

**Super-heavy nuclei:
Production and decay properties**

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Collaborators on this and other projects,
see coauthors on papers posted on URL below.

More details about masses, other projects (beta-decay, fission),
associated ASCII data files and figures are at

<http://t2.lanl.gov/nis/molleretal/>

Nuclear **POTENTIAL ENERGY** BW (1939)

$$B(N, Z) =$$

$$+a_v A \quad (\text{Volume energy})$$

$$-a_s A^{2/3} B_s(\beta) \quad (\text{Surface energy})$$

$$-a_c \frac{Z^2}{A^{1/3}} B_C(\beta) \quad (\text{Coulomb energy})$$

$$-a_I \frac{(N - Z)^2}{A} \quad (\text{Symmetry energy})$$

$$-\delta(A) \quad (\text{Pairing energy})$$

Nuclear Deformation Energy

Let the nuclear surface be described by

$$r(\theta, \phi) = R_0 [1 + \alpha_2 P_2(\cos \theta)]$$

The surface energy lowest order Taylor expansion:

$$E_s = E_s^0 \left(1 + \frac{2}{5} \alpha_2^2\right)$$

The Coulomb energy lowest order Taylor expansion

$$E_C = E_C^0 \left(1 - \frac{1}{5} \alpha_2^2\right)$$

The energy at deformation α_2 relative to spherical shape

$$E_{\text{def}}(\alpha_2) = E_C(\alpha_2) + E_s(\alpha_2) - (E_C^0 + E_s^0)$$

If E_{def} is negative then the system has no barrier wrt fission

$$E_{\text{def}}(\alpha_2) = \frac{2}{5} \alpha_2^2 E_s^0 - \frac{1}{5} \alpha_2^2 E_C^0 < 0$$

$$1 < \frac{E_C^0}{2E_s^0} = x$$

The surface energy for a sphere

$$E_s^0 = 17.80 A^{2/3}$$

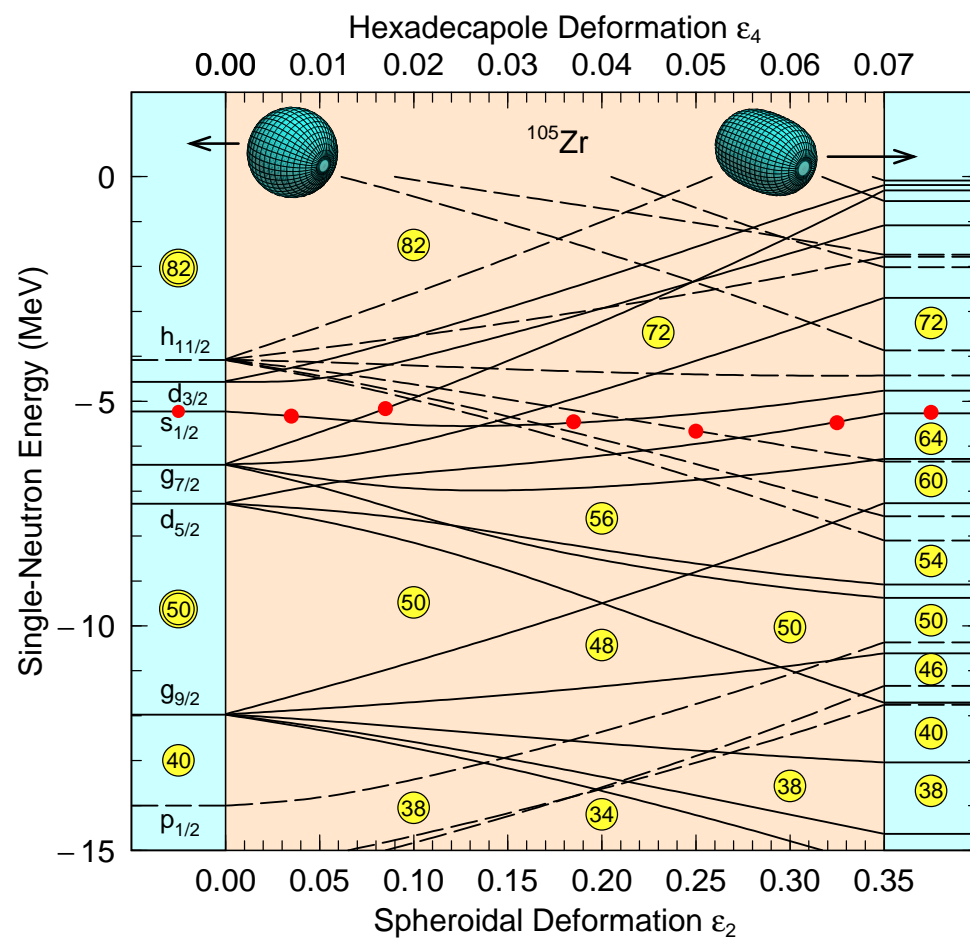
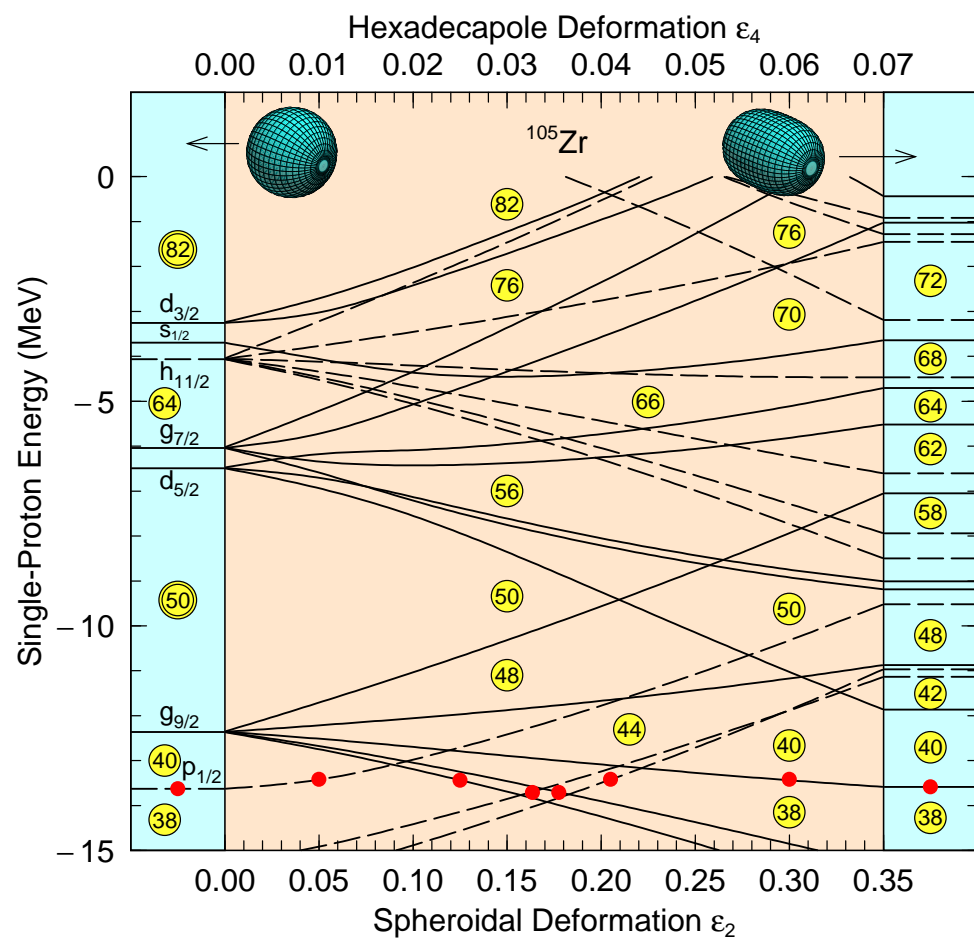
The Coulomb energy for a sphere

$$E_C^0 = 0.7103 \frac{Z^2}{A^{1/3}}$$

The fissility parameter x :

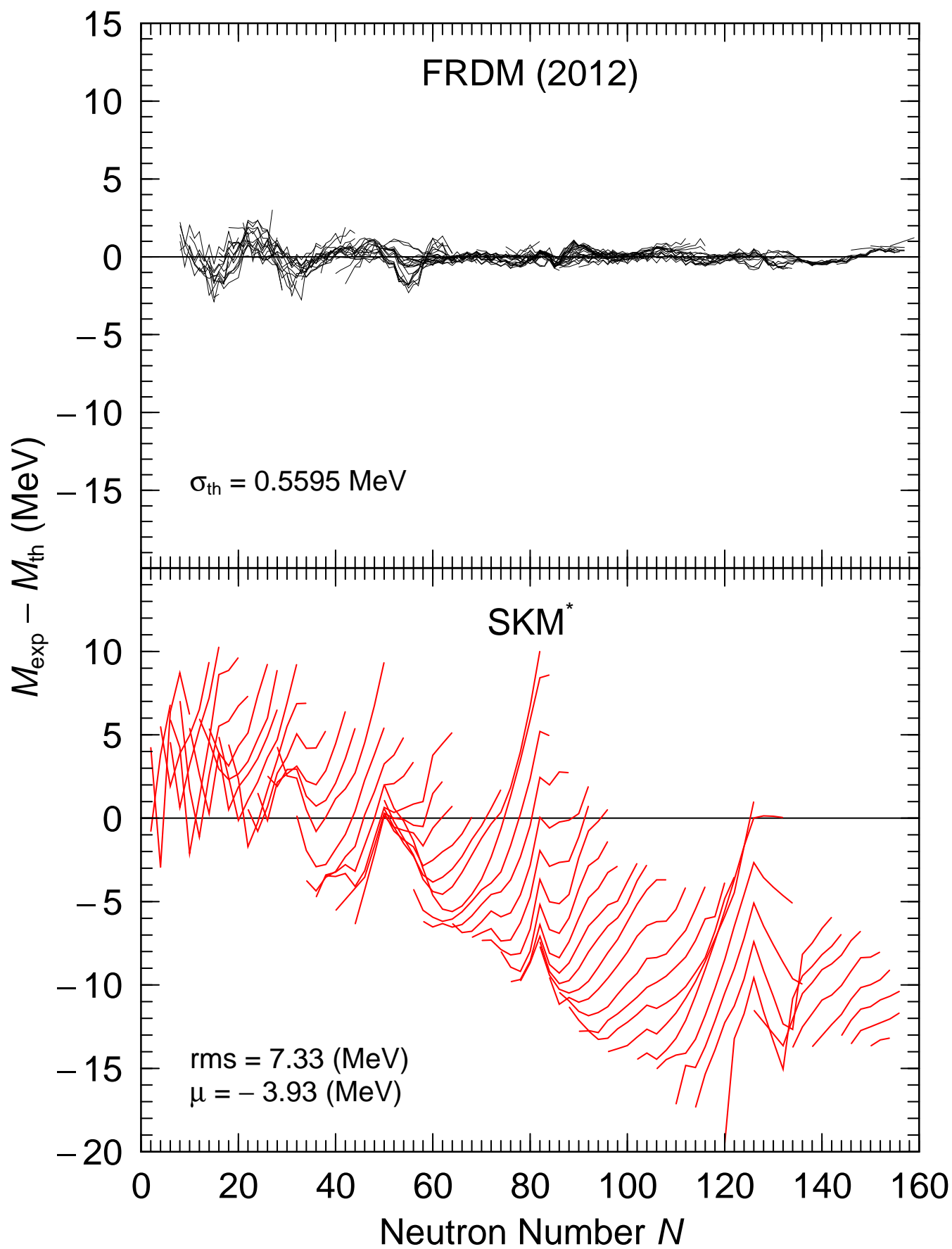
$$x = \frac{Z^2}{50.13 A}$$

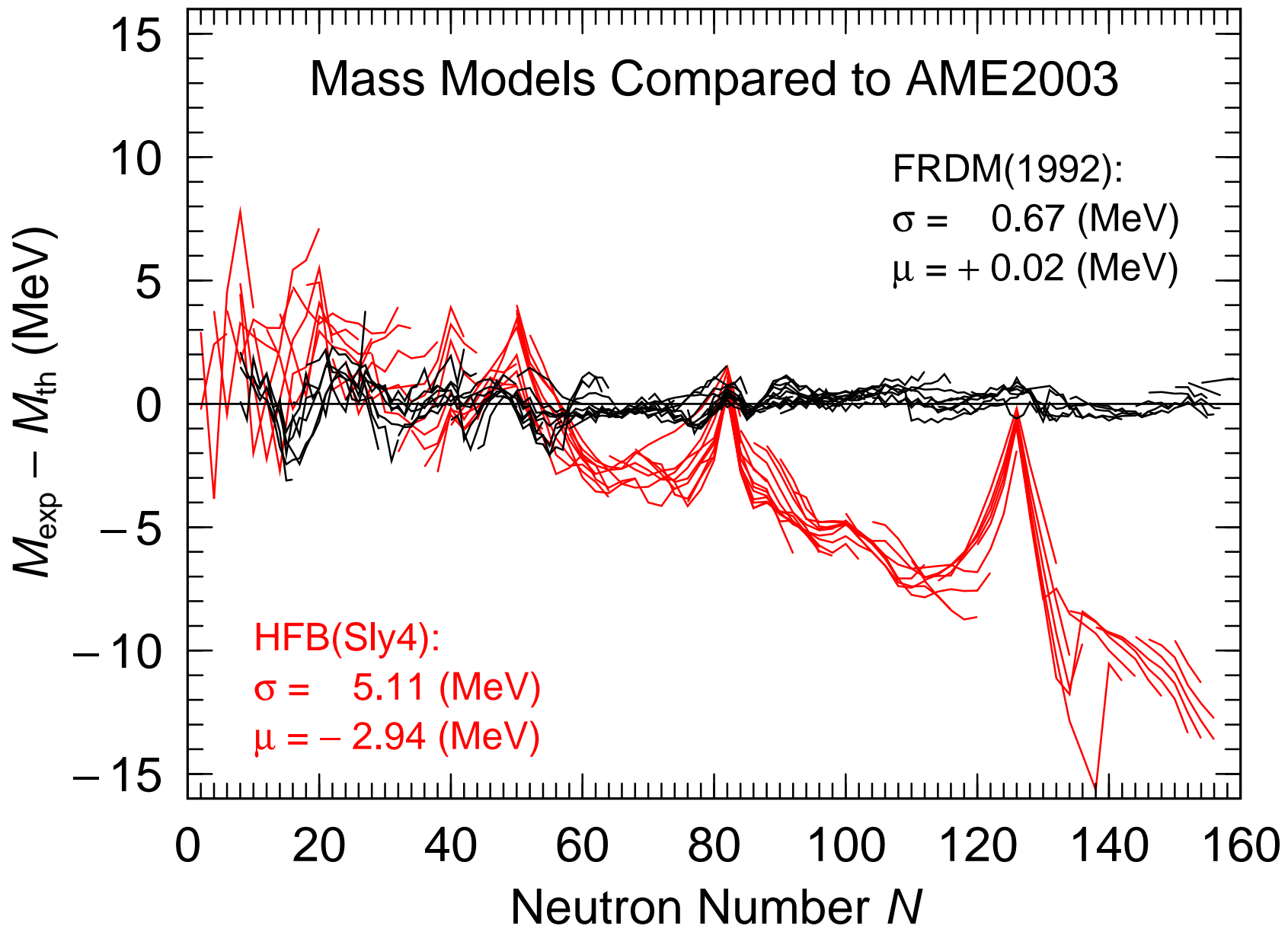
Z	A	x
50	124	0.402
82	208	0.645
92	138	0.709
100	252	0.792
114	298	0.870
125	328	0.950
130	335	1.006

