Non-perturbative Renormalization Group for Dirac Fermions; interactions and quasi-periodic disorder

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- Need to take into account large orders effects and irrelevant terms
- a)quasi-periodic disorder;+interaction b)universality in graphene or Hall insulators+ interactions; c)bulk-edge correspondence+interaction

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- Disorder can be random or quasi-periodic (more suitable for cold atoms experiments)
- Fermions on a 3d cubic lattice (Weyl semimetal) What is the effect of quasiperiodic disorder?

Dirac Fermions with quasi-periodic disorder

- $H_0 = \sum_{x\in\Lambda}\{\sum_{j=1}^{2}(-1)^{j-1}\big[(\zeta-1)a^\dagger_{x,j}a_{x,j}+\frac{1}{2}a^\dagger_{x,j}(-\Delta a)_{x,j}\big]+$

$$\frac{it_1}{2}\big[a^\dagger_{x,1}(a_{x+e_1,2}-a_{x-e_1,2})+a^\dagger_{x,2}(a_{x+e_1,1}-a_{x-e_1,1})\big]+$$

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- $H_0 = \int \frac{dk}{(2\pi)^3}\,\hat a^\dagger_k h(k)\hat a_k$ with $h(k) = \begin{pmatrix}\alpha(k) & \beta(k)\\ \beta^*(k) & -\alpha(k)\end{pmatrix}$ where $k\in(0,2\pi]^3$, $\alpha(k) = 2+\zeta-\cos k_1-\cos k_2-\cos k_3$ and $\beta(k) = t_1\sin k_1 - it_2\sin k_2$.

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- $\zeta\in[0,1)$, in which case $\hat h(k)$ is singular at $k=\pm p_F$, with $p_F=(0,0,\arccos\zeta)$.

Dirac Fermions with quasi-periodic disorder

- In the vicinity of $\pm p_F$, $k = q \pm p_F$, Dirac fermions
  $\hat{H}^0(q \pm p_F) = t_1\sigma_1 q_1 + t_2\sigma_2 q_2 \pm \sin p_F \sigma_3 q_3 + O(q^2)$

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- We include now a many body interaction and quasiperiodic disorder writing

$$H = H_0 + \varepsilon \sum_x \phi_x (a^+_{x,1} a^-_{x,1} - a^+_{x,2} a^-_{x,2}) + \lambda \sum_{x,y} v(x-y)\rho_x \rho_y$$

  where $v(x-y)$ is a short range potential, $\rho_x = a^+_{x,1} a^-_{x,1} + a^+_{x,2} a^-_{x,2}$
- Quasi-periodic disorder, ω_i irrational (rational=periodic)

$$\phi_x = \sum_n \hat{\phi}_n e^{i2\pi(\omega_1 n_1, x_1 + \omega_2 n_2 x_2 + \omega_3 n_3 x_3)}$$

  with $n \in \mathbb{Z}^3$, $\hat{\phi}_n = \hat{\phi}_{-n}$ and $|\hat{\phi}_n| \leq C e^{-\xi(|n_1|+|n_2|+|n_3|)}$.
- Compute the $S(x,y)$ the $T = 0$ 2 -point function

The Aubry-Andre' model

- Quasi-periodic disorder has been deeply investigated in $d = 1$; almost Mathieu equation

$$-\varepsilon\psi(x+1) - \varepsilon\psi(x-1) + u\cos(2\pi(\omega x + \theta))\psi(x) = E\psi(x)$$

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- Small divisors problem; in perturbation theory one gets $1/(E_n - E_m)$ with $E_n = \cos(k - 2\pi\omega n)$. If ω is irrational it can be arbitrarily small. They typically produce factorials in the expansions which can break convergence.

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- What happens in $d > 1$ and with interaction?

RG for Weyl semimetals

- In the present case one as an higher dimensional situation, and a many body interaction
- The propagator is $g(\mathbf{x}) = \frac{1}{L^3\beta} \sum_{\mathbf{k}} e^{i\mathbf{k}\mathbf{x}}(-ik_0 I + h(k))^{-1}$

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- We perform a multiscale decompositions $g(\mathbf{x}) = g^{(1)}(\mathbf{x}) + \sum_{\rho=\pm} g_\rho^{(\leq 0)}(\mathbf{x})$ with $\hat{g}_\rho^{(\leq 0)}(\mathbf{k})$ non vanishing in region around in ρp_F, $\rho = \pm$

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- We further write $\hat{g}_\rho^{(\leq 0)}(\mathbf{k}) = \sum_{h=-\infty}^{0} \hat{g}_\rho^{(h)}(\mathbf{k})$ non vanishing in $\gamma^{h-1} \leq |\mathbf{k} - \rho\mathbf{p}_F| \leq \gamma^{h+1}$ with $\gamma > 1$.

