



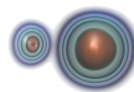
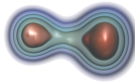
Probing the evolution of the fission modes with a microscopic approach

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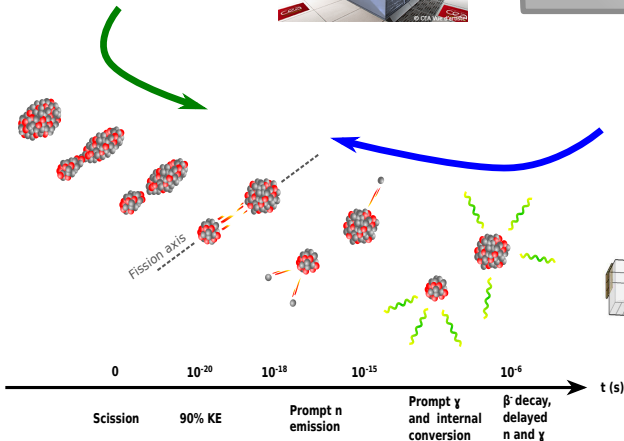


Probing the fission dynamics

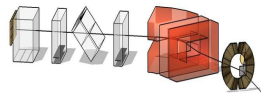
Many-body calculations
based on energy density
functional



Fission time scale ?
Scission neutron ?
Fragments characteristics ?
Fragments deexcitation ?

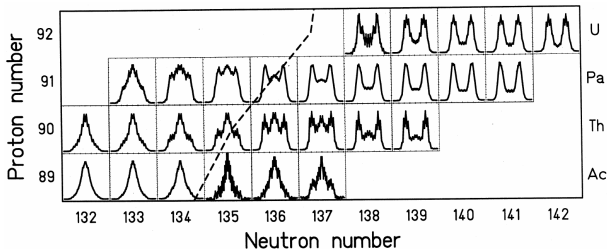
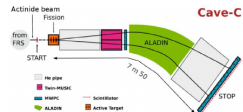


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SOFIA@GSI
SPIDER@LANL
Fission@ALTO
...



Symmetric/asymmetric yields transitions

Systematic studies of fission yields with **inverse kinematics measurements** sheds light on numerous yields transitions.



Fragment charge distribution measured by inverse kinematics at GSI, [K.H Schmidt et al., Nucl. Phys. A 665, 221-267 \(2000\)](#)

- How well can EDF based approaches reproduce these yields transitions ?
- What fission modes do these approaches predict in the **SHE region** ?

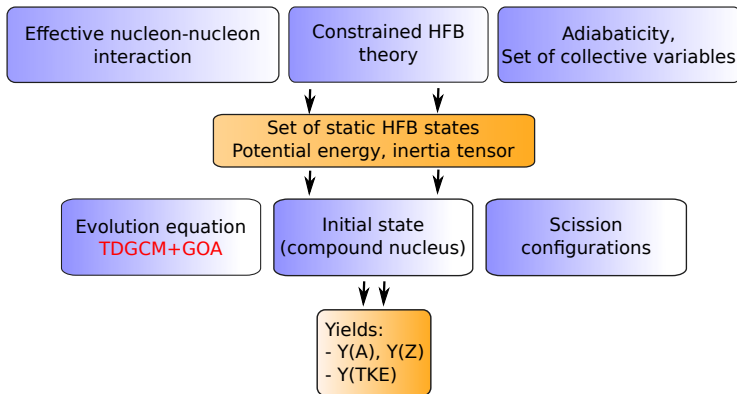
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- 2 Yields transitions in the Fm and Th isotopic chains
- 3 Going beyond 2-dimensional TDGCM+GOA
- 4 Outlook & Perspectives

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Time Dependent Generator Coordinate Method (Multi-reference DFT)



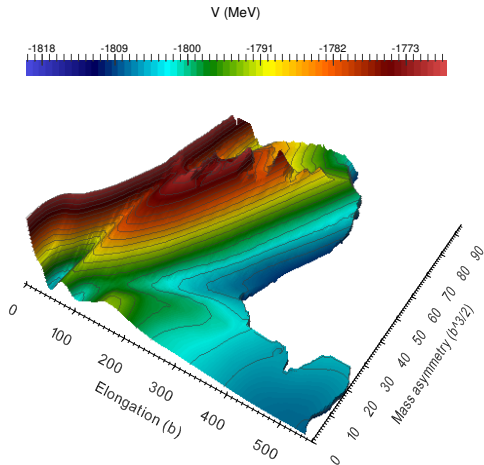
A **fully quantum-mechanical** description of the time evolution

Gives the amplitude of probability for the nucleus to have a given shape at time t .

