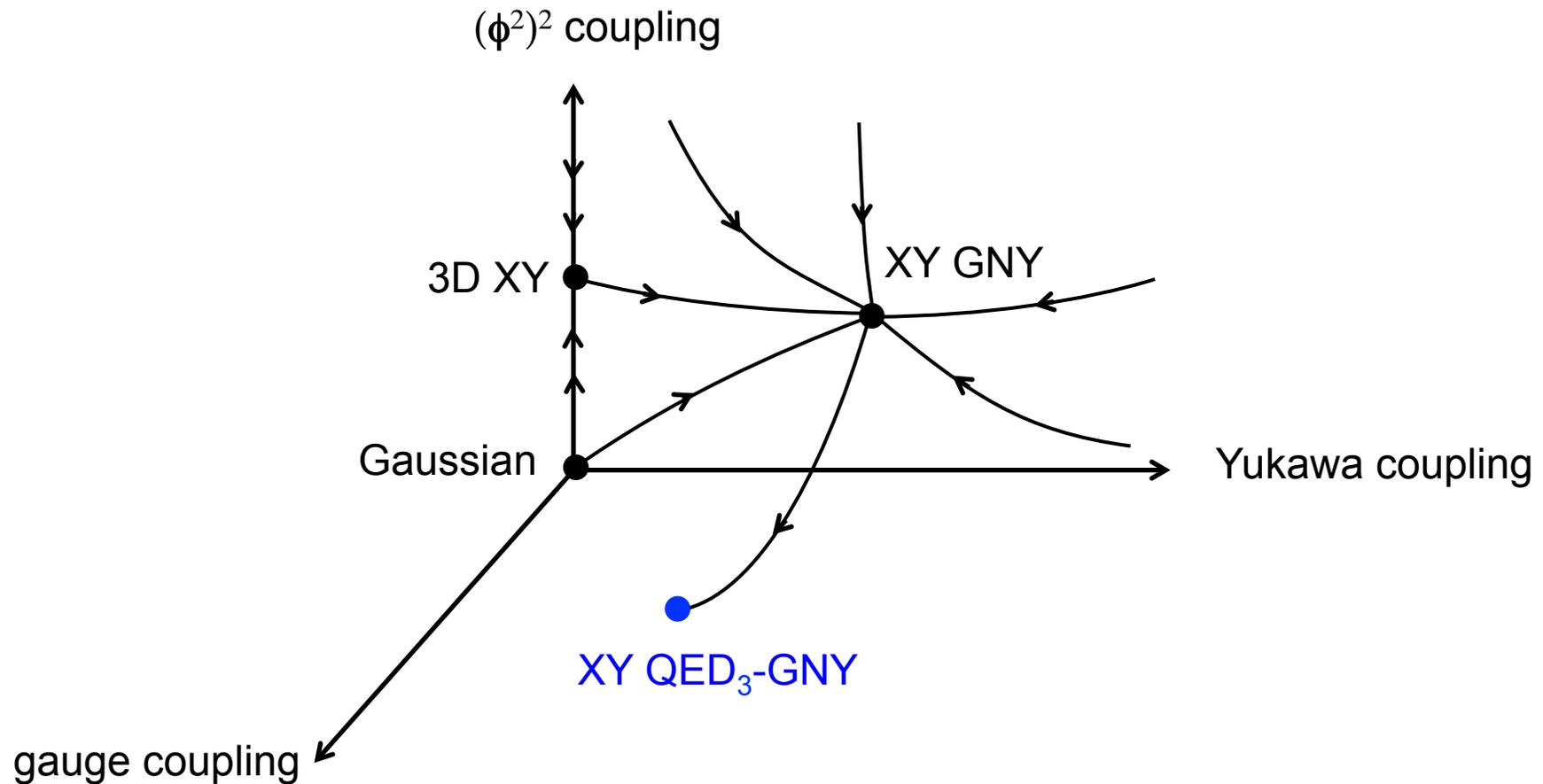
	η_ϕ	$1/\nu$	ν	Δ_{AF}
$N_f = 4$	1.473	0.3245	3.081	1.662
$N_f = 6$	1.315	0.5497	1.819	1.775
$N_f = 8$	1.236	0.6623	1.510	1.831

$$\chi_{\text{VBS}}(\mathbf{q}) \sim |\mathbf{q}|^{-(2-\eta_\phi)}$$

$$\chi_{\text{AF}}(\mathbf{q}) \sim |\mathbf{q}|^{2\Delta_{\text{AF}}-3}$$

4- ϵ expansion

- Stable fixed point for all N_f : continuous transition
- C_4 anisotropy irrelevant at criticality

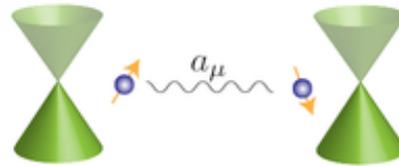


Zerf, Boyack, Marquard, Gracey, JM, PRD 101, 094505 (2020)
Janssen, Wang, Scherer, Meng, Xu, PRB 101, 235118 (2020)

PHYSICAL REVIEW B 72, 104404 (2005)

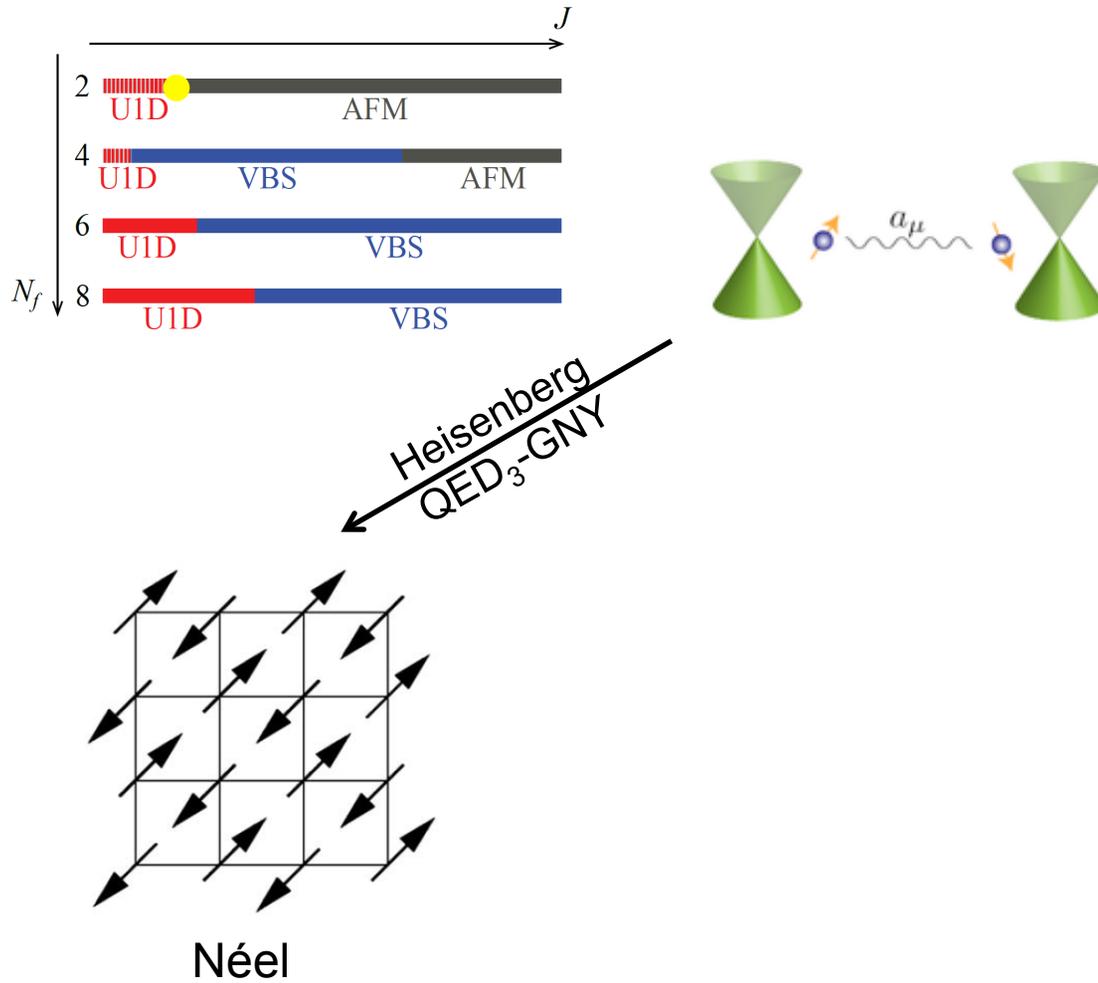
Algebraic spin liquid as the mother of many competing orders

