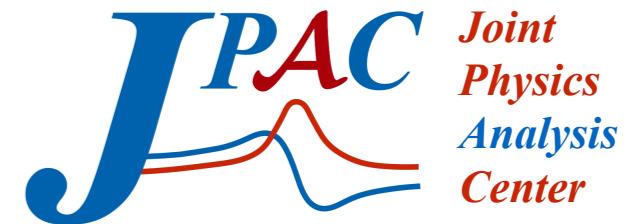


Photoproduction of 1 and 2 Mesons

Vincent MATHIEU

U. Complutense Madrid
Joint Physics Analysis Center

JPAC Collaboration Workshop
ECT*, December 2019



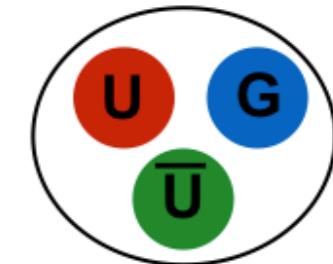
The Factorization Hypothesis



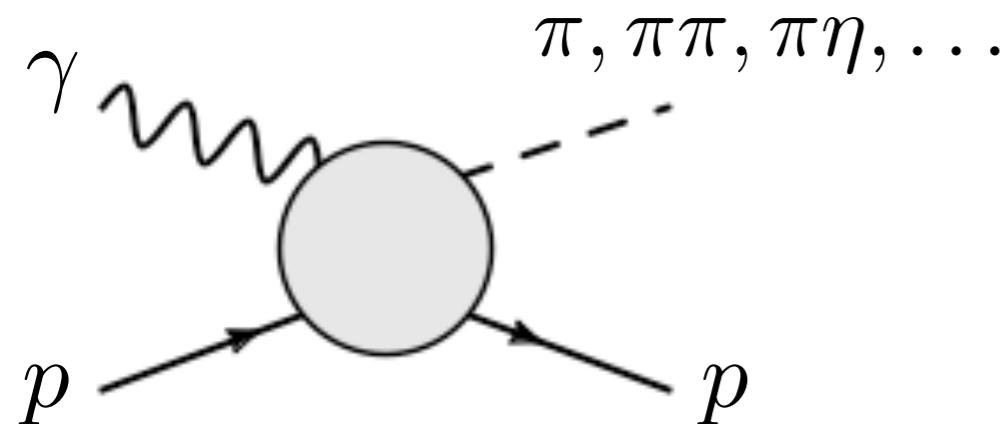
Photoproduction of mesons at $E_\gamma = 6 - 12 \text{ GeV}$

Study photoproduction of mesons

Search for exotic resonances



Special interest in mesons:



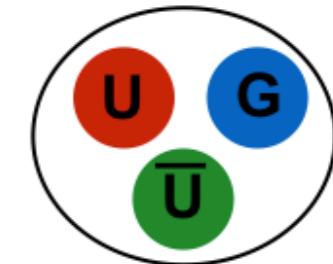
The Factorization Hypothesis



Photoproduction of mesons at $E_\gamma = 6 - 12 \text{ GeV}$

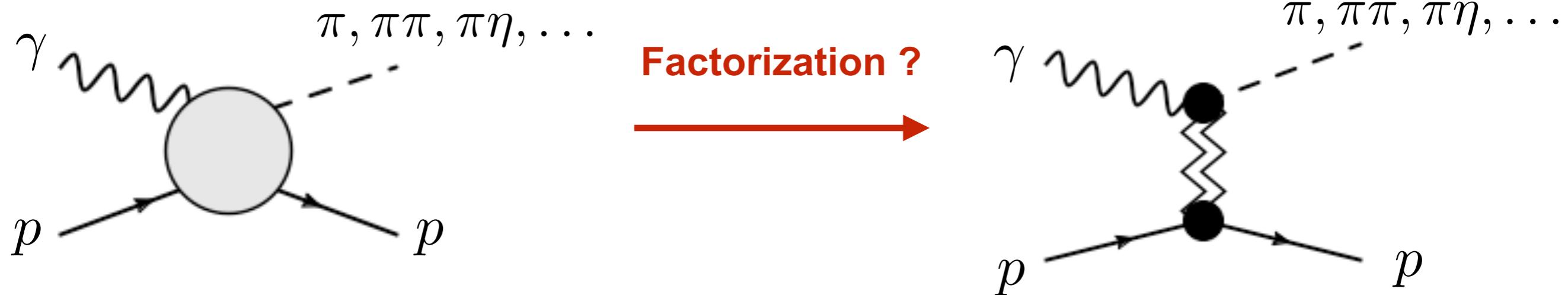
Study photoproduction of mesons

Search for exotic resonances



Special interest in mesons:

Does the target decouple at JLab energies ?



JPAC publications on JLab physics

Single Meson Photoproduction:

$\vec{\gamma}p \rightarrow \pi N$ Mathieu et al **PRD92 074013 (2015)**
 Mathieu et al **PRD98 014041 (2018)**

$\vec{\gamma}p \rightarrow \eta p$ Nys et al **PRD95 034014 (2017)**
 Mathieu et al **EPL122 41001 (2018)**

$\vec{\gamma}p \rightarrow \pi \Delta$ Nys et al **PLB779 77 (2018)**

$\vec{\gamma}p \rightarrow \eta' p$ Mathieu et al **PLB774 362 (2017)**

Vector Meson Photoproduction:

$\vec{\gamma}p \rightarrow \rho^0 p$
 $\vec{\gamma}p \rightarrow \omega p$
 $\vec{\gamma}p \rightarrow \phi p$

Mathieu et al
PRD97 094003 (2018)

$\vec{\gamma}p \rightarrow J/\psi p$ Hiller Blin et al **PRD94 034002 (2016)**
 Winney et al **PRD100 034019 (2019)**

Double Mesons Photoproduction:

$\vec{\gamma}p \rightarrow \pi^0 \eta p$ Mathieu et al **PRC100 0540 (172019)**

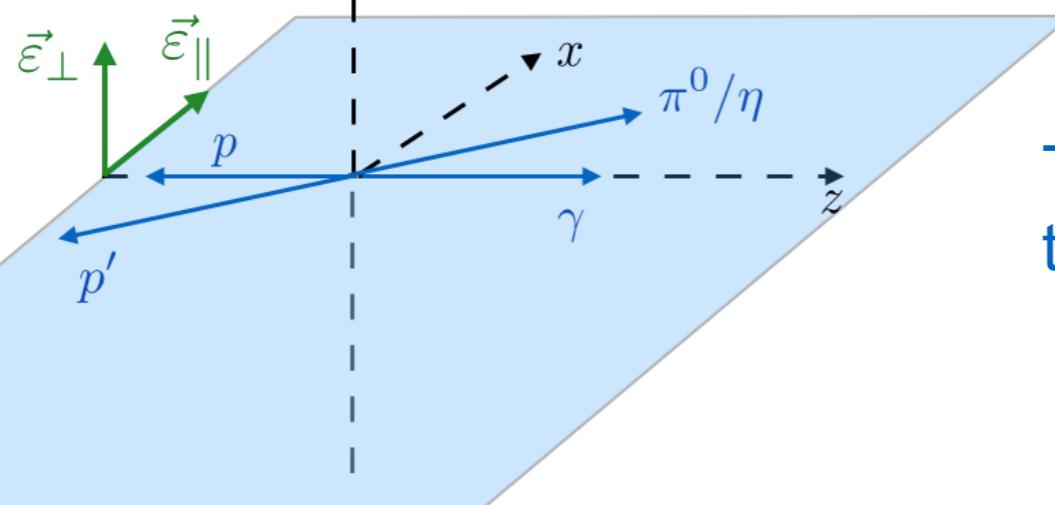
Inclusive Electroproduction:

$e^- p \rightarrow e^- X$ Hiller Blin et al **PRC100 035201 (2019)**

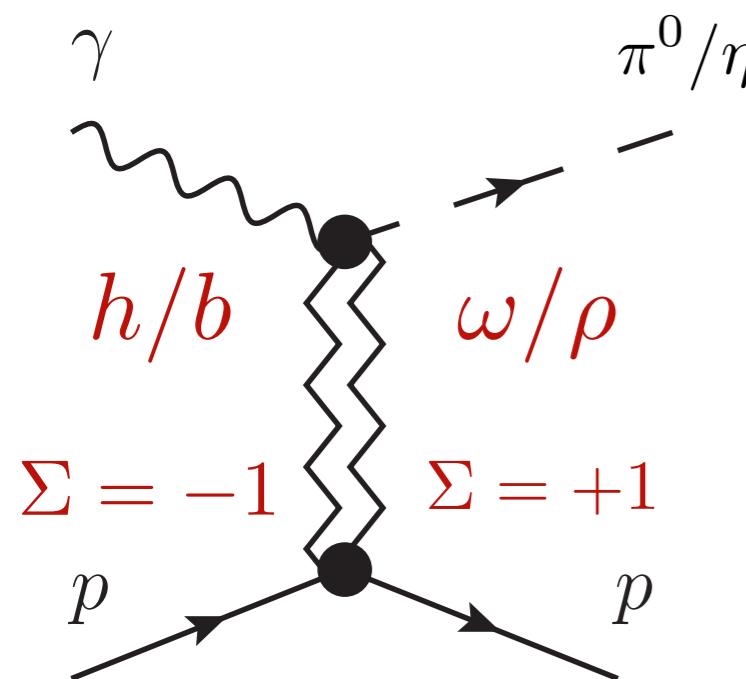
Simulations and codes available: <http://www.ceem.indiana.edu/~jpac/>

Single Meson Photoproduction

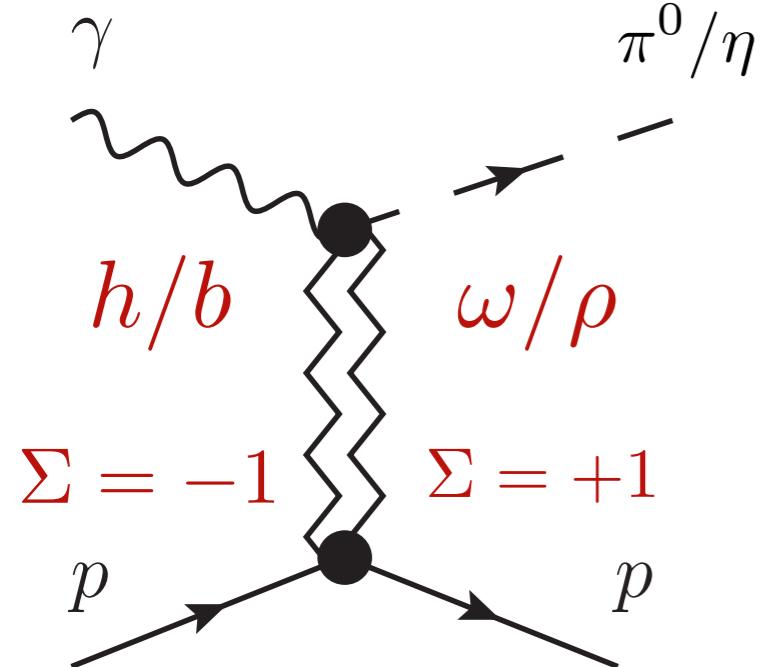
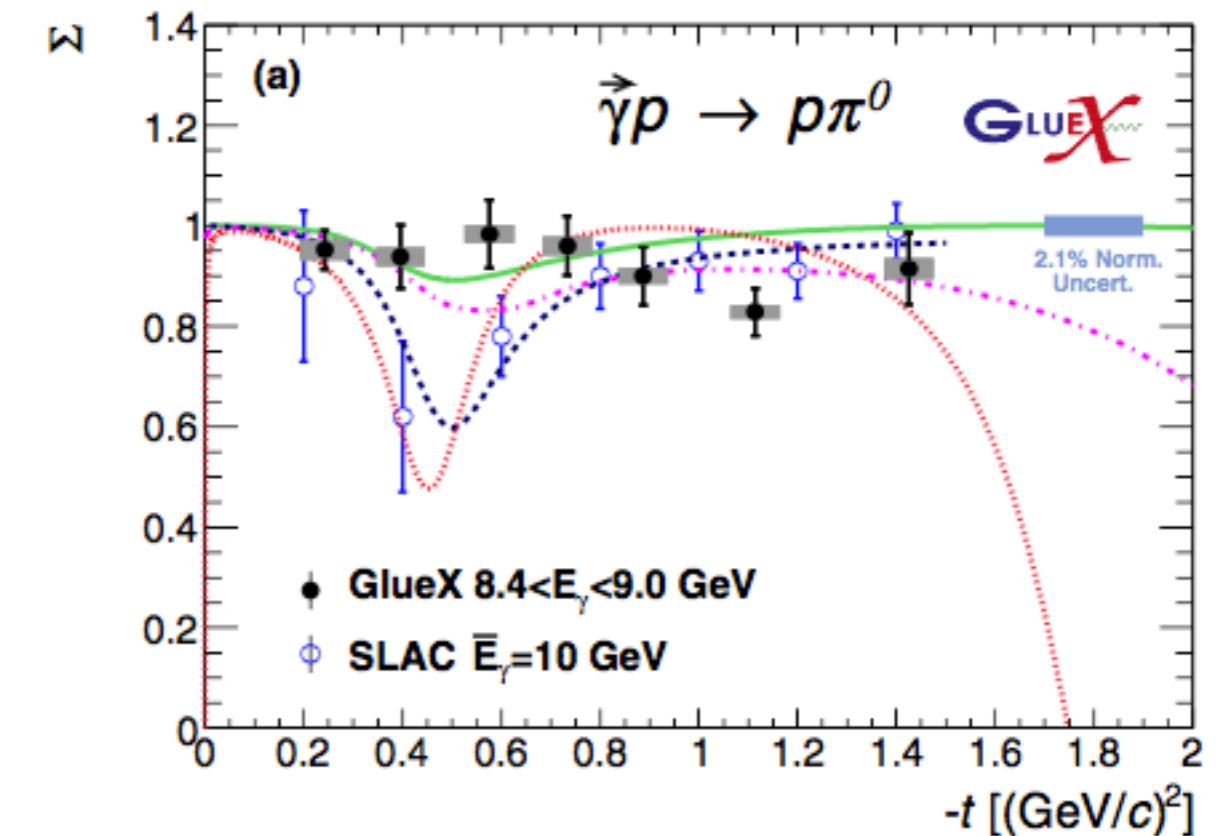
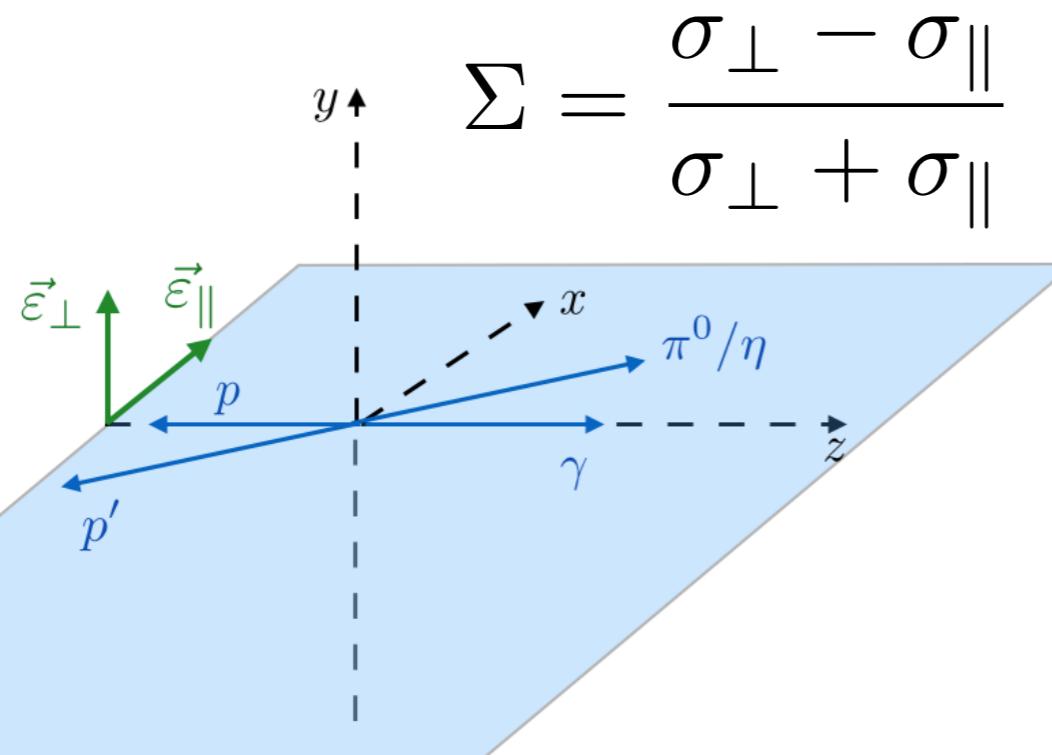
$$\Sigma = \frac{\sigma_{\perp} - \sigma_{\parallel}}{\sigma_{\perp} + \sigma_{\parallel}}$$



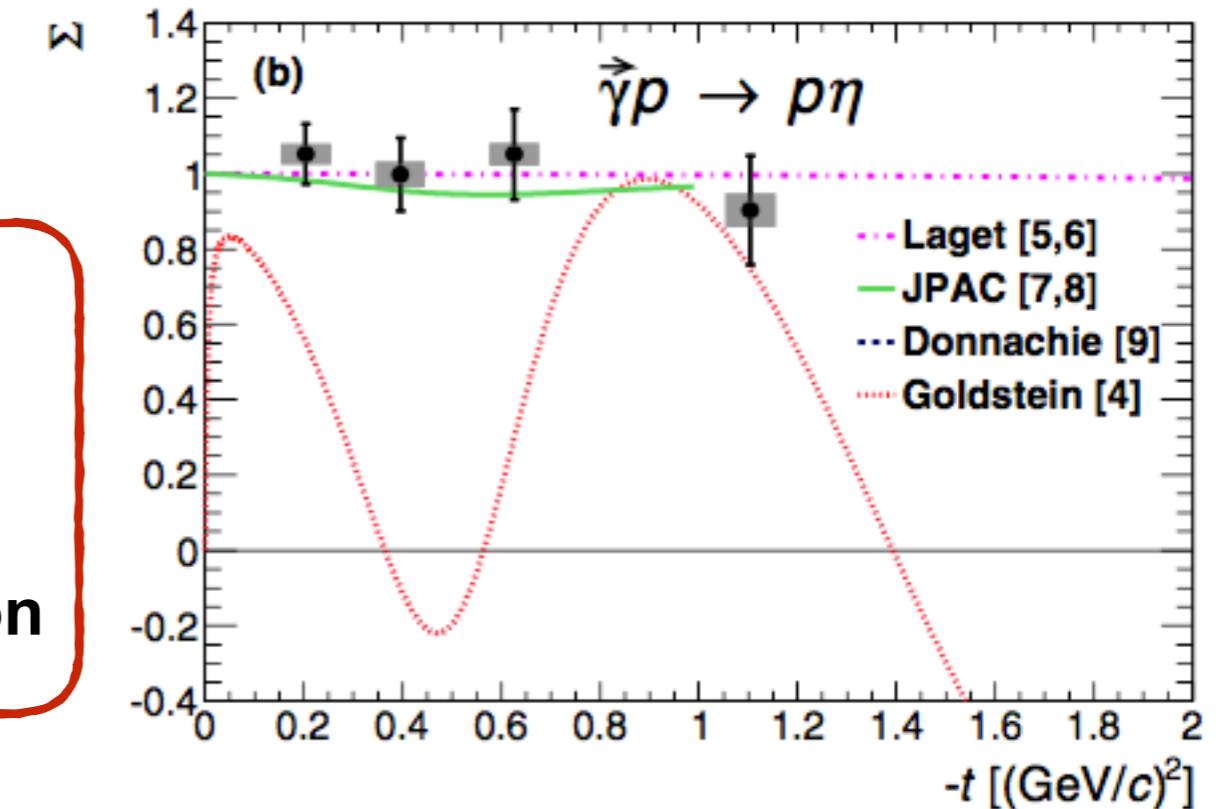
The beam asymmetry Σ is related to the reflection through the reaction plane