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- We can integrate scale by scale obtaining that the generating function is

$$e^{W(\phi)} = \int P(d\psi^{(\leq h)}) e^{V^{(h)}(\psi^{(\leq h)},\phi)}$$

with $V = \sum_n \int d\mathbf{x}_1 ... d\mathbf{x}_n W_n^h \psi_{\mathbf{x}_1} ... \psi_{\mathbf{x}_n}$; monomials of any order are generated

- $$\mathcal{E}_h^T(\widetilde{\psi}^{(h)}(\tilde{P}_1),\ldots,\widetilde{\psi}^{(h)}(\tilde{P}_n)) =$$
  $$\sum_T \prod_{l\in T} g^{(h)}(x_l - y_l) \int dP_T(\mathbf{t}) \det G^{h,T}(\mathbf{t}) ,$$

  where $\widetilde{\psi}^{(h)}(\tilde{P}_i)$ are monomials in the fields, T is a tree connecting the set $P_1, P_2, ..$, the $g^{(h)}(x_l - y_l)$ are the propagators of the tree T, $\det G^{h,T}(\mathbf{t})$ contain all the propagators of the fields not in the tree. dP is a normalized probability measure.

Truncated expectation

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- Updated derivation Gentile Mastropietro Phys Rep (2000) or Giuliani Mastropietro Ryckov JHEP 2020)

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$$q_{i,1} - q_{2i,} + 2\omega_i n_i \pi + 2l_i \pi + (\varepsilon_1 - \varepsilon_2)p_F = 0$$

The factor $2\omega_i n_i \pi$ is the momentum exchanged with the quasiperiodic disorder while the factor $2l_i\pi$ is exchanged with the lattice (Umklapp).

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- In the q-periodic case are relevant or irrelevant?

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- Diophantine numbers have full measure

- $k = q \pm p_F$ in quadratic term $|q_a| \le \gamma^h$, $|q_b| \le \gamma^h$; We call $N = (N_1, N_2, N_3)$, $N = \sum_i n_i$ (N non vanishing) where n_i is the momentum associated with each ε vertex in the subgraph; $k_a - k_b = 2\pi(N_1\omega_1, N_2\omega_2, N_3\omega_3)$ $2\gamma^h \ge |q_a|_T + |q_b|_T \ge |q_a - q_b|_T$

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- This inequality says that if the denominators of propagators are very small, than the momentum transferred is very large
- The exponentiial decay implies the irrelevance of non resonant terms (complications in full proof for overlapping divergences; differnt with slow decay)

Stability of quasi-periodic disorder

- As the interaction in general moves the location of the Weyl momentum, we write $\xi = \cos p_F + \nu$ and we fix the interacting p_F

Theorem 1.

(Mastropietro PRB 2021) For λ, ε small enough and imposing F+Diophantine conditions

$$S(\mathbf{q} \pm \mathbf{p}_F) = \frac{1}{Z} \begin{pmatrix} -iq_0 \pm v_3 q_3 & v_1 q_1 - i v_2 q_2 \\ v_1 q_1 + i v_2 q_2 & -iq_0 \mp v_3 q_3 \end{pmatrix}^{-1} (1 + O(\mathbf{q}))$$

with $Z = 1 + O(\lambda, \varepsilon)$, $v_1 = t_1 + O(\lambda, \varepsilon)$, $v_2 = t_2 + O(\lambda, \varepsilon)$, $v_3 = \sin p_F + O(\lambda, \varepsilon)$

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- With slow decay in momentum space could be relevant
- Other application in 2d is the Hofstadter model (PRB 2019) or interacting Aubry-Andre' or XXZ with q-periodic disorder (PRL 2015)

Graphene

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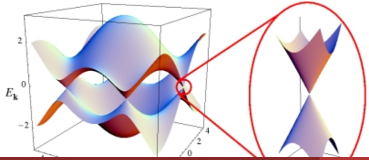
- Is the optical conductivity of graphene perfectly universal?
- The Hamiltonian of the Hubbard model on the honeycomb lattice is

$$H = H_0 + U \sum_{\vec{x} \in \Lambda_A \cup \Lambda_B} \left(n_{\vec{x},\uparrow} - \frac{1}{2} \right) \left(n_{\vec{x},\downarrow} - \frac{1}{2} \right) ,$$

$$H_0 = -t \sum_{\vec{x} \in \Lambda_A, i=1,2,3} \sum_{\sigma=\uparrow\downarrow} \left(a^+_{\vec{x},\sigma} b^-_{\vec{x}+\vec{\delta}_i,\sigma} + b^+_{\vec{x}+\vec{\delta}_i,\sigma} a^-_{\vec{x},\sigma} \right)$$

$a^\pm_{\vec{x}}, b^\pm_{\vec{x}}$ fermionic operators,
$\vec{\delta}_1 = (1,0)$, $\vec{\delta}_2 = \frac{1}{2}(-1, \sqrt{3})$, $\vec{\delta}_3 = \frac{1}{2}(-1, -\sqrt{3})$, Λ_A periodic triangular lattice.



