Parameterization of amplitudes, finding resonances and unitarity, peculiarities and traps

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Matter To The Deepest, Katowice IX 2019

$\rho(770)$

$I^G(J^{PC}) = 1^+(1^{--})$

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$\rho(770)$ MASS

We no longer list S-wave Breit-Wigner fits, or data with high comb background.

NEUTRAL ONLY, e^+e^-

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMM
775.26±0.25 OUR AVERAGE					
775.02±0.35		[1] LEES	12G	BABR	e^+e^-
775.97±0.46±0.70	900k	[2] AKHMETSHIN	07		e^+e^-
774.6 ±0.4 ±0.5	800k	[3,4] ACHASOV	06	SND	e^+e^-
775.65±0.64±0.50	114k	[5,6] AKHMETSHIN	04	CMD2	e^+e^-
775.9 ±0.5 ±0.5	1.98M	[7] ALOISIO	03	KLOE	1.02 e π^+
775.8 ±0.9 ±2.0	500k	[7] ACHASOV	02	SND	1.02 e π^+
775.9 ±1.1		[8] BARKOV	85	OLYA	e^+e^-
• • • We do not use the following data for averages, fits, limits, etc. • •					
775.8 ±0.5 ±0.3	1.98M	[9] ALOISIO	03	KLOE	1.02 e π^+
775.9 ±0.6 ±0.5	1.98M	[10] ALOISIO	03	KLOE	1.02 e π^+
775.0 ±0.6 ±1.1	500k	[11] ACHASOV	02	SND	1.02 e π^+
775.1 ±0.7 ±5.3		[12] BENAYOUN	98	RVUE	e^+e^- μ^+
770.5 ±1.9 ±5.1		[13] GARDNER	98	RVUE	0.28–(π^+
764.1 ±0.7		[14] O'CONNELL	97	RVUE	e^+e^-
757.5 ±1.5		[15] BERNICHA	94	RVUE	e^+e^-
768 ±1		[16] GESHKEN...	89	RVUE	e^+e^-

CHARGED ONLY, τ DECAYS and e^+e^-

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	CHG	COM
775.11±0.34 OUR AVERAGE						
774.6 ±0.2 ±0.5	5.4M	[17,18] FUJIKAWA	08	BELL	±	τ^-
775.5 ±0.7		[18,19] SCHAEL	05C	ALEP		τ^-
775.5 ±0.5 ±0.4	1.98M	[7] ALOISIO	03	KLOE		1.02 π
775.1 ±1.1 ±0.5	87k	[20,21] ANDERSON	00A	CLE2		τ^-
• • • We do not use the following data for averages, fits, limits, etc. • •						
774.8 ±0.6 ±0.4	1.98M	[10] ALOISIO	03	KLOE	−	1.02 π
776.3 ±0.6 ±0.7	1.98M	[10] ALOISIO	03	KLOE	+	1.02 π
773.9 ±2.0 $^{+0.3}_{-1.0}$		[22] SANZ-CILLERO	03	RVUE		τ^-
774.5 ±0.7 ±1.5	500k	[7] ACHASOV	02	SND	±	1.02 π
775.1 ±0.5		[23] PICH	01	RVUE		τ^-

MIXED CHARGES, OTHER REACTIONS

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	CHG	COMMENT
763.0±0.3±1.2	600k	[24] ABELE	99E	CBAR	0±	0.0 $\overline{p}p \to \pi^+\pi^-\pi^0$

CHARGED ONLY, HADROPRODUCED

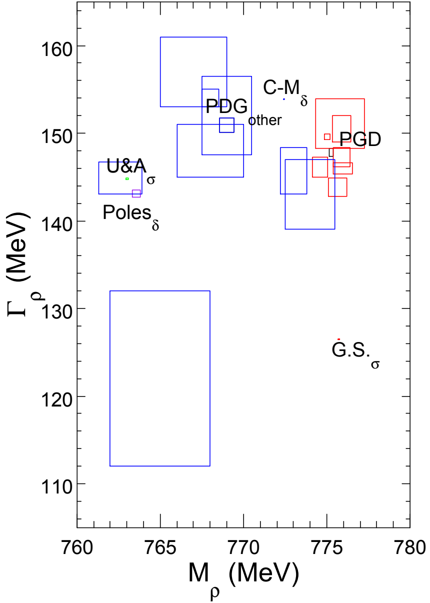
VALUE (MeV)	EVTS	DOCUMENT ID		TECN	CHG	COMMENT
766.5±1.1 OUR AVERAGE						
763.7±3.2		ABELE	97	CBAR		$\overline{p}n \to \pi^-\pi^0\pi^0$
768 ±9		AGUILAR-...	91	EHS		400 pp
767 ±3	2935	[25] CAPRARO	87	SPEC	−	200 π^- Cu $\to \pi^-\pi^0$ Cu
761 ±5	967	[25] CAPRARO	87	SPEC	−	200 π^- Pb $\to \pi^-\pi^0$ Pb
771 ±4		HUSTON	86	SPEC	+	202 $\pi^+ A \to \pi^+\pi^0 A$
766 ±7	6500	[26] BYERLY	73	OSPK	−	5 $\pi^- p$
766.8±1.5	9650	[27] PISUT	68	RVUE	−	1.7–3.2 $\pi^- p$, $t<10$
767 ±6	900	[25] EISNER	67	HBC	−	4.2 $\pi^- p$, $t<10$

NEUTRAL ONLY, PHOTOPRODUCED

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
769.0± 1.0 OUR AVERAGE					
771 ± 2 $^{+2}_{-1}$	63.5k	[28] ABRAMOWICZ	12	ZEUS	$ep \to e\pi^+\pi^- p$
770 ± 2 ±1	79k	[29] BREITWEG	98B	ZEUS	50–100 γp
767.6± 2.7		BARTALUCCI	78	CNTR	$\gamma p \to e^+e^- p$
775 ± 5		GLADDING	73	CNTR	2.9–4.7 γp
767 ± 4	1930	BALLAM	72	HBC	2.8 γp
770 ± 4	2430	BALLAM	72	HBC	4.7 γp
765 ±10		ALVENSLEB...	70	CNTR	γA, $t<0.01$
767.7± 1.9	140k	BIGGS	70	CNTR	<4.1 γC $\to \pi^+\pi^-$ C
765 ± 5	4000	ASBURY	67B	CNTR	γ + Pb
• • • We do not use the following data for averages, fits, limits, etc. • • •					
771 ± 2	79k	[30] BREITWEG	98B	ZEUS	50–100 γp

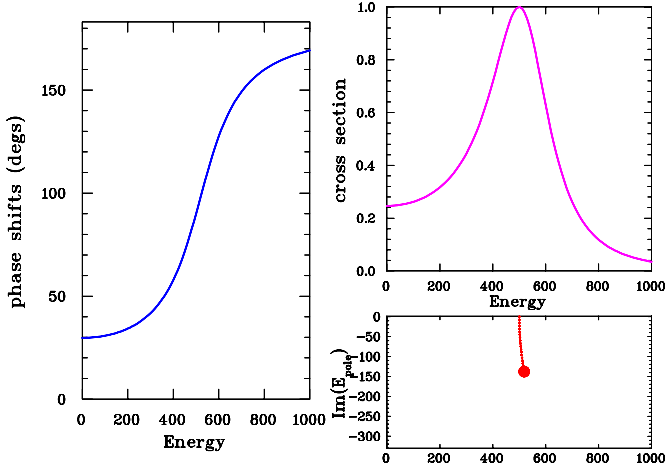
NEUTRAL ONLY, OTHER REACTIONS

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	CHG	COMMENT
769.0±0.9 OUR AVERAGE Error includes scale factor of 1.4. See the ideogram below.						
765 ±6		BERTIN	97C	OBLX		0.0 $\overline{p}p \to \pi^+\pi^-\pi^0$
773 ±1.6		WEIDENAUER	93	ASTE		$\overline{p}p \to \pi^+\pi^-\omega$
762.6±2.6		AGUILAR-...	91	EHS		400 pp
770 ±2		[31] HEYN	81	RVUE		Pion form factor
768 ±4		[32,33] BOHACIK	80	RVUE	0	



160
150
140
130
120
110
760
765
770
775
780
M_ρ (MeV)
Γ_ρ (MeV)
C-M_δ
PDG
other
PGD
U&A$_\sigma$
Poles$_\delta$
G.S.$_\sigma$

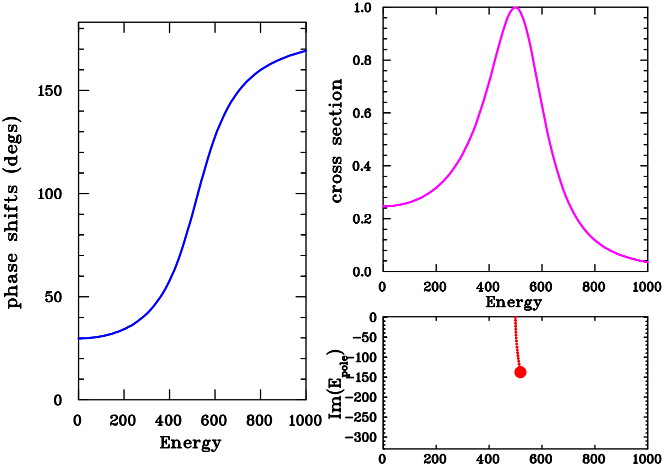
Resonances



phase shifts (degs)
0
50
100
150
0
200
400
600
800
1000
Energy
cross section
0.0
0.2
0.4
0.6
0.8
1.0
0
200
400
600
800
1000
Energy
Im(E_pole)
0
−50
−100
−150
−200
−250
−300
0
200
400
600
800
1000

Resonances: Breit-Wigner (BW) approximation:

$$Ampl = \frac{C}{M_{BW} - E - i\Gamma_{BW}/2}, \quad \delta = ArcTan\left(\frac{\Gamma/2}{M_{BW} - E}\right), \quad \sigma = \frac{C^2}{(M_{BW} - E)^2 + \Gamma^2/4}$$



Resonances: exercise - Breit Wigner parameterization

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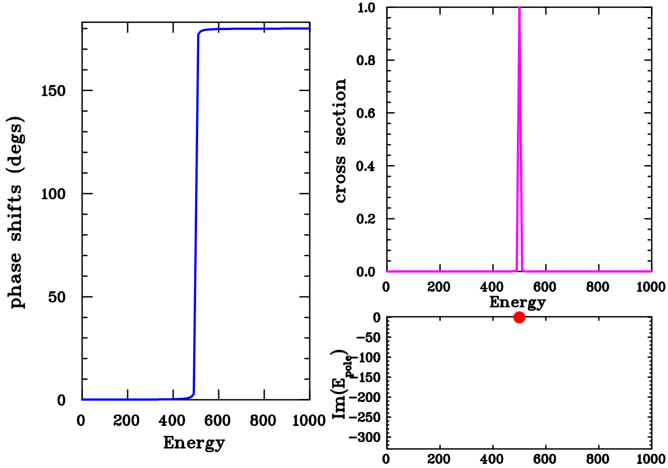

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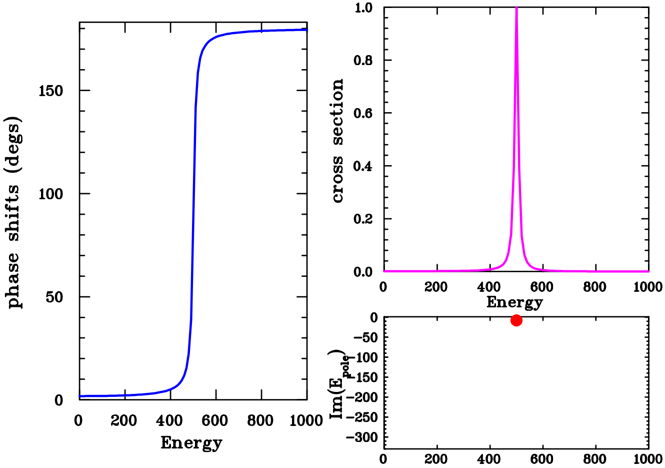

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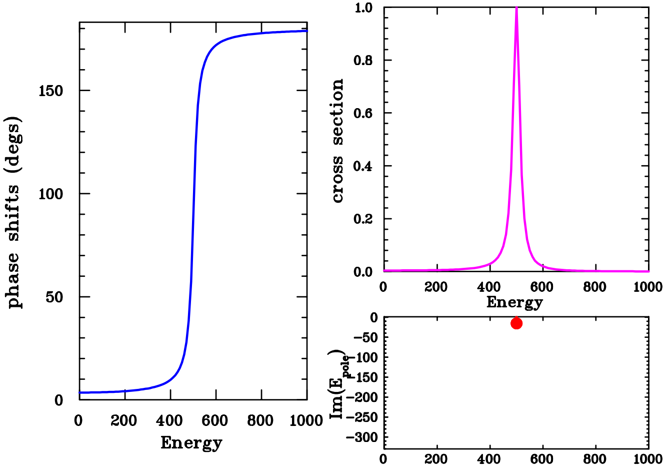