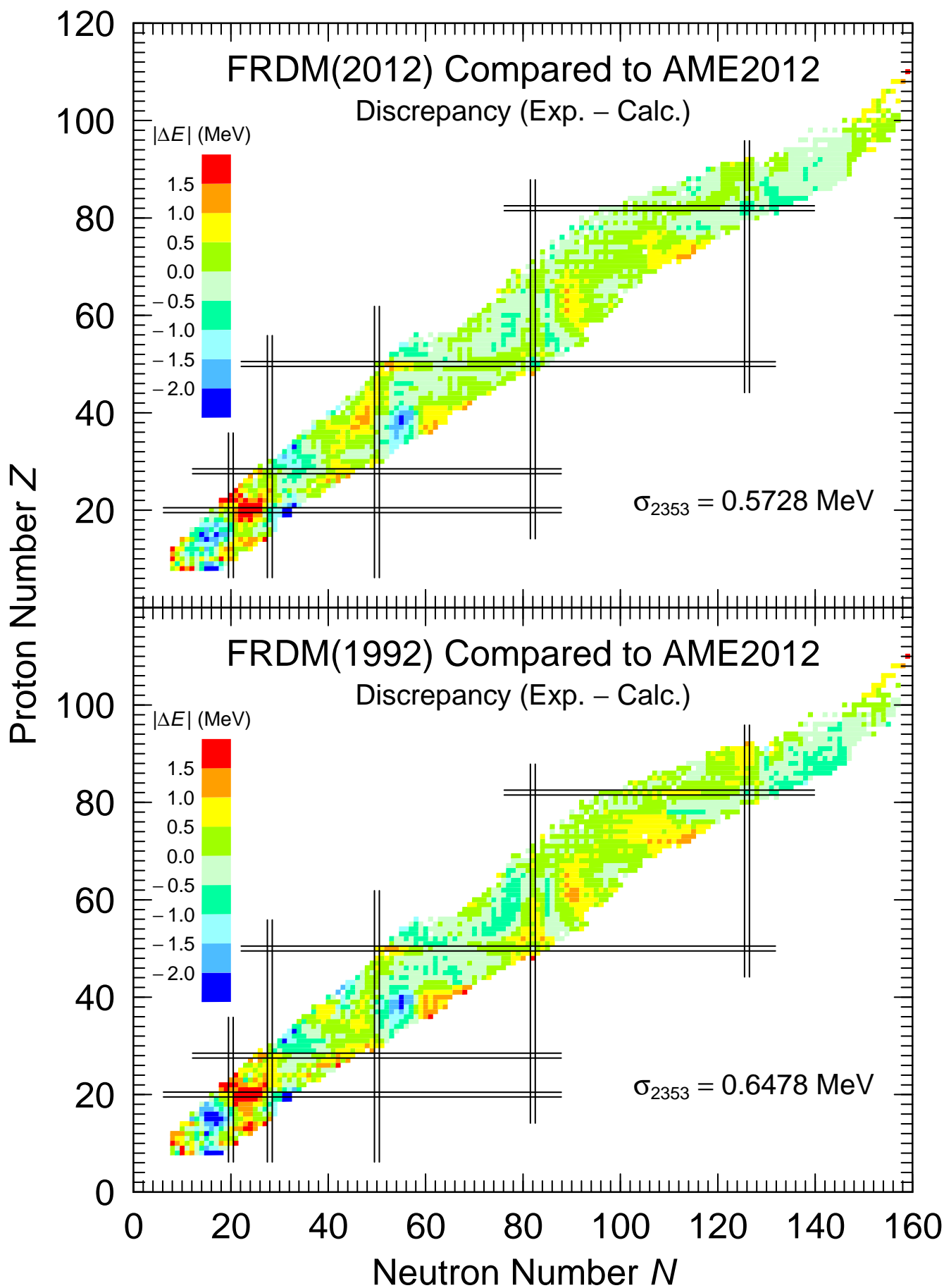


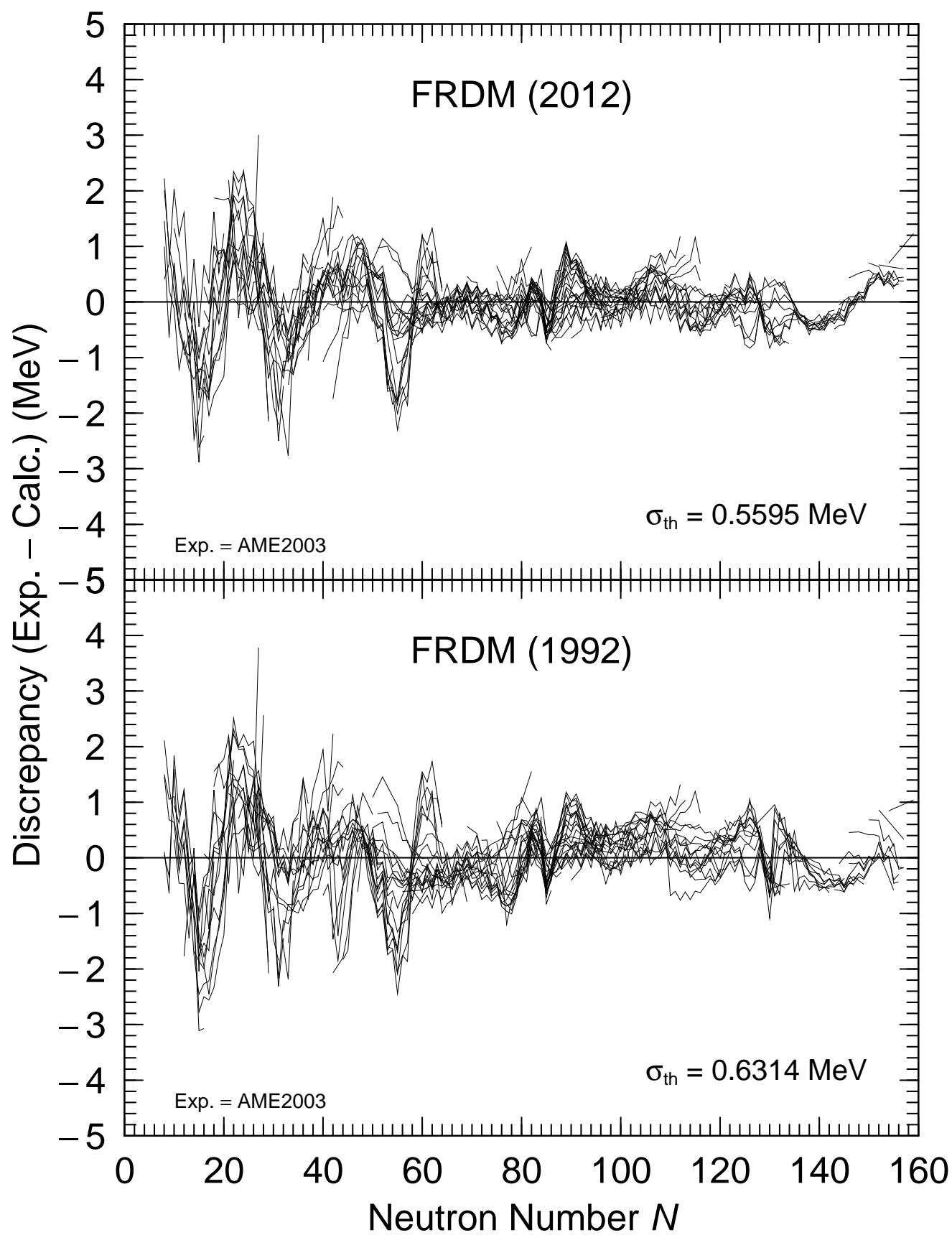
Feynman:

- I do not care how smart you are
- or how complicated your model is
- If it does not agree with experimental measurements it is wrong!

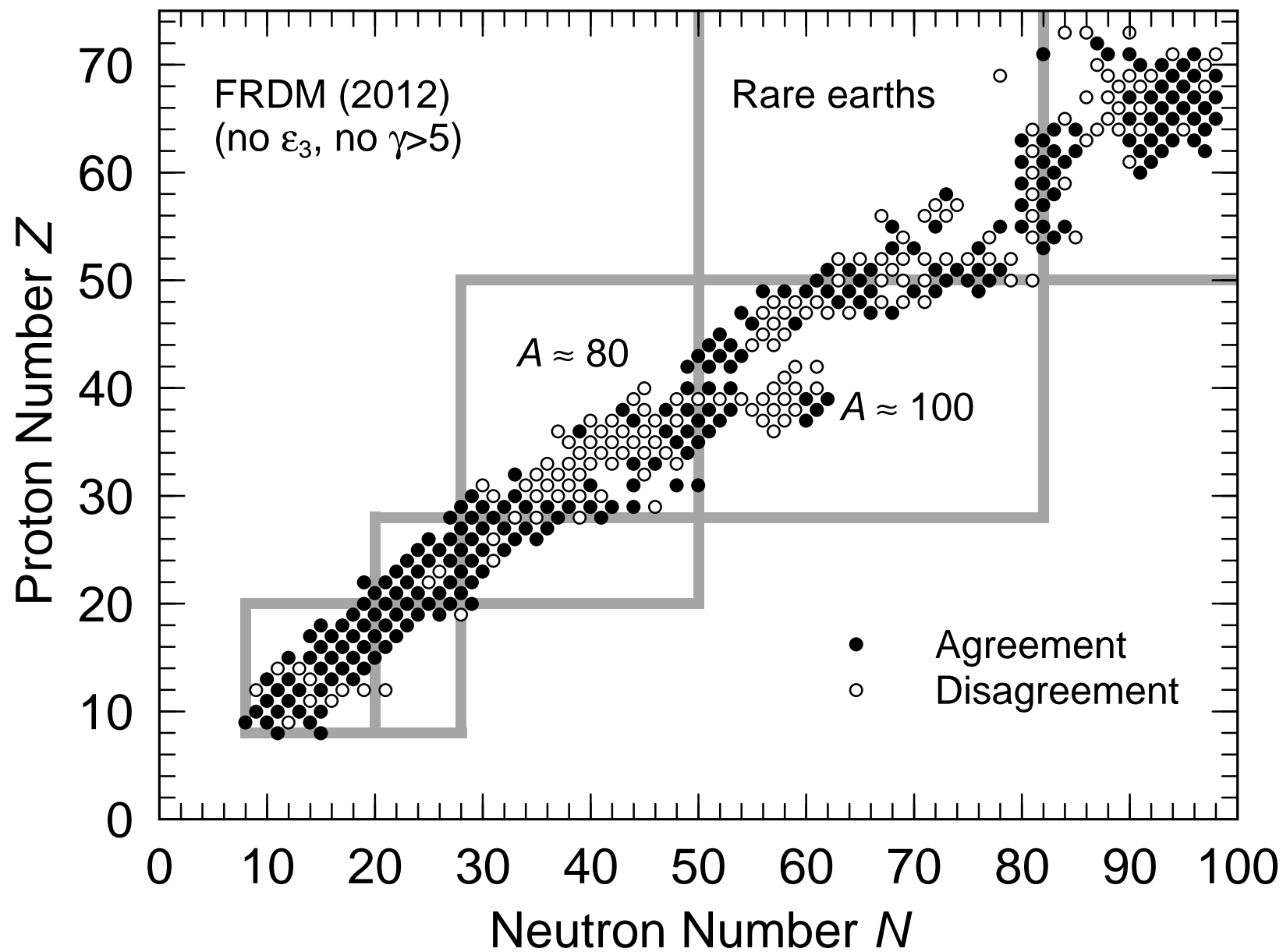




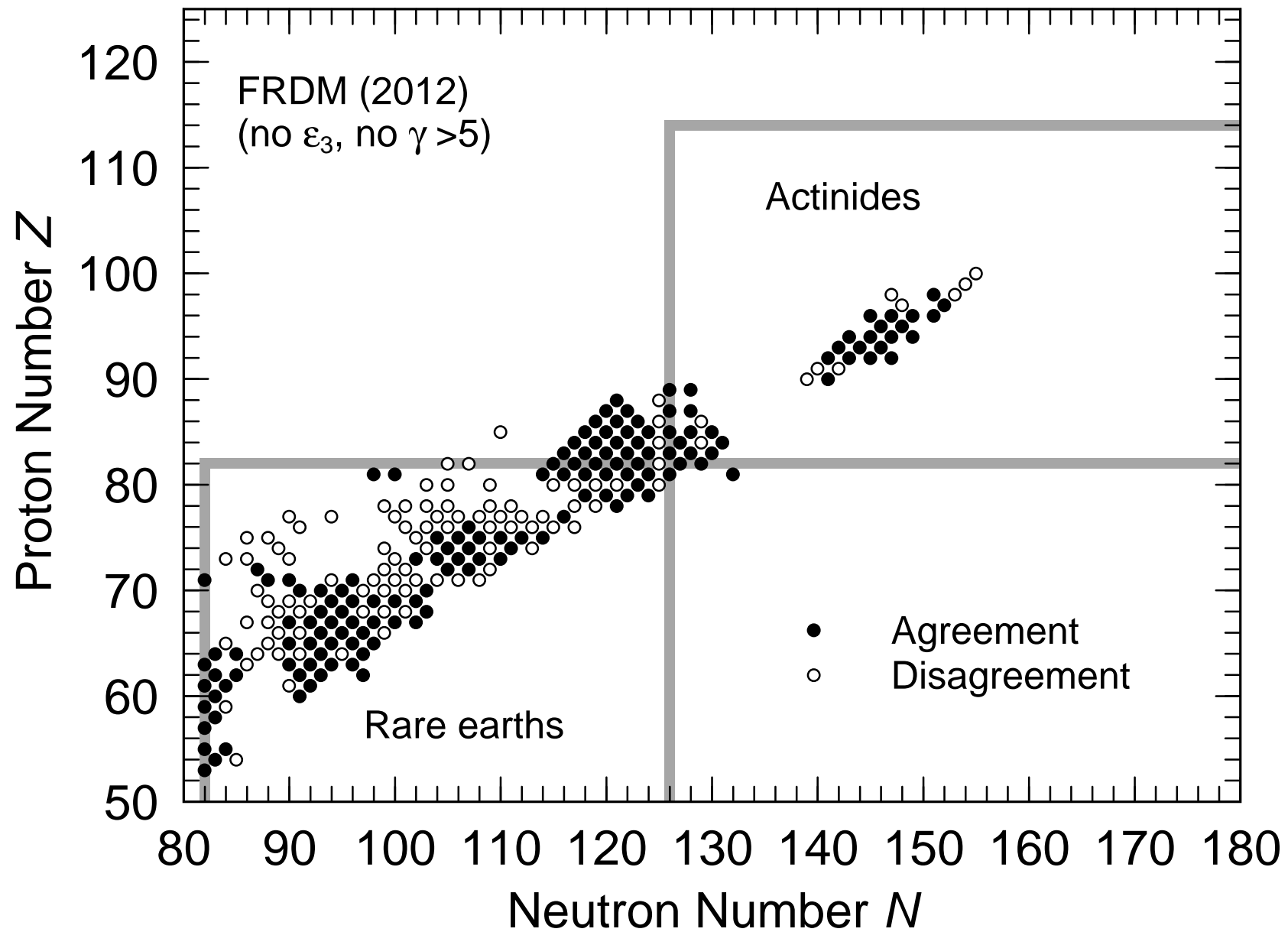


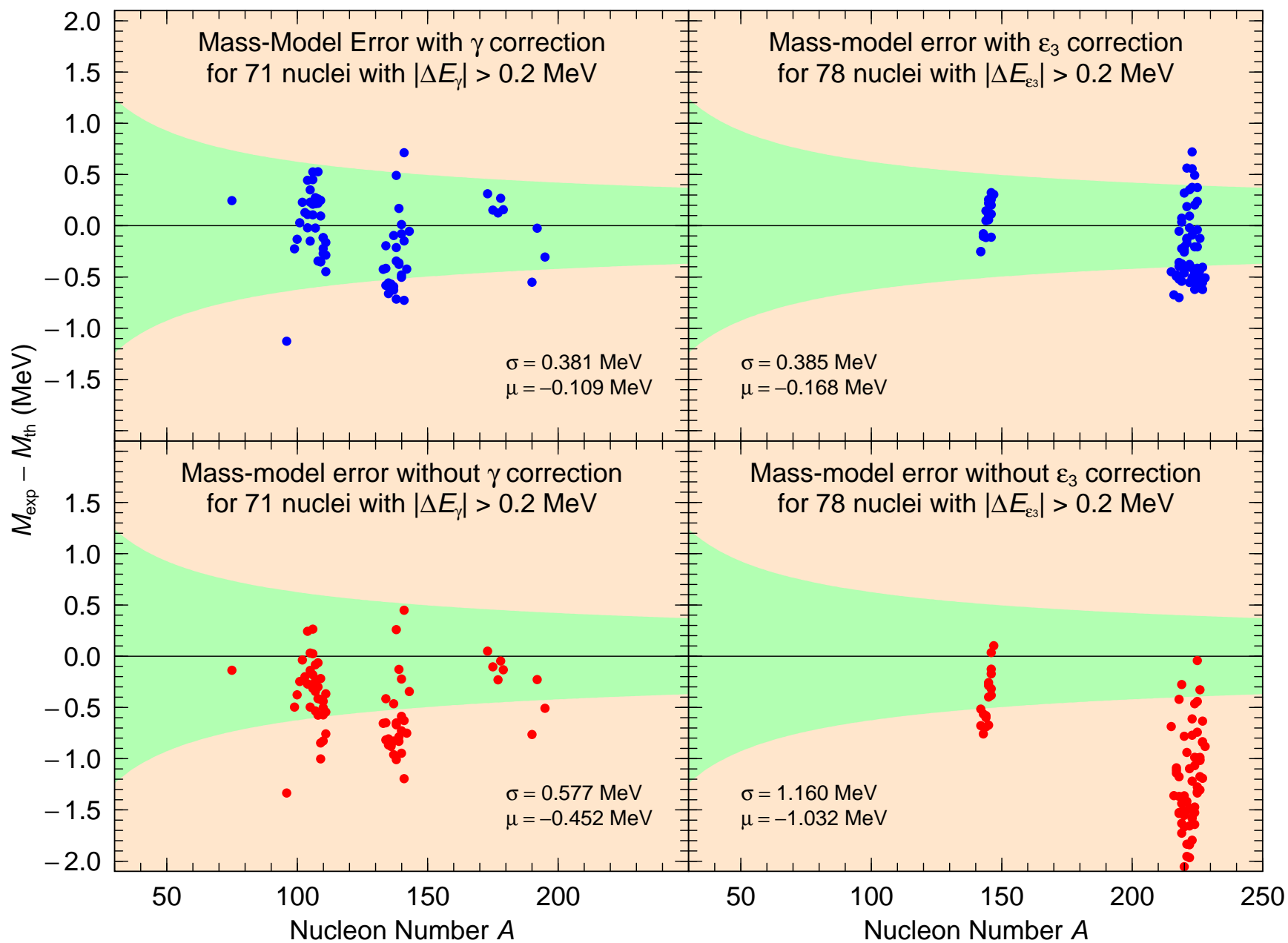


Calculated ground-state spin compared to experiment



Calculated ground-state spin compared to experiment





nuclear physics

By GERTRUDE SCHARFF-GOLDHABER

EVEN A SUPERFICIAL GLANCE backward will teach us that it is impossible to predict in detail the future of a fast-moving science like nuclear physics. It will remind us that often entirely unexpected events changed the direction of endeavor in this field. These events were either of an experimental nature, as, for instance, the finding that beta rays have continuous energy spectra, or they consisted in the formation of new concepts—as, for example, of the liquid-drop model of the nucleus.

It is, of course, similarly impossible to predict what extraneous happenings may in the future affect scientific progress as profoundly as two world wars and political persecution have affected it in the past. During the last twelve years the great importance attached to atomic energy has induced an unprecedented increase in the tempo of research, and new nuclear physics centers have sprung up all over the world. This development is viewed by many with delight while others are afraid that it may have a negative effect on nuclear physics as a pure science.

In spite of all the uncertainties mentioned, it is useful to interrupt from time to time one's preoccupation with the problem at hand to investigate the trends that current research seems to follow, both in experiment and theory, and to try to recognize how far these may serve to bring us closer to the solution of outstanding problems.

The central problem is to understand the nucleus in the same sense in which one might have said in 1926 that the atom was understood: one knew then not only that the forces between the nucleus and the atomic electrons were pure Coulomb forces, but also that the excited states in which the system could exist were governed by the laws of quantum mechanics, including the Pauli exclusion principle. (It is true, only the states of the simplest atom, hydrogen, could be exactly calculated, while already the helium atom presented such overwhelming mathematical difficulties that it took about another quarter century and development of computers to reach the same stage.)

					13	Al 26.98 #13
					12	Mg 24.32 #12
					11	Na 22.991 #11
					10	Ne 20.183 #10
						Ne 18 16 s #13.2 E 4.2
						Ne 19 18.5 s #12.2 E 3.2
					9	F 19.00 #9
						F 17 66 s #1.75 E 2.77
						F 18 1.87 h #1.65 E 1.87
					8	O 16.000 #8
						O 14 72 s #1.83 E 2.30 E 3.5
						O 15 2.1 m #1.7 E 2.7
						O 16 99.759 #1.00003 E 16.00005
						O 17 0.037 #1.00003 E 16.00005
						N 12 0.012 s #1.67 E 17.7
						N 13 10.0 m #1.20 E 2.22
						N 14 99.63 #1.00002 E 16.00002
						N 15 0.37 #1.00002 E 16.00002
						N 16 7.4 s #1.004 E 10.4
						C 10 19 s #1.9 E 7.2, 10.3 E 3.6
						C 11 20.5 m #1.96 E 1.96
						C 12 98.89 #1.00002 E 16.00002
						C 13 1.1 #1.00008 E 16.00008
						C 14 5600 y #1.55 E 1.55
						C 15 2.3 s #1.43, 9.9 E 5.3 E 9.6
						B 9 1.8
						B 10 1.8
						B 11 1.8
						B 12 1.025 s

If we want to study events happening inside the nucleus, its finite extension and the distribution of charge and current inside it, we have a task of a higher degree of difficulty than in the atomic case: (a) We do not know the exact nature of the force between two nucleons, nor do we know whether a potential exists for this force. (b) One cannot consider, in first approximation, the interaction of two nucleons only, because all nucleons are close to each other. (c) We do not know whether the laws of quantum mechanics are sufficient to describe a nucleus completely.

Quantum mechanics may need to be modified, e.g., by the introduction of the concept of a fundamental length, before nuclear phenomena can be explained

On the other hand, we have two important clues on the nature of nuclear forces:

1. Apart from the lightest ones, all nuclei have the same density $\rho = 1.7 \times 10^{28}$ nucleons/cm³, at least in their central part, so that it is reasonable to speak of "nuclear matter." Hence, if it were not for the Coulomb repulsion between the protons, nuclei of arbitrarily large size would exist. We therefore speak of the "saturation" of nuclear forces, which prevents nuclear matter from collapsing to a density less than ρ and from flying apart.

2. Experiments have shown that the forces between two protons, corrected for the effect of Coulomb repulsion, are the same as between two neutrons, i.e., charge symmetry prevails, and probably the forces are also the same between a proton and a neutron (charge independence).