Honeycomb lattice

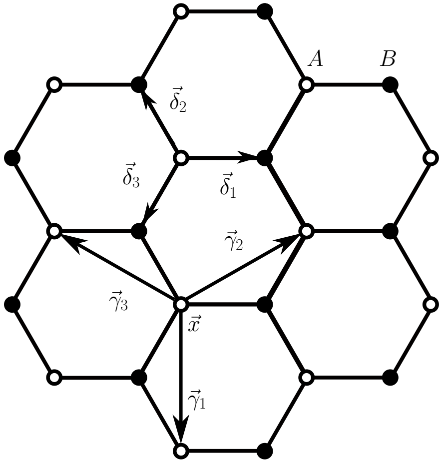


Figure:

Interacting 2 point function

- Giuliani, Mastropietro Phys. Rev. B 79, 201403(R) (2009); Comm. Math. Phys. 293, 301 (2010). For $|U| \leq U_0$

$$S(\mathbf{k}' + \mathbf{p}_F^{\pm}) = \frac{1}{Z} \begin{pmatrix} ik_0 & v_R(ik'_1 \mp k'_2) \\ v_R(-ik'_1 \mp k'_2) & ik_0 \end{pmatrix}^{-1} (1 + R(\mathbf{k})),$$

$$Z = 1 + O(U^2) \qquad v_R = \frac{3t}{2} + dU + O(U^2)$$

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  $d = 0.3707$. (for n n interaction), $|R(\mathbf{k})| \leq C|U||\mathbf{k}'|$
- The correlations are analytic and U_0 is finite.
- Non universal finite value of velocity and wave function (not constrained by Lorentz invariance).

The optical conductivity

- The currents are (spin is understood)

$$\vec{\hat{J}}_{\vec{p}} = iet \sum_{\vec{x}\in\Lambda,j} e^{-i\vec{p}\vec{x}} \vec{\delta}_j \eta^j_{\vec{p}} \left(a^+_{\vec{x}} b^-_{\vec{x}+\vec{\delta}_j} - b^+_{\vec{x}+\vec{\delta}_j} a^-_{\vec{x}} \right) = v_F^{(0)} \vec{\hat{j}}_{\vec{p}}$$

with $\eta^j_{\vec{p}} = \frac{1-e^{-i\vec{p}\vec{\delta}_j}}{i\vec{p}\vec{\delta}_j}$; sum of the three bond currents

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  with $\eta^j_{\vec{p}} = \frac{1-e^{-i\vec{p}\vec{\delta}_j}}{i\vec{p}\vec{\delta}_j}$; sum of the three bond currents

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- If $U = 0$ than $\sigma_{lm} = \delta_{lm} \frac{\pi e^2}{2h}$
- Are there or not corrections?

- Giuliani, Mastropietro, Porta. PRB 83, 195401 (2011); CMP 311,317 (2012).

Theorem 2.

For $|U| \leq U_0$

$$\sigma_{lm} = \frac{e^2}{h}\frac{\pi}{2}\delta_{lm} \; .$$

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- While the Fermi velocity and the wave function renormalization are renormalized $v_F(U) > v_F(0)$ the conductivity is protected.
- Still open the long range case, as non perturbative results are difficult with fermions (perturbative results Herbut-Mastropietro (PRB 2014))

Ward Identities

- By the continuity equation, defining $\hat{G}_\mu(\mathbf{k},\mathbf{p})$ the FT of $< \mathbf{T}\, J_{\vec{p},\mu}(x_0); \hat{\Psi}^-_{\vec{k}+\vec{p},\sigma}(y_0)\hat{\Psi}^+_{\vec{k},\sigma} >$ we get Ward Identities

$$\sum_{\mu=0}^{2}(i)^{\delta_{\mu,0}} p_\mu \hat{G}_\mu(\mathbf{k},\mathbf{p}) = \hat{S}(\mathbf{k}+\mathbf{p})M_0(\vec{p}) - M_0(\vec{p})\hat{S}(\mathbf{k}).$$

and $\left[M_0(\vec{p})\right]_{\rho,\rho'} = \delta_{\rho,\rho'} e^{-i\vec{p}\vec{\delta}_1(\rho-1)}$. Similar WI hold for the current and density correlations.

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- WI based on total phase holds also beyond the Dirac description; the chiral ones only in the Dirac description.

Proof.

- Conductivity properties are related to differentiability properties of the corrent correlations. If $\hat{K}_{lm}(\mathbf{p})$ is the FT of $\langle J_{l,\mathbf{x}}; J_{m,\mathbf{y}}\rangle$ and $\hat{K}_{0m}(\mathbf{p})$ is the FT of $\langle \rho_{\mathbf{x}}; J_{m,\mathbf{y}}\rangle$, from the bound

$$|K_{\mu,\nu}(\mathbf{x})| \leq \frac{C}{1+|\mathbf{x}|^4} \ ,$$

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- $K_{l,m}(\mathbf{p})$ is even: if the derivative were continuous the conductivity vanishes. But is not. (CFR $1D$ $\hat{K}_{l,m}$ non continuous $\sigma(0)=\infty$)