Time Dependent Generator Coordinate Method (TD-GCM)

Example of a $n + {}^{239}\text{Pu}$ fission

- 1 Choose the collective variables:
 - elongation (Q_{20} in b),
 - mass asymmetry (Q_{30} in $b^{3/2}$)
- 2 Calculate potential energy surface and inertia tensor
- 3 Define initial wave packet for the probability amplitude
- 4 Compute time evolution of probability amplitude
- 5 Extract fission fragment distribution by computing the flux of the probability amplitude across the scission line

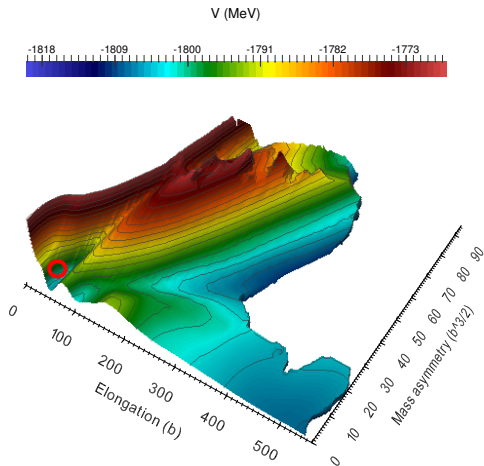


Interpolated potential energy surface for $(n+{}^{239}\text{Pu})$ fission

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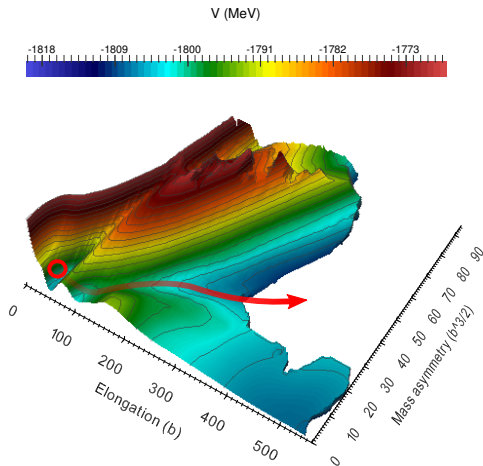


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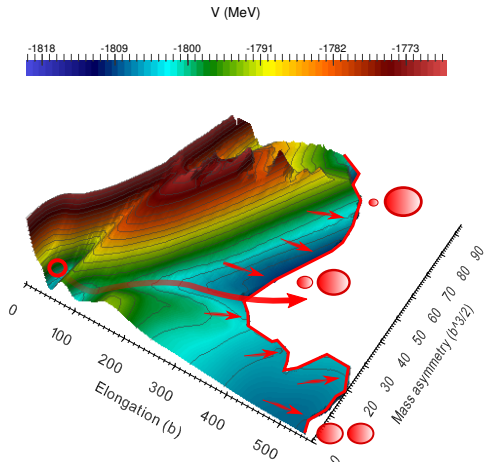


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Interpolated potential energy surface for $(n+{}^{239}\text{Pu})$ fission

Development of this microscopic approach

2005: First calculation for ^{238}U

H. Goutte *et al.*, *Phys. Rev. C* **71**, 024316

2012: Fission yields of ^{236}U and ^{240}Pu

W. Younes *et al.*, LLNL-TR-586678

- Promising results
- High numerical costs

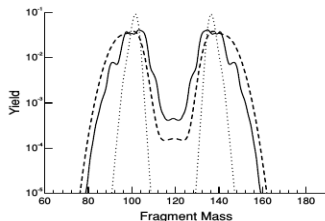
2D PES	40000 HFB states
Dynamics	10 zs (10^{-21}s)