In recent years a number of theoretical physicists, under the leadership of K. Brueckner, have tried to understand the nuclear phenomena by treating the nucleus as a many-body system assuming that nuclear forces can be derived from a two-body potential. The simplified case of infinite

surprising that only one such close-lying triplet has been found so far, namely, in Cd^{114} . The method used here was the analysis of conversion electrons accompanying neutron capture in Cd^{113} (H. Motz). A systematic search for triplets by means of suitably chosen experiments should throw light on the question of the nature of the vibrational even-even nuclei and, in turn, of those odd- A nuclei of which they form the core.

As is well known, the collective model has been enormously successful in describing not only the level characters and energy ratios, but also the transition probabilities of electromagnetic transitions, the $\log ft$ values, etc., in strongly deformed nuclei.

Recent efforts to interpret the level scheme of F^{19} have led to an interesting discovery: the results of one group of physicists who applied a shell-model analysis agreed surprisingly well with those of another group who applied a collective-model interpretation. Ironically, the theoretical values agreed even more closely among each other than with the experimental values. The reason for the good agreement is by no means obvious and is now being studied by a number of theoreticians. It allows one to conclude, however, that it will be possible to set up a unified model of the nucleus, in which the individual particle motions and the collective motions are self-consistent as in molecules. Important attempts in this direction have been made by Peierls and Yoccoz and by Wheeler and Griffin.

Now a few words about higher-energy nuclear reactions: As was mentioned above, the optical model is successful in describing the energy dependence of the absorption and scattering cross sections of the particles impinging on a nucleus for a considerable energy range. However, a theory has still to be evolved which will give the probabilities and angular correlations for the decomposition of the system target nucleus plus bombarding particle into the various energetically allowed end products. It may be added that the optical model is based on the assumption that the energy spectrum of the incoming particles overlaps many resonance levels in the target nucleus. It is likely that with increasing energy definition this approach will give way to a renewed interest in the fine structure, which is of particular importance in the fission process.

We can look forward to definitive progress toward understanding the bewildering variety of phenomena observed in fission

A few thoughts on beta-decay theory: the recent revolution in thought brought about by the discovery that parity conservation and the conservation of charge conjugation do not hold in weak interactions has attracted great interest to this field. Although at present the nature of the interactions for the nucleon-electron-neutrino system is not known, it is very probable that by the end of the coming decade it will be quite well understood. This will be brought about by studies of polarization of the electrons emitted in various types of beta decay, by further efforts to detect double beta decay and by the refinement of present neutrino-detection experiments.

The experimental determination of nuclear properties will increase in accuracy and scope as the equipment and methods grow in diversity and ingenuity.

For example, the steady improvement in resolution and efficiency of spectrometers will facilitate the determination of level energies.

For character assignments to nuclear energy levels there are now a number of methods at our disposal, which we have just begun to exploit. For short-lived states preceded by some previous radiation there is the delayed-coincidence method, which Sunyar has recently developed into a form that permits one to measure lifetimes as low as a few times 10^{-11} sec. For electric dipole or quadrupole transitions leading to the ground state the Coulomb-excitation method is able to cover a wide lifetime range. As a final example, the use of molecular-beam and paramagnetic-resonance methods for the spin determination of radioactive nuclei will doubtless increase and serve to check assignments made on the basis of decay-scheme studies. Further, it may be expected that the refinement of the theoretical interpretation of experimental results, e.g., a better theory of stripping, of internal conversion and of the angular distribution of inelastically scattered particles, will make level-character assignments of excited states more definite.

Radioactive nuclei will be found further away from the stability region, owing to the use of heavier bombarding particles and faster detection methods. The study of the binding energies of nuclei of this type may throw further light on the nature of nucleon-nucleon forces. Also, the number of known elements will certainly be increased.

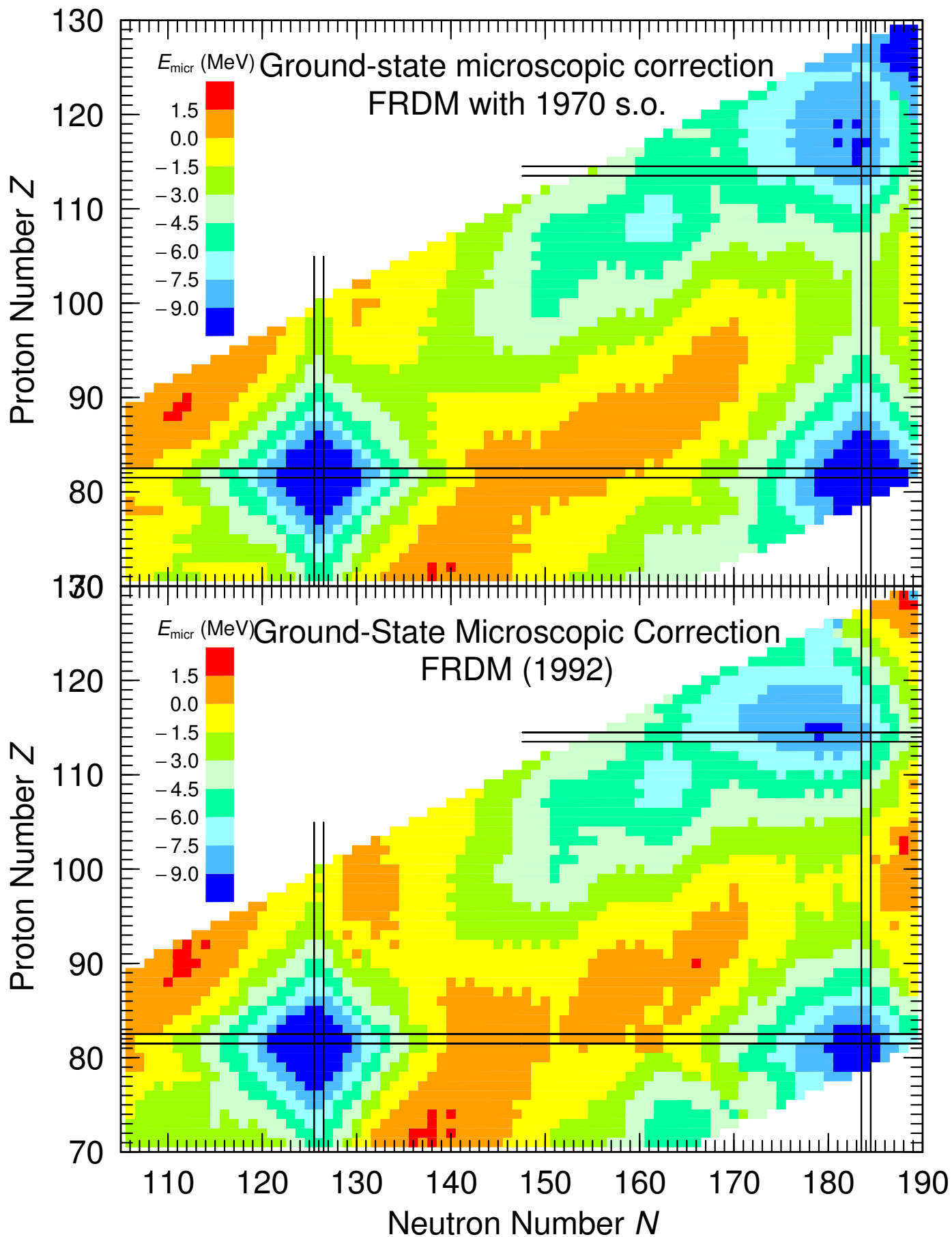
Relatively long-lived isotopes may well be found among the far-transuranic nuclides because of magic-number stability

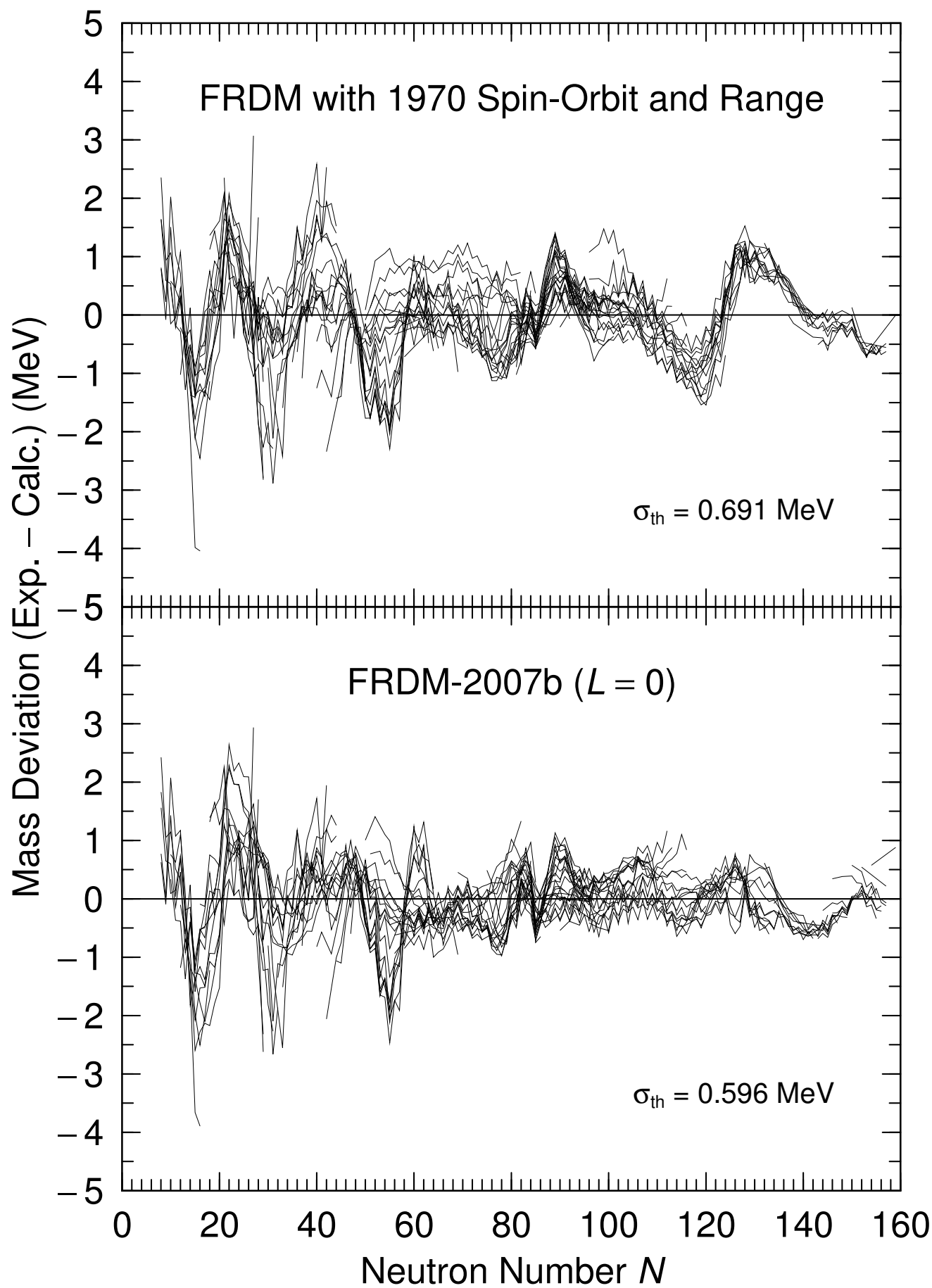
There may be, for instance, another region of relative stability at the doubly magic nucleus $^{126}\text{X}^{310}$ (the closing of the j neutron shell).

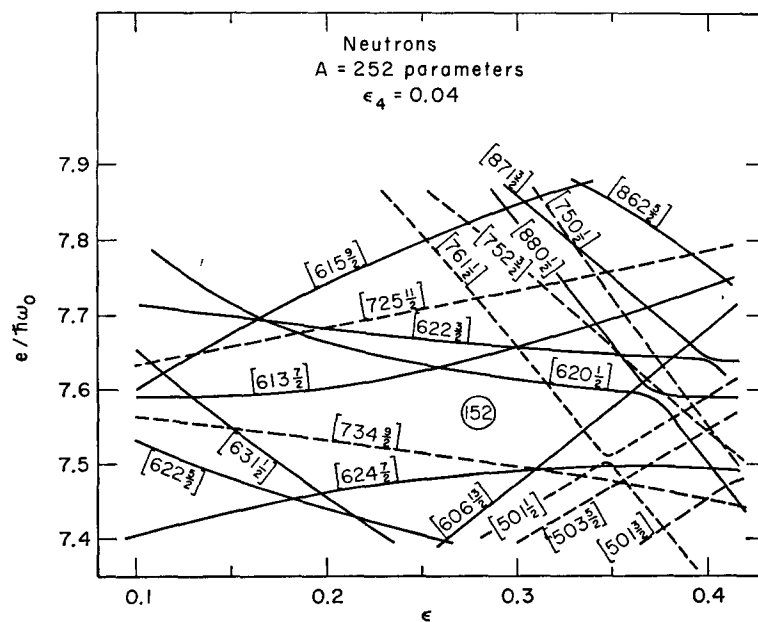
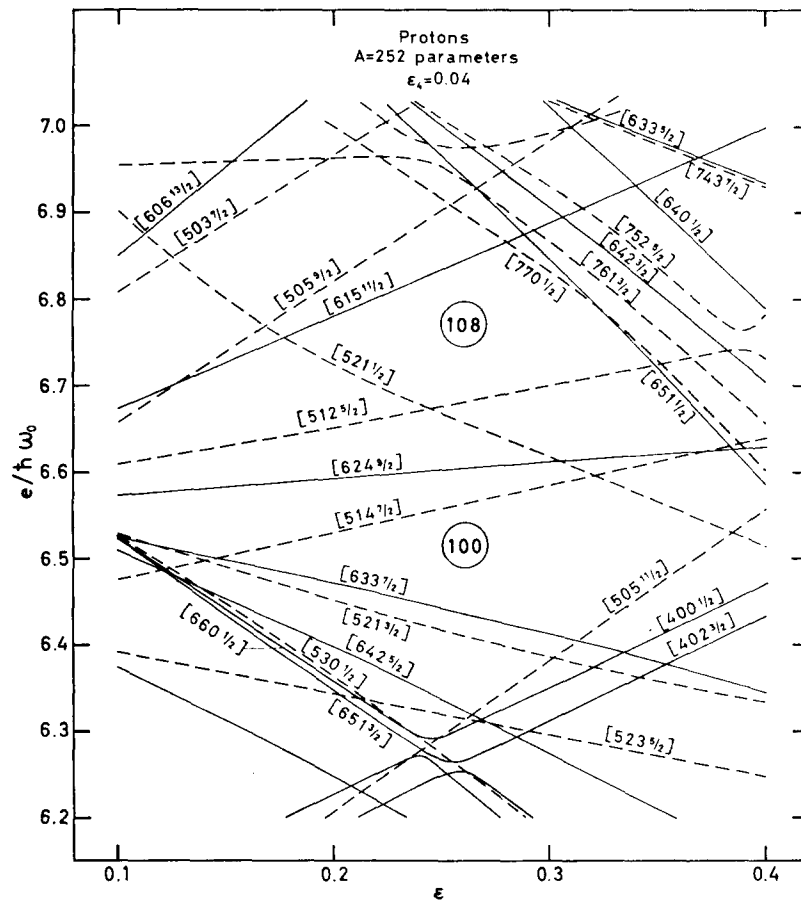
New accelerators like variable-energy cyclotrons and tandem Van de Graaffs capable of producing beams of well defined and sufficiently high energies will make it possible to study radiation and particle widths of excited states of light nuclei with accuracy, thus testing the shell-model wave functions, and to explore the level schemes of medium-weight nuclei ($50 < A < 150$).

Atomic-beam methods and possibly the study of μ -mesic X-rays will yield new data on electric and magnetic moments of higher order. The boldness of some thinkers in this field is indicated by the title of a recent theoretical paper: "Nuclear Hexadecapole Moments." It may even be possible to get a better idea of the charge and current distribution within the proton by means of the 6-Bev electron synchrotron now being constructed at Cambridge, Massachusetts, which will permit an extension of the very successful work carried out at Stanford.

One important goal of nuclear-physics research is the deduction of nuclear forces from meson fields. Among these fields the role of the π -meson field will probably be first understood, but it is clear that any ultimate theory will not be able to ignore the role of the "strange particles" (K mesons and hyperons). In the meantime, the new field of "hypernuclear" physics is likely to develop considerably and to help indirectly in the understanding of nuclear phenomena.







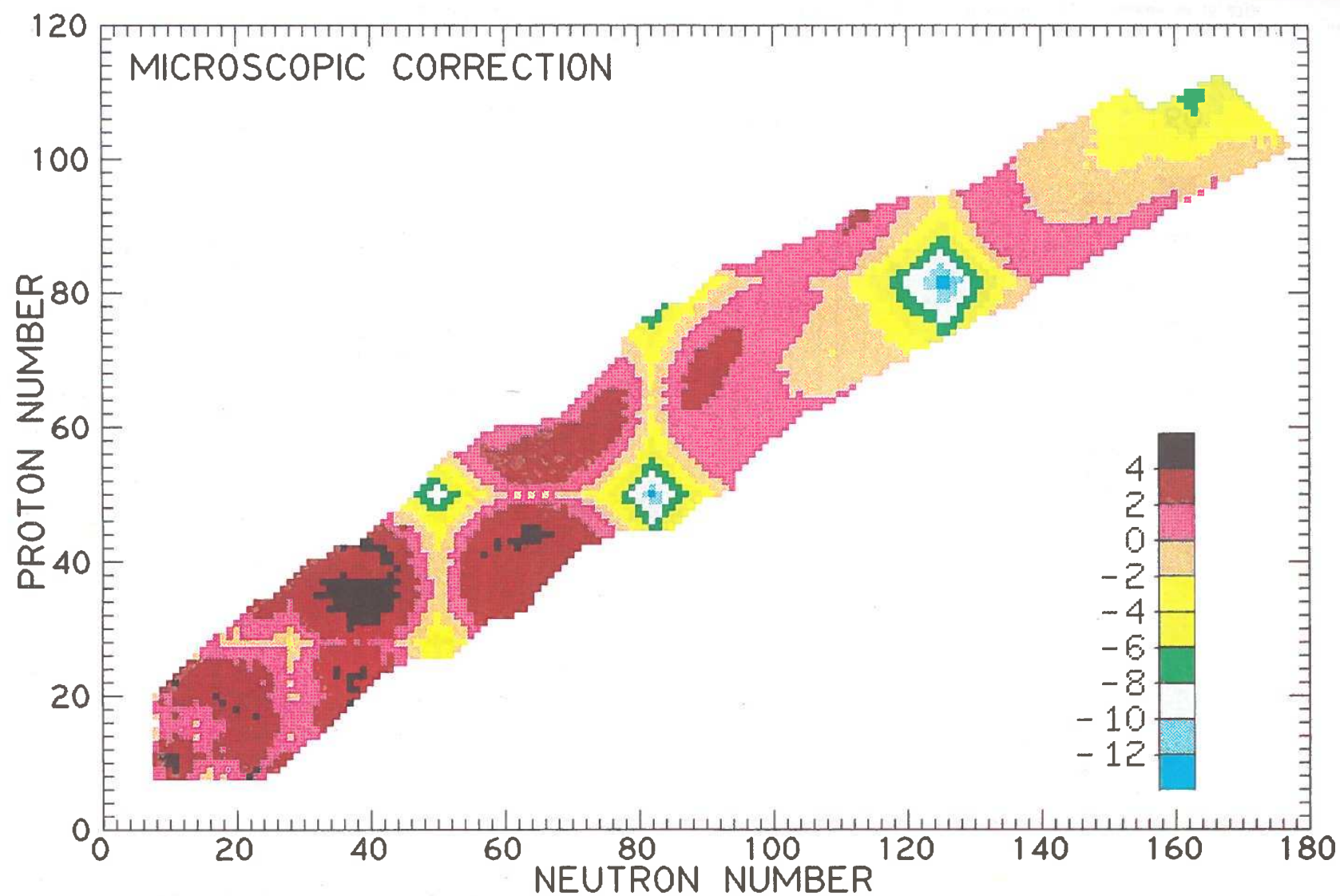
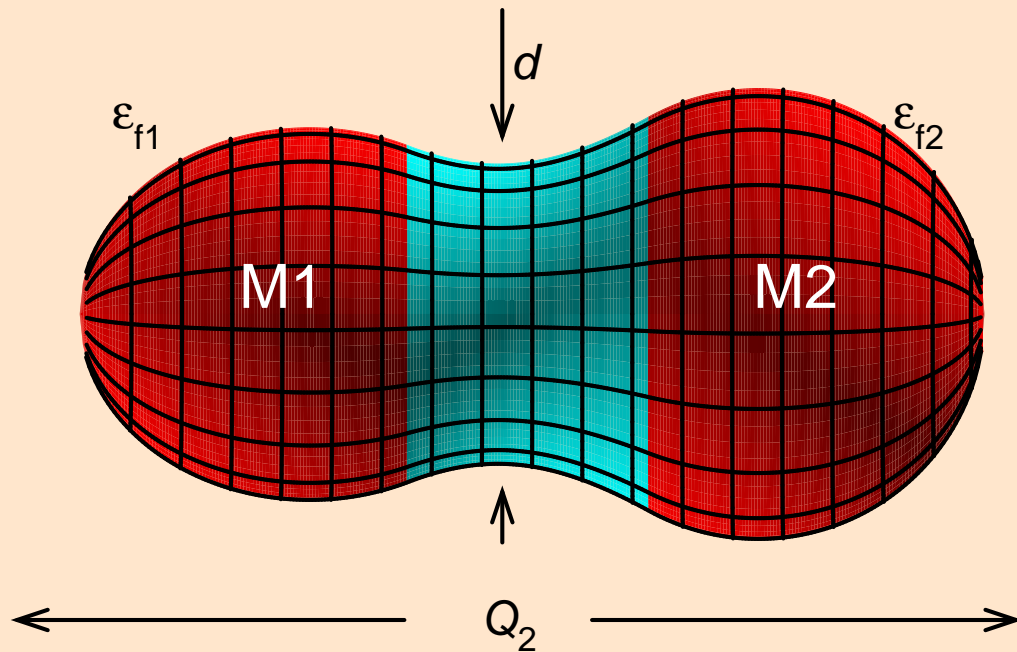


Fig. 2c. Plot of the ground-state microscopic correction, as calculated in [6] for 4023 nuclei. The fluctuations in the shell correction are larger in the heavier region than in the lighter region.

