# Super-heavy nuclei: Production and decay properties

#### Peter Möller

P. Moller Scientific Computing and Graphics, Inc.
P. O. Box 1440 Los Alamos NM 87544
PRESENTATION ECT TRENTO,
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Collaborators on this and other projects, see coauthors on papers posted on URL below.

More details about masses, other projects (beta-decay, fission), associated ASCII data files and figures are at

http://t2.lanl.gov/nis/molleretal/

# **Nuclear POTENTIAL ENERGY BW (1939)**

$$B(N,Z)=$$
 $+a_{V}A$  (Volume energy)
 $-a_{S}A^{2/3}B_{s}(\beta)$  (Surface energy)
 $-a_{C}\frac{Z^{2}}{A^{1/3}}B_{C}(\beta)$  (Coulomb energy)
 $-a_{I}\frac{(N-Z)^{2}}{A}$  (Symmetry energy)
 $-\delta(A)$  (Pairing energy)

## **Nuclear Deformation Energy**

Let the nuclear surface be described by

$$r(\theta, \phi) = R_0 \left[ 1 + \alpha_2 P_2(\cos \theta) \right]$$

The surface energy lowest order Taylor expansion:

$$E_{\rm s} = E_{\rm s}^0 (1 + \frac{2}{5}\alpha_2^2)$$

The Coulomb energy lowest order Taylor expansion

$$E_{\rm C} = E_{\rm C}^0 (1 - \frac{1}{5} \alpha_2^2)$$

The energy at deformation  $\alpha_2$  relative to spherical shape

$$E_{\text{def}}(\alpha_2) = E_{\text{C}}(\alpha_2) + E_{\text{s}}(\alpha_2) - (E_{\text{C}}^0 + E_{\text{s}}^0)$$

If  $E_{\mathrm{def}}$  is negative then the system has no barrier wrt fission

$$E_{\text{def}}(\alpha_2) = \frac{2}{5}\alpha_2^2 E_{\text{s}}^0 - \frac{1}{5}\alpha_2^2 E_{\text{C}}^0 < 0$$
$$1 < \frac{E_{\text{C}}^0}{2E_{\text{s}}^0} = x$$

The surface energy for a sphere

$$E_{\rm s}^0 = 17.80A^{2/3}$$

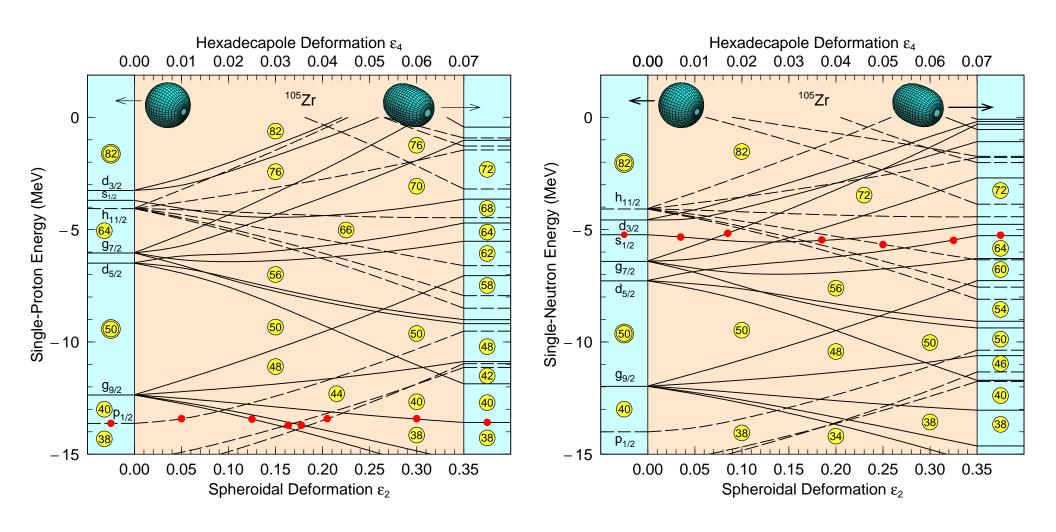
The Coulomb energy for a sphere

$$E_{\rm C}^0 = 0.7103 \frac{Z^2}{A^{1/3}}$$

The fissility parameter x:

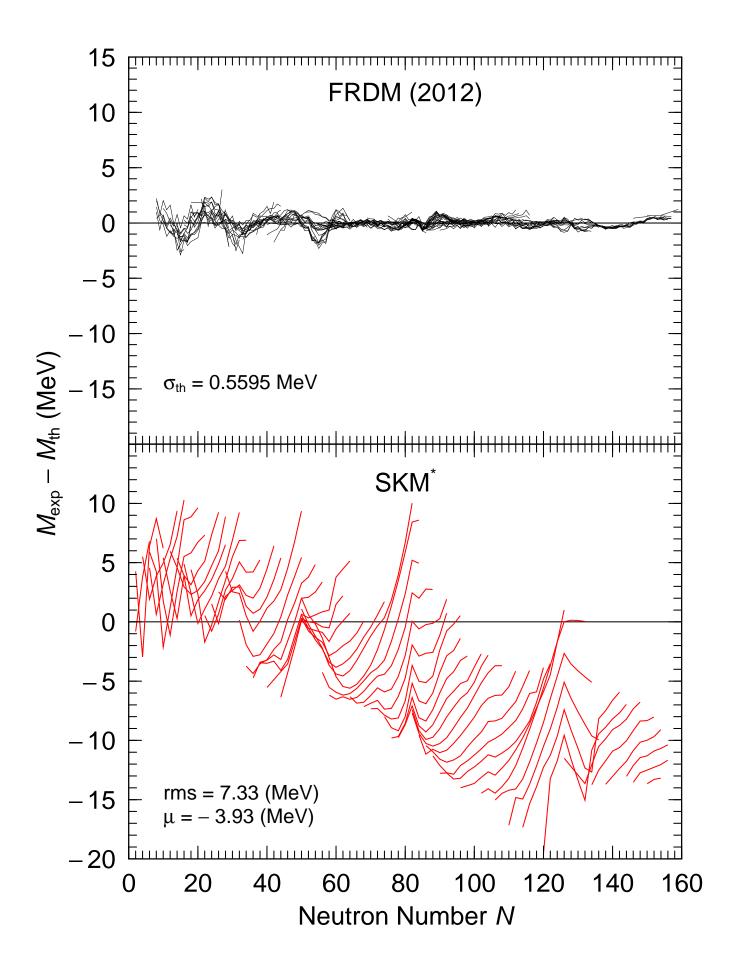
$$x = \frac{Z^2}{50.13A}$$

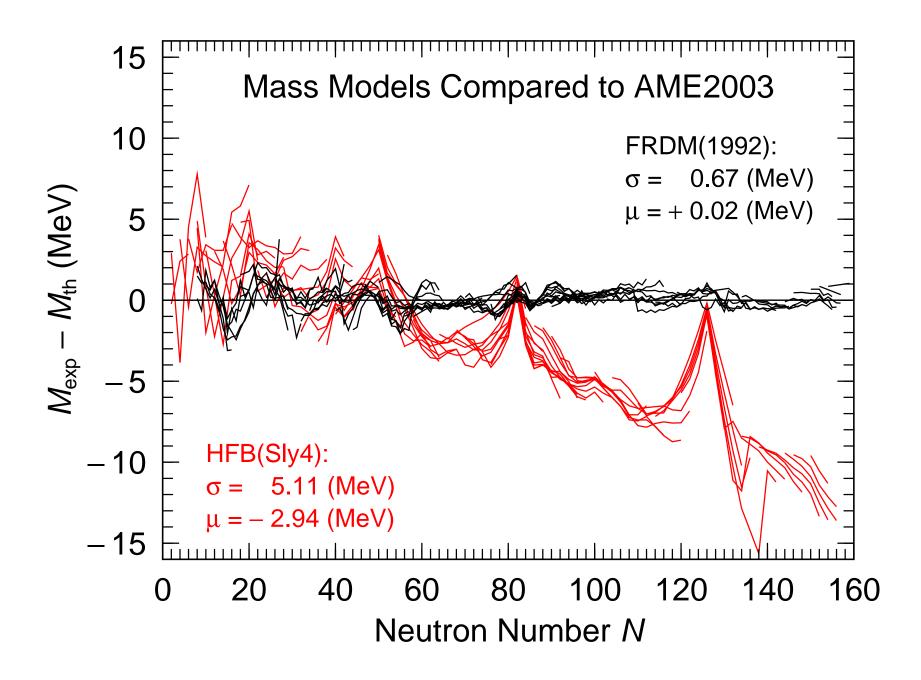
x	A	Z
0.402	124	50
0.645	208	82
0.709	138	92
0.792	252	100
0.870	298	114
0.950	328	125
1.006	335	130