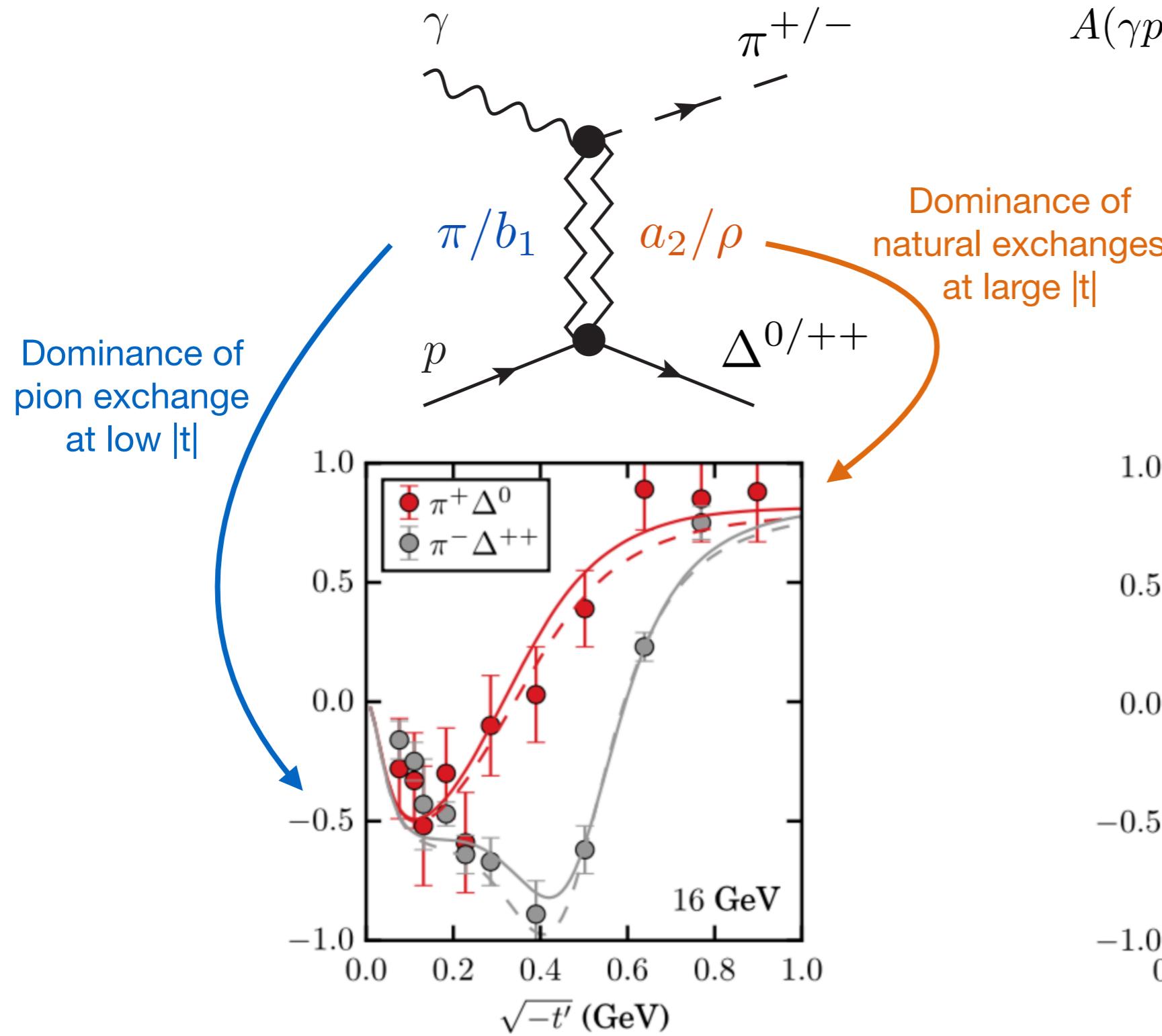
Single Meson Photoproduction



Dominance of vector meson exchange in both π^0/η photoproduction



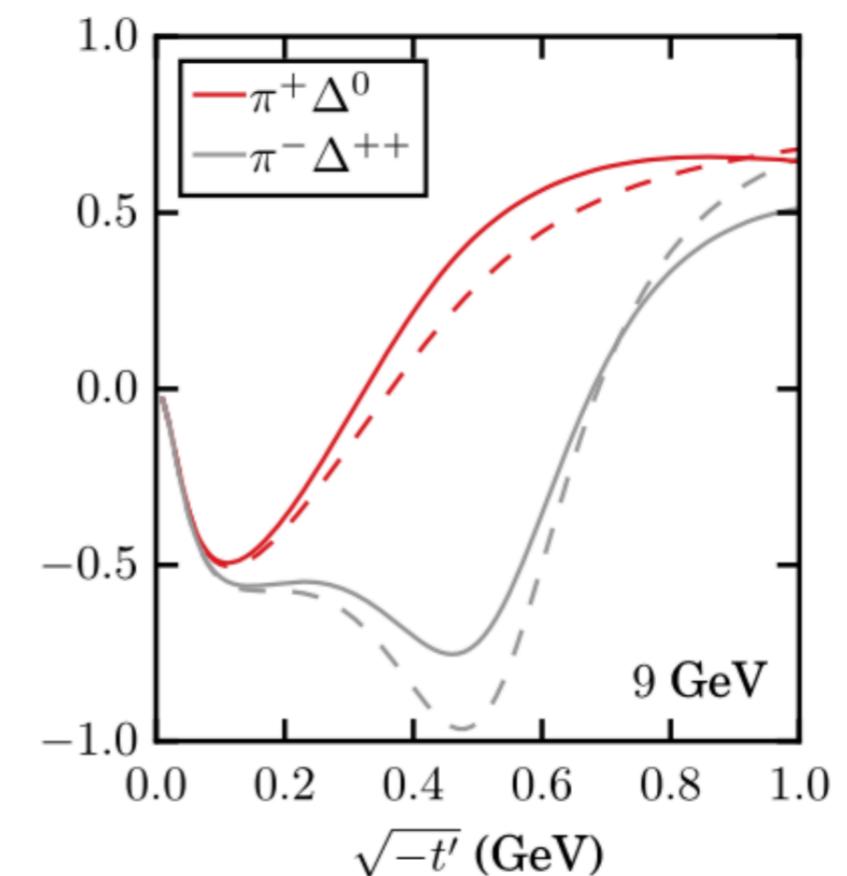
Single Meson Photoproduction



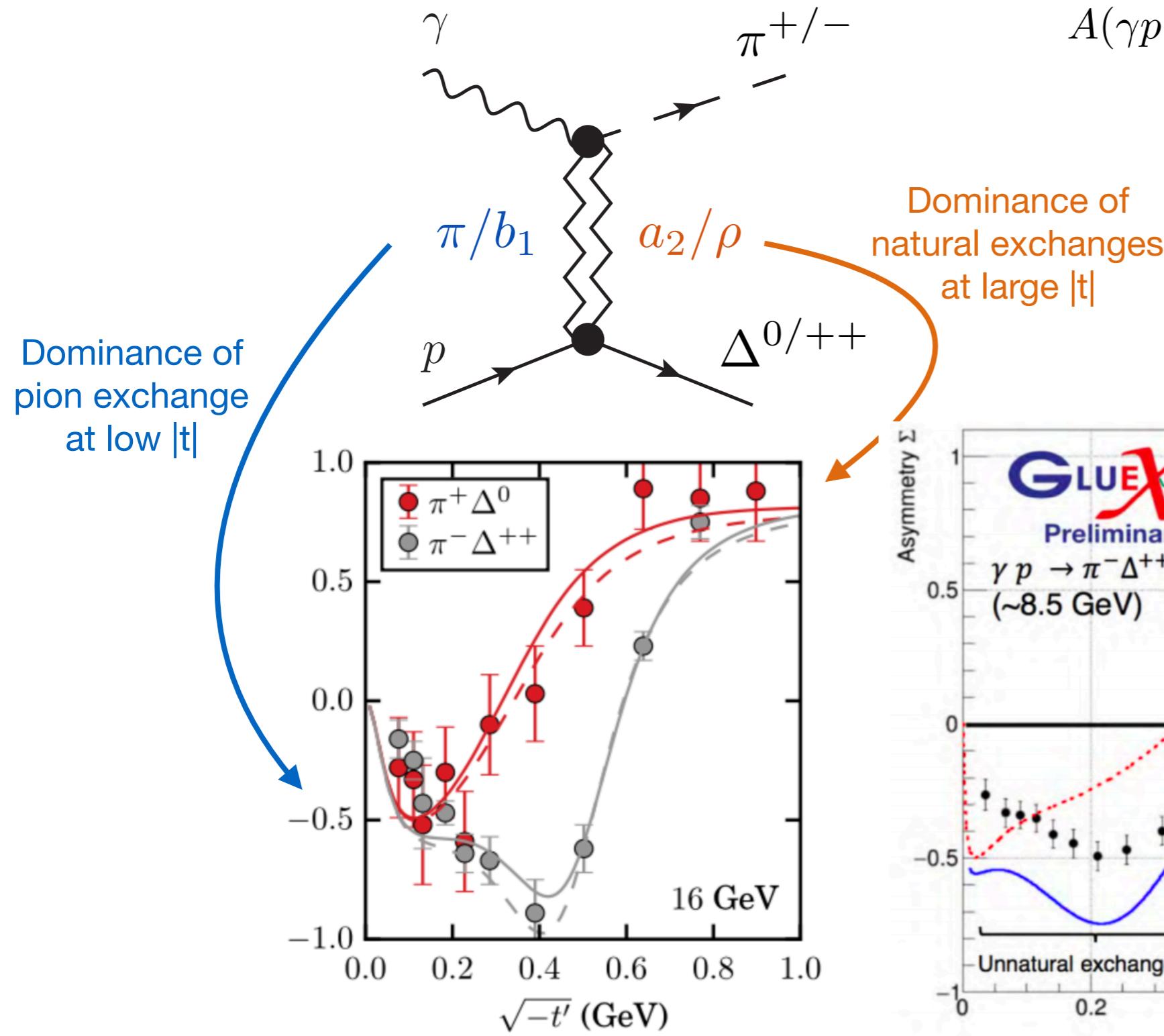
$$\sqrt{3}A(\gamma p \rightarrow \pi^+ \Delta^0) = A^{\rho+b_1} + A^{a_2+\pi}$$

$$A(\gamma p \rightarrow \pi^- \Delta^{++}) = A^{\rho+b_1} - A^{a_2+\pi}$$

Prediction for GlueX

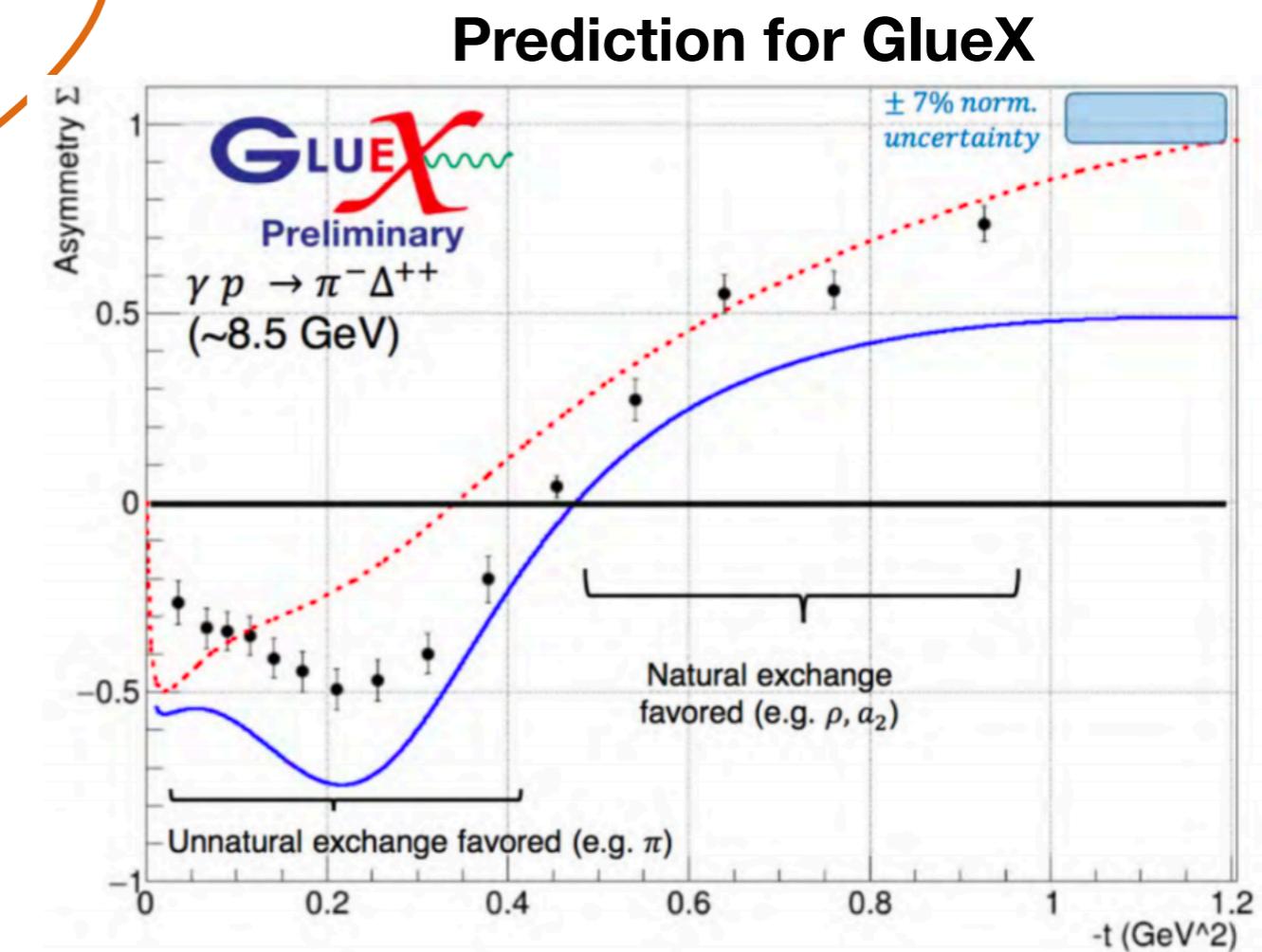


Single Meson Photoproduction

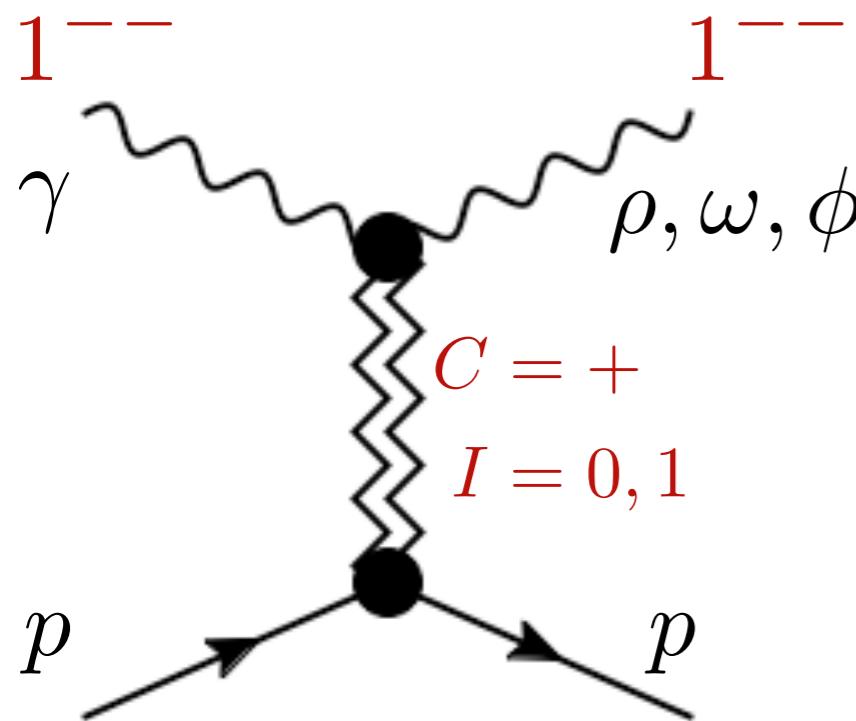


$$\sqrt{3}A(\gamma p \rightarrow \pi^+ \Delta^0) = A^{\rho+b_1} + A^{a_2+\pi}$$

$$A(\gamma p \rightarrow \pi^- \Delta^{++}) = A^{\rho+b_1} - A^{a_2+\pi}$$



Vector Meson Photoproduction



Probe different exchanges by combined analysis of ρ, ω, ϕ

Pomeron dominates at high energies

Use the angular distribution of the vector to extract spin density matrix elements

$$\frac{8\pi}{3} \frac{d\sigma}{d\Omega} = 1 - \rho_{00}^0 + (3\rho_{00}^0 - 1) \cos^2 \theta - 2\sqrt{2} \operatorname{Re} \rho_{10}^0 \sin 2\theta \cos \phi - 2\rho_{1-1}^0 \sin^2 \theta \cos 2\phi$$

9 SDME accessible with linearly polarized beam

$$\rho_{00}^0$$

$$\operatorname{Re} \rho_{10}^0$$

$$\rho_{1-1}^0$$

$$\rho_{11}^1$$

$$\operatorname{Re} \rho_{10}^1$$

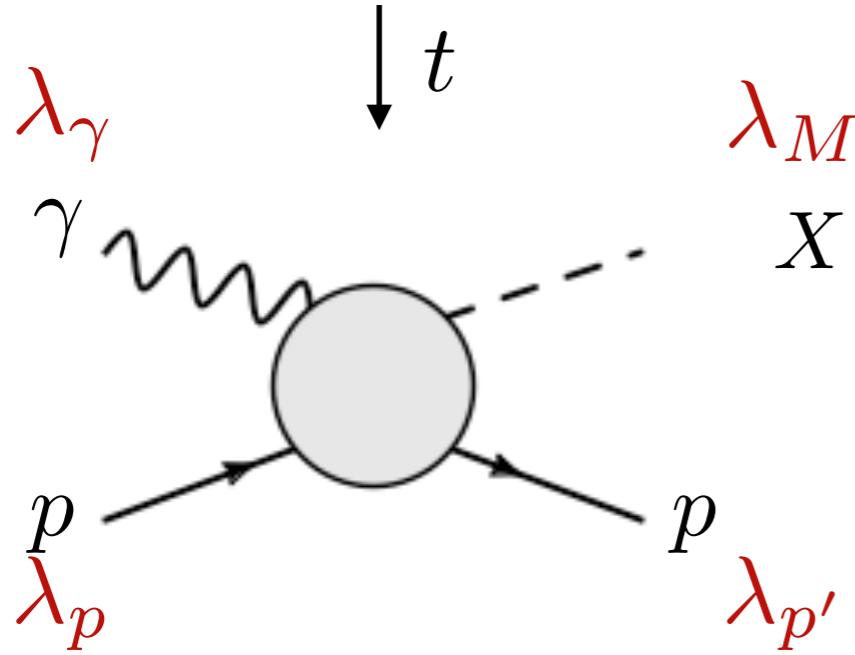
$$\rho_{11}^0$$

$$\rho_{1-1}^1$$

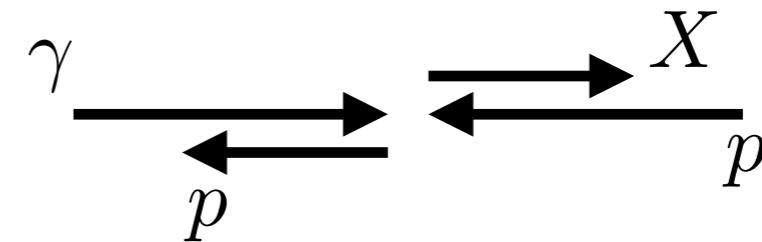
$$\operatorname{Im} \rho_{10}^2$$

$$\operatorname{Im} \rho_{1-1}^2$$

Factorization



Angular mom. conservation in forward direction:

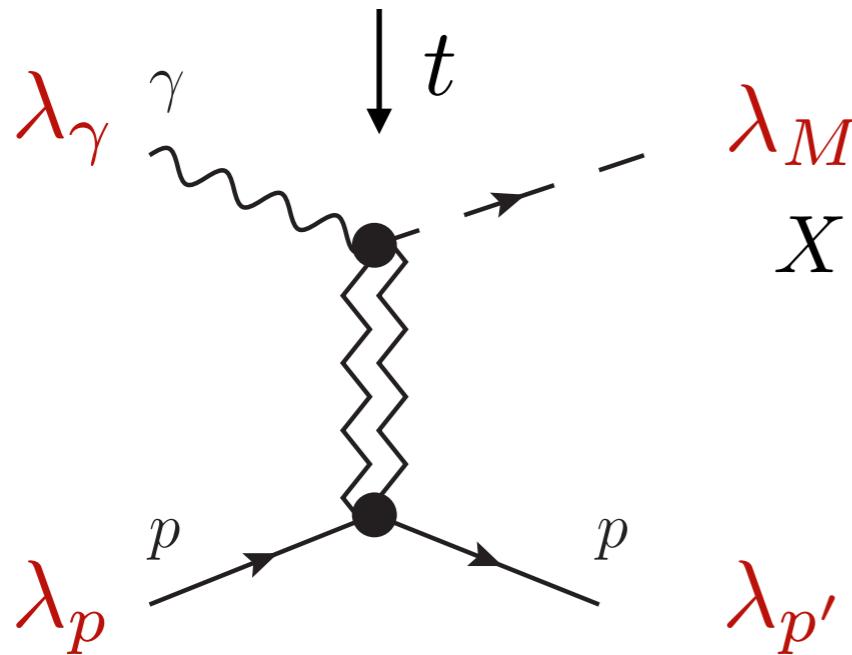


$$A_{\lambda_p \lambda_{p'}}^{\lambda_\gamma \lambda_M} \propto \underbrace{(\sin \theta/2)}_{\sqrt{-t}}^{|\lambda_\gamma - \lambda_M| - |\lambda_p - \lambda_{p'}|}$$

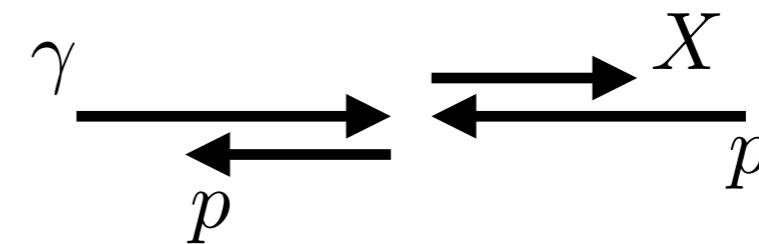
Leading order in the energy :

$$A_{\lambda_p \lambda_{p'}}^{\lambda_\gamma \lambda_M} \propto \gamma(t) (\sqrt{-t})^{|\lambda_\gamma - \lambda_M| - |\lambda_p - \lambda_{p'}|}$$

Factorization



Angular mom. conservation in forward direction:



$$A_{\lambda_p \lambda_{p'}}^{\lambda_\gamma \lambda_M} \propto \underbrace{(\sin \theta/2)}_{\sqrt{-t}}^{|\lambda_\gamma - \lambda_M|} |^{\lambda_\gamma - \lambda_p + \lambda_{p'} - \lambda_M|}$$

Leading order in the energy :

$$A_{\lambda_p \lambda_{p'}}^{\lambda_\gamma \lambda_M} \propto \gamma(t) (\sqrt{-t})^{|\lambda_\gamma - \lambda_M| - |\lambda_p - \lambda_{p'}|}$$

Factorization implies angular mom. conservation at each vertex:

$$A_{\lambda_p \lambda_{p'}}^{\lambda_\gamma \lambda_M} \propto \gamma(t) (\sqrt{-t})^{|\lambda_\gamma - \lambda_M|} \times (\sqrt{-t})^{|\lambda_p - \lambda_{p'}|}$$

top vertex

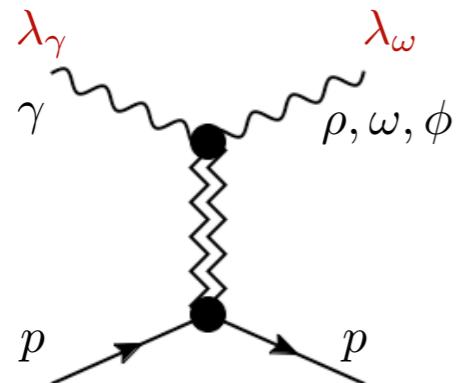
bottom vertex

Spin Density Matrix Elements

8

Use the angular distribution of the vector
to extract spin density matrix elements

$$\frac{8\pi}{3} \frac{d\sigma}{d\Omega} = 1 - \rho_{00}^0 + (3\rho_{00}^0 - 1) \cos^2 \theta - 2\sqrt{2} \operatorname{Re} \rho_{10}^0 \sin 2\theta \cos \phi - 2\rho_{1-1}^0 \sin^2 \theta \cos 2\phi$$



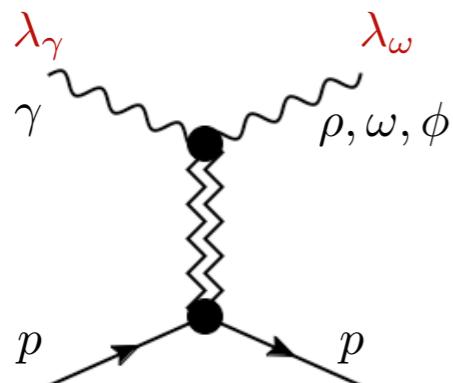
Structure at the top vertex:

$$T_{\lambda_\gamma \lambda_\omega} = \beta_0 \left(\delta_{\lambda_\gamma}^{\lambda_\omega} + \beta_1 \frac{\sqrt{-t}}{m_\omega} \delta_0^{\lambda_\omega} + \beta_2 \frac{-t}{m_\omega^2} \delta_{-\lambda_\gamma}^{\lambda_\omega} \right)$$

Spin Density Matrix Elements

Use the angular distribution of the vector
to extract spin density matrix elements

$$\frac{8\pi}{3} \frac{d\sigma}{d\Omega} = 1 - \rho_{00}^0 + (3\rho_{00}^0 - 1) \cos^2 \theta - 2\sqrt{2} \operatorname{Re} \rho_{10}^0 \sin 2\theta \cos \phi - 2\rho_{1-1}^0 \sin^2 \theta \cos 2\phi$$



Structure at the top vertex:

$$T_{\lambda_\gamma \lambda_\omega} = \beta_0 \left(\delta_{\lambda_\gamma}^{\lambda_\omega} + \beta_1 \frac{\sqrt{-t}}{m_\omega} \delta_0^{\lambda_\omega} + \beta_2 \frac{-t}{m_\omega^2} \delta_{-\lambda_\gamma}^{\lambda_\omega} \right)$$

$$\rho_{00}^0 = \frac{2}{N} \sum_{\lambda, \lambda'} \left| T_{\lambda, \lambda'}^{1,0} \right|^2$$

$$\operatorname{Re} \rho_{10}^0 = \frac{1}{N} \operatorname{Re} \sum_{\lambda, \lambda'} \left(T_{\lambda, \lambda'}^{1,1} - T_{\lambda, \lambda'}^{-1,-1} \right) T_{\lambda, \lambda'}^{*,1,0} \longrightarrow \operatorname{Re} \rho_{10}^0 \propto \frac{1}{2} \beta_1 \frac{\sqrt{-t}}{m_\omega}$$