The current-current function

- Mutiscale analysis provide the following formula (resummation of the naive perturbative expansion)

$$\hat{K}_{lm}(\mathbf{p}) = \frac{Z_l Z_m}{Z^2} \langle \hat{j}_{\mathbf{p},l}; \hat{j}_{-\mathbf{p},m} \rangle_{0,v_F} + \hat{R}_{lm}(\mathbf{p})$$

where $\langle \cdot \rangle_{0,v_F}$ is the average associated to a non-interacting system with Fermi velocity

$$v_F(U) = \frac{3}{2}t + dU + .. \quad Z_\mu = \frac{3t}{2} + aU + ..$$

and

$$|R_{lm}(\mathbf{x},\mathbf{y})| \le \frac{C}{1 + |\mathbf{x}-\mathbf{y}|^{4+\theta}}$$

with $0 < \theta < 1$ (power counting improvement due to irrelevance), so that $\hat{R}_{lm}(\omega, \vec{0})$ is continuous and differentiable at $\mathbf{p} = \mathbf{0}$ (this is not true with marginal interactions).

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3. Note that $\hat{K}_{lm}(\mathbf{p})$ is even

- Finally

$$\sigma_{11} = -\frac{2}{3\sqrt{3}} \lim_{\omega\to 0^+} \frac{1}{\omega}\Big[\big(\hat{R}_{11}(\omega,\vec{0}) - \hat{R}_{lm}(0,\vec{0})\big)$$
$$+\big(v_F^2 \langle \hat{j}_{(\omega,\vec{0}),l}; \hat{j}_{(-\omega,\vec{0}),m}\rangle_{0,v_F} - v_F^2 \langle \hat{j}_{\mathbf{0},l}; \hat{j}_{\mathbf{0},m}\rangle_{0,v_F}\big)\Big]\ .$$

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- The first term is differentiable and even hence vanishing, while the first term is identical to the free one so it does not depend from v_F ■

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- The first term is differentiable and even hence vanishing, while the first term is identical to the free one so it does not depend from v_F ∎
- In order to establish this result the crucial point are the non-perturbative bounds for the decay of correlations and the cancellations due to WI. The lattice plays a crucial role (in a linear model with a cut-off the WI have corrections).

Interacting Hall insulators: Bulk

- Graphene + nnn hopping + staggered potential (Haldane model).

$$H_0 = \sum_{\vec{x}\in\Lambda_A}\sum_{\sigma=\uparrow\downarrow}\sum_{j=1,2,3}\Big[-t_1\Big(a^+_{\vec{x},\sigma}b^-_{\vec{x}+\vec{\delta}_j,\sigma}+b^+_{\vec{x}+\vec{\delta}_j,\sigma}a^-_{\vec{x},\sigma}\Big)$$
$$-t_2\sum_{\alpha=\pm}\Big(e^{i\alpha\phi}a^+_{\vec{x},\sigma}a^-_{\vec{x}+\alpha\vec{\ell}_j,\sigma}+e^{-i\alpha\phi}b^+_{\vec{x}+\vec{\delta}_1,\sigma}b^-_{\vec{x}+\vec{\delta}_1+\alpha\vec{\ell}_j,\sigma}\Big)$$
$$+\frac{M}{3}\Big(a^+_{\vec{x},\sigma}a^-_{\vec{x},\sigma}-b^+_{\vec{x}+\vec{\delta}_j,\sigma}b^-_{\vec{x}+\vec{\delta}_j,\sigma}\Big)\Big]\,.$$

- Gapped system. Gaps: $\Delta_\pm = |m_\pm|\,,\quad m_\pm = M\pm 3\sqrt{3}t_2\sin\phi$
- IQHE without net external magnetic flux (σ_{ij} defined as for graphene starting from Kubo):

$$\sigma_{12}=\frac{2e^2}{h}\nu\,,\qquad \nu=\frac{1}{2}[\mathrm{sgn}(m_-)-\mathrm{sgn}(m_+)]$$

- What is the effect of many-body interactions? Do new phases appear close to transition lines? $H=H_0+U\sum_{\vec{x}}\rho_{\vec{x},+}\rho_{\vec{x},-}$. Instead of topological arguments we use RG+WI as before

Phase transitions in interacting Hall insulators

- Giuliani, Mastropietro, Porta CMP 2015, JSP 2018

Theorem 3.

For small U there exists two renormalized critical curves intersecting $m^R_\pm = M \pm 3\sqrt{3} t_2 \sin\phi + \delta_\omega = 0$ such that

$$\sigma_{12} = \frac{2e^2}{h}\nu \,, \qquad \nu = \frac{1}{2}[\mathit{sgn}(m^R_-) - \mathit{sgn}(m^R_+)]$$

Moreover on the critical lines $\sigma^{cr}_{ii} = \frac{e^2}{h}\frac{\pi}{4}$ while $\sigma^{cr}_{ii} = \frac{e^2}{h}\frac{\pi}{2}$ at $(\phi, W) = (0,0), (\pi, 0)$.

