

# Machine Learning Prediction and Compression of Lattice QCD Observables

Boram Yoon

(Los Alamos National Laboratory)

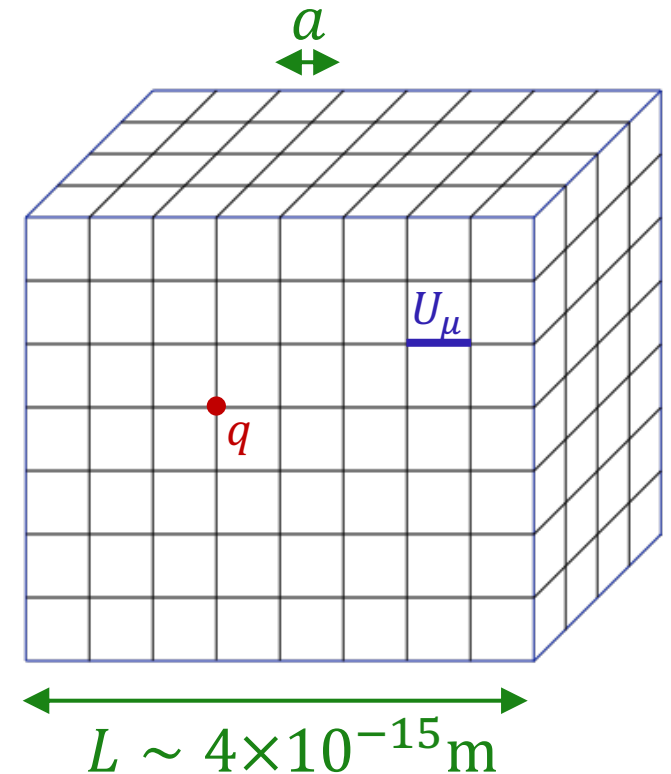
# Contents

- 1) Correlations in Lattice QCD Observables
- 2) Prediction of Lattice QCD Observables using ML
- 3) ML Regression Algorithm using D-Wave Quantum Annalaer
- 4) Lossy Data Compression Algorithm for Lattice QCD Data
- 5) Summary

# Correlations in Lattice QCD Observables

# Lattice QCD

- Non-perturbative approach to solving QCD on **discretized Euclidean space-time**
  - Hypercubic lattice
  - Lattice spacing  $a$
  - Quark fields placed on sites
  - Gluon fields on the links between sites;  $U_\mu$
- Numerical lattice QCD calculations using Monte Carlo methods
  - Computationally intensive
  - Use supercomputers
- Continuum results are obtained in  $a \rightarrow 0$
- Has been successful for many QCD observables
  - Some results are with less than 1% error



# Lattice QCD

- Correlation functions

$$\begin{aligned}\langle O \rangle &= Z^{-1} \int dU dq d\bar{q} O(U, q, \bar{q}) e^{-S_g - \bar{q}(D + m_q)q} \\ &= Z^{-1} \int dU \left[ O \left( U, (D + m_q)^{-1} \right) e^{-S_g} \det(D + m_q) \right]\end{aligned}$$

- Monte-Carlo integration

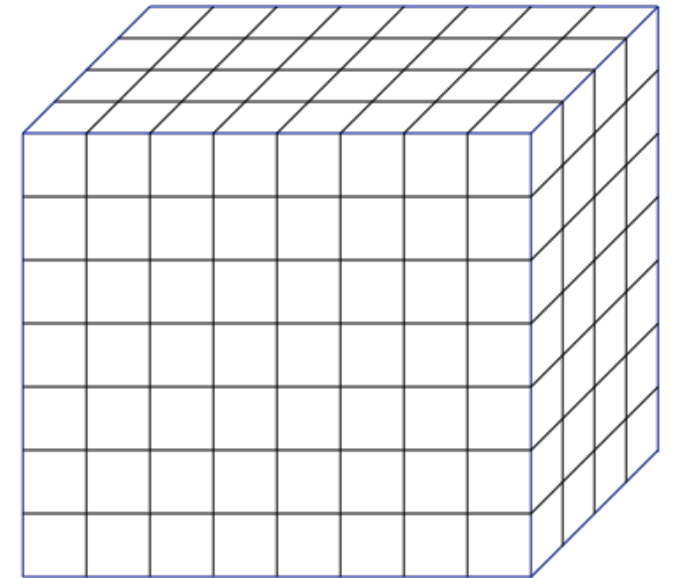
- Integration variable  $U$  is huge

$$N_s^3 \times N_t \times 4 \times 8 \sim 10^9$$

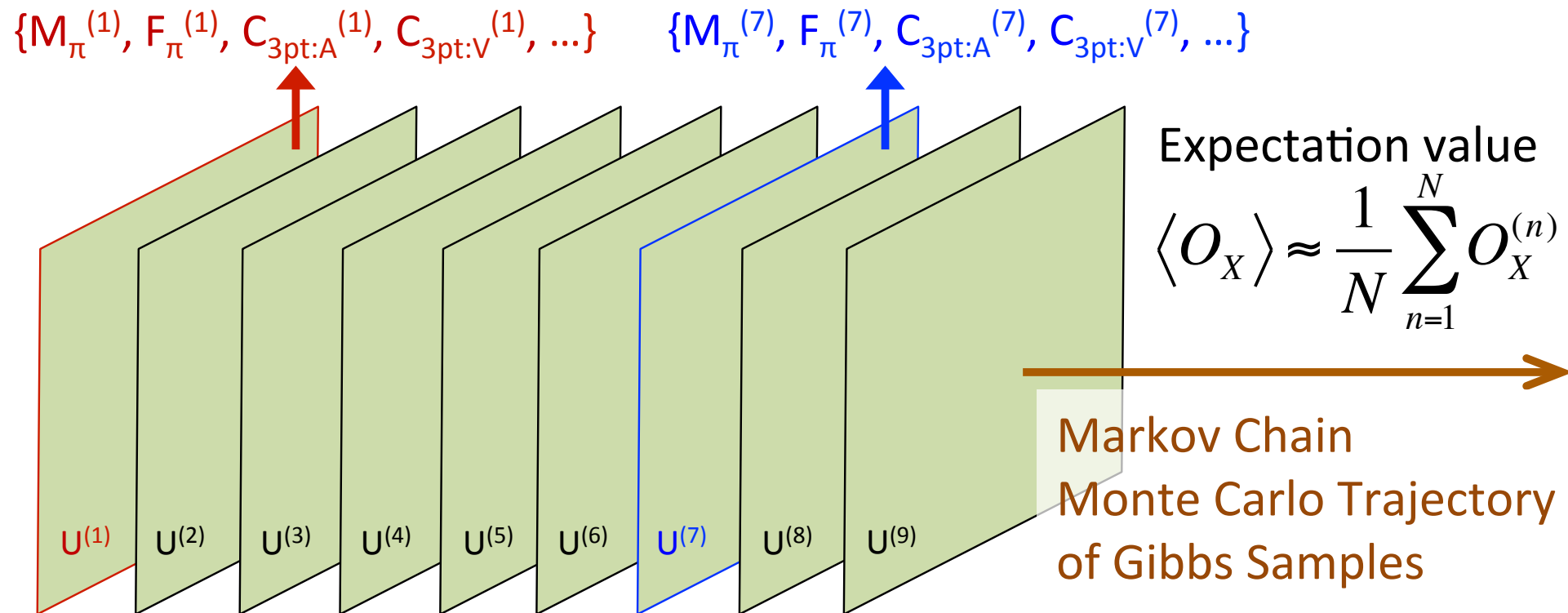
- Generate Markov chain of gauge configurations  $U$
- Calculate average as expectation value

$$\langle O \rangle \approx \frac{1}{N} \sum_i O_i \left( U, (D + m_q)^{-1} \right)$$

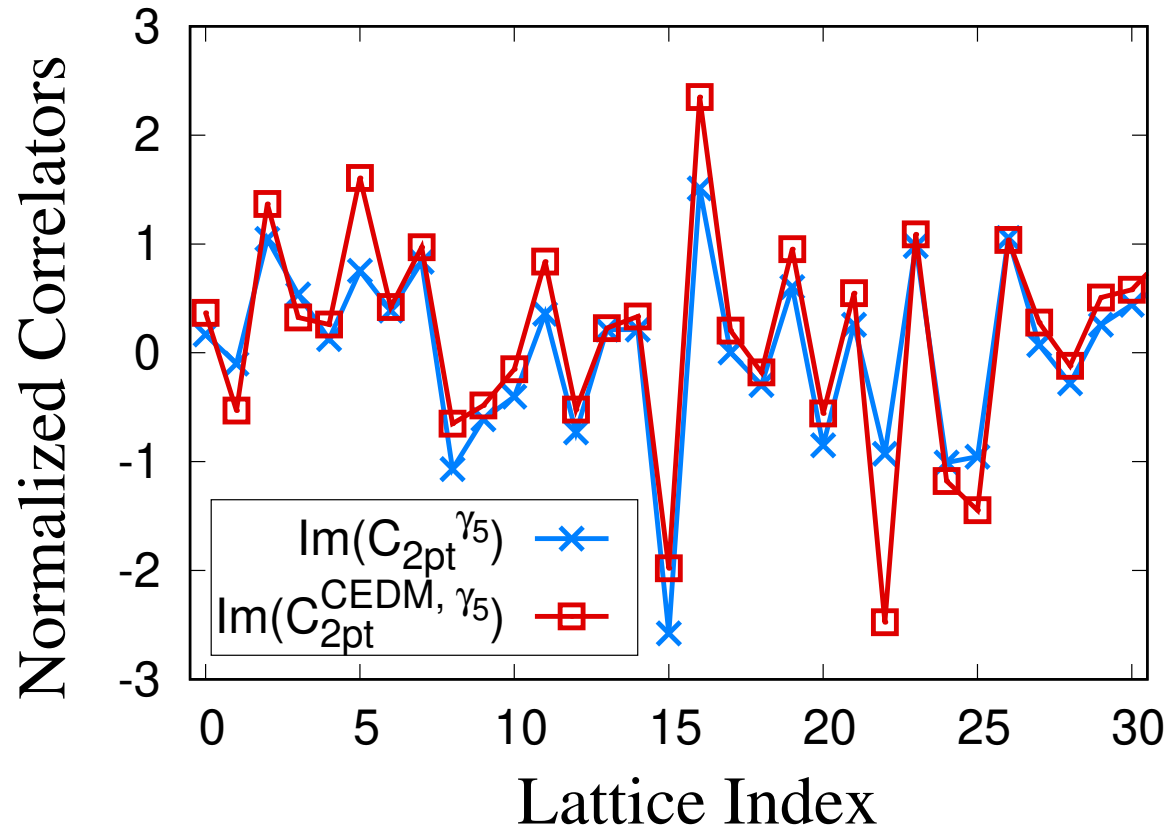
- Calculation of  $O_i \left( U, (D + m_q)^{-1} \right)$ : measurement
- $(D + m)^{-1}$  is computationally expensive



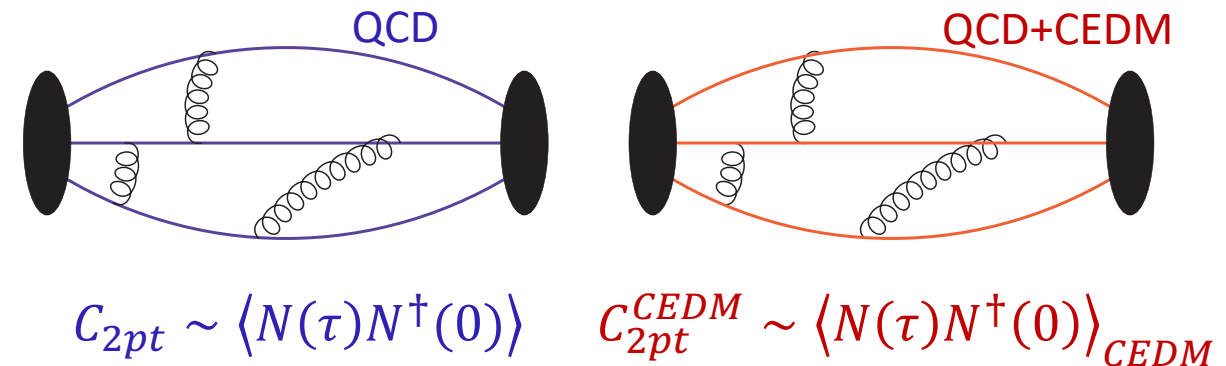
# Lattice QCD Observables are Correlated