Five Essential Fission Shape Coordinates

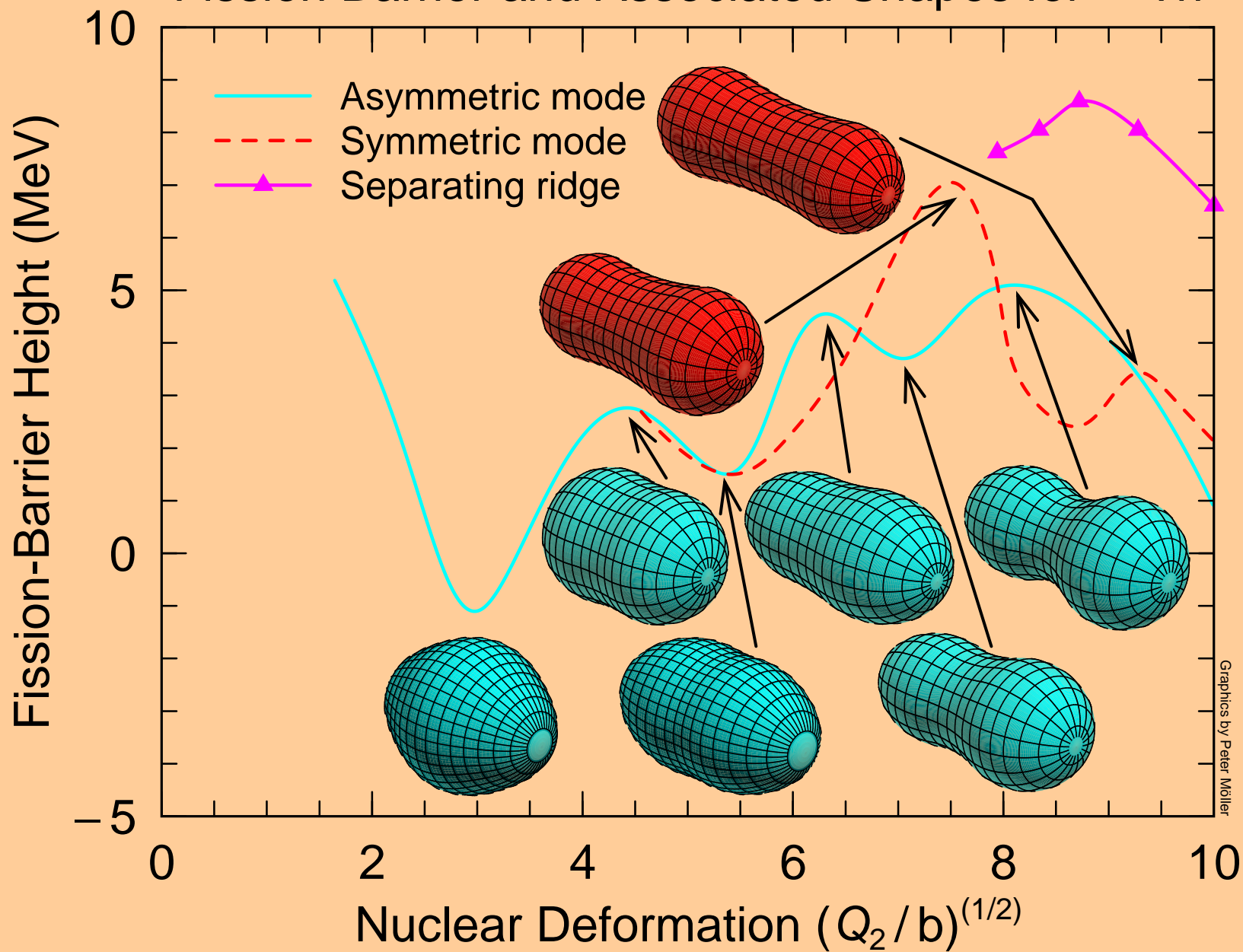


45	$Q_2 \sim$ Elongation (fission direction)
⊗	
35	$\alpha_g \sim (M1-M2)/(M1+M2)$ Mass asymmetry
⊗	
15	$\epsilon_{f1} \sim$ Left fragment deformation
⊗	
15	$\epsilon_{f2} \sim$ Right fragment deformation
⊗	
15	$d \sim$ Neck

\Rightarrow 5 315 625 grid points – 306 300 unphysical points

\Rightarrow 5 009 325 physical grid points

Fission Barrier and Associated Shapes for ^{232}Th



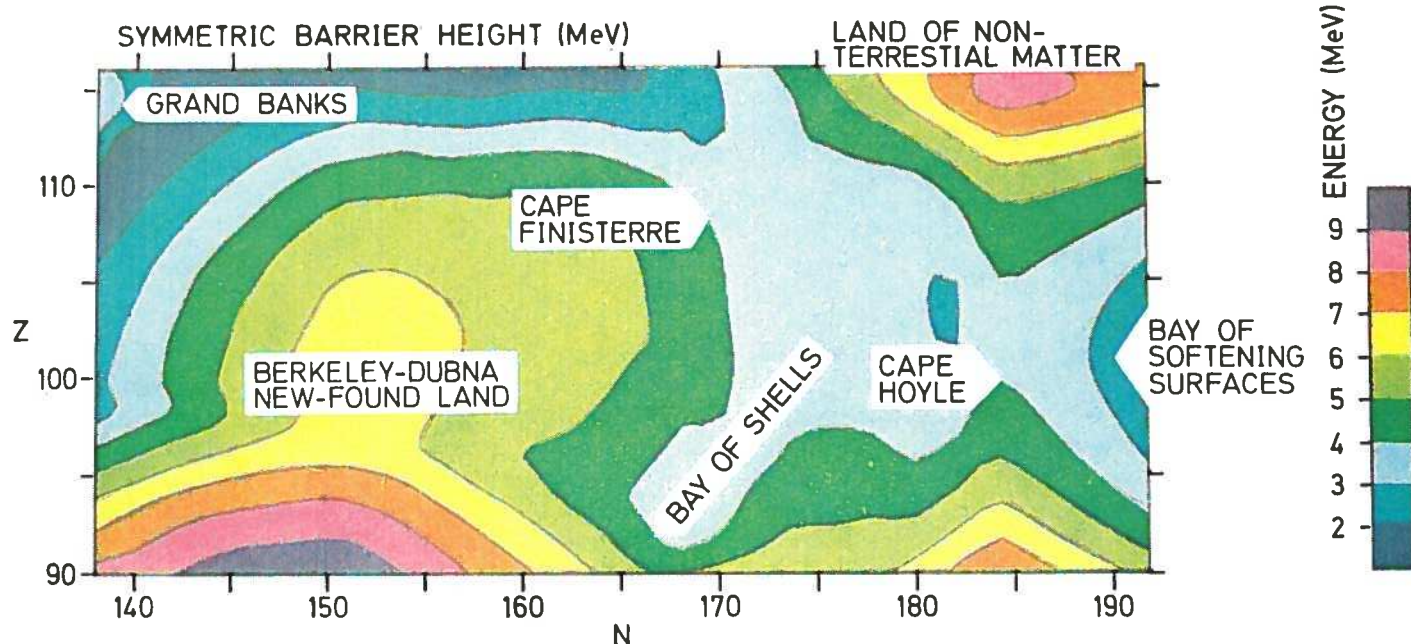
Nobel Symposium 27

Physics

Super-Heavy Elements— Theoretical Predictions and Experimental Generation

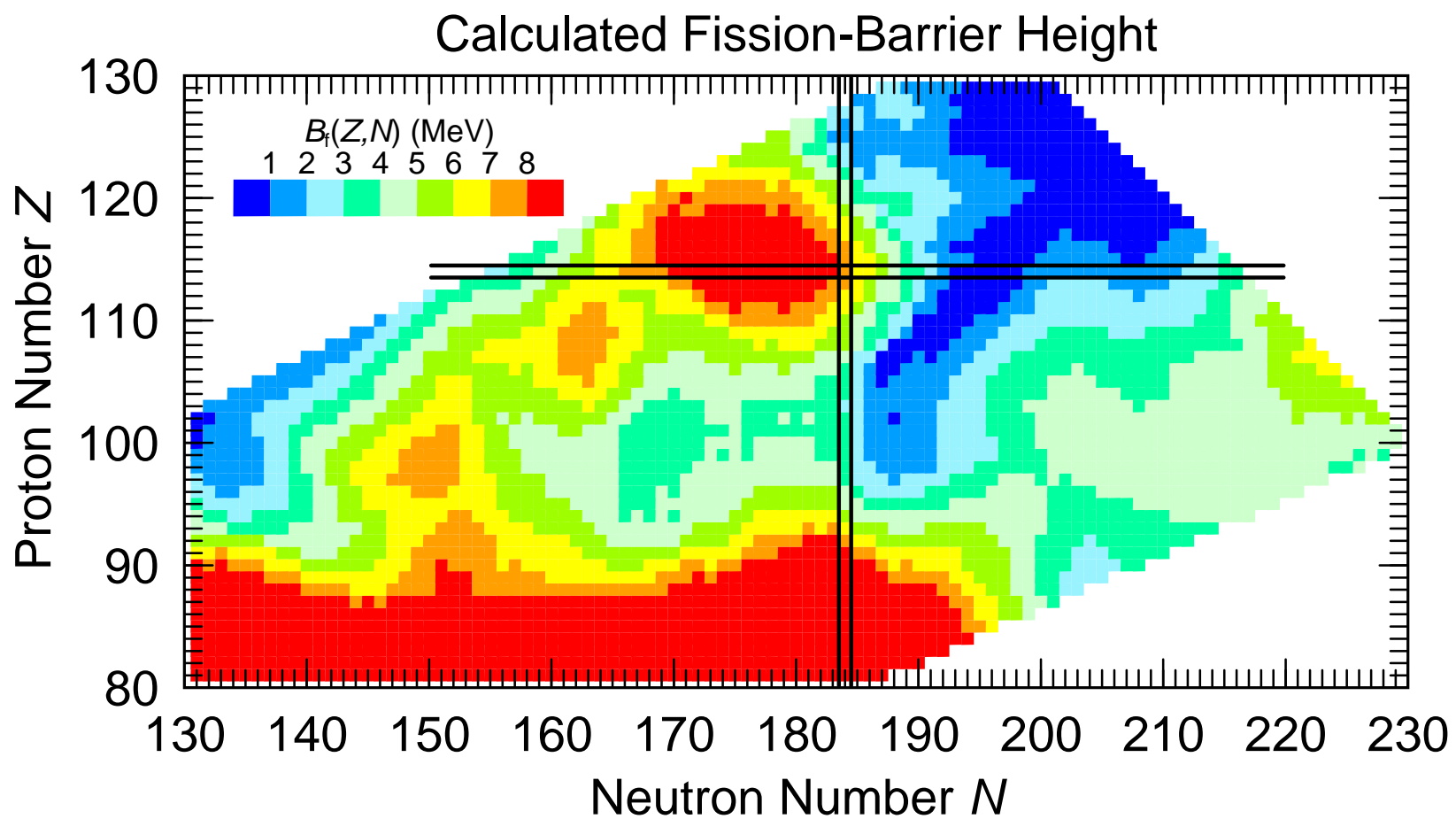
Proceedings of the Twenty-Seventh Nobel Symposium
held at Ronneby, Sweden, June 11–14, 1974