The interaction moves the critical lines; no new phases appears (at strong coupling an intermediate phase was conjectured)

Renormalized transition curves

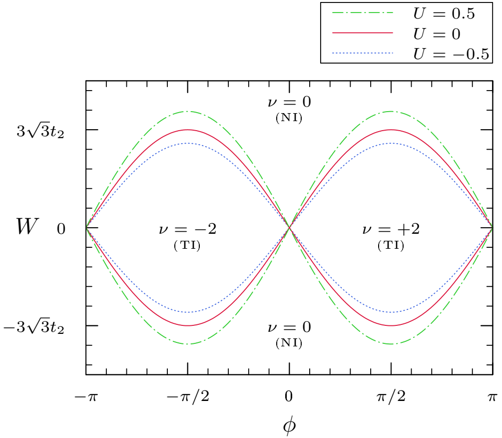

Figure: Red: $U = 0$. Blue: $U > 0$.

- The topological region is enlarged

Phase transitions in the interacting Haldane model

- New intermediate phases are rigorously excluded at weak coupling and the universality class of the topological transition remain unchanged.
- For U small enough

$$\hat{S}_2(k_0, \vec{p}_F^{\,\omega} + \vec{k}') \simeq$$
$$\simeq \begin{pmatrix} -ik_0 Z_{1,R} + m_{\omega,R} & -v_R(-ik_1' + \omega k_2') \\ -v_R(ik_1' + \omega k_2') & -ik_0 Z_{2,R} - m_{\omega,R} \end{pmatrix}^{-1} (1+R)\ ,$$

with $v_R(U) \neq v_R(0)$ and $Z_{1,R} \neq Z_{2,R}$

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  with $v_R(U) \neq v_R(0)$ and $Z_{1,R} \neq Z_{2,R}$
- Despite $v_R(U), Z_{1,R}(U), Z_{2,R}(U)$ are non universal, they are related by lattice Ward Identities, and in the computation of the conductivity they exactly compensate (even with this unusual form of the interacting propagator).

Kane-Mele-Hubbard model

- The (spin conserving) KMH model is obtained considering a fermions with spin $\sigma = \pm$ with Hamiltonian given by the Haldane Hamiltonian for fermion with spin σ H_σ with ϕ and $-\phi$ respectively, coupled by an Hubbard interaction
$$H = H_+ + H_- + \lambda \sum_{\vec{x}} \rho_{\vec{x},+} \rho_{\vec{x},-}$$
where $\rho_{\vec{x},\sigma}$ is the density.

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where $\rho_{\vec{x},\sigma}$ is the density.
- In the non-interacting case, if we choose the chemical potential in the gap and so that $m_{+,\sigma}$ has opposite sign to $m_{-,\sigma}$, $m_{\pm,\sigma} = W \pm 3\sqrt{3}\sin\sigma\phi$, then the charge Hall conductivity $\sigma^c_{12} = \sigma_\uparrow - \sigma_\downarrow = 0$ and the spin Hall conductivity $\sigma^s_{12} = \sigma_\uparrow + \sigma_\downarrow = \pm\frac{1}{\pi}$ $(e = \hbar = 1)$.

Edge states

- Consider cylindric boundary conditions, by taking periodic boundary conditions in the 1 direction and Dirichlet boundary conditions in the 2 direction.

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- The edge conductance remains quantized in presence of interaction? Is the BE correspondence valid? No topological arguments. The theory now is critical, hence even a weak perturbation can produce dramatic effects. In contrast to graphene, the interaction is marginal and not irrelevant.

Edge states

- We define, for $h = \rho^c, \rho^s, j_1^c, j_1^s$:

$$G^a_{h_1,h_2}(\underline{p}) = \sum_{x_2=0}^{a} [\sum_{y_2=0}^{\infty} \langle \mathbf{T}\, \hat{h}_{1,\underline{p},x_2}\,; \hat{h}_{2,-\underline{p},y_2} \rangle_\infty - i\Delta_{h_1,h_2}(x_2)]$$

$\Delta_{h_1,h_2}(x_2) = \langle [X_1, j^c_{1,\vec{x}}] \rangle_\infty$ if $h_1 = h_2 = j^c, j^s$ and zero otherwise. The edge spin conductance is

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$$\sigma^s = \lim_{a\to\infty} \lim_{p_0\to 0^+} \lim_{p_1\to 0} G^a_{\rho^c, j^s_1}(\underline{p})$$

- It measures the variation of the spin current after introducing a shift of the chemical potential supported in a region of width a from the $x_2 = 0$ edge. In the non interacting case this quantity is quantized due to topological reasons.