# Correlation Map of Nucleon Observables

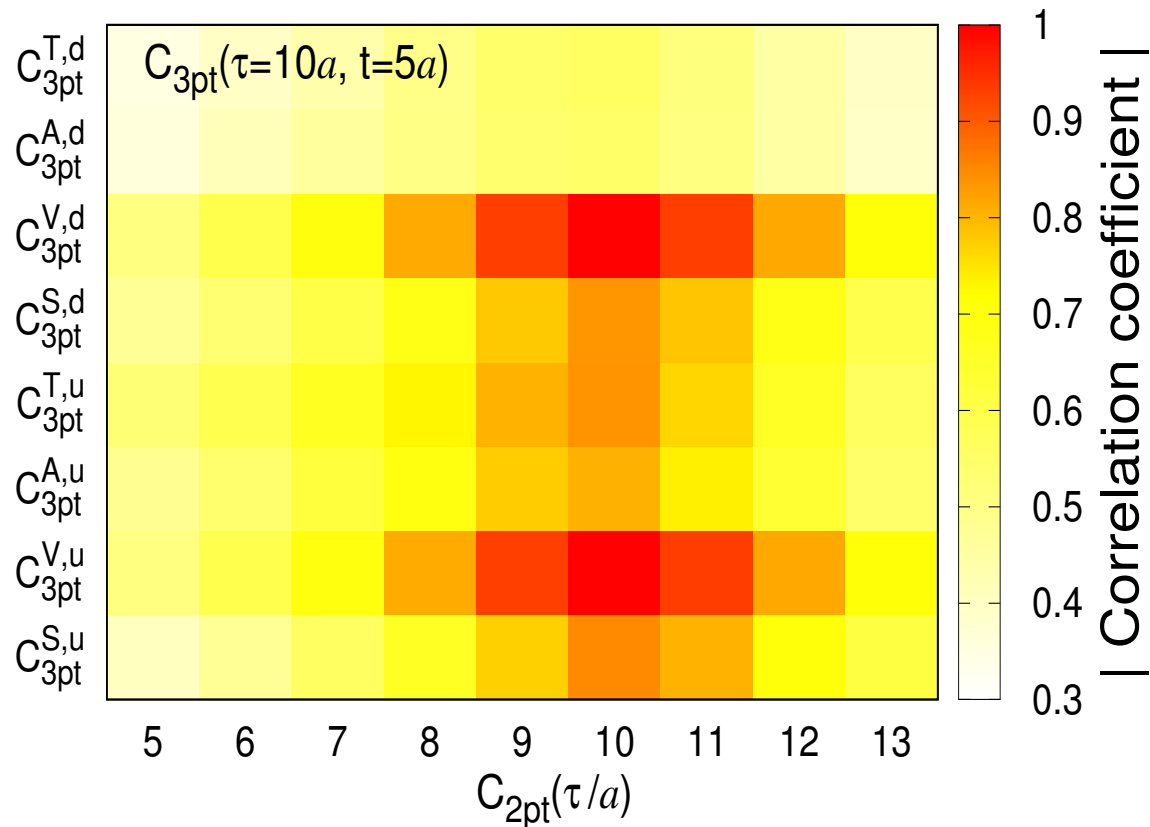


- Correlation between proton(uud) 2-pt correlation function and that calculated in presence of CEDM interaction

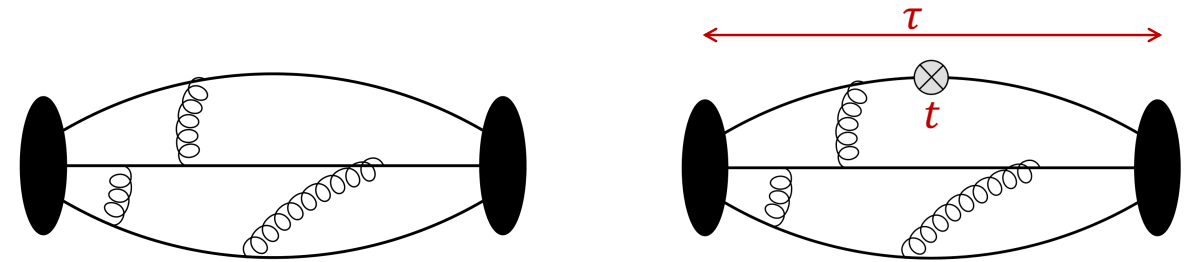


- QCD:  $D_{clov}$   
 QCD+CEDM:  $D_{clov} + \frac{i}{2} \varepsilon \sigma^{\mu\nu} \gamma_5 G_{\mu\nu}$

# Correlation Map of Nucleon Observables



- Correlation between proton(uud) 3-pt and 2-pt correlation functions



$$C_{2pt} \sim \langle N(\tau) N^\dagger(0) \rangle \quad C_{3pt}^{A,S,T,V} \sim \langle N(\tau) O(t) N^\dagger(0) \rangle$$

- Using these correlations,  $C_{3pt}$  can be estimated from  $C_{2pt}$  on each configuration



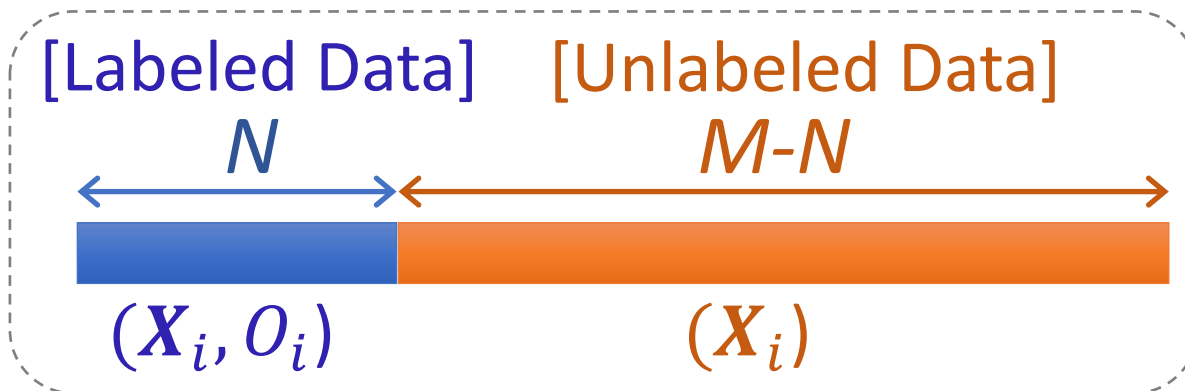
# Prediction of Lattice QCD Observables using ML

Measured and computationally cheap observables

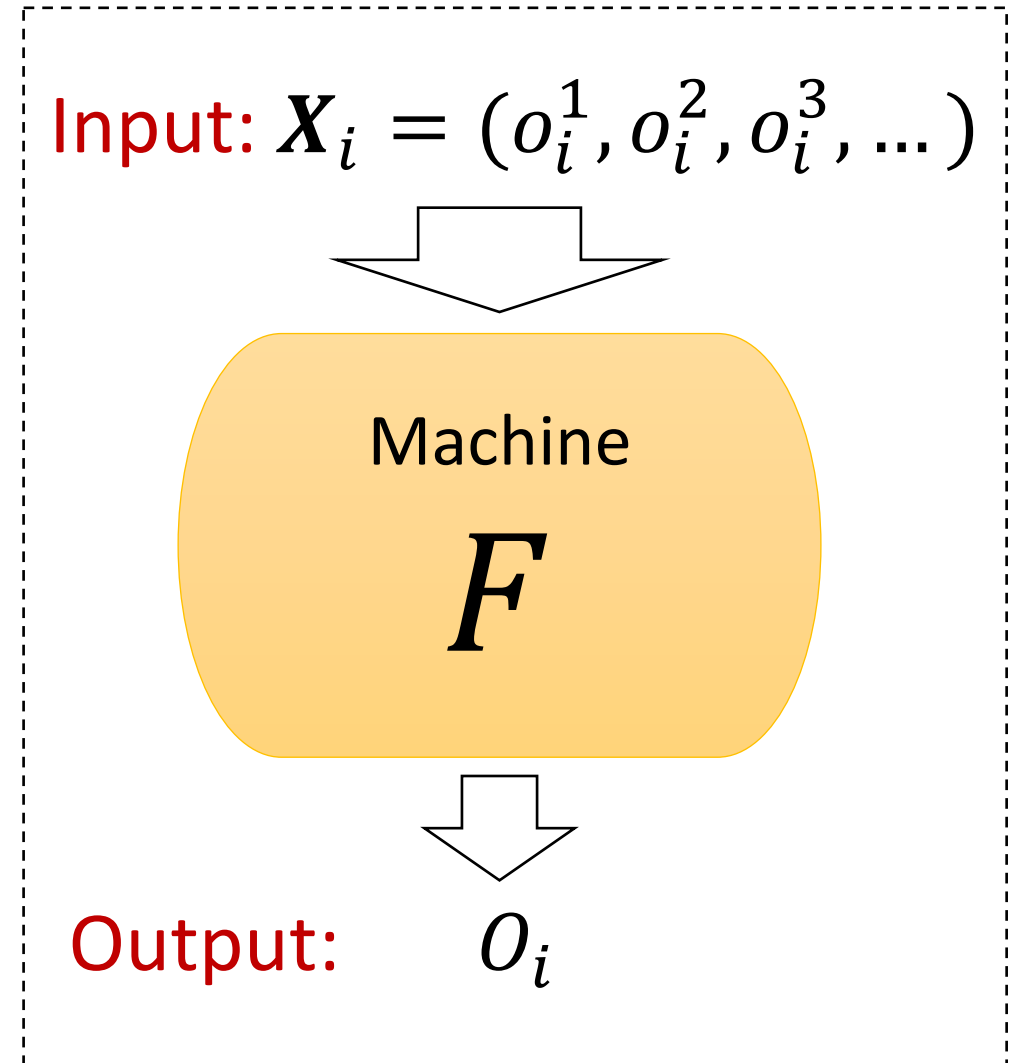
→ Prediction of unmeasured and computationally expensive observables

# Prediction of Lattice QCD Observables using ML

- Assume  $M$  indep. measurements
- Common observables  $\mathbf{X}_i$  on all  $M$   
Target observable  $O_i$  on first  $N$



- 1) **Train** machine  $F$  to yield  $O_i$  from  $\mathbf{X}_i$  on the Labeled Data
- 2) **Predict**  $O_i$  of the Unlabeled data from  $\mathbf{X}_i$   
$$F(\mathbf{X}_i) = O_i^P \approx O_i$$