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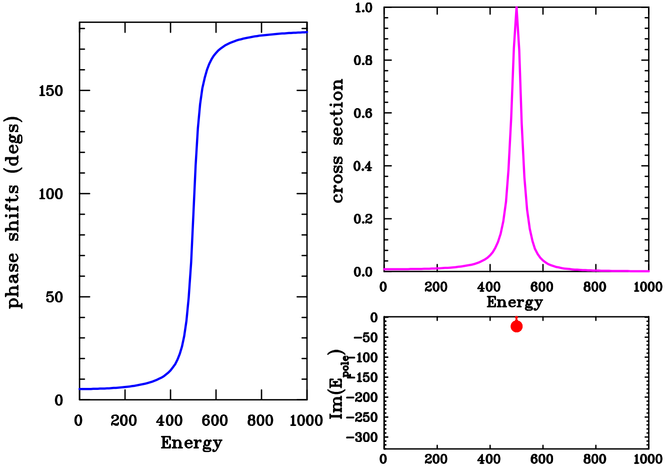

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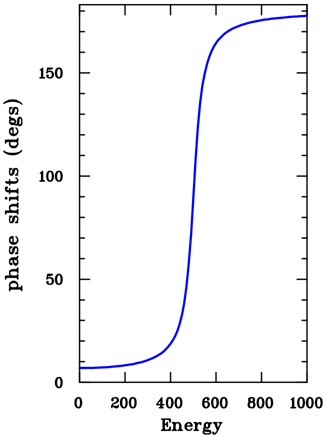

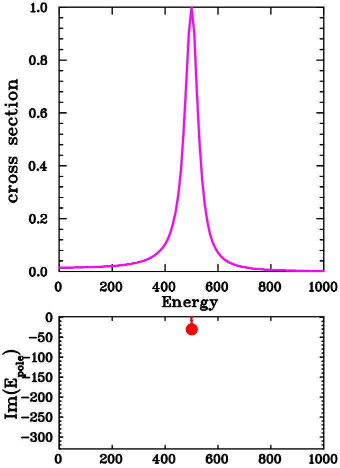

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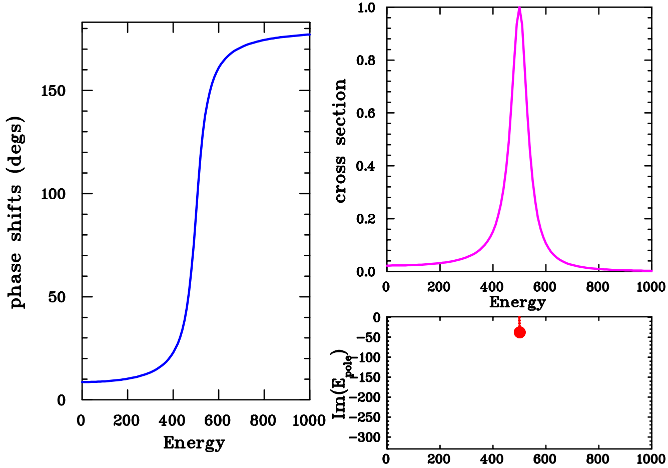

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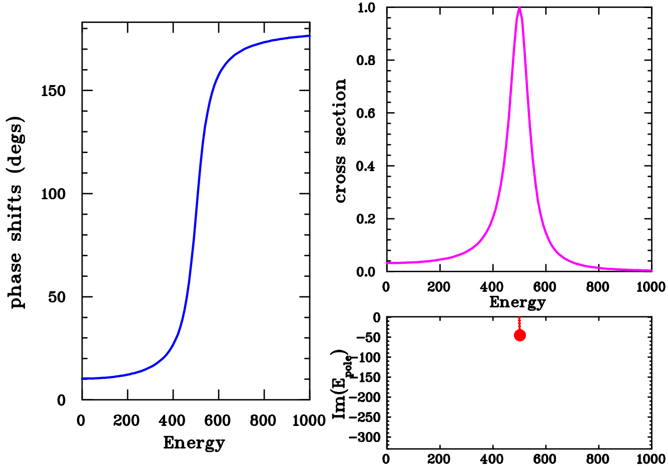

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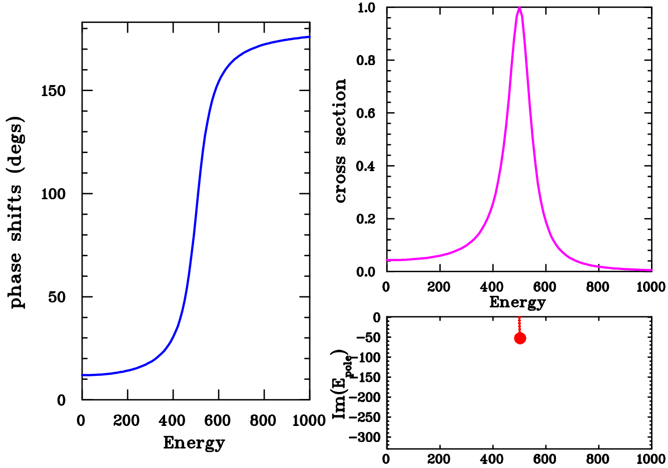

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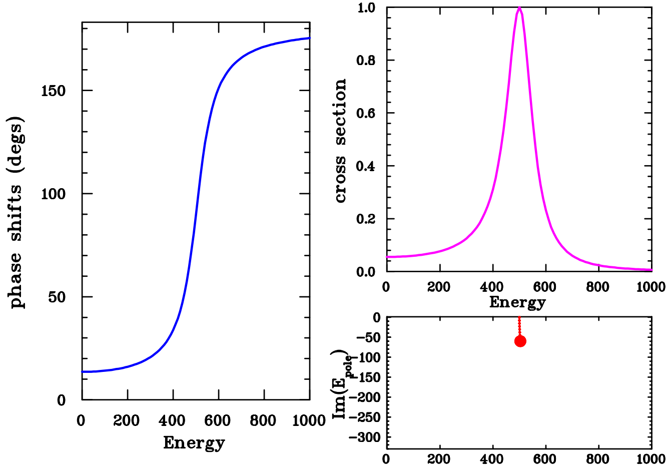

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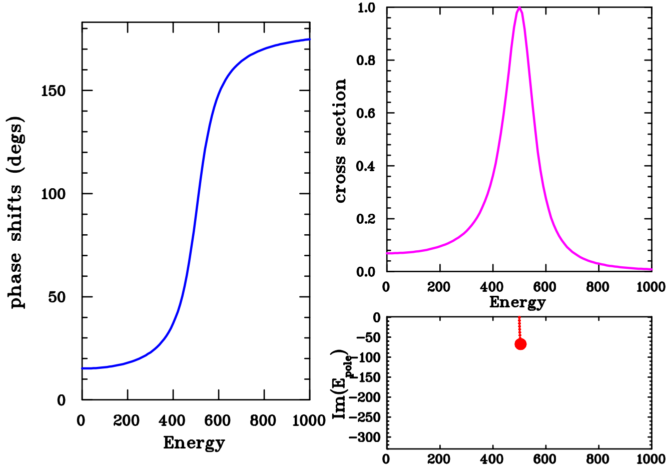

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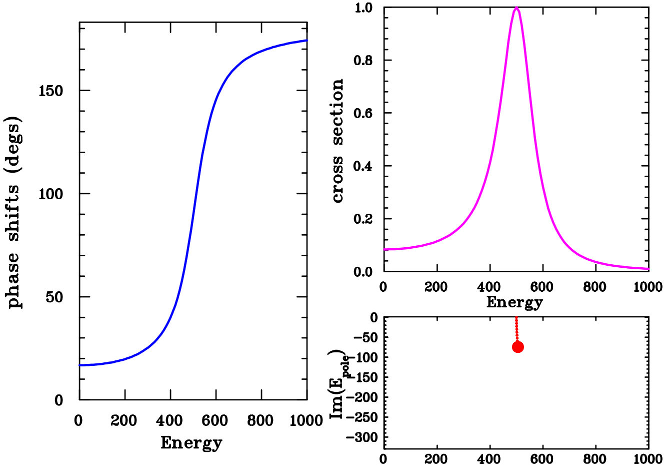

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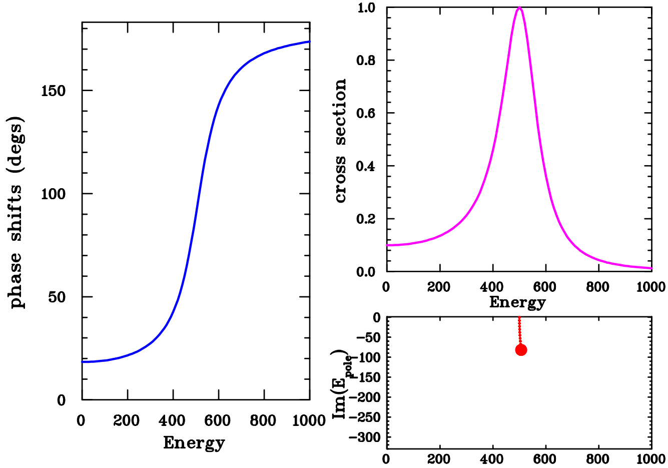

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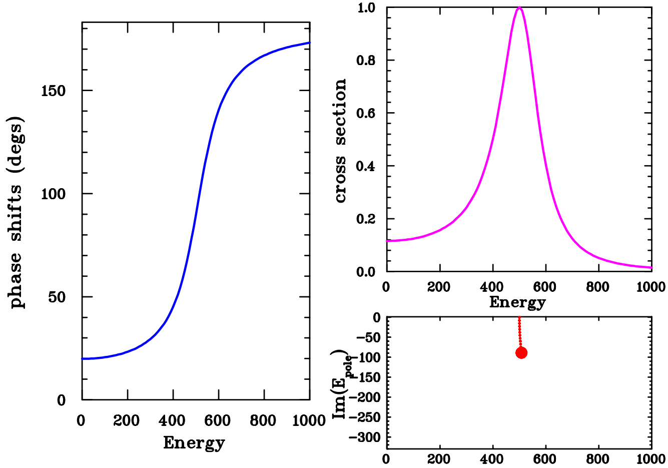

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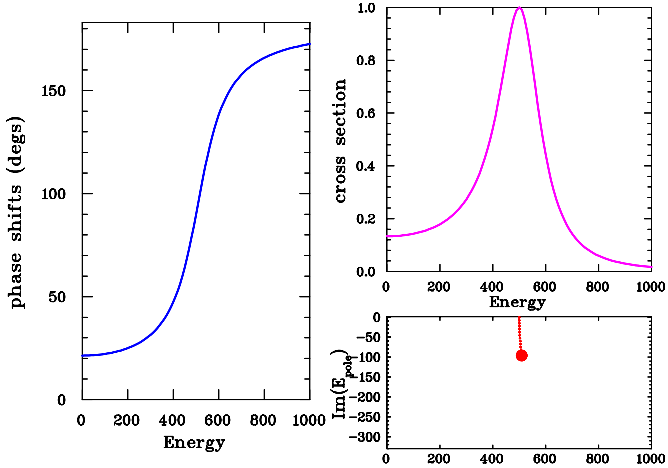

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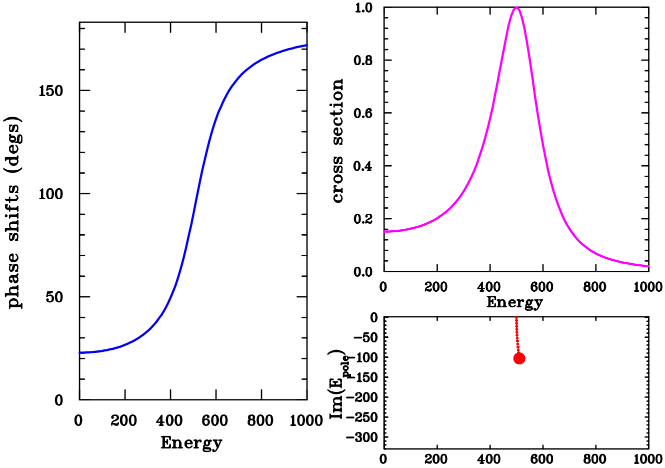

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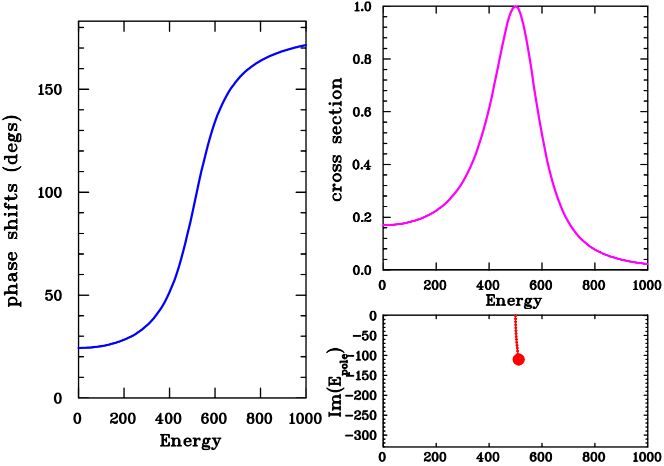