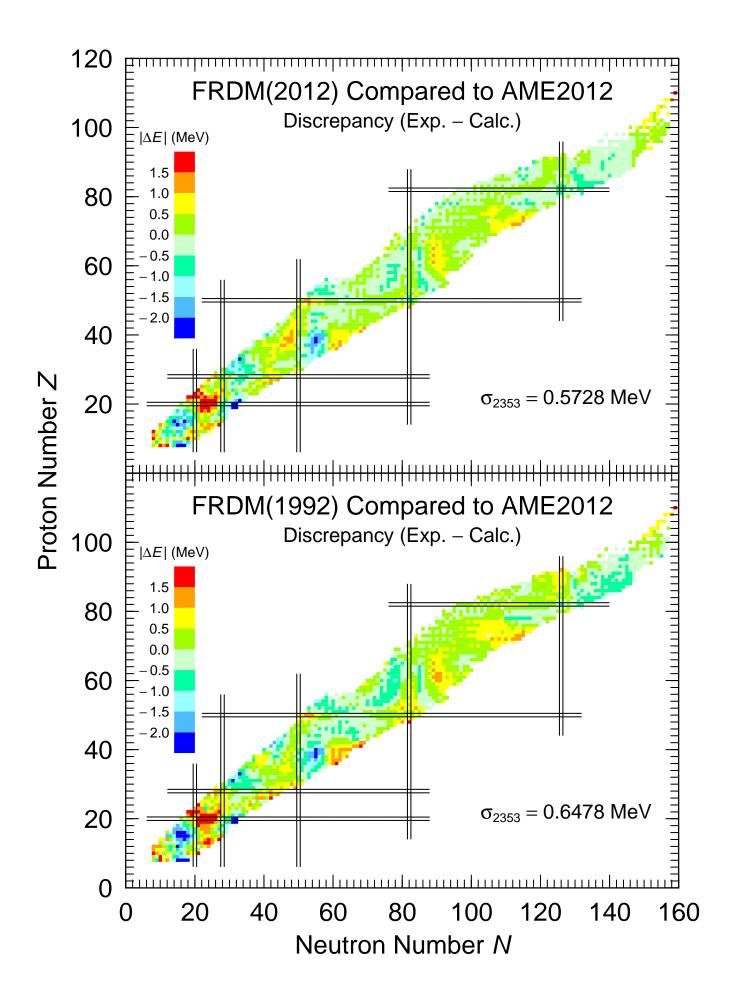


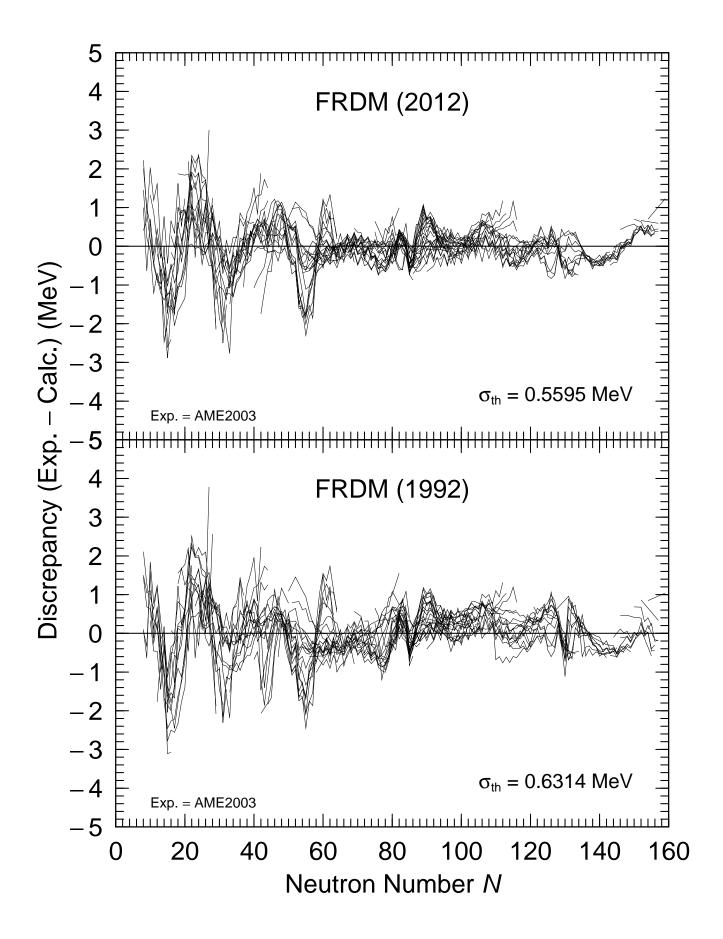
### Feynman:

- I do not care how smart you are
- or how complicated your model is
- If it does not agree with experimental measurements it is wrong!

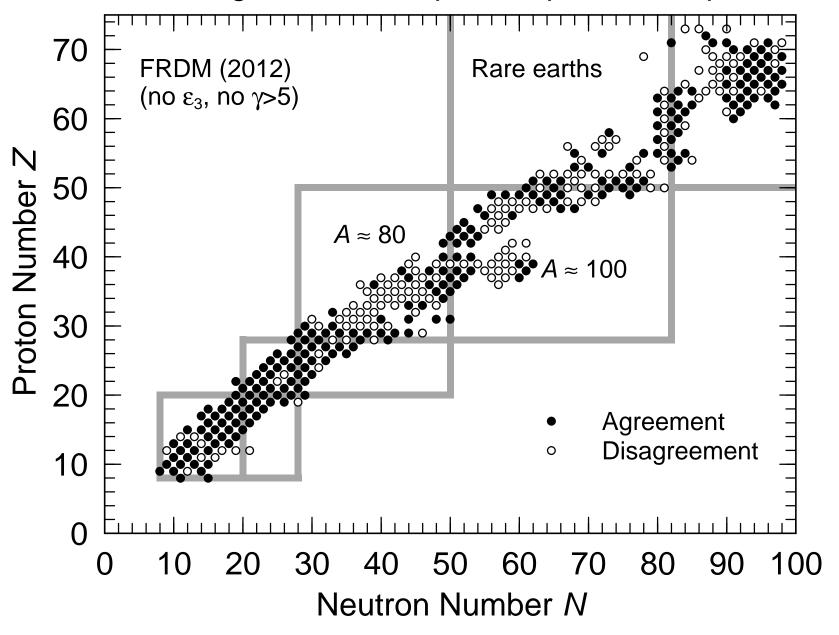




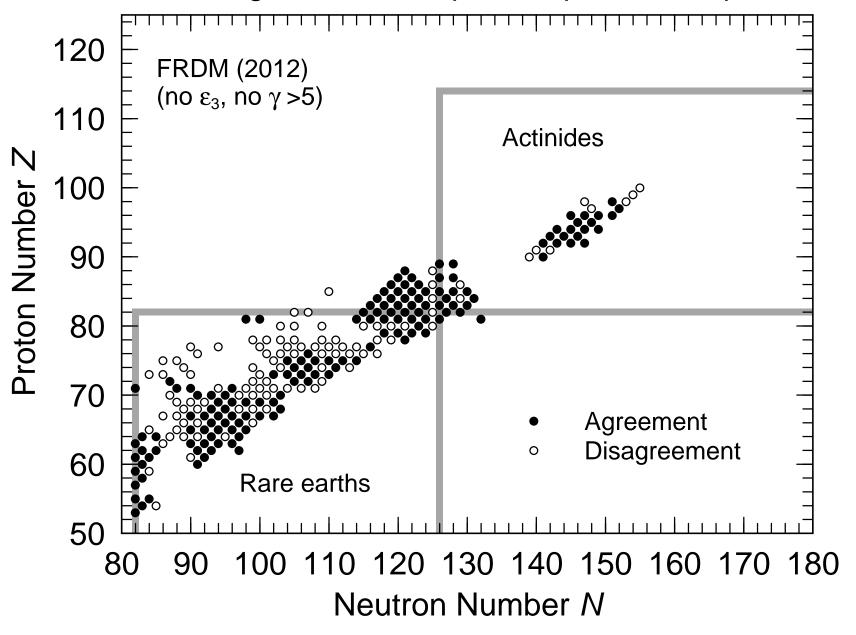


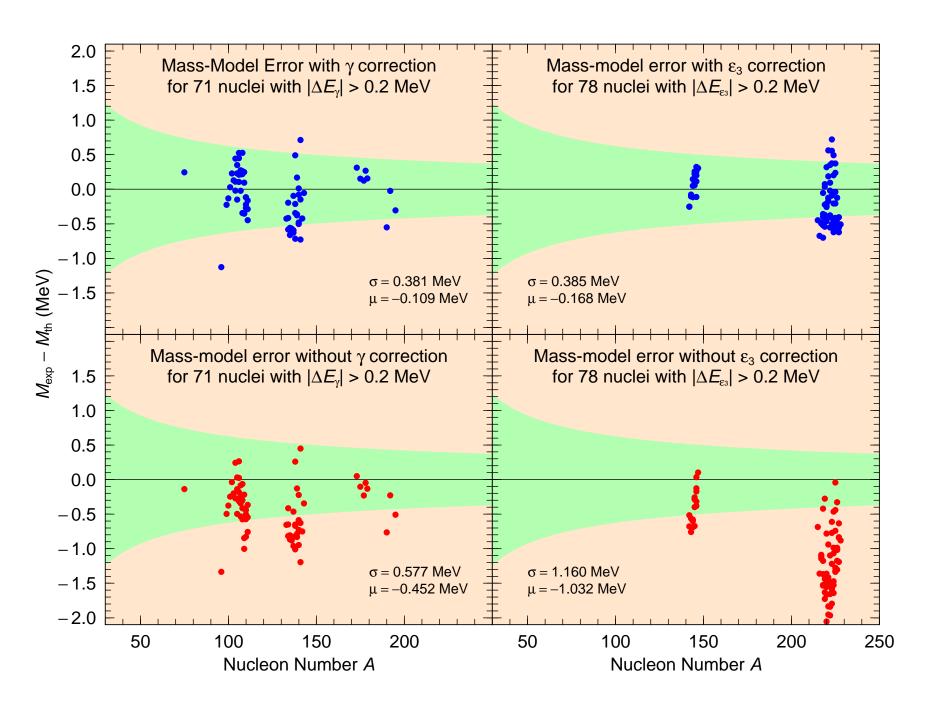


## Calculated ground-state spin compared to experiment



## Calculated ground-state spin compared to experiment





# nuclear physics

By GERTRUDE SCHARFF-GOLDHABER

EVEN A SUPERFICIAL GLANCE backward will teach us that it is impossible to predict in detail the future of a fast-moving science like nuclear physics. It will remind us that often entirely unexpected events changed the direction of endeavor in this field. These events were either of an experimental nature, as, for instance, the finding that beta rays have continuous energy spectra, or they consisted in the formation of new concepts—as, for example, of the liquid-drop model of the nucleus.

It is, of course, similarly impossible to predict what extraneous happenings may in the future affect scientific progress as profoundly as two world wars and political persecution have affected it in the past. During the last twelve years the great importance attached to atomic energy has induced an unprecedented increase in the tempo of research, and new nuclear physics centers have sprung up all over the world. This development is viewed by many with delight while others are afraid that it may have a negative effect on nuclear physics as a pure science.

In spite of all the uncertainties mentioned, it is useful to interrupt from time to time one's preoccupation with the problem at hand to investigate the trends that current research seems to follow, both in experiment and theory, and to try to recognize how far these may serve to bring us closer to the solution of outstanding problems.

The central problem is to understand the nucleus in the same sense in which one might have said in 1926 that the atom was understood: one knew then not only that the forces between the nucleus and the atomic electrons were pure Coulomb forces, but also that the excited states in which the system could exist were governed by the laws of quantum mechanics, including the Pauli exclusion principle. (It is true, only the states of the simplest atom, hydrogen, could be exactly calculated, while already the helium atom presented such overwhelming mathematical difficulties that it took about another quarter century and development of computers to reach the same stage.)

					13	A) 26.98
		12	Mg 24.32 •.063			
		П	Na 22.991 #.53			No 20 0.3%
		10	Ne 20.183		Ne 18 1.6 s #3.2 £4.2	Ne 19 18.51 4*22 53.2
9	F 19.00 •.009				F 17 66s #1,75 E 2,77	F 18 1.87h
8	0 16.000 #4.0002		0  4 72 s 4'1.83 72.50 E 5.15	0 I5 2.I m p*17 E 2.7	0 16 99,759 * 4,00002 HL00000	017 0.037 *** 3 1500493
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If we want to study events happening inside the nucleus, its finite extension and the distribution of charge and current inside it, we have a task of a higher degree of difficulty than in the atomic case: (a) We do not know the exact nature of the force between two nucleons, nor do we know whether a potential exists for this force. (b) One cannot consider, in first approximation, the interaction of two nucleons only, because all nucleons are close to each other. (c) We do not know whether the laws of quantum mechanics are sufficient to describe a nucleus completely.

Quantum mechanics may need to be modified, e.g., by the introduction of the concept of a fundamental length, before nuclear phenomena can be explained

On the other hand, we have two important clues on the nature of nuclear forces:

1. Apart from the lightest ones, all nuclei have the same density  $\rho=1.7\times10^{38}$  nucleons/cm³, at least in their central part, so that it is reasonable to speak of "nuclear matter." Hence, if it were not for the Coulomb repulsion between the protons, nuclei of arbitrarily large size would exist. We therefore speak of the "saturation" of nuclear forces, which prevents nuclear matter from collapsing to a density less than  $\rho$  and from flying apart.