Upgrade numerical methods

FELIX-1.0 D. Regnier *et al.*, *Comput. Phys. Commun.* **122**, 350-363 (2016)

FELIX-2.0 D. Regnier *et al.*, *Comput. Phys. Commun.* **225**, 180-191 (2018)

Machine learning for PES N. Martin *et al.*, work in progress



Pre-neutron mass yields for ^{238}U at 2.4 MeV above the fission barrier (H. Goutte *et al.*).
solid line: dynamics calculation
dashed line: What evaluation (2002)

Recent applications

Fission of ^{240}Pu , ^{252}Cf , ^{226}Th

D. Regnier *et al.*, *Phys. Rev. C* **93**, 054611 (2016)

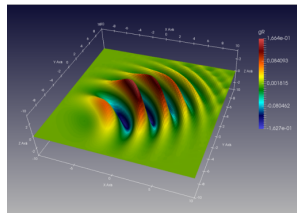
A. Zdeb *et al.*, *Phys. Rev. C* **95**, 054608 (2017)

H. Tao *et al.*, *Phys. Rev. C* **96**, 024319 (2017)

FELIX: a generic solver for the TDGCM+GOA

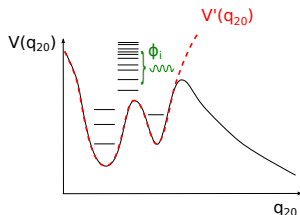
Features:

- Capable of handling $N \geq 2$ collective variables
- Arbitrary collective degree of freedom
- Scalable numerical methods
- Solving the static GCM+GOA



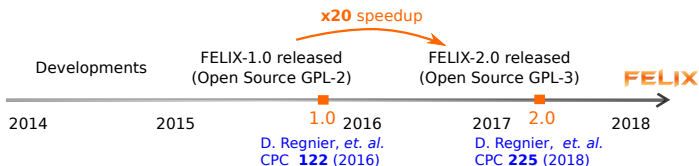
Numerical methods for 2.0:

- Spectral finite element (space)
- Krylov propagator (time)

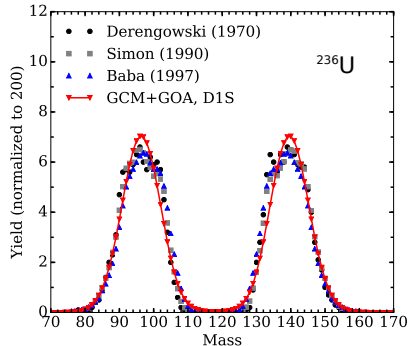
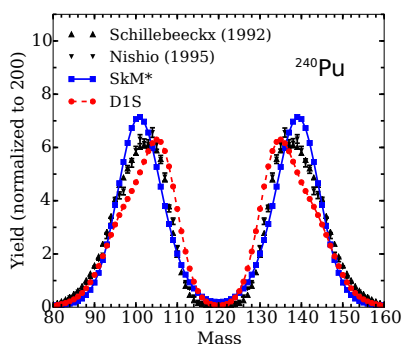


