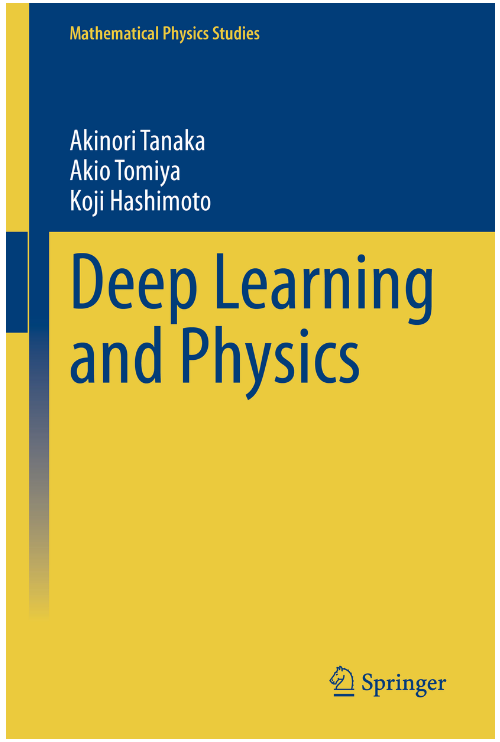
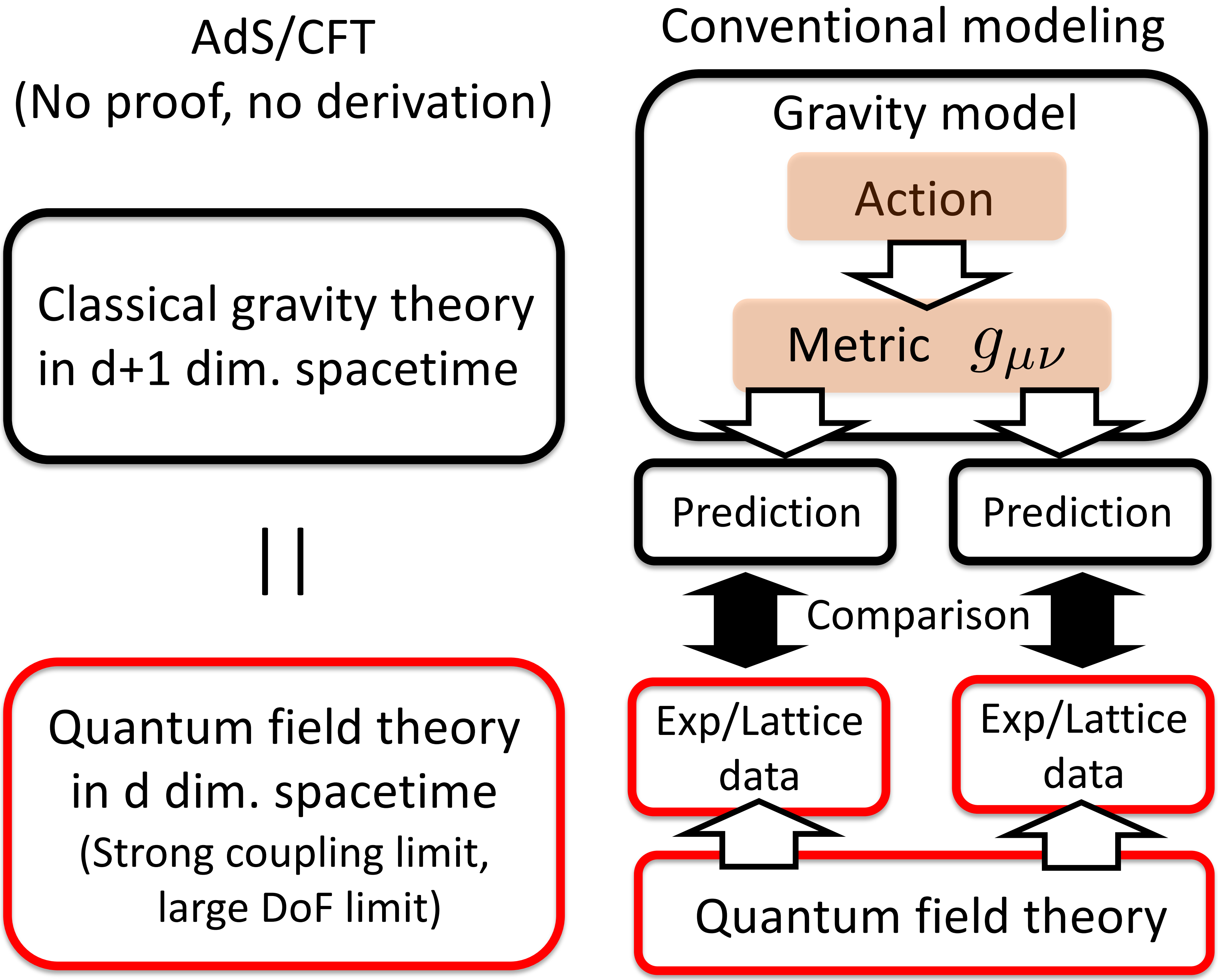
Workshop "Machine learning for high energy physics, on and off the lattice", ECT*, 30 Sep 2021

Deep Learning and Holographic QCD

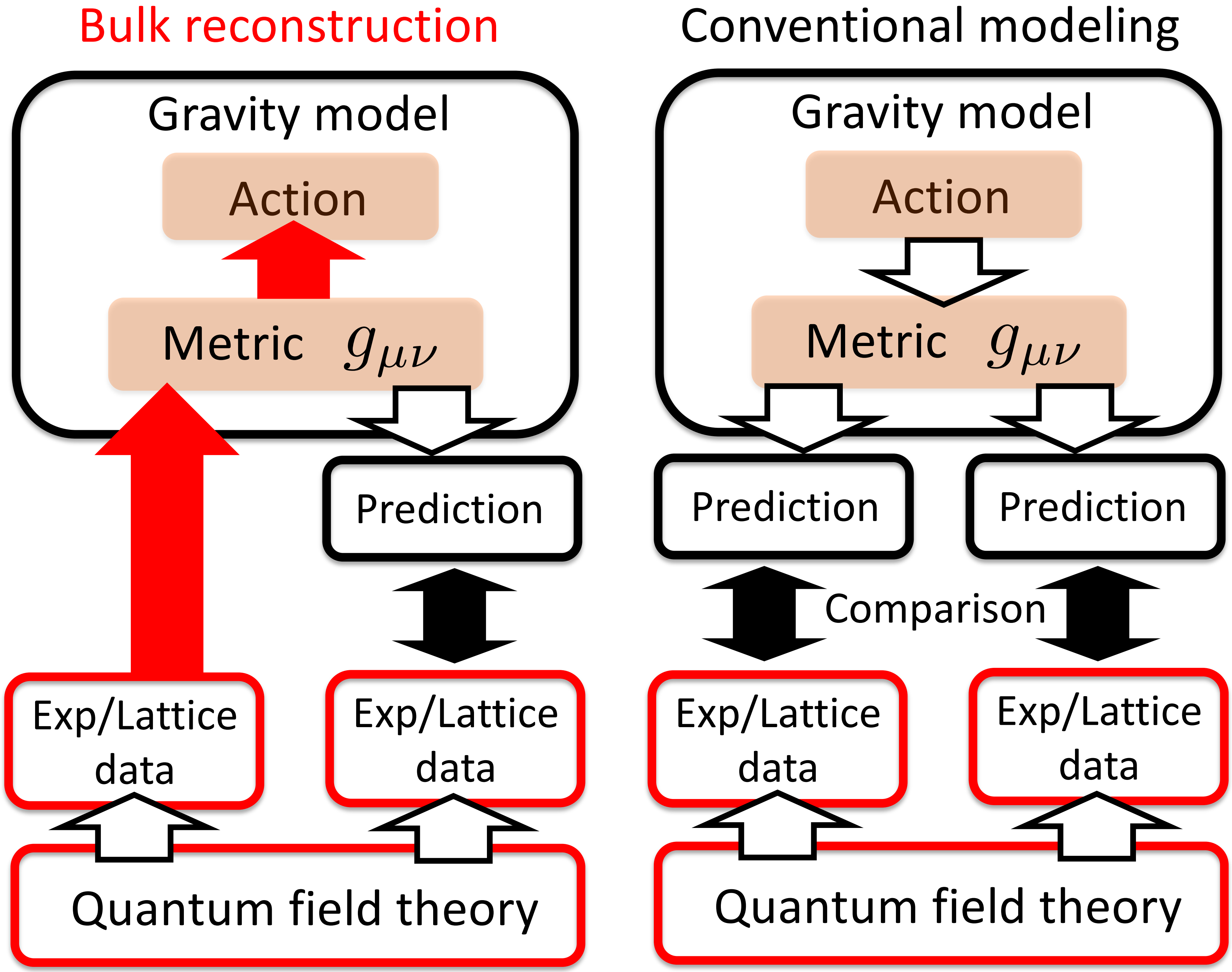
Koji Hashimoto (Kyoto U.)