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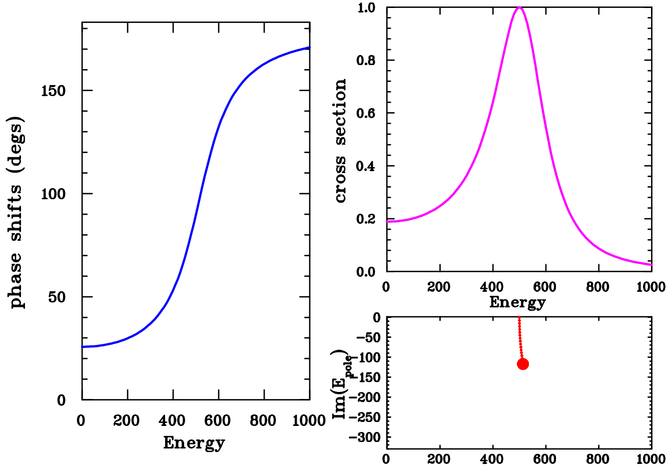

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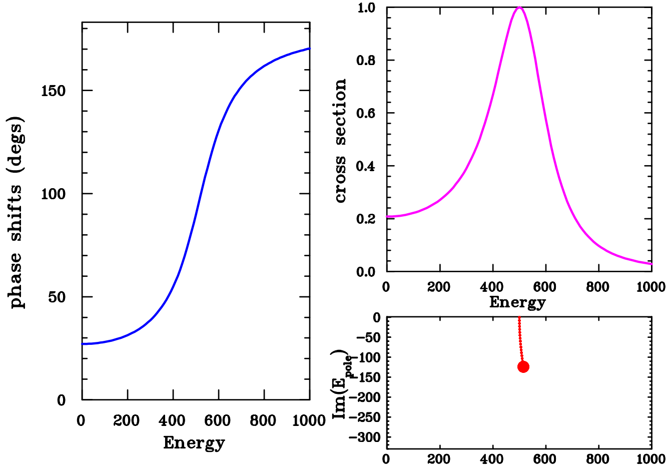

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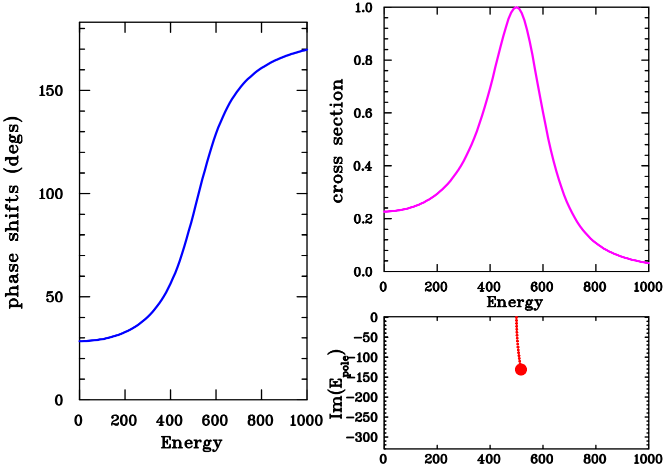

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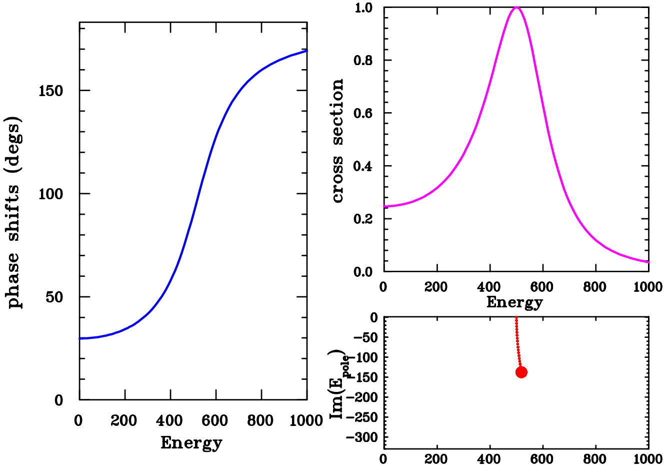

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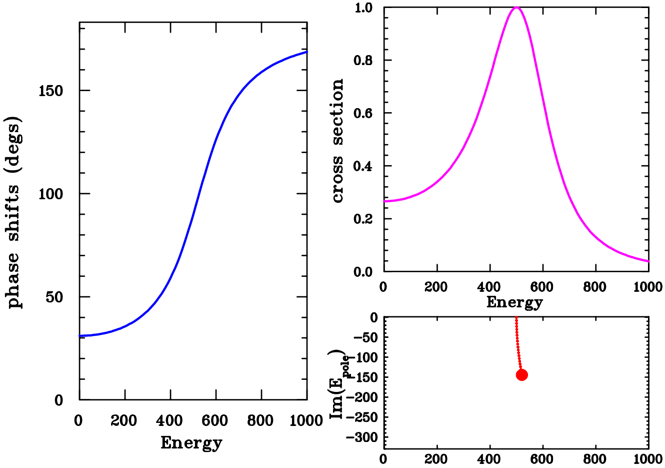

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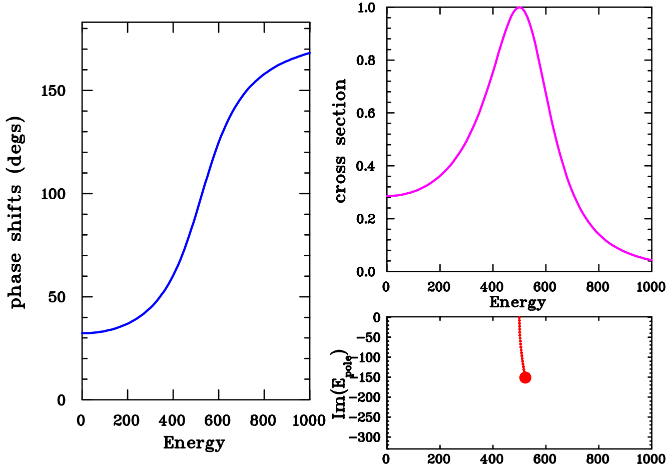

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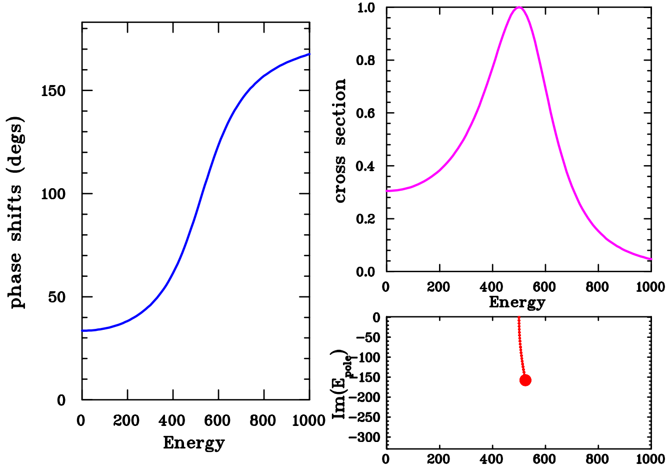

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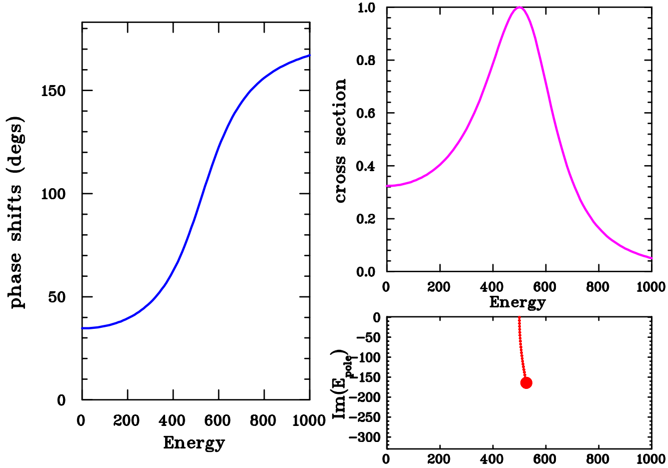

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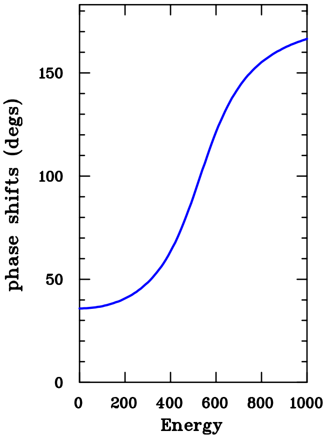

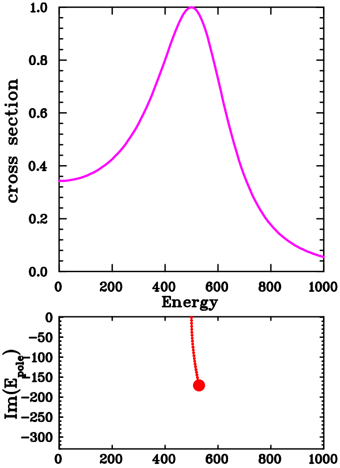

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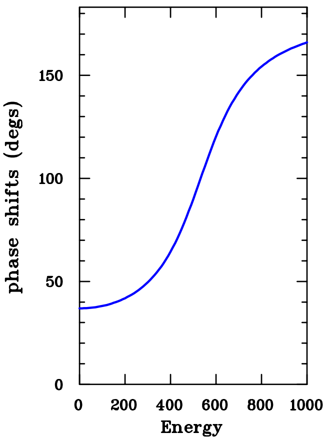

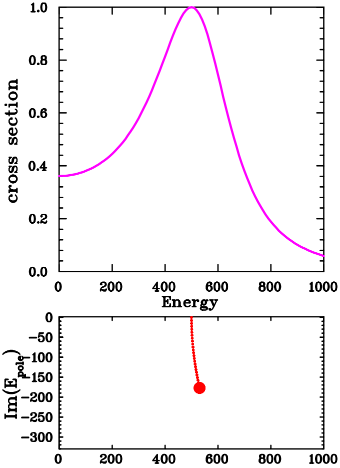

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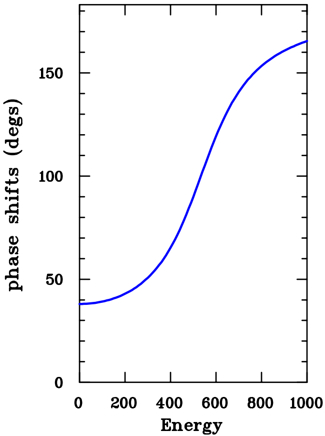

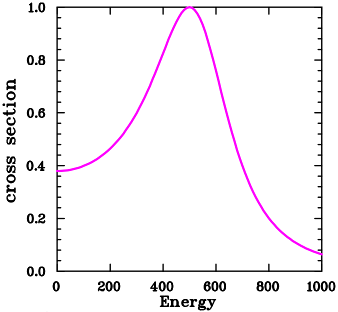

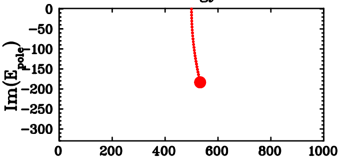

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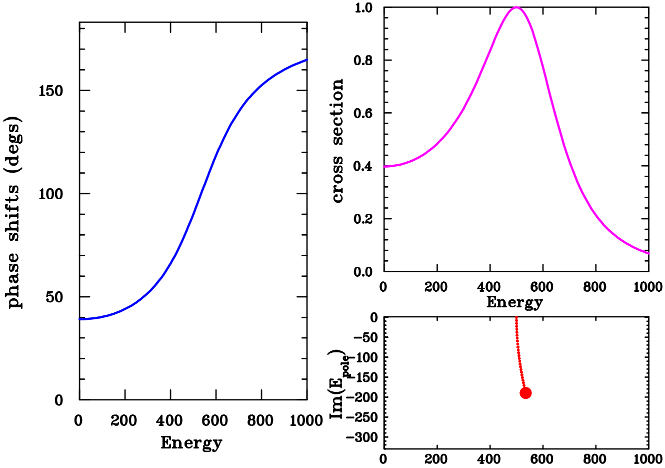

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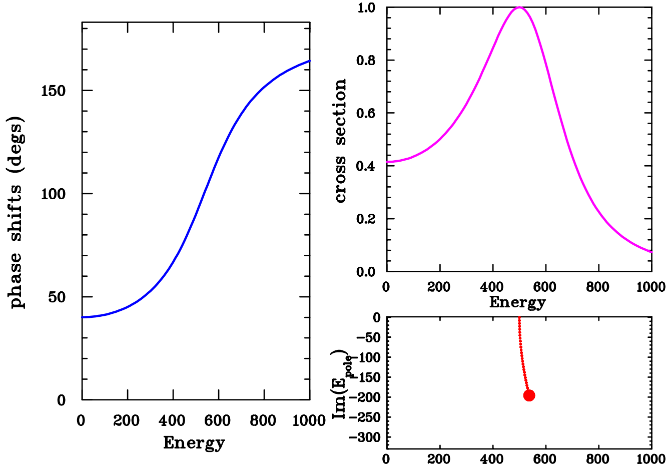