2. Experiments have shown that the forces between two protons, corrected for the effect of Coulomb repulsion, are the same as between two neutrons, i.e., charge symmetry prevails, and probably the forces are also the same between a proton and a neutron (charge independence).

In recent years a number of theoretical physicists, under the leadership of K. Brueckner, have tried to understand the nuclear phenomena by treating the nucleus as a manybody system assuming that nuclear forces can be derived from a two-body potential. The simplified case of infinite

	β*<8,5,6.5 y1,38~7.2 (e~2) E14	β <sup>+</sup> 3.24 E4.26	E4.23 71.82,72, 2.9 E40	# 23 26.99000	#2.87 y1.78 E 4.65	P Y
	Mg 23 125 173.0 E4.0	Mg 24 76.8 - 03 23.27254	Mg 25/ 10.1 - 27 /24.99375	Mg 26	Mg 27 - 9.5m 8°15,157 7.84,107,18 E 2,59	EI.
No 21 231	No 22 26y 1 34	Na 23 100 53 12297705	Na 24 15.0 h #139, y2/53,1368, E 5.51	Na 25 60 s 8 40.2,6,3.4 7.98,58,38, 1.60 £4.0		
20	Ne 21 0.26 20,00080	Ne 22 8.9 21.99635	Ne 23 40 s 8 a.2,3.8, 7.44,1.69,- E4.2	Ne 24 3.4 m 8 1,90,1-4.31 9.470,88,		
19 390	F 20 II a F 5.42 y 1.63 E 7.05	F21 5s 85 85,7		14	200	
118	019 291 8-32.44 7.870,137,30 £4,79		By permissi General Ele	on of ectric Co.		
N 17 4,146 F17 F104 F104		12				
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nuclear matter is considered first. A few months ago P. S. Signell and R. E. Marshak and independently J. Gammel and R. Thaler showed that the phase shifts derived from nucleon-nucleon scattering experiments up to 150 Mev can be fitted remarkably well by a charge-independent potential including a spin-orbit term of the same sign as in shell theory. This potential has already been introduced in a Brueckner-type theory by de Dominicis, and reasonable answers for the binding energy of infinite nuclear matter have been obtained.

One would hope to apply similar methods to the more complicated ease of the finite nucleus, thereby demonstrating the validity of the shell model for such a system. The next decade may see an understanding of complex nuclei based on interactions derived from nucleon-nucleon scattering experiments.

While the solution of these very difficult fundamental problems progresses slowly, experimental nuclear physicists are guided in their research by a number of more phenomenological nuclear models, each one of which is limited in its application. Usually these models are not born in a finished form but have to be modified continuously to fit the facts. Sometimes, they even seem to contradict fundamental principles, as when the basic assumption of the shell model that a nucleon may be considered to move in a central potential seemed at first to violate the idea that the mean free path of a nucleon is of the order of its diameter. J. H. D. Jensen, in a talk given in 1956 at the International Congress on Theoretical Physics in Seattle, recounted how Maria Mayer and independently Haxel, Jensen and Suess revived and modified—by the introduction of strong spin-orbit coupling—the early shell-model ideas (which in turn were conceived in analogy to the atomic case) merely as a working hypothesis. A possibility of removing the apparent contradiction was first pointed out by Weisskopf, who emphasized the importance of the role of the exclusion principle in simplifying the particle motions in the presence of strong forces. Later quantitative studies of Brueckner, Eden and Bethe have shown that this conjecture is correct and that the shell model can be used to give a useful first approximation to the nucleus up to  $\sim 10$  MeV.

Two other models which have played an important role in recent years are the collective model developed by A. Bohr and B. Mottelson, which is likewise valid at low energies only, and the optical model, which successfully describes nuclear absorption and scattering processes, e.g., the so-called mountain resonances for neutrons and protons.

Let us now consider some of the aspects of these models which are likely to lead to further progress in nuclear research: The shell model will, of course, continue to be useful for the construction of level schemes and in giving the characters (spins and parities) of levels not only of stable nuclei, but also of radioactive nuclei, in providing a means for the classification of beta decays and for the explanation of the occurrence of nuclear isomers, etc. Of more farreaching importance are the possibilities for a quantitative analysis of the energies of nuclear levels. In the near future such an analysis will be restricted to the immediate neighborhood of doubly magic nuclei, e.g., O<sup>16</sup> and Pb<sup>268</sup>.

The successful analysis by Elliott and Flowers, by D. Kurath and by B. French and others not only of the level energies of some light nuclei, but also of the transition rates of a number of beta transitions, and of several electromagnetic transitions—mainly of dipole character—justifies the expectation of further progress in this field.

As an example of a field where much progress can be expected in the near future, and in which the author has been particularly interested, a few words may be said about those even-even nuclei which have vibrational level schemes and which lie between the "magic" nuclei and the rotational region. The vibrational level schemes have been tentatively interpreted on the basis of the Bohr-Mottelson model in the region of weak to moderate coupling. Such a model predicts a triplet, of characters 0+, 2+, 4+, at about twice the energy of the first 2+ state. Since states differing in spin by 4 would escape detection with most of the usual methods of determining level schemes, it is not



G. Scharff-Goldhaber 🖣

Physics Department, Brookhaven National Laboratory; has worked on nuclear energy levels, neutron and electron diffraction, spontaneous fission, and ferromagnetism.

surprising that only one such close-lying triplet has been found so far, namely, in Cd<sup>114</sup>. The method used here was the analysis of conversion electrons accompanying neutron capture in Cd<sup>113</sup> (H. Motz). A systematic search for triplets by means of suitably chosen experiments should throw light on the question of the nature of the vibrational even-even nuclei and, in turn, of those odd-A nuclei of which they form the core.

As is well known, the collective model has been enormously successful in describing not only the level characters and energy ratios, but also the transition probabilities of electromagnetic transitions, the log ft values, etc., in strongly deformed nuclei.

Recent efforts to interpret the level scheme of F<sup>19</sup> have led to an interesting discovery: the results of one group of physicists who applied a shell-model analysis agreed surprisingly well with those of another group who applied a collective-model interpretation. Ironically, the theoretical values agreed even more closely among each other than with the experimental values. The reason for the good agreement is by no means obvious and is now being studied by a number of theoreticians. It allows one to conclude, however, that it will be possible to set up a unified model of the nucleus, in which the individual particle motions and the collective motions are self-consistent as in molecules. Important attempts in this direction have been made by Peierls and Yoccoz and by Wheeler and Griffin.

Now a few words about higher-energy nuclear reactions: As was mentioned above, the optical model is successful in describing the energy dependence of the absorption and scattering cross sections of the particles impinging on a nucleus for a considerable energy range. However, a theory has still to be evolved which will give the probabilities and angular correlations for the decomposition of the system target nucleus plus bombarding particle into the various energetically allowed end products. It may be added that the optical model is based on the assumption that the energy spectrum of the incoming particles overlaps many resonance levels in the target nucleus. It is likely that with increasing energy definition this approach will give way to a renewed interest in the fine structure, which is of particular importance in the fission process.

#### We can look forward to definitive progress toward understanding the bewildering variety of phenomena observed in fission

A few thoughts on beta-decay theory: the recent revolution in thought brought about by the discovery that parity conservation and the conservation of charge conjugation do not hold in weak interactions has attracted great interest to this field. Although at present the nature of the interactions for the nucleon-electron-neutrino system is not known, it is very probable that by the end of the coming decade it will be quite well understood. This will be brought about by studies of polarization of the electrons emitted in various types of beta decay, by further efforts to detect double beta decay and by the refinement of present neutrino-detection experiments.

The experimental determination of nuclear properties will increase in accuracy and scope as the equipment and methods grow in diversity and ingenuity.

For example, the steady improvement in resolution and efficiency of spectrometers will facilitate the determination of level energies.

For character assignments to nuclear energy levels there are now a number of methods at our disposal, which we have just begun to exploit. For short-lived states preceded by some previous radiation there is the delayed-coincidence method, which Sunyar has recently developed into a form that permits one to measure lifetimes as low as a few times 10<sup>-11</sup> sec. For electric dipole or quadrupole transitions leading to the ground state the Coulomb-excitation method is able to cover a wide lifetime range. As a final example, the use of molecular-beam and paramagnetic-resonance methods for the spin determination of radioactive nuclei will doubtless increase and serve to cheek assignments made on the basis of decay-scheme studies. Further, it may be expected that the refinement of the theoretical interpretation of experimental results. e.g., a better theory of stripping, of internal conversion and of the angular distribution of inelastically scattered particles, will make level-character assignments of excited states more definite.

Radioactive nuclei will be found further away from the stability region, owing to the use of heavier bombarding particles and faster detection methods. The study of the binding energies of nuclei of this type may throw further light on the nature of nucleon-nucleon forces. Also, the number of known elements will certainly be increased.

# Relatively long-lived isotopes may well be found among the far-transuranic nuclides because of magic-number stability

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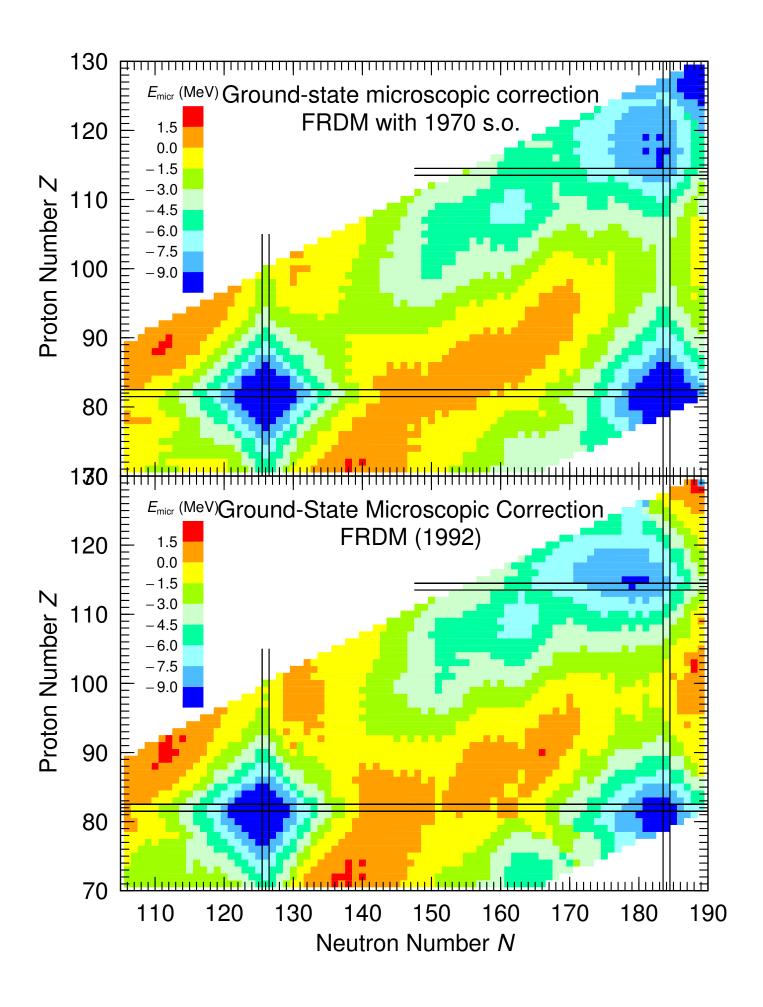
There may be, for instance, another region of relative stability at the doubly magic nucleus  $_{126}X^{310}$  (the closing of the j neutron shell).