Michael Hermele,¹ T. Senthil,^{2,3} and Matthew P. A. Fisher⁴



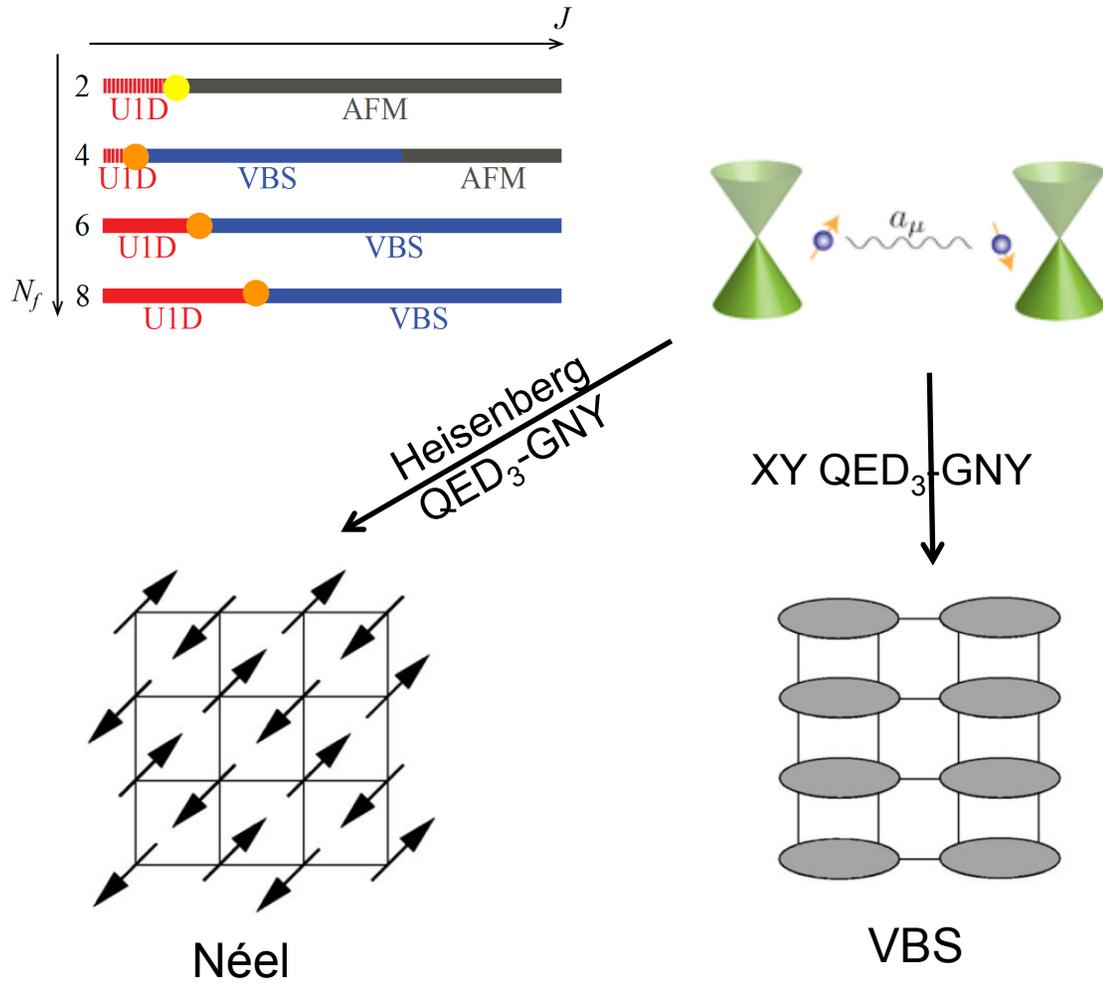
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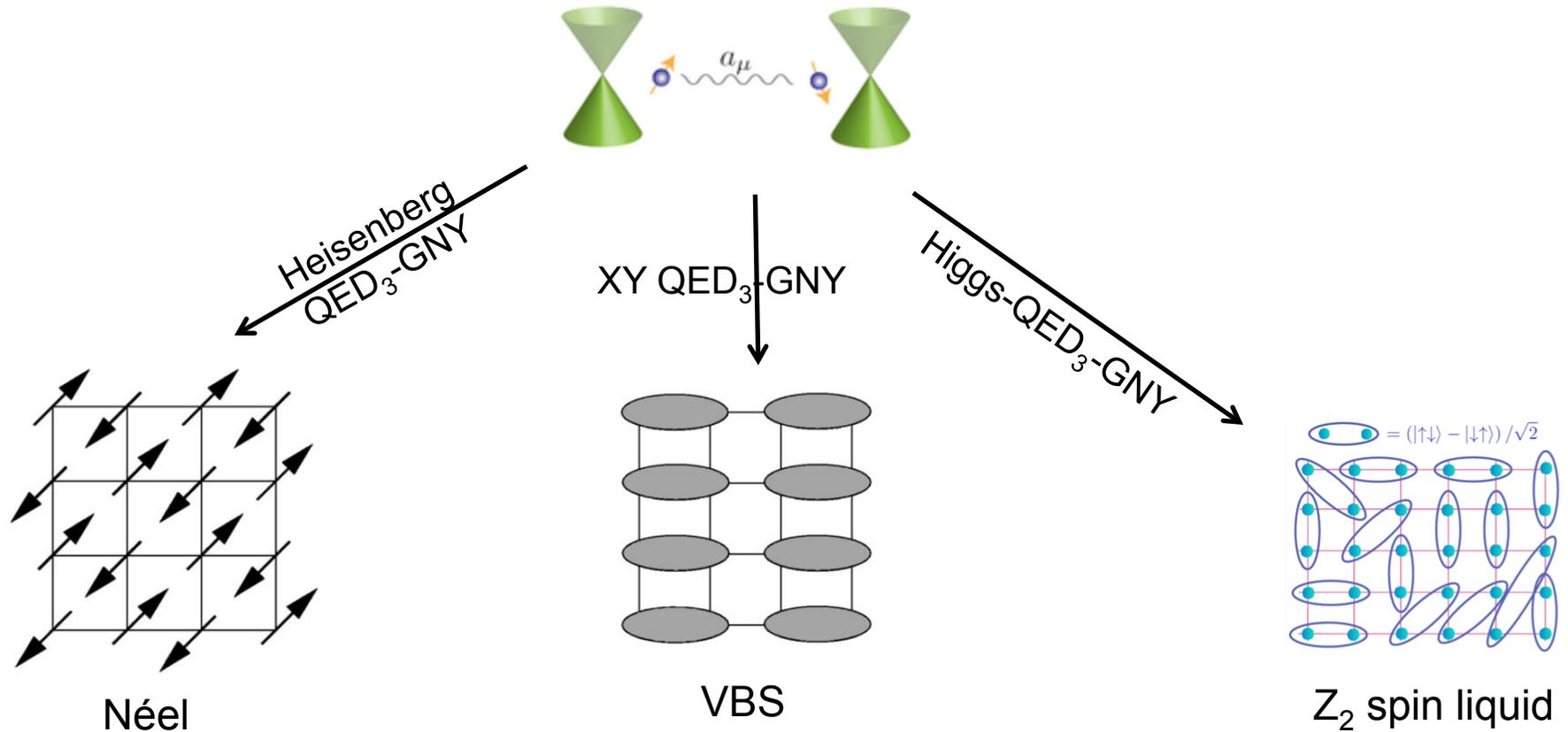
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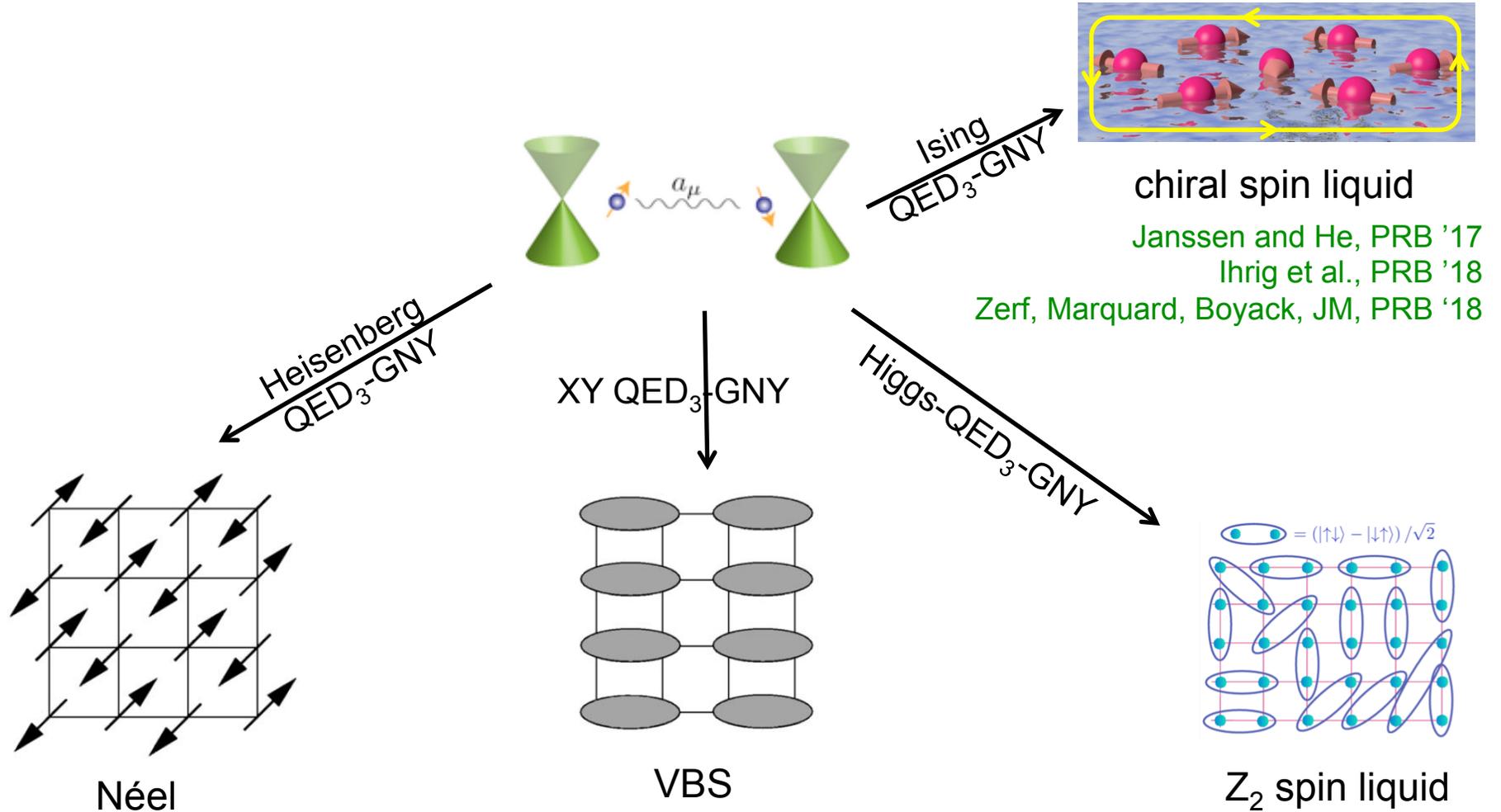