Validation

- Unitary tests
- Analytical benchmarks



Primary fragments mass yields for low energy fission of actinides

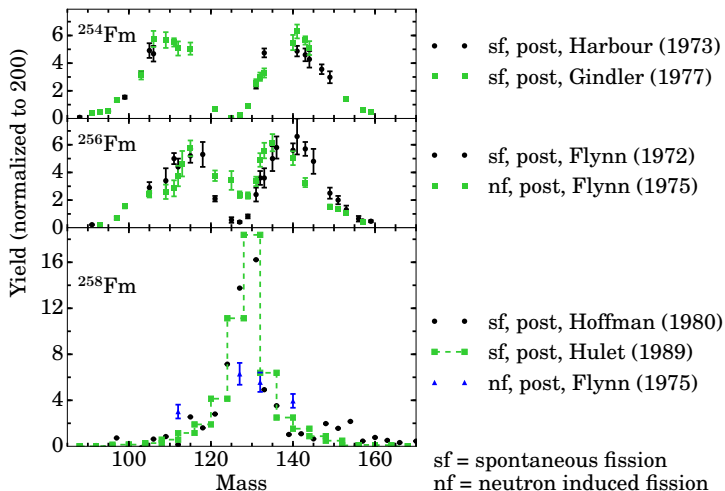


- The initial energy is taken 1 MeV above the fission barrier.
- The raw flux results are convoluted with a Gaussian of width $\sigma = 4$.
- The qualitative reproduction of the asymmetric fission of actinides is robust.
- A better modeling of several physics effects (initial state, fragment separation) is **necessary to reach a $\simeq 10\%$ accuracy.**

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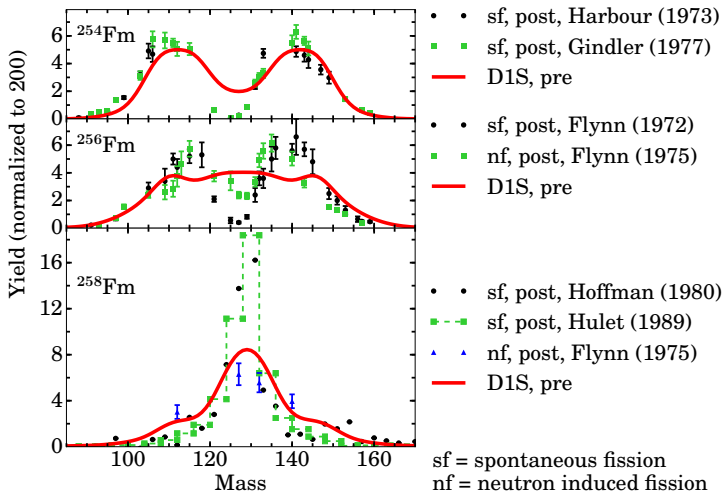
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Fission yields in neutron rich Fermium isotopes



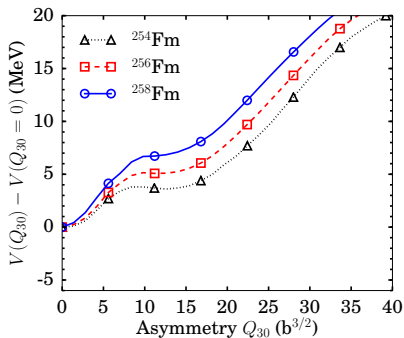
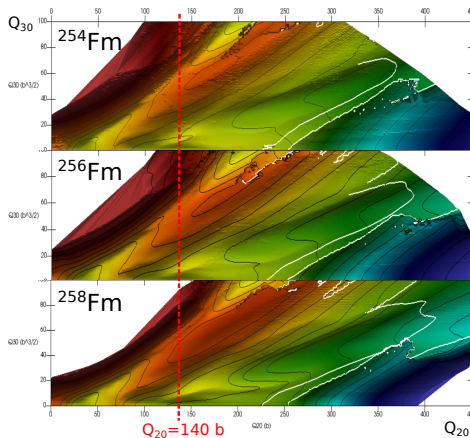
	^{253}Fm	^{254}Fm	^{255}Fm	^{256}Fm	^{257}Fm	^{258}Fm
T1/2	3.00 d	3.240 h	20.07 h	157.6 min	100.5 d	370 μs
SF (%)		0.0592	$2.4 \cdot 10^{-5}$	91.9	0.210	
$\sigma(n, f)$ (b)			3200		2950	

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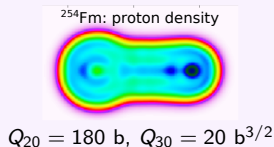