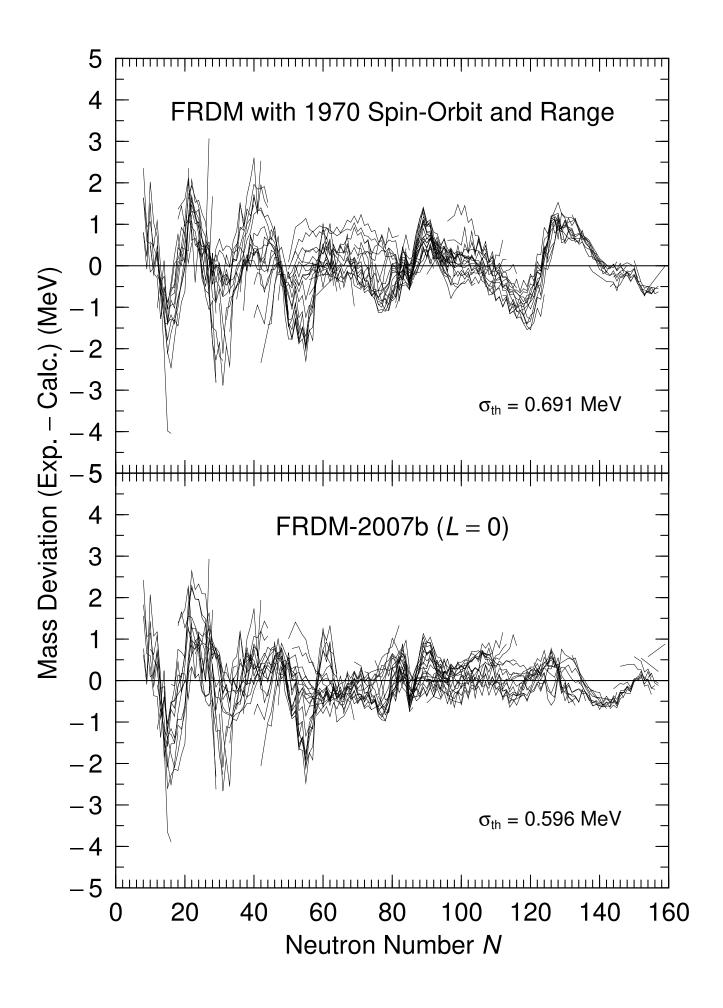
New accelerators like variable-energy cyclotrons and tandem Van de Graaffs capable of producing beams of well defined and sufficiently high energies will make it possible to study radiation and particle widths of excited states of light nuclei with accuracy, thus testing the shell-model wave functions, and to explore the level schemes of medium-weight nuclei (50 < A < 150).

Atomic-beam methods and possibly the study of  $\mu$ -mesic X-rays will yield new data on electric and magnetic moments of higher order. The boldness of some thinkers in this field is indicated by the title of a recent theoretical paper: "Nuclear Hexadecapole Moments." It may even be possible to get a better idea of the charge and current distribution within the proton by means of the 6-Bev electron synchrotron now being constructed at Cambridge Massachusetts, which will permit an extension of the very successful work carried out at Stanford.

One important goal of nuclear-physics research is the deduction of nuclear forces from meson fields. Among these fields the role of the  $\pi$ -meson field will probably be first understood, but it is clear that any ultimate theory will not be able to ignore the role of the "strange particles" (K mesons and hyperons). In the meantime, the new field of "hypernuclear" physics is likely to develop considerably and to help indirectly in the understanding of nuclear phenomena.

September, 1957 - NUCLEONICS





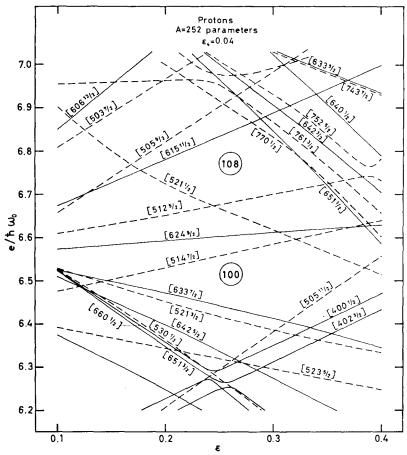


Fig. 2g. Single-proton levels 250 < A < 270;  $\kappa$  = 0.0569,  $\mu$  = 0.656,  $\varepsilon_4$  = 0.04.

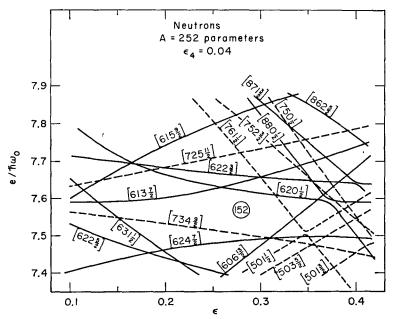


Fig. 2h. Single-neutron levels 250 < A < 270;  $\kappa = 0.0635$ ,  $\mu = 0.314$ ,  $\varepsilon_2 = 0.04$ .

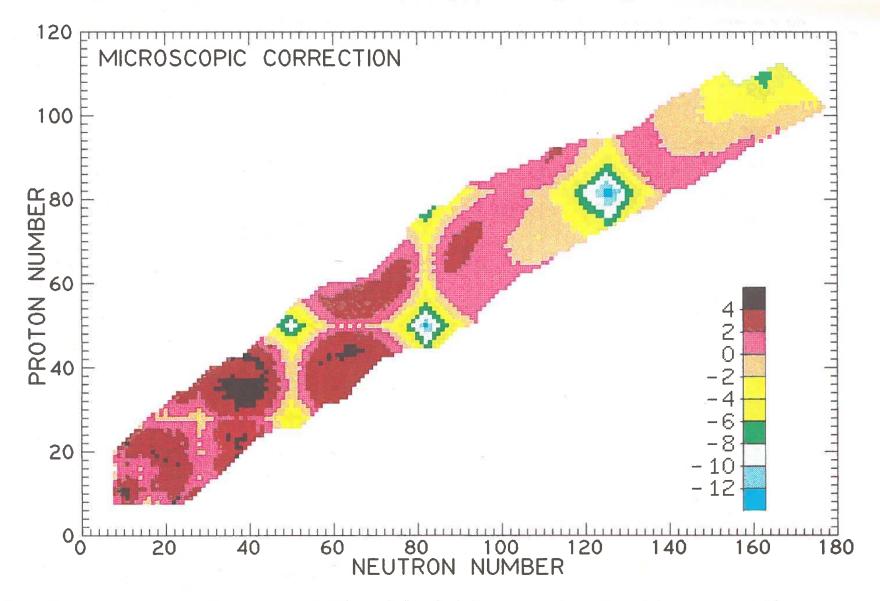
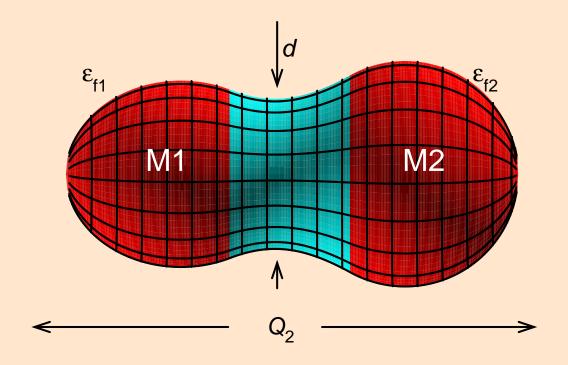


Fig. 2c. Plot of the ground-state microscopic correction, as calculated in [6] for 4023 nuclei. The fluctuations in the shell correction are larger in the heavier region than in the lighter region.

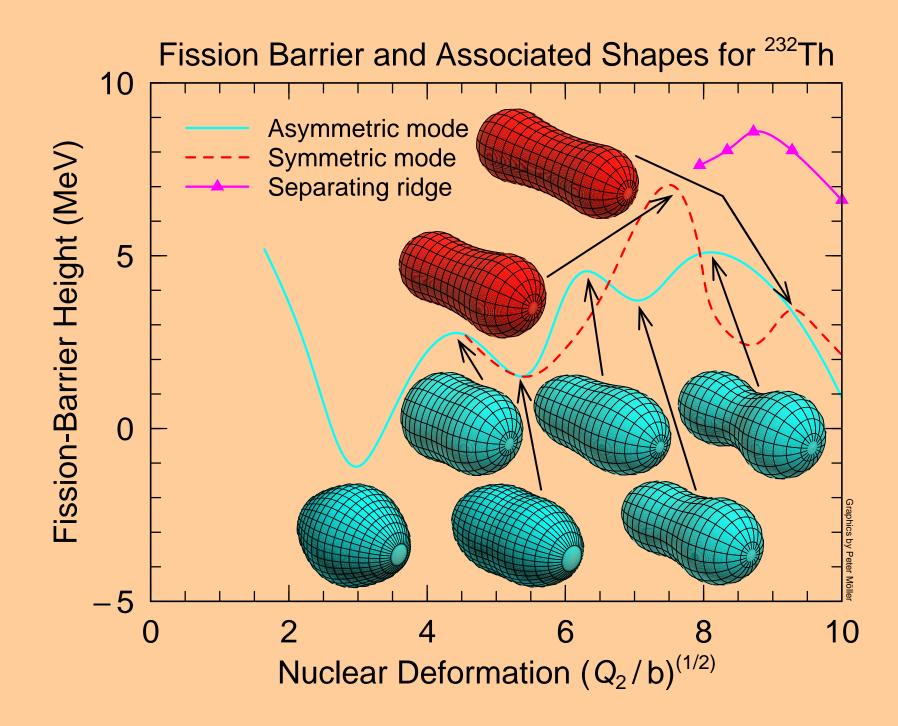
## Five Essential Fission Shape Coordinates



45	$Q_2 \sim Elongation$ (fission direction)
⊗ 35	$\alpha_{\rm g}$ ~ (M1-M2)/(M1+M2) Mass asymmetry
⊗ 15	ε <sub>f1</sub> ~ Left fragment deformation
⊗ 15	ε <sub>f2</sub> ~ Right fragment deformation
⊗ 15	d ~ Neck

 $<sup>\</sup>Rightarrow$  5 315 625 grid points – 306 300 unphysical points

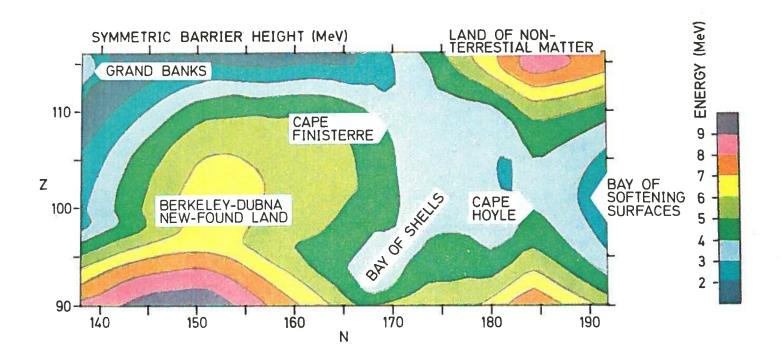
<sup>⇒ 5 009 325</sup> physical grid points