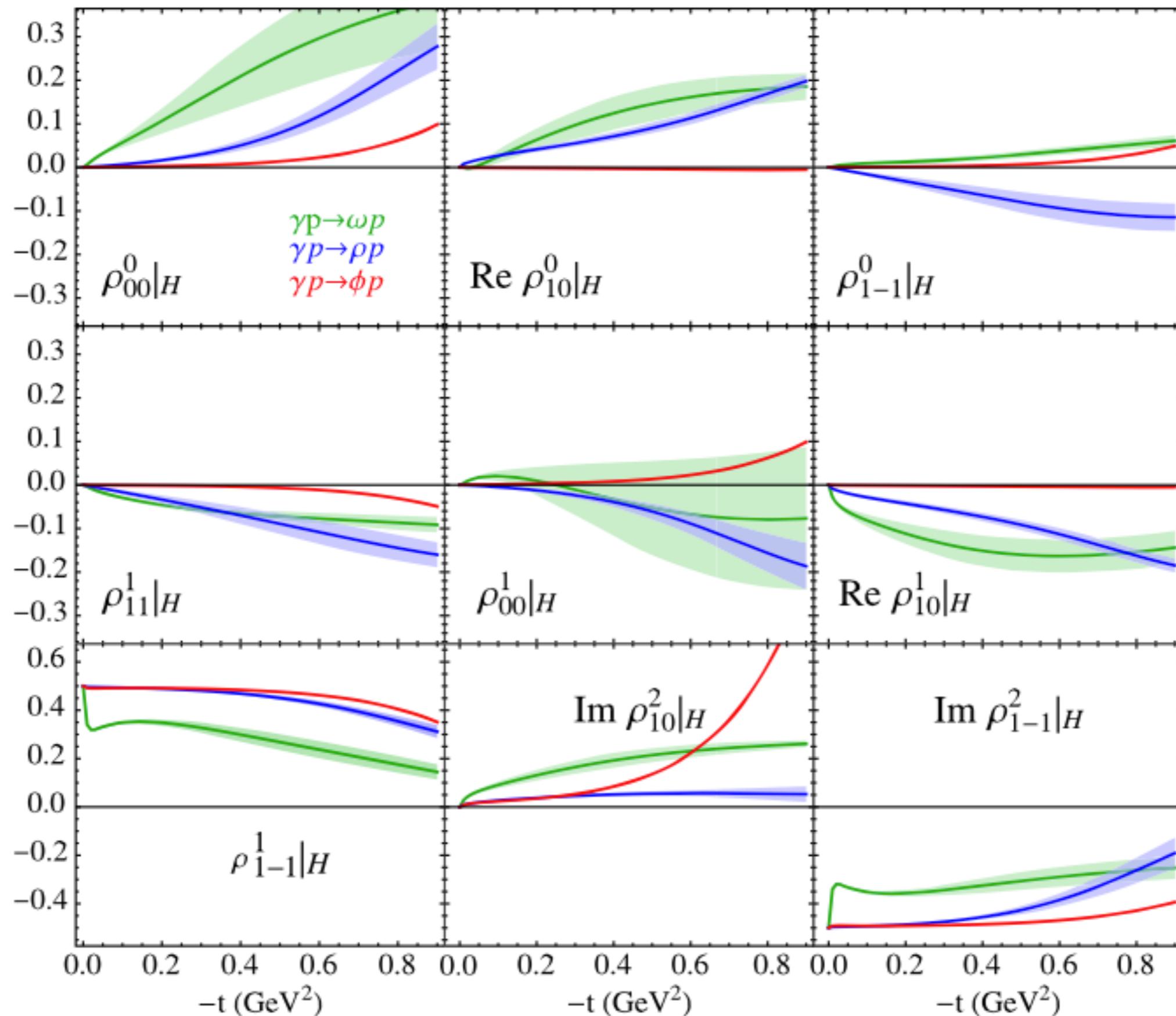
$$\rho_{1-1}^0 = \frac{2}{N} \operatorname{Re} \sum_{\lambda, \lambda'} T_{\lambda, \lambda'}^{1,1} T_{\lambda, \lambda'}^{*,1,-1}$$

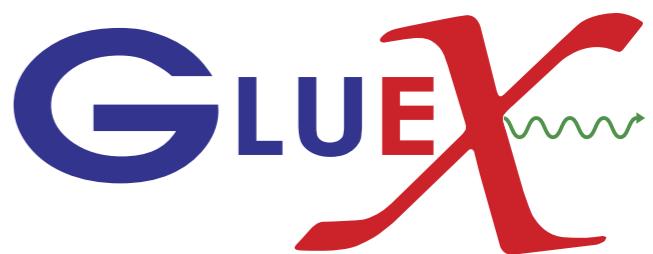
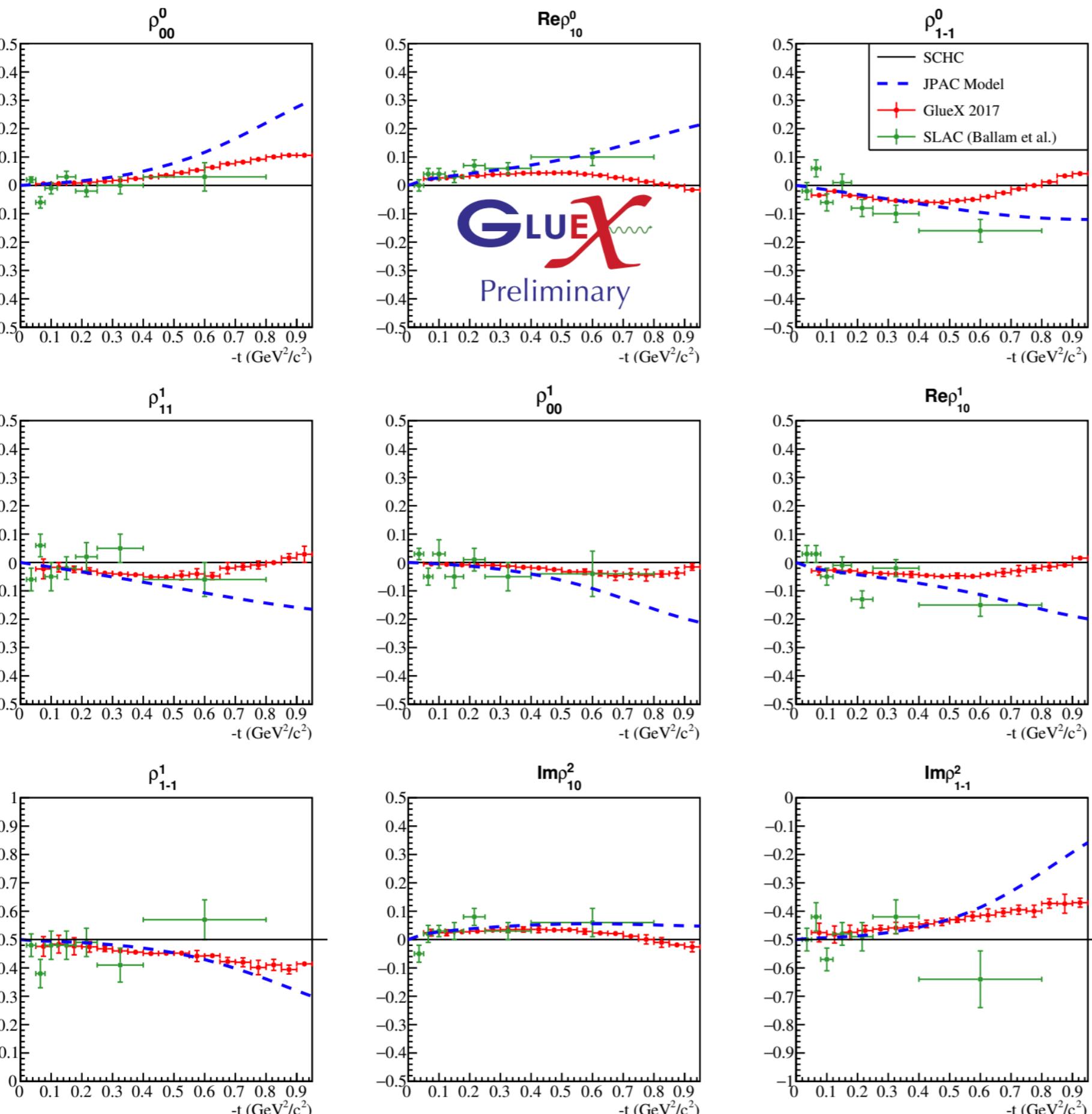
$$\rho_{00}^0 \propto \beta_1^2 \frac{-t}{m_\omega^2}$$

$$\rho_{1-1}^0 \propto \beta_2 \frac{-t}{m_\omega^2}$$

Predictions for Vector Meson SDME

VM et al (JPAC), PRD97 (2018)





Preliminary
A. Austregesilo 1908.07275

kinematics expanded

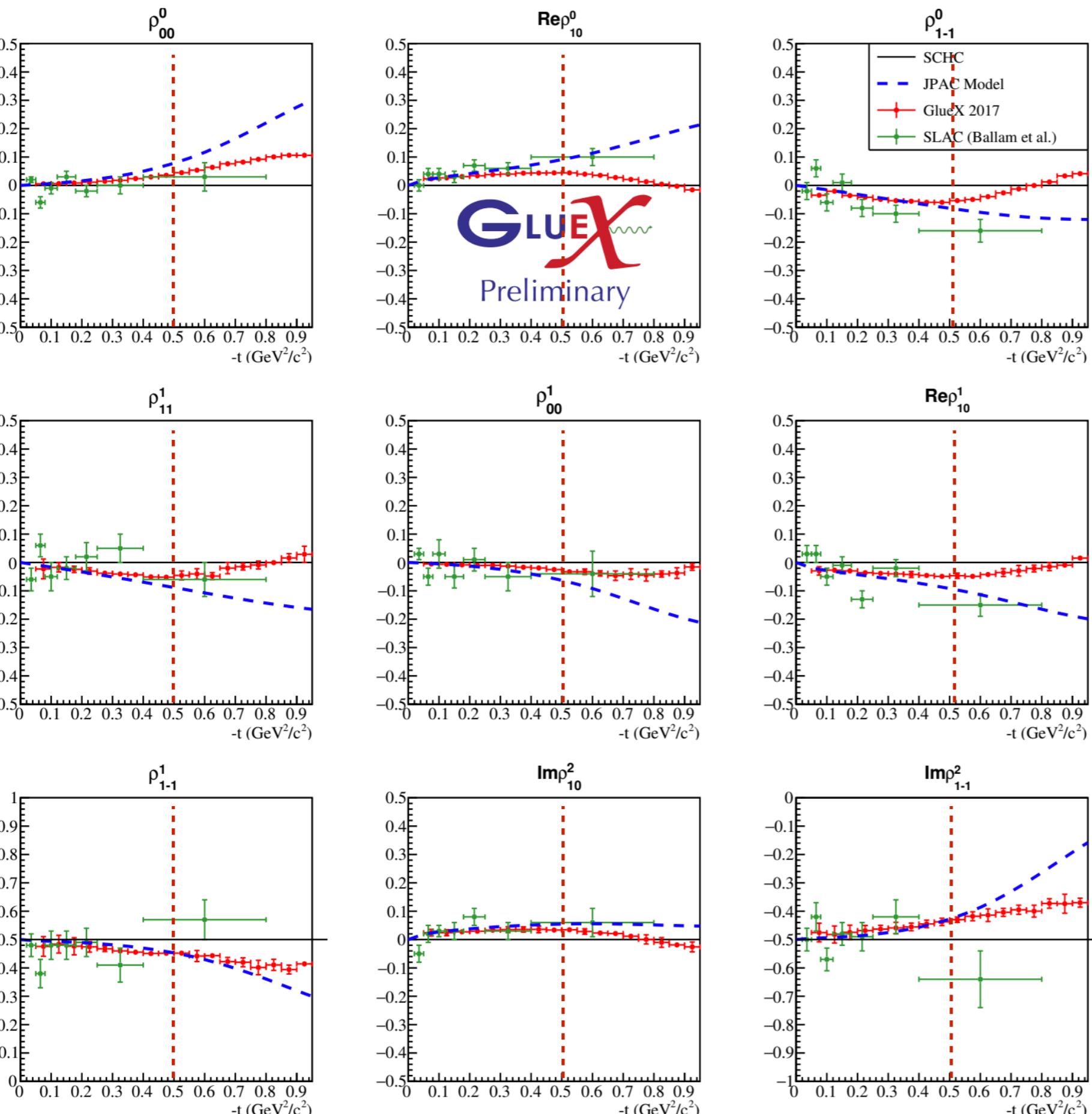
in power of

$$\frac{-t}{m_\rho^2}$$

$$\rho_{1-1}^1 = \pm \frac{1}{2} + \mathcal{O}(t^2)$$

$$Im \rho_{1-1}^2 = \mp \frac{1}{2} + \mathcal{O}(t^2)$$

top sign for natural exchange
bottom sign for unnatural exch.



GLUEX

Preliminary

A. Austregesilo 1908.07275

kinematics expanded

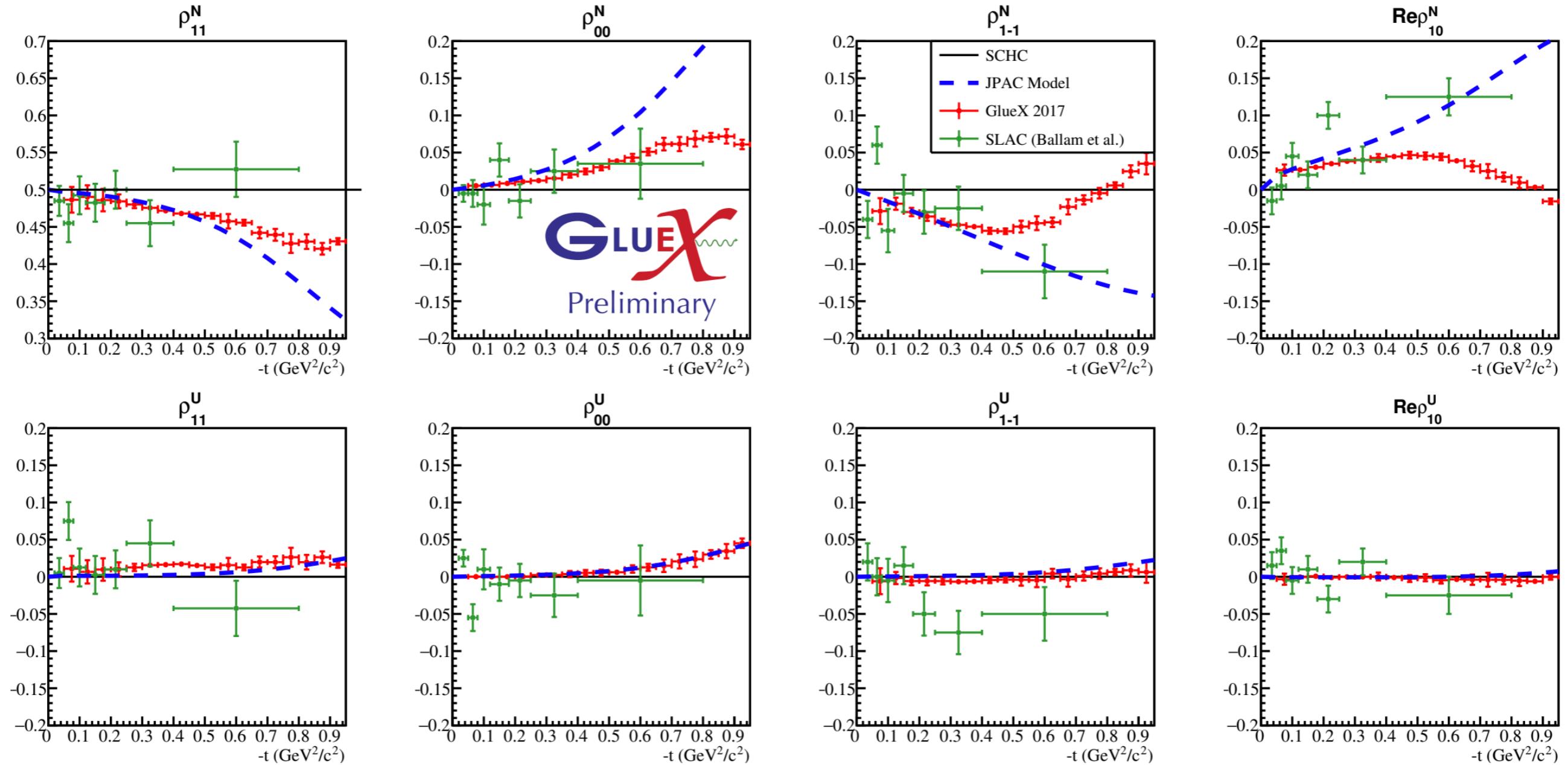
in power of

$$\frac{-t}{m_\rho^2}$$

$$\rho_{1-1}^1 = \pm \frac{1}{2} + \mathcal{O}(t^2)$$

$$\text{Im } \rho_{1-1}^2 = \mp \frac{1}{2} + \mathcal{O}(t^2)$$

top sign for natural exchange
bottom sign for unnatural exch.



Clear domination of natural (Pomeron and tensor 2++) exchanges over unnatural (pseudo-scalar 0+ and 1++) ones

GLUEX

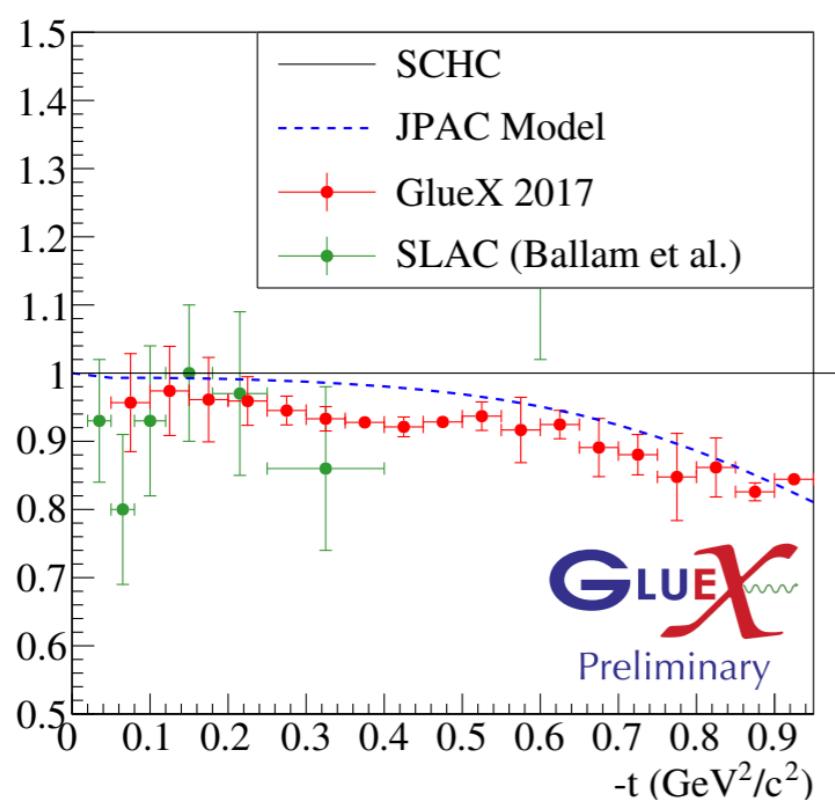
Preliminary

Vector Meson SDME: Parity Asymmetry

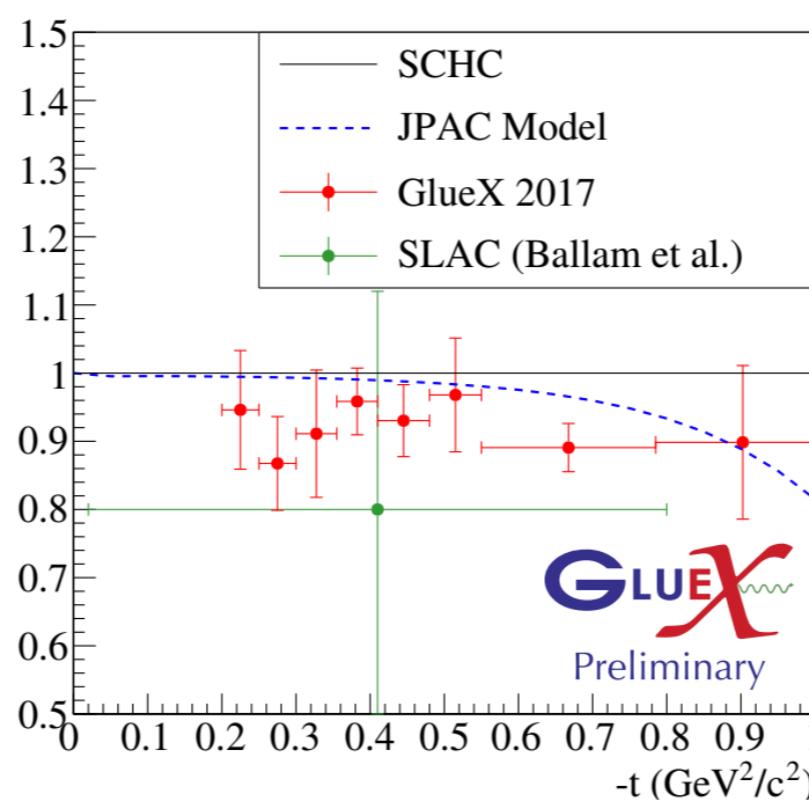
12

$$P_\sigma = 2\rho_{1-1}^1 - \rho_{00}^1 = \frac{\sigma^N - \sigma^U}{\sigma^N + \sigma^U}$$

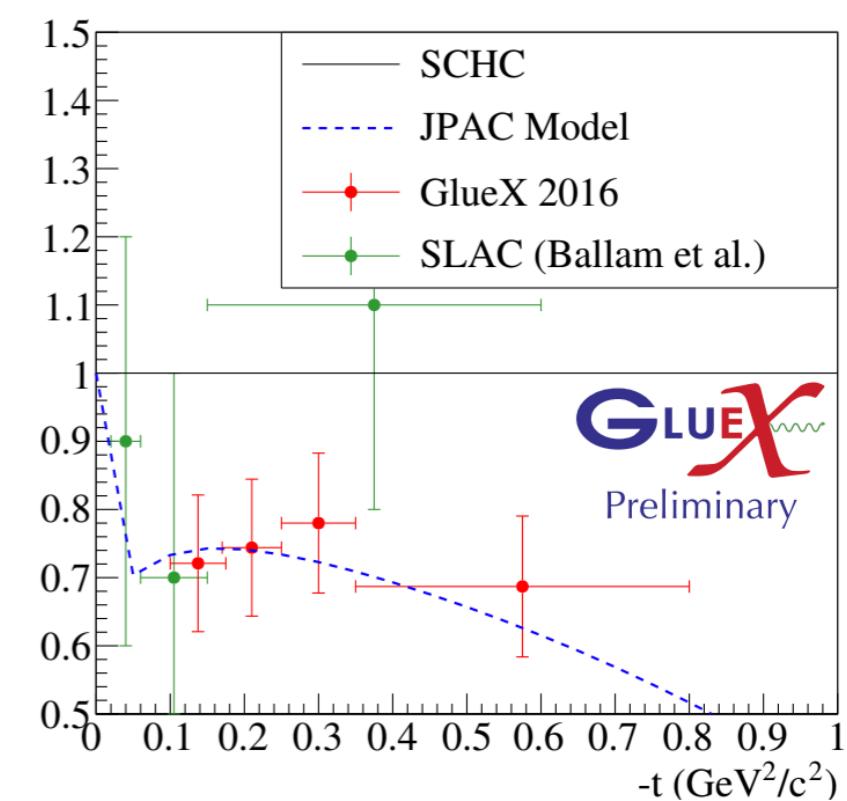
$\rho(770)$



$\phi(1020)$



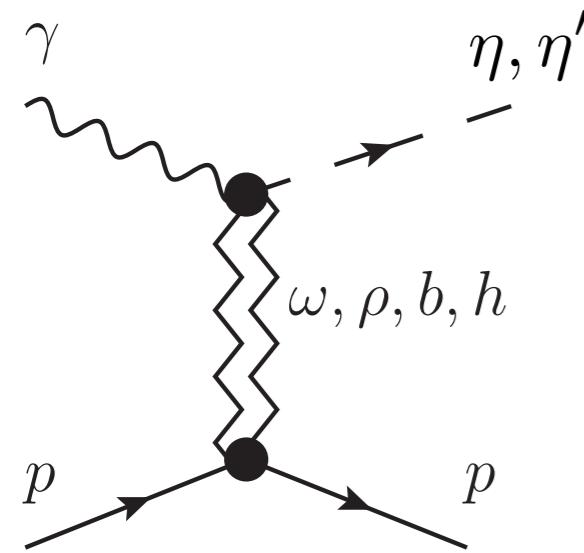
$\omega(782)$



Clear domination of natural (Pomeron and tensor 2++) exchanges
over unnatural (pseudo-scalar 0++ and 1++) ones
Except at low $|t|$ for the Omega (pion exchange)