Edge transport

- Similarly, $\sigma^c = \lim_{a\to\infty} \lim_{p_0\to 0^+} \lim_{p_1\to 0} G^a_{\rho^c, j_1^c}(\underline{p})$ is the edge charge conductance,

Edge transport

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- The charge or spin susceptibility and Drude weight are, for $\sharp = c, s$:

$$\kappa^\sharp = \lim_{a\to\infty} \lim_{p_1\to 0} \lim_{p_0\to 0^+} G^a_{\rho^\sharp, \rho^\sharp}(\underline{p})$$

and

$$D^\sharp = -\lim_{a\to\infty} \lim_{p_0\to 0^+} \lim_{p_1\to 0} G^a_{j_1^\sharp, j_1^\sharp}(\underline{p})$$

- We decompose the Euclidean propagator g as $g^{(\text{edge})} + g^{(\text{bulk})}$

$$g^{(\text{edge})}(\mathbf{x}, \mathbf{y}) = \frac{\delta_{\sigma\sigma'}}{\beta L} \sum_{\underline{k}} \frac{e^{-i\underline{k}\cdot(\underline{x}-\underline{y})}\xi^{\sigma}(k_1; x_2)\overline{\xi^{\sigma}(k_1; y_2)}\chi_{\sigma}(\underline{k})}{-ik_0 + \varepsilon_{\sigma}(k_1) - \mu}$$

where $\chi_{\pm}(\underline{k})$ is a smooth, compactly supported cutoff function supported around $\underline{k}_F^{\pm} = (0, \pm k_F)$.

RG ANALYSIS

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- $g^{(\text{bulk})}$ decays exponentially. Instead, the edge propagator $g^{(\text{edge})}$ decays exponentially in the 2-direction, and as $|\underline{x}-\underline{y}|^{-1}$ in the other variables.
- Notice that $\varepsilon_{\sigma}(k_1' + k_F^{\sigma}) - \mu = \sigma v_+ k_1' + O({k_1'}^2)$. Neglecting $O({k_1'}^2)$ effective description in terms of chiral fermions, with chirality locked to the spin.

RG

- We decompose the Gaussian integration as:
$P(d\Psi) = P(d\Psi^{(\mathsf{edge})})P(d\Psi^{(\mathsf{bulk})})$. The bulk field can be integrated out, and one is left with a new Grassmann integral with Gaussian integration $\int P(d\Psi^{(\mathsf{edge})})$.

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- The next crucial step is to realize that $\hat{\Psi}^{(\text{edge})}_{(\underline{k},x_2),\sigma,\rho}$ can be written as $\hat{\psi}_{\underline{k},\sigma}\xi^{\sigma}_{x_2}(k_1;\rho)$, with $\hat{\psi}^{\pm}_{\underline{k},\sigma}$ a one-dimensional Gaussian Grassmann field with propagator $\hat{g}_\sigma(\underline{k}) = \frac{\chi_\sigma(\underline{k})}{-ik_0+\varepsilon_\sigma(k_1)-\mu_\sigma}$.
- The effective interaction has the form $V^{(0)}(\psi) = \int d\underline{x}\lambda_0\psi^+_{\underline{x},\uparrow}\psi^-_{\underline{x},\uparrow}\psi^+_{\underline{x},\downarrow}\psi^-_{\underline{x},\downarrow}$ plus irr. terms.
- The effective source $B^{(0)}(\psi;A) = \int d\mathbf{x}\, Z^\sharp_\mu(x_2)A^\sharp_{\mu,\mathbf{x}}\rho^\sharp_{\mu,\underline{x}}$, for some $Z^\sharp_{0,\mu}(x_2)$, $\rho^c_{0,\underline{x}} = \sum_\sigma \psi^+_{\underline{x},\sigma}\psi^-_{\underline{x},\sigma}$, $\qquad \rho^c_{1,\underline{x}} = \sum_\sigma \sigma\psi^+_{\underline{x},\sigma}\psi^-_{\underline{x},\sigma}$, $\rho^s_{0,\underline{x}} = \rho^c_{1,\underline{x}}$, $\qquad \rho^s_{1,\underline{x}} = \rho^c_{0,\underline{x}}$; $\mu = 0,1$ respectively for density and current.

Theorem 4.

(Mastropietro,Porta PRB 2017) For $|\lambda| < \lambda_0$

$$\sigma^s = \frac{1}{\pi}\ .$$

$$\kappa^c = \frac{K}{\pi v}\ , \quad D^c = \frac{vK}{\pi}\ , \quad \kappa^s = \frac{1}{\pi v K}\ , \quad D^s = \frac{v}{\pi K}$$

with $K = 1 + O(\lambda) \neq 1$, $v = v_+ + O(\lambda) \neq v_+$. *Finally, the* 2*-point function decays with anomalous exponent* $\eta = (K + K^{-1} - 2)/2$.