# Nobel Symposium 27 Physics

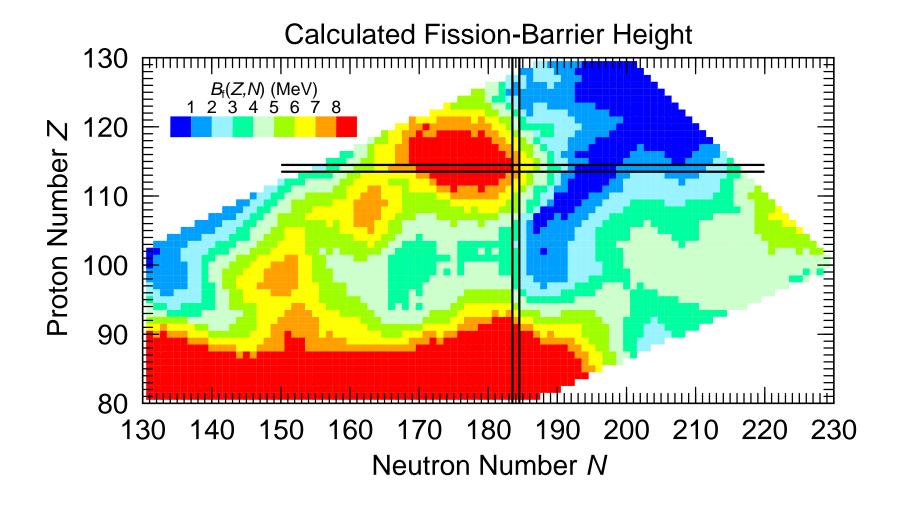
# Super-Heavy Elements— Theoretical Predictions and Experimental Generation

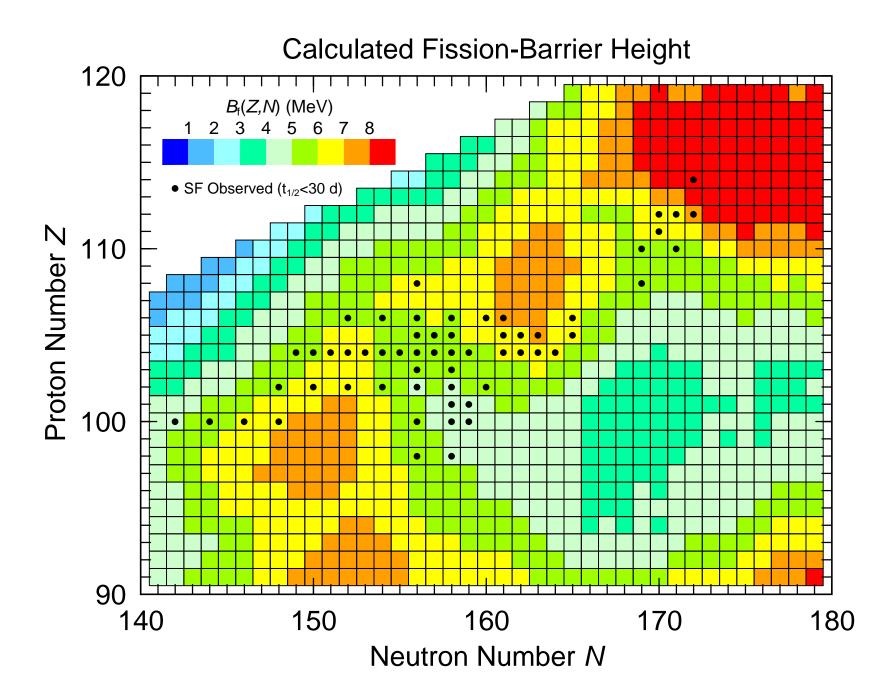
Proceedings of the Twenty-Seventh Nobel Symposium held at Ronneby, Sweden, June 11–14, 1974
Editors Sven Gösta Nilsson and Nils Robert Nilsson



Nobel Foundation, Stockholm

Almqvist & Wiksell International, Stockholm, Sweden





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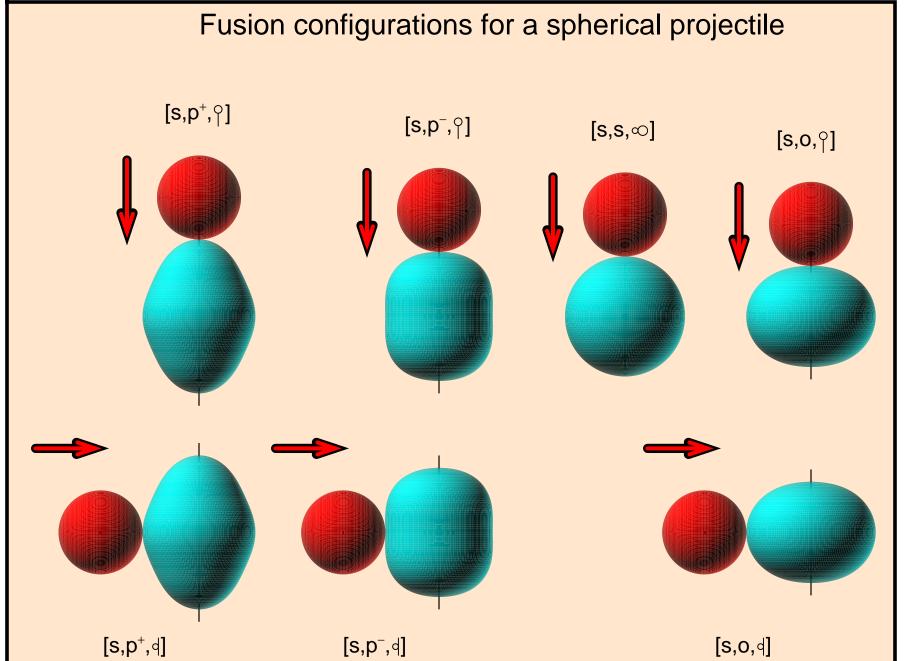


Figure 1

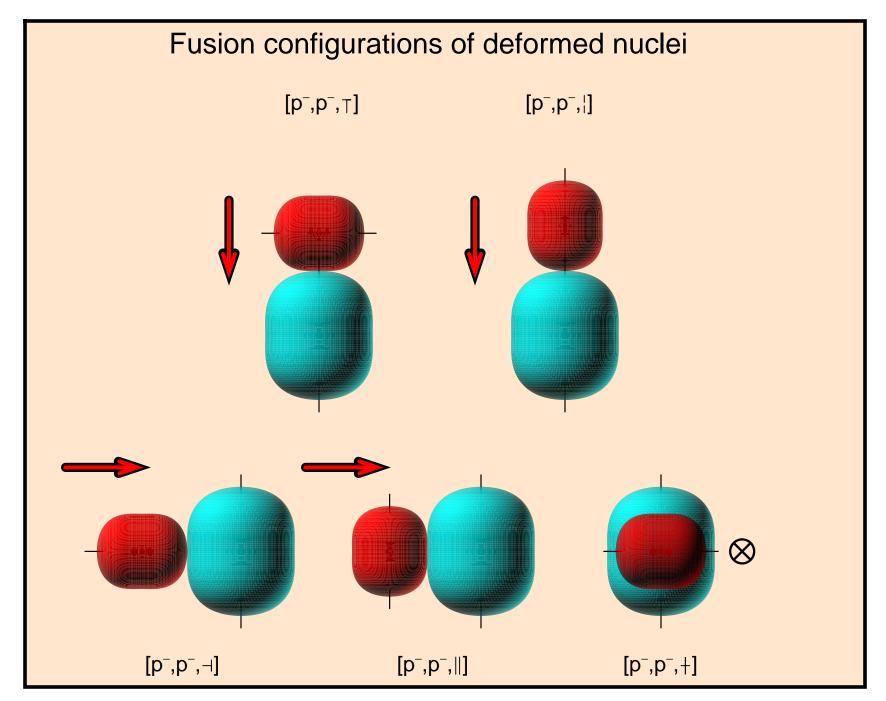
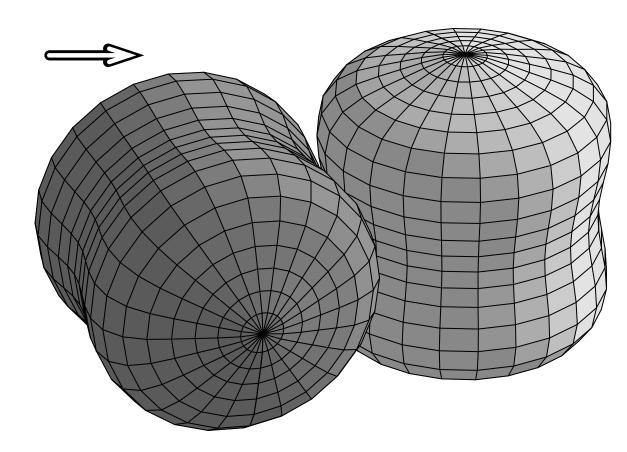
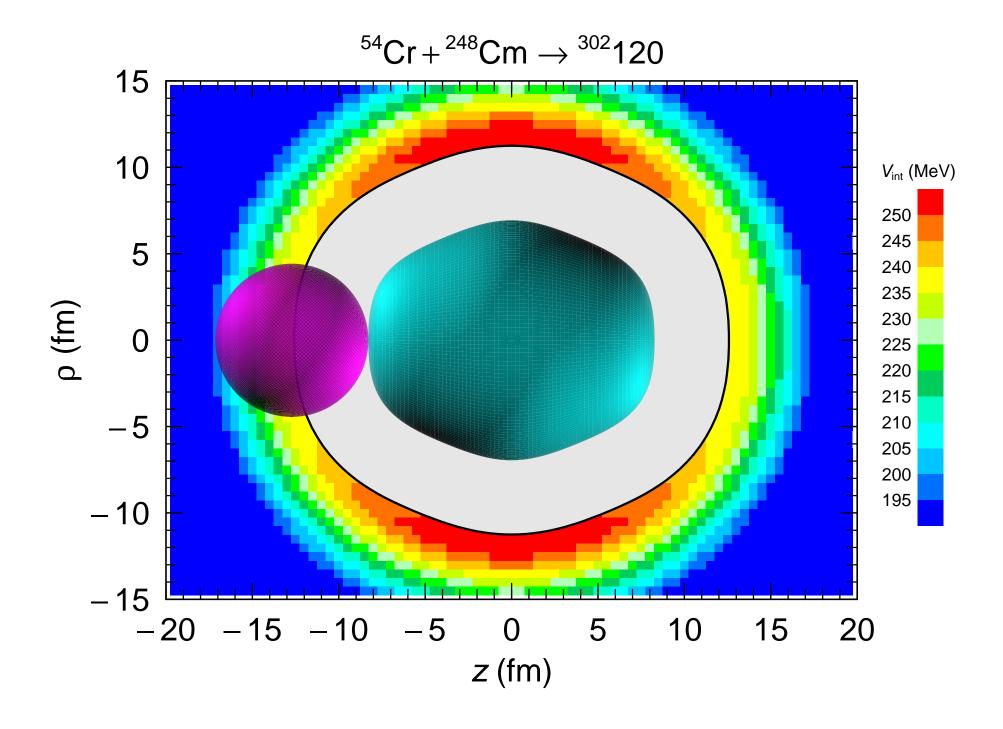