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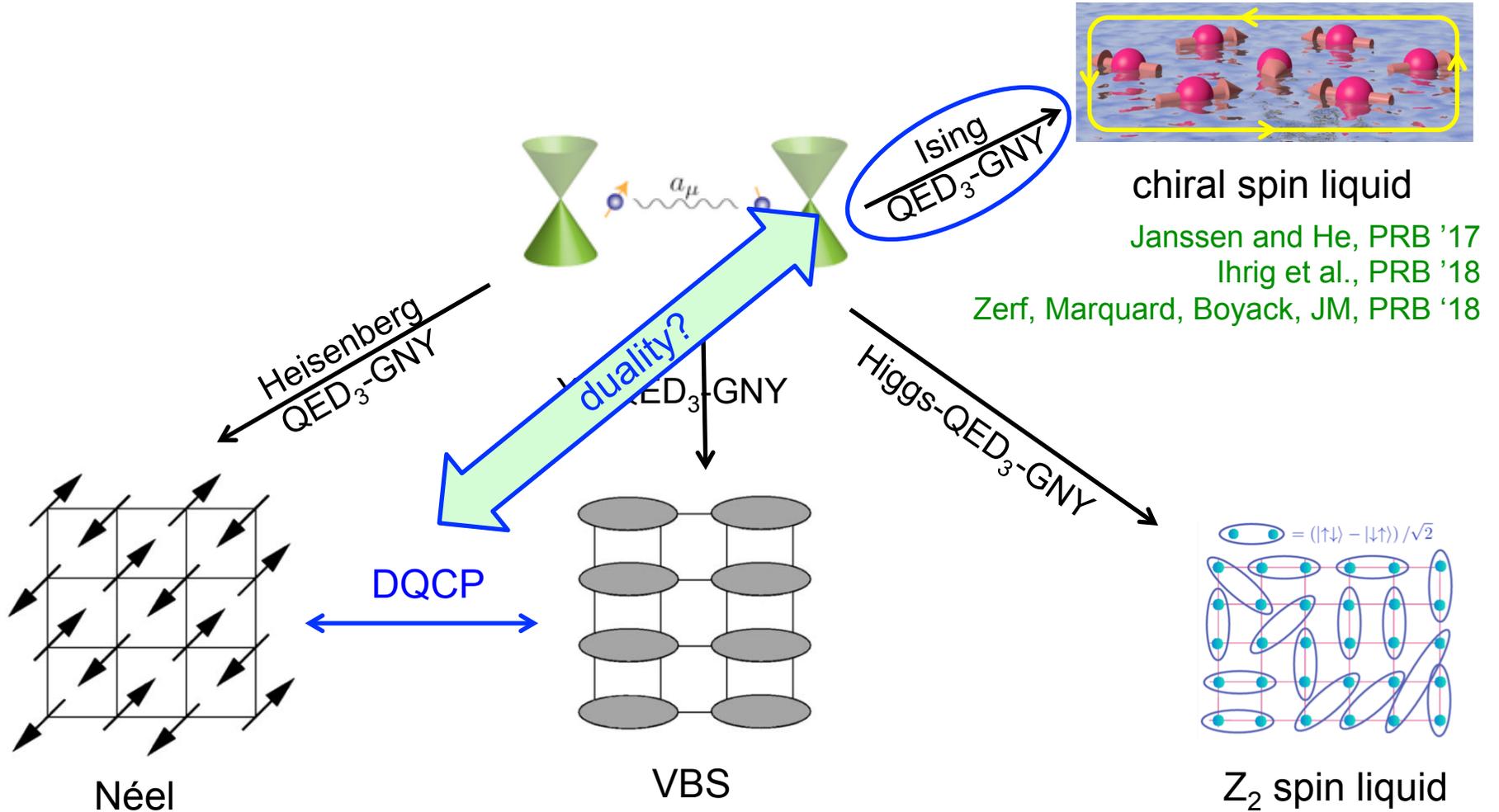
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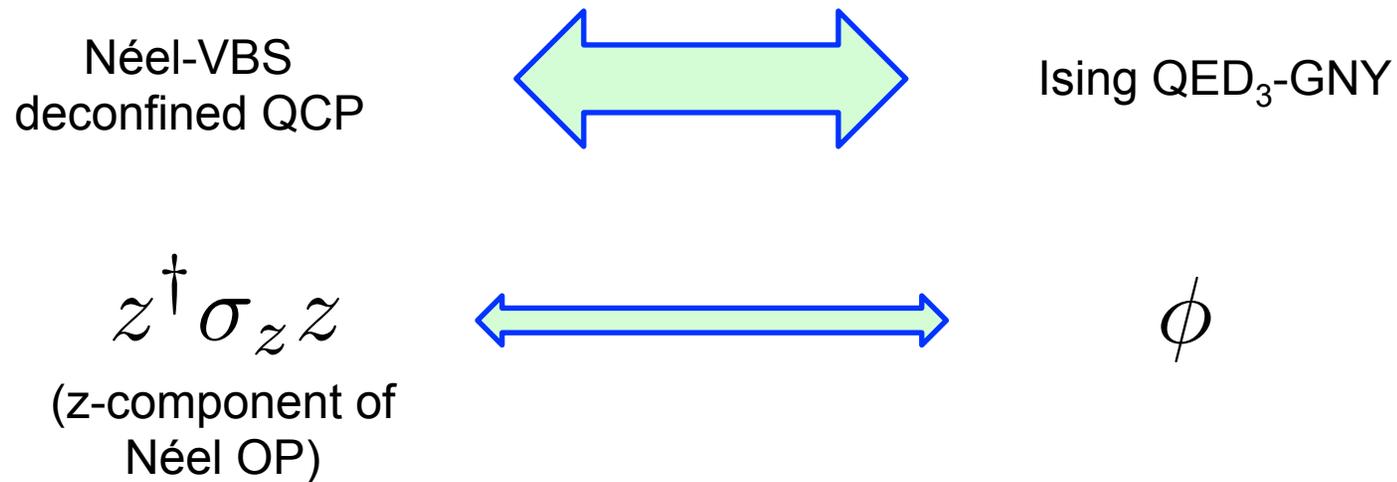
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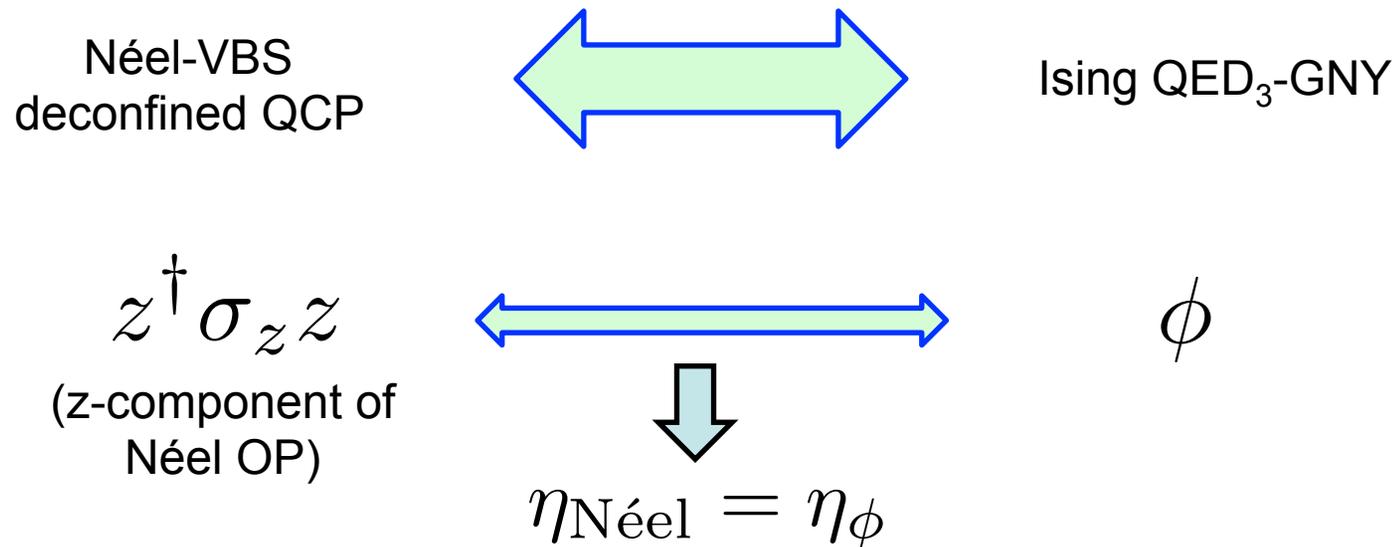
Deconfined Quantum Critical Points: Symmetries and Dualities