Editors *Sven Gösta Nilsson* and *Nils Robert Nilsson*

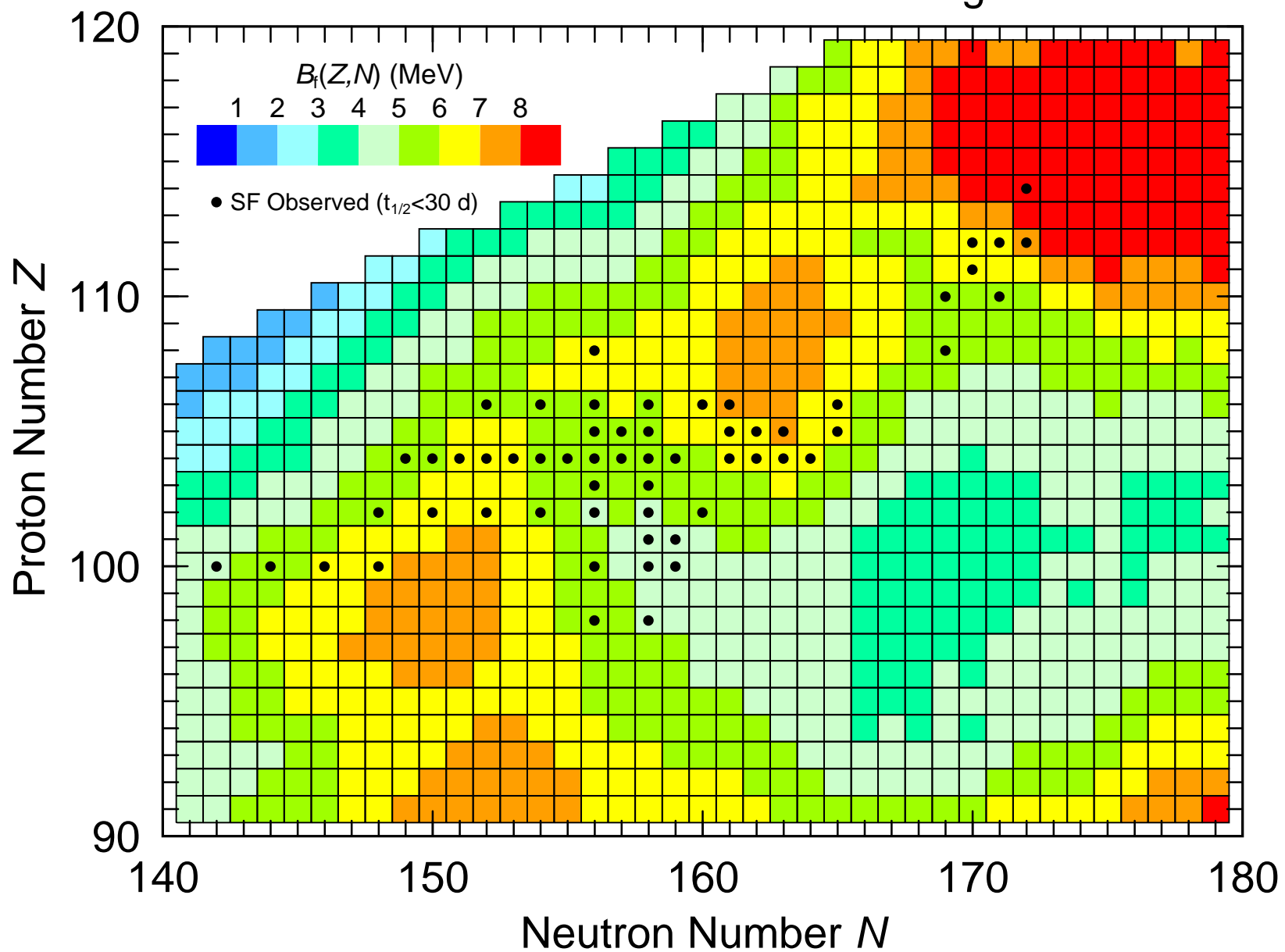


Nobel Foundation, Stockholm

Almqvist & Wiksell International, Stockholm, Sweden



Calculated Fission-Barrier Height



Fusion configurations for a spherical projectile

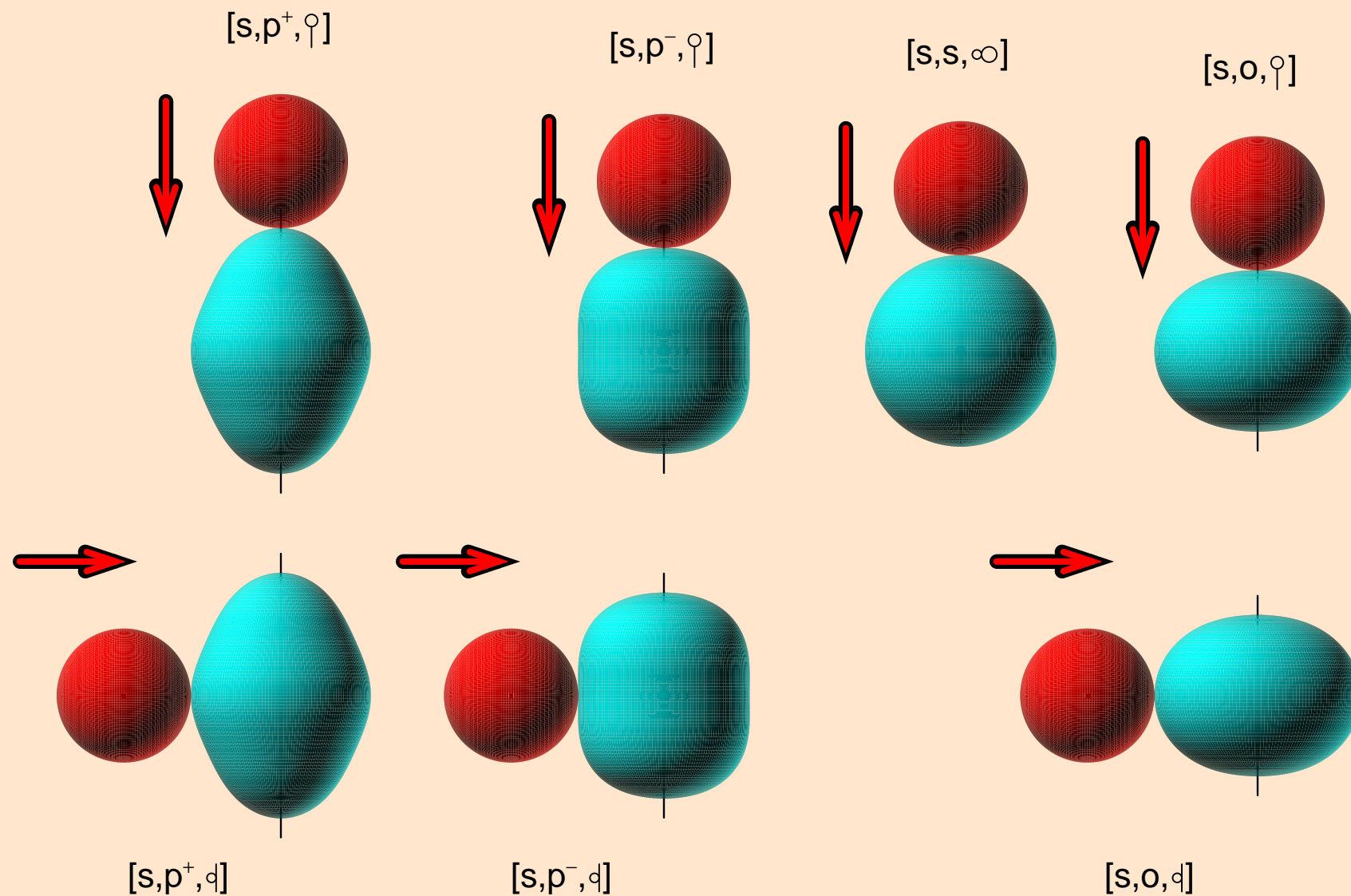


Figure 1

Fusion configurations of deformed nuclei

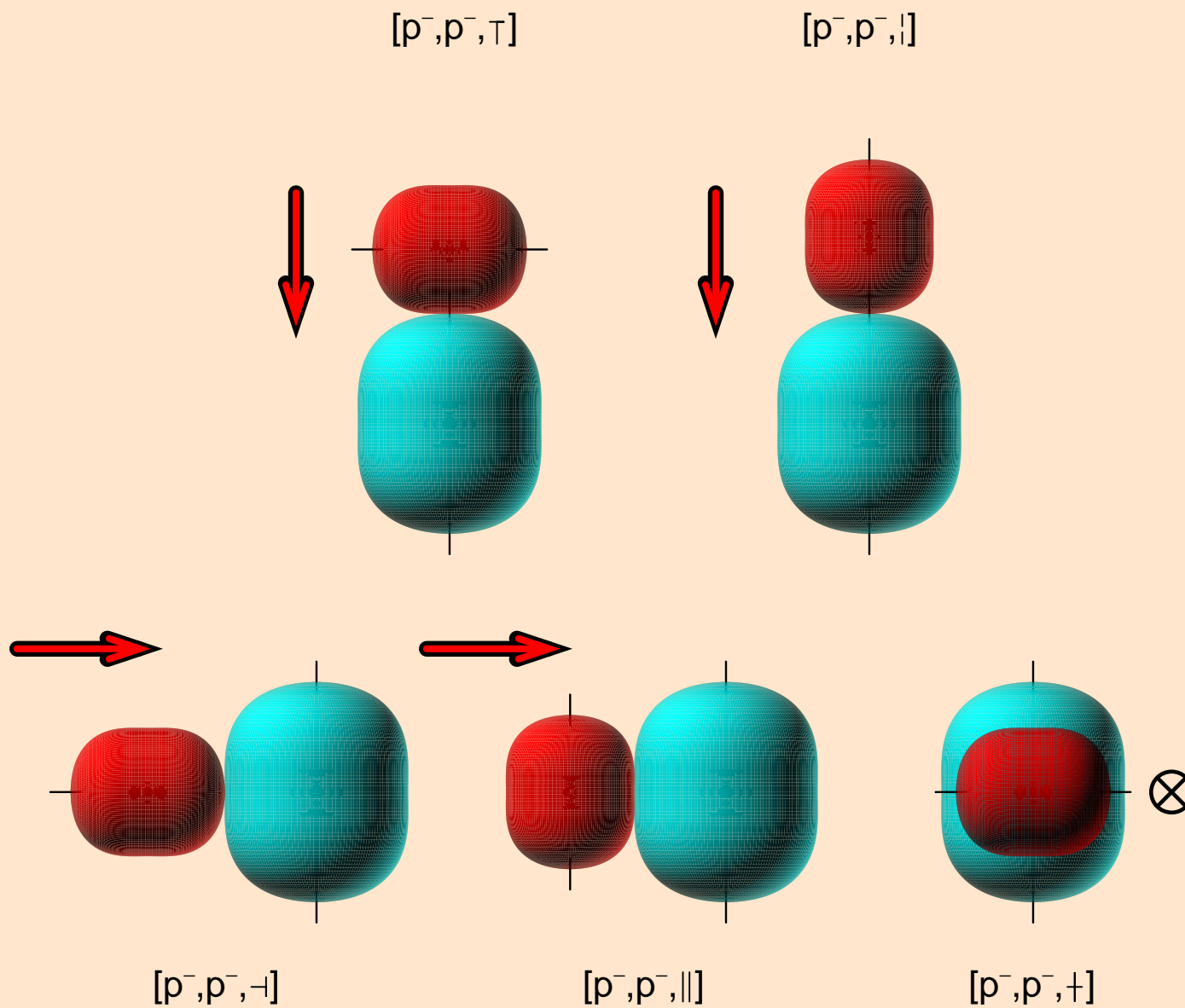
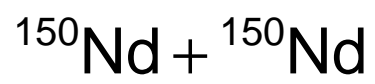
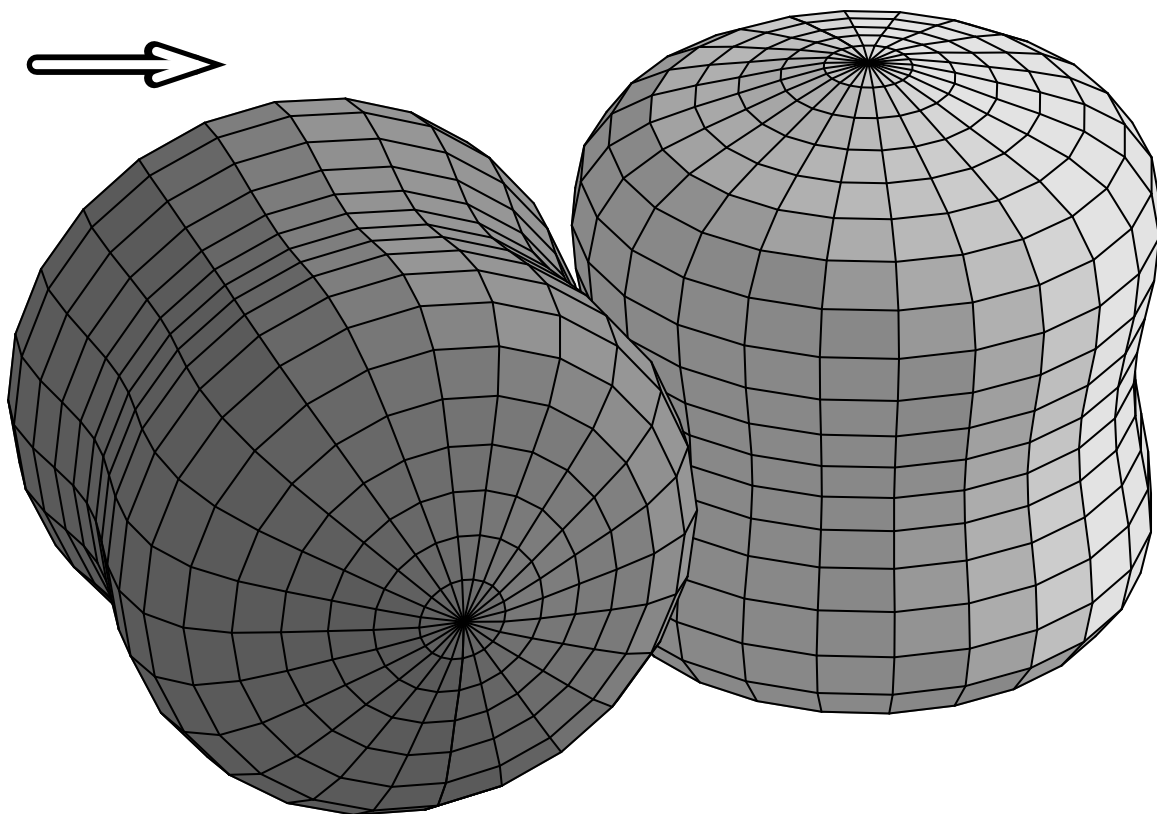
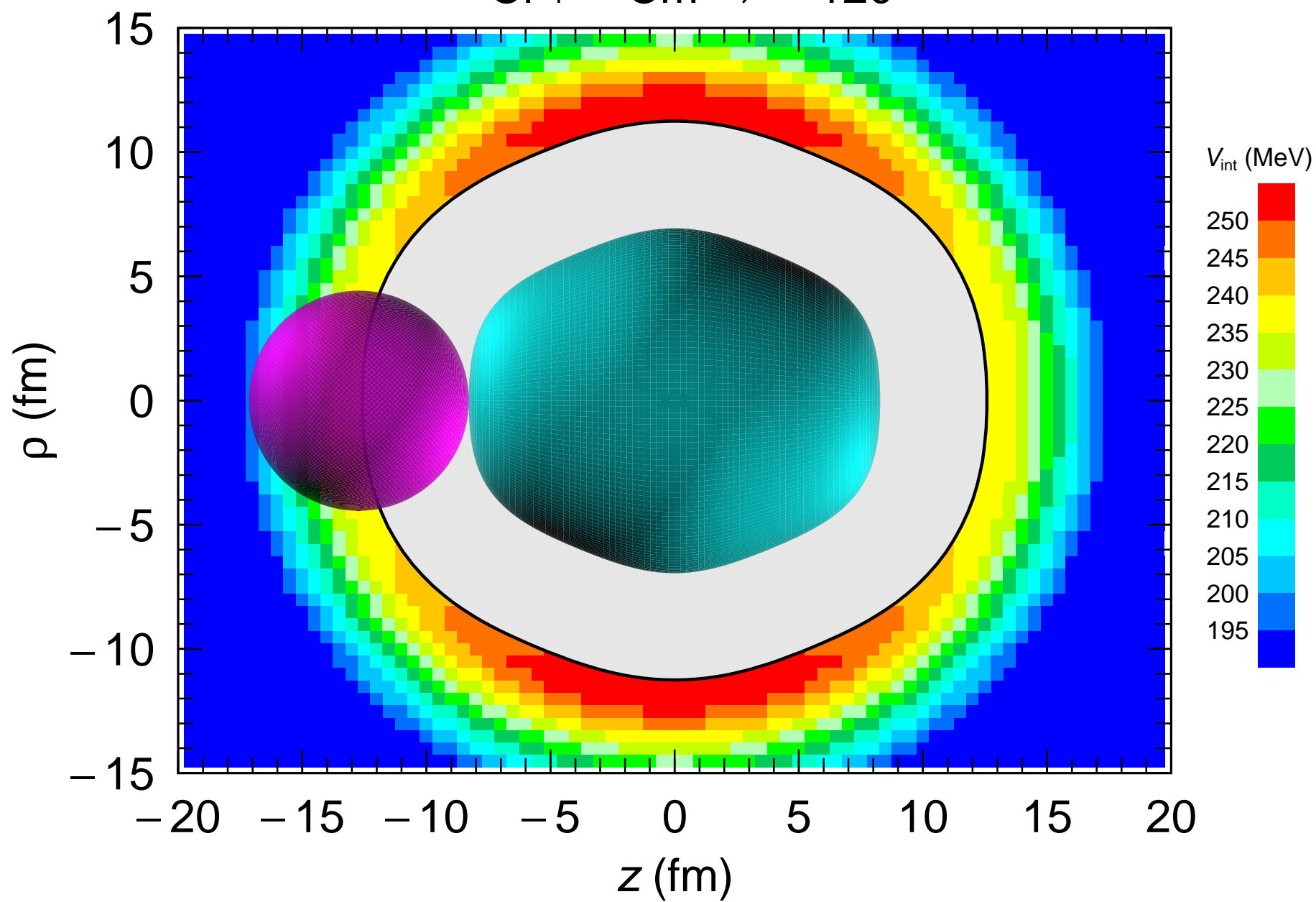