"Deriving dilaton potential in improved holographic QCD from meson spectrum" 2108.08091
w/ K.Ohashi (Keio), T.Sumimoto (Osaka u)
"Neural ODE and Holographic QCD" 2006.00712
w/ H.Y.Hu, Y.Z.You (UCSD)
"Deep Learning and AdS/QCD" 2005.02636
w/ T. Akutagawa, T. Sumimoto (Osaka u)
"Deep Boltzmann Machine and AdS/CFT" 1903.04951
"Deep Learning and Holographic QCD" 1809.10536
w/ S. Sugishita (Kentucky), A. Tanaka, A. Tomiya (RIKEN)
"Deep Learning and AdS/CFT" 1802.08313
w/ S. Sugishita (Kentucky), A. Tanaka, A. Tomiya (RIKEN)

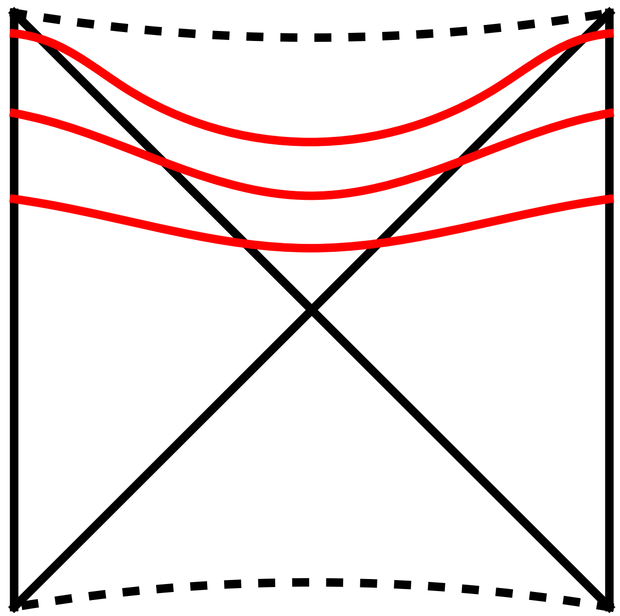




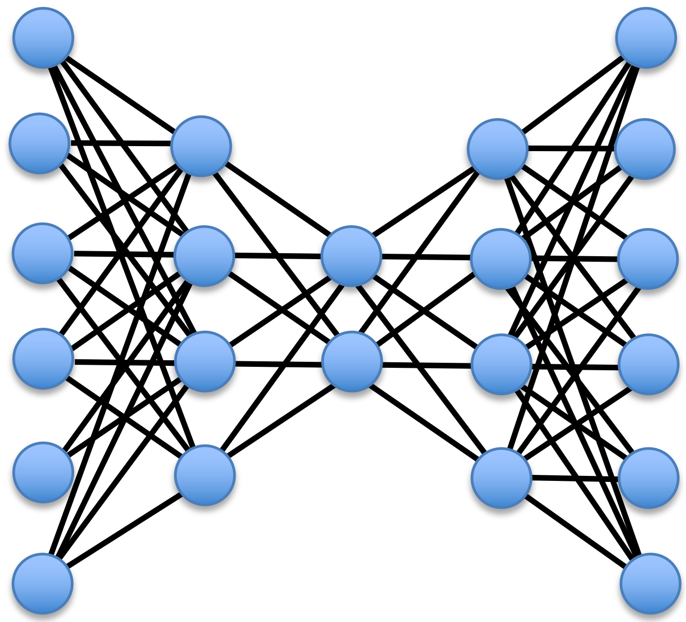
AdS/CFT
(No proof, no derivation)
Classical gravity theory in d+1 dim. spacetime
||
Quantum field theory in d dim. spacetime
(Strong coupling limit, large DoF limit)
Conventional modeling
Gravity model
Action
Metric $g_{\mu\nu}$
Prediction
Prediction
Comparison
Exp/Lattice data
Exp/Lattice data
Quantum field theory


Bulk reconstruction
Conventional modeling
Gravity model
Action
Metric $g_{\mu\nu}$
Prediction
Comparison
Exp/Lattice data
Quantum field theory

Similarity!?



Wormholes in Penrose diagram
of maximally extended eternal
AdS Schwarzschild black hole
[Iizuka, Sugishita, KH `17]



Deep Autoencoder

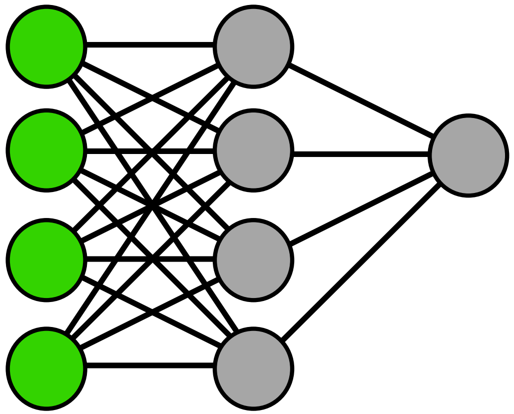
Spacetime is a Neural Network

1. Space is a NN
2. Time is a NN — 1802.08313
3. Holography is a DBM — 1903.04951
4. Holographic space is a NN — 1802.08313, 1809.10536, 2006.00712
5. Holographic spacetime is a NN — 2005.02636
6. Gravity reconstructed — 2108.08091