# Prediction Bias

- $F(X_i) = O_i^P \approx O_i$
- Simple average

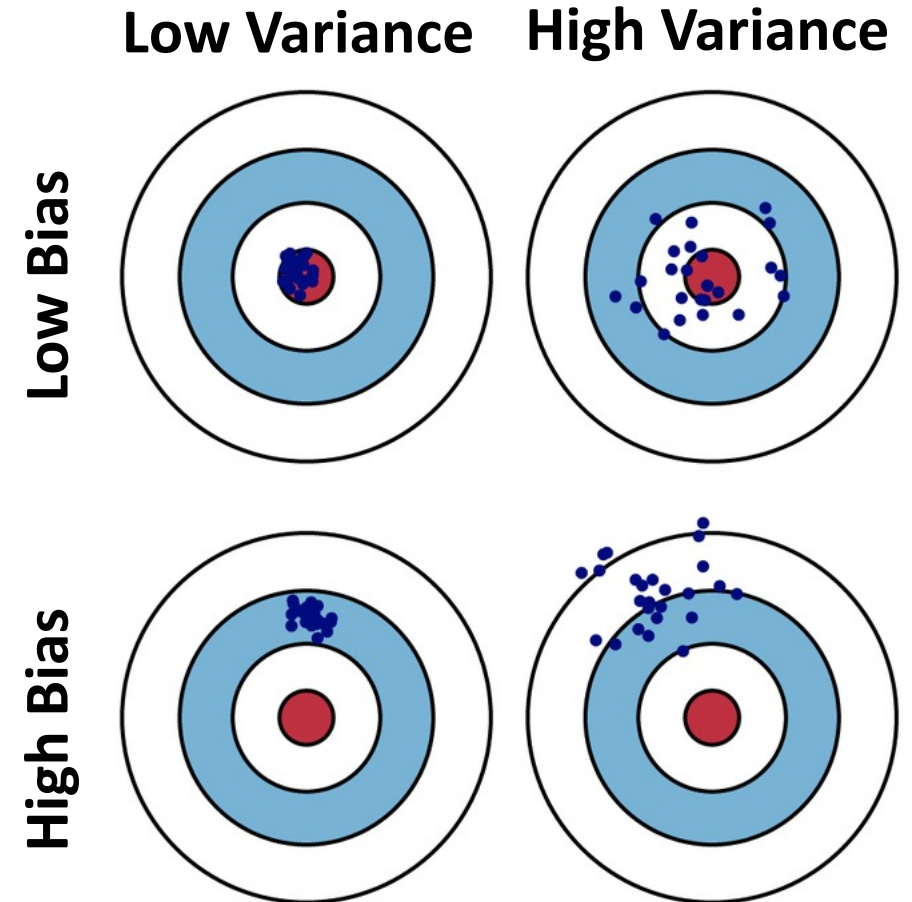
$$\bar{O} = \frac{1}{M} \sum_{i \in \text{Unlabeled}} O_i^P$$

is not correct due to **prediction bias**

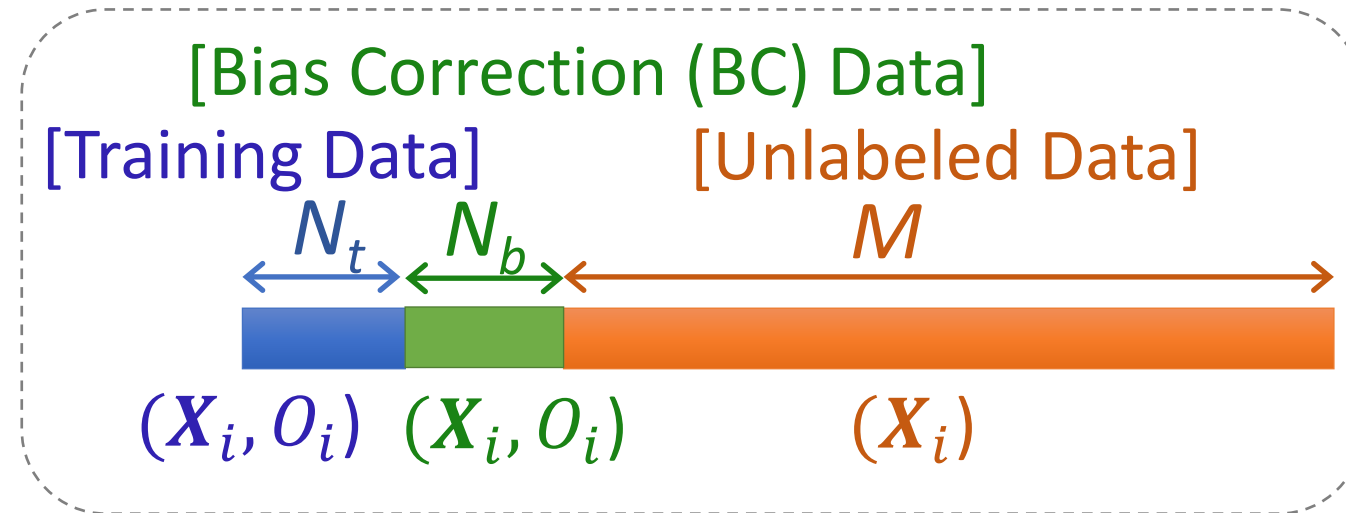
- **Prediction** = **TrueAnswer** + **Noise** + **Bias**
- ML prediction may have bias

$$\langle O^P \rangle \neq \langle O \rangle$$

$$\text{Bias} = \langle O^P \rangle - \langle O \rangle$$



# Bias Correction and Error Quantification



- Split labeled data  $N = N_t + N_b$
- Average of predictions on test data with bias correction

$$\bar{O}_{BC} = \frac{1}{M} \sum_{i \in \text{Unlabeled}} O_i^P + \frac{1}{N_b} \sum_{i \in BC} (O_i - O_i^P)$$

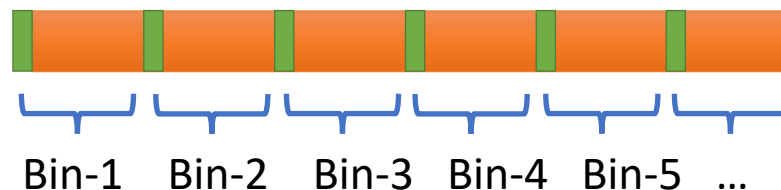
- Expectation value,  $\langle \bar{O}_{BC} \rangle = \langle O^P \rangle + \langle O - O^P \rangle = \langle O \rangle$
- BC term converts **systematic error of prediction** to **statistical uncertainty**

# Incorporating Labeled Data

- Include directly measured values  $O_i$  from labeled data

$$\bar{O}_{BC}^{\text{imp}} = w_1 \times \left( \frac{1}{N} \sum_{i \in \text{Labeled}} O_i \right) + w_2 \times \left( \frac{1}{M} \sum_{i \in \text{Unlabeled}} O_i^P + \frac{1}{N_b} \sum_{i \in BC} (O_i - O_i^P) \right)$$

- $w_1, w_2$ : weights determined based on the (co)variance of two terms
- If you need more than just a simple average in data analysis
  - two different data,  $O_i$  on labeled and  $O_i^P$  on unlabeled samples
  - simultaneous fit on these two data sets with the same fit parameters
  - $O_i$  and  $O_i^P$  have the same mean after BC but may have different variance
- Statistical errors can be estimated using Bootstrap resampling
- Binning and BC for each bin is another option for complicated data analysis



# Quality of Prediction

- Bias-corrected average

$$\bar{o}_{BC} = \frac{1}{M} \sum_{i \in \text{Unlabeled}} o_i^P + \frac{1}{N_b} \sum_{i \in BC} (o_i - o_i^P)$$

- Statistical error of the unbiased average

$$\begin{aligned} \sigma_{\bar{o}_{BC}}^2 &\approx \frac{1}{M} \sigma_{o^P}^2 + \frac{1}{N_{bc}} \sigma_{o-o^P}^2 \\ &\approx \frac{\sigma_o^2}{M} \left( 1 + \frac{M}{N_{bc}} \frac{\sigma_{o-o^P}^2}{\sigma_o^2} \right) \equiv \frac{\sigma_o^2}{M} \left( 1 + \frac{M}{N_{bc}} Q^2 \right); \quad Q^2 \equiv \frac{\sigma_{o-o^P}^2}{\sigma_o^2} \end{aligned}$$

for  $N_{bc}/M = 0.2$

approximations ( $\approx$ ) for small correlation between the two terms and a good prediction algorithm that gives  $\sigma_o^2 \approx \sigma_{o^P}^2$

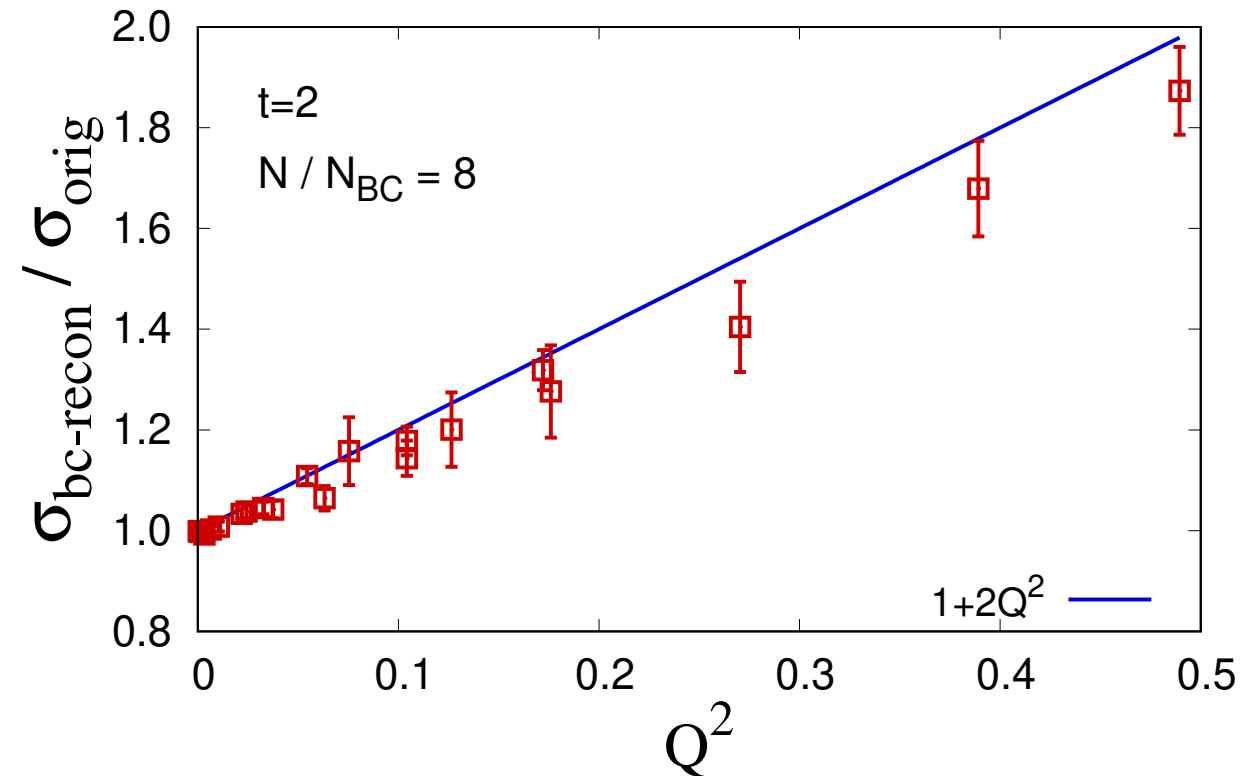
$$\frac{\sigma_{\bar{o}_{BC}}^2}{\sigma_o^2} \approx 1 + \frac{M}{N_{bc}} Q^2; \quad \frac{\sigma_{\bar{o}_{BC}}}{\sigma_o} \approx 1 + \frac{M}{2N_{bc}} Q^2$$

Q	Error Increase
0.5	62.5%
0.3	22.5%
0.1	2.5%

- Q-value shows the expected error-increase due to the ML prediction error
- In practice, BC data have less autocorrelation than full data, because of the many measurements per configuration, so  $\sigma_{\bar{o}_{BC}}$  gives smaller error than expected above