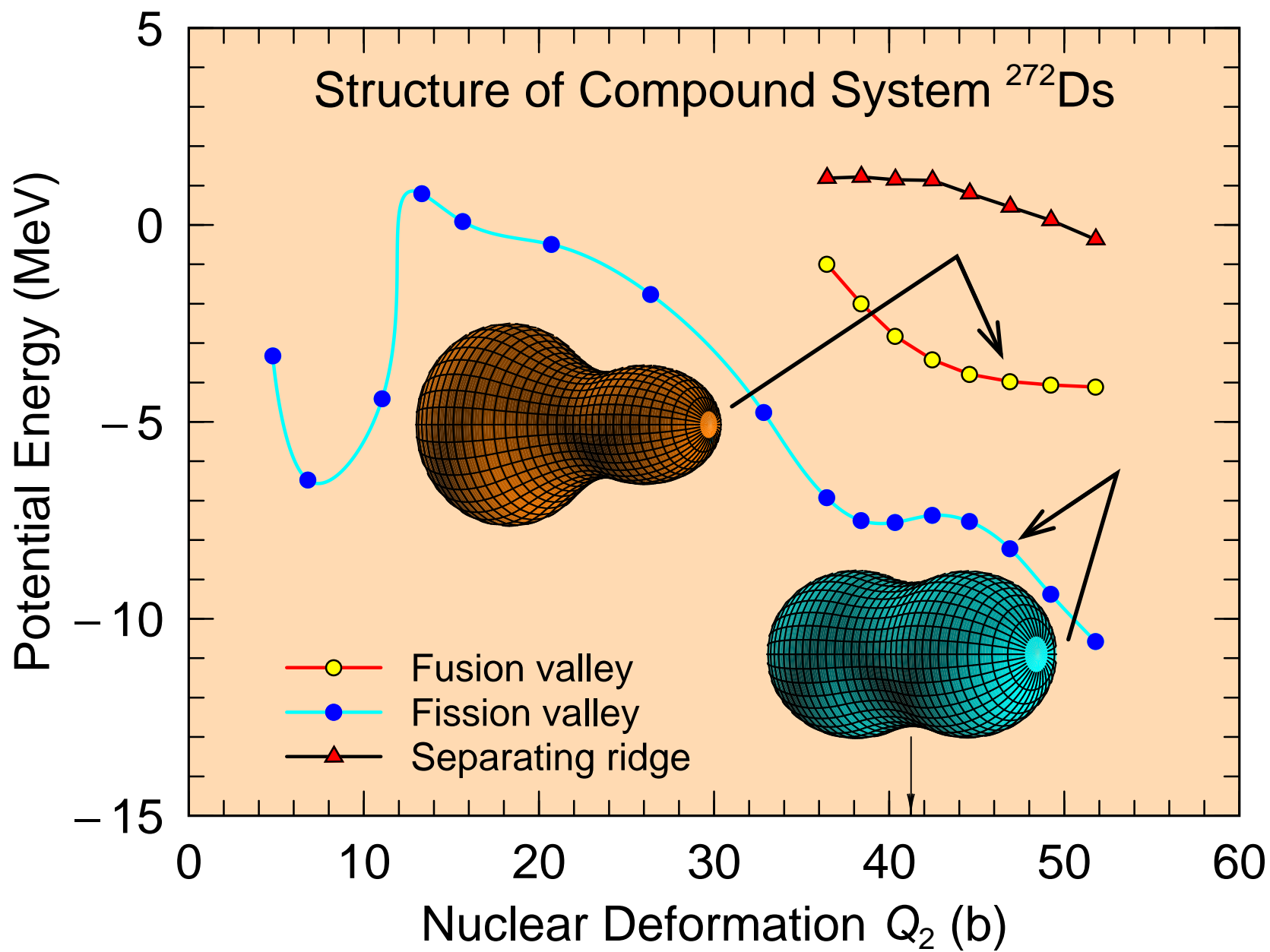
Figure 6

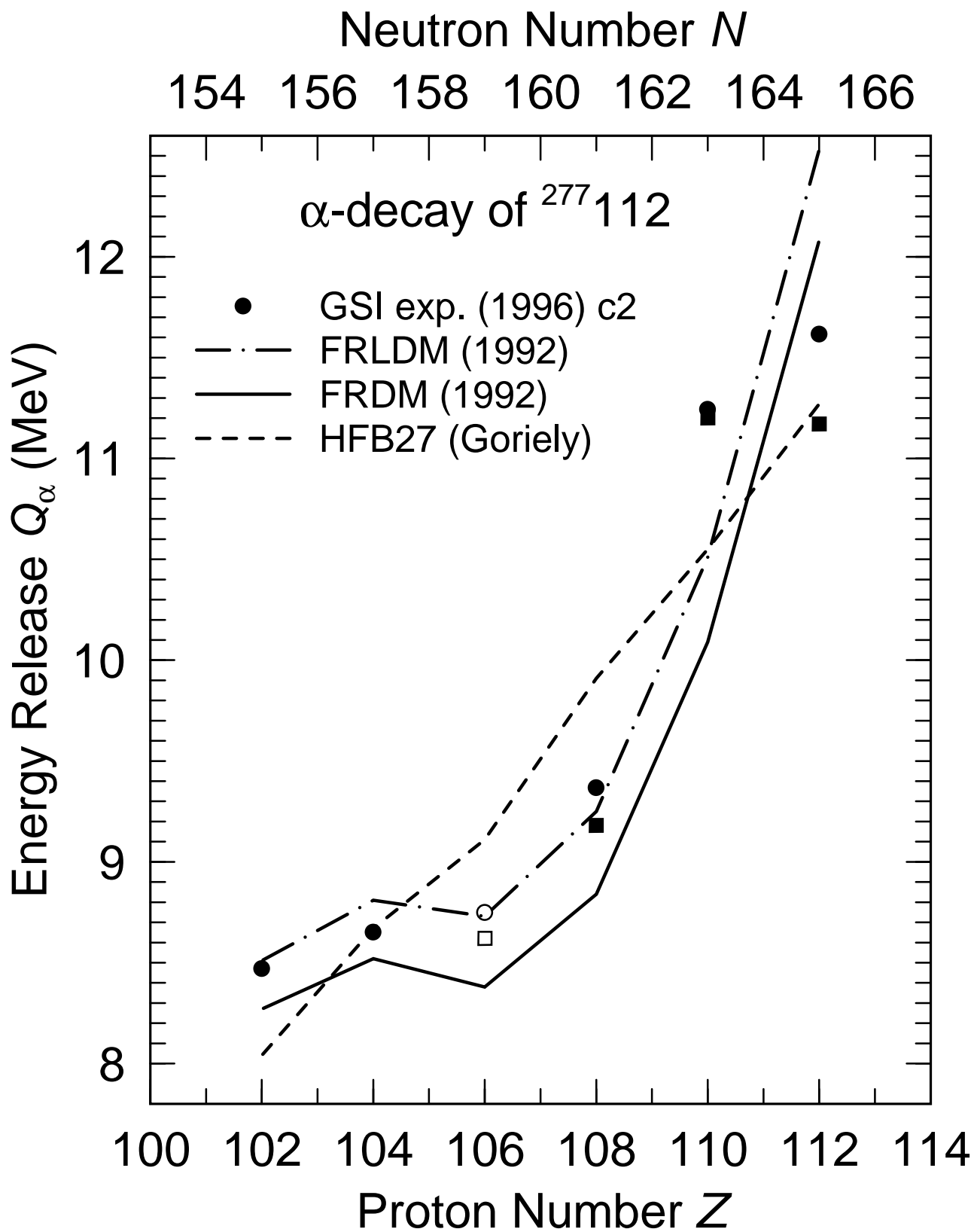


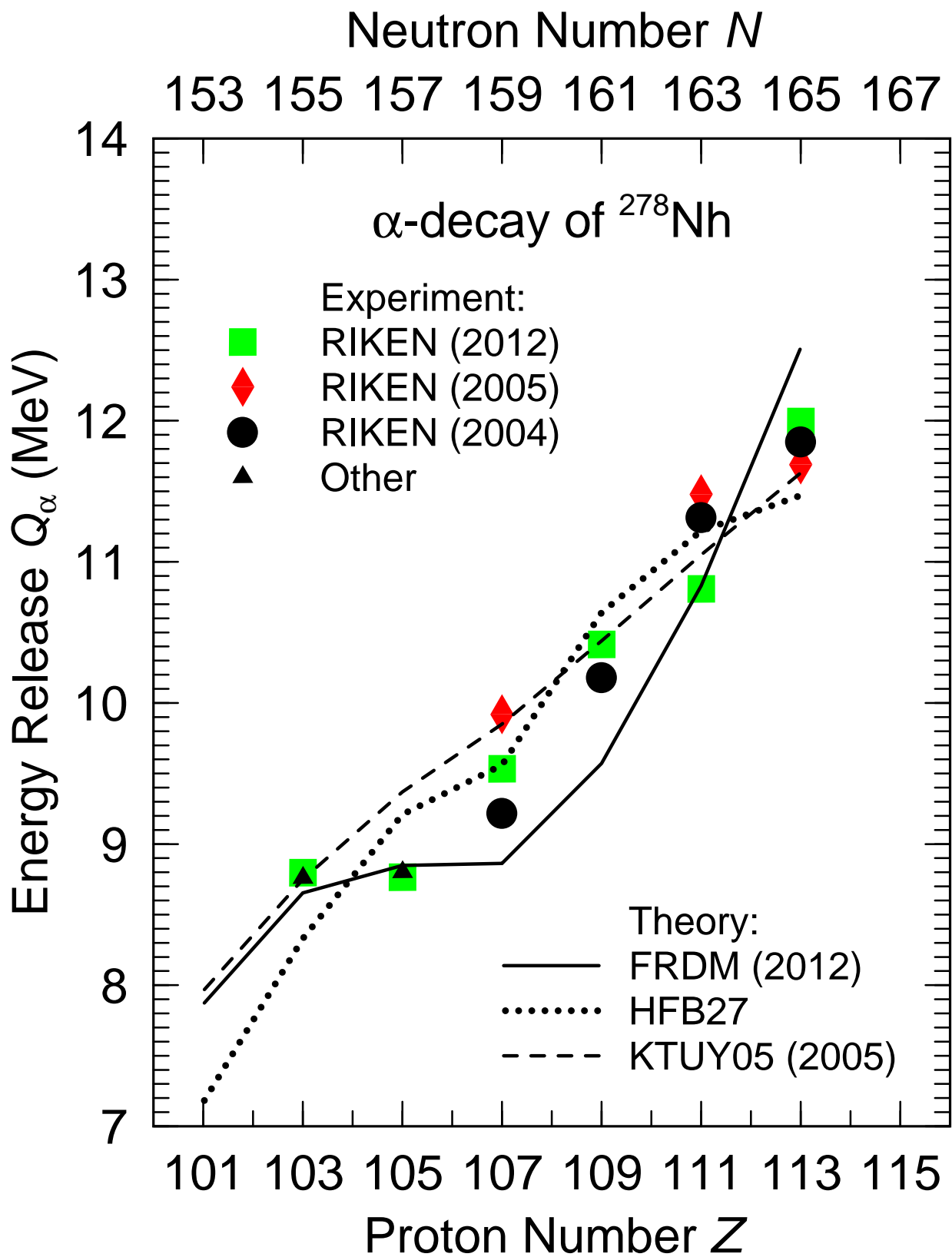
Shapes with large negative hexadecapole moments

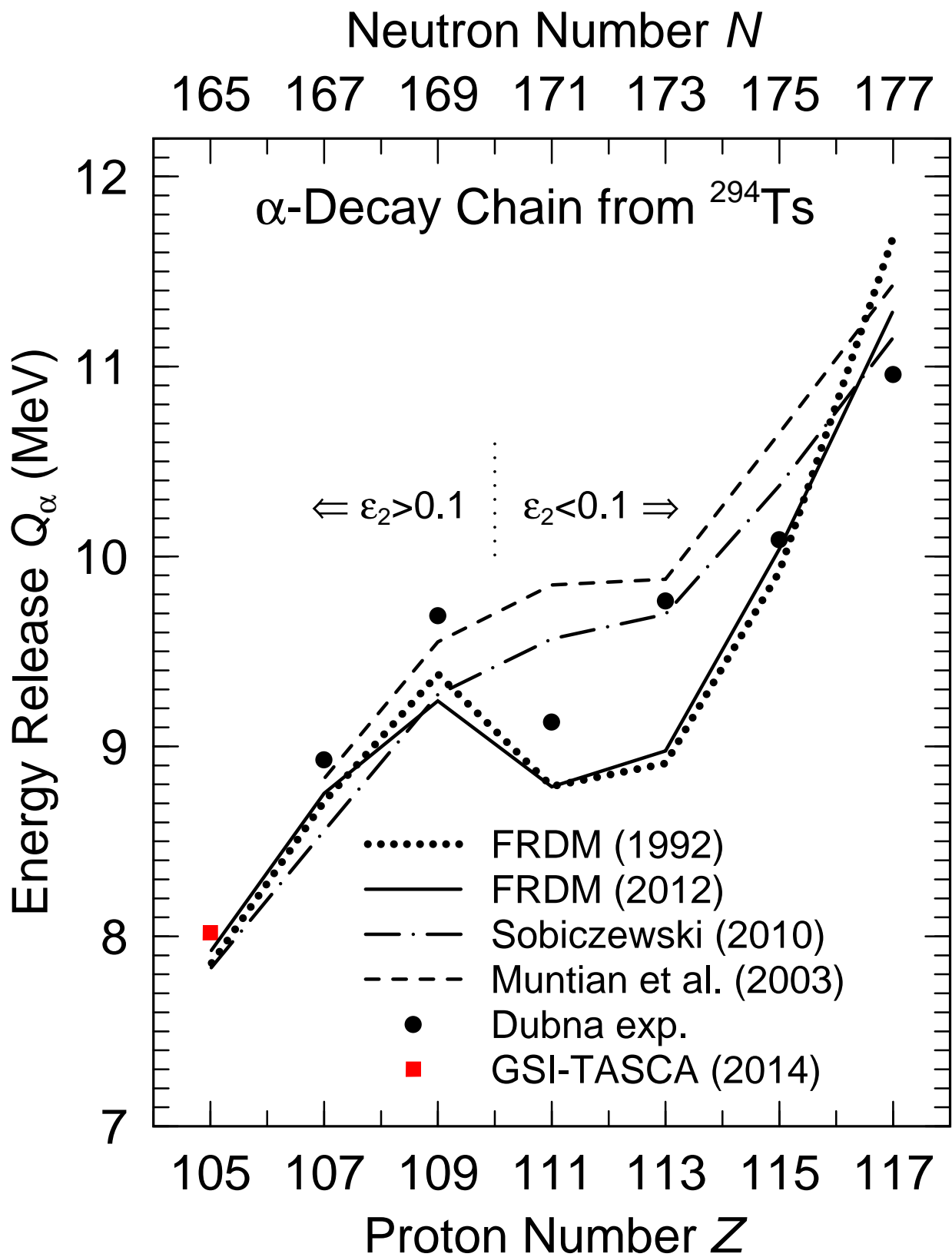


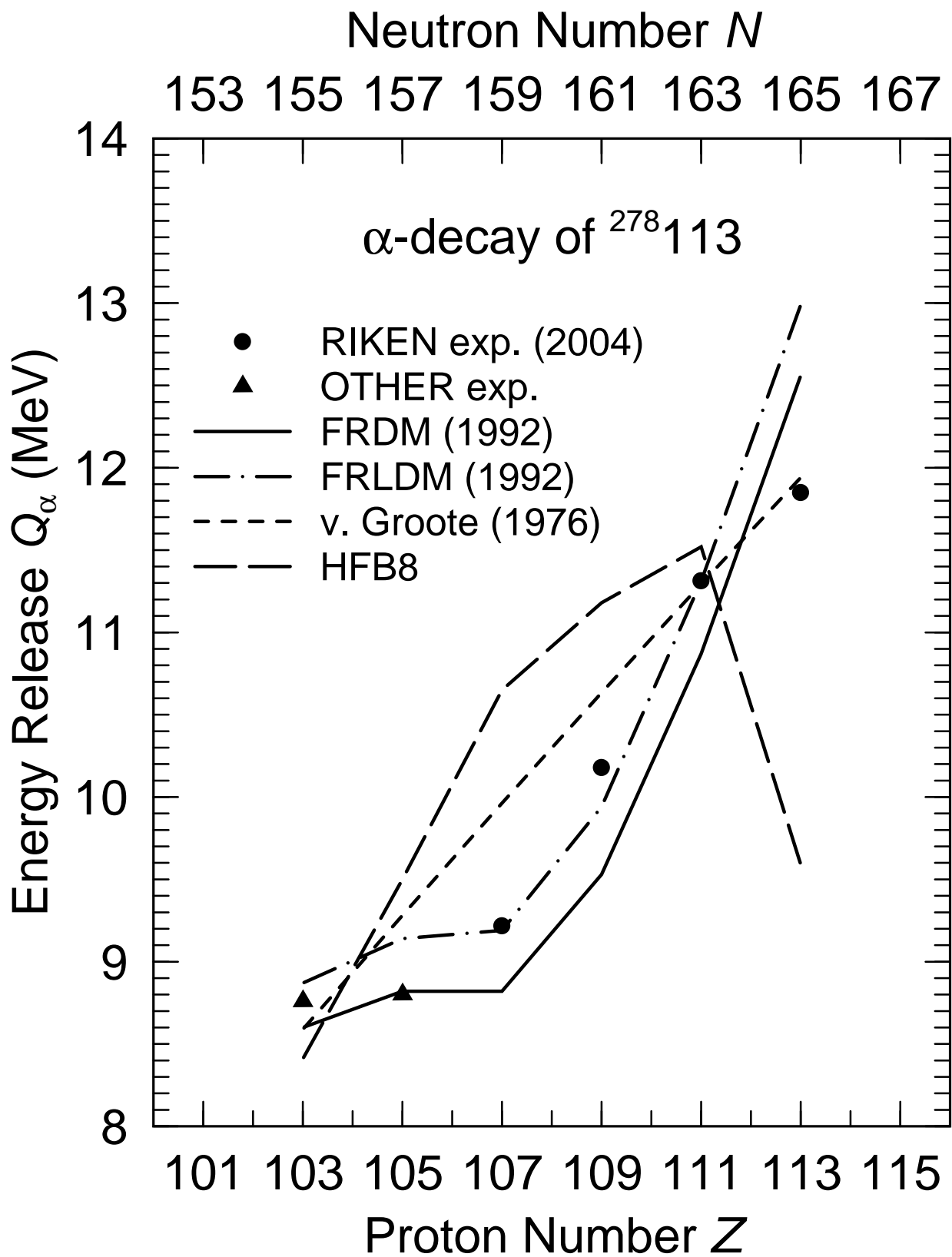






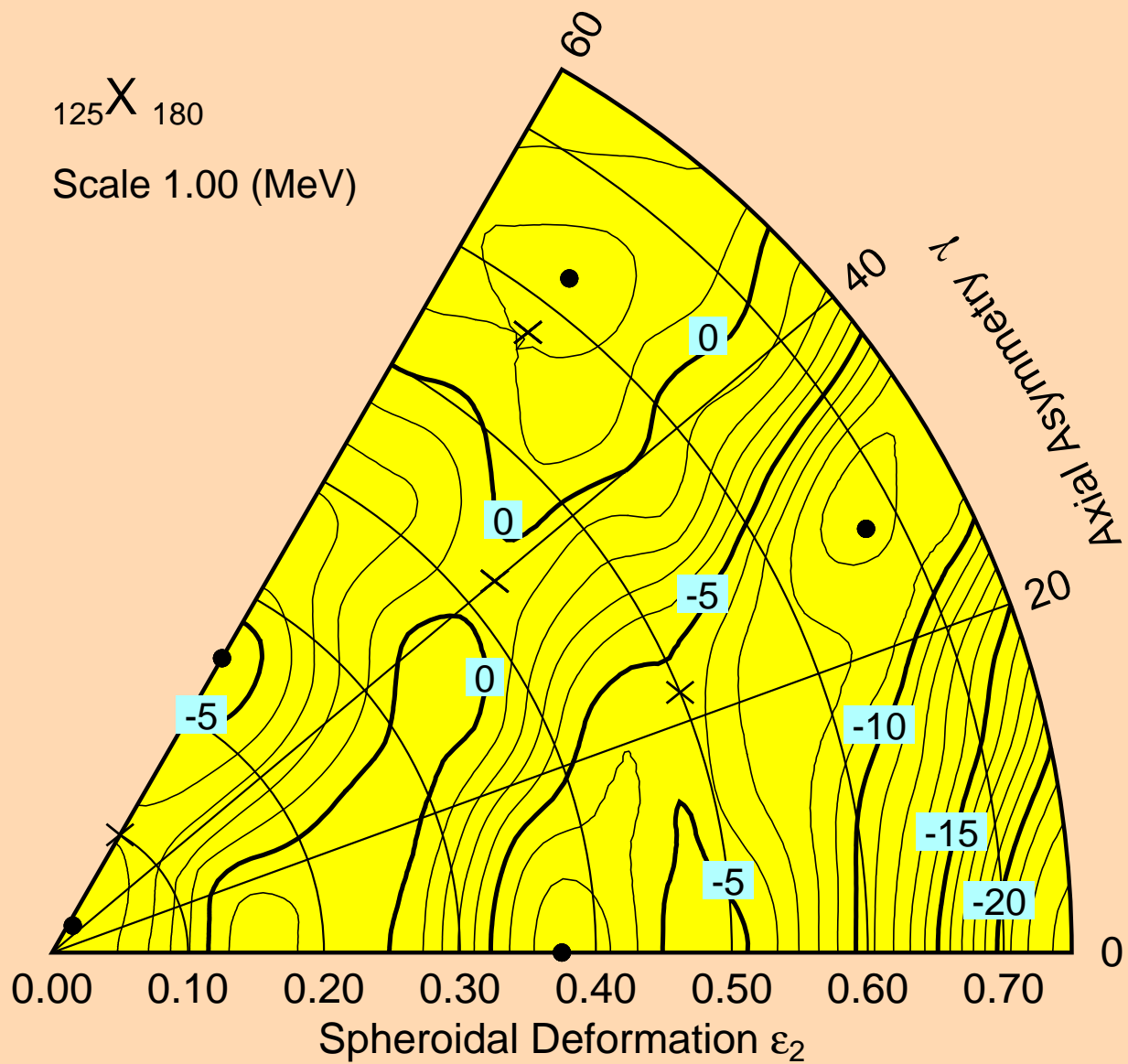






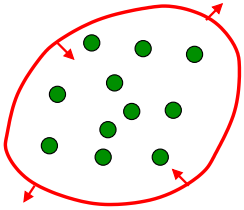
$^{125}\text{X}_{180}$

Scale 1.00 (MeV)



Q_α Deviations beyond $N = 126$

Region	Model	Nuclei	RMS (MeV)
$Z > 82$	SkM*	46	2.6
$Z > 82$	Sly4	46	2.6
$Z > 82$	HFB21	145	0.409
$Z > 82$	FRDM(1992)	145	0.463
$Z > 82$	FRDM(2012)	145	0.326
$Z > 88$	SkM*	36	1.7
$Z > 88$	Sly4	36	2.2
$Z > 88$	HFB21	101	0.367
$Z > 88$	FRDM(1992)	101	0.448
$Z > 88$	FRDM(2012)	101	0.274



Brownian shape motion

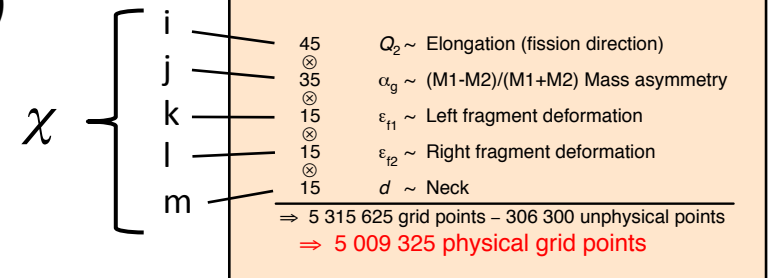
Nuclear deformation energy: $E_{\text{def}}(i,j,k,l,m)$