1. Space is a NN

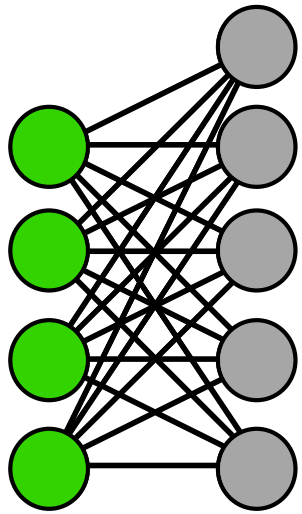
General NN is not a space

No notion of which unit is close to which



Perceptron model
[Rosenblatt 1958]
[Rumelhart, McClelland 1986]

$$f(x_i) = W_j^{(2)} \varphi \left(W_{jk}^{(1)} x_k \right)$$



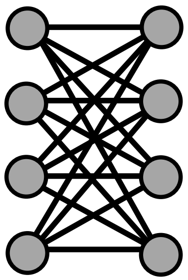
Boltzmann machine
[Ackley, Hinton, Sejnowski 1985]

$$P(x_i) = \sum_{h_j \in \{0,1\}} \exp \left[-\sum_{ij} w_{ij} x_i h_j \right]$$

1. Space is a NN

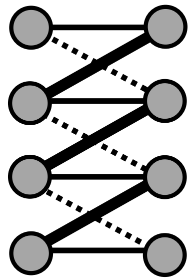
Need of sparsity for NN to be a space

No locality



Fully connected

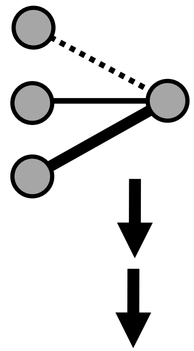
Locality imposed



Convolutional layer

[K. Fukushima `80]

=



Parallelly translated

Input: $\phi(n\Delta x)$

Output:

$$a\phi(n\Delta x) + b\partial_x\phi(n\Delta x) + c\partial_x^2\phi(n\Delta x) + \cdots$$

2. Time is a NN

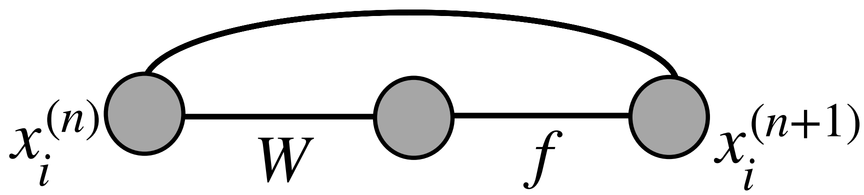
ResNET and ODE

ResNET (Residual network) : easily trained deep model

[K.He et al.,1512.03385]

$$x_i^{(n+1)} = f(W_{ij} x_j^{(n)}) + \underline{x_i^{(n)}}$$

"Skip connection"



Equivalence to discretized dynamical system

$$\dot{x}_i = f_i(x(t)) \quad \Longrightarrow \quad x_i(t_{n+1}) = \underline{x_i(t_n) +} \, \Delta t \cdot f_i(x(t_n))$$

$$t_{n+1} = t_n + \Delta t$$

Discretized time

2. Time is a NN

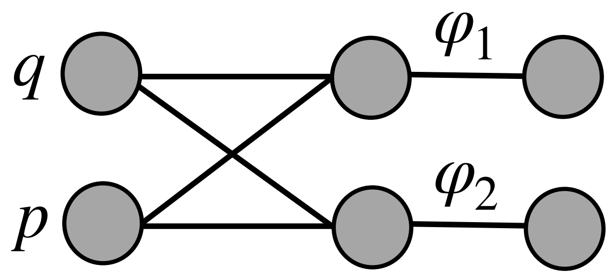
Hamilton dynamics is a NN

1802.08313

$$\dot{q} = \frac{\partial H}{\partial p}, \qquad \dot{p} = -\frac{\partial H}{\partial q}$$

Trial NN representation:

$$\begin{cases} q(t+\Delta t) = \varphi_1(W_{11}q(t) + W_{12}p(t)) \\ p(t+\Delta t) = \varphi_2(W_{21}q(t) + W_{22}p(t)) \end{cases}$$



$\Delta t \to 0$

Consistency requires

$$\begin{cases} W_{ij} = \delta_{ij} + \Delta t w_{ij} \\ \varphi_i(x) = x + \Delta t g_i(x) \end{cases}$$

$$\begin{cases} \dot{q} = w_{11}q + w_{12}p + g_1(q) \\ \dot{p} = w_{21}q + w_{22}p + g_2(p) \end{cases}$$

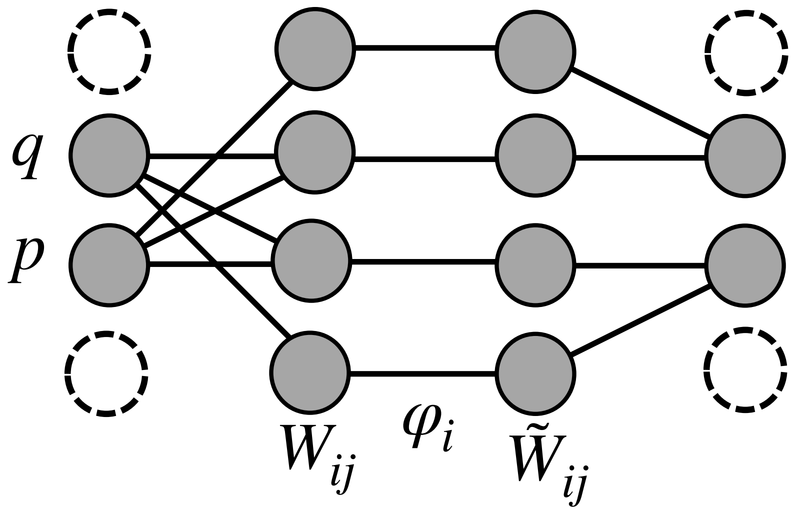
Trivial linear Hamiltonian…

2. Time is a NN

Hamilton dynamics is a NN

$$H = w_{11}pq + \frac{1}{2}w_{12}p^2 - \frac{1}{2}w_{21}q^2 + \frac{\lambda_1}{v}F(vp) - \frac{\lambda_2}{u}G(uq)$$

$$(F' = f, \quad G' = g)$$



$$W = \begin{pmatrix} 0 & 0 & v & 0 \\ 0 & 1+\Delta t w_{11} & \Delta t w_{12} & 0 \\ 0 & \Delta t w_{21} & 1+\Delta t w_{12} & 0 \\ 0 & u & 0 & 0 \end{pmatrix}$$

$$\varphi_i = \begin{pmatrix} \Delta t f(x) \\ 1 \\ 1 \\ \Delta t g(x) \end{pmatrix} \qquad \tilde{W} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \lambda_1 & 1 & 0 & 0 \\ 0 & 0 & 1 & \lambda_2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