- There is universality in the edge conductance and Luttinger liquid relations hold

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- We compare with a 1d theory; the cut-off is not only on space momenta so cannot be solved by bosonization but can be solved by WI. The choice of cut-off is important as we need to take into account irrelevant terms
- For $N \gg 1$: $e^{\mathcal{W}^{(\mathsf{H})}(A)} =$

$$\int P_N(d\psi) e^{-\lambda^{(\mathsf{H})} Z^{(\mathsf{H})2} \int d\underline{x} d\underline{y}\, v(\underline{x}-\underline{y}) \rho_{\vec{x},+}\rho_{\vec{y},-} + B(\psi;A)}$$

where $P_N(d\psi)$ is a Grassmann Gaussian integration with propagator:

$$g^{(\mathsf{H})}_\sigma(\underline{x},\underline{y}) = \frac{1}{Z^{(\mathsf{H})}} \int \frac{d\underline{k}}{(2\pi)^2}\, e^{-i\underline{k}\cdot(\underline{x}-\underline{y})} \frac{\chi_N(\underline{k})}{-ik_0 + \sigma v^{(\mathsf{H})} k_1}$$

with $\chi_N(\underline{k})$ a smooth ultraviolet cutoff function with support $|\underline{k}| \leq 2^{N+1}$, and where $B(\psi;A) = \sum_{x_2=0}^{\infty} \int d\underline{x}\, Z^{\sharp,(\mathsf{H})}(x_2) A^\sharp_{\mu,\mathbf{x}} \rho^\sharp_{\mu,\underline{x}}$. The potential $v(\underline{x}-\underline{y})$ is short ranged.

Emerging QFT description

- The correlations of the KMH model can be written in terms of the correlations of that model HL model, up to multiplicative and additive renormalizations (depending on the microscopic details of the KMH model, in particular the lattice irr terms): $\langle \mathbf{T}\, j^{\sharp}_{\mu,\underline{p},x_2} j^{\sharp'}_{\nu,-\underline{p},y_2}\rangle_\infty =$

$$Z^{\sharp,(\mathsf{H})}_{\mu}(x_2) Z^{\sharp,(\mathsf{H})}_{\nu}(y_2) \langle \rho^{\sharp}_{\mu,\underline{p}} \rho^{\sharp'}_{\nu,-\underline{p}}\rangle^{(\mathsf{H})} + \hat{H}^{\sharp,\sharp'}_{\mu,\nu}(\underline{p}, x_2, y_2)$$

  where $\langle \cdot \rangle^{(\mathsf{H})}$ denotes the average of the HL model, and $\hat{H}^{\sharp,\sharp'}_{\mu,\nu}(\underline{p}; x_2, y_2)$ is continuous in $\underline{p}$ while the first term is not continuous (the continuity comes from the fact that irrelevant terms contribute). $|Z^{\sharp,(\mathsf{H})}_{\mu}(x_2)| \le C e^{-c x_2}$.

Chiral anomaly

- Chiral WI $D_\sigma(\underline{p})\langle\hat{\rho}_{\underline{p},\sigma}\,;\hat{\rho}_{-\underline{p},\sigma}\rangle^{(\mathsf{H})} =$

$$-\frac{1}{4\pi|v^{(\mathsf{H})}|{Z^{(\mathsf{H})}}^2}D_{-\sigma}(\underline{p}) + \tau D_{-\sigma}(\underline{p})\hat{v}(\underline{p})\langle\hat{\rho}_{\underline{p},-\sigma}\,;\hat{\rho}_{-\underline{p},\sigma}\rangle^{(\mathsf{H})}$$

where $\mathrm{D}_\sigma(\underline{p}) = -ip_0 + \sigma vp$ and $\tau = \frac{\lambda^{(\mathsf{H})}}{4\pi|v^{(\mathsf{H})}|}$ is the chiral anomaly. Note that the anomaly is linear in $\lambda^{(\mathsf{H})}$ Adler-Bardeen non-renormalization. (cfr massless QED_{3+1})

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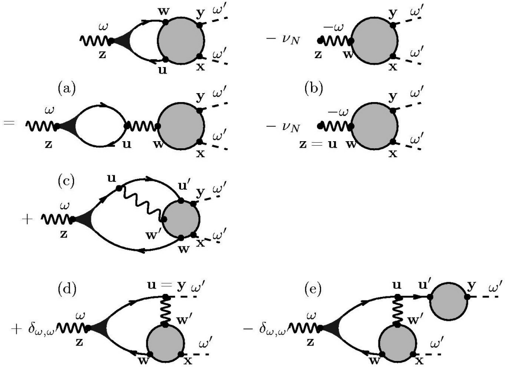
- This allows to get an exact expression

$$\langle\hat\rho_{\underline{p},\sigma}\hat\rho_{-\underline{p},\sigma}\rangle^{(\mathsf{H})} = \frac{-1}{4\pi|v^{(\mathsf{H})}|Z^{(\mathsf{H})2}}\frac{1}{1-\tau^2}\frac{D_{-\sigma}(\underline{p})}{D_\sigma(\underline{p})}$$

$$\langle\hat\rho_{\underline{p},-\sigma}\hat\rho_{-\underline{p},\sigma}\rangle^{(\mathsf{H})} = \frac{-1}{4\pi|v^{(\mathsf{H})}|Z^{(\mathsf{H})2}}\frac{\tau}{1-\tau^2}$$

WI



At finite N one gets an extra term in the WI (the dot is $\chi_N(\chi_N - 1)$; the contribution to the vertex function can be decomposed (without breaking the determinants)