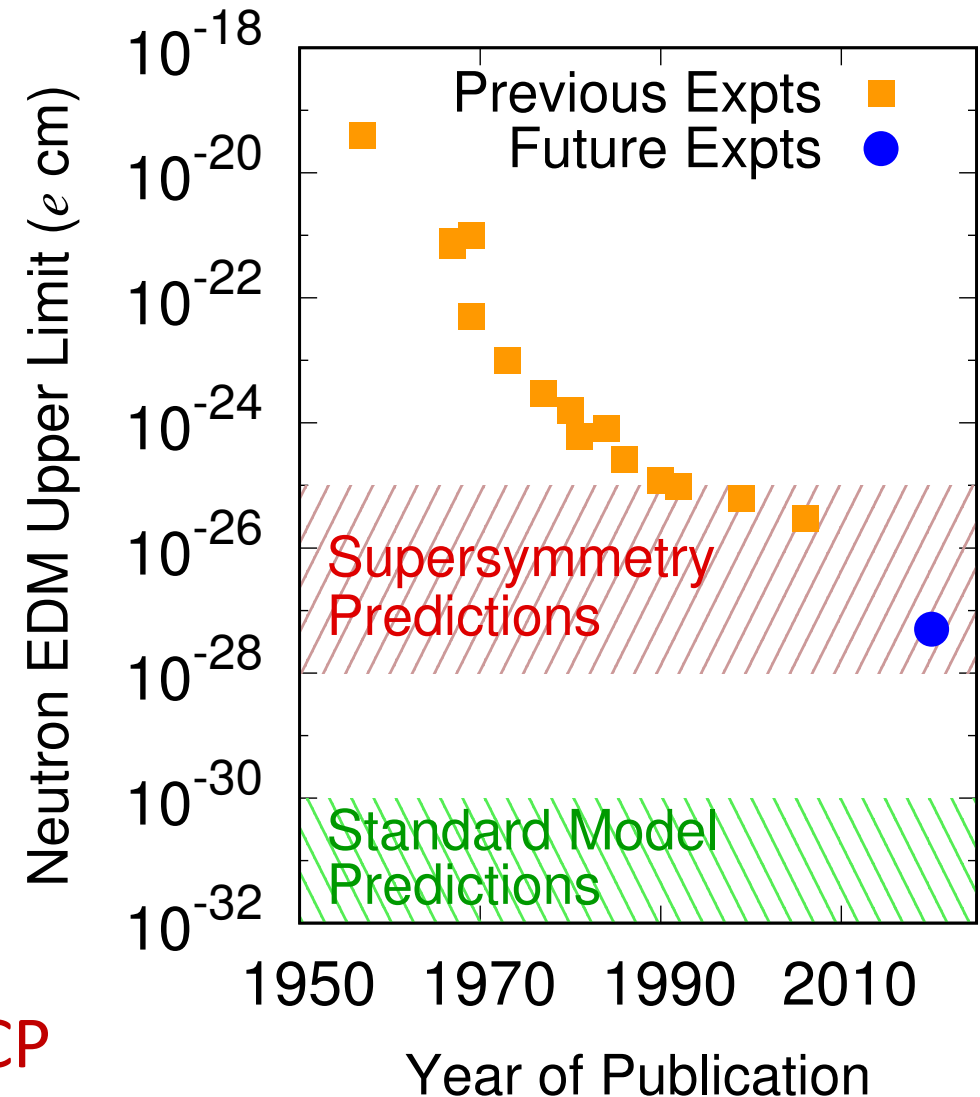
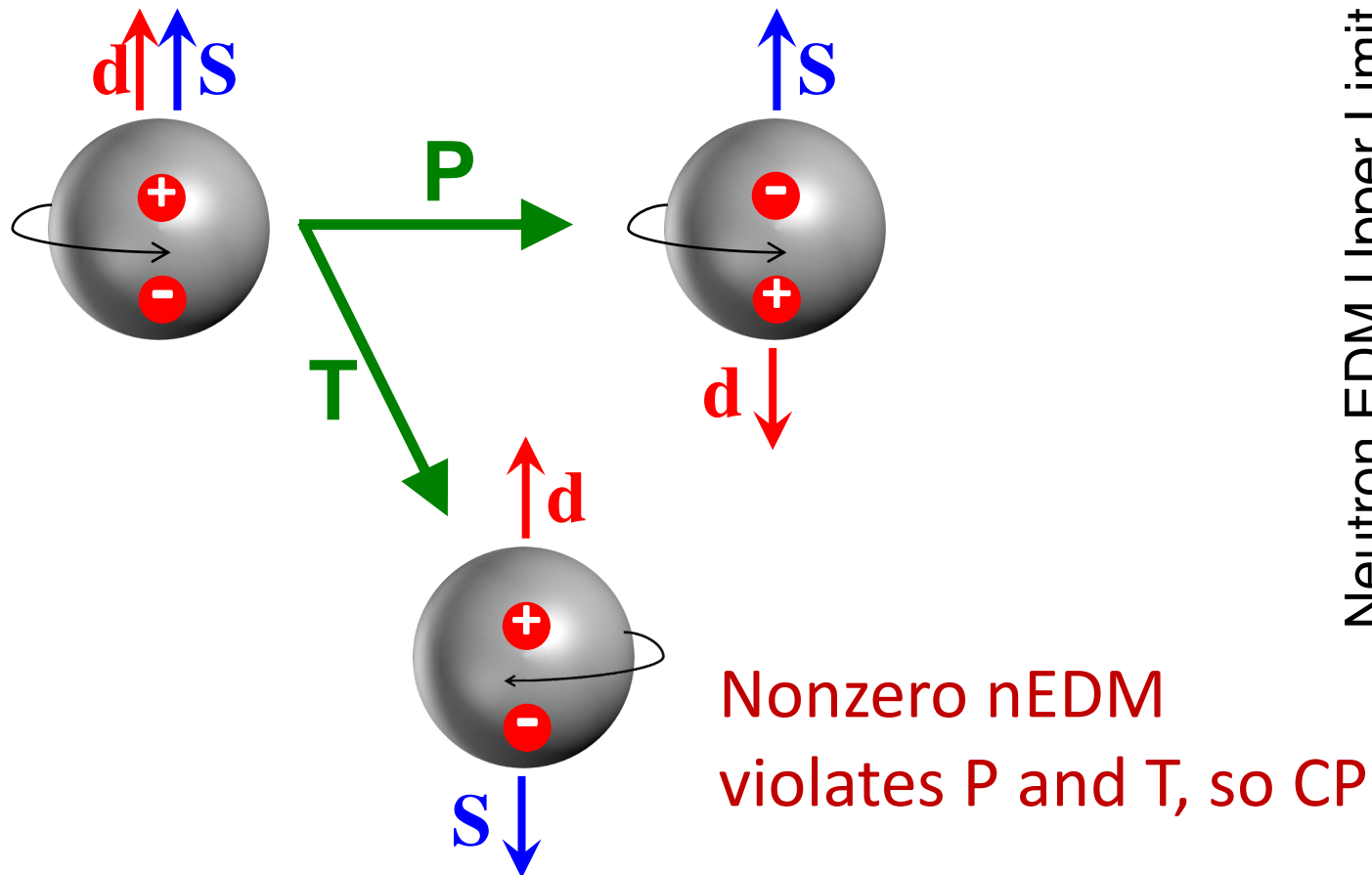
# Statistical Error Increase for Different Q-values

- The statistical error increase is proportional to  $Q^2 \equiv \frac{\sigma_{O-O^P}^2}{\sigma_O^2}$
- For *independent data*, the error increase ratio due to bias correction is expected to be  $1 + \frac{M}{2N_{bc}} Q^2$
- *Correlation between the data samples* makes it  $1 + \alpha \frac{M}{2N_{bc}} Q^2$  with  $0 < \alpha < 1$



# Neutron EDM and CP Violation

- Measures separation between centers of (+) and (-) charges





# Effective CPV Lagrangian

$$\mathcal{L}_{\text{CPV}}^{d \leq 6} = -\frac{g_s^2}{32\pi^2} \bar{\theta} G \tilde{G}$$

dim=4 QCD  $\theta$ -term

$$-\frac{i}{2} \sum_{q=u,d,s} d_q \bar{q} (\sigma \cdot F) \gamma_5 q$$

dim=5 Quark EDM (qEDM)

$$-\frac{i}{2} \sum_{q=u,d,s} \tilde{d}_q g_s \bar{q} (\sigma \cdot G) \gamma_5 q$$

dim=5 Quark Chromo EDM (CEDM)

$$+ d_w \frac{g_s}{6} G \tilde{G} G$$

dim=6 Weinberg 3g operator

$$+ \sum_i C_i^{(4q)} O_i^{(4q)}$$

dim=6 Four-quark operators

# Quark Chromo EDM (cEDM)

- Simulation in presence of CPV cEDM interaction

$$S = S_{QCD} + S_{cEDM}$$

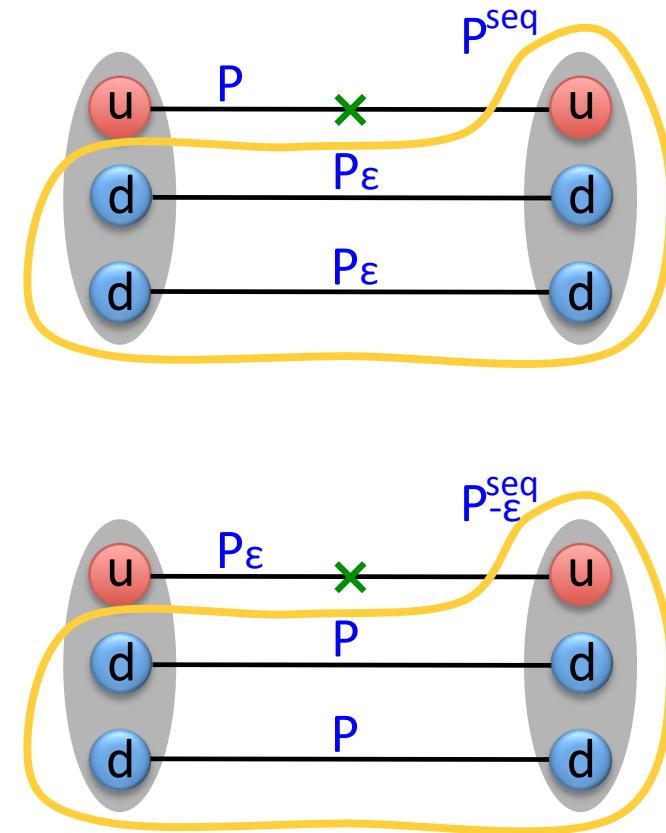
$$S_{cEDM} = -\frac{i}{2} \int d^4x \tilde{d}_q g_s \bar{q} (\sigma \cdot G) \gamma_5 q$$

- Schwinger source method

Include cEDM term in valence quark propagators  
by modifying Dirac operator

$$D_{\text{clov}} \rightarrow D_{\text{clov}} + i\varepsilon \sigma^{\mu\nu} \gamma_5 G_{\mu\nu}$$

- cEDM contribution to nEDM can be obtained by calculating vector form-factor  $F_3$  with propagators including cEDM &  $O_{\gamma_5} = \bar{q} \gamma_5 q$