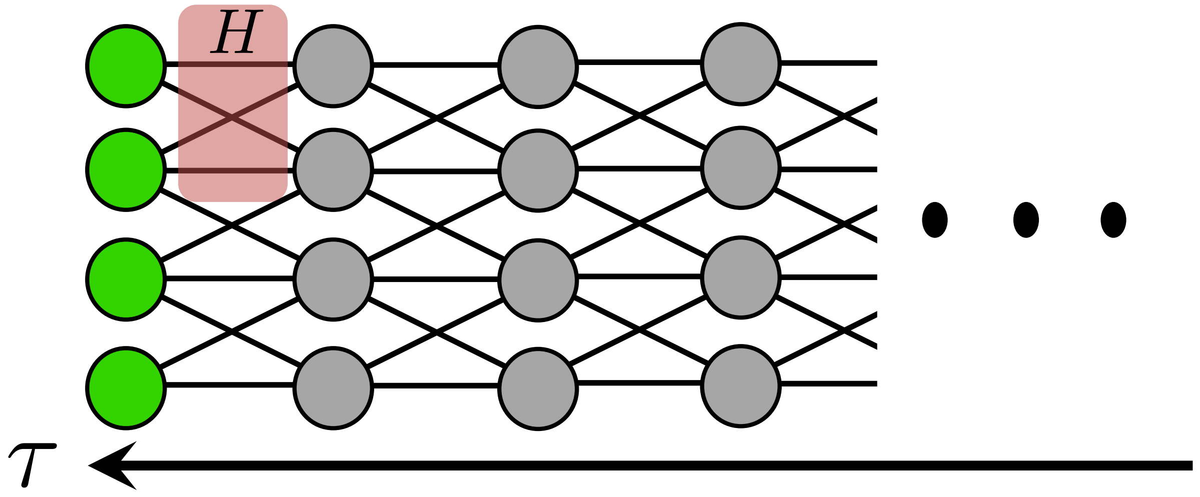
2. Time is a NN

Deep Boltzmann Machine has Euclidean time

Ground state wave function for given Hamiltonian
is identified as a deep Boltzmann machine

[Carleo, Nomura, Imada `18], ..

$$|\psi\rangle = \lim_{\tau \to \infty} e^{-\tau H} |\text{any}\rangle = e^{-\Delta\tau H} e^{-\Delta\tau H} \cdots |\text{any}\rangle$$



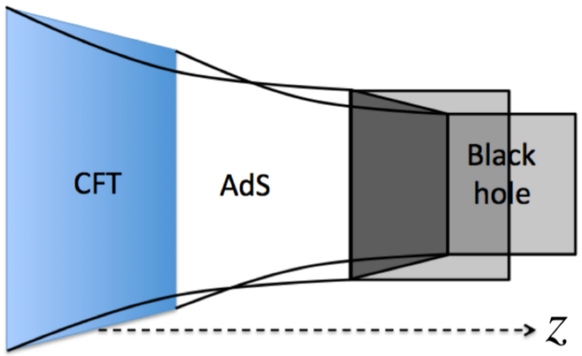
$$\psi(x_i) = \sum_{h_j^{(n)} \in \{0,1\}} \exp\left[-\sum_{ij} w_{ij}^{(0)} x_i h_j - \sum_n \sum_{ij} w_{ij}^{(n)} h_i^{(n)} h_j^{(n+1)} \right]$$

3. Holography is a DBM

AdS/CFT is a Deep Boltzman machine

AdS/CFT

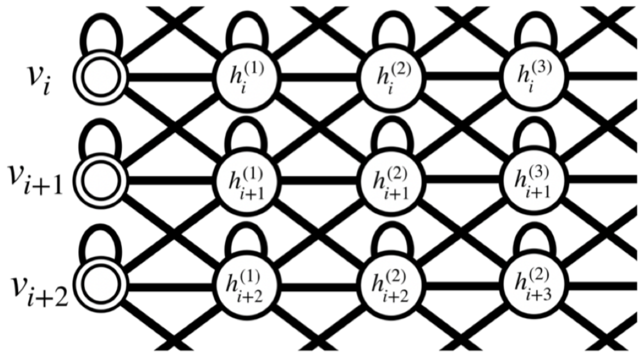
[Maldacena 1997]



$$Z_{\mathrm{QFT}}[J] = \int_{\phi(z=0)=J} \mathcal{D}\phi \, \exp\left(-S_{\mathrm{gravity}}[\phi]\right)$$

Deep Boltzman machine

[Salakhutdinov, Hinton 2009]



$$P(v_i) = \sum_{h_i \in \{0,1\}} \exp\left[-\mathcal{E}(v_i, h_i)\right]$$

[KH `19] [You,Yang,Qi `18] (See also [Gan,Shu `17][Howard `18])

3. Holography is a DBM

Detailed mapping possible

Scalar field theory in curved spacetime

$$S = \int d^d x dz \, \frac{1}{2} \left[a(z)(\partial_z \phi)^2 + b(z) \sum_{I=1}^{d-1} (\partial_I \phi)^2 + d(z)(\partial_\tau \phi)^2 + c(z) m^2 \phi^2 \right]$$

Proper discreti-zation

$$(\partial_z \phi)^2 = \lim_{\Delta z \to 0} \frac{\left(\phi(z_{k+1}) - \phi(z_k)\right)^2}{(\Delta z)^2}$$

$$(\partial_\tau \phi)^2 = \lim_{\Delta \tau, \Delta z \to 0} \left[\frac{\phi(x_{i,l+1}, z_k) - \phi(x_{i,l}, z_k)}{\Delta \tau} \cdot \frac{\phi(x_{i,l+1}, z_{k+1}) - \phi(x_{i,l}, z_{k+1})}{\Delta \tau} \right]$$

$$h_{i,l}^{(k)} \equiv \phi(x_{i,l}, z_k)$$

Energy function of deep Boltzmann

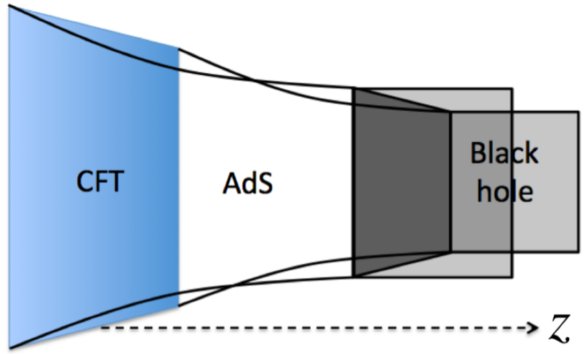
$$S = \mathcal{E} \equiv \sum_k \left[\sum_{i,j} \sum_{l,m} \left\{ w_{ij,lm}^{(k)} h_{i,l}^{(k)} h_{j,m}^{(k+1)} + \widetilde{w}_{ij,lm}^{(k)} h_{i,l}^{(k)} h_{j,m}^{(k)} \right\} \right]$$