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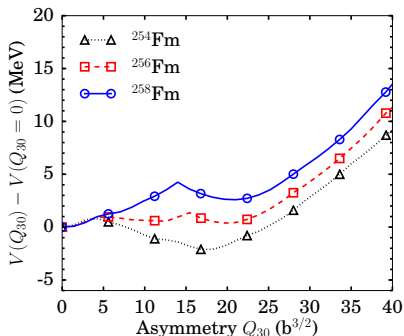
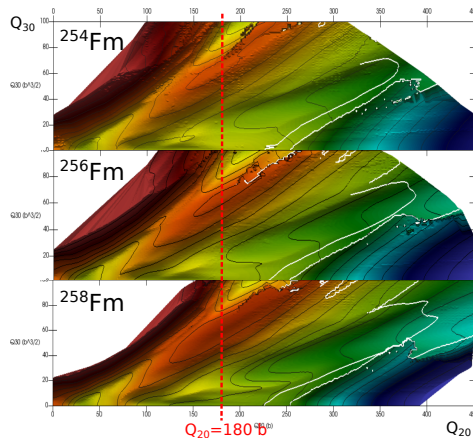
Competition between collective potential valleys



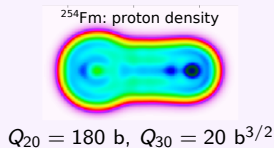
- Main modes of the fission yields driven by the static potential energy
- Dominant mode decided at rather low elongation $Q_{20} \simeq 180$ b



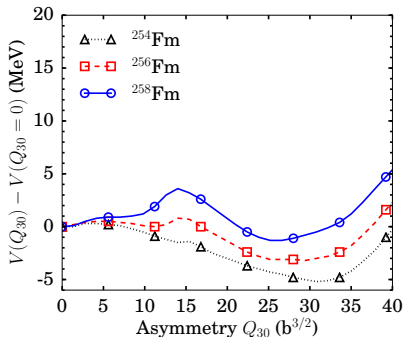
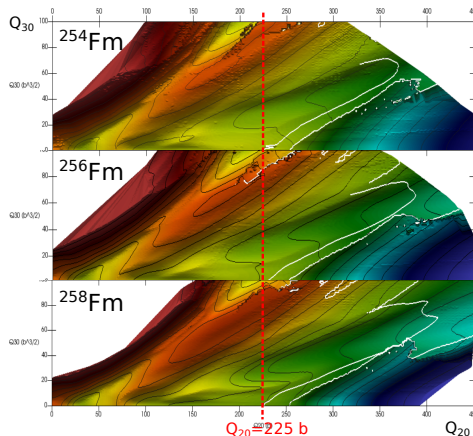
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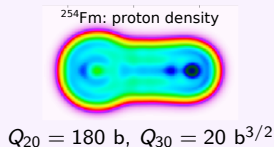
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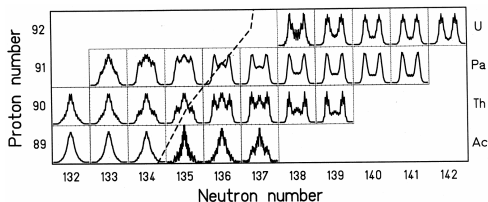


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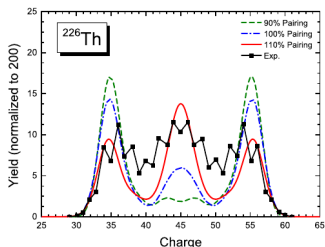
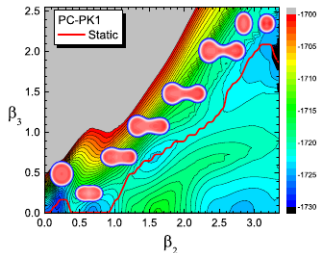


Fission yields in proton rich Thorium isotopes

- Fragment charge distribution, K.H Schmidt *et al.*, Nucl. Phys. A 665, 221-267 (2000)