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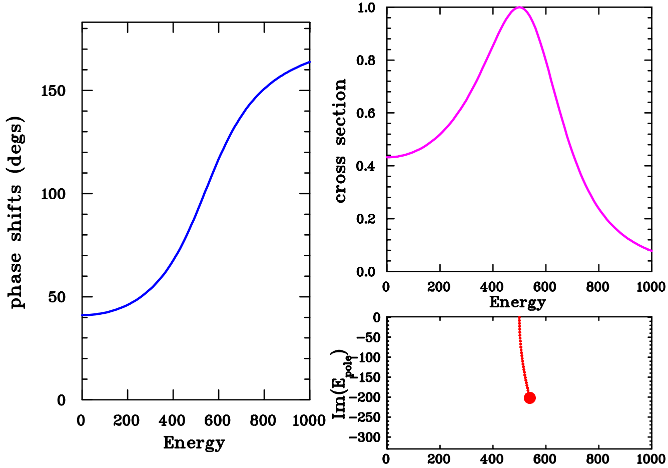

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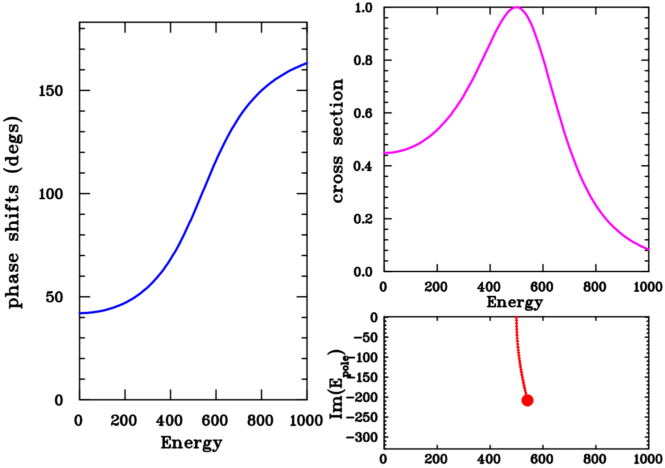

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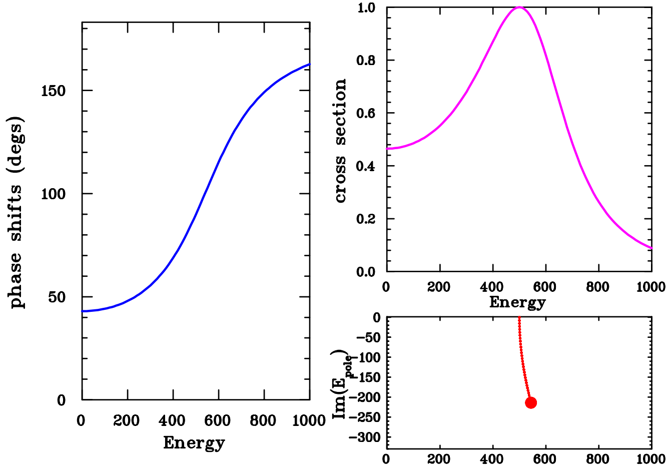

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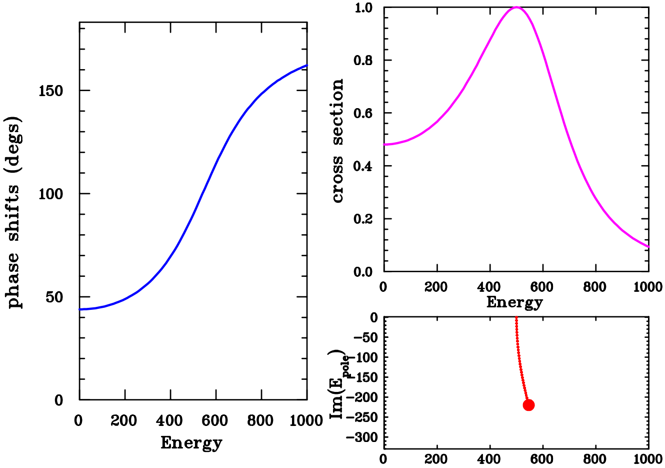

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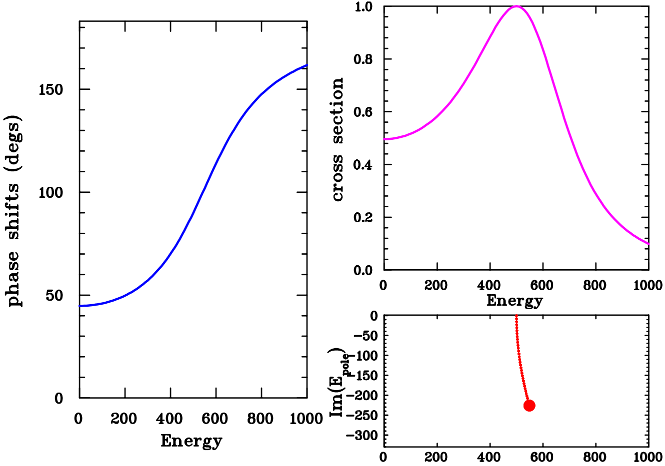

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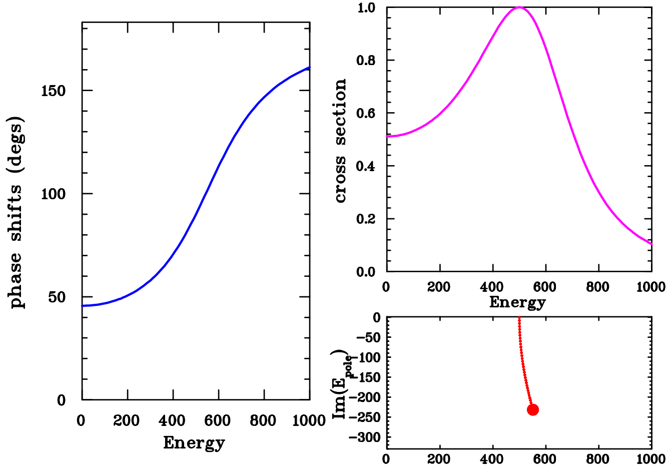

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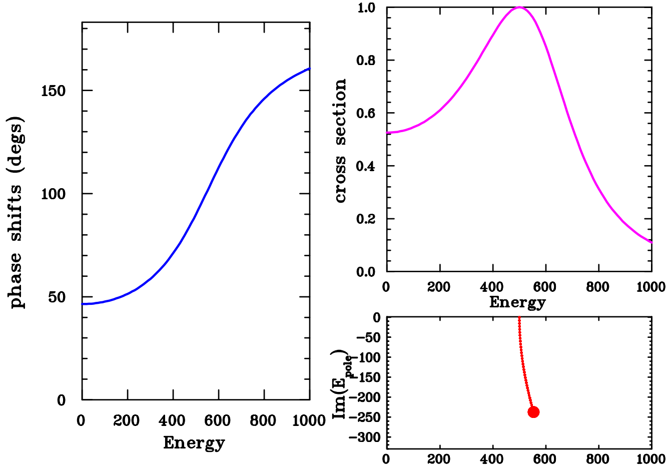

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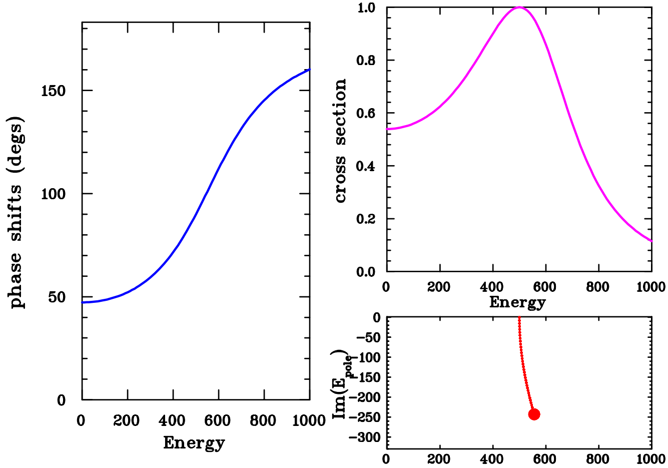

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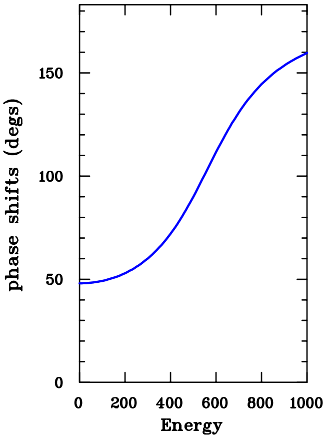

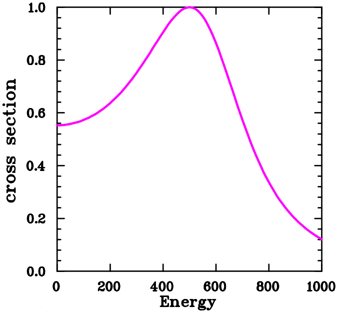

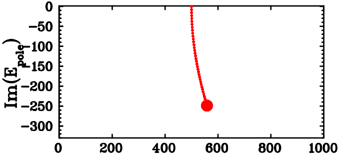

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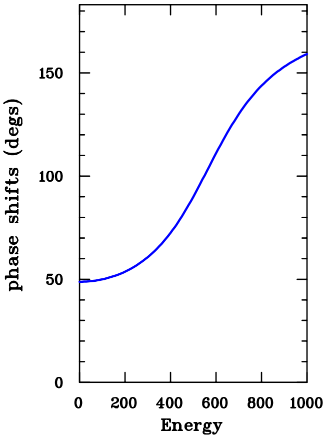

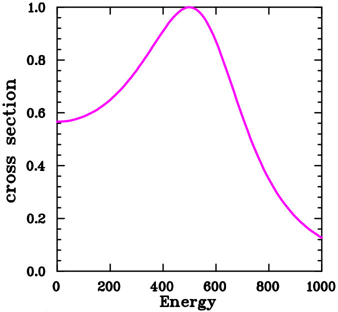

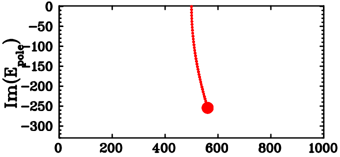

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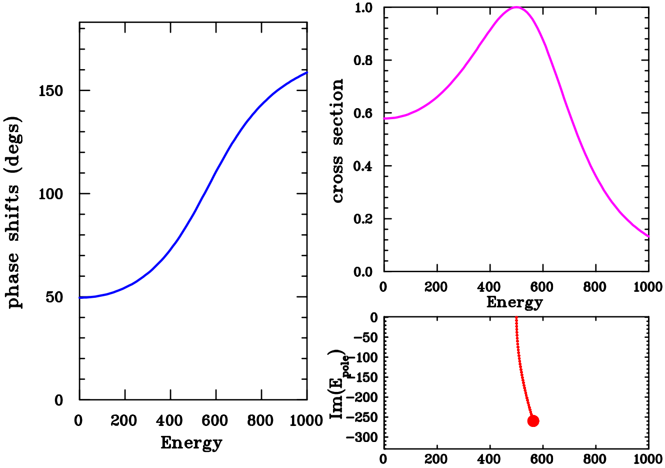

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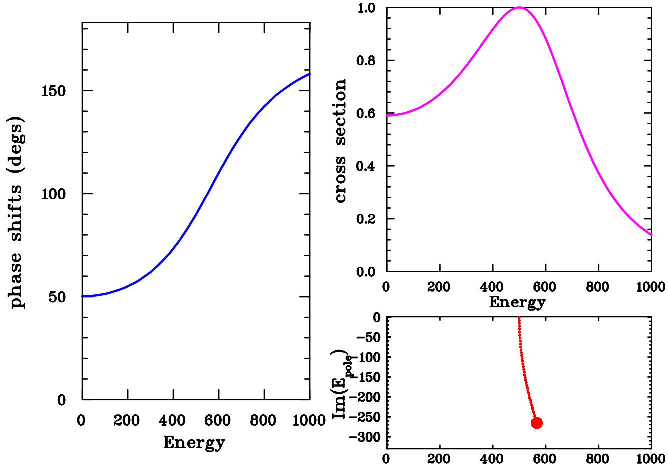

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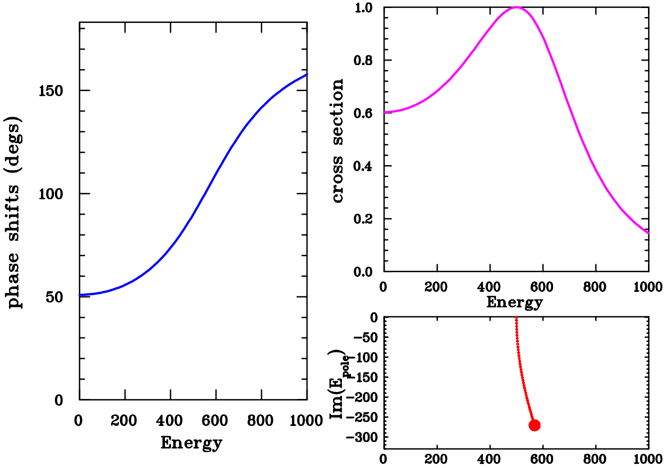