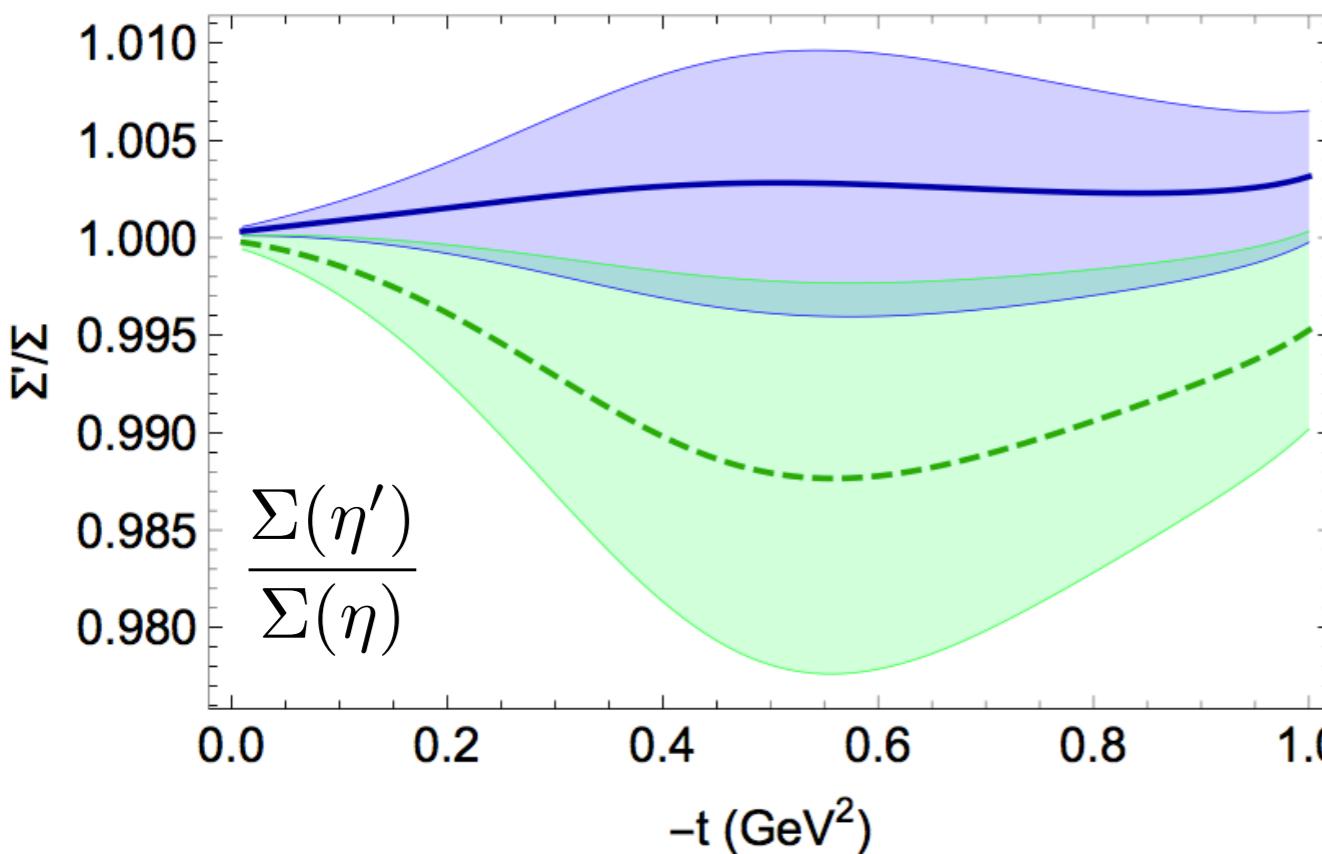
Pseudoscalar Meson Beam Asymmetry

13



$$\Sigma(\eta) = \frac{|\rho + \omega|^2 - |b + h|^2}{|\rho + \omega|^2 + |b + h|^2}$$
$$= \Sigma(\eta')$$

$b_1 \rightarrow \gamma\eta^{(')}$ not known



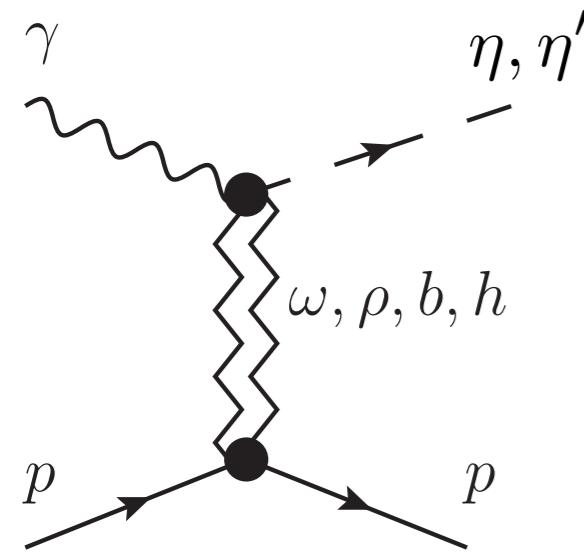
Beam asymmetry Difference probes strange exchanges contribution and deviation from quark model

blue and green models represent the estimation of systematic errors

13

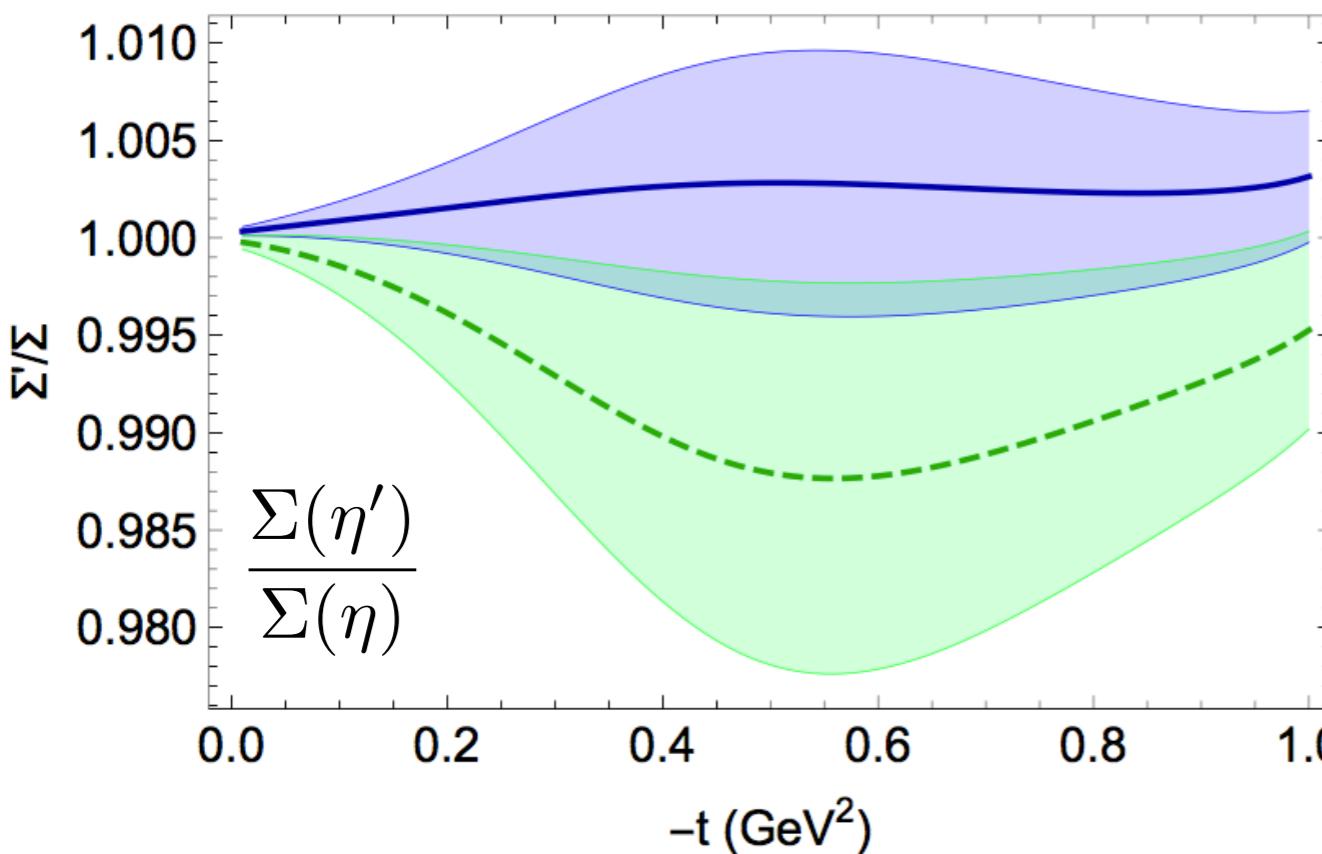
Pseudoscalar Meson Beam Asymmetry

13



$$\Sigma(\eta) = \frac{|\rho + \omega + \phi|^2 - |b + h + h'|^2}{|\rho + \omega + \phi|^2 + |b + h + h'|^2}$$
$$\neq \Sigma(\eta')$$

$b_1 \rightarrow \gamma\eta^{(')}$ not known



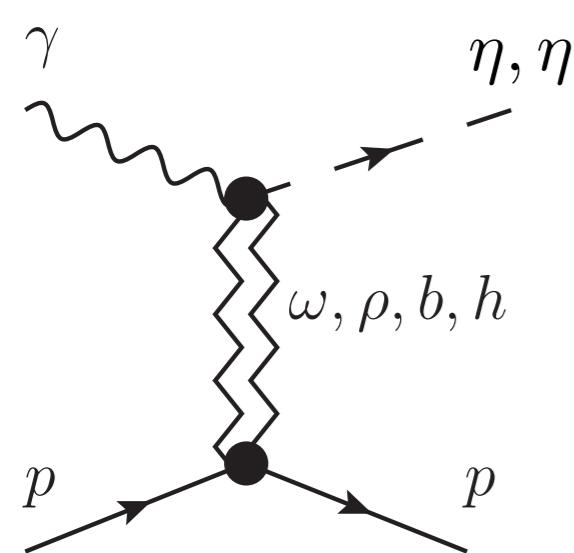
Beam asymmetry Difference probes strange exchanges contribution and deviation from quark model

blue and green models represent the estimation of systematic errors

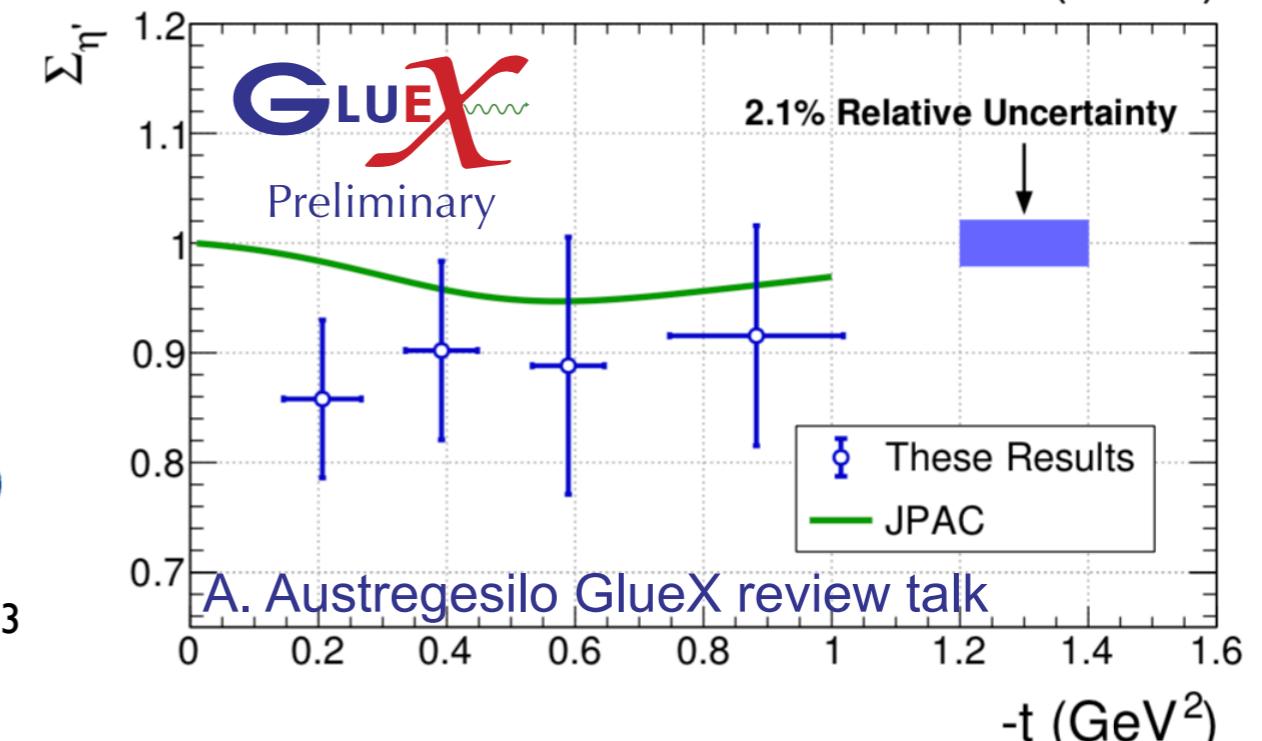
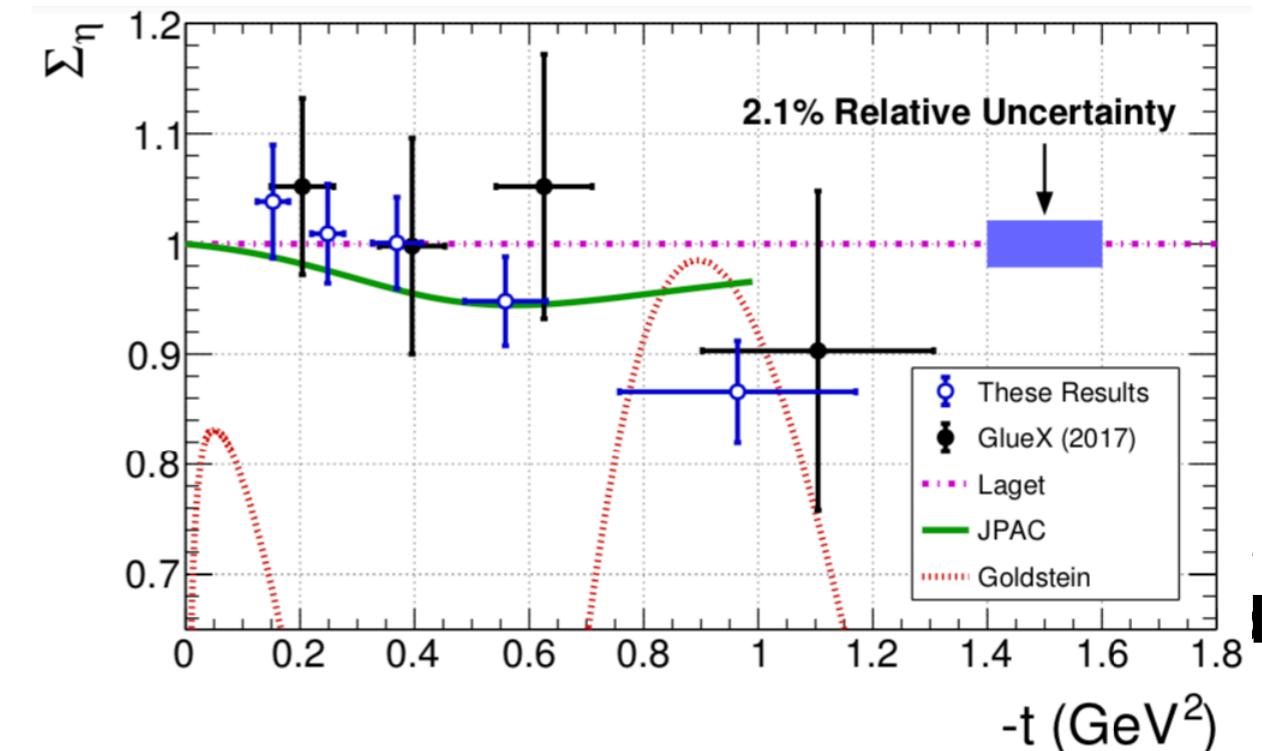
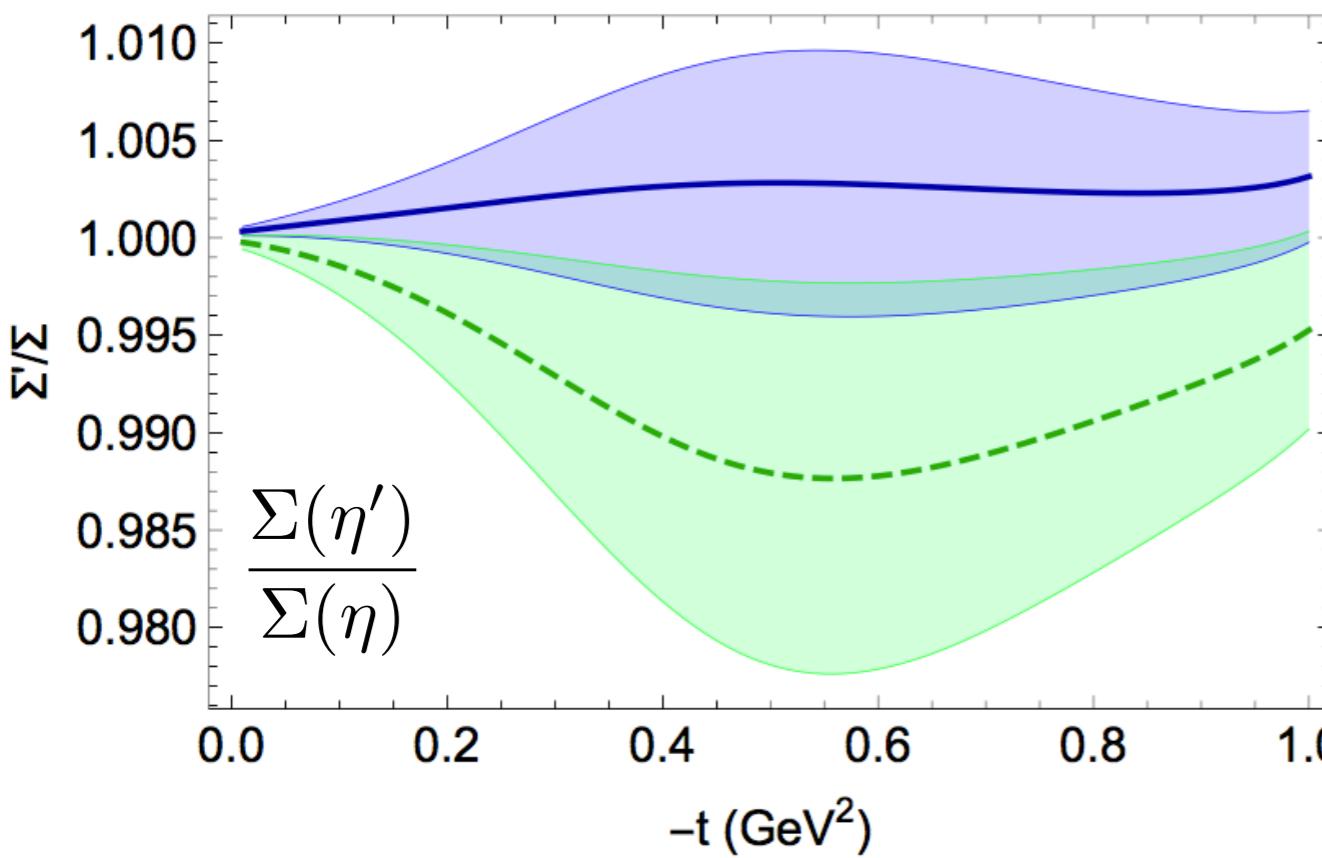
13

Pseudoscalar Meson Beam Asymmetry

13



$$\Sigma(\eta) = \frac{|\rho + \omega + \phi|^2 - |b + h + h'|^2}{|\rho + \omega + \phi|^2 + |b + h + h'|^2}$$



Single Meson Photoproduction:

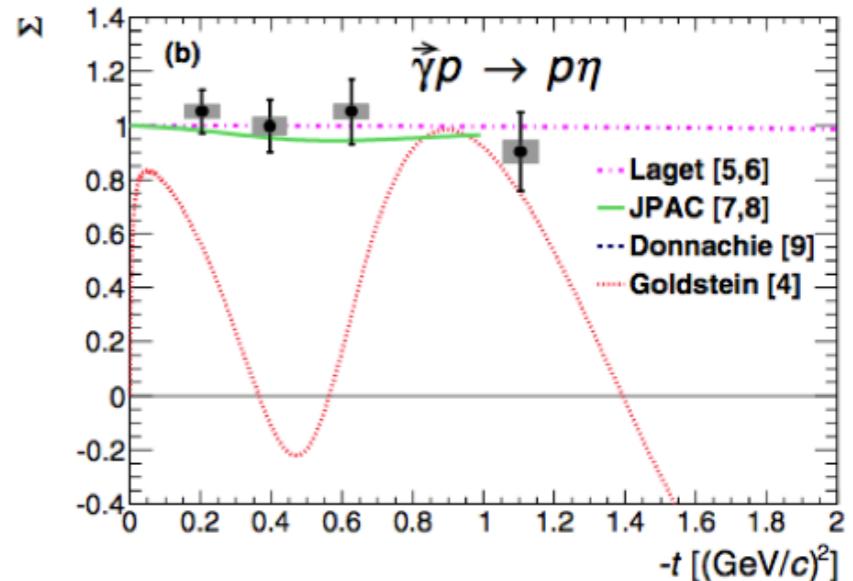
$$\vec{\gamma}p \rightarrow \pi^0 p$$

$$\vec{\gamma}p \rightarrow \eta p$$

$$\vec{\gamma}p \rightarrow \pi \Delta$$

Dominance of natural exch. in both π^0/η photoproduction

Significant π^\pm exch. at low t



$\rho(770)$

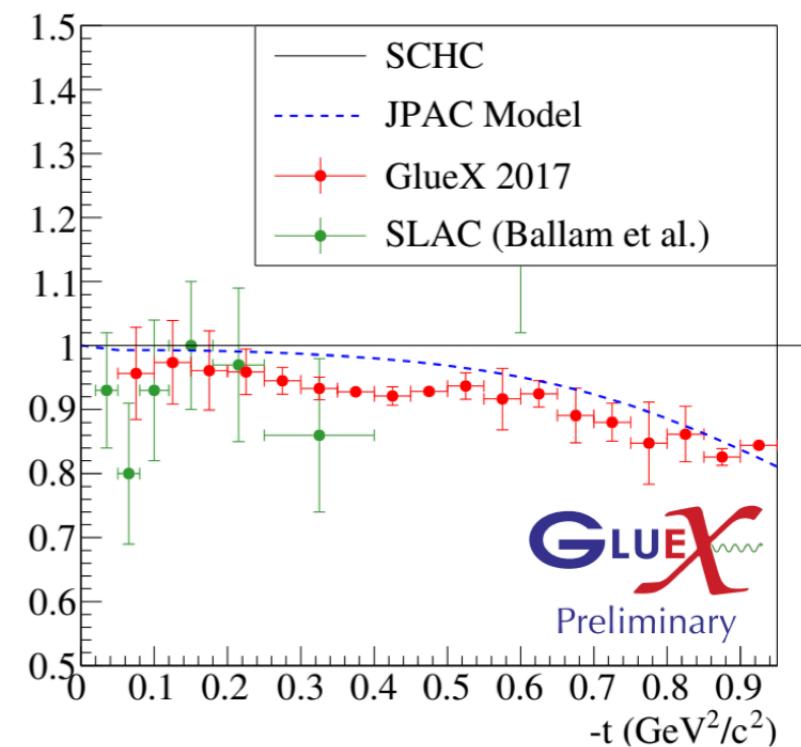
Vector Meson Photoproduction:

$$\vec{\gamma}p \rightarrow \rho^0 p$$

$$\vec{\gamma}p \rightarrow \omega p$$

$$\vec{\gamma}p \rightarrow \phi p$$

Consistent with factorization
Dominance of natural exchanges



Double Mesons Photoproduction:

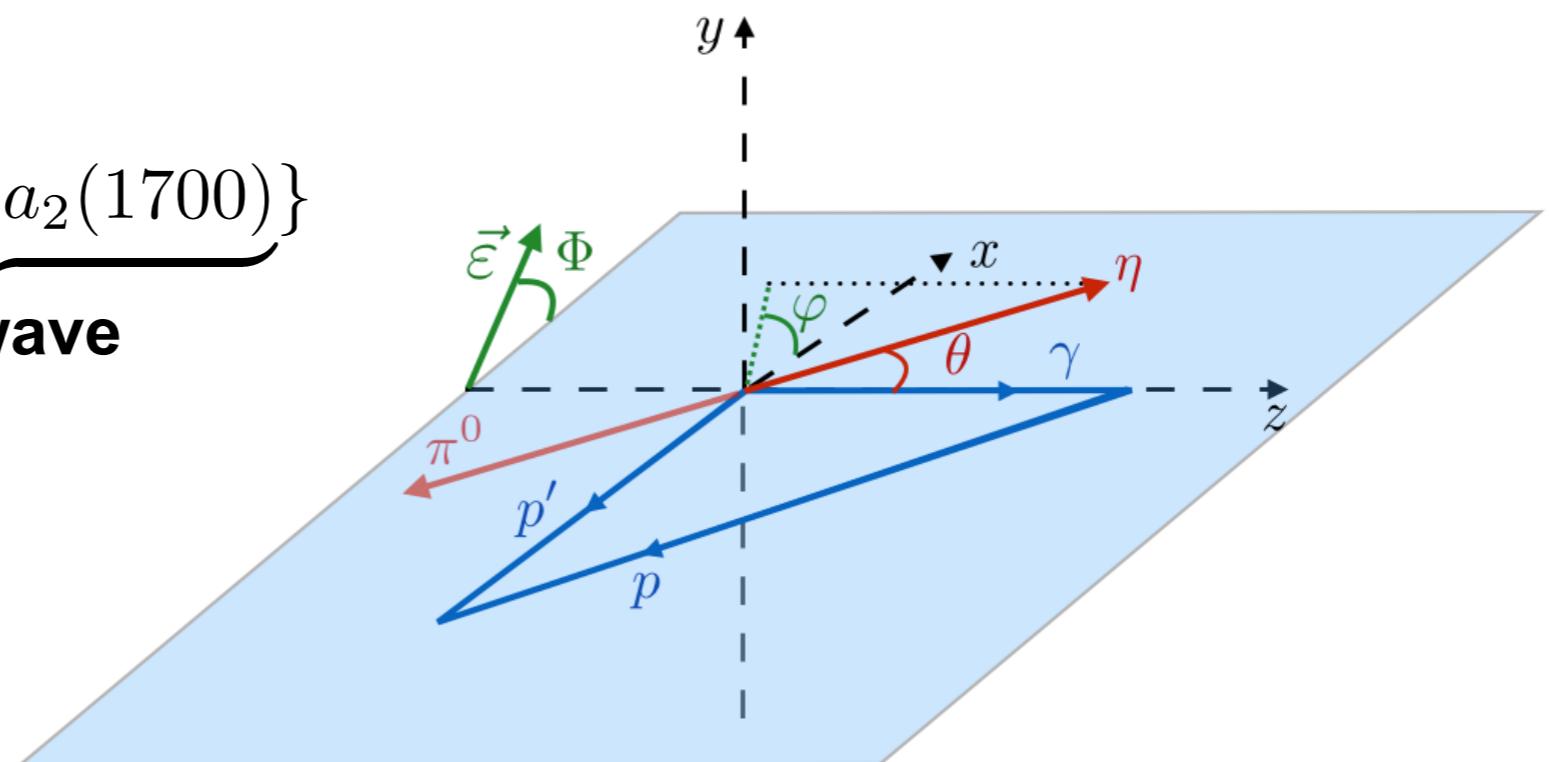
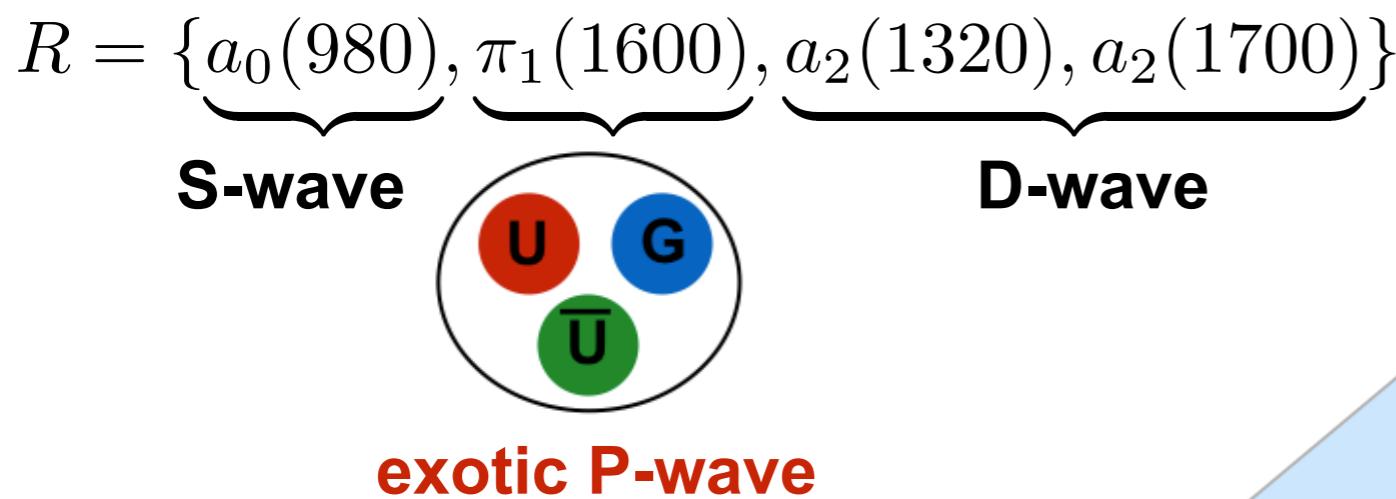
$$\vec{\gamma}p \rightarrow \pi^0 \eta p$$

Observables: Moments of Angular distribution

15

$$I(\Omega, \Phi) = I^0(\Omega) - P_\gamma I^1(\Omega) \cos 2\Phi - P_\gamma I^2(\Omega) \sin 2\Phi$$

$$H^0(LM) = \frac{1}{2\pi} \int I(\Omega, \Phi) d_{M0}^L(\theta) \cos M\phi \, d\Omega d\Phi$$



Observables: Moments of Angular distribution

15

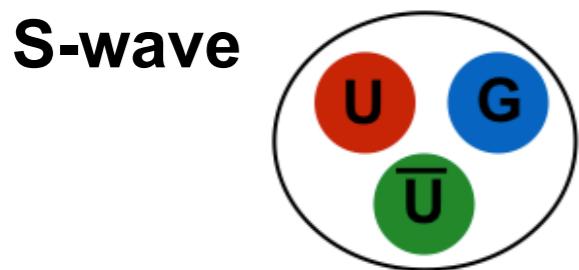
$$I(\Omega, \Phi) = I^0(\Omega) - P_\gamma I^1(\Omega) \cos 2\Phi - P_\gamma I^2(\Omega) \sin 2\Phi$$

$$H^0(LM) = \frac{1}{2\pi} \int I(\Omega, \Phi) d_{M0}^L(\theta) \cos M\phi \, d\Omega d\Phi$$

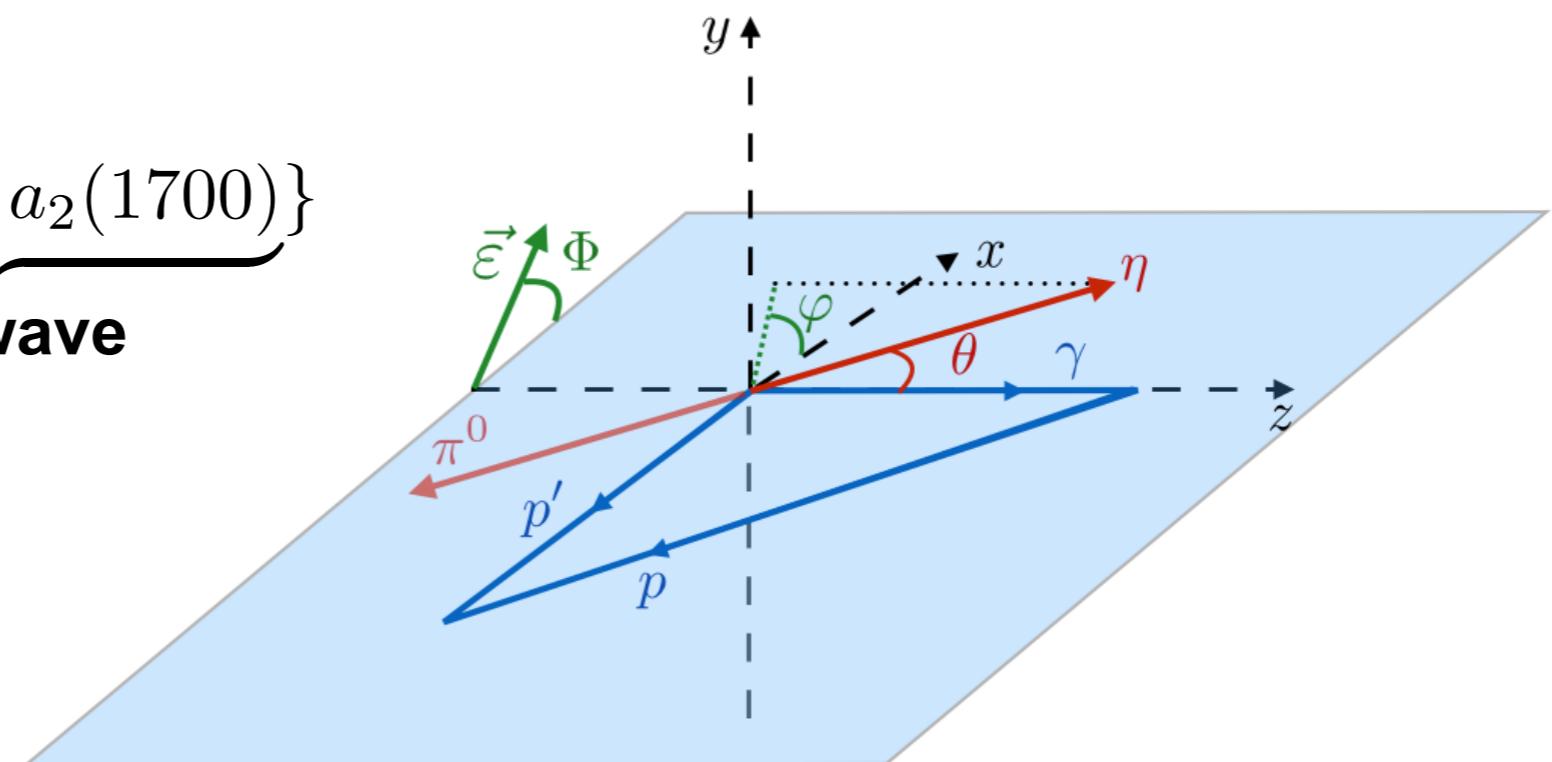
$$H^1(LM) = \frac{-1}{\pi P_\gamma} \int I(\Omega, \Phi) \boxed{\cos 2\Phi} d_{M0}^L(\theta) \cos M\phi \, d\Omega d\Phi$$

$$\text{Im } H^2(LM) = \frac{1}{\pi P_\gamma} \int I(\Omega, \Phi) \boxed{\sin 2\Phi} d_{M0}^L(\theta) \sin M\phi \, d\Omega d\Phi$$

$$R = \underbrace{\{a_0(980), \pi_1(1600)\}}_{\textbf{S-wave}}, \underbrace{\{a_2(1320), a_2(1700)\}}_{\textbf{D-wave}}$$



exotic P-wave



Observables: Moments of Angular distribution

$$\begin{aligned}
 H^0(LM) &= \frac{1}{2\pi} \int I(\Omega, \Phi) d_{M0}^L(\theta) \cos M\phi \, d\Omega d\Phi \\
 H^1(LM) &= \frac{-1}{\pi P_\gamma} \int I(\Omega, \Phi) \cos 2\Phi d_{M0}^L(\theta) \cos M\phi \, d\Omega d\Phi \\
 \text{Im } H^2(LM) &= \frac{1}{\pi P_\gamma} \int I(\Omega, \Phi) \sin 2\Phi d_{M0}^L(\theta) \sin M\phi \, d\Omega d\Phi
 \end{aligned}$$

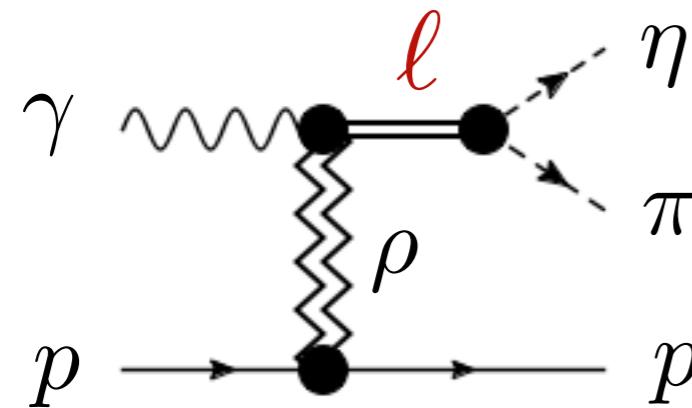
Moments are unambiguously extracted and are related to partial waves (interferences)

$${}^{(+)}H^0(00) = 2 \left[|S_0^{(+)}|^2 + |P_{-1}^{(+)}|^2 + |P_0^{(+)}|^2 + |P_1^{(+)}|^2 + |D_{-2}^{(+)}|^2 + |D_{-1}^{(+)}|^2 + |D_0^{(+)}|^2 + |D_1^{(+)}|^2 + |D_2^{(+)}|^2 \right]$$

$${}^{(+)}H^0(10) = \frac{4}{\sqrt{3}} \operatorname{Re} \left(S_0^{(+)} P_0^{(+)*} \right) + \frac{8}{\sqrt{15}} \operatorname{Re} \left(P_0^{(+)} D_0^{(+)*} \right) + \frac{4}{\sqrt{5}} \operatorname{Re} \left(P_1^{(+)} D_1^{(+)*} \right) + \frac{4}{\sqrt{5}} \operatorname{Re} \left(P_{-1}^{(+)} D_{-1}^{(+)*} \right)$$

$$\begin{aligned}
 {}^{(+)}H^0(20) &= \frac{4}{5} |P_0^{(+)}|^2 - \frac{2}{5} \left(|P_1^{(+)}|^2 + |P_{-1}^{(+)}|^2 \right) + \frac{4}{7} |D_0^{(+)}|^2 + \frac{2}{7} \left(|D_1^{(+)}|^2 + |D_{-1}^{(+)}|^2 \right) \\
 &\quad - \frac{4}{7} \left(|D_2^{(+)}|^2 + |D_{-2}^{(+)}|^2 \right) + \frac{4}{\sqrt{5}} \operatorname{Re} \left(S_0^{(+)} D_0^{(+)*} \right)
 \end{aligned}$$

$${}^{(+)}H^0(30) = \frac{12}{7\sqrt{5}} \operatorname{Re} \left(\sqrt{3} P_0^{(+)} D_0^{(+)*} - P_1^{(+)} D_1^{(+)*} - P_{-1}^{(+)} D_{-1}^{(+)*} \right)$$



$$R = \{ \underbrace{a_0(980)}, \underbrace{\pi_1(1600)}, \underbrace{a_2(1320)}, \underbrace{a_2(1700)} \}$$

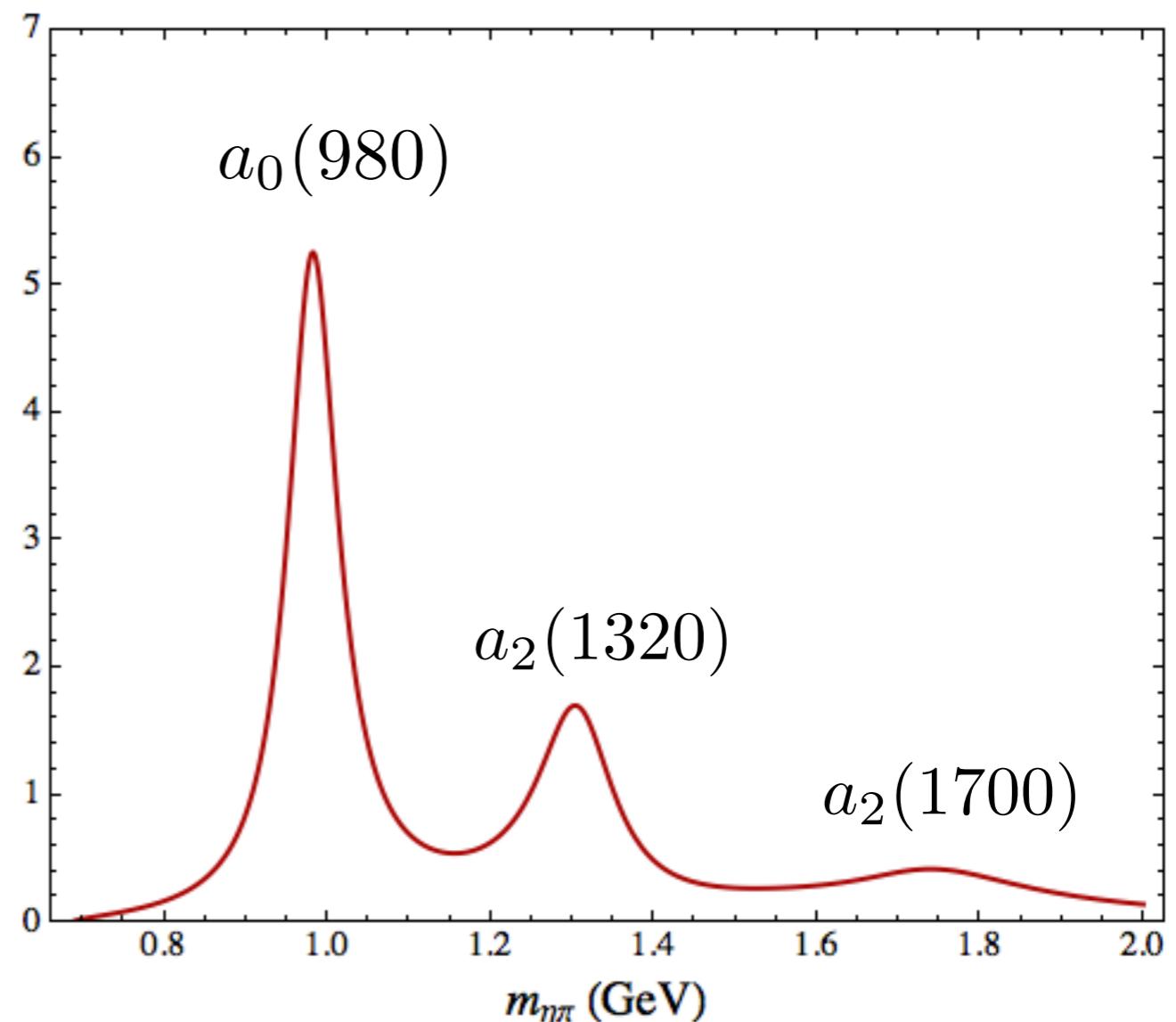
$S_0^{(+)}$ $P_{0,1}^{(+)}$ $D_{0,1,2}^{(+)}$

production: natural exchanges

line shape: Breit-Wigner form

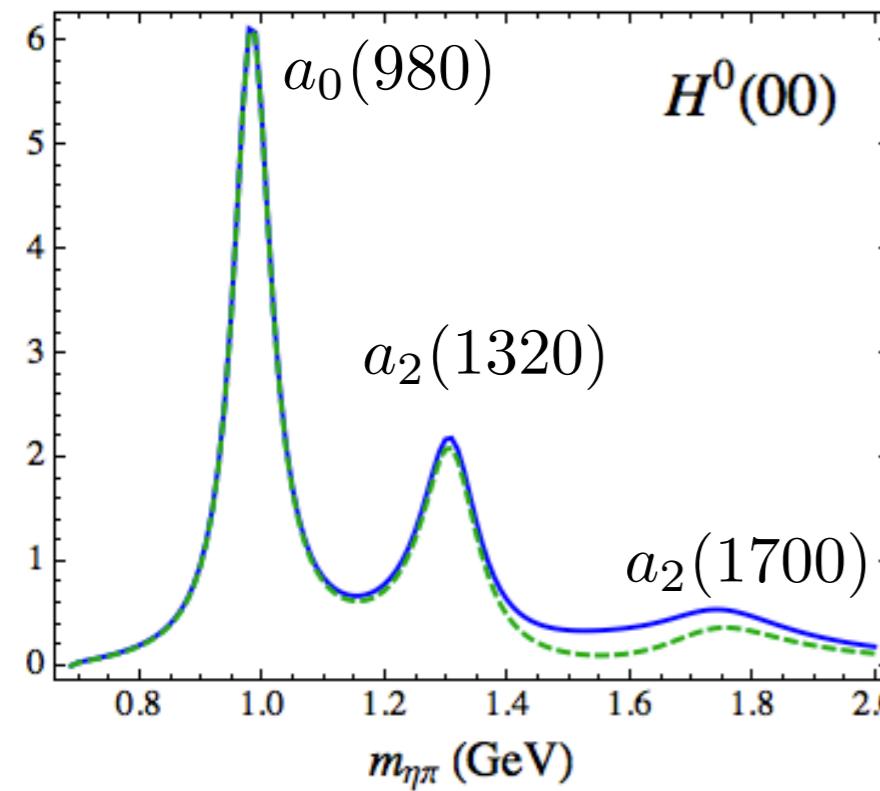
parameters: arbitrary

Small exotic wave,
not apparent in the diff. cross. section



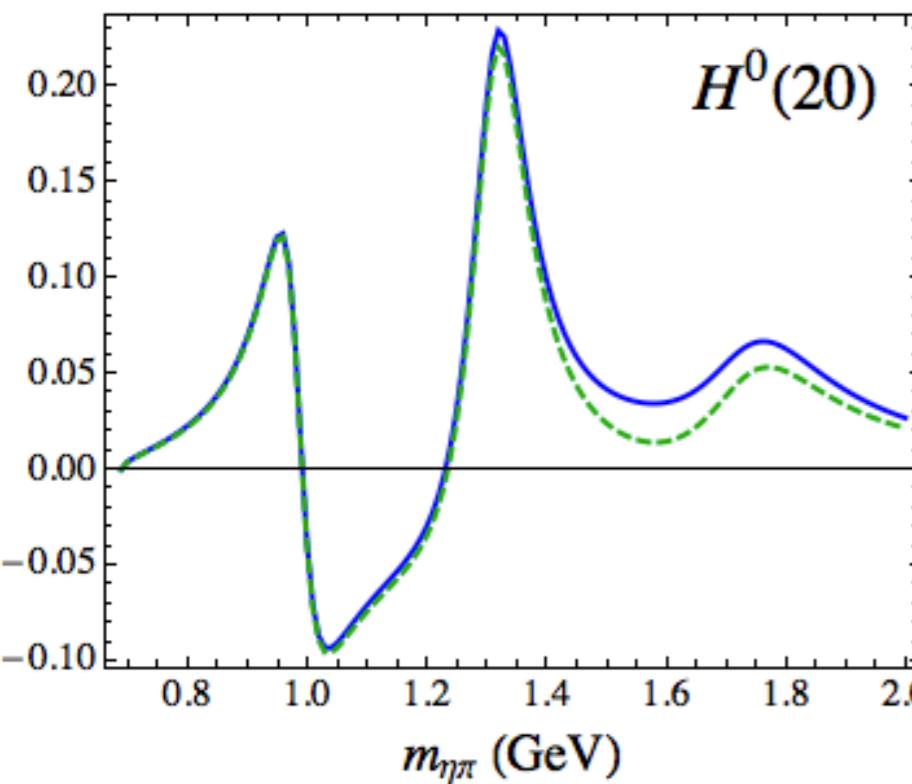
Moments

18



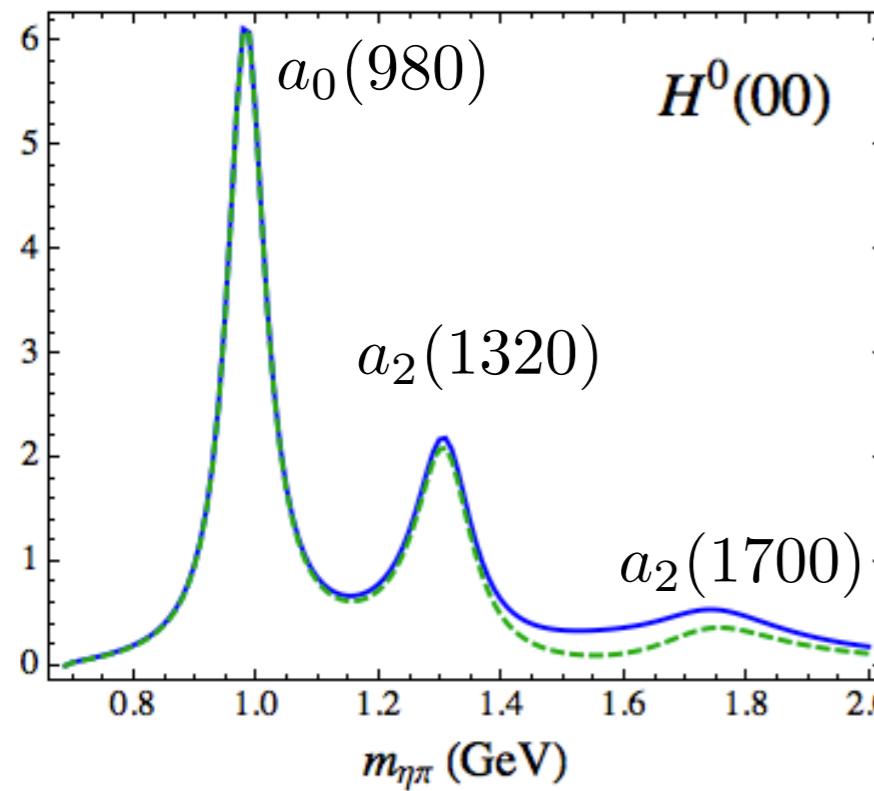
VM et al (JPAC), PRD100 (2019) 054017
solid lines: $S + \underline{P} + D$ waves
dashed lines: $S + D$ waves