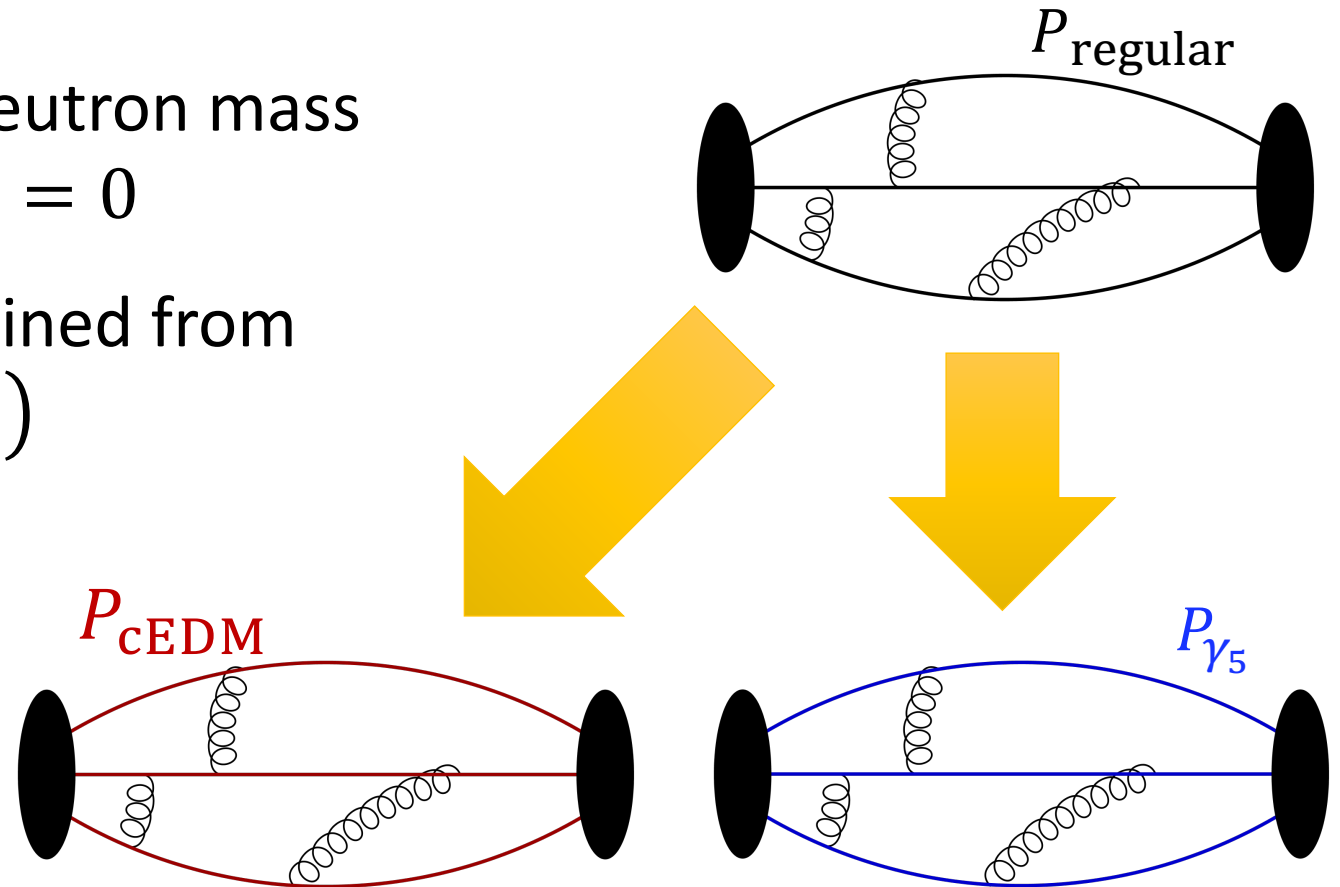
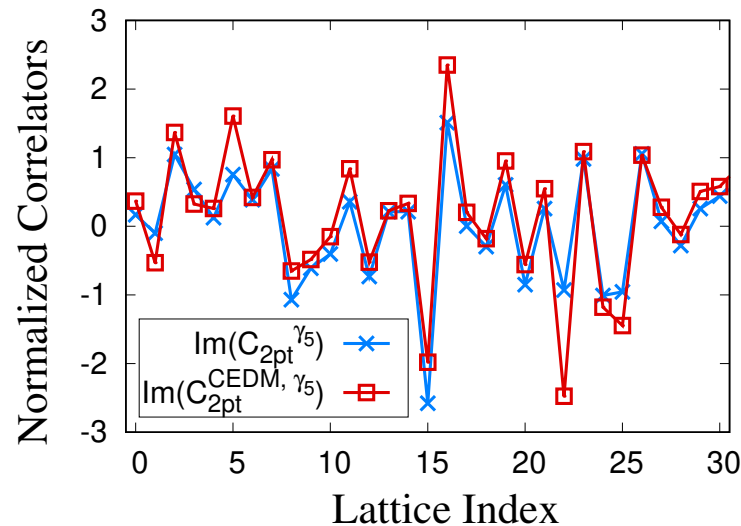


# Prediction of $C_{2pt}^{\text{CPV}}$ from $C_{2pt}$

- Predict  $C_{2pt}$  for **cEDM** and  $\gamma_5$  insertions from  $C_{2pt}$  without CPV
- **CPV interactions**  $\rightarrow$  **phase** in neutron mass  

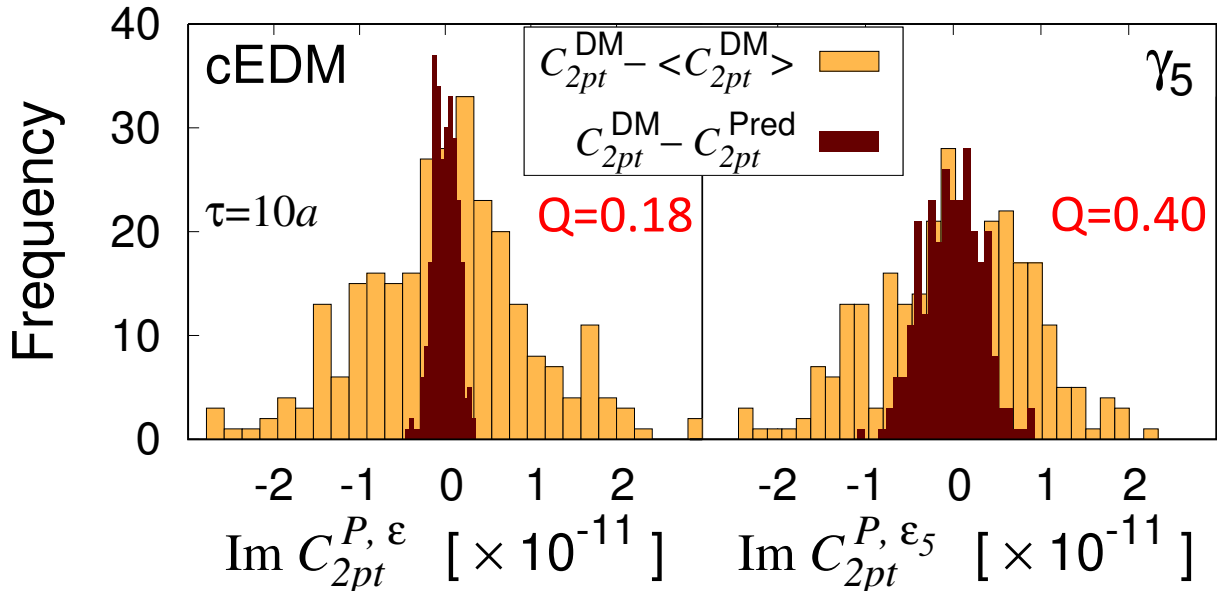
$$(ip_\mu \gamma_\mu + me^{-2i\alpha\gamma_5})u_N = 0$$
- At leading order,  $\alpha$  can be obtained from  

$$C_{2pt}^P \equiv \text{Tr}(\gamma_5 \langle NN^\dagger \rangle)$$



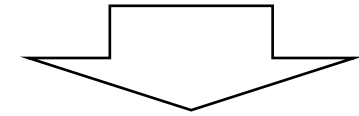
# Prediction of $C_{2pt}^{\text{CPV}}$ from $C_{2pt}$

- Training and Test performed for
  - $a = 0.12$  fm,  $M_\pi = 305$  MeV
  - Measurements: 400 confs  $\times$  64 srcs
- # of training data: 70 confs
- # of BC data: 50 confs
- # of unlabeled data: 280 confs

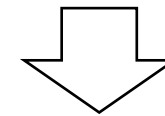


Input:

$$\mathbf{X}_i = \{\text{Re}, \text{Im}[C_{2pt}^{S,P}(0 \leq \tau/a \leq 16)]\}$$

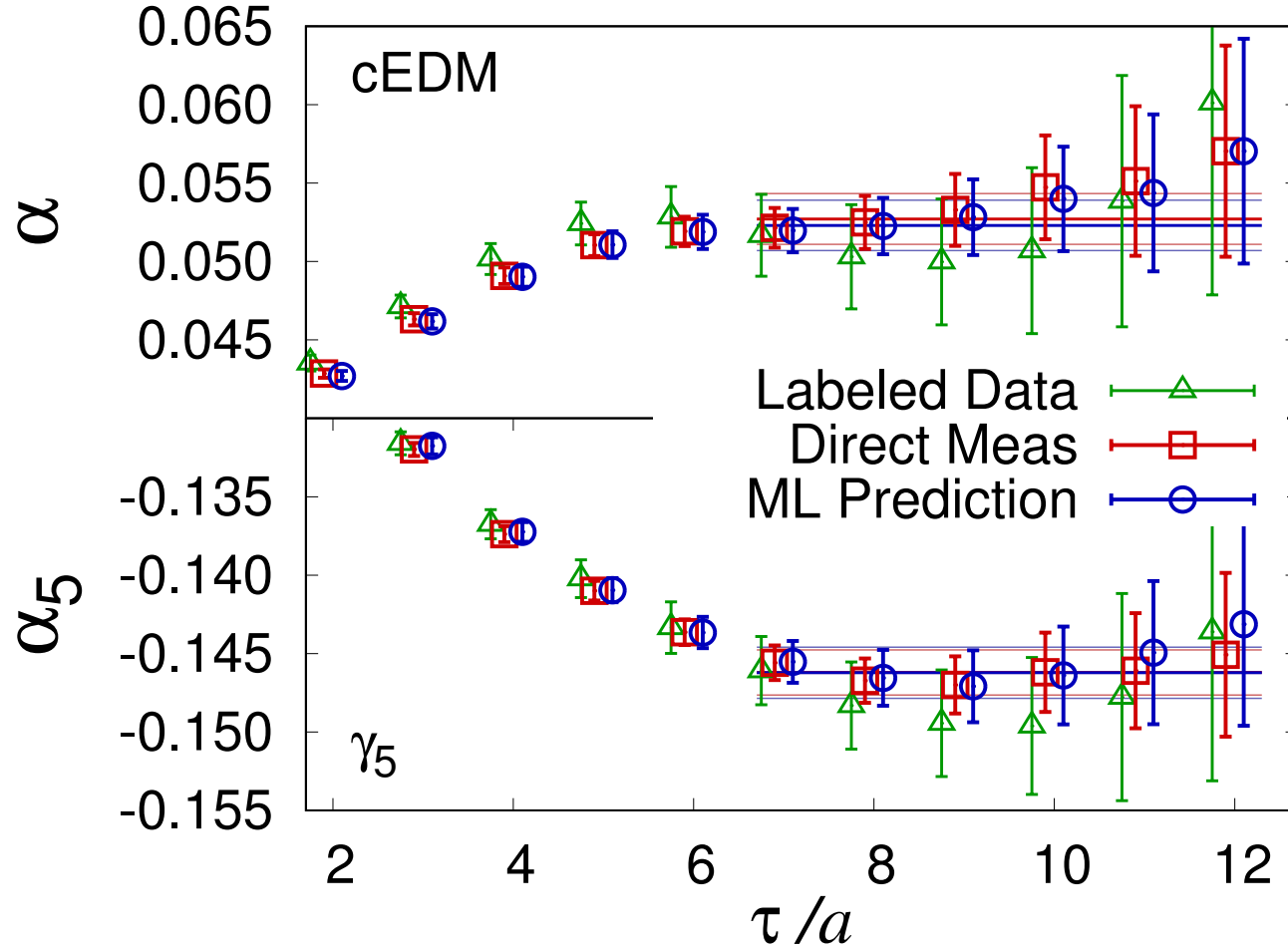


Boosted  
Decision Tree  
Regression



Output:  $\text{Im} \left[ C_{2pt}^{P(\text{cEDM}, \gamma_5)}(\tau) \right]$

# Prediction of $C_{2pt}^{\text{CPV}}$ from $C_{2pt}$



## • $\alpha$ (cEDM)

DM: 0.0527(17)