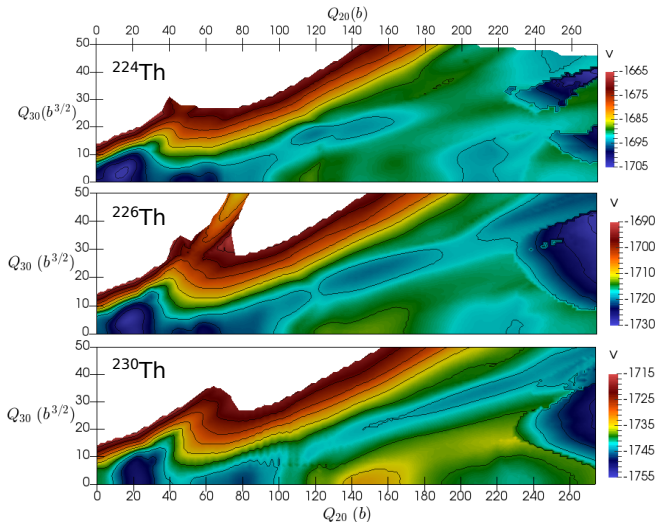


- First TDGCM+GOA calculation performed with relativistic density functional PC-PK1 H. Tao *et al.*, Phys. Rev. C 96, 024319 (2017)



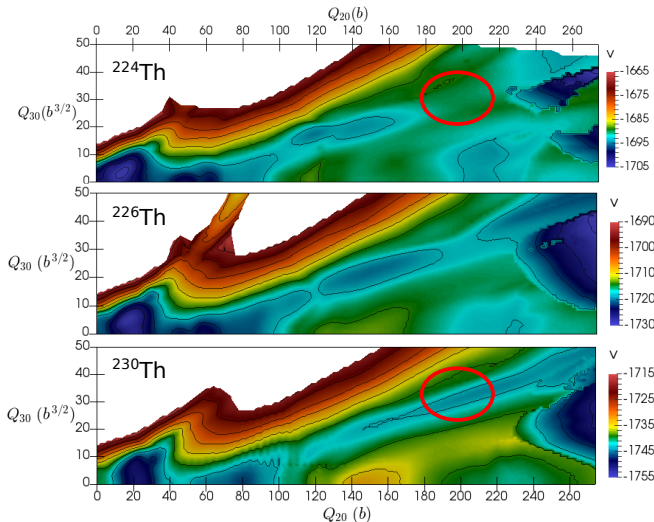
- Difficulty to get the correct ratio between modes
- Important sensitivity to the pairing strength

Preliminary Thorium PES obtained with D1S



- Main modes of the fission yields driven by the static potential energy
- Clear change of topology around $Q_{20} \simeq 200$

Preliminary Thorium PES obtained with D1S



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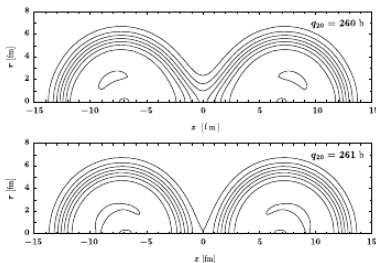
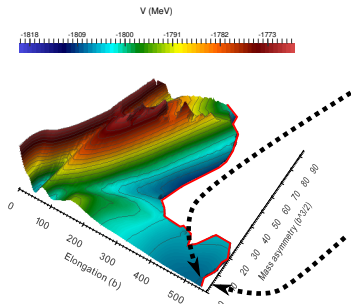
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Discontinuities in the set of generator states

Each point of the PES:

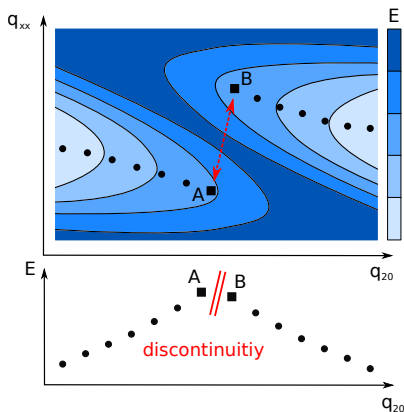
- is a solution of the constrained Hartree Fock Bogoliubov (HFB) equation,
- is determined by **minimization of the energy** among a set of deformed states



Problem

- 1 Two solutions close in the collective subspace are not close in the full deformation space.