Chong Wang,^{1,2} Adam Nahum,^{3,4} Max A. Metlitski,^{3,5,2} Cenke Xu,^{6,2} and T. Senthil³



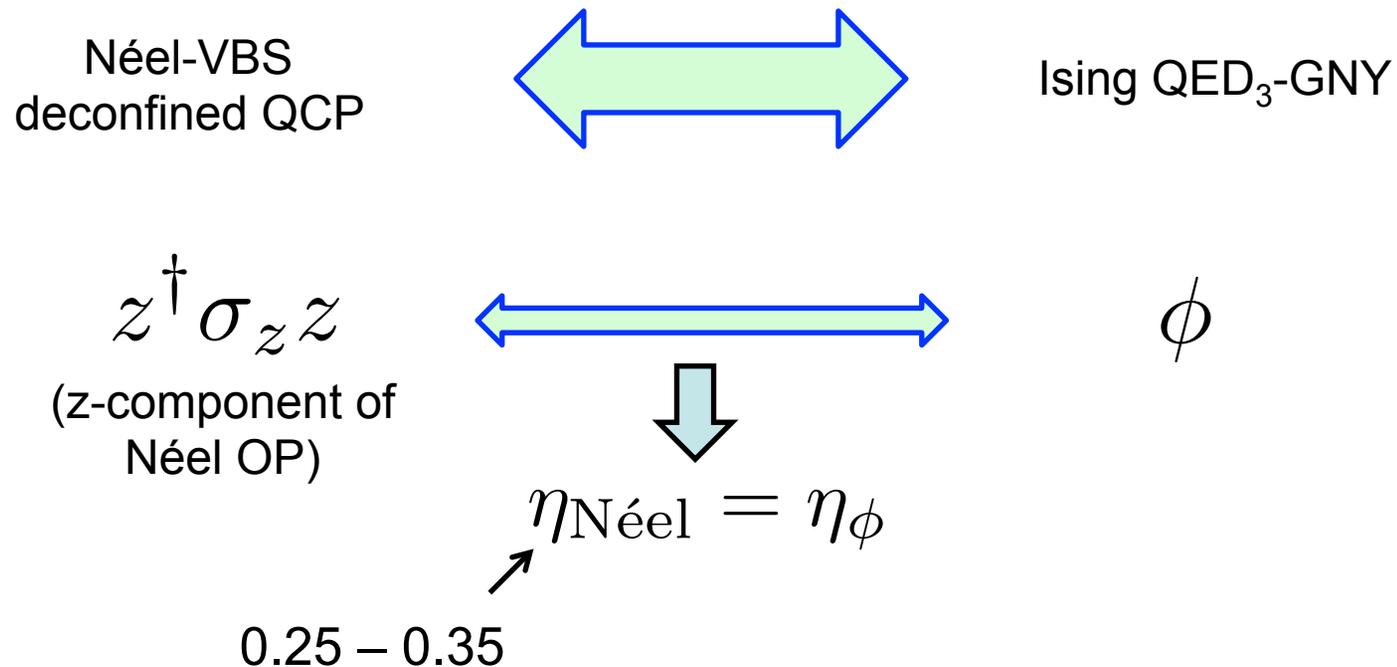
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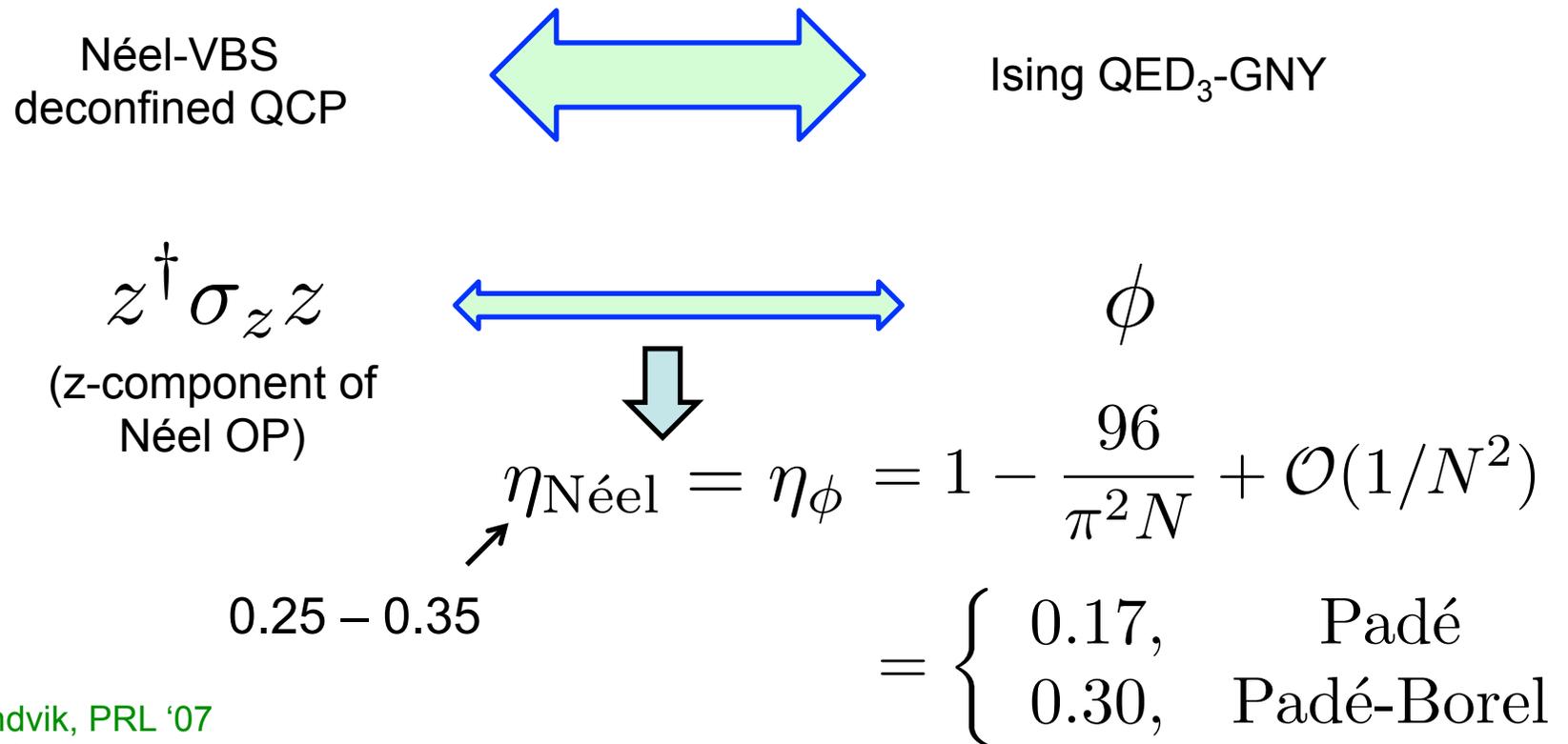
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Sandvik, PRL '07
Melko & Kaul, PRL '08
Nahum et al., PRX '15

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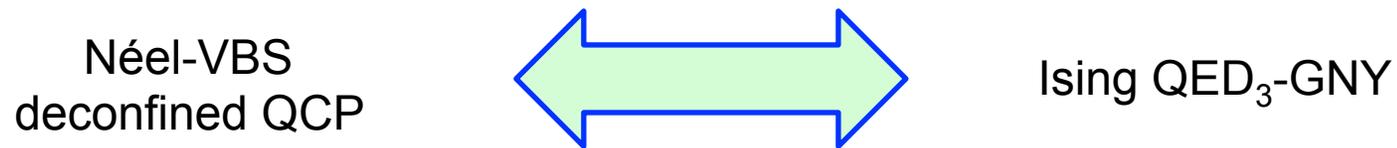


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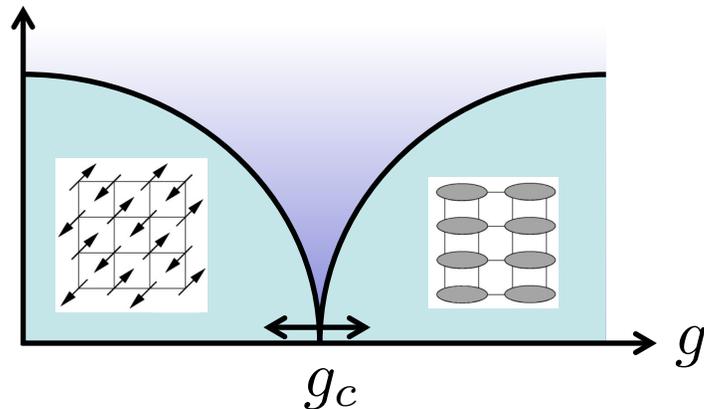
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$z^\dagger z$
(relevant operator)

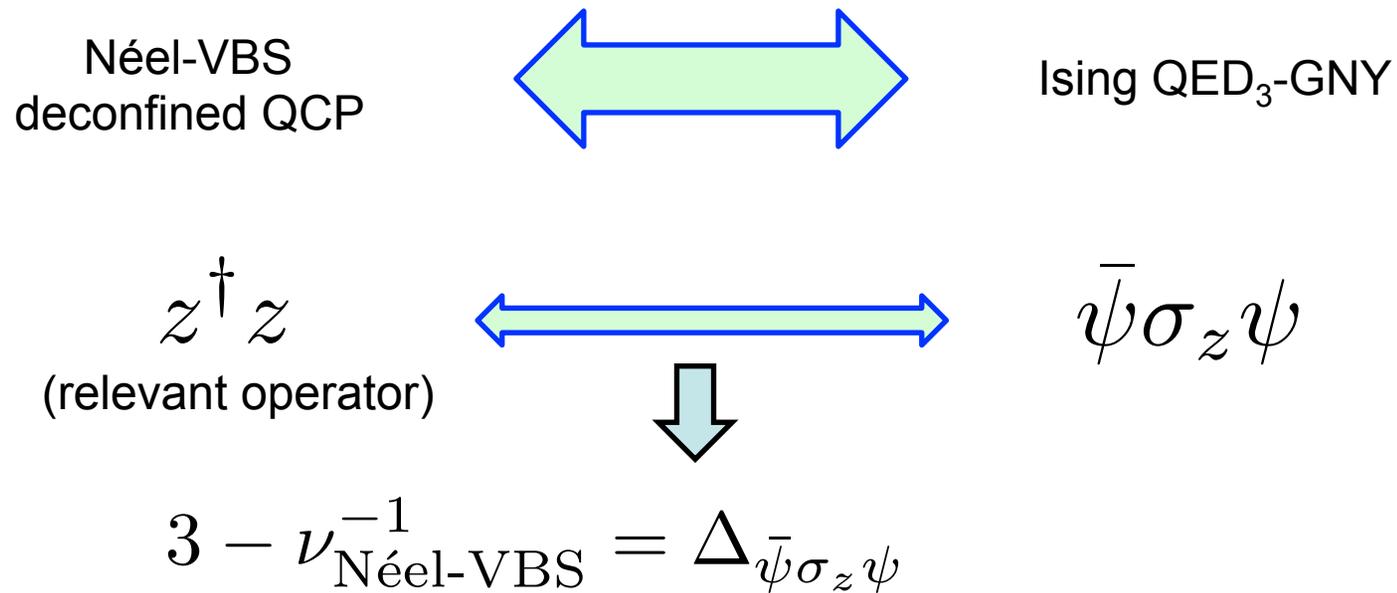


$\bar{\psi} \sigma_z \psi$



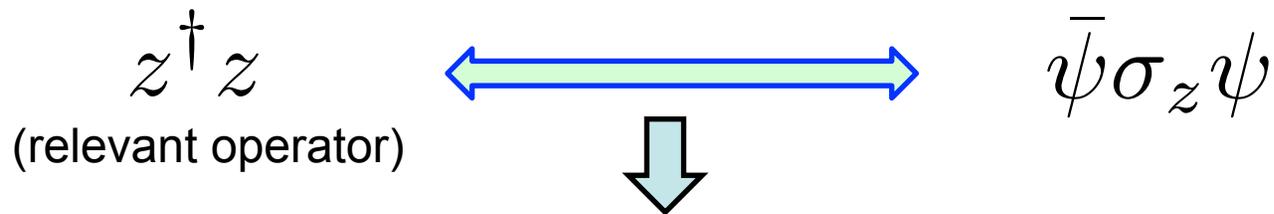
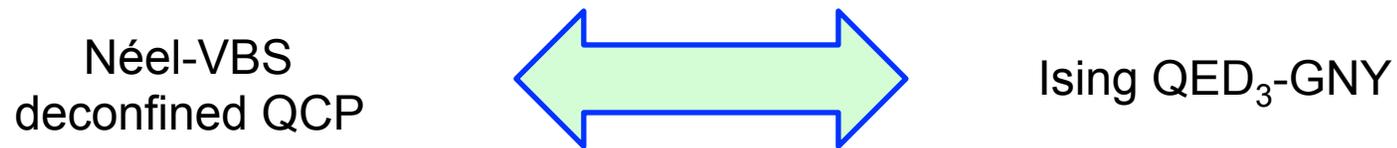
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$$3 - \nu_{\text{Néel-VBS}}^{-1} = \Delta \bar{\psi} \sigma_z \psi$$