$$\begin{array}{lll} |S|^2 & |D|^2 & |P|^2 \\ (S+D)(S+D)^* & & \end{array}$$

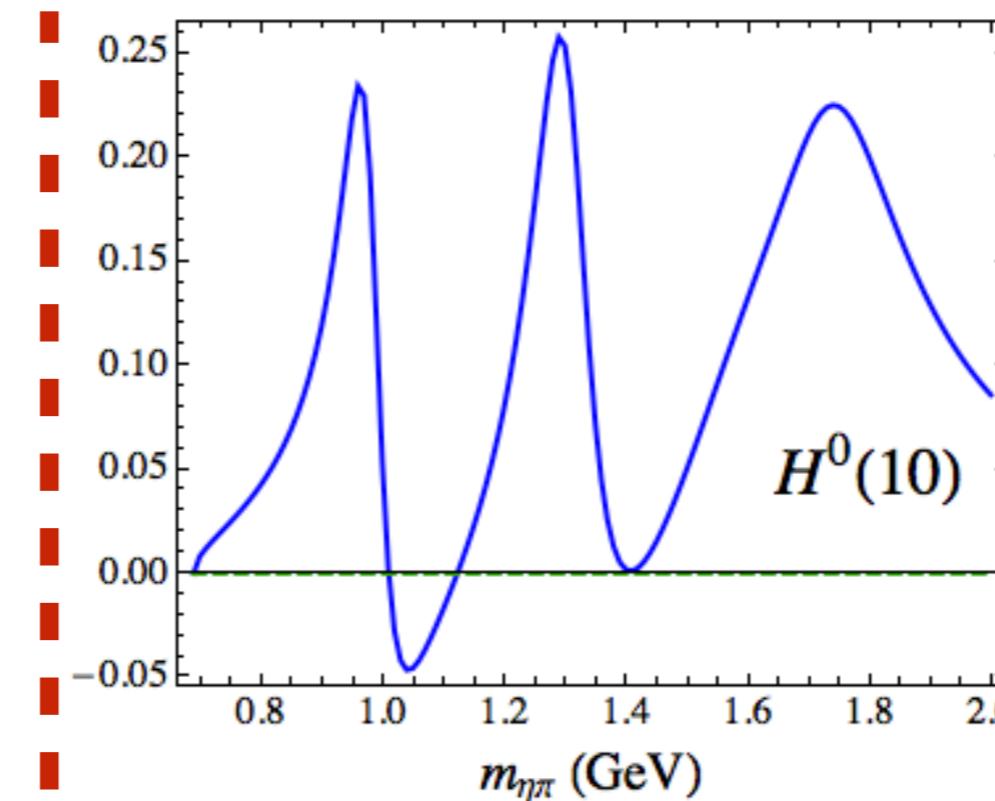


Moments

18

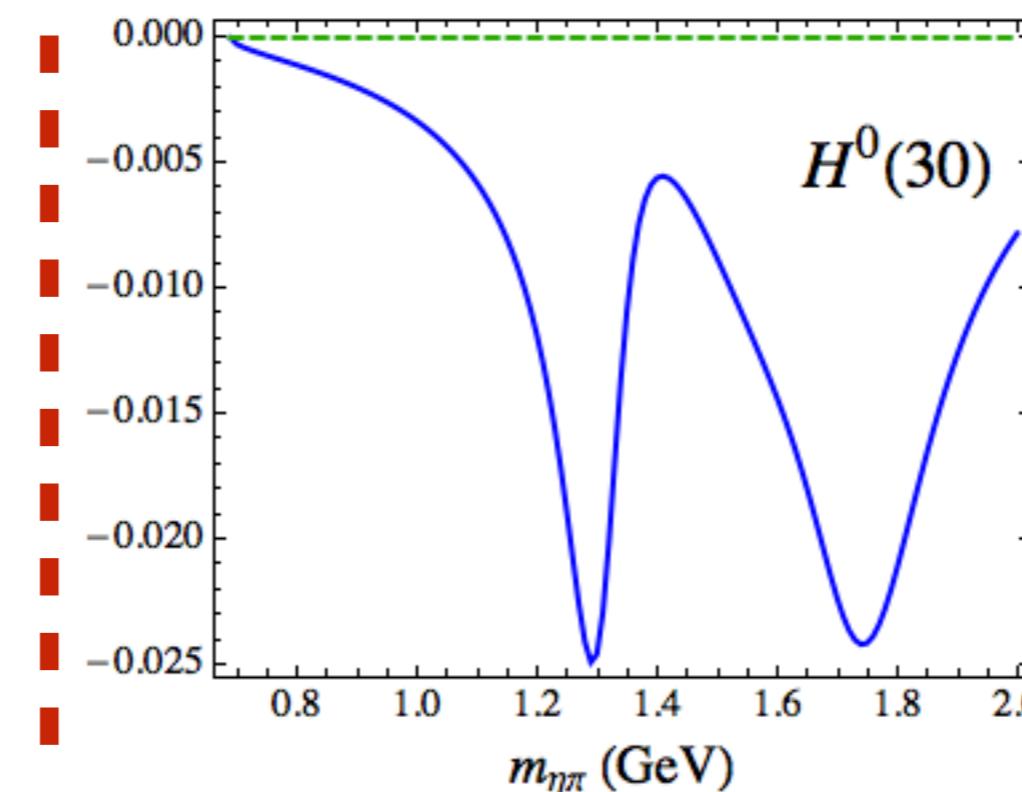
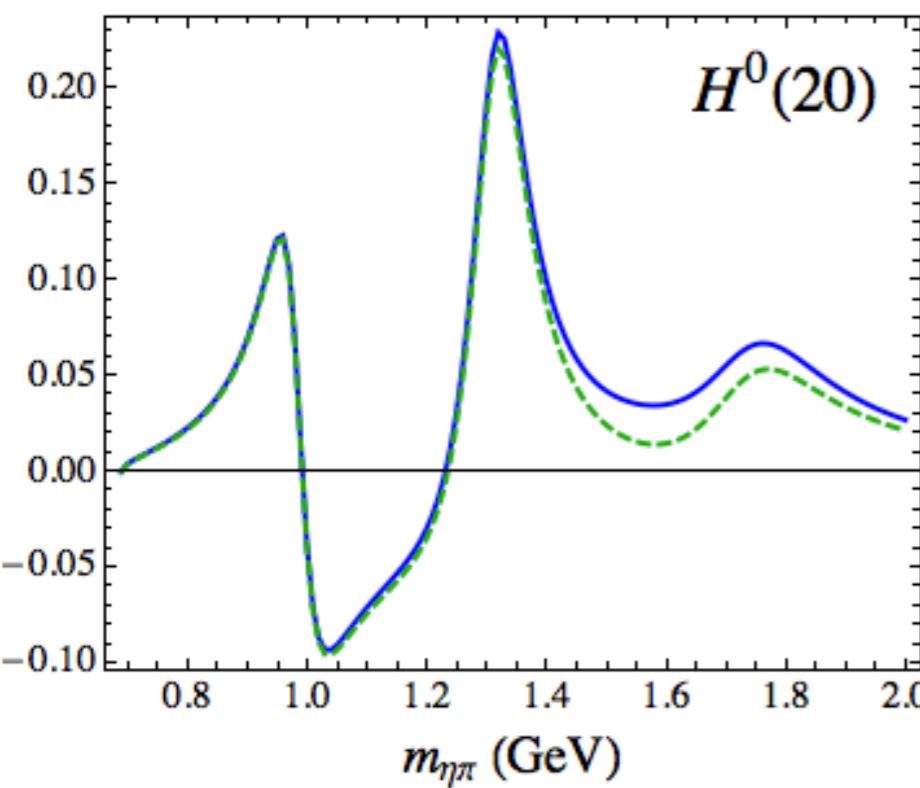


$$\begin{array}{ccc} |S|^2 & |D|^2 & |P|^2 \\ (S+D)(S+D)^* & & \end{array}$$



VM et al (JPAC), PRD100 (2019) 054017
solid lines: $S + P + D$ waves
dashed lines: $S + D$ waves

P- wave apparent as an interference in odd moments but not in even moments

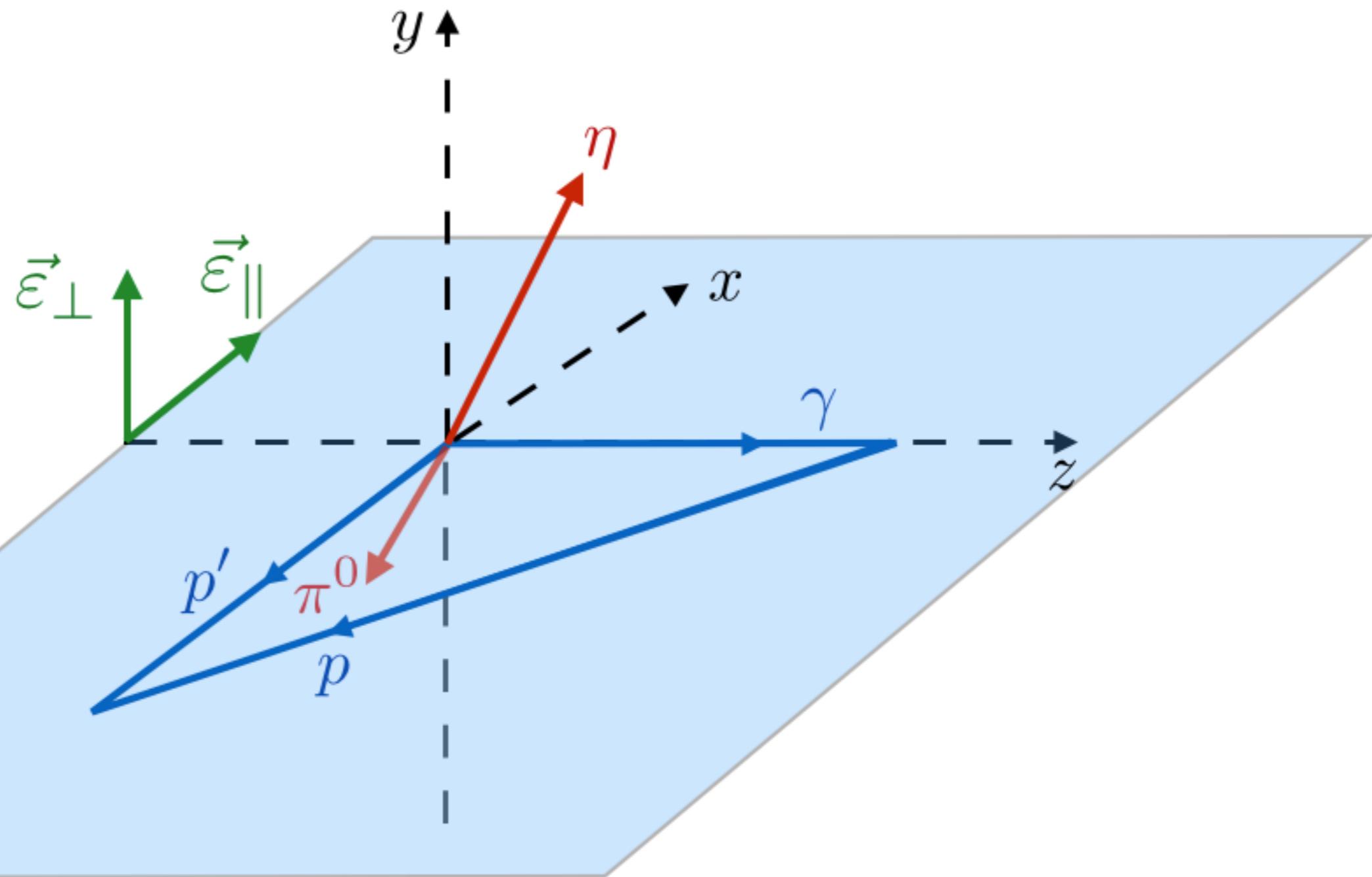


Beam Asymmetries

19

VM et al (JPAC), PRD100 (2019) 054017

$$\Sigma_{\mathcal{D}} = \frac{1}{P_\gamma} \frac{\int_{\mathcal{D}} I^{\parallel}(\Omega) - I^{\perp}(\Omega) d\Omega}{\int_{\mathcal{D}} I^{\parallel}(\Omega) + I^{\perp}(\Omega) d\Omega}$$

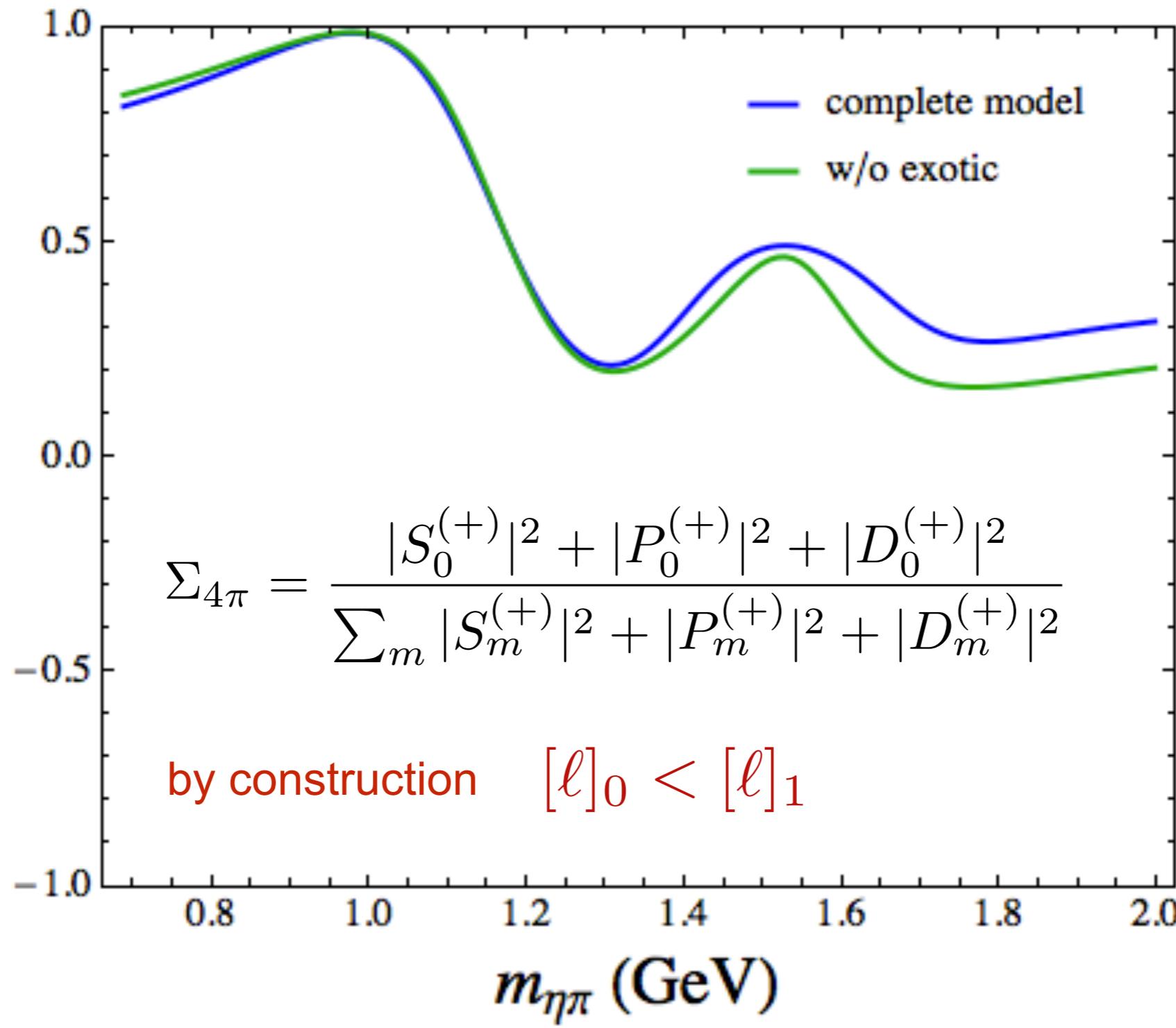
 $\Sigma_{4\pi} = \text{fully integrated}$ 

Beam Asymmetries

19

VM et al (JPAC), PRD100 (2019) 054017

$$\Sigma_{\mathcal{D}} = \frac{1}{P_\gamma} \frac{\int_{\mathcal{D}} I^{\parallel}(\Omega) - I^{\perp}(\Omega) d\Omega}{\int_{\mathcal{D}} I^{\parallel}(\Omega) + I^{\perp}(\Omega) d\Omega}$$

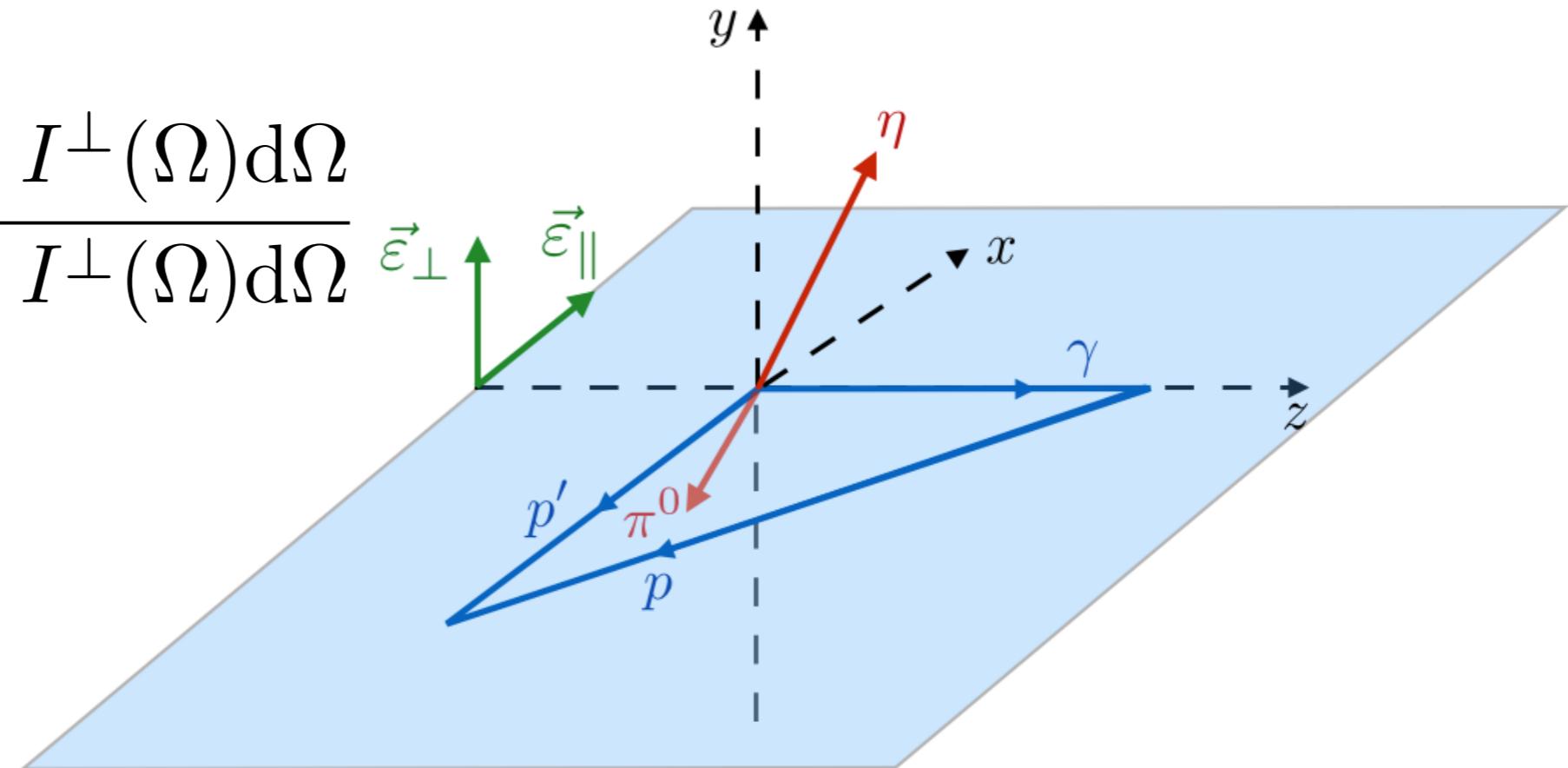
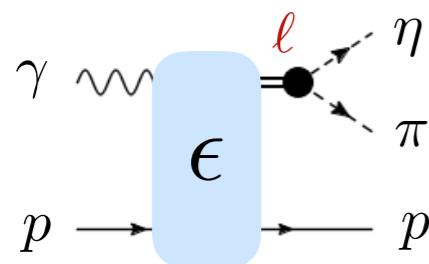
 $\Sigma_{4\pi} = \text{fully integrated}$ 