Origin of the problem



Exemple of a discontinuity in a **1D Potential Energy Surface (PES)** for the collective variable q_{20} :

- Point A and B may represent very different systems
- Point A and B energy are close to each other
- Due to the energy minimization involved, they are neighbors in the 1D potential energy surface

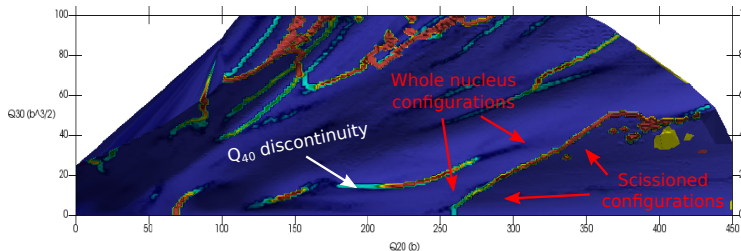
Consequence

- The 1D PES is missing a part of the physics: underestimated barriers.
- In principle, the system should not cross a discontinuity during the dynamics

Pinpoint the discontinuities: example of ^{256}Fm

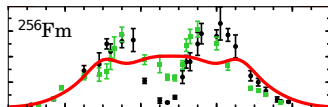
Simple criteria to detect discontinuities:

$$d(\rho, \rho') = \int_{\mathbf{r}} d\mathbf{r} |\rho(\mathbf{r}) - \rho'(\mathbf{r})|$$



Maximum density distance between closest neighbors

- A higher potential ridge should separate the symmetric and asymmetric valleys (Q_{40} discontinuity)
- States around scission are missing in the GCM description



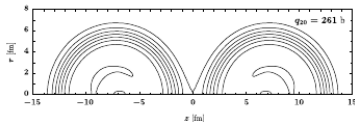
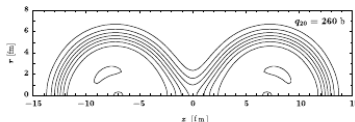
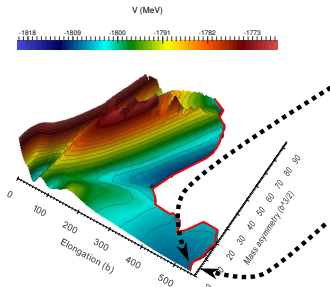
Discontinuity around scission

Consequences:

- The dynamics has to be **stopped before scission**
 \implies Prevent predictions of a number of observables (e.g. TKE)

- Presence of a **neck in the final states**
 \implies The number of particles in the fragments are defined up to the number of particles in the neck (Q_{neck}).

Actinides	$Q_{neck} \simeq 4$
Proton rich Th	$Q_{neck} \simeq 12$

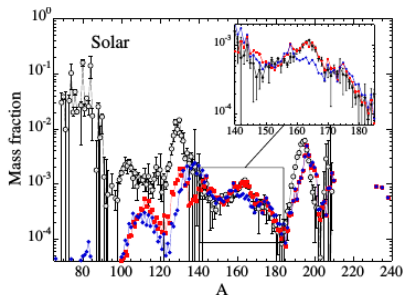


Discontinuities in the neutron rich ^{278}Cf

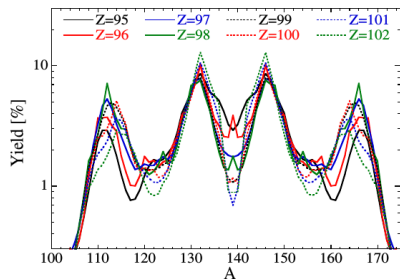
Motivation:

Prediction of a **very asymmetric fission modes** with a static approach (SPY) that impacts the element abundances in the rare earth region.