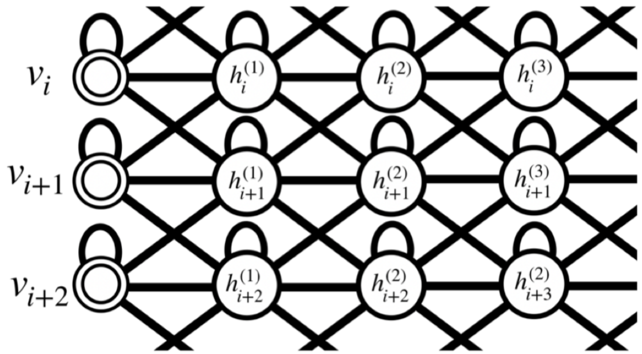
$$w_{ij,lm}^{(k)} \equiv -\frac{a_k}{(\Delta z)^2} \delta_i^j \delta_l^m + \frac{b_k}{2(\Delta x)^2} \left(2\delta_i^j \delta_l^m - \delta_{i+1}^j \delta_l^m - \delta_i^{j+1} \delta_l^m \right) + \frac{d_k}{2(\Delta \tau)^2} \left(2\delta_i^j \delta_l^m - \delta_i^j \delta_{l+1}^m - \delta_i^j \delta_l^{m+1} \right)$$

$$\widetilde{w}_{ij,lm}^{(k)} \equiv \left(\frac{a_k + a_{k-1}}{2(\Delta z)^2} + m^2 \frac{c_k}{2} \right) \delta_i^j \delta_l^m$$

3. Holography is a DBM

Dictionary: Spacetime is a NN





AdS/CFT	Deep Boltzmann machine
Bulk coordinate z	Hidden layer label k
QFT source $J(x)$	Input value v_i
Bulk field $\phi(x,z)$	Hidden variables $h_i^{(k)}$
QFT generating function $Z[J]$	Provability distribution $P(v_i)$
Bulk action $S[\phi]$	Energy function $\mathcal{E}(v_i, h_i^{(k)})$

4. Holographic space is a NN

Simplest holographic model

Classical scalar field probing 5-dim. curved spacetime

$$S = \int d\eta d^4x \sqrt{\det g} \left[(\partial_\eta \phi)^2 - V(\phi) \right]$$

1802.08313
1809.10536

$$ds^2 = -f(\eta)dt^2 + d\eta^2 + g(\eta)(dx_1^2 + \cdots + dx_{d-1}^2)$$

- AdS boundary ($\eta \sim \infty$) : $f \sim g \sim \exp[2\eta/L]$
- Black hole horizon ($\eta \sim 0$) : $f \sim \eta^2, \; g \sim \text{const.}$

Solve eq. of motion to get response $\langle \bar{\psi}\psi \rangle_{m_q}$. [Klebanov, Witten `98]

- AdS boundary ($\eta \sim \infty$) : $\phi = m_q e^{-\eta} + \langle \bar{\psi}\psi \rangle e^{-3\eta}$
- Black hole horizon ($\eta \sim 0$) : $\partial_\eta \phi \Big|_{\eta=0} = 0$

4. Holographic space is a NN

AdS scalar field as a feedforward NN

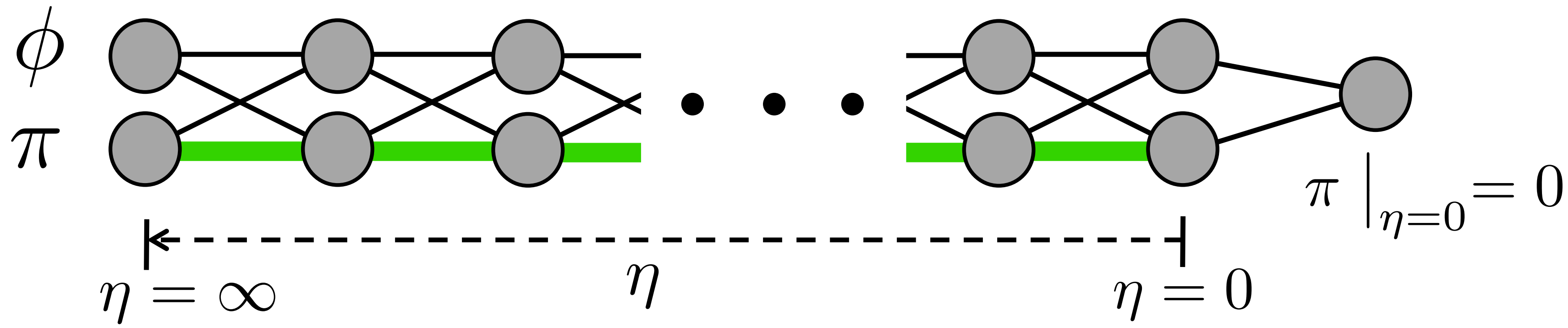
Eq. of motion $\partial_\eta^2\phi + \underline{\underline{h(\eta)}}\partial_\eta\phi - \dfrac{\delta V[\phi]}{\delta\phi} = 0$

metric $h(\eta) \equiv \partial_\eta \left[\log\sqrt{f(\eta)g(\eta)^{d-1}}\right]$

Discretization
Hamilton form

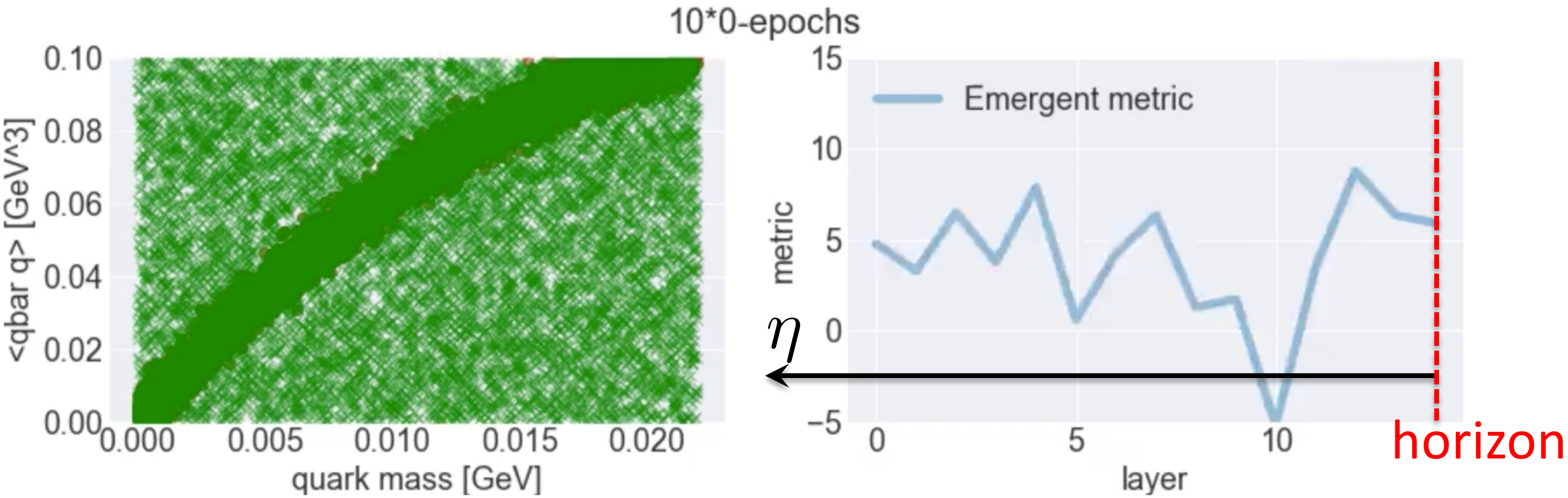
$$\begin{cases} \phi(\eta+\Delta\eta) = \phi(\eta) + \Delta\eta\,\pi(\eta) \\ \pi(\eta+\Delta\eta) = \pi(\eta) + \Delta\eta\left(h(\eta)\pi(\eta) - \dfrac{\delta V(\phi(\eta))}{\delta\phi(\eta)}\right) \end{cases}$$