Bias potential: $V_{\text{bias}}(i) = V_0 (Q_0/Q_2)^2$

Level density parameter: $a_A = A/(8 \text{ MeV})$

Temperature T : $E^* - E_{\text{def}} = a_A T^2$

$$\Rightarrow V(\chi) = E_{\text{def}} + V_{\text{bias}}$$



P. Möller *et al*, Nature 409 (2001) 785

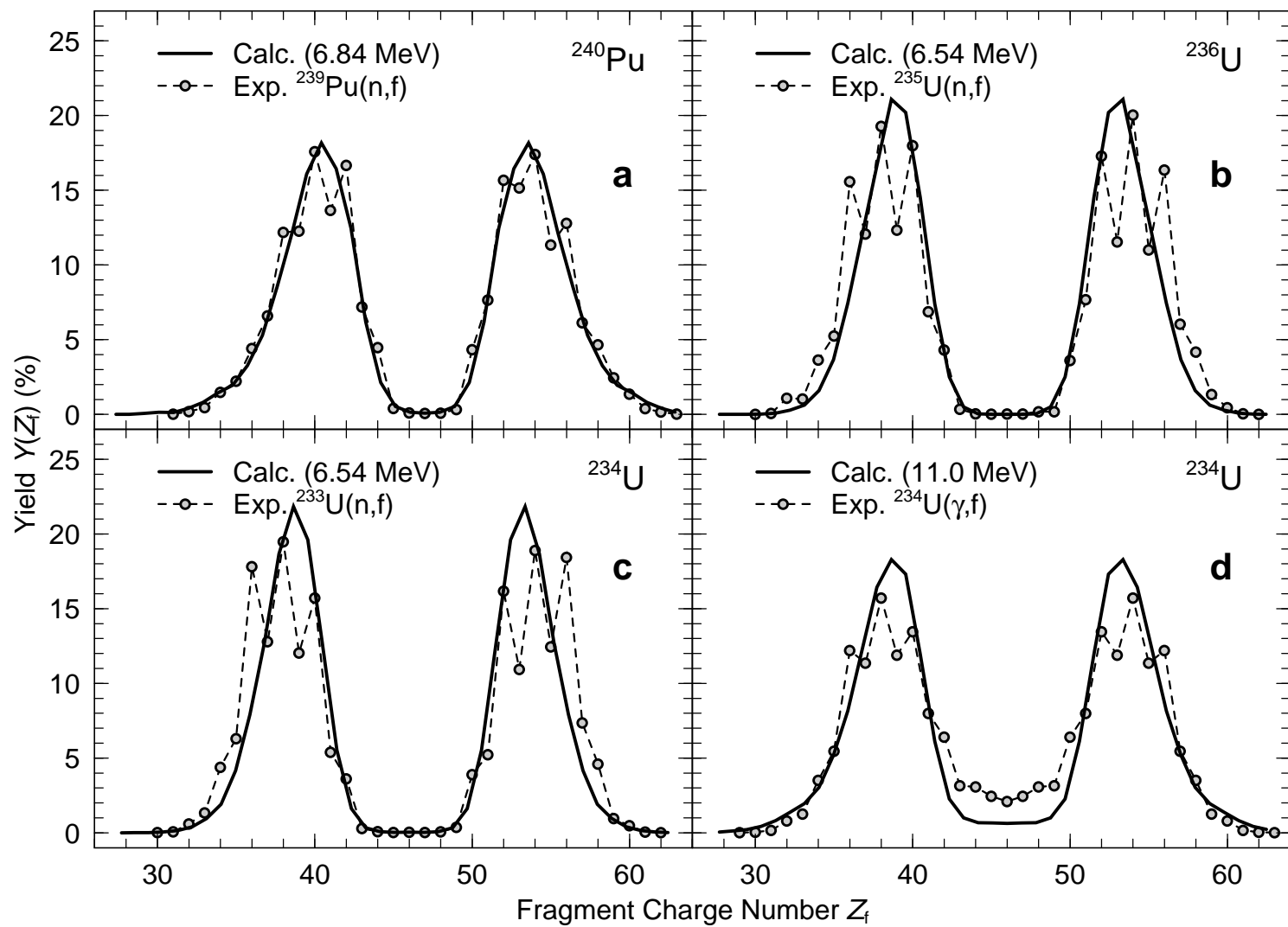
Metropolis walk:

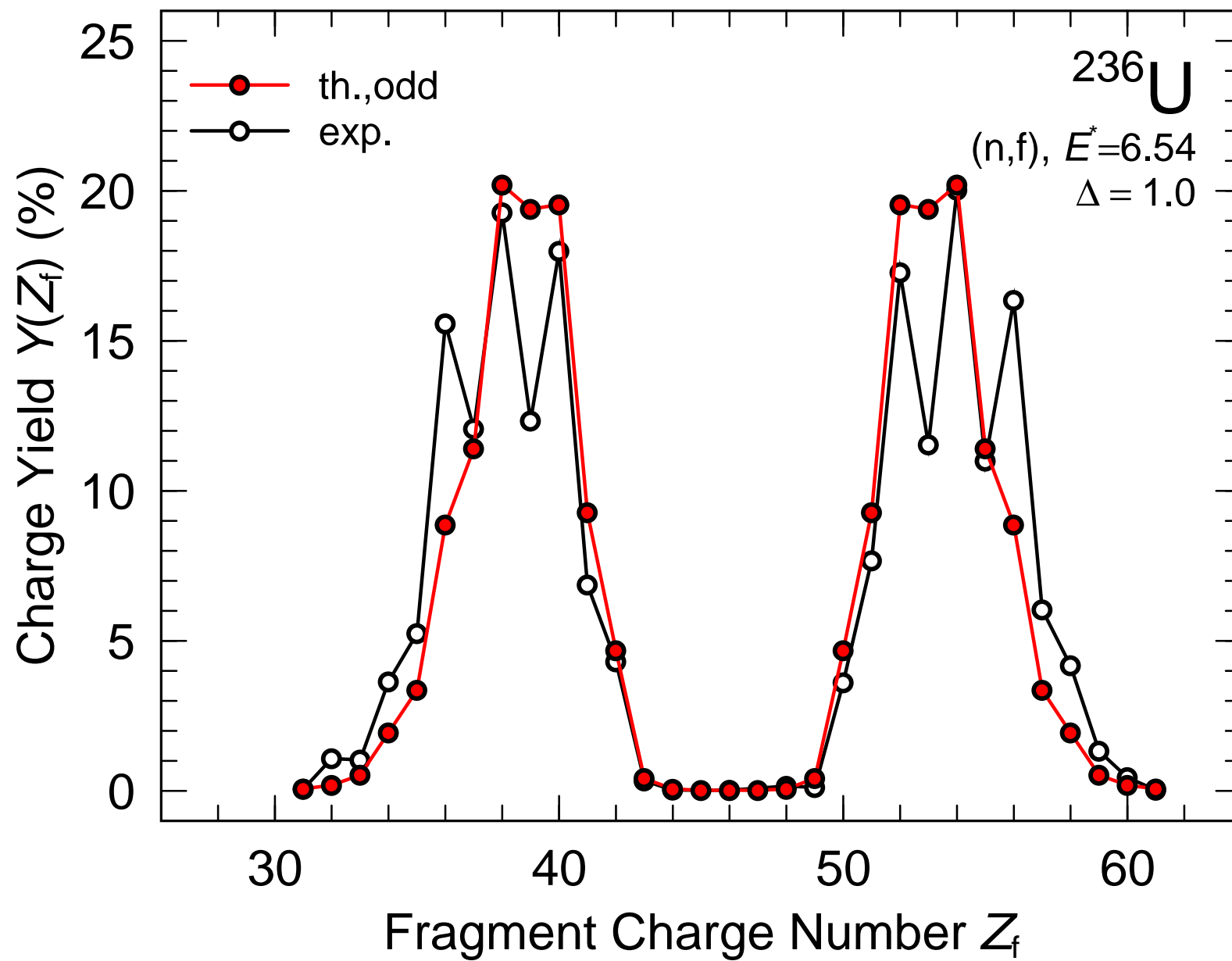
N. Metropolis *et al*, J Chem Phys 26 (1953) 1087

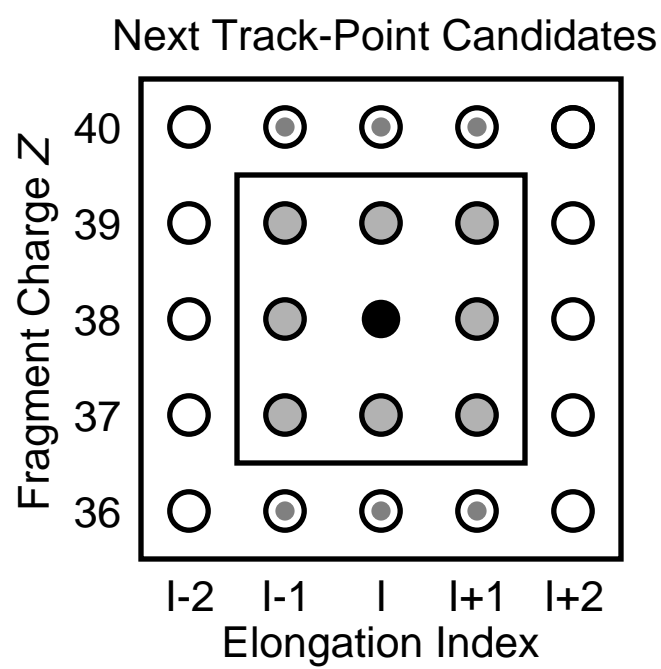
Change shape: $\chi \rightarrow \chi'$?

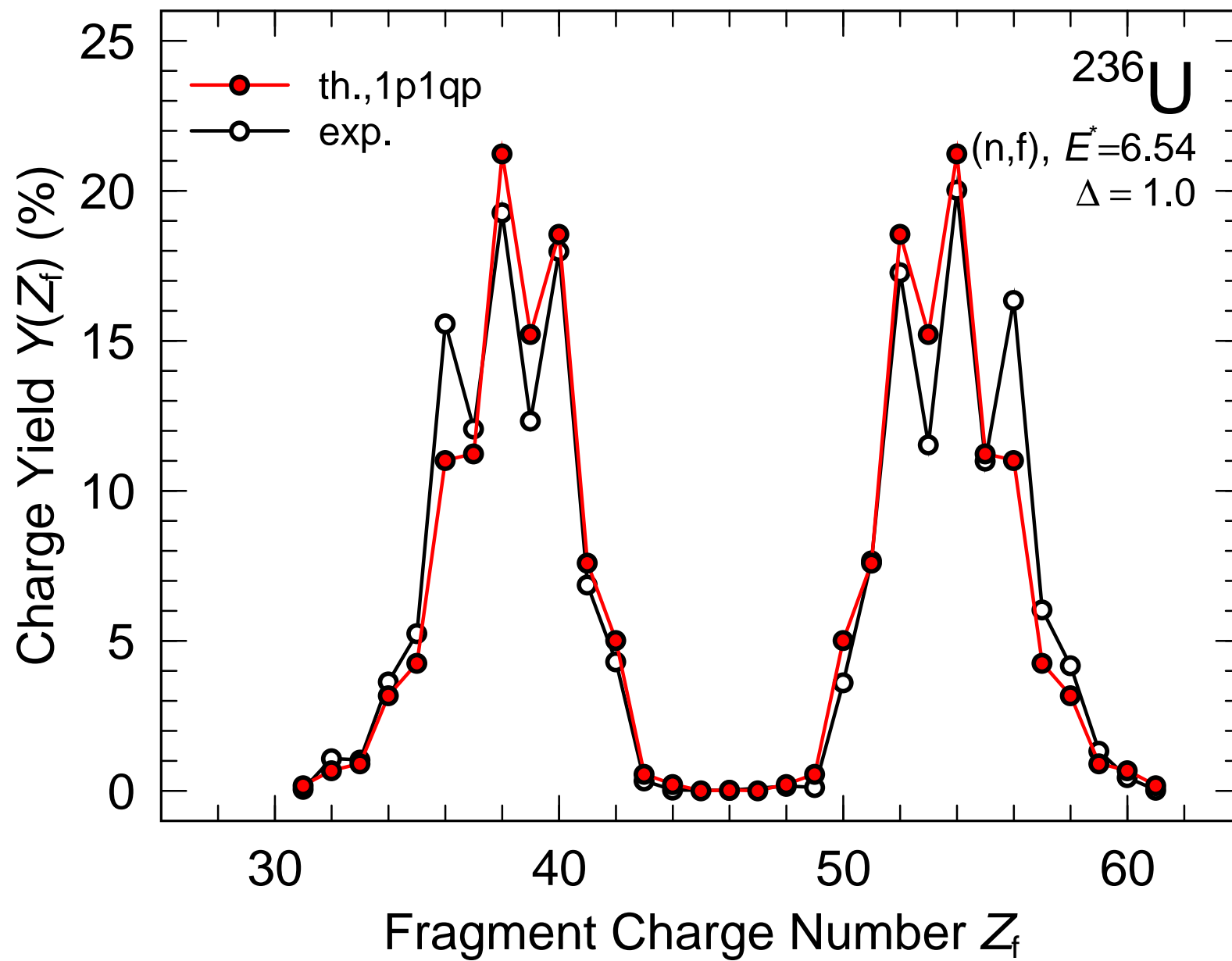
$$\begin{cases} V(\chi') < V(\chi): \text{ move with } P = 1 \\ V(\chi') > V(\chi): \text{ move with } P = \exp(-\Delta V/T) \end{cases}$$

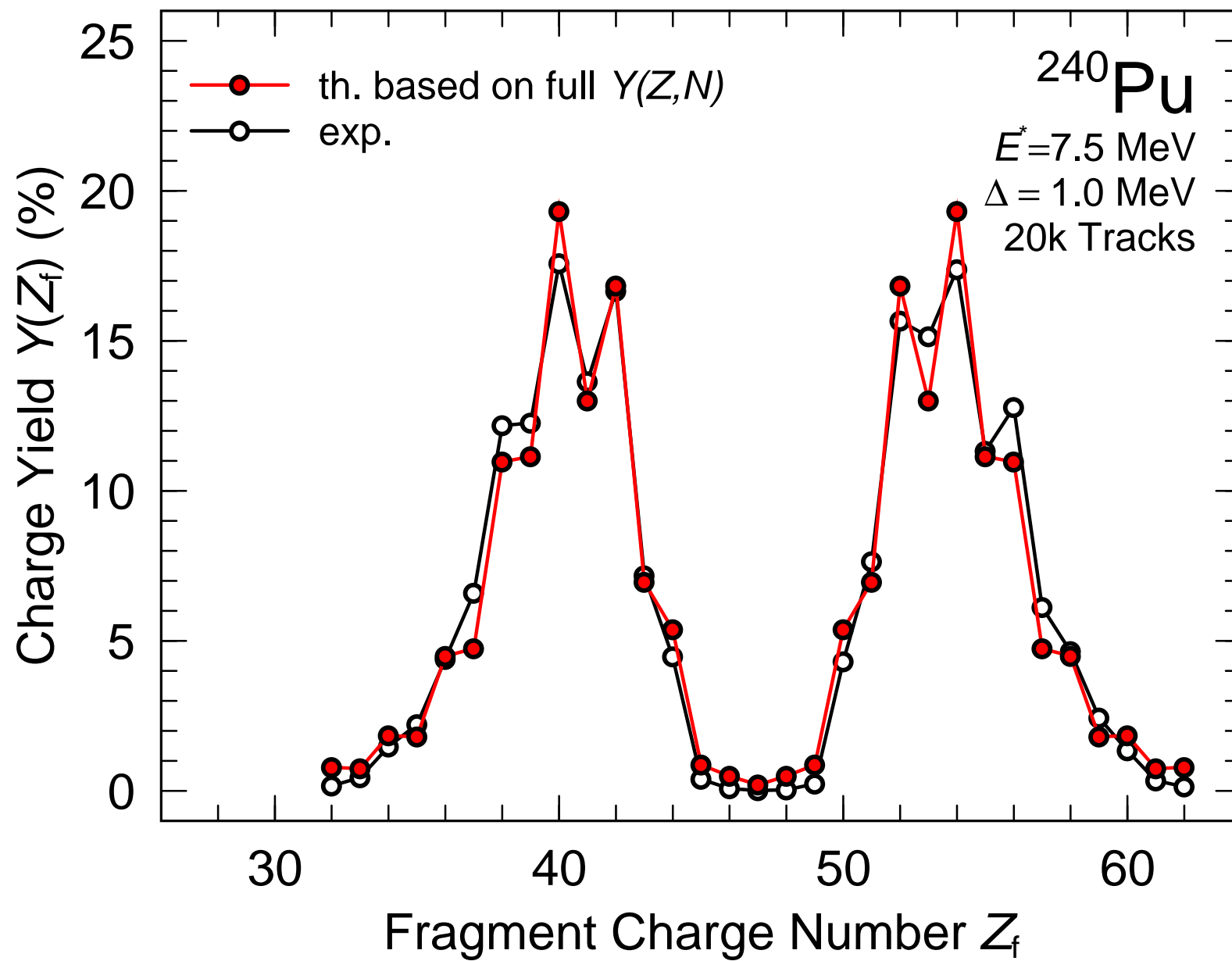
Scission: Critical neck radius $c_0 \approx 2.5 \text{ fm}$

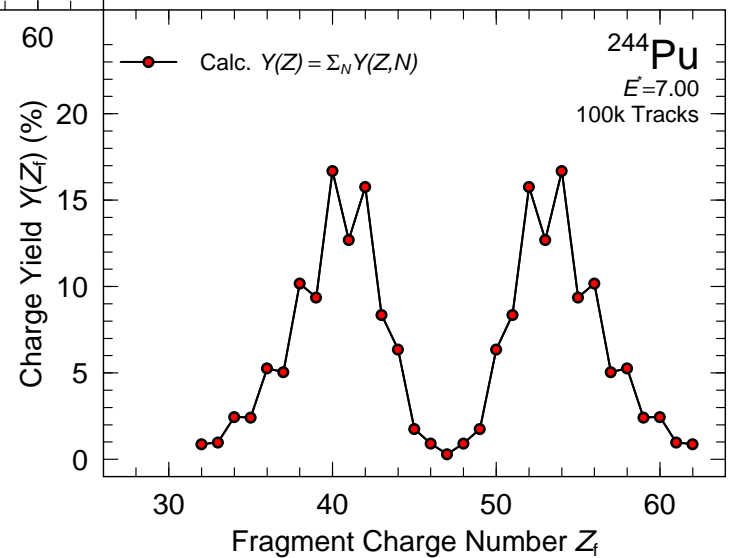
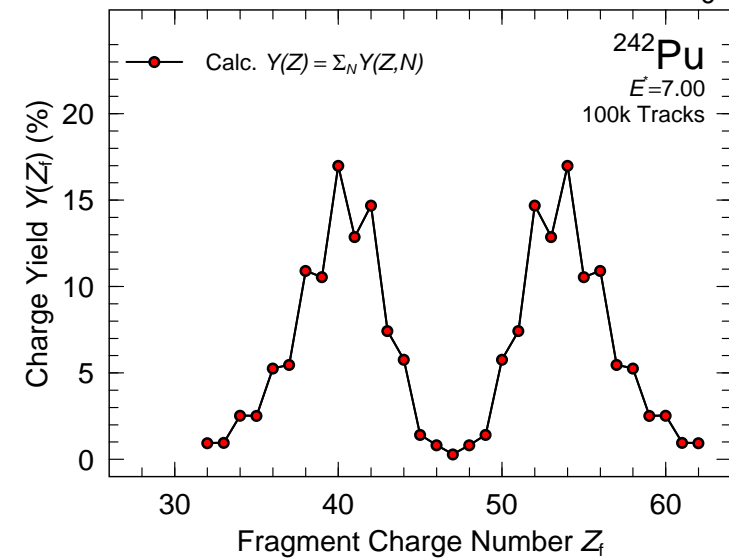
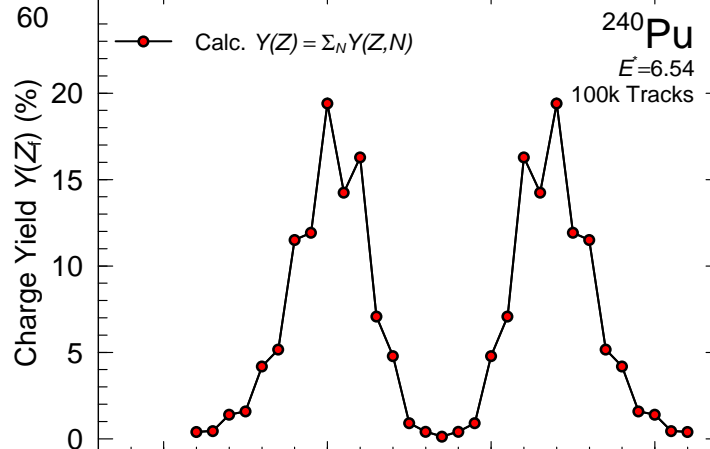
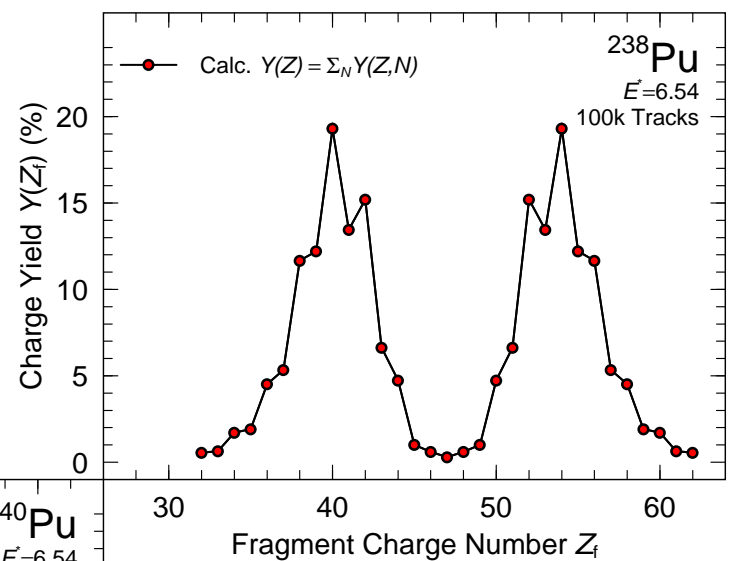
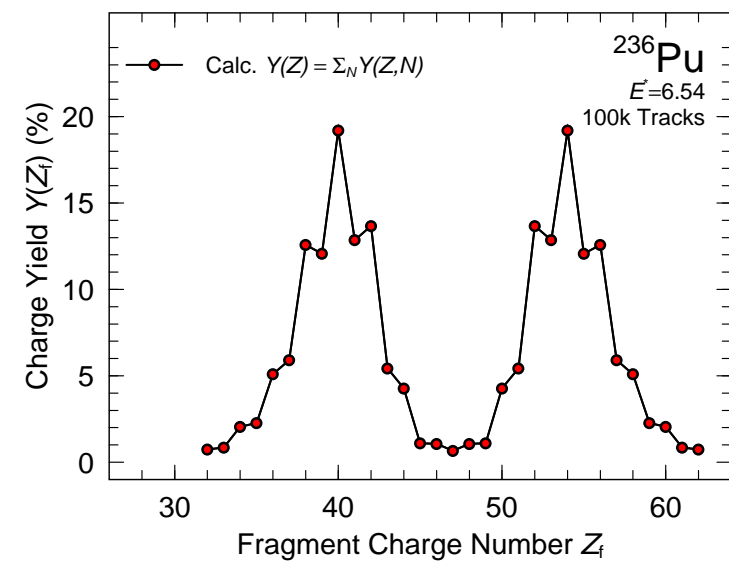