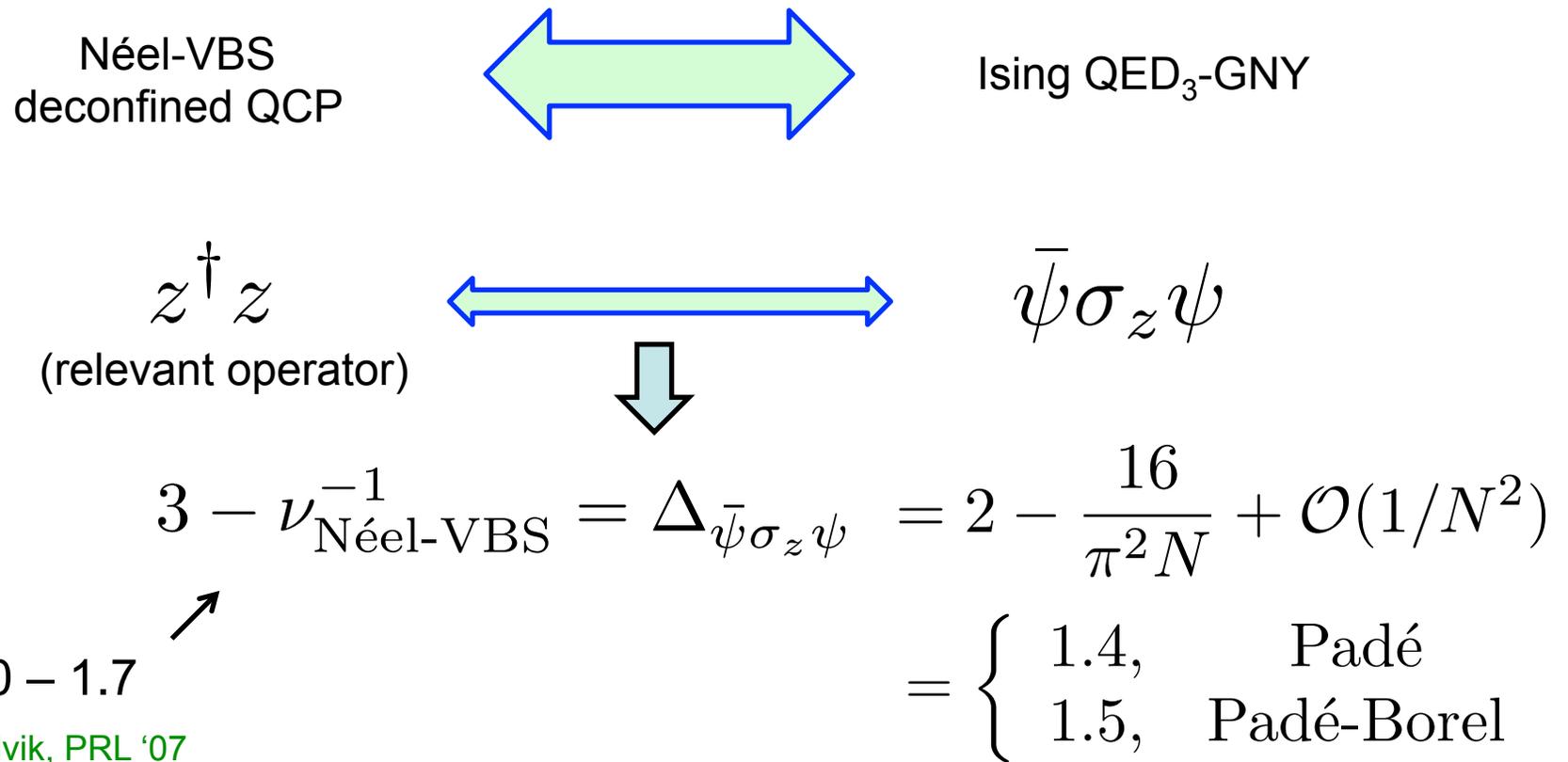
1.0 – 1.7



Sandvik, PRL '07
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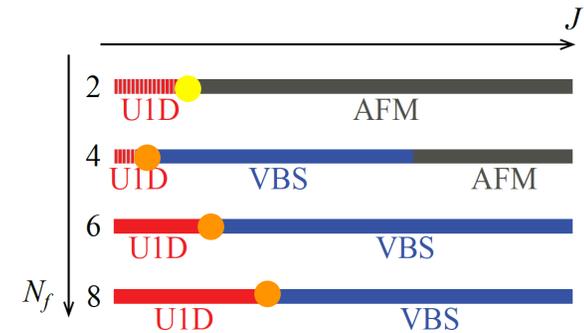


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Melko & Kaul, PRL '08
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Boyack, Rayyan, JM, PRB '19

Summary (part I)

- U(1) deconfined phase / Dirac spin liquid observed in QMC as critical phase of matter



- Gauge fluctuations at U1D-Néel and U1D-VBS transitions produce new O(3) and O(2) universality classes in 2+1 dimensions
- Future: test critical exponent predictions in QMC

Collaborators (part II)



G. Shankar
(Alberta)

G. Shankar & JM, arXiv:2103.11717

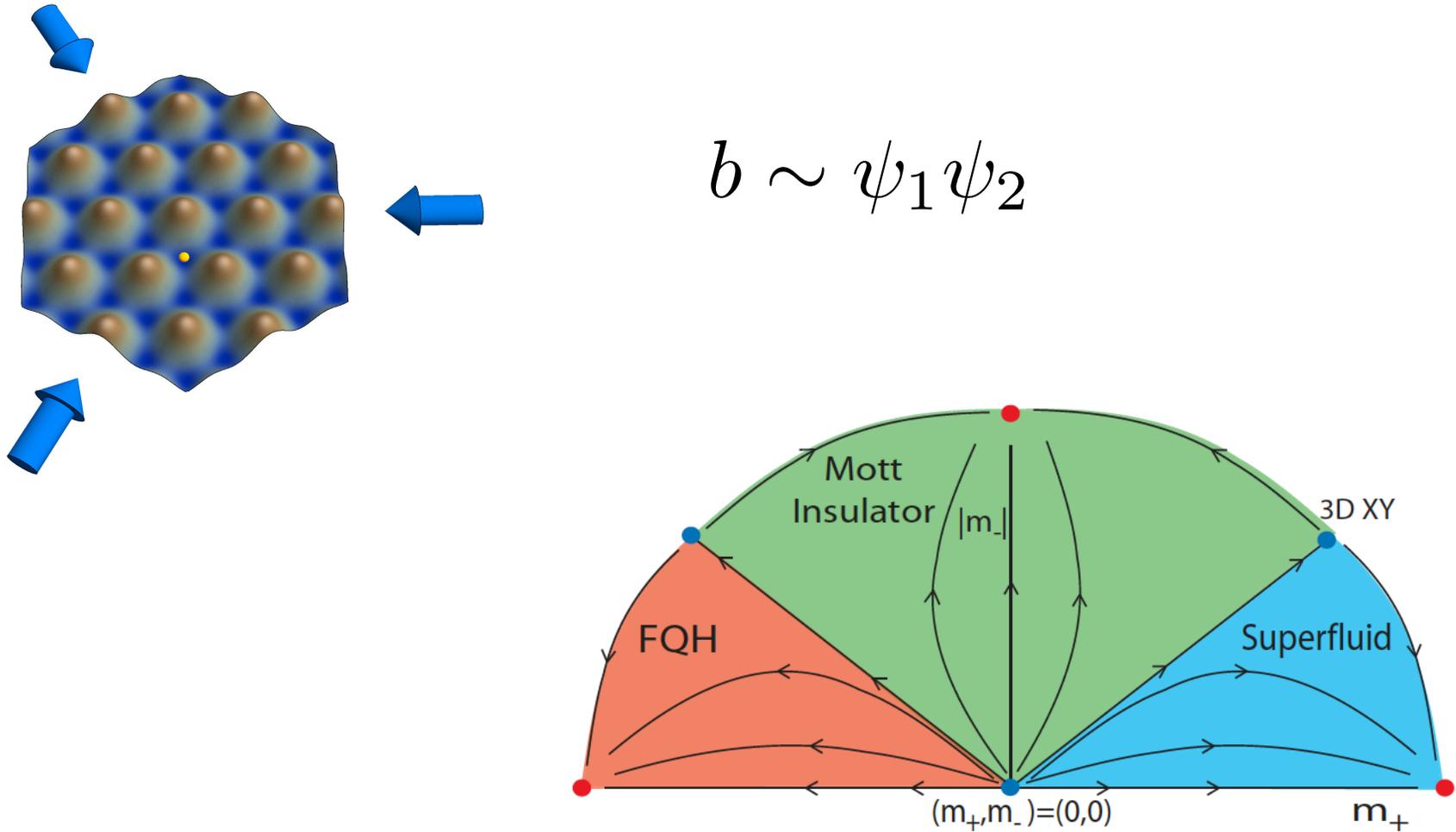
👉 see also “*Symmetry-Breaking Effects in Parton Gauge Theories*”,
poster session

Continuous transition between fractional quantum Hall and superfluid states

Maissam Barkeshli¹ and John McGreevy²

¹Microsoft Station Q, Elings Hall, University of California, Santa Barbara, CA 93106, USA

²Department of Physics, University of California at San Diego, La Jolla, CA 92093, USA