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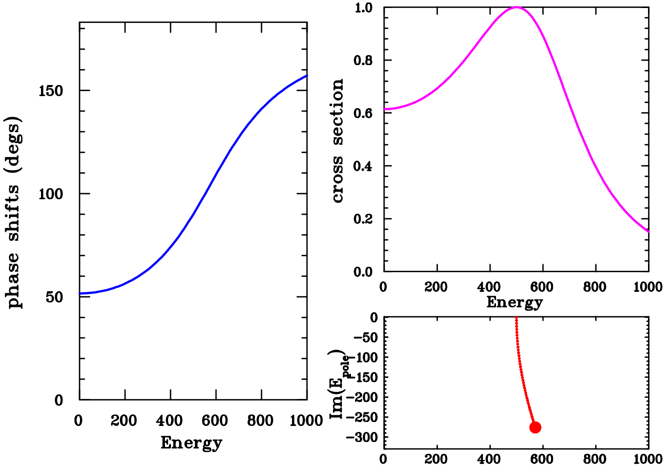

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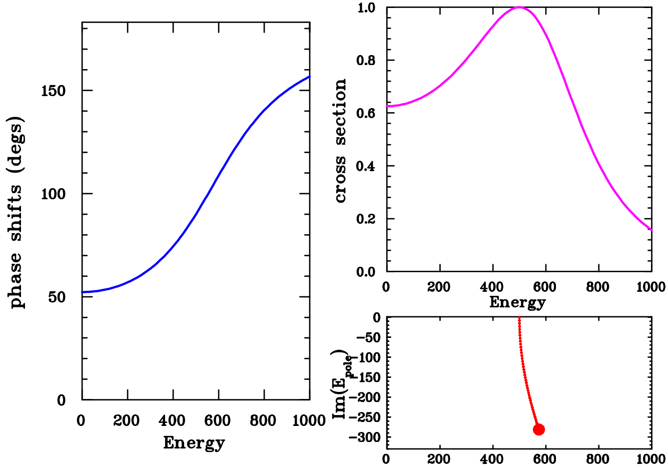

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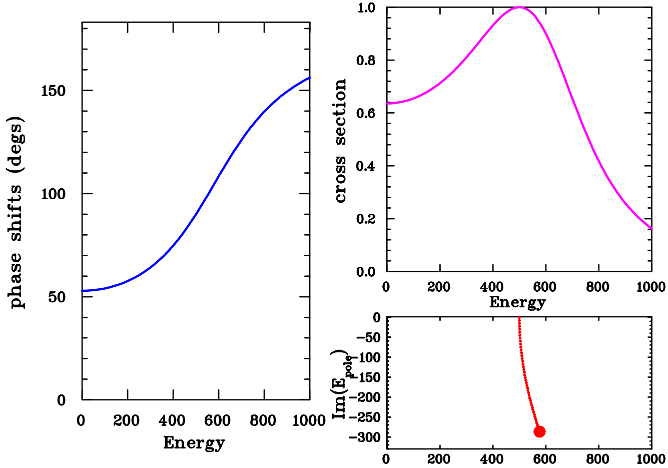

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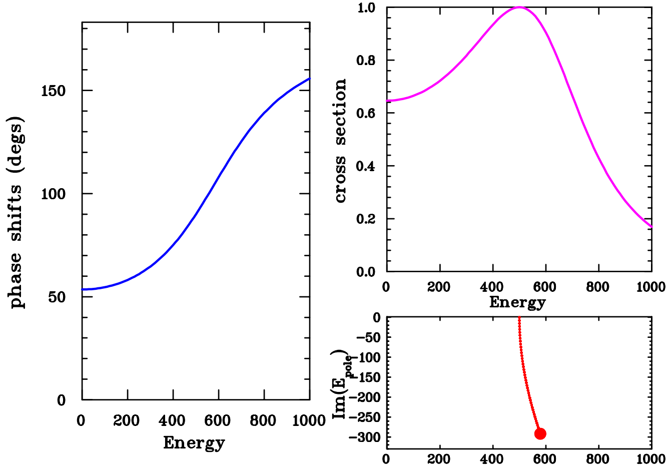

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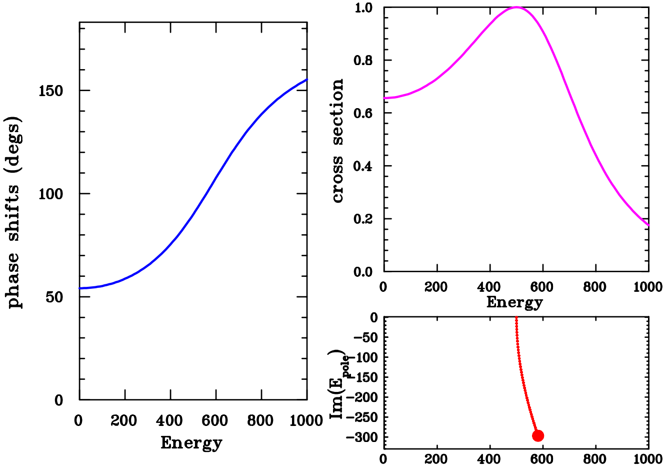
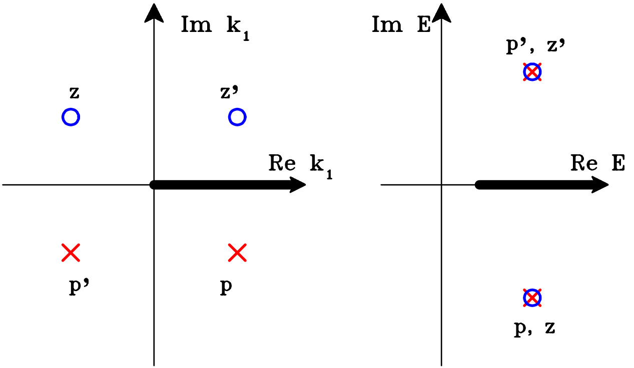


Complex momentum and energy space frame

$$E = 2\sqrt{(\pm k)^2 + m^2}$$



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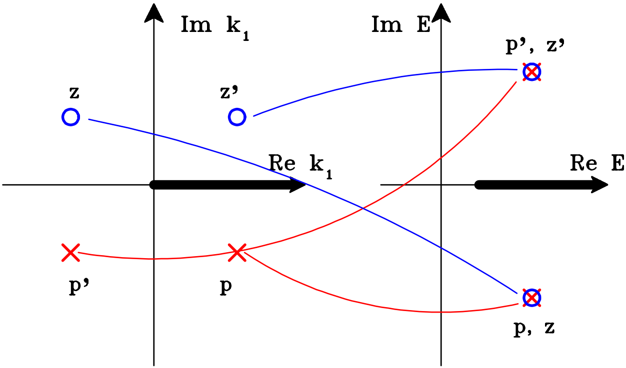

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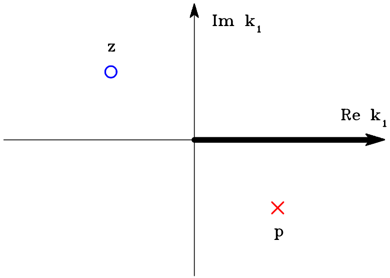

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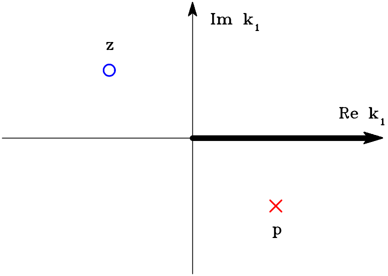

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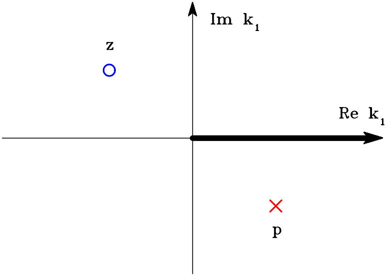

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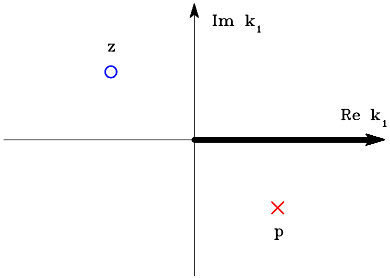

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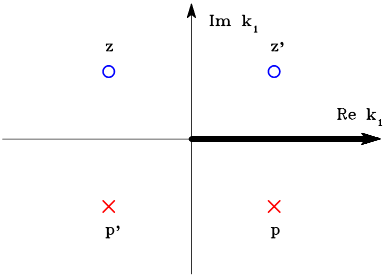

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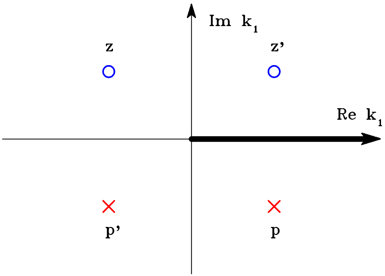

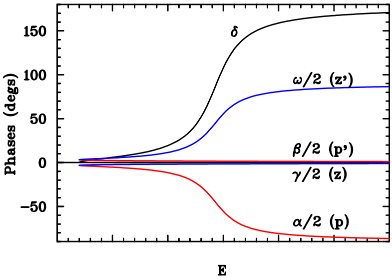

Breit Wigner approximation

- $BW(E) = \frac{\Gamma/2k}{M_{BW}-E-i\Gamma/2}$
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- $S_l(E) = \frac{E-E_j^*}{E-E_j}$ and
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- $BW(E) = \frac{\Gamma/2k}{M_{BW} - E - i\Gamma/2}$
- $\sigma_l(E) = 4\pi(2+1)|BW(E)|^2 = \frac{\pi}{k^2}(2l+1)\frac{\Gamma^2}{(M_{BW}-E)^2+\Gamma^2/4}$
- $BW(E) = \frac{S_l(E)-1}{2ik}$
- $S_l(E) = \frac{M_{BW}-E+i\Gamma/2}{M_{BW}-E-i\Gamma/2}$
- Defining $E_j = M_{BW} - i\Gamma/2$ we get
- $S_l(E) = \frac{E-E_j^*}{E-E_j}$ and
- because $E = \sqrt{(\pm k)^2/4 + m^2}$ there are two poles and two zeroes and
- $|S_l(E)| = 1$
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- Let's imagine good fit of an amplitude to the data —> mass M_{BW} at $\delta = 90^o$
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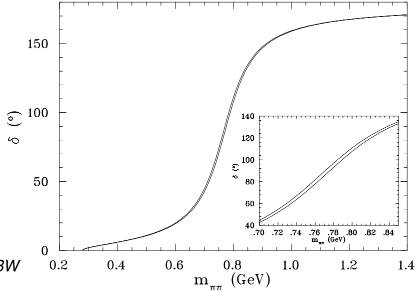
Let's check it for $\rho(770)$:
$M_{BW} = 775.26 \pm 0.25$ MeV (PDG'2016),
$\Gamma = 149.1 \pm 0.8$ MeV (PDG'2016),
$2\sqrt{a^2+m^2} < M_{BW}$ by ≈ 9 MeV !!!