Feedfoward neural network representation



4. Holographic space is a NN

Training with QCD data: chiral condensate



Learned value of (AdS radius)$^{-1}$: 1/L = 237(3)[MeV]

bulk scalar self coupling : λ/L = 0.0127(6)

5. Holographic spacetime is a NN

Algebraize PDE by Fourier transformation

[Karch, Kaz, Son, Stephanov `06]

Classical gauge theory in 5-d dilaton gravity background

$$S = \int d^4x dz\, e^{-\Phi}\sqrt{-g}\,\left(F_{MN}\right)^2$$

Dilaton $\Phi(z)$, metric $ds^2 = e^{2A(z)}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$

AdS boundary ($z \sim 0$) : $B(z) \equiv \Phi(z) - A(z) \sim \log z$

Solve EoM for gauge field $A_\mu(z, x^\mu) = v_n(z)\rho_\mu(x^\mu)$

$$\frac{\partial}{\partial z}\left(e^{-B}\frac{\partial}{\partial z}v_n\right) + \omega^2 e^{-B} v_n = 0$$

When frequency takes a proper discrete value $\omega^2 \sim m_n^2$,
gauge field is normalizable : vector meson spectra.

5. Holographic spacetime is a NN

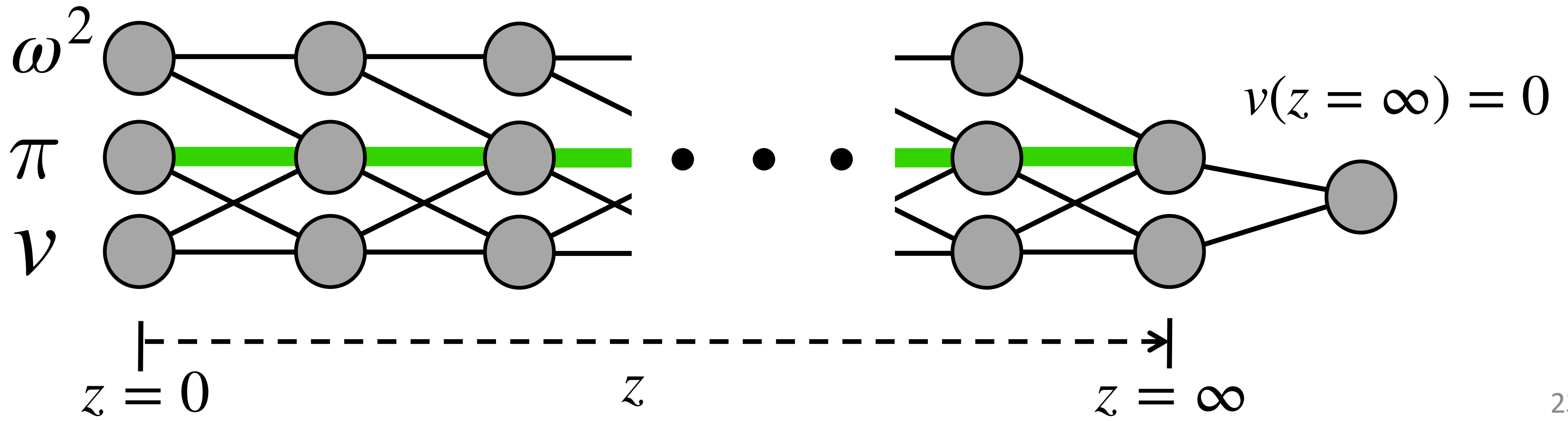
Algebraize PDE by Fourier transformation

Bulk EoM
$$\frac{\partial}{\partial z}\left(e^{-B}\frac{\partial}{\partial z}v_n\right)+\omega^2 e^{-B}v_n=0$$

Discretization

Hamilton form
$$\begin{cases} v_n(z+\Delta z)=v_n(z)+\Delta z\,\pi_n(z) \\ \pi_n(z+\Delta z)=\pi_n(z)+\Delta z\left(B'(z)\pi_n(z)-\omega^2 v_n(z)\right) \end{cases}$$