Beam Asymmetries

20

$$\Sigma_{\mathcal{D}} = \frac{1}{P_\gamma} \frac{\int_{\mathcal{D}} I^{\parallel}(\Omega) - I^{\perp}(\Omega) d\Omega}{\int_{\mathcal{D}} I^{\parallel}(\Omega) + I^{\perp}(\Omega) d\Omega}$$

amplitude:
production x decay



Beam asymmetry sensitive to reflection through the reaction plane

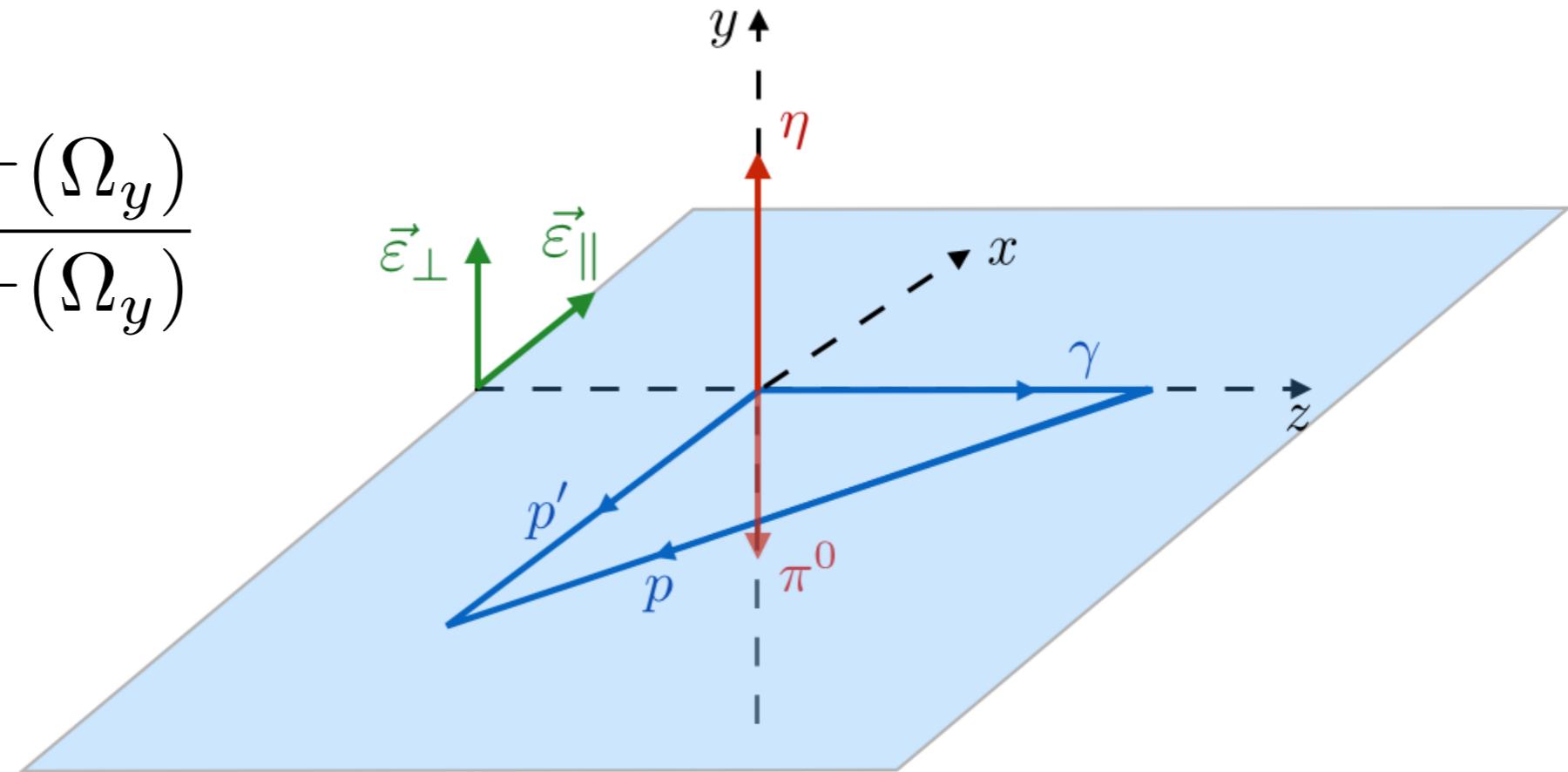
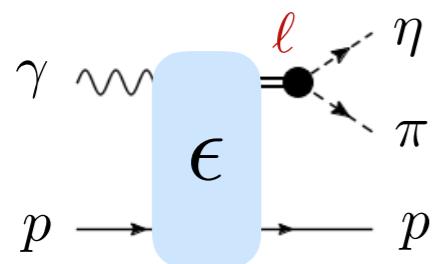
use reflection operator = parity followed by 180° rotation around Y-axis

Beam Asymmetries

20

$$\Sigma_y = \frac{1}{P_\gamma} \frac{I^{\parallel}(\Omega_y) - I^{\perp}(\Omega_y)}{I^{\parallel}(\Omega_y) + I^{\perp}(\Omega_y)}$$

amplitude:
production x decay



Beam asymmetry sensitive to reflection through the reaction plane

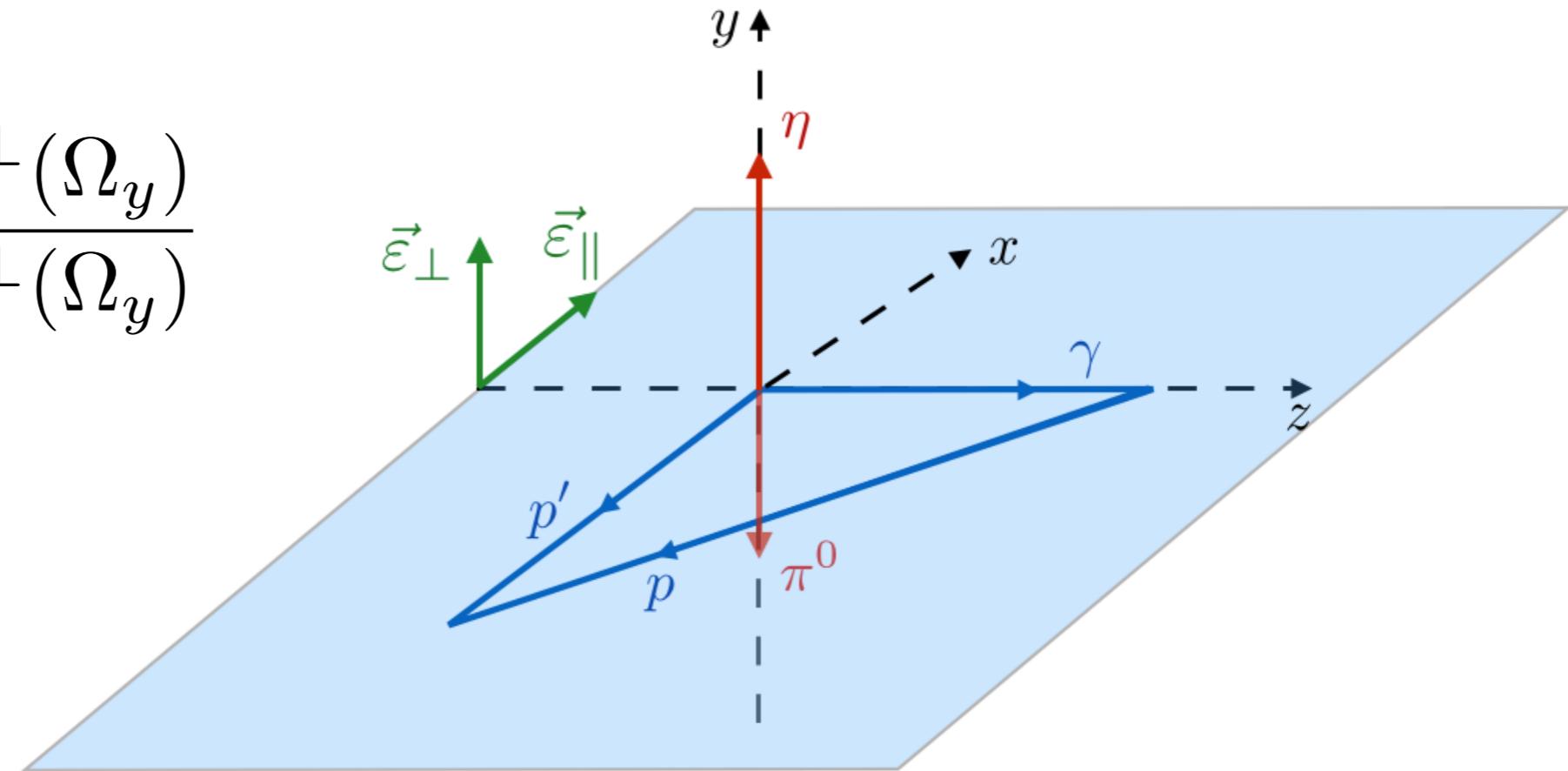
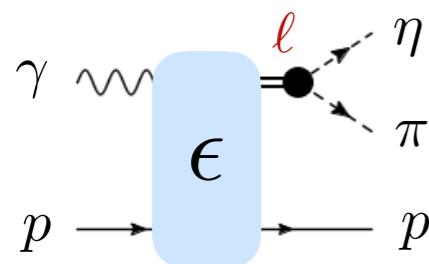
use reflection operator = parity followed by 180° rotation around Y-axis

Beam Asymmetries

20

$$\Sigma_y = \frac{1}{P_\gamma} \frac{I^\parallel(\Omega_y) - I^\perp(\Omega_y)}{I^\parallel(\Omega_y) + I^\perp(\Omega_y)}$$

**amplitude:
production x decay**



Beam asymmetry sensitive to reflection through the reaction plane

use reflection operator = parity followed by 180° rotation around Y-axis

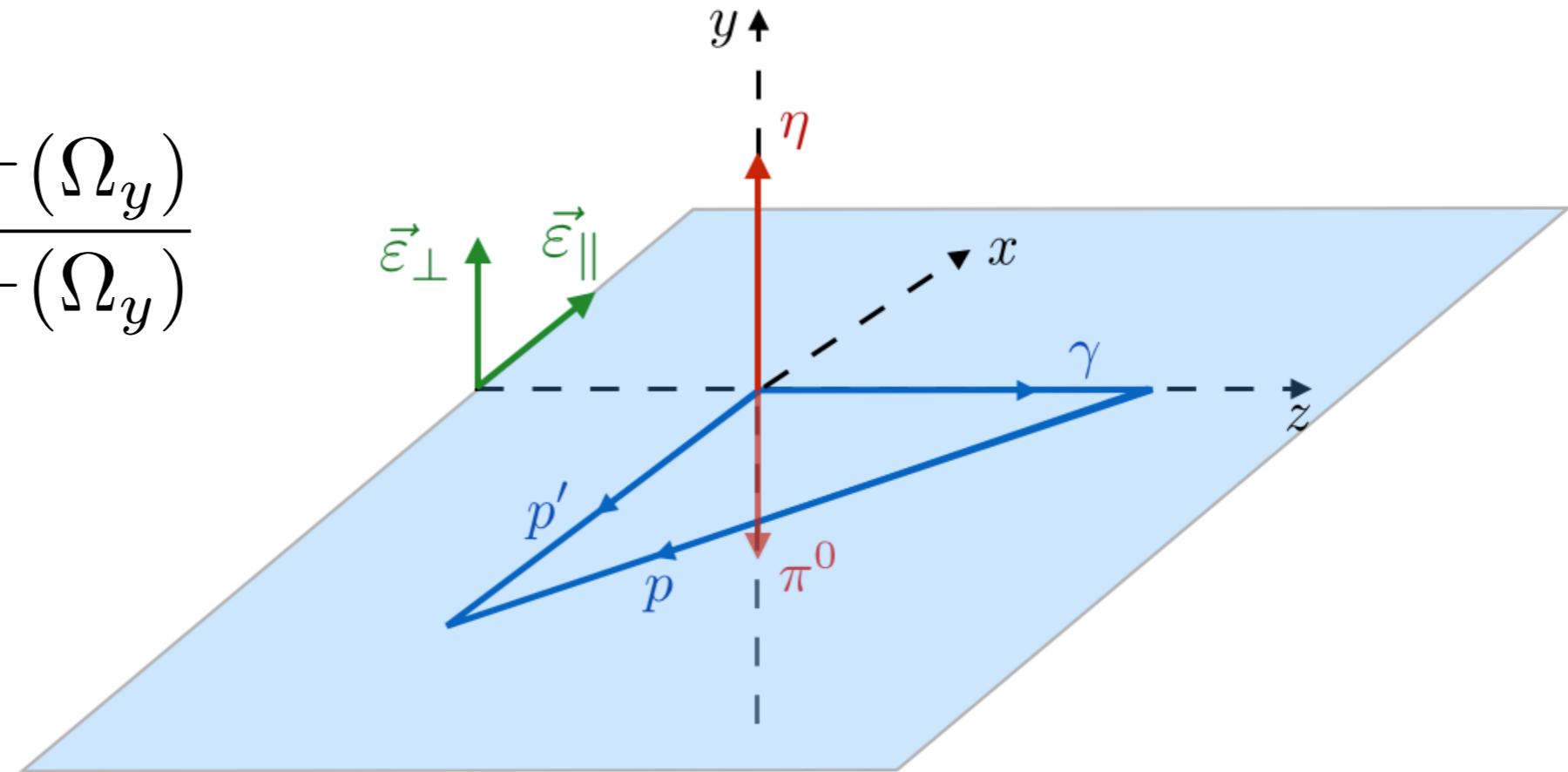
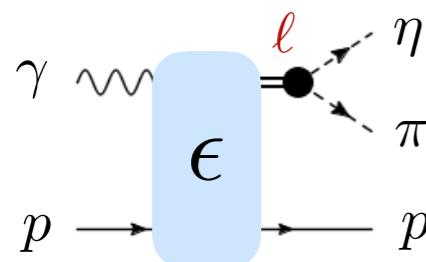
$$[\ell]_m^{(\epsilon)} \longrightarrow \Sigma_y = \epsilon(-1)^\ell$$

Odd waves change sign!!!

Beam Asymmetries

$$\Sigma_y = \frac{1}{P_\gamma} \frac{I^{\parallel}(\Omega_y) - I^{\perp}(\Omega_y)}{I^{\parallel}(\Omega_y) + I^{\perp}(\Omega_y)}$$

**amplitude:
production x decay**



Beam asymmetry sensitive to reflection through the reaction plane

use reflection operator = parity followed by 180° rotation around Y-axis

$$[\ell]_m^{(\epsilon)} \longrightarrow \Sigma_y = \epsilon(-1)^\ell$$

Odd waves change sign!!!

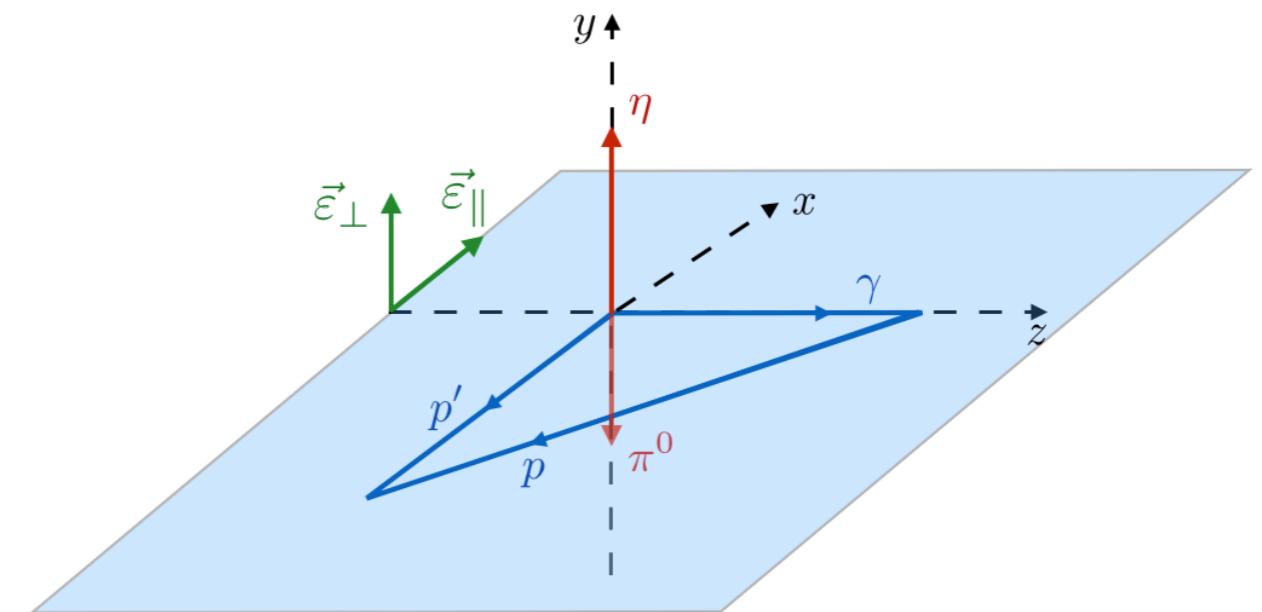
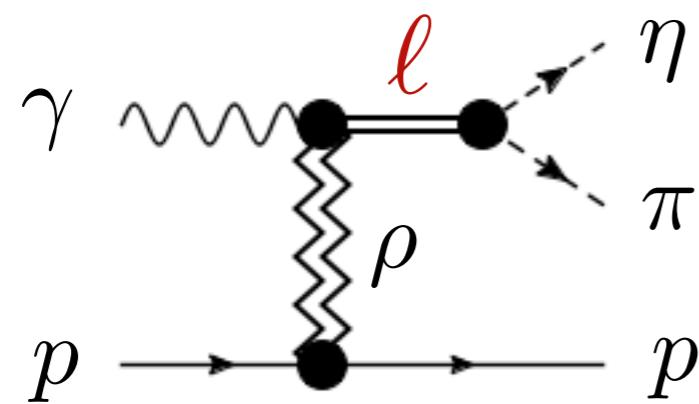
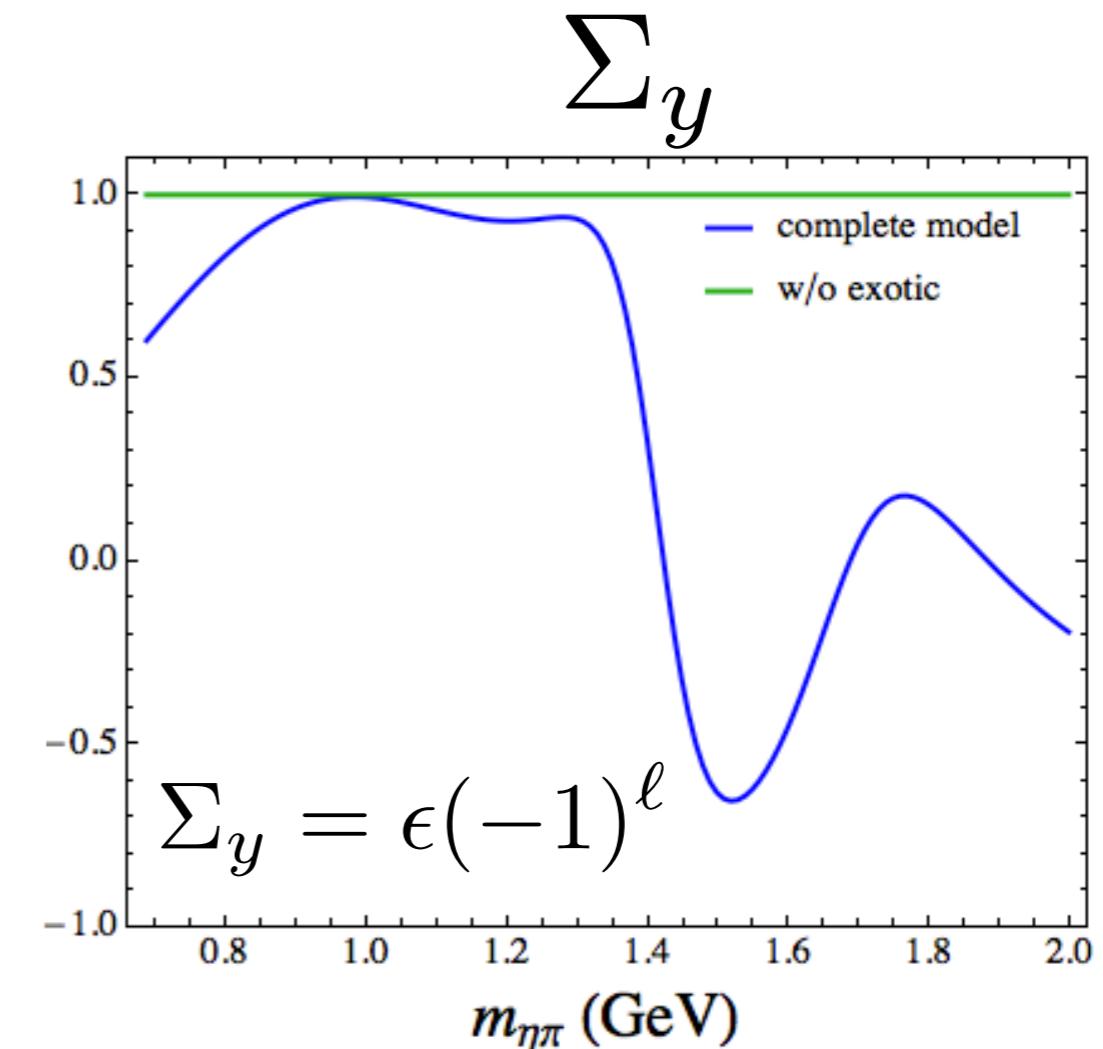
For $\pi^0\pi^0, \eta\eta$ Only even waves $\longrightarrow \Sigma_y = \epsilon$

Beam Asymmetries

$$\Sigma_y = \frac{1}{P_\gamma} \frac{I(\Omega_y, 0) - I(\Omega_y, \frac{\pi}{2})}{I(\Omega_y, 0) + I(\Omega_y, \frac{\pi}{2})} = -\frac{I^1(\Omega_y)}{I^0(\Omega_y)}$$

Intensities can be computed from moments:

$$I^0(\Omega_y) = H^0(00) - \frac{5}{2}H^0(20) - 5\sqrt{\frac{3}{2}}H^0(22) \\ + \frac{27}{8}H^0(40) + \frac{9}{2}\sqrt{\frac{5}{2}}H^0(42) + \frac{9}{4}\sqrt{\frac{35}{2}}H^0(44)$$