Left (upper) line:
A_{BW} fitted to the data

Right (lower) line:
A_S fitted to the data with $2\sqrt{a^2+m^2} = M_{BW}$



More resonances (but still one channel)

Adding resonances (for simplicity two resonances, both with $S = e^{2i\delta}$):

- Isobar model: adding amplitudes (even unitary ones) violates unitarity:
  $T_{1,2} = T_1 + T_2 = \frac{S_1 - 1}{2ik} + \frac{S_2 - 1}{2ik} \to S_1 + S_2 = e^{2i\delta_1} + e^{2i\delta_2}$
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- Product of S matrices: $|S_1 S_2| = 1$ in elastic case and $|S_1 S_2| < 1$ in inelastic case ($S = \eta e^{2i\delta}$)
  For example $S_{1,2} = \frac{(-k-k_1)(-k+k_1^*)(-k-k_2)(-k+k_2^*)}{(k-k_1)(k+k_1^*)(k-k_2)(k+k_2^*)}$
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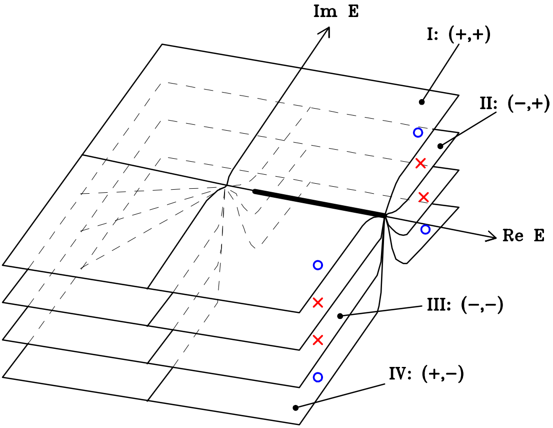

Multiplication and displacement of S matrix poles

- Multiplication:

  1 pole $\longrightarrow$ 2^{n-1} poles due to $(\pm k)^2$ ambiguity and 2^n Riemann sheets

- Displacement:

  $S_{11} = \frac{D_1(-k_1)D_2(k_2)}{D_1(k_1)D_2(k_2)}$ in decoupled case

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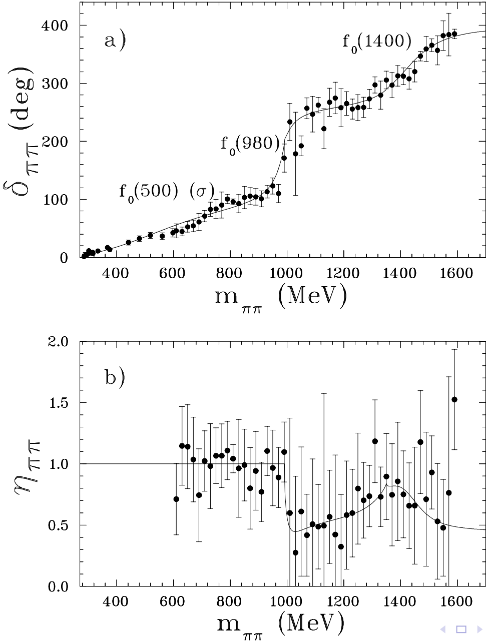
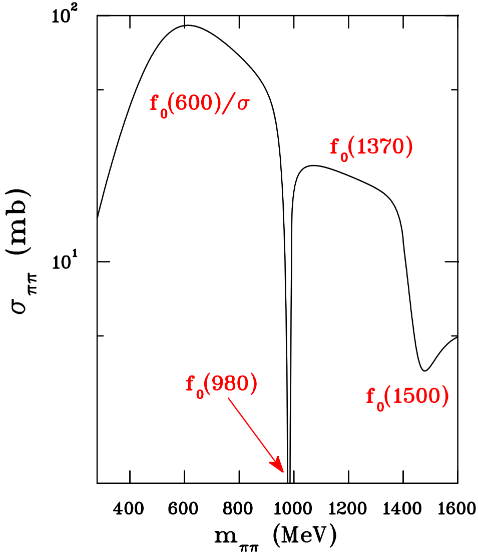
$$S = \begin{pmatrix} \eta e^{2i\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix} = \begin{pmatrix} \frac{D(-k_1,k_2)}{D(k_1,k_2)} & S_{12} \\ S_{21} & \frac{D(k_1,-k_2)}{D(k_1,k_2)} \end{pmatrix}$$

where $S_{12}^2 = S_{21}^2 = S_{11}S_{22} - \frac{D(-k_1,-k_2)}{D(k_1,k_2)}$

Example for two channels: $JI = S0$ wave



Puzzling $(J I)$ $S0$ wave $\pi\pi$ cross section



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Pole	ReE_{pole} MeV	ImE_{pole} MeV	R. sheet
1	639.6	-323.9	$(-,-)$: *III*
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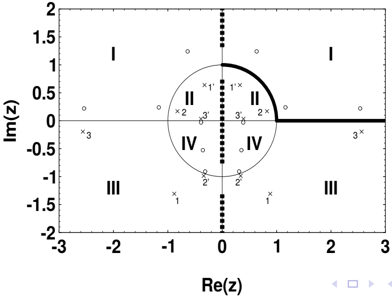
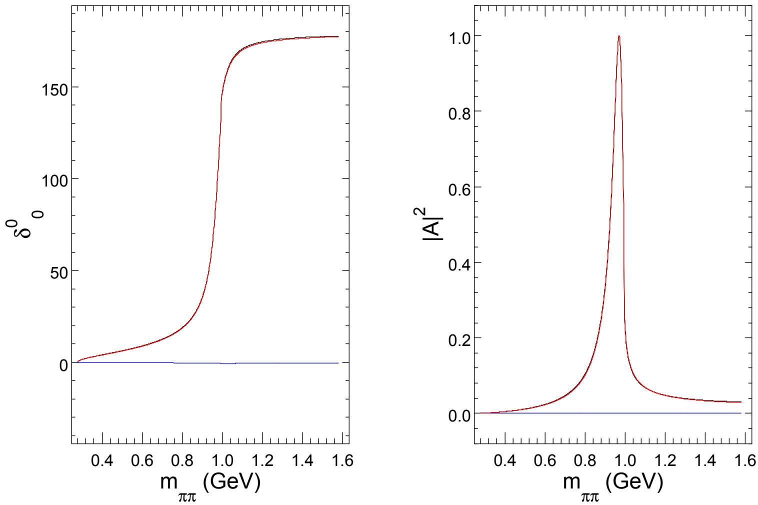
Rysunek 16: Położenie biegunów (krzyże) i zer (kółka) elementu macierzowego S_{11} macierzy rozpraszania dla dopasowania do zestawu $\mathrm{D_{CKM}}$ A. Gruba linia ciągła oznacza obszar fizyczny rozpraszania w kanałach sprzężonych $\pi\pi$ i $K\overline{K}$. Grubą linią przerywaną przedstawione jest położenie cięć funkcji Josta. Cienką linią zaznaczony jest okrąg $|z| = 1$. Numeracja poszczególnych płatów i biegunów została wyjaśniona w tekście.



For two channels" TWO POLES 2 and 2’ ($f_0(980)$)

2: 982.0-i*36.9 2’: 432.4-i*8.4 both 2 and 2’



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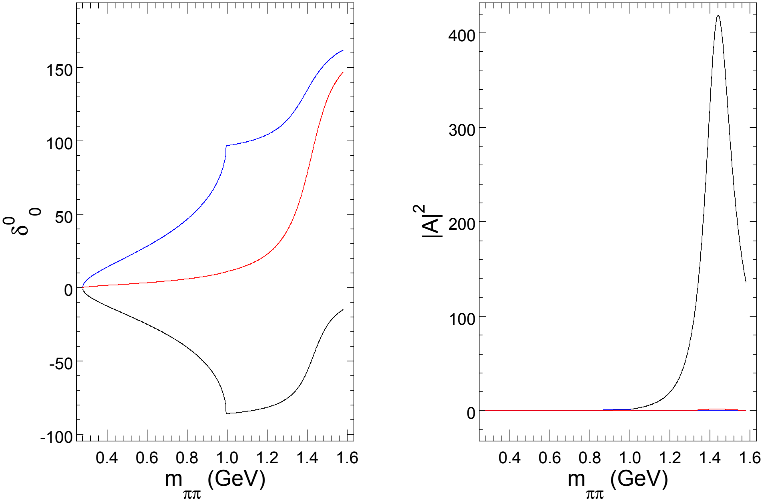

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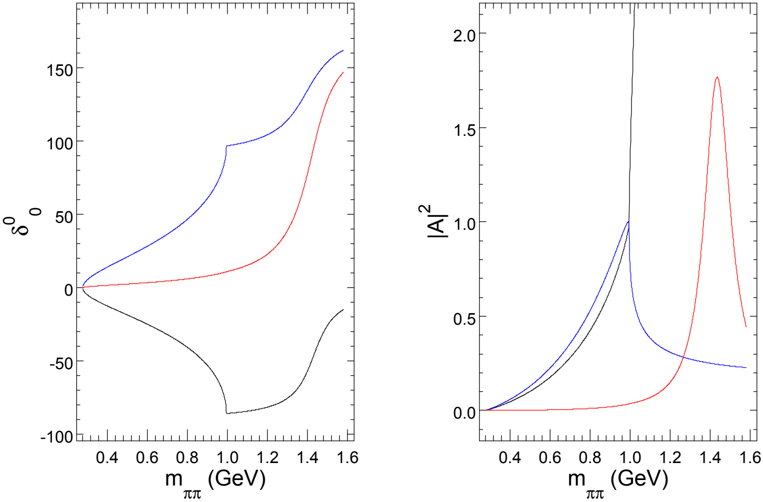

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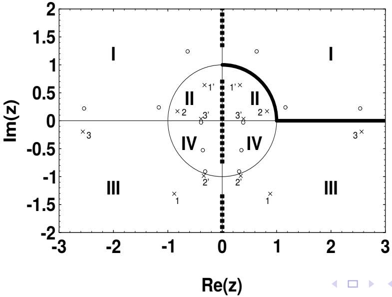


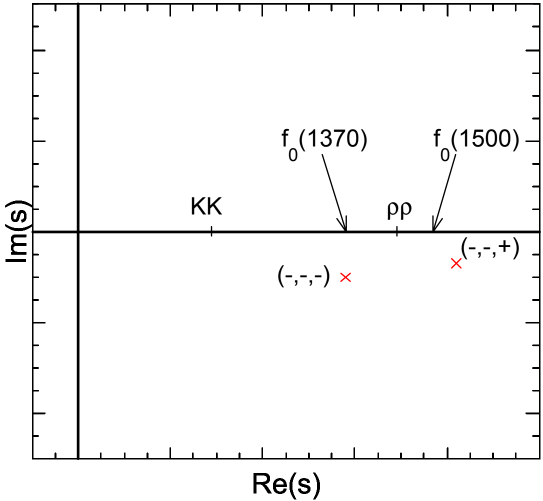
More than 2 channels

- more poles and more Riemann sheets (2^n)
- no similar "z" variable

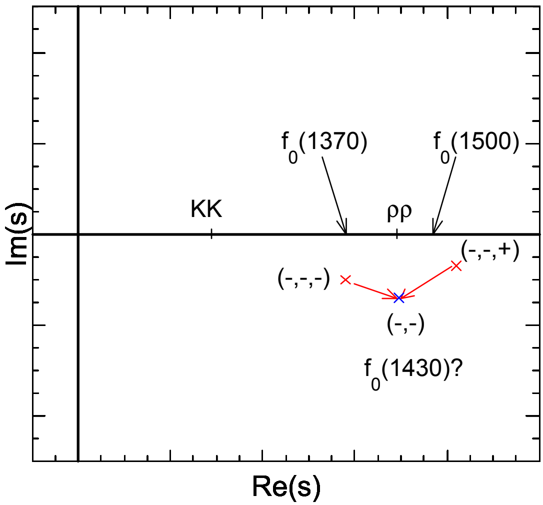
2^n Riemann sheets for n channels

channel	$C=0$		$C=1$		sign	sheet
	ReE	*ImE*	*ReE*	*ImE*	Imk_π, Imk_K, Imk_3	
$\pi\pi$	658	-607	564	-279	$-,-,-$	VI
			518	-261	$-,+,+$	II
			211	0	$-,+,-$	VII
			532	-315	$-,-,+$	III
			235	0	$+,+,-$	VIII
$\pi\pi$	1346	-275	1405	-74	$-,-,-$	VI
			1445	-116	$-,+,+$	II
			1424	-94	$-,+,-$	VII
			1456	-47	$-,-,+$	III
$K\bar{K}$	881	-498	170	0	$+,-,-$	V
			159	0	$-,-,-$	VI
			418	-10	$-,-,+$	III
			1038	-204	$-,+,-$	VII
			988	-31	$-,+,+$	II
$\sigma\sigma$	118	-2227	4741	-4688	$-,-,-$	VI
			3687	-2875	$-,+,-$	VII
			3626	-3456	$+,-,-$	V
			3533	-579	$+,+,-$	VIII

$f_0(1370)$ and $f_0(1500)$: positions of poles, $C = 1$



$f_0(1370)$ and $f_0(1500)$: positions of poles, $C = 0$



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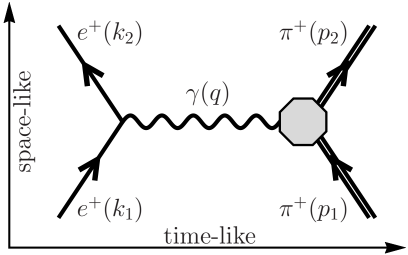
Pion electromagnetic form factor in the P wave

What are the correct $\rho(770)$ meson mass and width values?

PRD 96, 113004 (2017)

Erik Bartoŝ, Stanislav Dubniĉka, Andrej Liptaj Anna Zuzana Dubniĉkova and RK

$$\langle \pi^+(p_2)|J^\mu_\pi(0)|\pi^-(p_1)\rangle = e\,(p_1+p_2)^\mu\, F_\pi(q^2)$$



Gounaris-Sakurai pion EM FF and $\rho(770)$

Gounaris-Sakurai pion electromagnetic form factor at the elastic region.