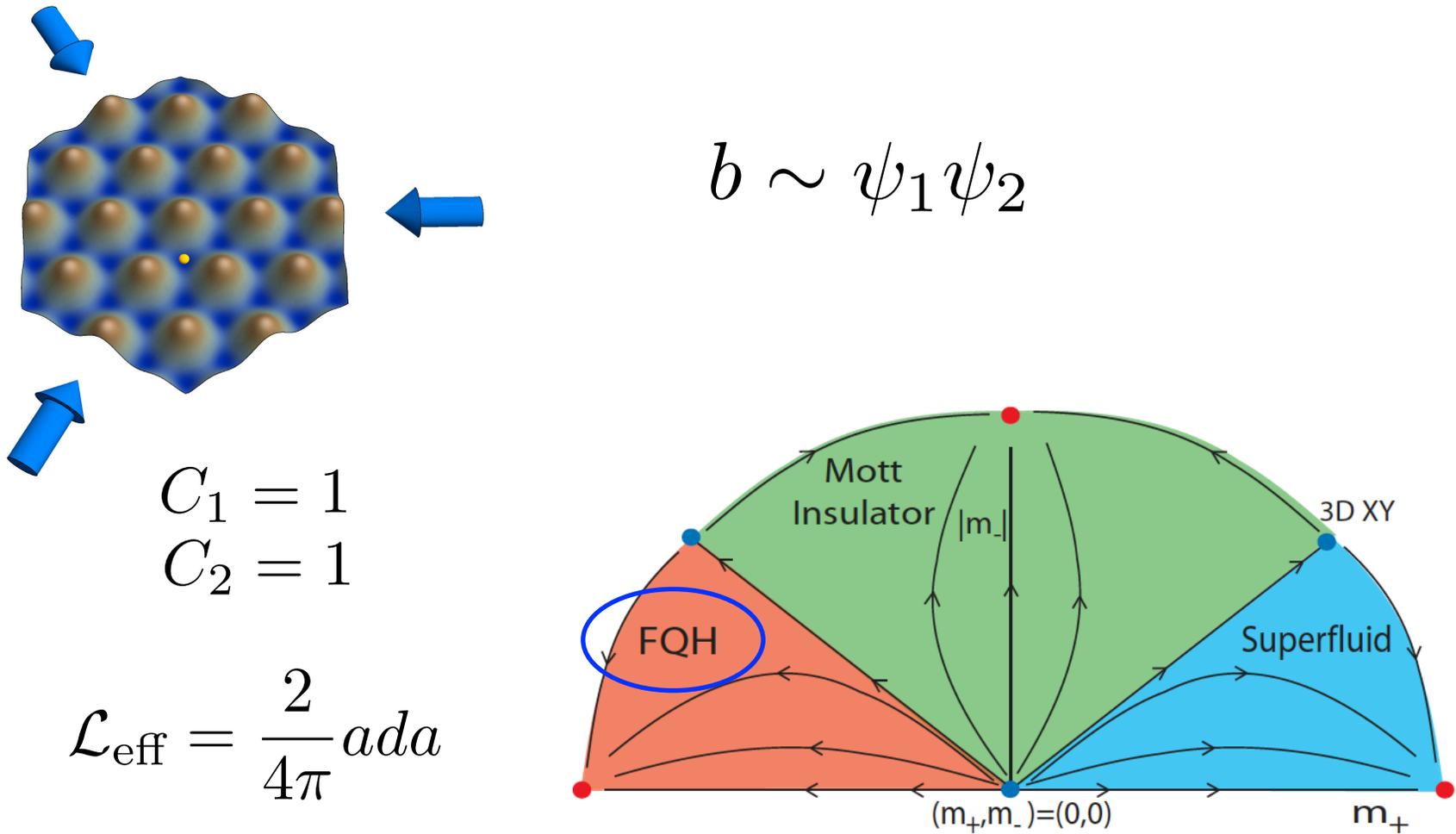


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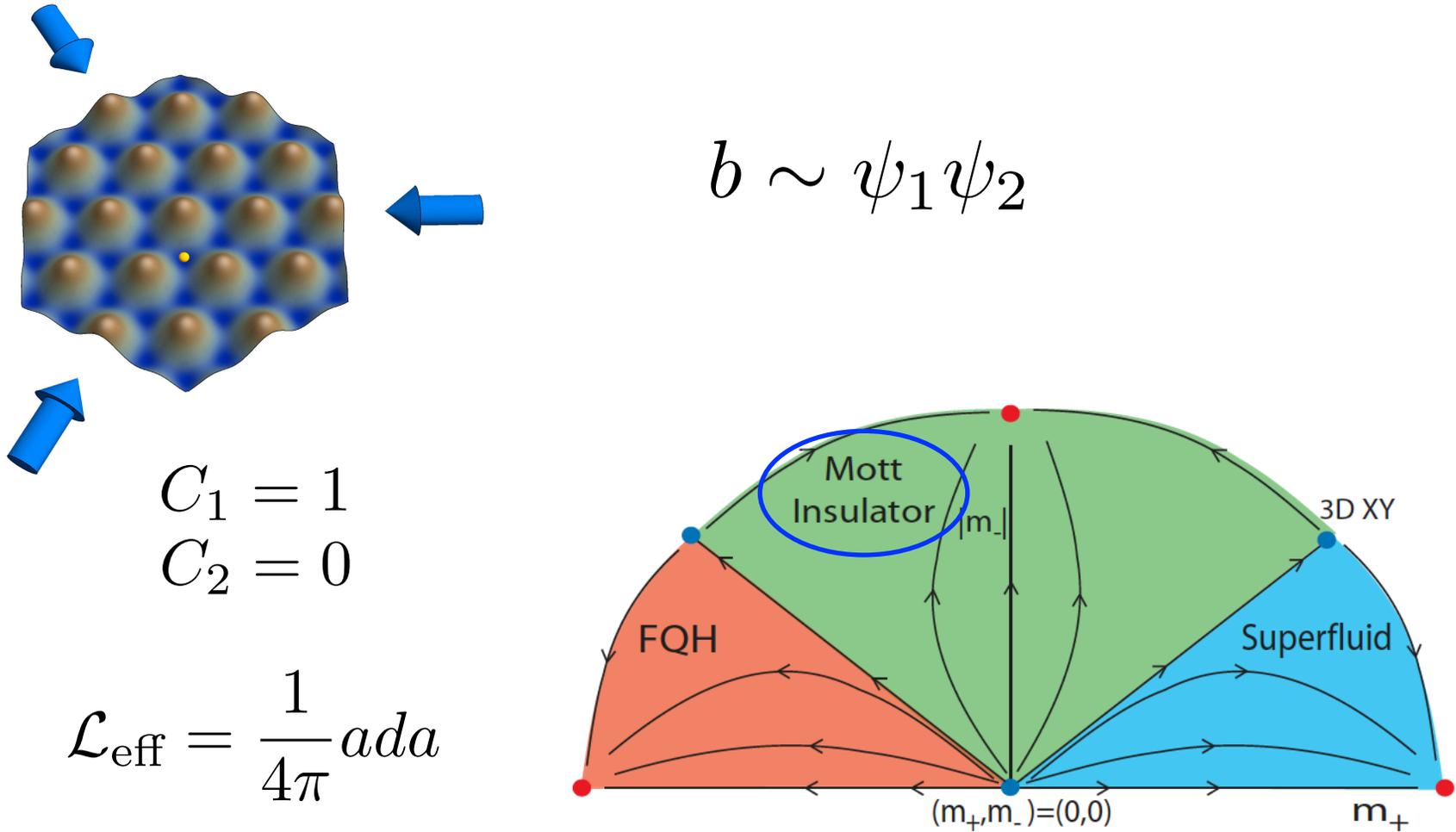


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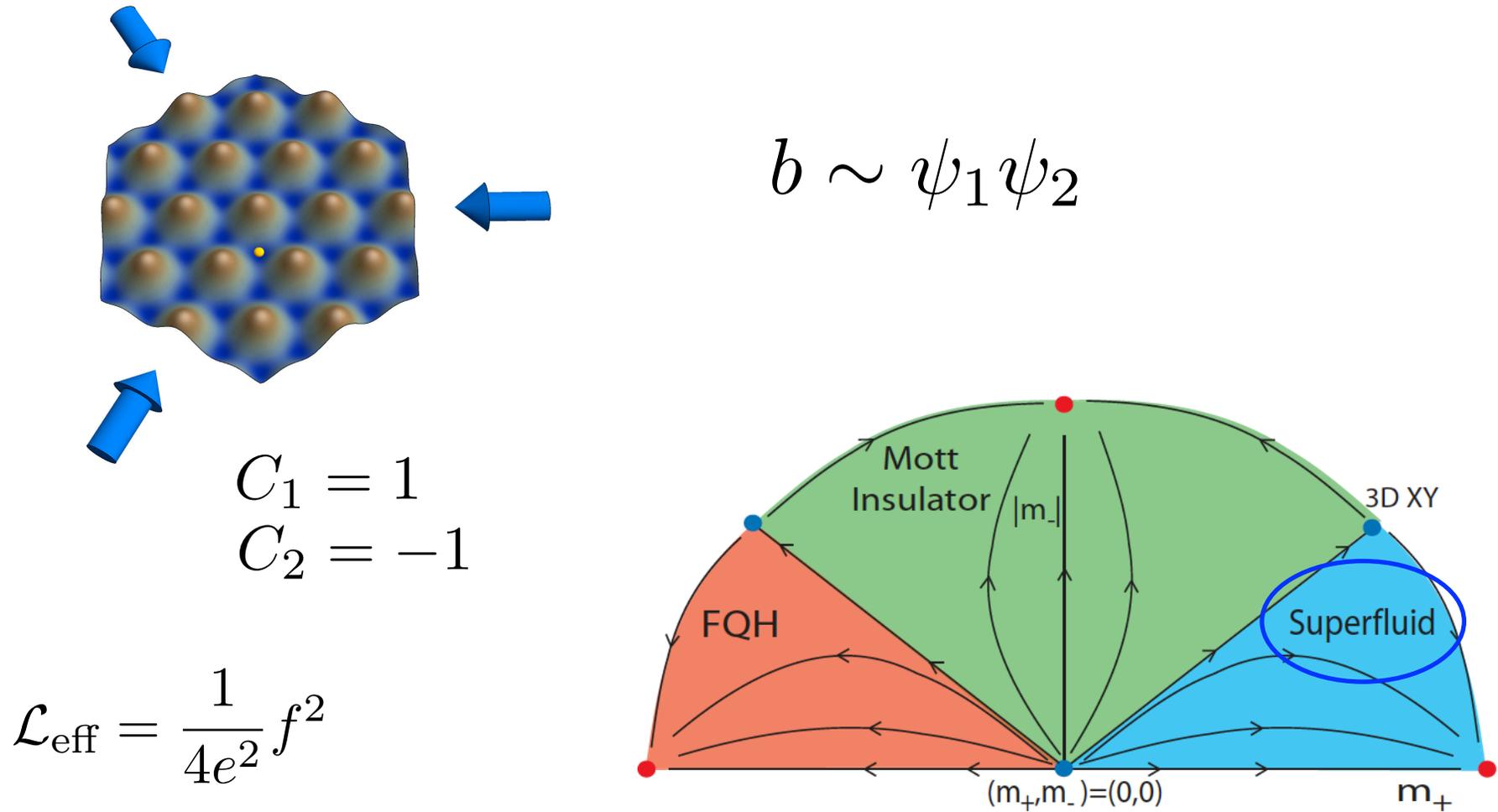