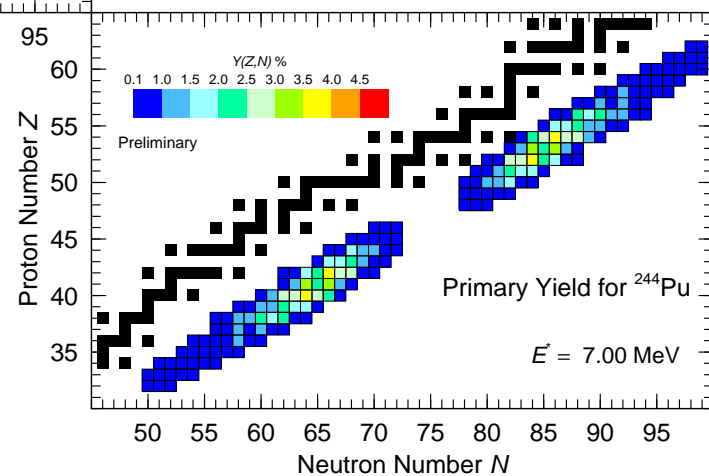
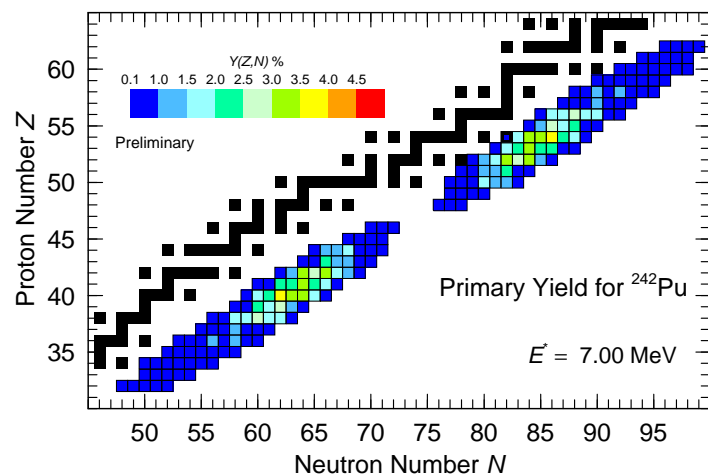
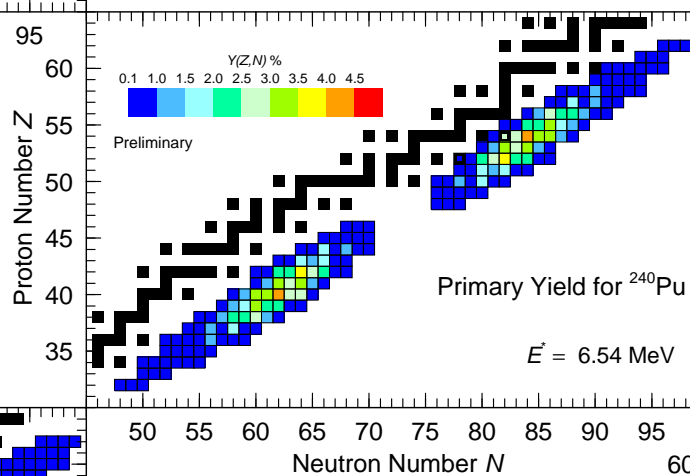
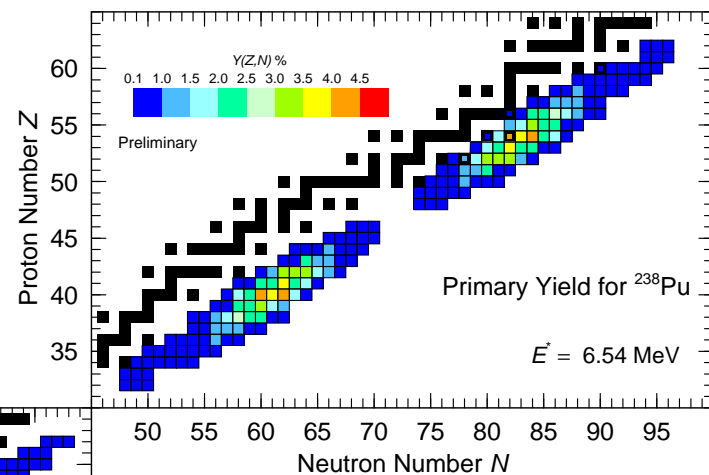
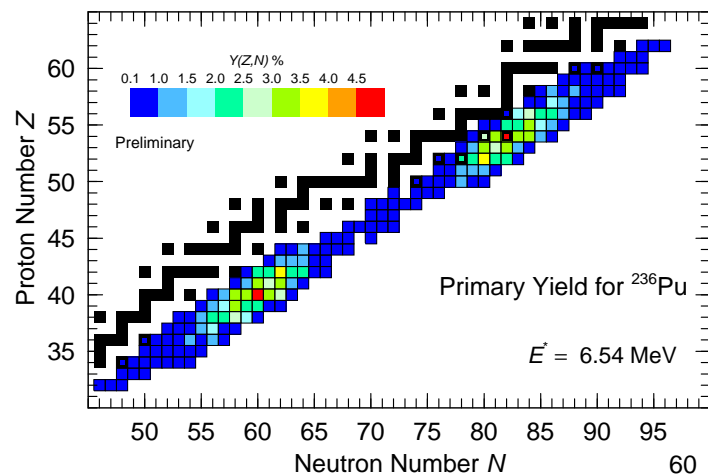


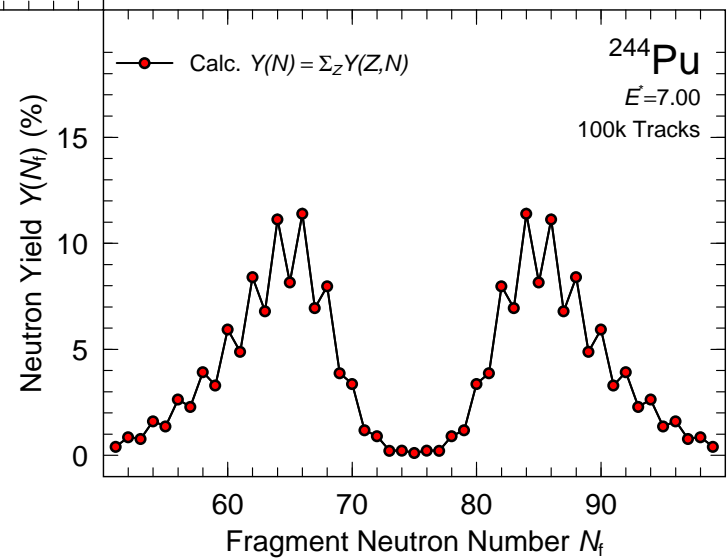
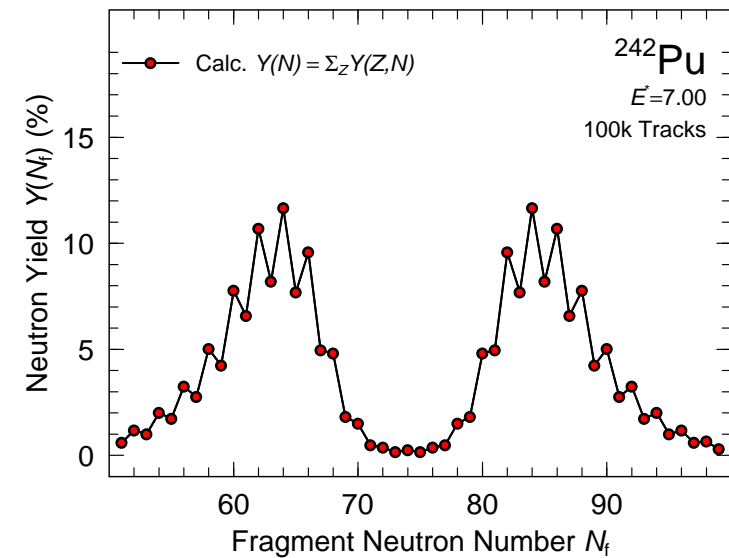
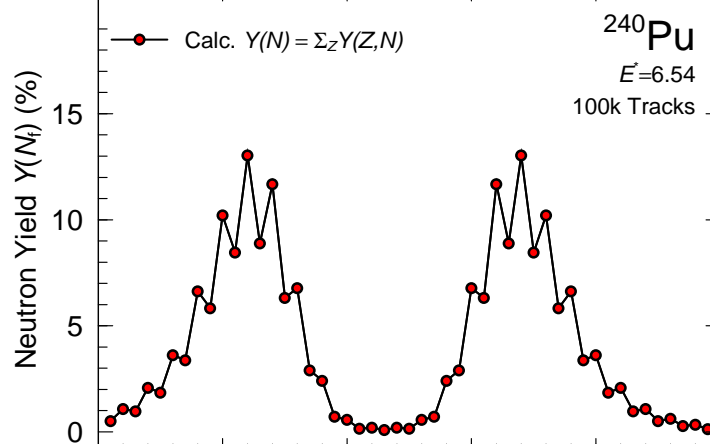
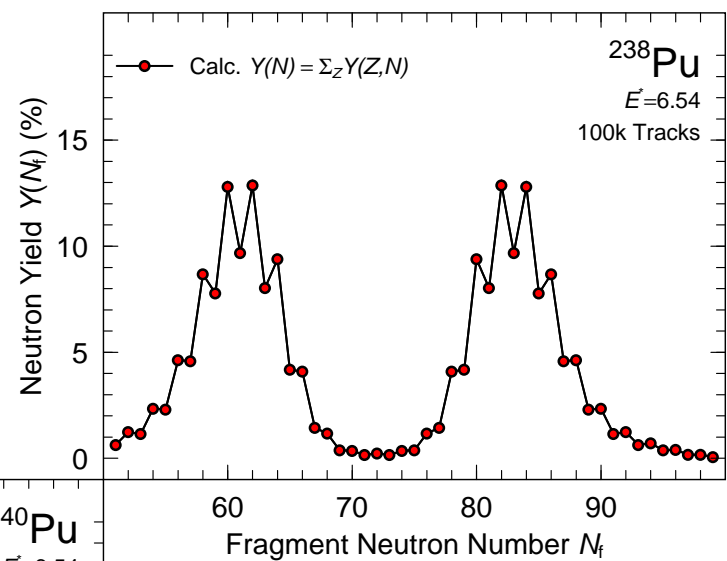
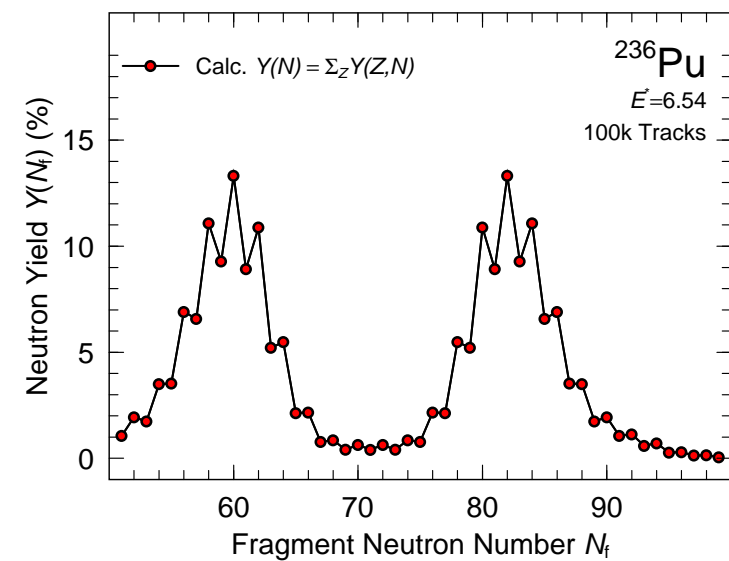




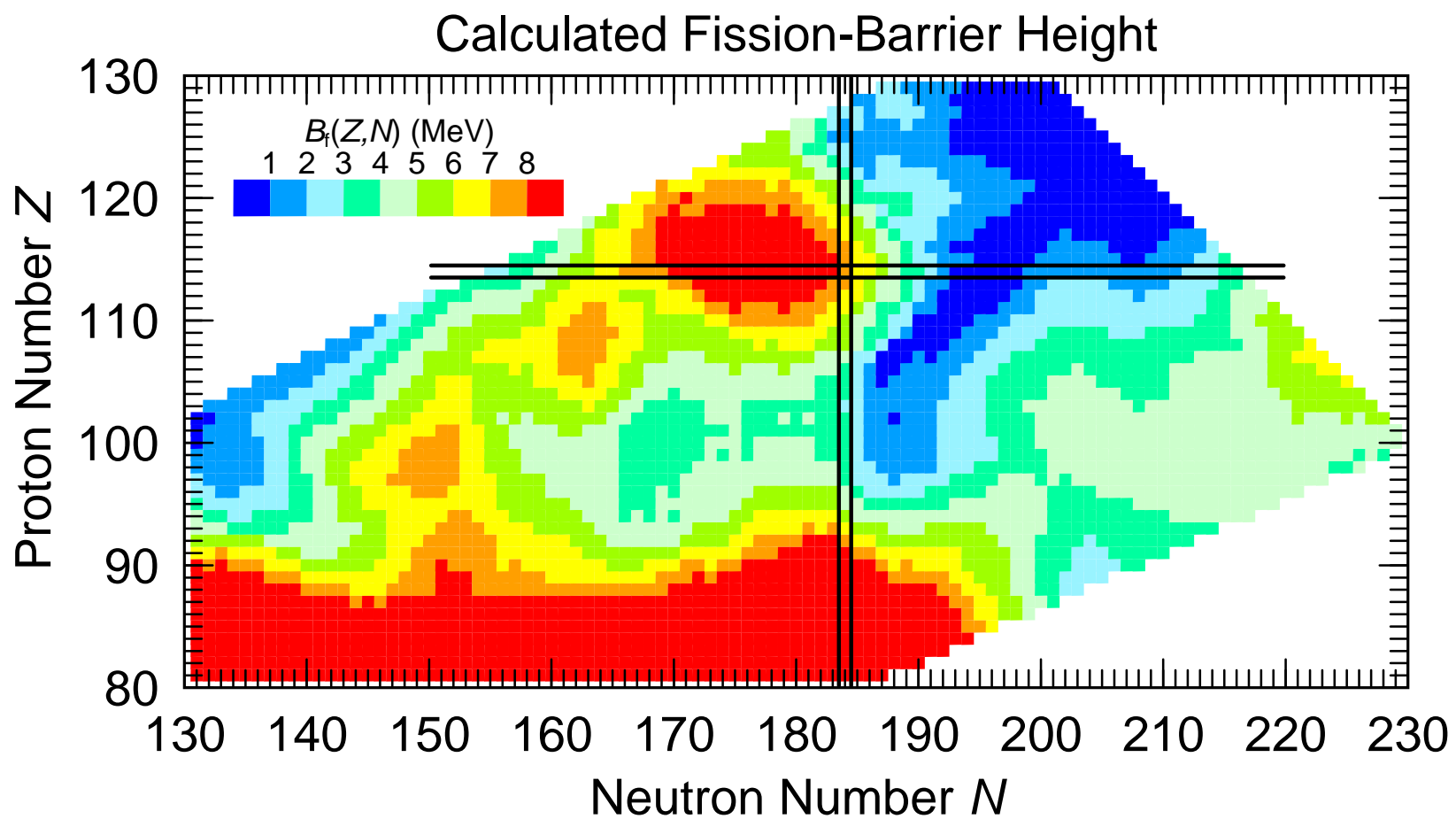








First in chain		fissioning nucl at chain end			TKE
Z	A	Z	A	N	
114	289	110	281	171	212
114	288	112	284	172	216
114	287	112	283	171	206
114	288	112	284	172	202
114	287	110	279	169	SF<10%
		108	275	167	SF>90%
		106	271	165	228
114	286	112	282	170	212 ~60%
116	293	110	281	171	197
116	292	112	284	172	190
116	291	110	279	169	no data
		104	267	163	240
116	290	114	286	172	?
		112	282	170	209
112	283	110	279	169	185,194,1
116	290	112	282	170	209
118	294	112	282	170	202
113	282	104	266	162	203
115	288	104	268	164	203
115	287	105	267	162	206



C O N C L U S I O N S

- All global nuclear-structure models are simple representations of nuclear properties. To expect infinite accuracy with global models is unrealistic.
- Both Wood-Saxon and folded-Yukawa based models give properties of SHE elements to useful accuracy.
- Remaining differences between these models and between the models and experiment must be considered unavoidable model uncertainties.
- Obviously less deviations can be achieved by local adjustments of parameters, but for those of us who strive to improve global model accuracy, this would be a null results.
- Most HFB models have poor results for known nuclei, therefore their predictions in the SHE region are irrelevant.
- For heavy systems it is not the lowest minimum that is the most stable, it is the minimum with the highest fission barrier, a fact usually ignored in HFB calculations.

F U T U R E

- Study additional isotopes to establish Z , N of maximum stability.
- More events to obtain fission-fragment mass distributions and TKE distributions.
- Investigate additional projectile-target possibilities (hugging, transfer, ...)