What are the correct $\rho(770)$ *meson mass and width values?*
Erik Bartoŝ, Stanislav Dubniĉka, Andrej Liptaj Anna Zuzana Dubniĉkova and RK
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$$\sigma_{tot}(e^+e^- \to \pi^+\pi^-) = \frac{\pi\alpha^2(0)}{3s}\beta_\pi^3(s)\left|F_\pi^{EM,I=1}(s) + Re^{i\phi}\frac{m_\omega^2}{m_\omega^2 - s - im_\omega\Gamma_\omega}\right|^2$$

where pion "velocity" $\beta_\pi(s) = \sqrt{\frac{s-4m_\pi^2}{s}}$, R - amplitude for $\rho - \omega$ interference (free parameter), phase $\phi = ArcTan\frac{m_\rho\Gamma_\rho}{m_\rho^2 - m_\omega^2}$ is the $\delta_1^1 = \delta_\rho$ fixed at $s = m_\omega^2$

Fit to data for $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$
- M. Ablikin et al. (BESIII Collaboration), Phys. Lett. B 753, 629 (2016).
- J. P. Lees et al. (BABAR Collaboration), Phys. Rev. D 86, 032013 (2012).

Gounaris-Sakurai pion electromagnetic form factor

G. J. Gounaris and J. J. Sakurai, Phys. Rev. Lett. 21, 244 (1968)

Assumption: $\frac{q^3}{\sqrt{s}} Cotg\delta_1^1(s) = a + bq^2 + h(s)q^2$ where $h(s) = \frac{2q}{\pi\sqrt{s}} Log\left(\frac{\sqrt{s}+2q}{2m_\pi}\right)$
Then $F_\pi^{GS}(s) = \frac{\sqrt{s}}{q^3} \frac{1}{Cotg\delta_1^1(s) - i}$ - no direct dependence on $\rho(770)$ parameters, however taking into account two conditions:

- $Cotg\delta_1^1(s)\Big|_{s=m_\rho^2} = 0$ and
- $F_\pi^{BW}(s) = \frac{m_\rho^2}{m_\rho^2 - s - im_\rho\Gamma_\rho} \rightarrow \delta_1^1(s) = ArcTan\frac{m_\rho\Gamma_\rho}{m_\rho^2 - s} \rightarrow \frac{d\delta_1^1(s)}{ds}\Big|_{s=m_\rho^2} = \frac{1}{m_\rho\Gamma_\rho}$

$$a = \frac{4q_\rho^5}{m_\rho^2\Gamma_\rho} + 4q_\rho^4 h'(m_\rho^2)$$

$$b = -\frac{4q_\rho^3}{m_\rho^2\Gamma_\rho} - 4q_\rho^4 h'(m_\rho^2) - h(m_\rho^2)$$

$$F_\pi^{GS}(s) = \frac{m_\rho^2 + m_\rho\Gamma_\rho\left[\frac{3m_\pi^2}{\pi q_\rho^2} log\left(\frac{m_\rho + 2mq_\rho}{2m_\pi}\right) + \frac{m_\rho}{2\pi q_\rho} - \frac{m_\pi^2 m_\rho}{\pi q_\rho^3}\right]}{(m_\rho^2 - s) + \Gamma_\rho\frac{m_\rho^2}{q_\rho^3}\left[q^2\left(h(s) - h(m_\rho^2)\right) + q_\rho^2 h'(m_\rho^2)(m_\rho^2 - s)\right] - im_\rho\Gamma_\rho\left(\frac{q}{q_\rho}\right)^3\frac{m_\rho}{\sqrt{s}}}$$

Fit to unified BESIII-BABAR data at the elastic region using G-S model

- $\chi^2 = 40.6$ pdf
- $m_\rho = (775.73 \pm 0.10)$ MeV
- $\Gamma_\rho = (126.51 \pm 0.13)$ MeV
- PDG'2017:

  $m_\rho = 775.26 \pm 0.25$ MeV (from $e^+e^- \to \pi^+\pi^-$)
  $m_\rho = 769.0 \pm 0.9$ MeV (other)

  $\Gamma_\rho = 147.8 \pm 0.9$ MeV (from $e^+e^- \to \pi^+\pi^-$)
  $\Gamma_\rho = 150.9 \pm 1.7$ MeV (other)

in Ref. [6].

Now, the $\rho^0(770)$ meson parameters will be determined by an application of the U&A model of the pion EM FF to an optimal description of the same data on the total cross section of the $e^+e^- \to \pi^+\pi^-$ process.

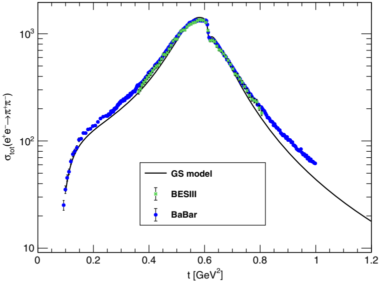

FIG. 2. Optimal description of the unified BESIII-*BABAR* data on $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ at the elastic region by the pion EM FF G.-S. model.

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Fit to unified BESIII-BABAR data at the elastic region using U&A model

- $F_{\pi}^{VDM}(s) = \frac{m_{\rho}^2 f_{\rho\pi\pi}/f_{\rho}}{m_{\rho}^2 - s}$
- $F_{\pi}^{VDM}(q) = \frac{(i-q_{\rho})(i+q_{\rho})}{(q-q_{\rho})(q+q_{\rho})} f_{\rho\pi\pi}/f_{\rho}$
- $F_{\pi}^{VDM}(q) = \frac{(q-q_Z)(i-q_P)(i-q_{\rho})(i+q_{\rho})}{(q-q_P)(i-q_Z)(q-q_{\rho})(q+q_{\rho})} f_{\rho\pi\pi}/f_{\rho}$
- $\chi^2 = 1.54$ pdf
- $m_{\rho} = (763.03 \pm 0.14)$ MeV
- $\Gamma_{\rho} = (144.8 \pm 0.23)$ MeV

The optimal description of the recent data [1,2] on the total cross section of the $e^+e^- \to \pi^+\pi^-$ process at the region of the ρ meson resonance by (21) (see Fig. 3), achieved with $\chi^2/ndf = 1.5443$, gives the ρ meson mass and width

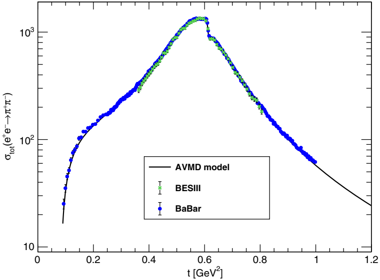
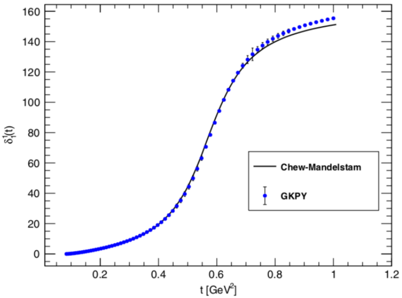


FIG. 3. Optimal description of the unified BESIII-*BABAR* data on $\sigma_{\rm tot}(e^+e^- \to \pi^+\pi^-)$ at the elastic region by the pion EM FF U&A model.

Fit to δ_1^1 data using Chew-Mandelstam type effective-range formula

- $\frac{q^3}{\sqrt{s}} Cotg\delta_1^1(s) = a + bq^2 + h(s)q^2$
  where $h(s) = \frac{2q}{\pi\sqrt{s}} Log\left(\frac{\sqrt{s}+2q}{2m_\pi}\right)$
- $a = 0.2860 \pm 0.0011$ MeV^2,
  $b = -2.7025 \pm 0.0089$
- $\chi^2 = 2.45$ pdf
- $m_\rho = (772.42 \pm 0.03)$ MeV
- $\Gamma_\rho = (153.85 \pm 0.11)$ MeV



Fit to δ_1^1 data

- $F_\pi^{EM,I=1}(s) =$

$$P_n(s)exp\left[\frac{s}{\pi}\int_4^\infty \frac{\delta_1^1(s')}{s'(s'-s)}ds'\right]$$

- $\delta_1^1(q) = ArcTan\frac{A_3q^3+A_5q^5+\dots}{1+A_2q^2+A_4q^4+\dots}$
- $\equiv Log\frac{(q-q_2)(q-q_3)(q-q_4)(q-q_5)}{(q-q_2^*)(q-q_3^*)(q-q_4^*)(q-q_5^*)}$
- $\chi^2 = 0.024$ pdf
- $m_\rho = (763.56 \pm 0.51)$ MeV
- $\Gamma_\rho = (143.09 \pm 0.82)$ MeV

$$\delta_1^1(q) = arctg\frac{A_3q^3+A_5q^5+\dots}{1+A_2q^2+A_4q^4+\dots}, \qquad (26)$$

where $A_1 \equiv 0$ in order to secure the threshold behavior of $\delta_1^1(q)$. An optimal description of the GKPY phase shift $\delta_1^1(q)$ data is achieved (see Fig. 5) with $\chi^2/ndf = 0.0244$ and four nonzero coefficients A_2, A_3, A_4, and A_5.

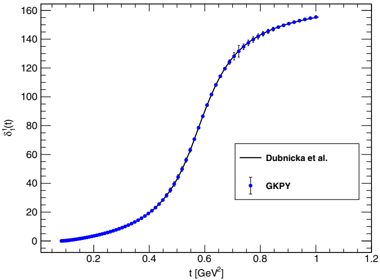


FIG. 5. Optimal description of the most accurate up-to-now P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ data with model-independent parametrization (26).

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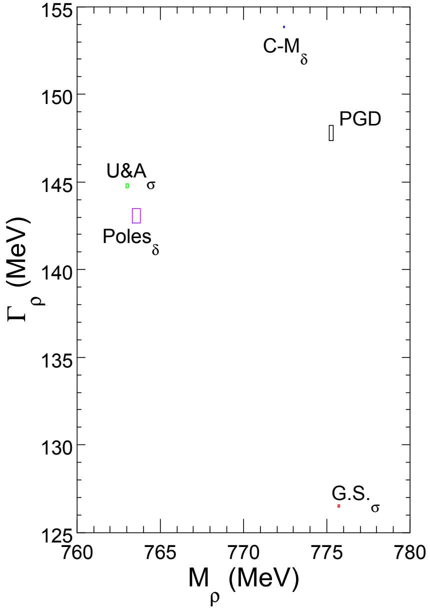
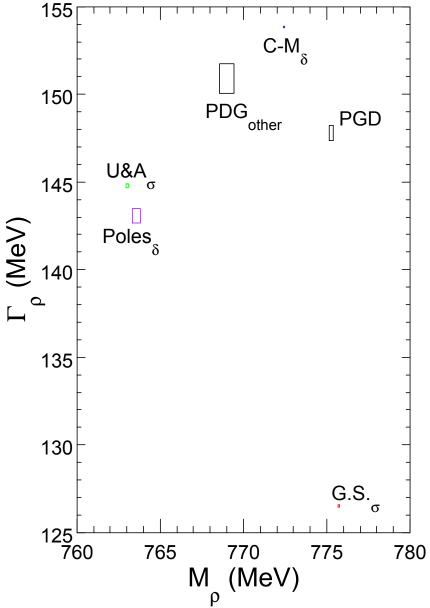


C-M$_\delta$
PGD
U&A$_\sigma$
Poles$_\delta$
G.S.$_\sigma$
Γ_ρ (MeV)
M_ρ (MeV)
155
150
145
140
135
130
125
760
765
770
775
780



C-M$_\delta$
PDG$_{other}$
PGD
U&A$_\sigma$
Poles$_\delta$
G.S.$_\sigma$
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150
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140
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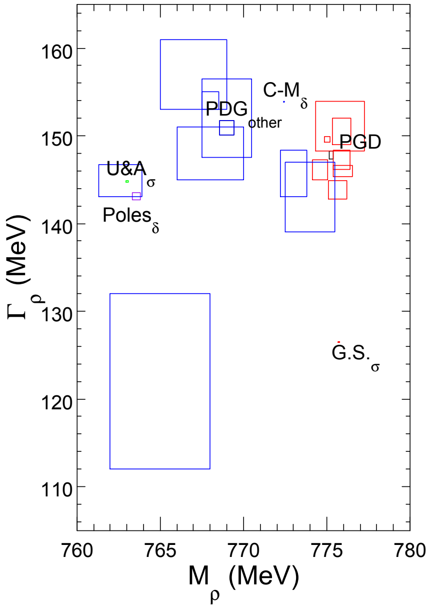
130
125
760
765
770
775
780
Γ_ρ (MeV)
M_ρ (MeV)



C-M$_\delta$
PDG$_{other}$
PGD
U&A$_\sigma$
Poles$_\delta$
G.S.$_\sigma$
160
150
140
130
120
110
760
765
770
775
780
Γ_ρ (MeV)
M_ρ (MeV)

Generalisation of Gounaris-Sakurai model to excited ρ mesons $\rho(1450)$ and $\rho(1700)$

- $F_\pi = \frac{1}{1+\beta+\gamma}\left[F^{GS}_{\rho(770)}(s)\left(1 + \delta \frac{s}{m^2_\omega} BW_\omega(s)\right) + \beta F^{GS}_{\rho(1450)}(s) + \gamma F^{GS}_{\rho(1700)}(s)\right]$
- $\chi^2 = 0.98$ pdf

Generalisation of U&A model to excited ρ mesons

- $F_\pi = \frac{\Pi(q - q_i)}{\Pi(q + q_i^*)}$
- $\chi^2 = 1.84$ pdf

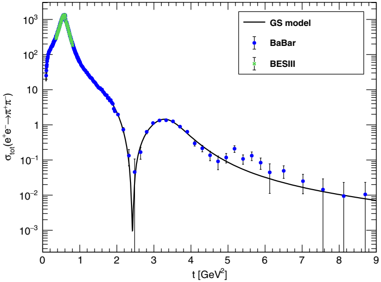


FIG. 6. Optimal description of the unified BESIII-*BABAR* complete data on $\sigma_{\rm tot}(e^+e^- \to \pi^+\pi^-)$ by the generalized pion EM FF G.-S. model.

determined by the original pion EM FF G.-S. model (13) to be valid only at the elastic region.