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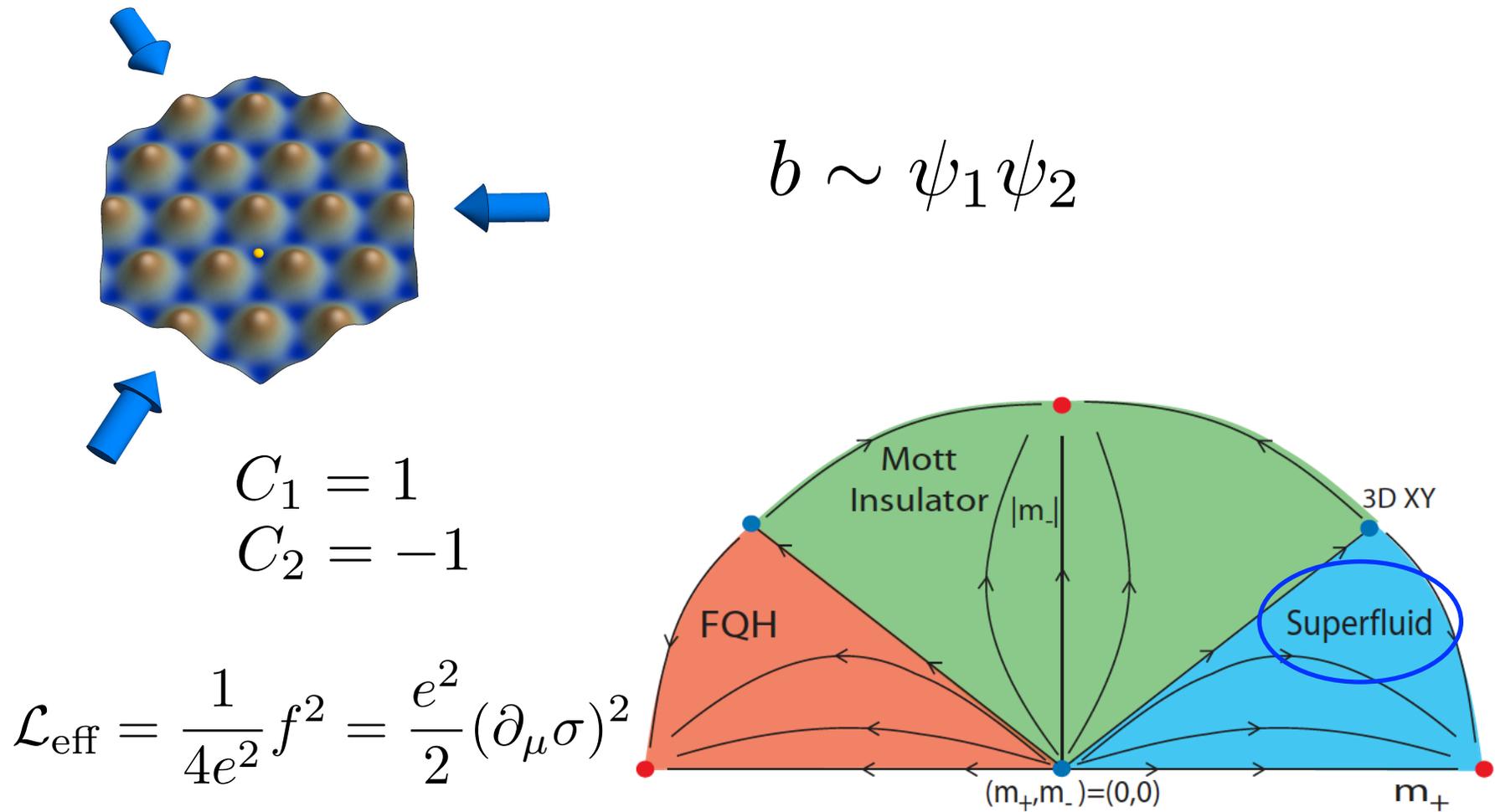


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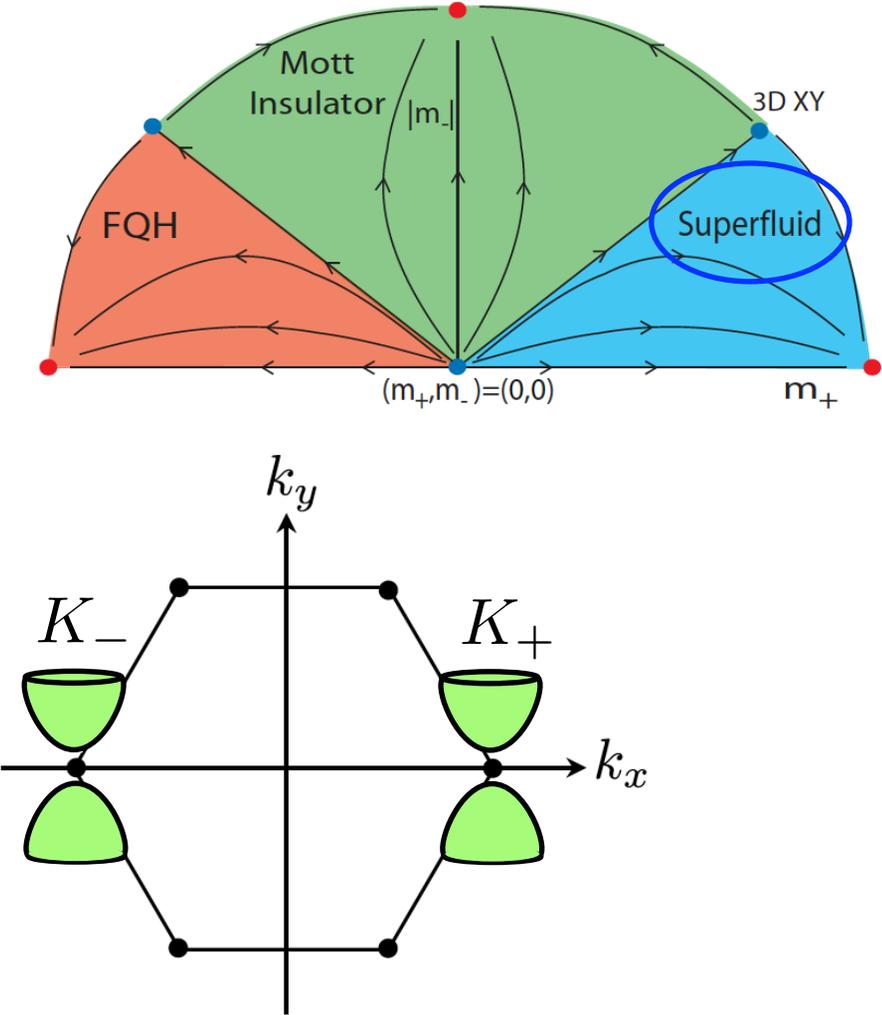
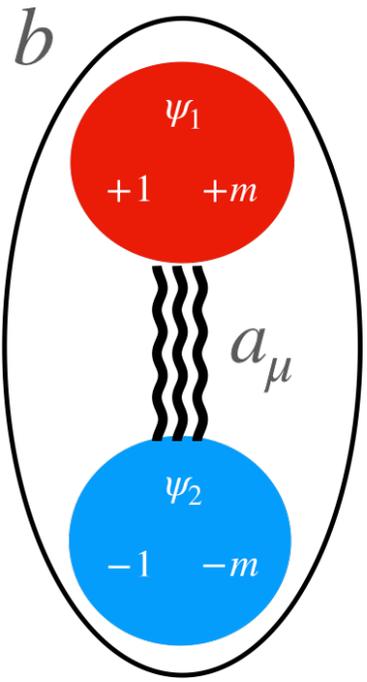
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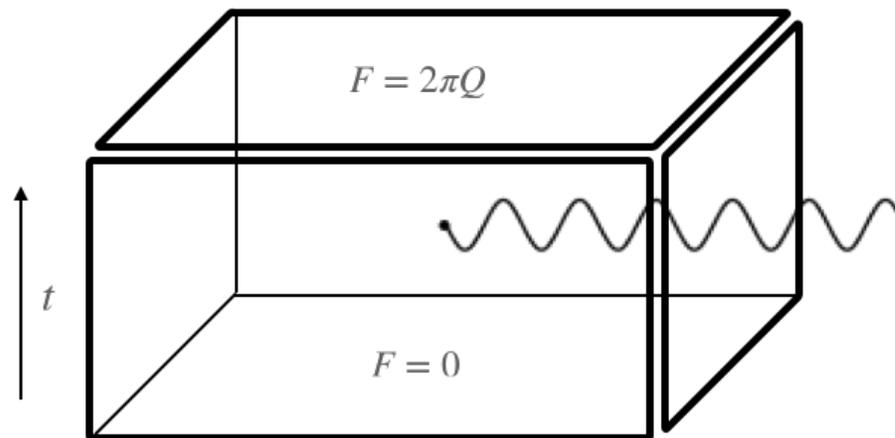
Massive QED₃



$$\mathcal{L} = \sum_{\alpha=\pm} [\psi_{1\alpha}(\not{\partial} - i\not{A} - i\not{\phi} + m)\psi_{1\alpha} + \psi_{2\alpha}(\not{\partial} + i\not{\phi} - m)\psi_{2\alpha}] + \frac{1}{4e^2} f^2$$

Instantons and confinement

- Superfluid phase ought to be a confined (non-fractionalized) phase
- Lattice QED₃: gauge field is compact, confinement via **instanton proliferation** in pure gauge theory



- Effect of fermionic matter on instanton proliferation?

Polyakov confinement

$$Z = \int D\sigma e^{-\frac{e^2}{2} \int d^3x (\partial_\mu \sigma)^2}$$

Dualized Maxwell: $\int Da \exp\left(-\int f^2\right)$

$$\times \sum_{N=0}^{\infty} \frac{1}{N!} \prod_{j=1}^N \int d^3z_j \sum_{Q_j=-\infty}^{\infty} e^{-\frac{\pi^2 Q_j^2}{2e^2}} e^{i2\pi Q_j \sigma(z_j)}$$

Sum over all monopole configurations

Action for $Q_j/2e$ Dirac monopole

Monopole operator $\mathcal{M}_{Q_j}(z_j)$

Polyakov confinement

$$Z = \int D\sigma e^{-\frac{e^2}{2} \int d^3x (\partial_\mu \sigma)^2}$$

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$$= e^{-\lambda \int d^3x \cos(2\pi \sigma)}$$

Sum over all monopole configurations

Action for $Q_j/2e$
Dirac monopole

Monopole operator
 $\mathcal{M}_{Q_j}(z_j)$

Polyakov with fermions?