WI

- The additive renormalization are fixed by lattice continuity equation $\sum_{z_2} \bar{Z}^{\alpha}_{z_2,\mu} = \bar{Z}^{\alpha}_{\mu}$,

$$G^{\infty}_{\rho^c, j_1^c}(\underline{p}) = \sum_{\sigma} \frac{Z_0^c Z_1^c}{Z^2(1-\tau^2)} \frac{\sigma}{2\pi v} \frac{-ip_0}{-ip_0 + \sigma v p_1}$$

$$G^{\infty}_{\rho^c, j_1^s}(\underline{p}) = -\frac{Z_0^c Z_1^s}{Z^2(1-\tau^2)} \frac{1}{\pi |v|} \frac{p_0^2}{p_0^2 + v^2 p_1^2}$$

$$G^{\infty}_{j_1^\sharp, j_1^\sharp}(\underline{p}) = -\frac{Z_1^\sharp Z_1^\sharp}{Z^2(1-\tau^2)} \frac{1}{\pi |v|} \frac{p_0^2}{p_0^2 + v^2 p_1^2}$$

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- All Z depend on λ and are not universal; how universality arise?

WI

- By comparing the WI for the reference and the lattice we get, $Z_0^\sharp = \sum_{x_2} Z_0^{\sharp(H)}(x_2)$

$$\frac{vZ_0^\sharp}{Z_1^\sharp} = 1 \, , \qquad \frac{Z_0^\sharp}{Z(1-\varepsilon^\sharp \tau)} = 1 \, .$$

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which provide the exact link between the emergent chiral anomaly and the finite renormalizations due to the lattice corrections.

- Setting $K^c = K$, $K^s = K^{-1}$:

$$\frac{Z_0^\sharp Z_1^\sharp}{Z^2(1-\tau^2)v} = K^\sharp \,, \quad \frac{Z_0^c Z_1^s}{Z^2(1-\tau^2)v} = 1 \,,$$
$$\frac{Z_1^\sharp Z_1^\sharp}{Z^2(1-\tau^2)v} = K^\sharp v \,, \quad \frac{Z_0^\sharp Z_0^\sharp}{Z^2(1-\tau^2)v} = \frac{K^\sharp}{v} \,.$$

- The second relation implies the quantization of σ^s. The last two imply the nonuniversality of $D^\sharp$, $\kappa^\sharp$, and the HL relation $D^\sharp = v^2\kappa^\sharp$; that is the LL relations.

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- Luttinger liquid relations uniformly up to the quantum critical point

$$D = Kv/\pi \quad \kappa = K/\pi v$$

where (r is the distance from criticality)

$$K = \frac{1-\tau}{1+\tau}, \tau = \lambda \frac{v(0) - v(2p_F)}{2\pi v} + O(\lambda^2 r) \quad v = \sin p_F (1 + O(\lambda r))$$

$\mu = \mu_R + \nu$, $\nu = \lambda v(o) p_F/\pi + O(\lambda)$, $\mu_R = -\cos p_F = \pm 1 \mp r$. τ is the anomaly of the emerging theory.

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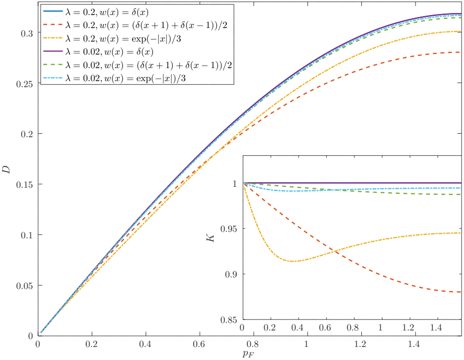
- Moreover at criticality one gets the non interacting values

$$K \to 1 \quad D/D_0 \to 1$$

as $r \to 0$. μ_c is shiifted by the interaction (see e.g. Zotos et al (2016)).

Spinless case



D and K as function of density (or magnetic field), both in Heisenberg or non solvable cases. D/D_0 and K tend to 1: Features found in the solvable case persists up to the criticapl point.

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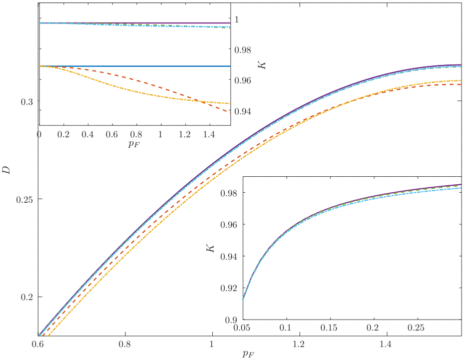
  ν_ρ, ν_4 are the anomalies of the emerging theory.(non perturbative)
- One cannot take the $r \to 0$ limit; however for λ small one can see that K does not tend to the non interacting value 1 but D becomes close to D_0.



Contrary to the spinless case, we cannot get $p_F = 0$. K show the tendency to a strongly interacting fixed point while D is close to the non interacting value. Cfr the behavior of the Hubbard model by Bethe ansatz

Conclusion

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- Important issues regards the effect of long range interaction for which the control of functional integral is more challenging
- RG takes into full account irrelevant terms at $T = 0$; challenging problem $T > 0$