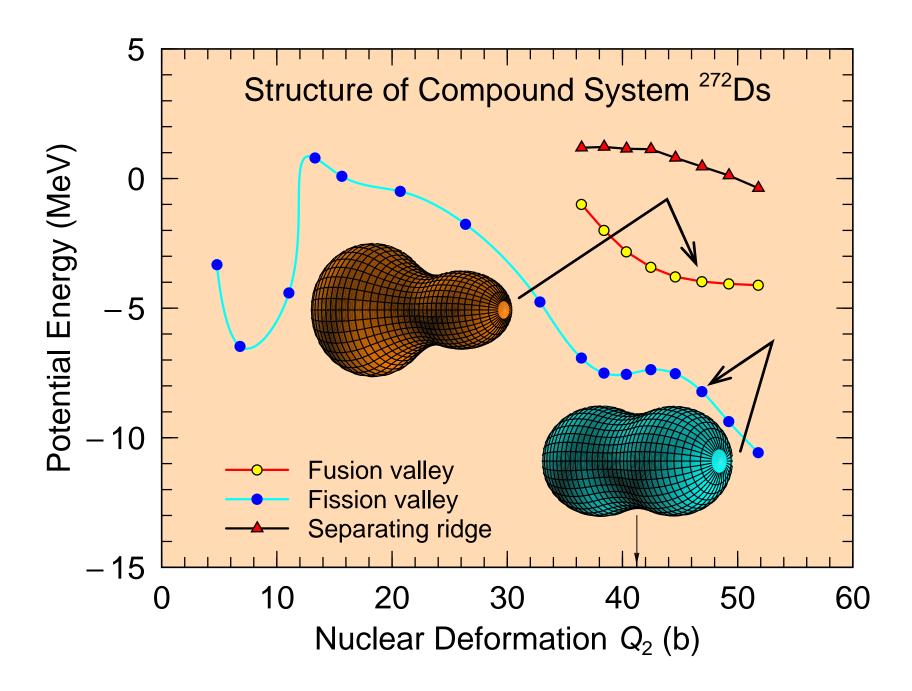
Figure 6

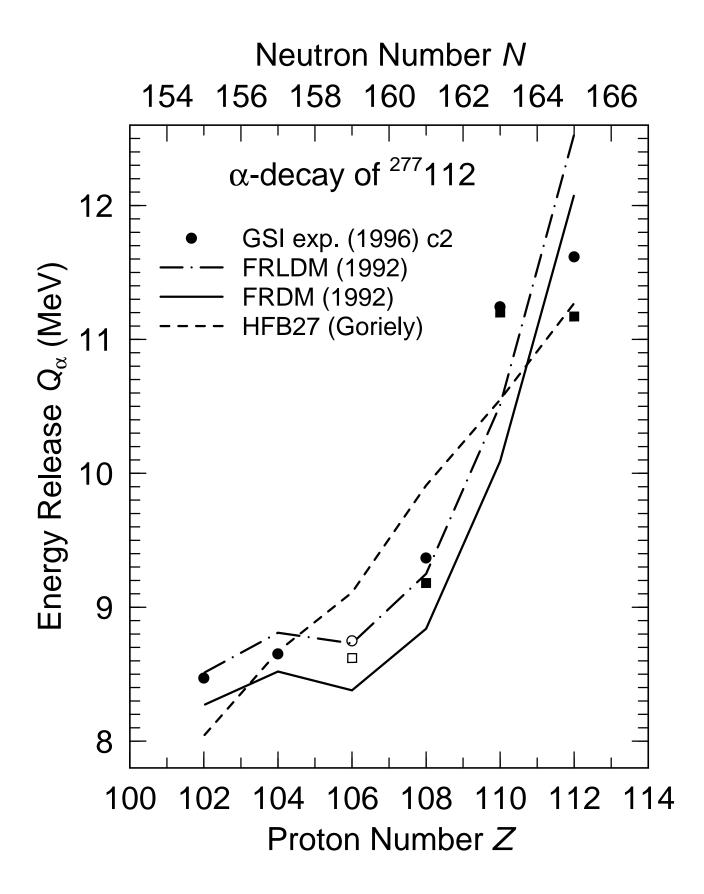
# $^{150}Nd + ^{150}Nd$

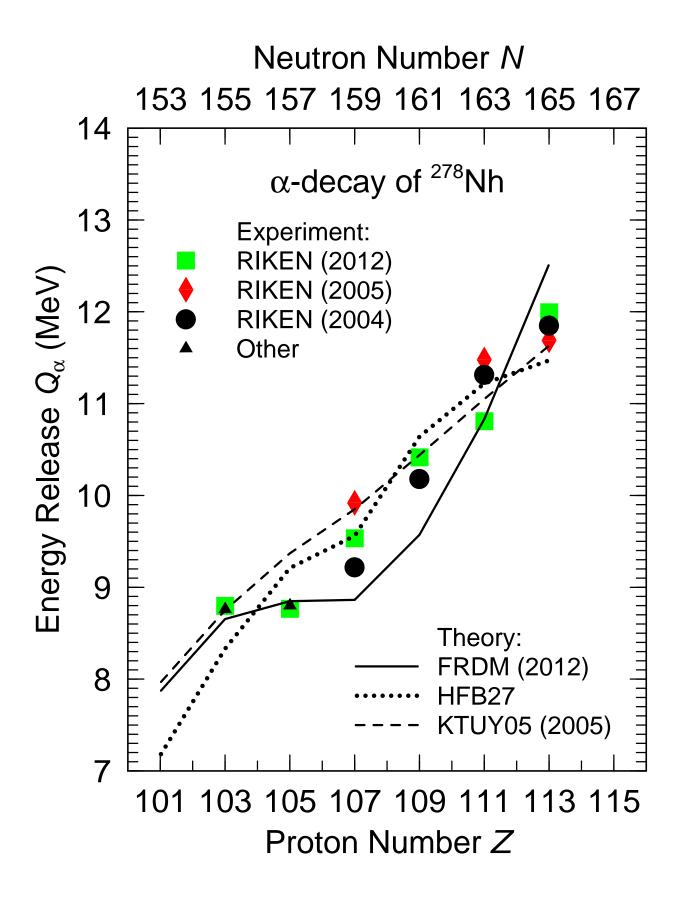
Shapes with large negative hexadecapole moments