Prediction: 0.0525(18)

## • $\alpha_5$ ( $\gamma_5$ )

DM: -0.1463(14)

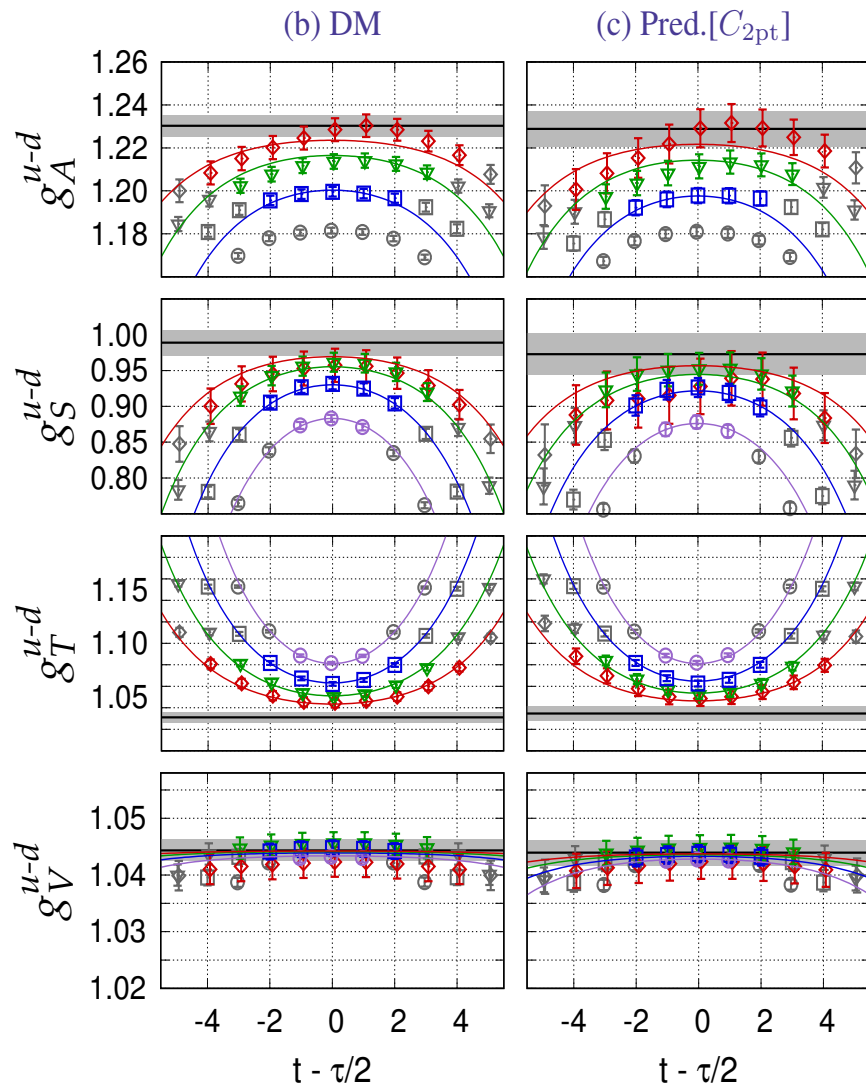
Prediction: -0.1460(17)

➤ DM: DM on 400 confs

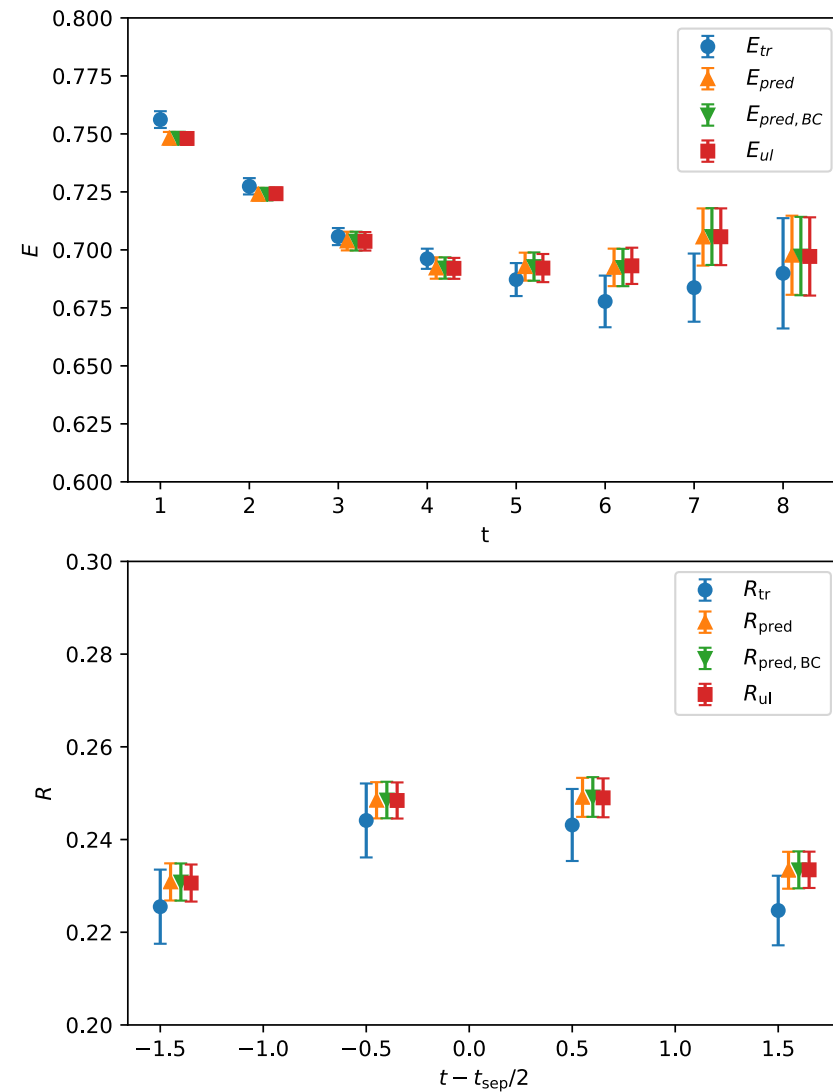
➤ Prediction: DM on 120 confs  
+ ML prediction on 280 confs

# Other Applications

BY, Tanmoy Bhattacharya, Rajan Gupta, PRD 100, 014504 (2019)  
 Rui Zhang, Zhouyou Fan, Ruizi Li, Huey-Wen Lin, BY, PRD 101, 034516 (2020)



Prediction of  $C_{3pt}$  from  $C_{2pt}$



Prediction of  $\eta_s$  distribution amplitude (upper) and Kaon quasi-PDF (lower)  $z = 4$  from  $z < 4$

# ML Regression Algorithm using D-Wave Quantum Annalaer

# ML Regression using D-Wave Quantum Annealer

- Most **ML algorithms involve optimization problems**; many of them rely on stochastic approaches, but expensive for large problems
- **D-Wave quantum annealer** can be used as a **fast** and **accurate optimizer** for ML optimization problems

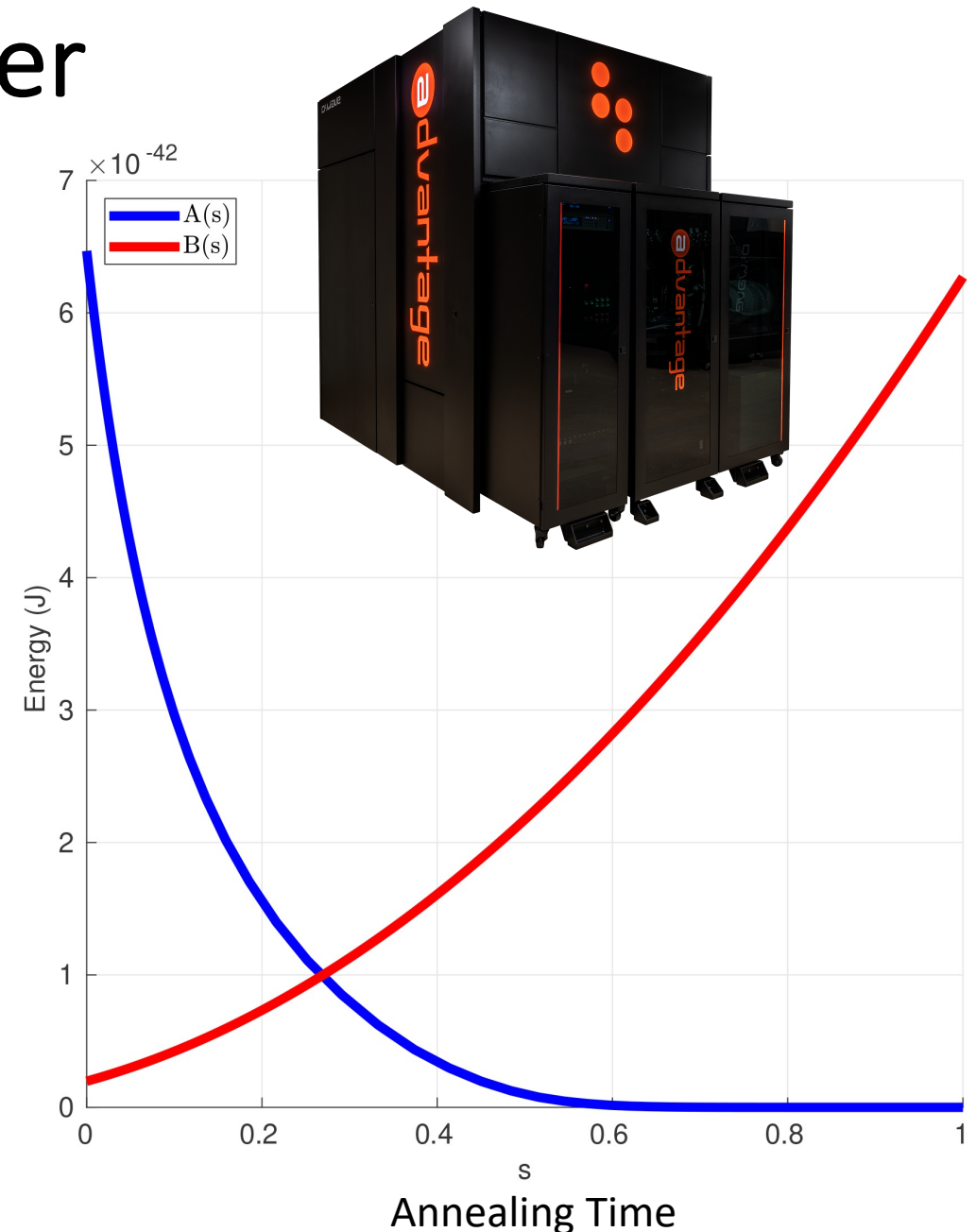


# D-Wave Quantum Annealer

- Hamiltonian

$$H = -\frac{A(s)}{2} \left( \sum_i \hat{\sigma}_x^{(i)} \right) + \frac{B(s)}{2} \left( \sum_i h_i \hat{\sigma}_z^{(i)} + \sum_{i>j} J_{i,j} \hat{\sigma}_z^{(i)} \sigma_z^{(j)} \right)$$