Neural-Network representation



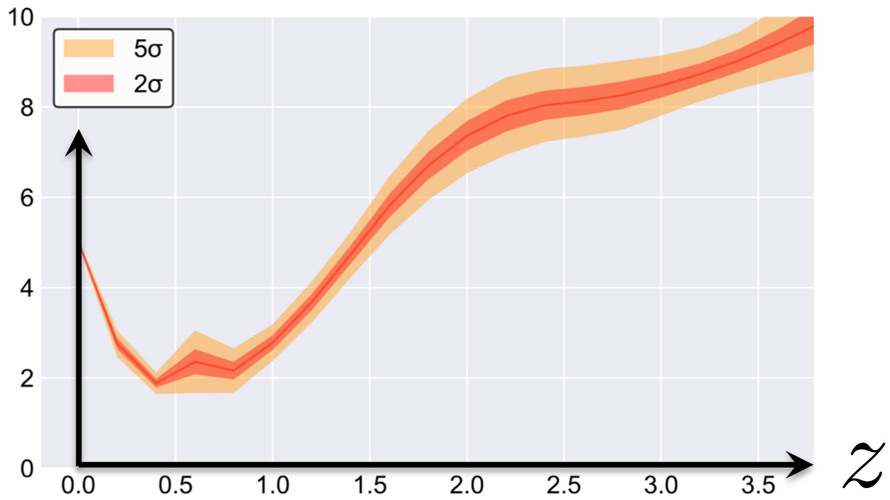
5. Holographic spacetime is a NN

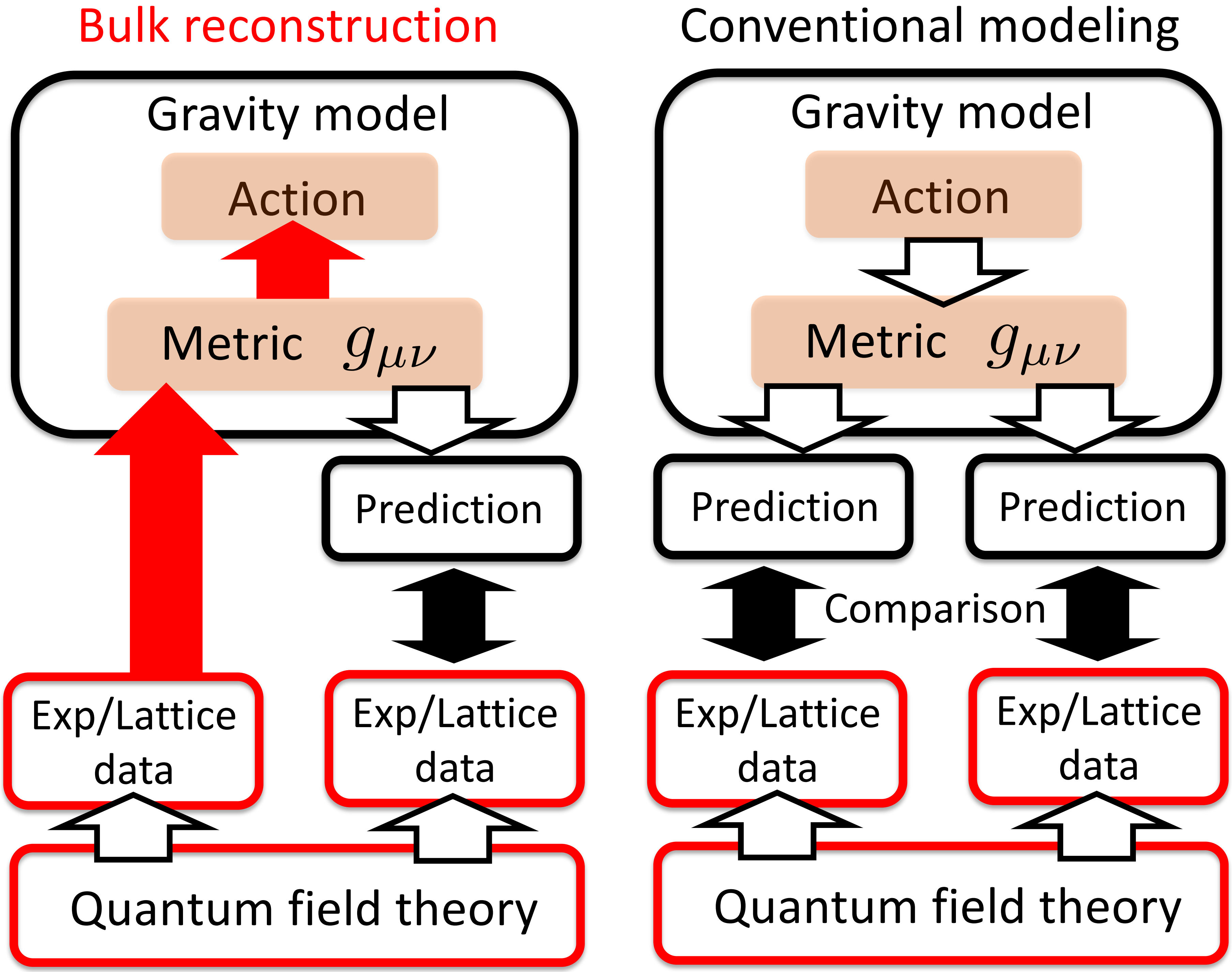
Training with QCD data: hadron spectra

Input : PDG data for rho meson mass

$$m_\rho^{(1)} = 0.77\ \text{GeV},\ m_\rho^{(2)} = 1.45\ \text{GeV}$$

Result: Emergent metric $B'(z) = \Phi'(z) - A'(z)$





Bulk reconstruction
Gravity model
Action
Metric $g_{\mu\nu}$
Prediction
Exp/Lattice data
Exp/Lattice data
Quantum field theory
Conventional modeling
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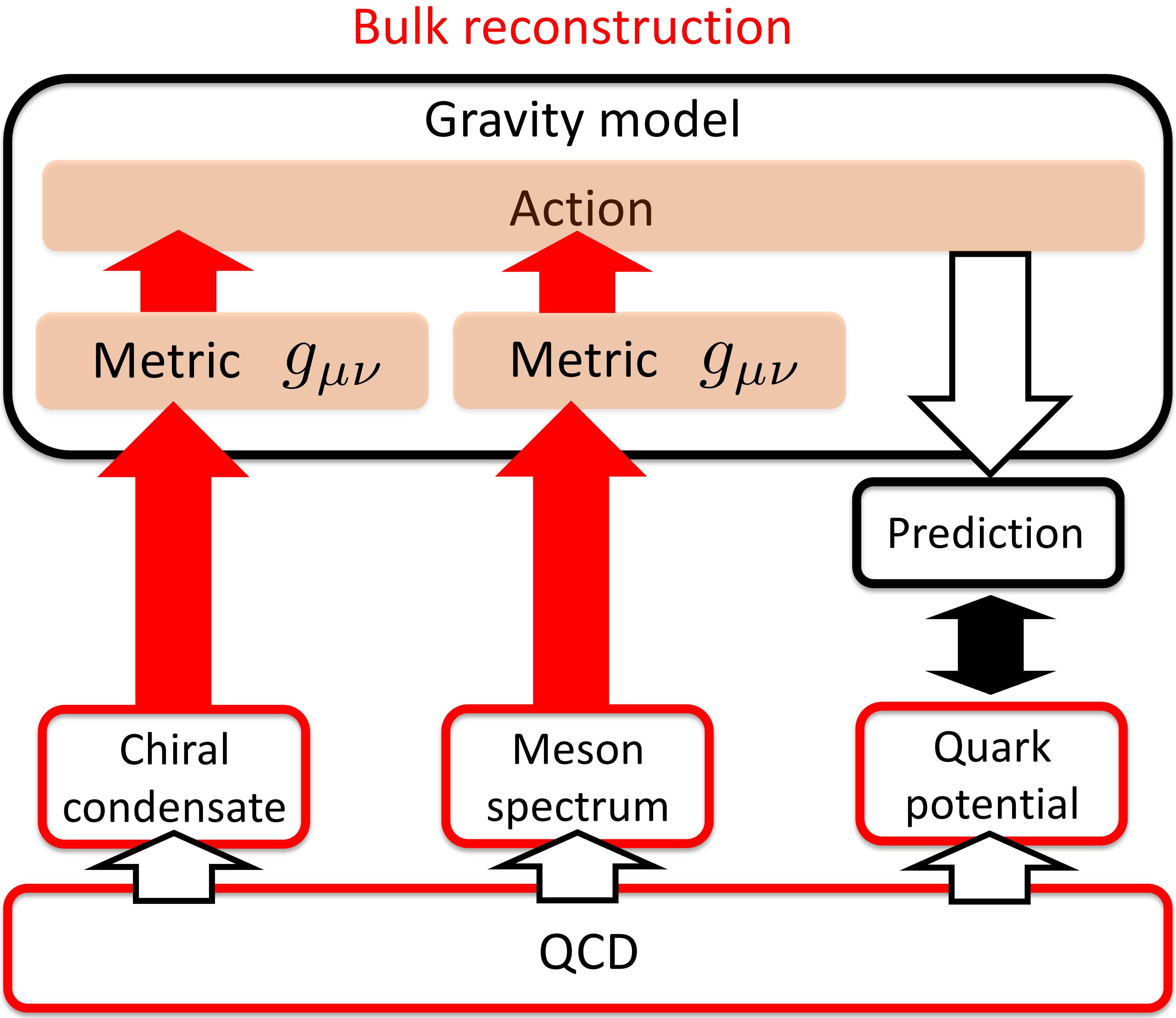


Bulk reconstruction
Gravity model
Action
Metric $g_{\mu\nu}$
Metric $g_{\mu\nu}$
Prediction
Chiral condensate
Meson spectrum
Quark potential
QCD