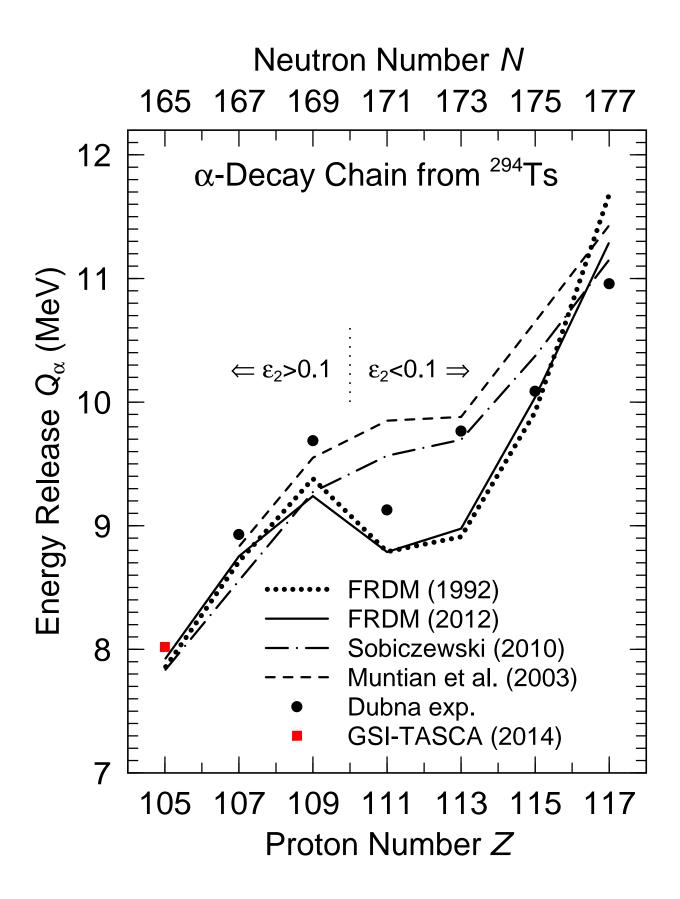


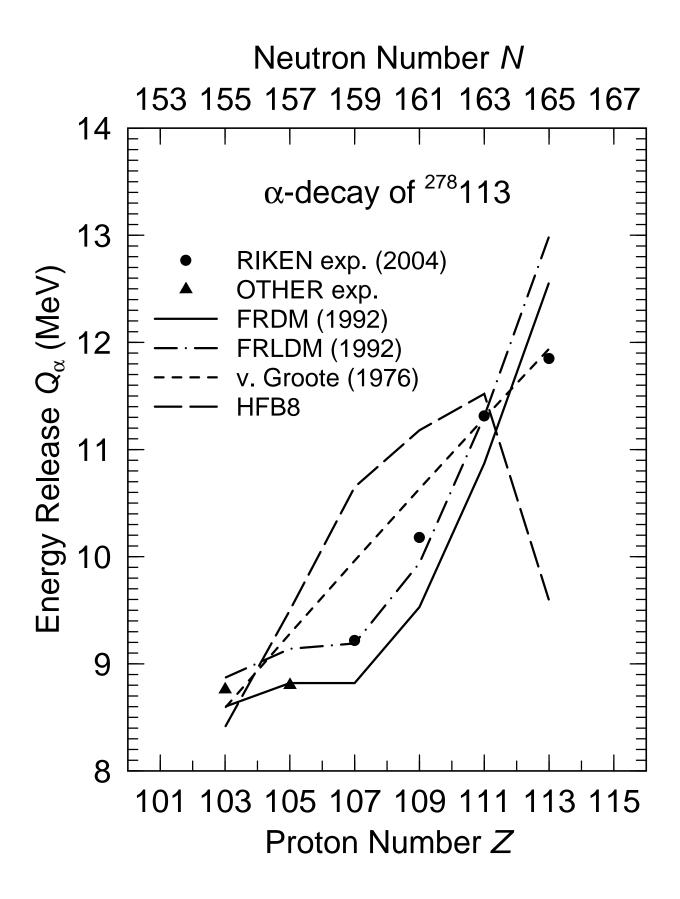


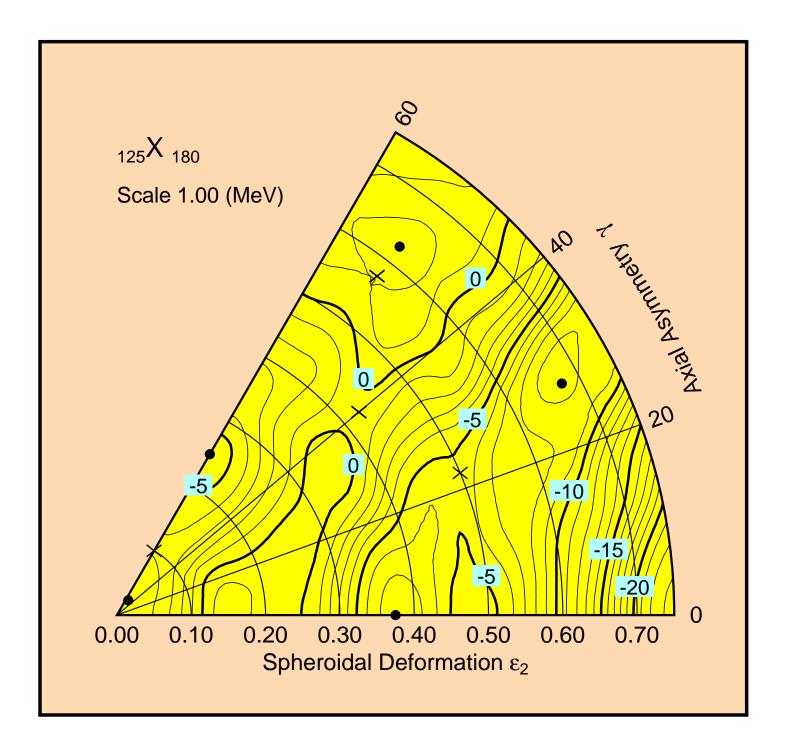






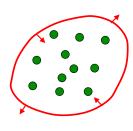






# $Q_{\alpha}$ Deviations beyond N=126

Region	Model	Nuclei	RMS
			(MeV)
Z > 82	SkM*	46	2.6
Z > 82	Sly4	46	2.6
Z > 82	HFB21	145	0.409
Z > 82	FRDM(1992)	145	0.463
Z > 82	FRDM(2012)	145	0.326
Z > 88	SkM*	36	1.7
Z > 88	Sly4	36	2.2
Z > 88	HFB21	101	0.367
Z > 88	FRDM(1992)	101	0.448
Z > 88	FRDM(2012)	101	0.274



## Brownian shape motion

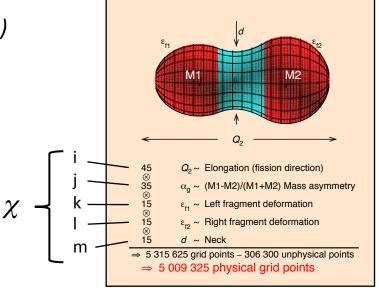
Nuclear deformation energy:  $E_{def}(i,j,k,l,m)$ 

Bias potential:  $V_{\text{bias}}(i) = V_0 (Q_0/Q_2)^2$ 

Level density parameter:  $a_A = A/(8 \text{ MeV})$ 

Temperature *T*:  $E^*-E_{def} = a_A T^2$ 

$$=> V(\chi) = E_{def} + V_{bias}$$



P. Möller et al, Nature 409 (2001) 785

Five Essential Fission Shape Coordinates

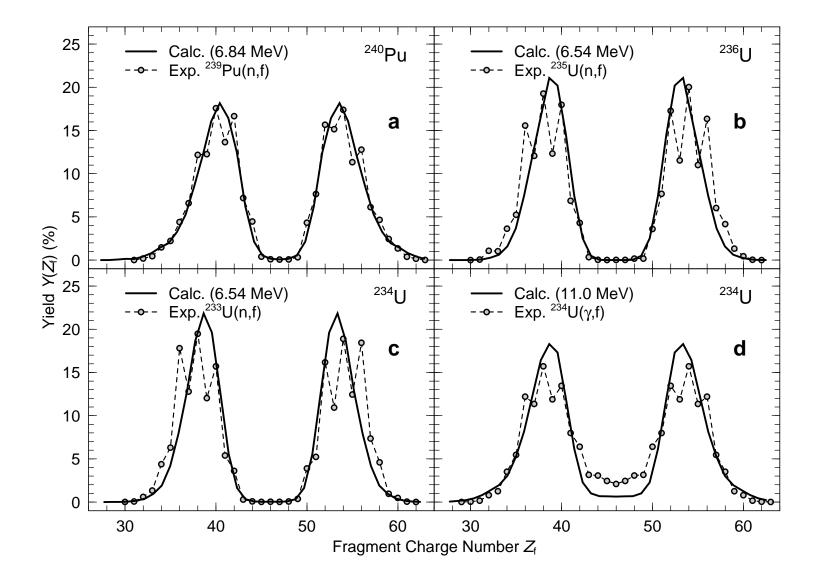
Metropolis walk:

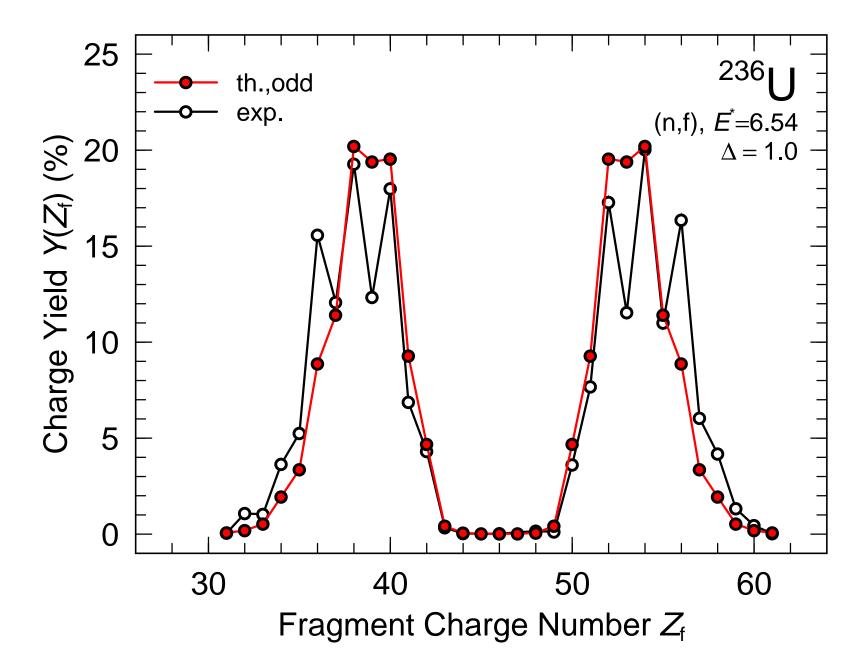
Change shape:  $\chi \rightarrow \chi'$ ?

N. Metropolis et al, J Chem Phys 26 (1953) 1087

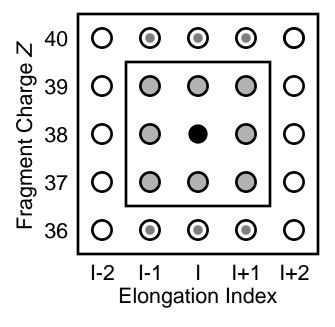
 $\begin{cases} V(\chi') < V(\chi): \text{ move with } P = 1 \\ V(\chi') > V(\chi): \text{ move with } P = \exp(-\Delta V/T) \end{cases}$ 

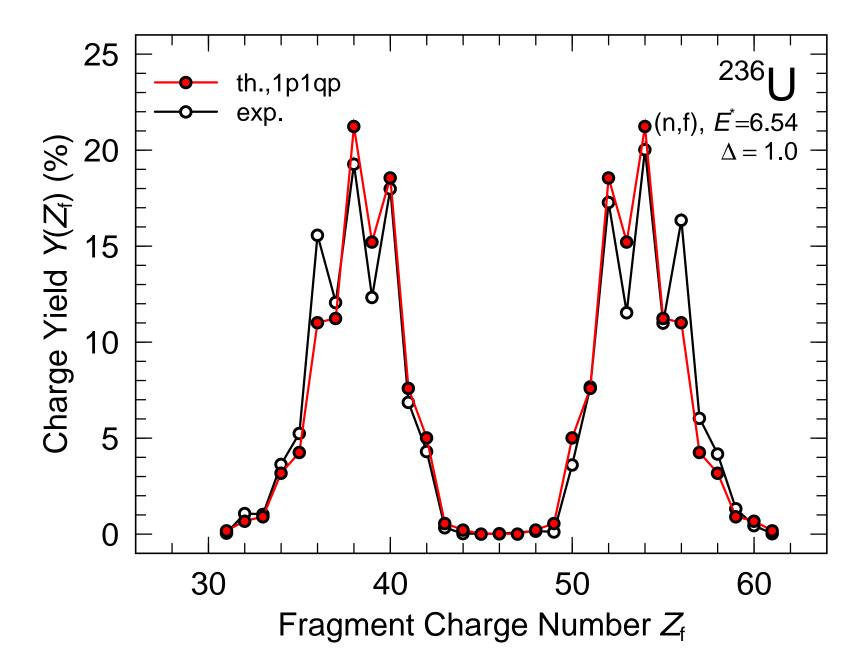
Scission: Critical neck radius  $c_0 \approx 2.5$  fm