- $h_i, J_{i,j}$ : biases and coupling strengths that **user can set** to their problem parameters
- After annealing at  $< 15$  mK, **QPU returns low-energy solution** (spin up/down of quantum bits) of the **Ising model Hamiltonian**
- Large number of reads is required to obtain minimum energy solution for large problems, but each read takes  $O(10)\mu s$
- **ML typically needs only near-optimal solution**



# Sparse Coding

$$\min_{\Phi} \sum_{k=1}^K \min_{\vec{a}^{(k)}} \left[ \frac{1}{2} \|\vec{X}^{(k)} - \Phi \vec{a}^{(k)}\|_2 + \lambda \|\vec{a}^{(k)}\|_0 \right]$$

- Unsupervised ML algorithm
- Find dictionary  $\Phi \in \mathbb{R}^{D \times N_q}$  and sparse representation  $\vec{a}^{(k)} \in \mathbb{R}^{N_q}$  from which input data  $\vec{X}^{(k)} \in \mathbb{R}^D$  can be reconstructed by
$$\boxed{\vec{X}^{(k)} \approx \Phi \vec{a}^{(k)}} = a_1^{(k)} \vec{v}_1 + a_2^{(k)} \vec{v}_2 + \cdots + a_{N_q}^{(k)} \vec{v}_{N_q}$$
- The representation is sparse because the  $\lambda$ -term enforces a minimal set of dictionary elements for the reconstruction of a given input data
- Optimization in  $\vec{a}^{(k)}$  of  $l^0$ -norm function is a highly non-convex problem

# Sparse Coding on D-Wave quantum annealer

$$\min_{\Phi} \sum_{k=1}^K \min_{\vec{a}^{(k)}} \left[ \frac{1}{2} \|\vec{X}^{(k)} - \Phi \vec{a}^{(k)}\|_2 + \lambda \|\vec{a}^{(k)}\|_0 \right]$$

- The **sparse coding** problem can be **mapped onto D-Wave** by

$$H(\vec{h}, \mathbf{Q}, \vec{a}) = \sum_i a_i h_i + \sum_{i < j} Q_{ij} a_i a_j$$
$$\vec{h} = -\Phi^T \vec{X} + \left( \lambda + \frac{1}{2} \right), \quad \mathbf{Q} = \frac{1}{2} \Phi^T \Phi$$

- On D-Wave,  $a_i$  is restricted to binary:  $\vec{a}^{(k)} \in \{0,1\}^{N_q}$
- D-Wave finds  $\vec{a}^{(k)}$  minimizing  $H$
- Optimization for  $\Phi$  is performed offline (on classical computers)

# Inpainting

Nvidia AI Playground - Inpainting



Ground Truth



Data with Missing Pixels

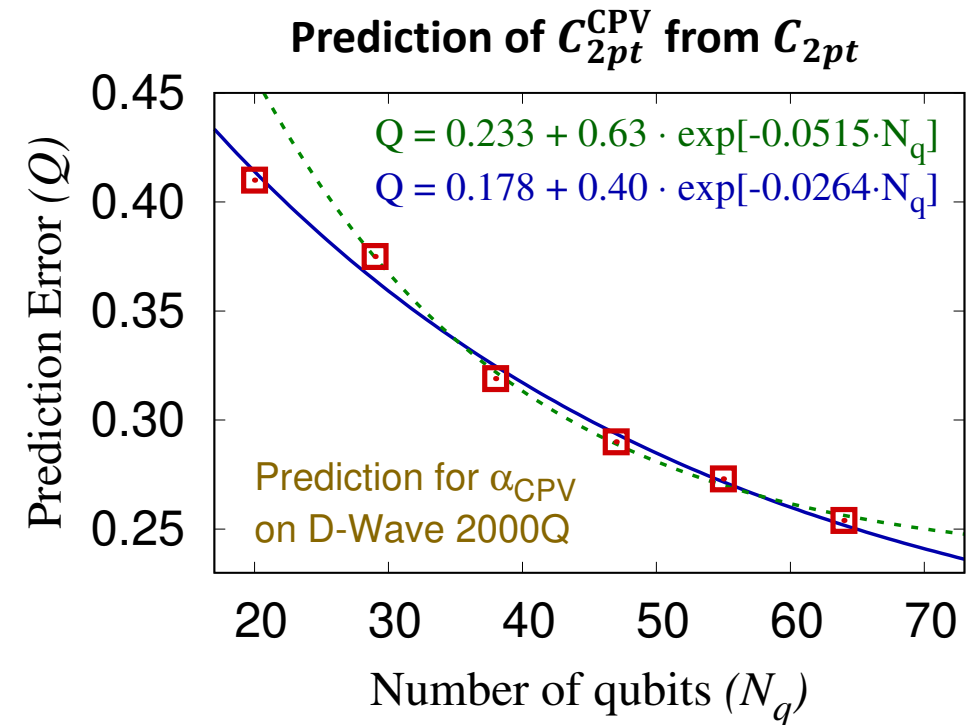


Inpainted Results

- **Inpainting**: restorative conservation where damaged, deteriorating, or missing parts of an artwork are reconstructed as it was originally created
- **Sparse coding** works as an inpainting algorithm because the **reconstruction**  $\vec{X}^{(k)} \approx \Phi \vec{a}^{(k)}$  fills up the missing pixels based on the **correlation pattern**  $\Phi$  learned

# Sparse Coding Regression on D-Wave

- Goal: prediction of  $y$  from  $\vec{x} = \{x_1, x_2, \dots, x_D\}$
- Procedure:
  - 1) Obtain  $\Phi_0 \in \mathbb{R}^{D \times N_q}$  of  $\vec{x}$  from unlabeled data
  - 2) Extend  $\Phi_0$  to  $\Phi \in \mathbb{R}^{(D+1) \times N_q}$  and encode correlation between  $\vec{x}$  and  $y$  in  $\Phi$  using augmented vector  $\{\vec{x}, y\}$
  - 3) For unknown  $y$ , reconstruct new vector  $\{\vec{x}, \bar{y}\}$  using  $\Phi$ ; reconstruction replaces  $\bar{y}$  with its prediction
- This approach is a semi-supervised learning as it utilizes unlabeled data to improve prediction
- D-Wave is used for optimization in  $\vec{a}^{(k)}$
- Currently, the performance is limited by the maximum number of qubits available on D-Wave, but the predictions applied on lattice QCD data look promising



# Lossy Data Compression Algorithm for Lattice QCD Data

# Lossy Data Compression for Lattice QCD

- Modern **lattice QCD simulations** produce  **$O(\text{PetaBytes})$  of data** that need to be stored for future analysis
- **Exploiting correlation** between the data components can reduce storage requirement → **Machine learning**
- **Reconstruction error** sufficiently **smaller than** the observables **statistical fluctuation** is good enough for most of the analysis → **Lossy compression**



# Lossy Data Compression Algorithm

- **Goal:** find  $\Phi \in \mathbb{R}^{D \times N_q}$  and  $\vec{a}^{(k)} \in \{0,1\}^{N_q}$  precisely reconstructing input vectors  $\vec{X}^{(k)} \in \mathbb{R}^D$  such that  $\vec{X}^{(k)} \approx \Phi \vec{a}^{(k)} \equiv \vec{X}'^{(k)}$ 
  - $\Phi$  is common for all  $k = 1, 2, 3, \dots, N$ , so memory usage is small
  - Each vector  $\vec{a}^{(k)}$  can be stored in  $N_q$  bits
  - Storing  $\left(\{\vec{a}^{(k)}\}_{k=1}^N, \Phi\right)$  for  $\{\vec{X}^{(k)}\}_{k=1}^N$ : compression of  $D$  floating-point numbers into  $N_q$  bits
  - Correlation between  $X_i$ , encoded in  $\Phi$ , allows precise reconstruction with  $N_q \ll 32D$
- Such solutions of  $\Phi$  and  $\vec{a}^{(k)}$  can be obtained by solving
$$\min_{\Phi} \sum_{k=1}^N \min_{\vec{a}^{(k)}} \left[ (\vec{X}^{(k)} - \Phi \vec{a}^{(k)})^2 \right]$$
  - Finding binary solution of  $\vec{a}^{(k)}$  is an NP-hard problem but can be solved using D-Wave
  - Finding  $\Phi$  is done on classical computers with stochastic optimizer
  - Iterate  $\vec{a}^{(k)}$ - and  $\Phi$ -optimizations until it reaches the minimum reconstruction error
  - Need standardization of  $\vec{X}^{(k)}$  beforehand if the data exhibits heteroskedasticity



# Bias Correction of Lossy Reconstruction

- Lossy reconstruction introduces error  $\vec{X}^{(k)} \neq \Phi \vec{a}^{(k)} \equiv \vec{X}'^{(k)}$   
Simple average is a biased estimator  $\langle f(\vec{X}) \rangle \neq \frac{1}{N} \sum_k f(\vec{X}'^{(k)})$
- Unbiased estimator of  $\langle f(\vec{X}) \rangle$  can be defined using small portion of original data

$$\bar{0} = \frac{1}{N} \sum_{k=1}^N f(\vec{X}'^{(k)}) + \frac{1}{N_{bc}} \sum_{k=1}^{N_{bc}} \left( f(\vec{X}^{(k)}) - f(\vec{X}'^{(k)}) \right)$$

- Quality of lossy-compression on statistical data

$$Q^2 \equiv \frac{1}{D} \sum_{i=1}^D \frac{\sigma_{X_i - X'_i}^2}{\sigma_{X_i}^2}$$

➤ Smaller  $Q^2$  indicates the better compression

➤ Increase of statistical error due to bias correction is proportional to  $\frac{N}{2N_{bc}} Q^2$

➤ eg) With 10% of bias correction data ( $N_{bc}/N=0.1$ ) and compression of  $Q^2 = 0.01$ , original data is typically reconstructed within 5% statical error increase

# Comparison with other Algorithms

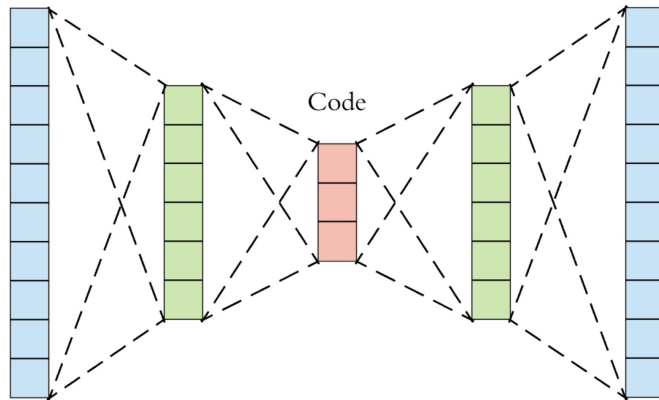
- **Binary compression using D-Wave**

- Find a set of vectors ( $\Phi$ ) and their binary coefficients ( $\mathbf{a}^{(k)}$ ) reconstructing  $\mathbf{X}^{(k)}$

$$\min_{\Phi} \sum_k \min_{\mathbf{a}^{(k)}} \left[ \sum_i \left( X_i^{(k)} - [\Phi \mathbf{a}^{(k)}]_i \right)^2 \right]$$

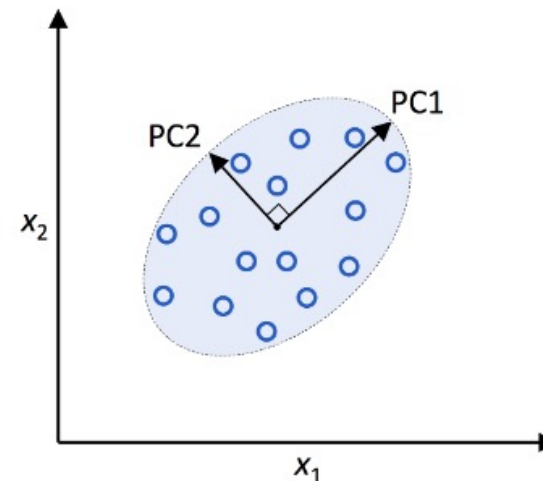
- **Bottle-neck Autoencoder (AE)**

- Fully connected NN with ReLU
- Encoder: (16, 128, 64, 32,  $N_z$ )
- Decoder: ( $N_z$ , 32, 64, 128, 16)