$$Z = \int D\sigma e^{-\frac{e^2}{2} \int d^3x (\partial_\mu \sigma)^2}$$

Dualized Maxwell: $\int Da \exp\left(-\int f^2\right)$

$$\times \sum_{N=0}^{\infty} \frac{1}{N!} \prod_{j=1}^N \int d^3z_j \sum_{Q_j=-\infty}^{\infty} e^{-\frac{\pi^2 Q_j^2}{2e^2}} e^{i2\pi Q_j \sigma(z_j)}$$

$$\int D(\bar{\psi}_\alpha, \psi_\alpha) e^{-S_F[a_\mu^{Q_j}]}$$

Sum over all monopole configurations

Action for $Q_j/2e$ Dirac monopole

Monopole operator $\mathcal{M}_{Q_j}(z_j)$

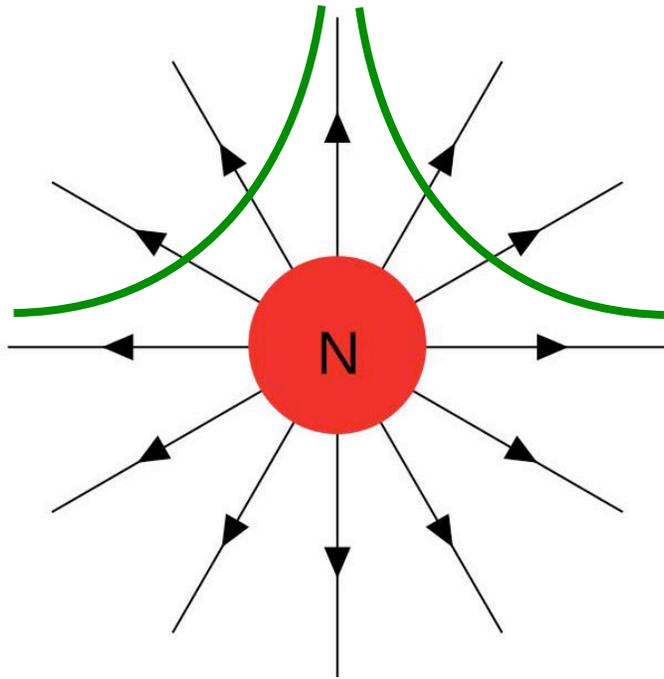
Z_F in fixed monopole background

Fermion zero modes

- Zero modes (ZM) of massless Dirac fermions on 4D Yang-Mills instanton give rise to explicit (anomalous) breaking of axial U(1) symmetry ('t Hooft, PRL '76, PRD '76)
- 3D Georgi-Glashow model with massless fermions: fermion ZM protected by Callias index theorem \rightarrow breaking of U(1) fermion number symmetry (Affleck, Harvey, Witten, Nucl. Phys. B '82)
- Massless QED₃: no Callias index theorem, no fermion ZM (Marston, PRL '90)
- Fermion zero modes in **massive** QED₃?

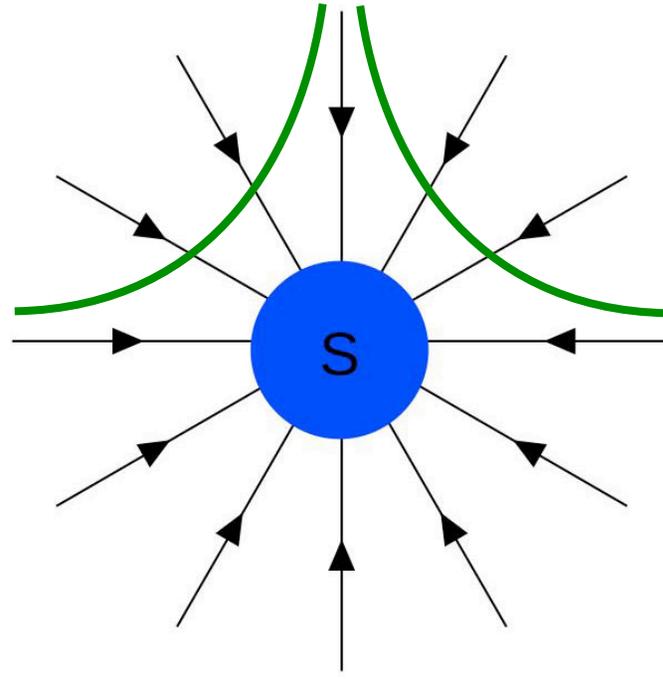
Fermion zero modes in massive QED₃

$$\psi_0 \sim \frac{e^{-mr}}{r} \in \ker \mathcal{D}$$



$$g = +1/2e$$

$$\tilde{\psi}_0 \sim \frac{e^{-mr}}{r} \in \ker \mathcal{D}^\dagger$$

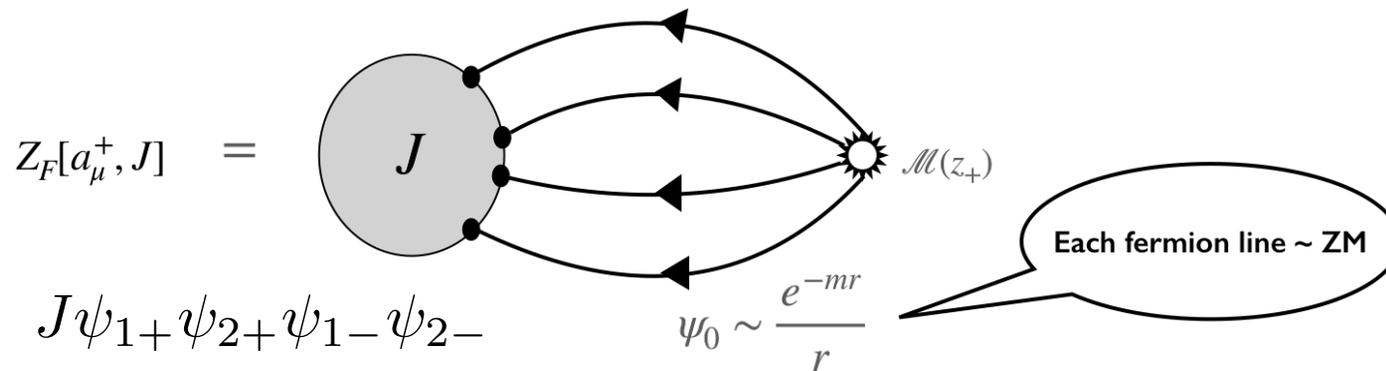


$$g = -1/2e$$

't Hooft vertex

$$Z_F[a_\mu^+] = \int D(\bar{\psi}_\alpha, \psi_\alpha) e^{-\int \bar{\psi}_\alpha \mathcal{D}_\alpha \psi_\alpha} \propto \prod_\alpha \det \mathcal{D}_\alpha = 0.$$

- ZM kill partition function due to unpaired Grassmann variables in functional integration measure, but can be “soaked up” by symmetry-breaking amplitudes (anomalous insertions)



Effective Lagrangian

$$\mathcal{L}_0 = (\partial\sigma)^2 + \bar{\psi}_{1\pm}(\not{\partial} - i\not{\phi} + m)\psi_{1\pm} + \bar{\psi}_{2\pm}(\not{\partial} + i\not{\phi} - m)\psi_{2\pm}$$

$$\mathcal{L}_{\text{inst}} = \frac{K}{m} \left[e^{-2\pi i\sigma} \psi_{1+}^\dagger \Delta \psi_{2+} \psi_{1-}^\dagger \Delta \psi_{2-} + e^{2\pi i\sigma} \bar{\psi}_{2+} \Delta \bar{\psi}_{1+} \bar{\psi}_{2-} \Delta \bar{\psi}_{1-} \right]$$

- Instantons break apparent $U(1)_{\text{top}} \times U(1)_b$ to diagonal $U(1)$ subgroup:

$$\psi_{1\pm} \rightarrow e^{i\beta} \psi_{1\pm}, \quad \psi_{2\pm} \rightarrow \psi_{2\pm}, \quad \sigma \rightarrow \sigma + \frac{\beta}{\pi} \pmod{1}$$

Effective Lagrangian

$$\mathcal{L}_0 = (\partial\sigma)^2 + \bar{\psi}_{1\pm}(\not{\partial} - i\not{a} + m)\psi_{1\pm} + \bar{\psi}_{2\pm}(\not{\partial} + i\not{a} - m)\psi_{2\pm}$$

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- If dual photon condenses:

$$\text{paired SF} \quad \langle \psi_1 \psi_2 \psi_1 \psi_2 \rangle \sim \langle b(x) b(x') \rangle \neq 0$$

Effective Lagrangian

$$\mathcal{L}_0 = (\partial\sigma)^2 + \bar{\psi}_{1\pm}(\not{\partial} - i\not{a} + m)\psi_{1\pm} + \bar{\psi}_{2\pm}(\not{\partial} + i\not{a} - m)\psi_{2\pm}$$

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- If dual photon condenses:

$$\text{paired SF} \quad \langle \psi_1 \psi_2 \psi_1 \psi_2 \rangle \sim \langle b(x) b(x') \rangle \neq 0$$



$$\text{ordinary SF} \quad \langle \psi_1 \psi_2 \rangle \sim \langle b \rangle \neq 0$$

Summary (part II)

- Effect of instantons (monopoles) in parton theory of hardcore bosons?
- Compact QED_3 with massive fermions has Euclidean fermion ZM bound to instantons
- Instanton proliferation + ZM = effective 4-fermion interaction ('t Hooft vertex)
- Possible phases: paired SF or conventional BEC
- Future:
 - Apply to instabilities of Dirac spin liquid; relation to Hamiltonian picture (monopole-operator ZM dressing)? (Song et al., Nat. Comm. '19, PRX '20)
 - Callias index thm for massive QED_3 ?

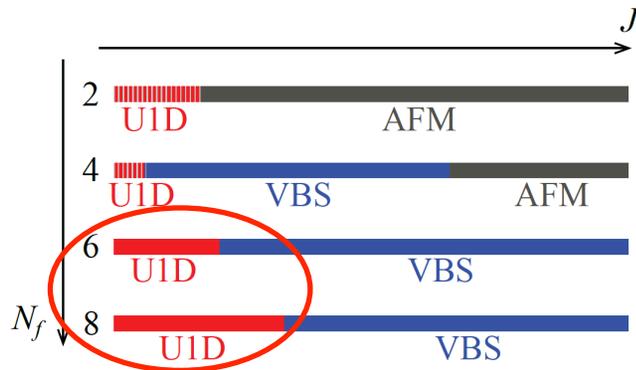
Thank you!

Monopole proliferation?

- Critical phase / point destroyed by monopole proliferation if $\Delta_M < 3$:

$$\Delta_M = aN_f + b + \mathcal{O}(1/N_f)$$

- U1D phase on square lattice (Pufu, PRD 89, 065016 (2014)): a, b known, suggest U1D phase stable for $N_f \geq 6$
- Néel and VBS QCPs: a, b unknown, but a known for $\mathbf{q} = 0$ noncollinear QCP on kagome lattice (Dupuis, Paranjape, Witczak-Krempa, PRB 100, 094443 (2019))



- Scaling laws in U1D phase & continuous QCPs in QMC at small N_f : quasicritical behavior possible for

$$L \ll L^* \sim ay_0^{-1/(3-\Delta_M)}$$

monopole fugacity