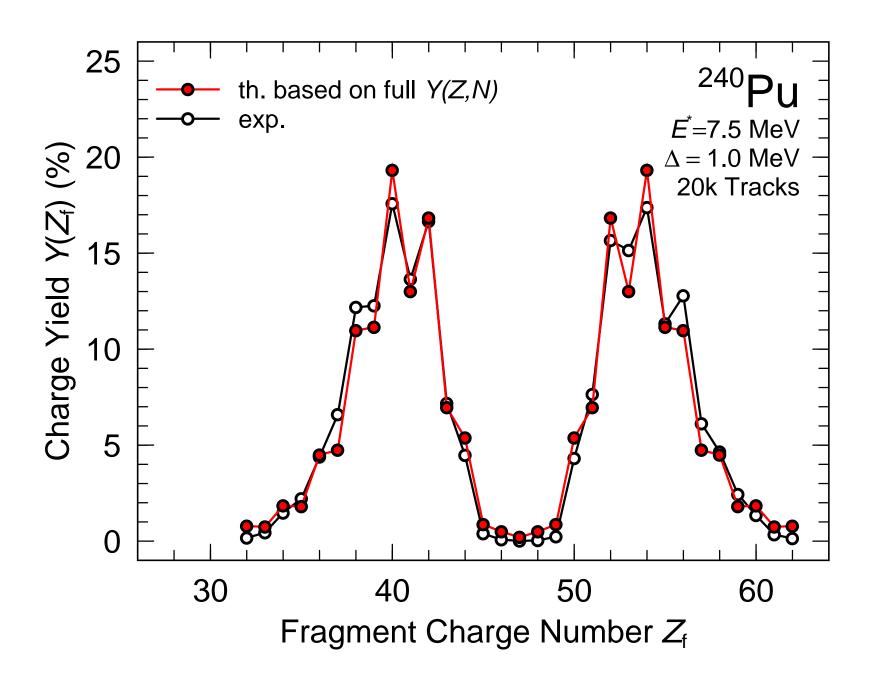


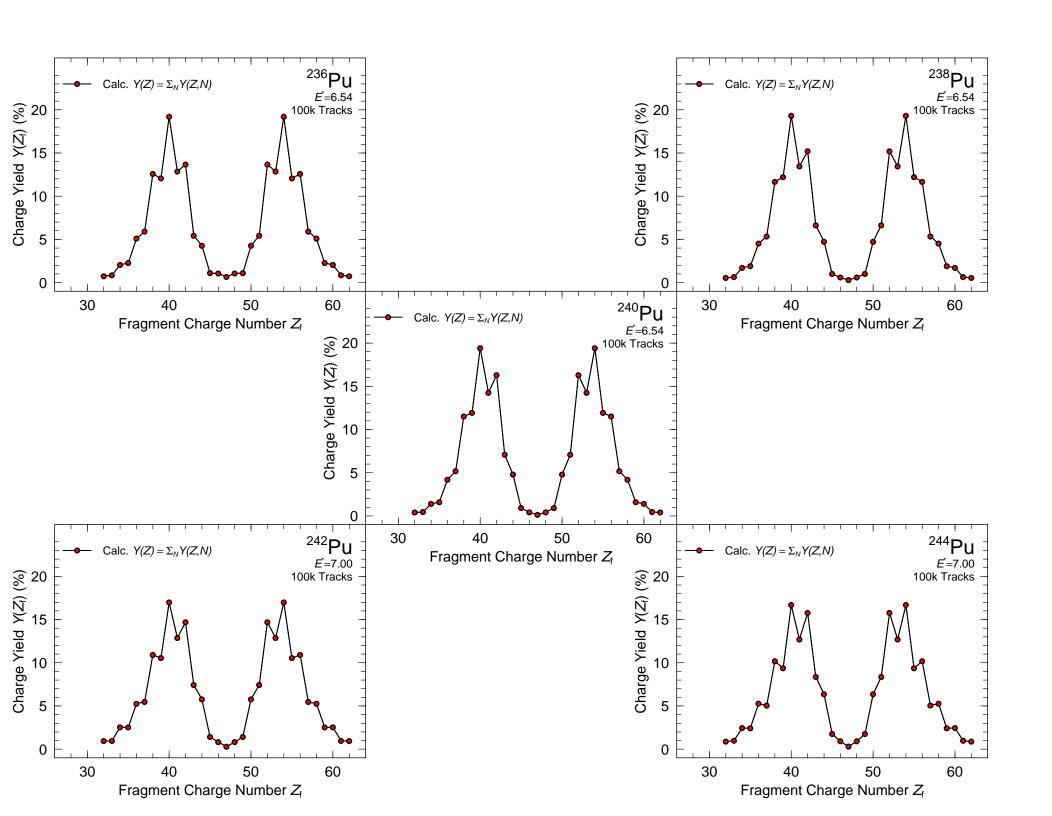


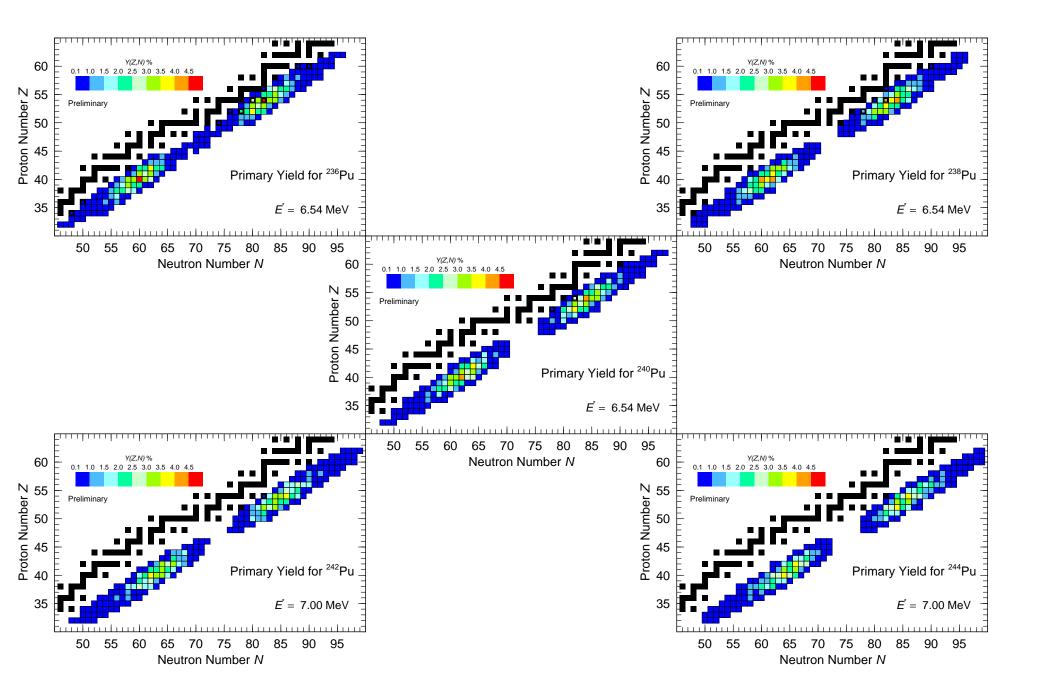
## **Next Track-Point Candidates**

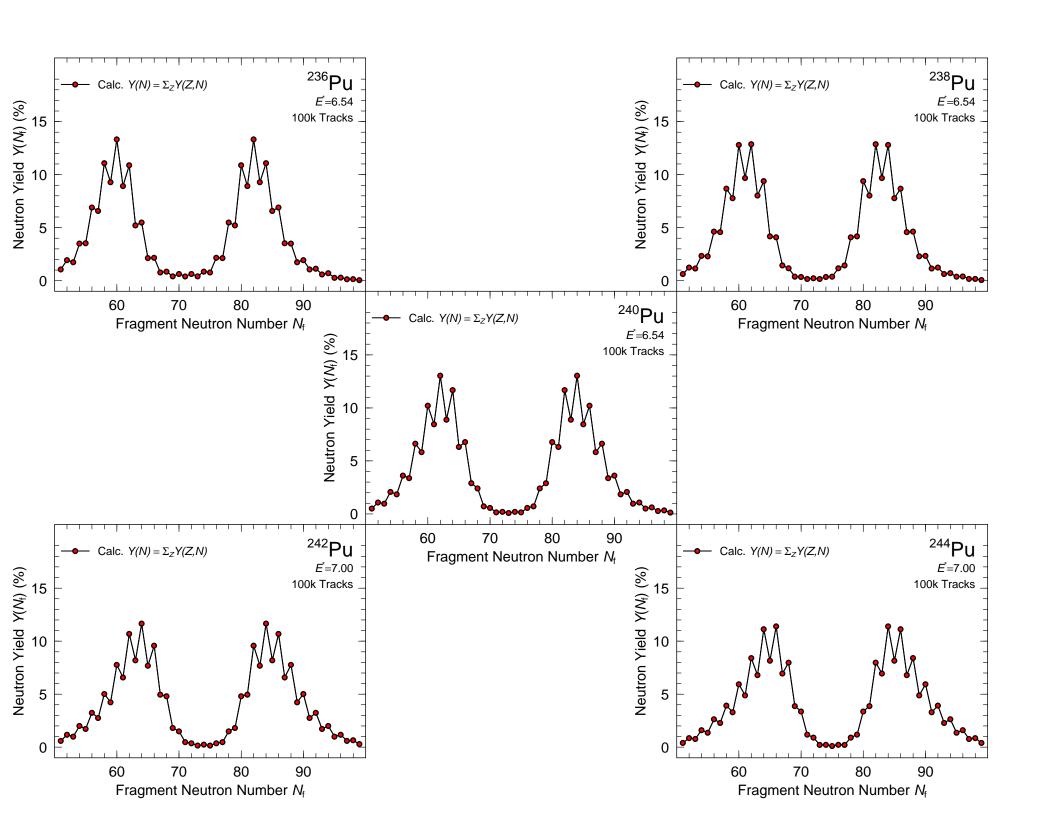




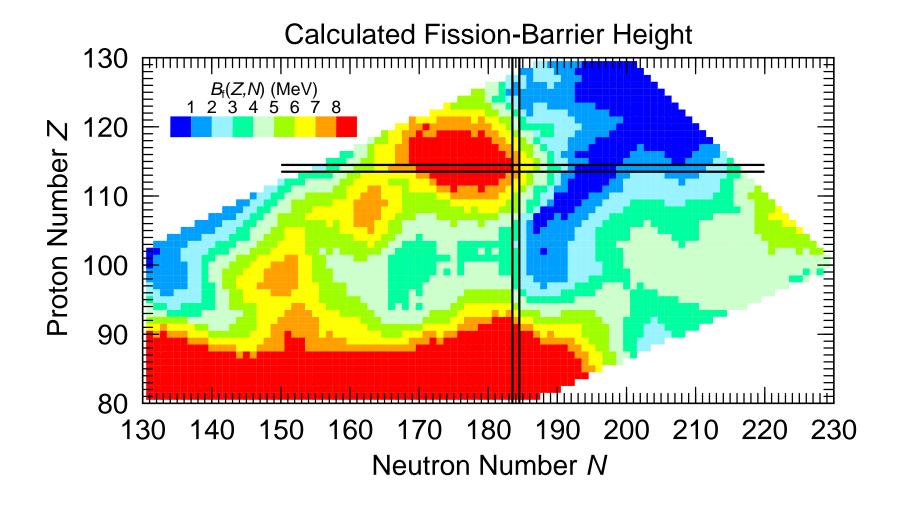








First in chain Z A	fissioning nucl at chai	n end TKE
114 289	110 281 171	212
114 288	112 284 172	216
114 287	112 283 171	206
114 288	112 284 172	202
114 287	110 279 169 108 275 167 106 271 165	SF<10% SF>90% 228
114 286	112 282 170	212 ~60%
116 293	110 281 171	197
116 292	112 284 172	190
116 291	110 279 169 104 267 163	no data 240
116 290	114 286 172 112 282 170	? 209
112 283	110 279 169	185,194,1
116 290	112 282 170	209
118 294	112 282 170	202
113 282	104 266 162	203
115 288	104 268 164	203
115 287	105 267 162	206



## CONCLUSIONS

- All global nuclear-structure models are simple representations of nuclear properties. To expect infinite accuracy with global models is unrealistic.
- Both Wood-Saxon and folded-Yukawa based models give properties of SHE elements to useful accuracy.
- Remaining differences between these models and between the models and experiment must be considered unavoidable model uncertainties.
- Obviously less deviations can be achieved by local adjustments of parameters, but for those of us who strive to improve global model accuracy, this would be a null results.
- Most HFB models have poor results for known nuclei, therefore their predictions in the SHE region are irrelevant.
- For heavy systems it is not the lowest minimum that is the most stable, it is the minimum with the highest fission barrier, a fact usually ignored in HFB calculations.

## FUTURE

- ullet Study additional isotopes to establish Z,N of maximum stability.
- More events to obtain fission-fragment mass distributions and TKE distributions.
- Investigate additional projectile-target possibilites (hugging, transfer, ...)