A totally different situation is in a generalization of the U&A pion EM FF model. Here, the contribution of all three vector mesons is at an equal level. Only now, the effective inelastic threshold, which is left as a free parameter of the model, has to be taken into account explicitly. Therefore, instead of the q variable, the W variable

Parameter	PDG MeV	G.S. MeV	U&A MeV
m_ρ	775.26 ± 0.25	774.81 ± 0.01	763.88 ± 0.04
$m_{\rho'}$	1465.00 ± 25.00	1497.70 ± 1.07	1326.35 ± 3.46
$m_{\rho''}$	1720.00 ± 20.00	1848.40 ± 0.09	1770.54 ± 5.49
Γ_ρ	149.10 ± 0.80	149.22 ± 0.01	144.28 ± 0.01
$\Gamma_{\rho'}$	400.00 ± 60.00	442.15 ± 0.54	324.13 ± 12.01
$\Gamma_{\rho''}$	250.00 ± 100.00	322.48 ± 0.69	268.98 ± 11.40
χ^2 pdf		0.98	1.84
		14 param.	11 param.

WHAT ARE THE CORRECT $\rho^0(770)$ MESON MASS …

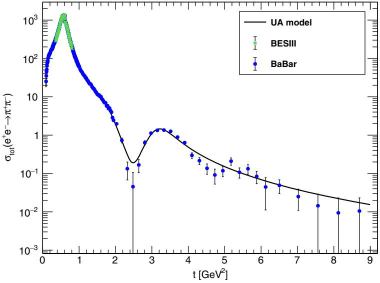


FIG. 7. Optimal description of the unified BESIII-*BABAR* complete data on $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ by the generalized pion

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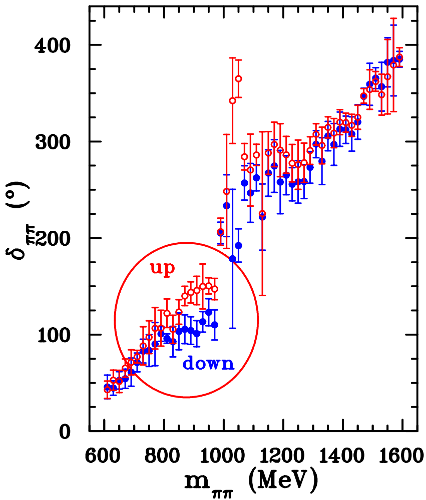
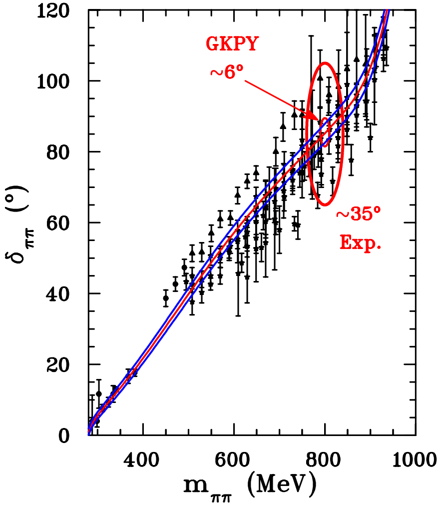
Experimental data for the $\pi\pi$ in the S0 wave (JI)

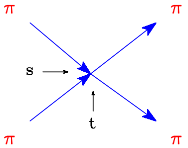
In PWA (CERN-Munich group'74) $A(s,t) \sim Cos(\theta_S - \theta_P)$





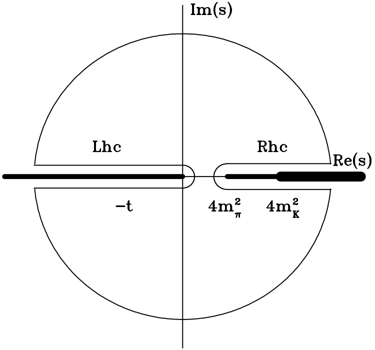
Dispersion relations with imposed crossing symmetry condition for $\pi\pi$ interactions theory $\longleftrightarrow$ experiment

crossing symmetry:



$\rightarrow \vec{T}_s(s,t) = \hat{C}_{st}\vec{T}_t(t,s)$

$\vec{T}(s,t)$ + crossing symmetry $\rightarrow$ dispersion relations for $\underline{4m_\pi^2 < s < \sim (1150\ \text{MeV})^2}$

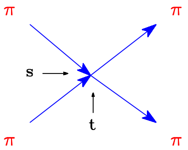


Once subtracted DR:

$$\begin{aligned}
\text{Re}\,\vec{F}(s,t) \quad &= \quad \text{Re}\,\vec{F}(s_0,t) + \frac{s-s_0}{\pi} \\
&\times \left[\int_{4m_\pi^2}^{\infty} ds' \frac{\text{Im}\,\vec{F}(s',t)}{(s'-s_0)(s'-s)} \right. \\
&+ \left. \int_{-t}^{-\infty} ds' \frac{\text{Im}\,\vec{F}(s',t)}{(s'-s_0)(s'-s)} \right]
\end{aligned}$$

Dispersion relations with imposed crossing symmetry condition for $\pi\pi$ interactions theory $\longleftrightarrow$ experiment

crossing symmetry:



$\rightarrow \vec{\boldsymbol{T}}_{\boldsymbol{s}}(s,t) = \hat{\boldsymbol{C}}_{st} \vec{\boldsymbol{T}}_{\boldsymbol{t}}(t,s)$

$\vec{\boldsymbol{T}}(\boldsymbol{s},\boldsymbol{t})$ + crossing symmetry $\rightarrow$ dispersion relations for $\underline{4m_\pi^2 < s < \sim (1150 \text{ MeV})^2}$

Once subtracted dispersion relations ("GKPY" for the S and P waves):

$$\boxed{\text{Re } t_\ell^{I(OUT)}(s) = \sum_{I'=0}^{2} C_{st}^{II'} a_0^{I'} + \sum_{I'=0}^{2} \sum_{\ell'=0}^{4} \; \fint_{4m_\pi^2}^{\infty} ds' K_{\ell\ell'}^{II'}(s,s') \text{ Im } t_{\ell'}^{I'(IN)}(s')}$$

$a_0^{I'}$ - subtraction constant $= \vec{\boldsymbol{T}}_{\boldsymbol{s}}(s = 4m_\pi^2, t = 0)$ - scattering lengths from only S wave

due to Re $t_\ell^I(k) = k^{2\ell}(a_\ell^I + b_\ell^I k^2 + O(k^4))$ $\qquad$ $\underline{\text{Re } t_\ell^{I(OUT)}(s) - \text{Re } t_\ell^{I(IN)}(s) \rightarrow 0}$

GKPY equations and $\pi\pi$ amplitudes

partial waves: ***Jl***

experiment

F1 D2

S0 D0

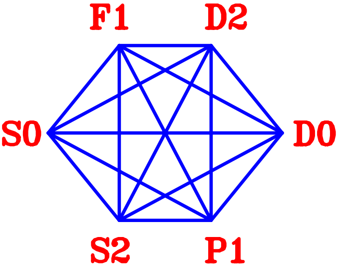
S2 P1

GKPY equations and $\pi\pi$ amplitudes

partial waves: *Jl*

experiment + theory (GKPY)

$$\int_{4m_\pi^2}^{\infty}$$



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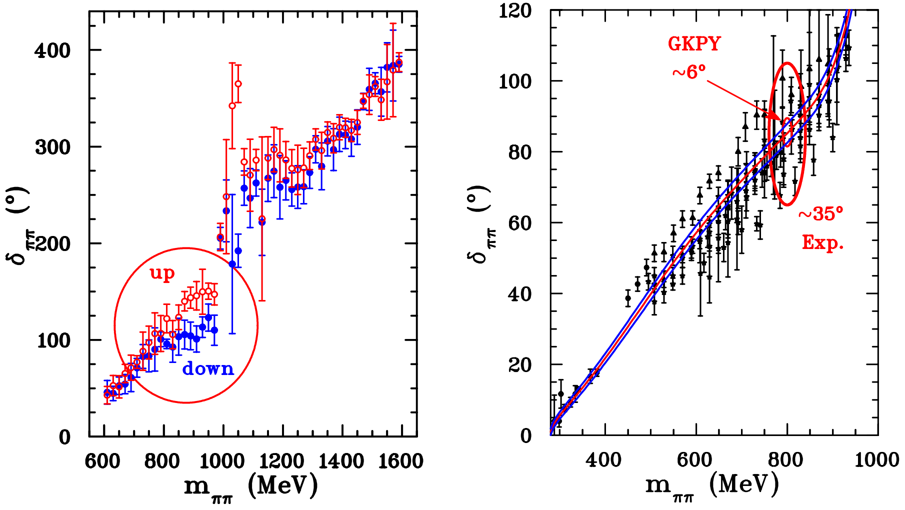
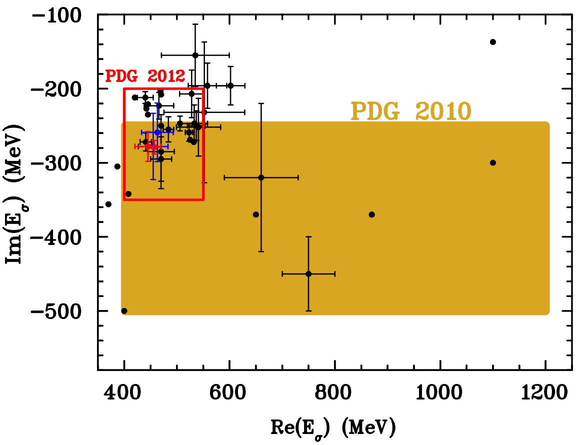


precise determination of $f_0(500)$ (σ) meson and threshold parameters

$f_0(500)$ (σ)

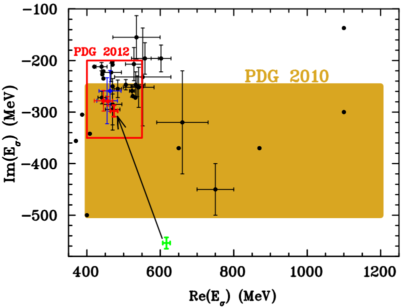
- PDG 2010:
  M = 400 – 1200 MeV
  $\Gamma = 2 \times (250 - 500)$ MeV
- PDG 2012:
  M = 400 – 550 MeV
  $\Gamma = 2 \times (200 - 350)$ MeV
- GKPY:
  $E_\sigma = 457 \pm 14 - i279^{+11}_{-7}$ MeV



threshold parameters, e.g. a_0^0:

- ChPT + Roy eqs (Bern group):
  $0.220 \pm 0.005\, m_\pi^{-1}$
- GKPY:
  $0.220 \pm 0.008\, m_\pi^{-1}$

what forces GKPY eqs to pull up-left the sigma pole?



Two things: trigonometry and crossing symmetry algebra lead to narrower and lighter σ.

Modified $\pi\pi$ amplitude with σ pole PRD 90, 116005 (2014) P. Bydzovský, 1, R. Kamiński, V. Nazari

Nothing more and nothing instead of it is needed.

Conclusions and what to do?

Conclusions:

- there is significant difference between parameters (especially mass) of a resonance using different methods of parameterisation of amplitudes,
- amplitudes should be unitary and analytic
- we should always determine what definition we use,
- the determination of the most significant poles in amplitudes becomes difficult for $n > 2$,

What to do?

- never use isobar model!
- anyway be carefull! = check the 2nd point of conclusions!
- possible parameterizations: made for wide energy range including many resonances we are interested in,
- please use only parameterization based on S- or K-matrix approach!
- possible degrees of freedom: poles themself! mesons
- on $\pi\pi$, $K\bar{K}$ ... amplitudes in all partial waves up to 1.8 GeV ask RK - they are done
- on other channels ask RK - they may be done

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Conclusions on scalar resonances

Experimental situation in the $0^{+}0^{++}$ sector is finally clear (although without new experiments!),

- $f_0(500)$ (σ) - loosely bound $\pi\pi$ system or/and $2q2\bar{q}$ state,
- $f_0(980)$ - dynamically generated $2q2\bar{q}$ state or/and $K\bar{K}$ quasi bound state,
- $f_0(1370)$ - ordinary $q\bar{q}$ state,
- $f_0(1500)$ - ordinary $q\bar{q}$ state + glueball,
- $f_0(1720)$ - glueball

Probably $K_0^*(800)$ (κ) is also dynamical effect of coupling of $K_0^*(1430)$