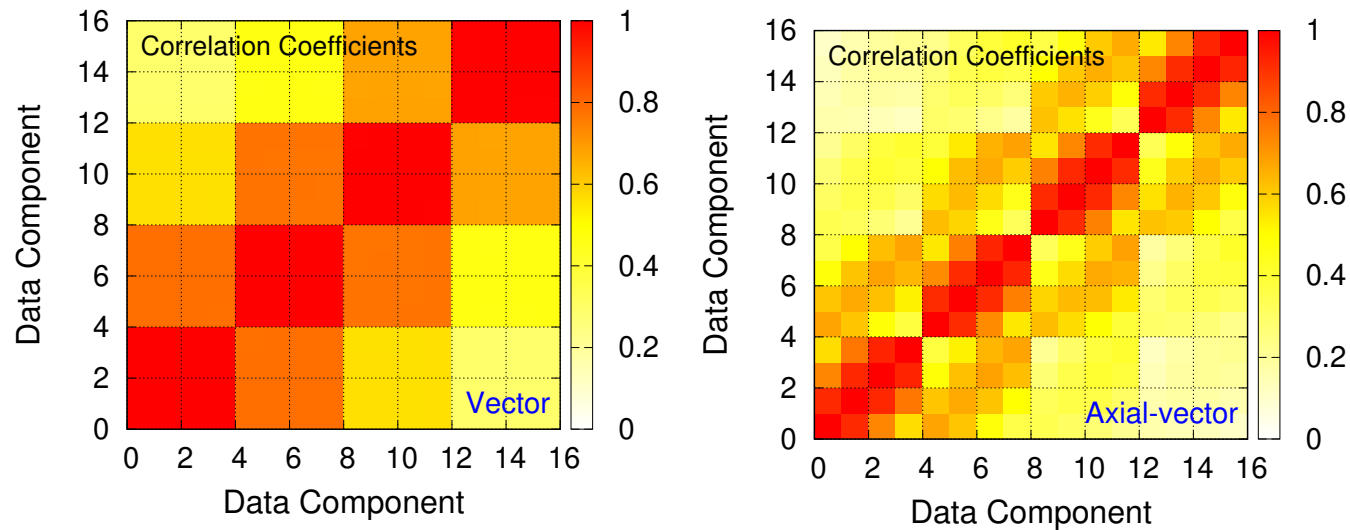


- **Principal Component Analysis (PCA)**

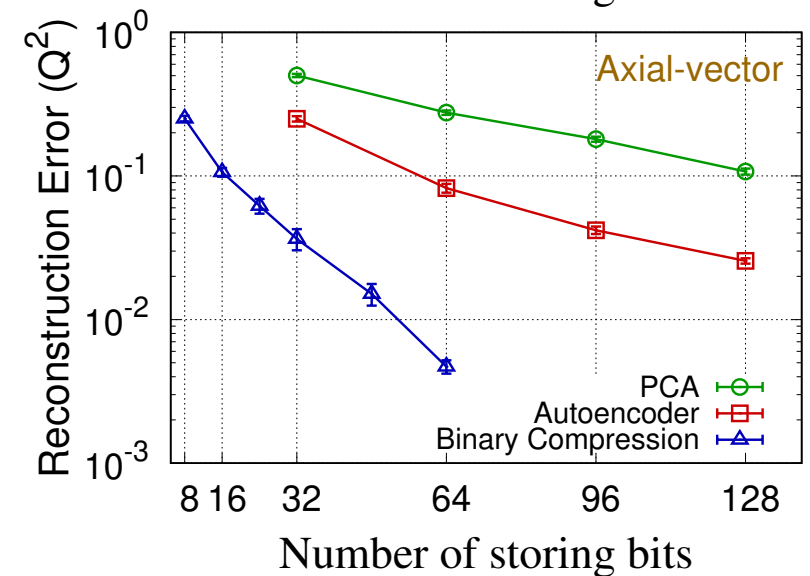
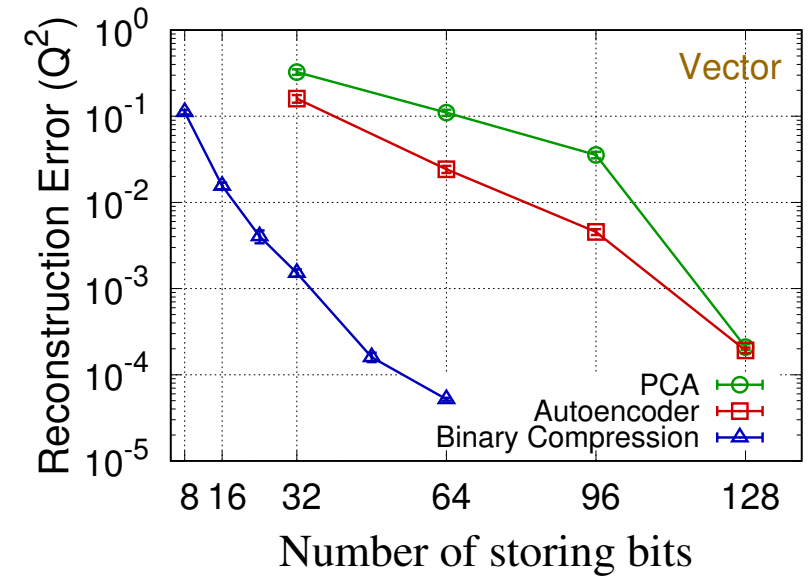
- Compression by saving the first  $N_z$  coefficients of the principal components



# Compression of Lattice QCD data

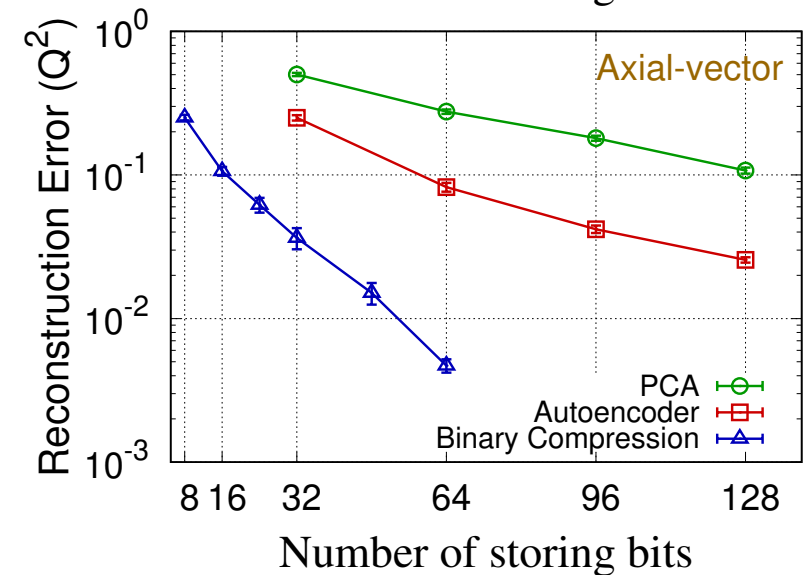
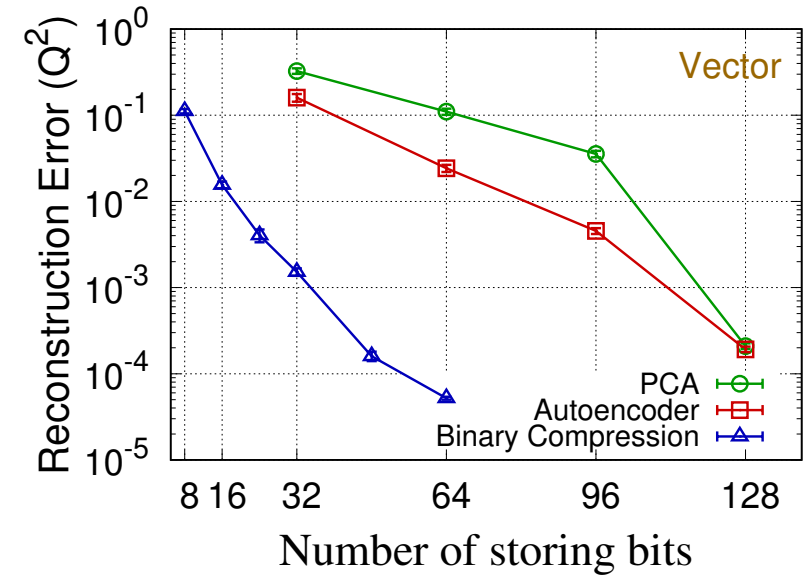


- Compression of “4 timeslices  $\times$  4 src-sink separations” of vector and axial-vector nucleon 3pt correlators
- Compression performance of the **new algorithm** outperforms those based on **principal component analysis (PCA)** or **neural-network autoencoder**
- Results from D-Wave simulated annealing; real QPU gives worse performance due to noise in  $h$  and  $J$  parameters
- PCA and NN-Autoencoder with single-precision (32bits) codes



# Estimated Error Increase

- $\frac{\sigma_{bc-recon}}{\sigma_{orig}} = 1 + \alpha \frac{N}{2N_{bc}} Q^2$  with  $0 < \alpha < 1$
- With 10% of bias correction data ( $N/N_{bc} = 10$ ) and  $\alpha = 0.5$ , expected error increase is  $1 + 2.5Q^2$
- When  $Q^2 = 10^{-2}$ , expected error increase is 2.5%
- When  $Q^2 = 10^{-3}$ , expected error increase is 0.25%
- For good lossy compression algorithms, error increase due to bias correction is negligibly small



# More Use of Binary Compression Algorithm

- **Outlier detection**

- An input data with large reconstruction error can be marked anomalous
- Could find events of new physics or data corruption

- **Cheaper operations in  $\vec{a}$ -space ( $\vec{X}^{(k)} \approx \Phi \vec{a}^{(k)}$ )**

- Operations on floating-point numbers  $\vec{X}^{(k)}$  can be replaced by those on single-bit coefficients  $\vec{a}^{(k)}$  with much cheaper computational cost

- eg 1) sum of vectors 
$$\sum_{k=1}^N \mathbf{X}^{(k)} \approx \sum_{k=1}^N \Phi \mathbf{a}^{(k)} = \Phi \left( \sum_{k=1}^N \mathbf{a}^{(k)} \right)$$

- eg 2) sum of  $l^2$ -norm squares

$$\begin{aligned} \sum_{k=1}^N \|\mathbf{X}^{(k)}\|^2 &\approx \sum_{k=1}^N \sum_{i=1}^D \left( \sum_{j=1}^{N_q} \phi_{ij} a_j^{(k)} \right)^2 \\ &= \sum_{i=1}^D \left[ \sum_{j=1}^{N_q} \phi_{ij}^2 \left( \sum_{k=1}^N a_j^{(k)} \right)^2 + 2 \sum_{l < m} \left( \sum_{k=1}^N a_l^{(k)} a_m^{(k)} \right) \phi_{il} \phi_{im} \right] \end{aligned}$$

# Summary

- Machine learning (ML) is employed to **predict unmeasured observables from measured observables**  
(Expensive lattice QCD calculation → Cheap ML estimators)
- **Bias correction** is used to quantify the ML prediction error
- Developed a **new regression algorithm utilizing quantum annealer** and showed promising prediction ability
- Developed a new ML-based **compression algorithm** using quantum annealer for binary optimization