S. Goriely et. al., PRL 111 (2013)

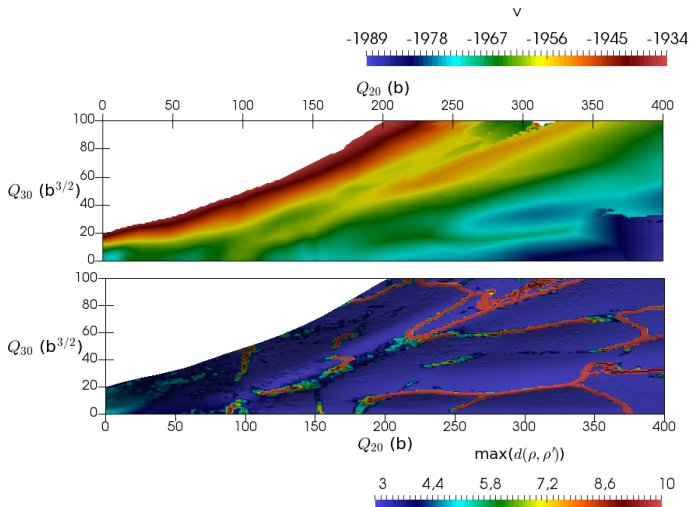


Impact of the fission yields model on r-process abundances. Blue= GEF, Red= SPY.



Mass yields of exotic systems with mass $A_{CN} = 278$ obtained with SPY

Discontinuities in the neutron rich ^{278}Cf



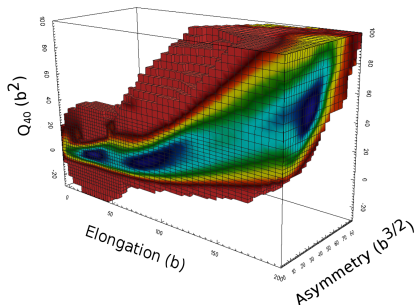
Patchwork of PES that are **disconnected**

\implies The 2D TDGCM+GOA approach cannot predict the yields

Adding collective variables to remove some discontinuities

Including the additional variable Q_{40} (partly related to the neck size)

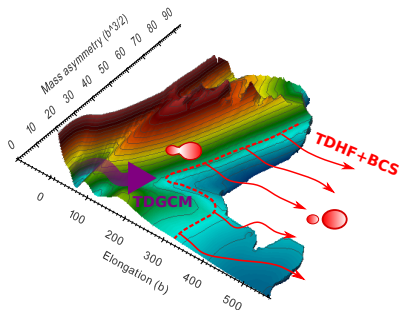
- would involve roughly $2 \cdot 10^6$ constrained HFB calculations,
- imply a numerical cost $\simeq 1 \cdot 10^6$ cpu.h for a full 3D PES.



One third of the full potential energy surface for a $n+^{239}\text{Pu}$ fission in the collective space (\hat{Q}_{20} , \hat{Q}_{30} , \hat{Q}_{40})

- Current bottleneck: **generation of the potential energy surface.**
- Work in progress: machine learning for HFB optimization and PES generation

Using TDHFB to describe the diabatic dynamics through scission

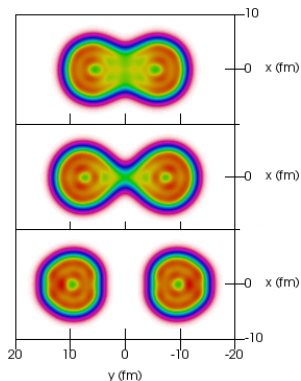


Hybrid approach to fission dynamics

Requires:

- TDBCS starting from octupolar deformed densities
- Proper connection between the two theories

Density of protons for a symmetric fission of ^{258}Fm



Impact: a variety of new characteristics of the fission fragments

- Improved resolution of for $Y(A)$, $Y(Z)$
- Distribution of kinetic and excitation energies
- Emission of neutrons at scission

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Outlook & Perspectives

Improved numerical methods and increased computational power provides **new opportunities** to bridge theory with the state of the art experiments.

Fission dynamics with TDGCM+GOA

- Development of a new generation of numerical tools
- Reproduction of the dominant mode of fission yields in neutron rich Fermium isotopes
- 2-dimensional description seems **insufficient** to describe transition nuclei and **explore the SHE region**

Challenge: **merging** the benefits of TDGCM+GOA with TDHFB

- Include diabatic dynamics through scission
- Prediction of new observables e.g. $Y(A, TKE)$
- Impact far beyond the fission process (heavy ion reactions, etc)

Thank you for your attention !