6. Gravity reconstructed

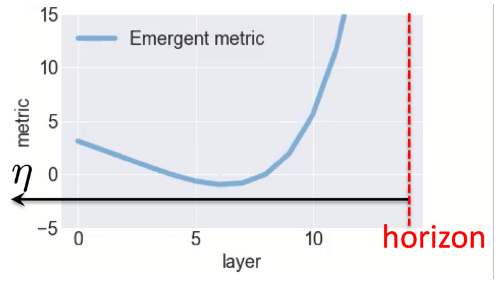
Deriving the dilaton potential

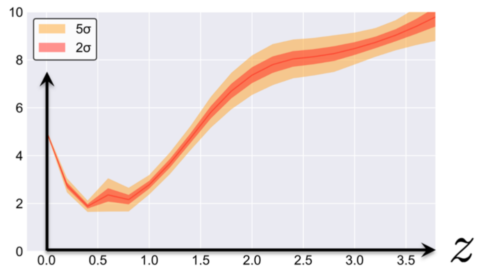
Gravity model

Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$

Metric $g_{\mu\nu}$

Metric $g_{\mu\nu}$





Chiral condensate

Meson spectrum

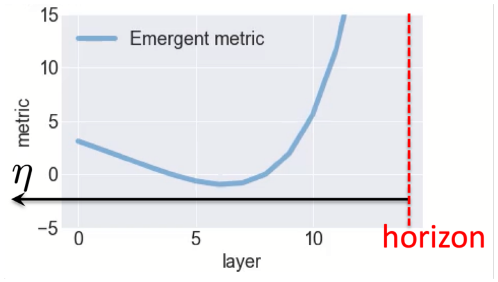
6. Gravity reconstructed

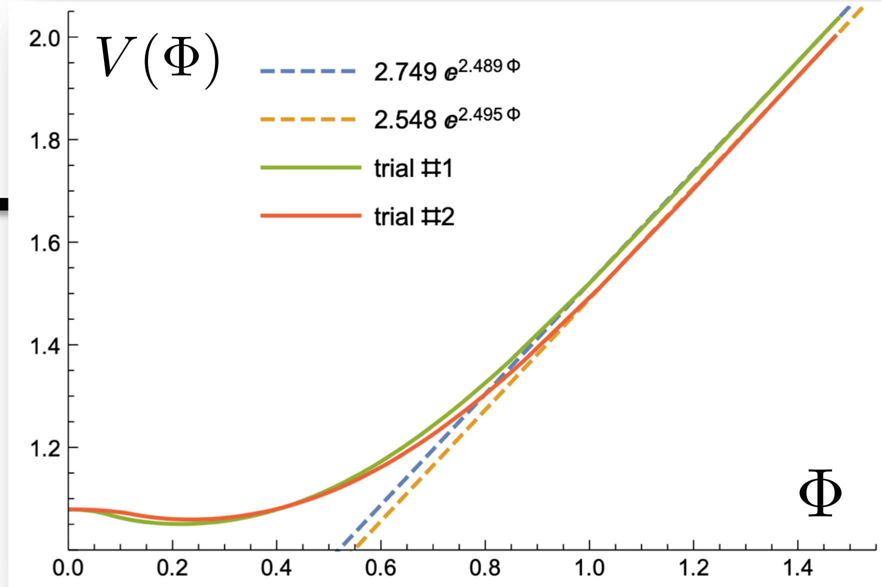
Deriving the dilaton potential

Gravity model

Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$

Metric $g_{\mu\nu}$

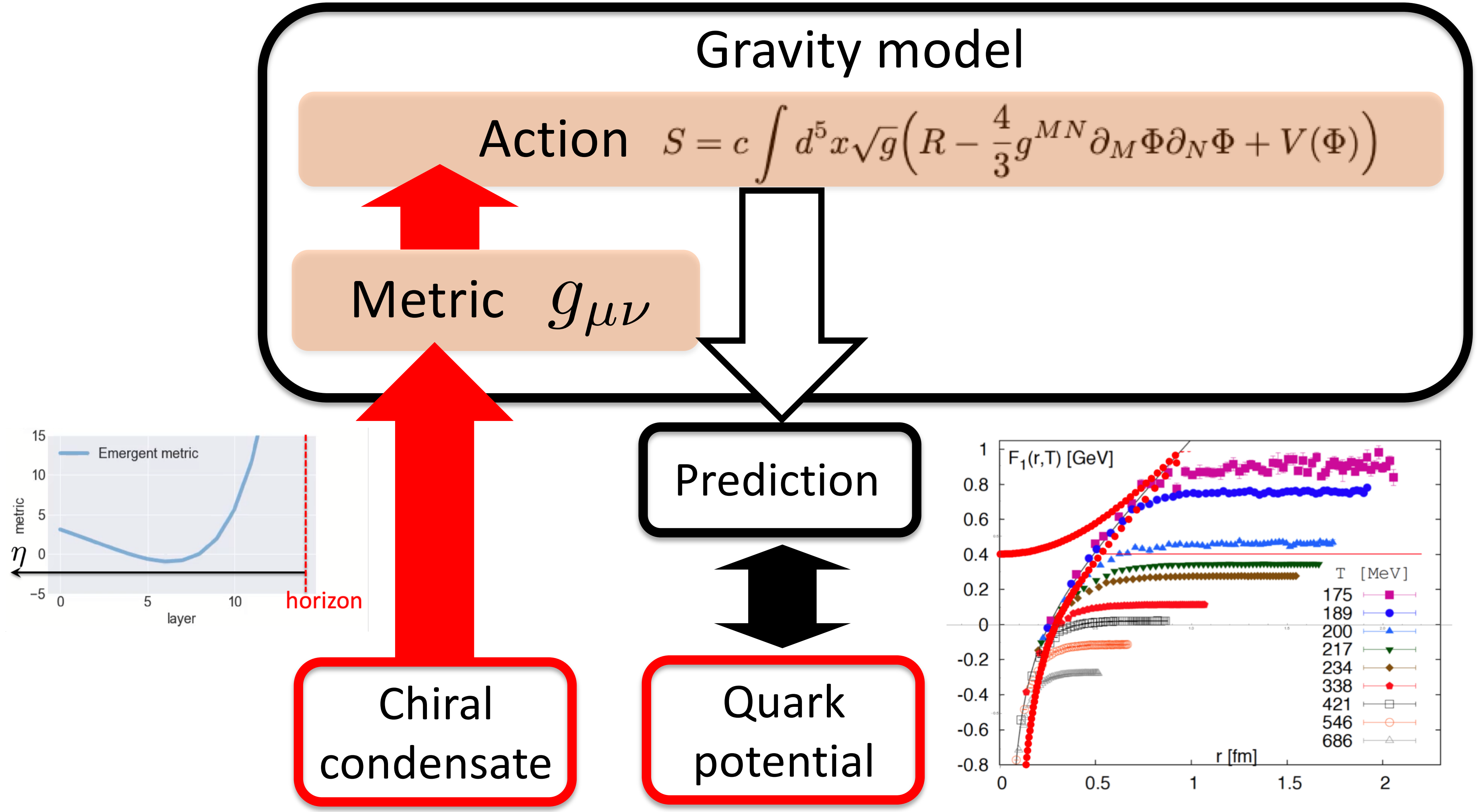




Chiral condensate

6. Gravity reconstructed

Prediction of string breaking



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