Outline

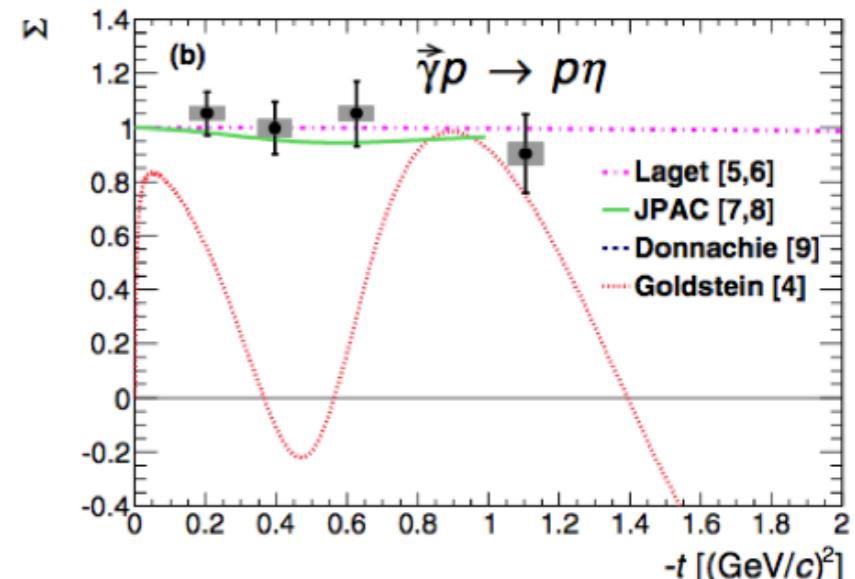
Conclusion

Single Meson Photoproduction:

$$\vec{\gamma}p \rightarrow \pi^0 p$$

$$\vec{\gamma}p \rightarrow \eta p$$

Dominance of natural exchanges in both π^0/η photoproduction



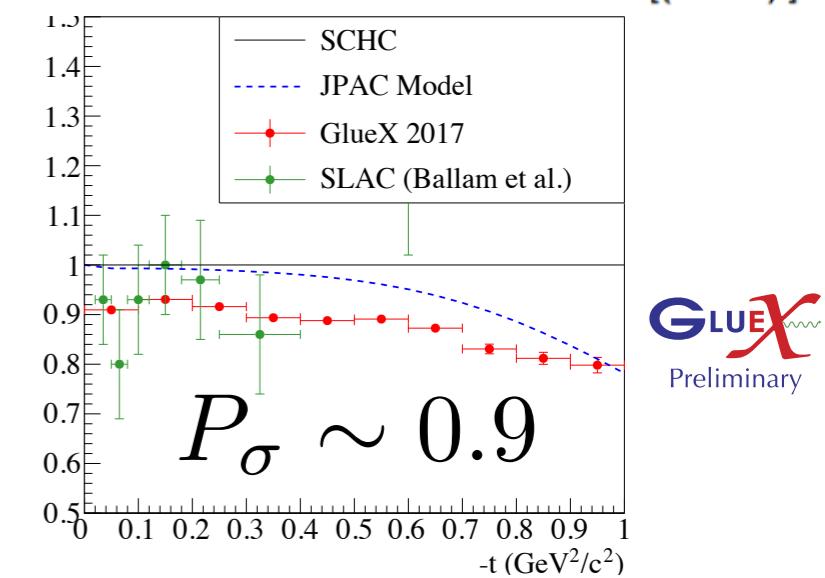
Vector Meson Photoproduction:

$$\vec{\gamma}p \rightarrow \rho^0 p$$

$$\vec{\gamma}p \rightarrow \omega p$$

$$\vec{\gamma}p \rightarrow \phi p$$

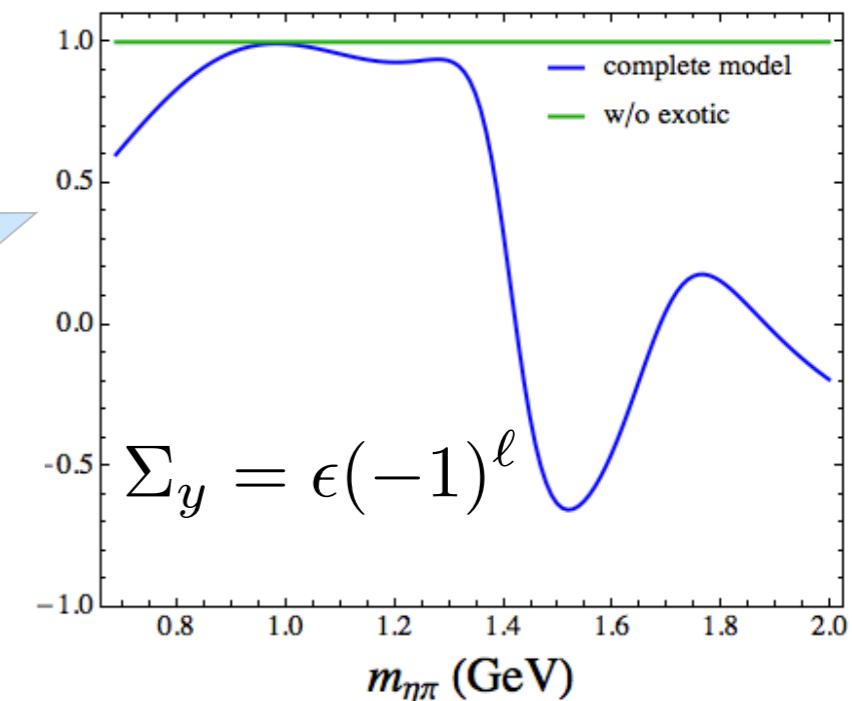
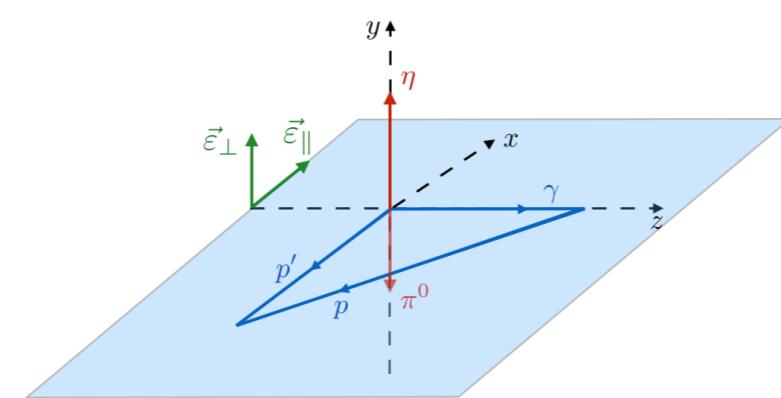
Consistent with factorization
Dominance of natural exchanges



Double Mesons Photoproduction:

$$\vec{\gamma}p \rightarrow \pi^0 \eta p$$

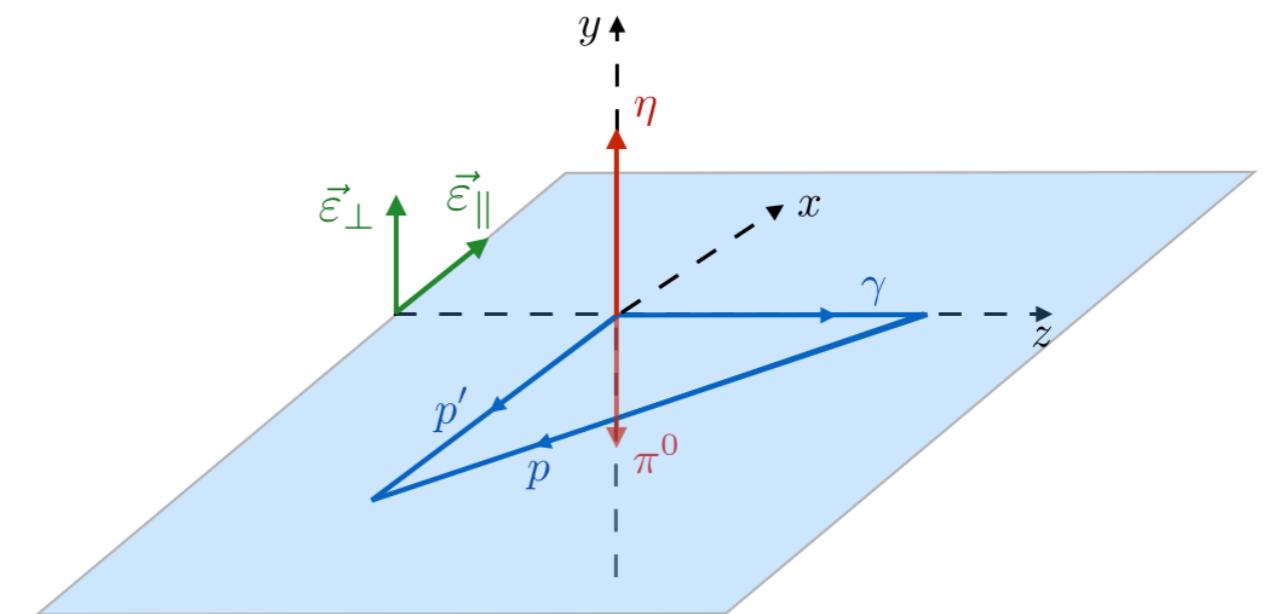
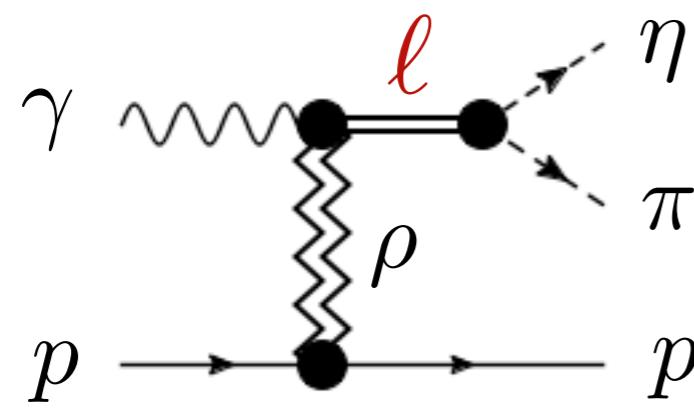
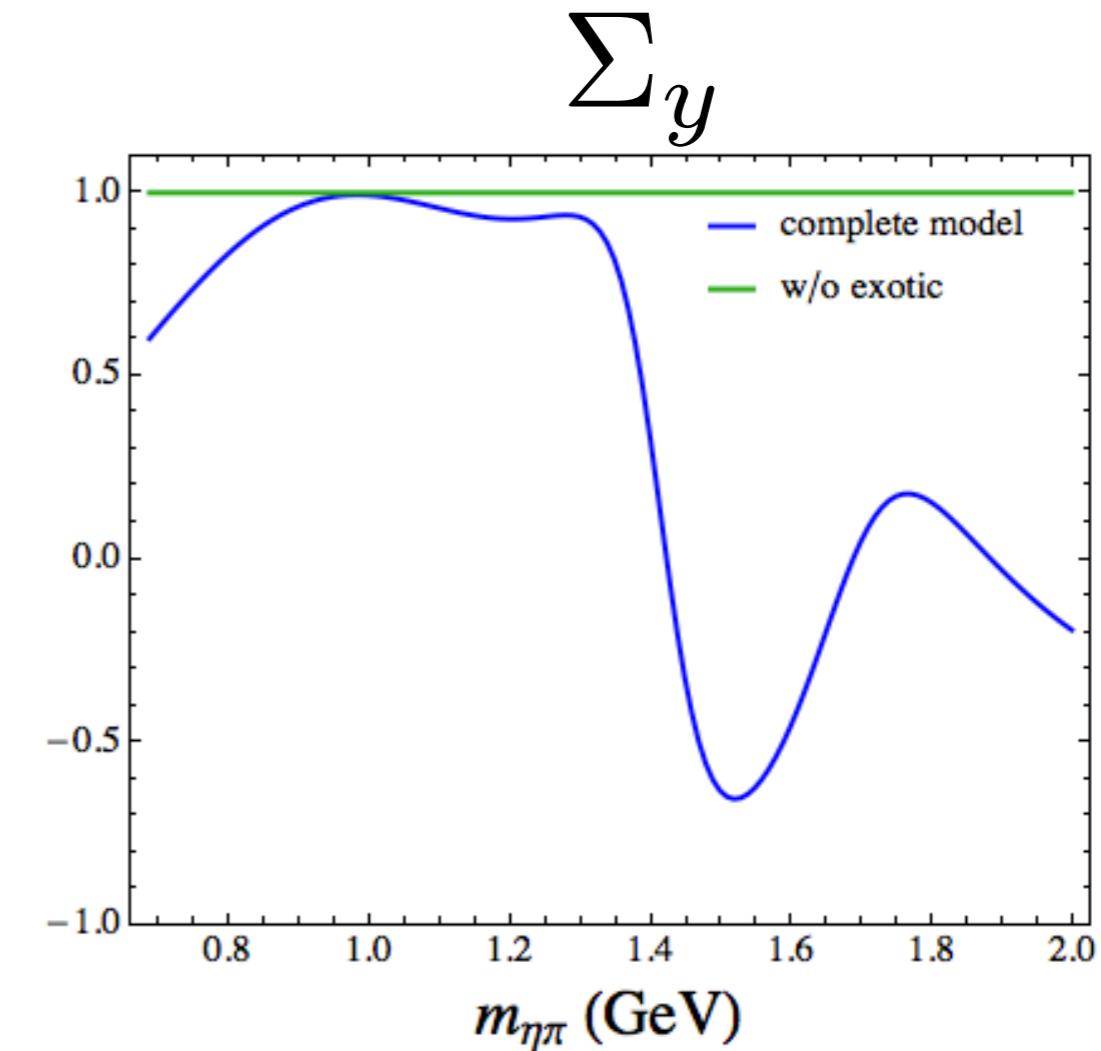
New observable sensitive to exotic production



Backup Slides

Beam Asymmetries

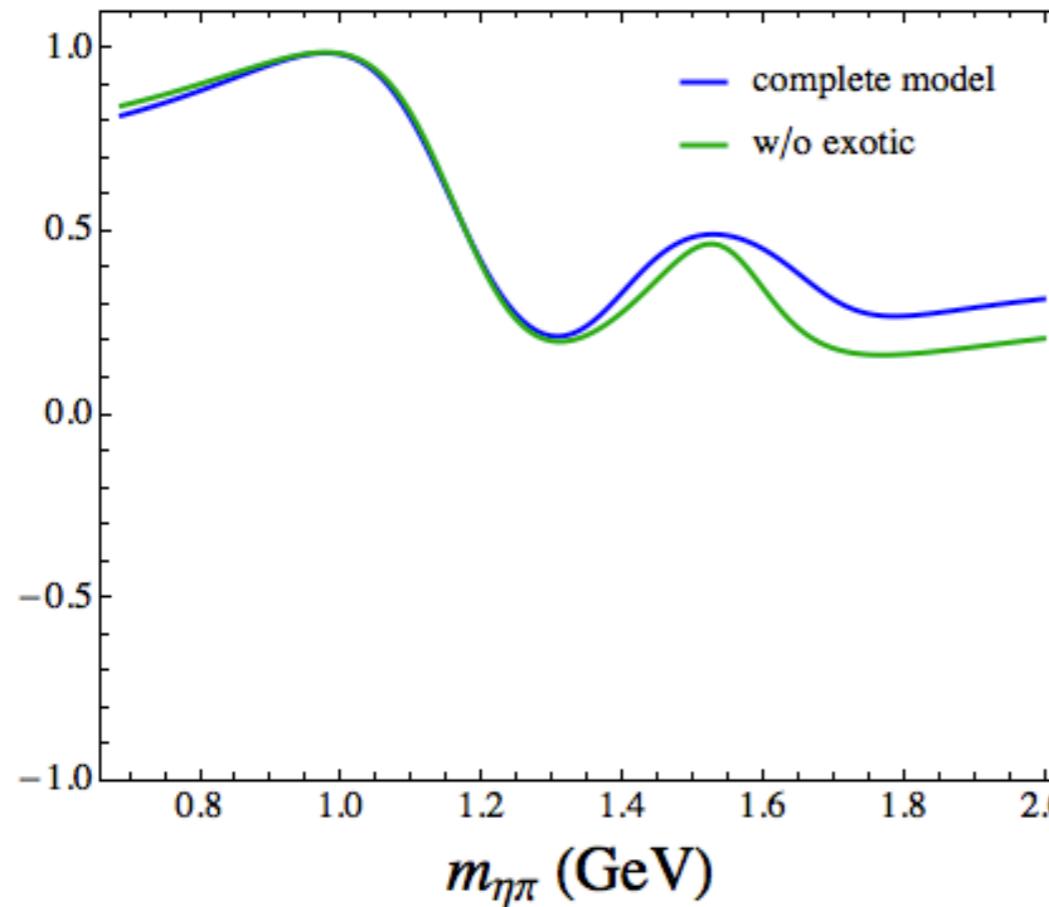
24



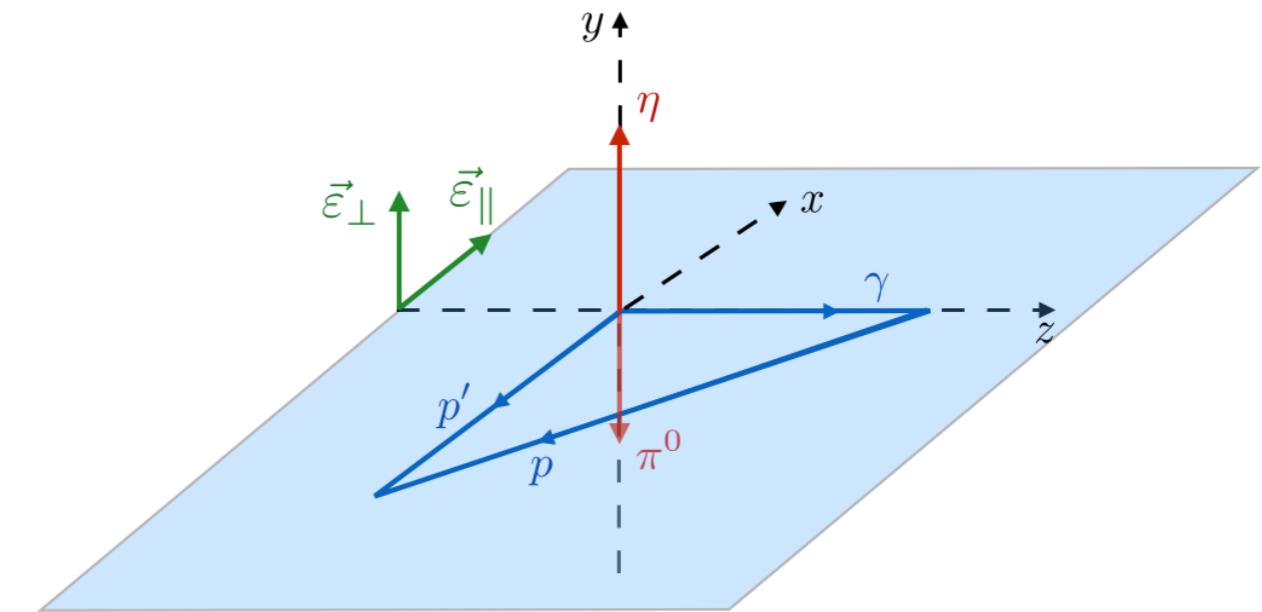
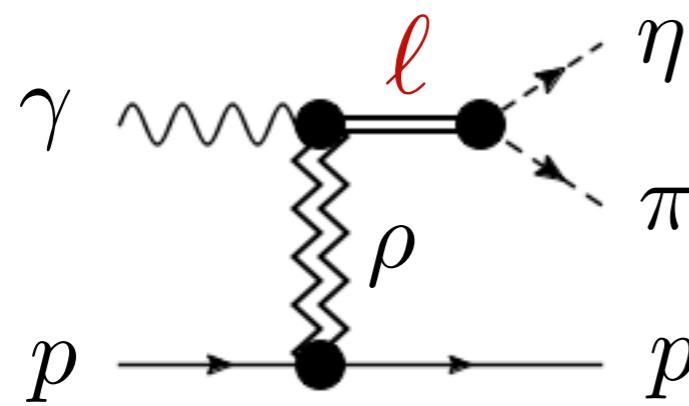
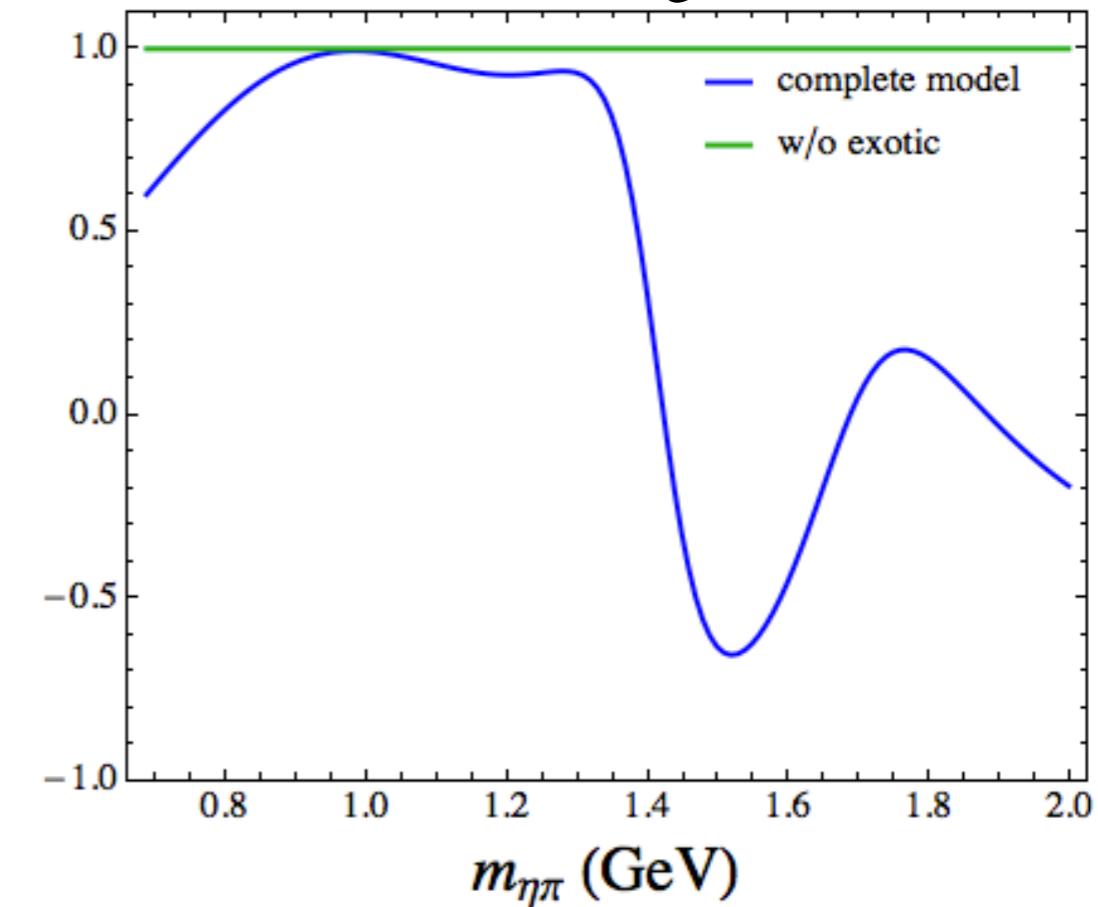
Beam Asymmetries

24

$$\Sigma_{4\pi}$$

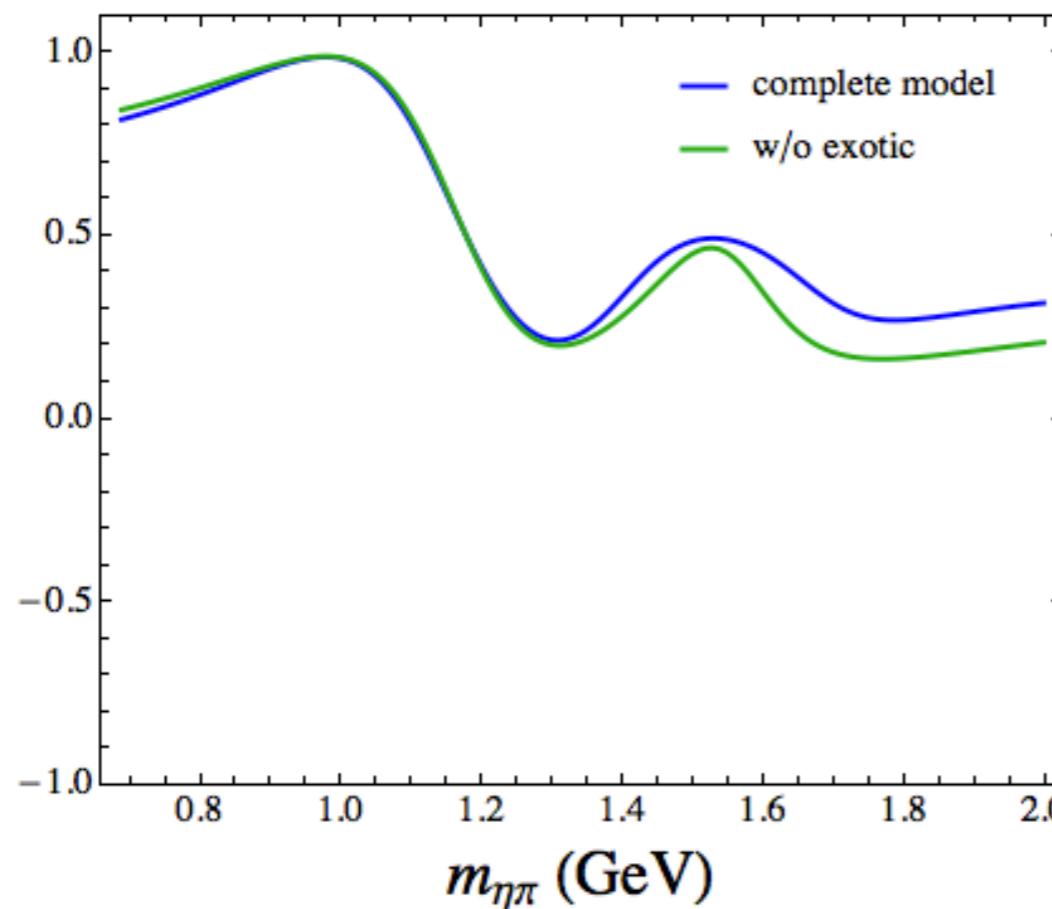
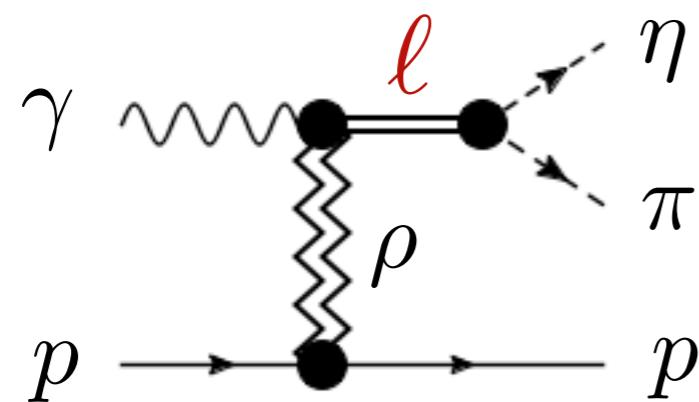
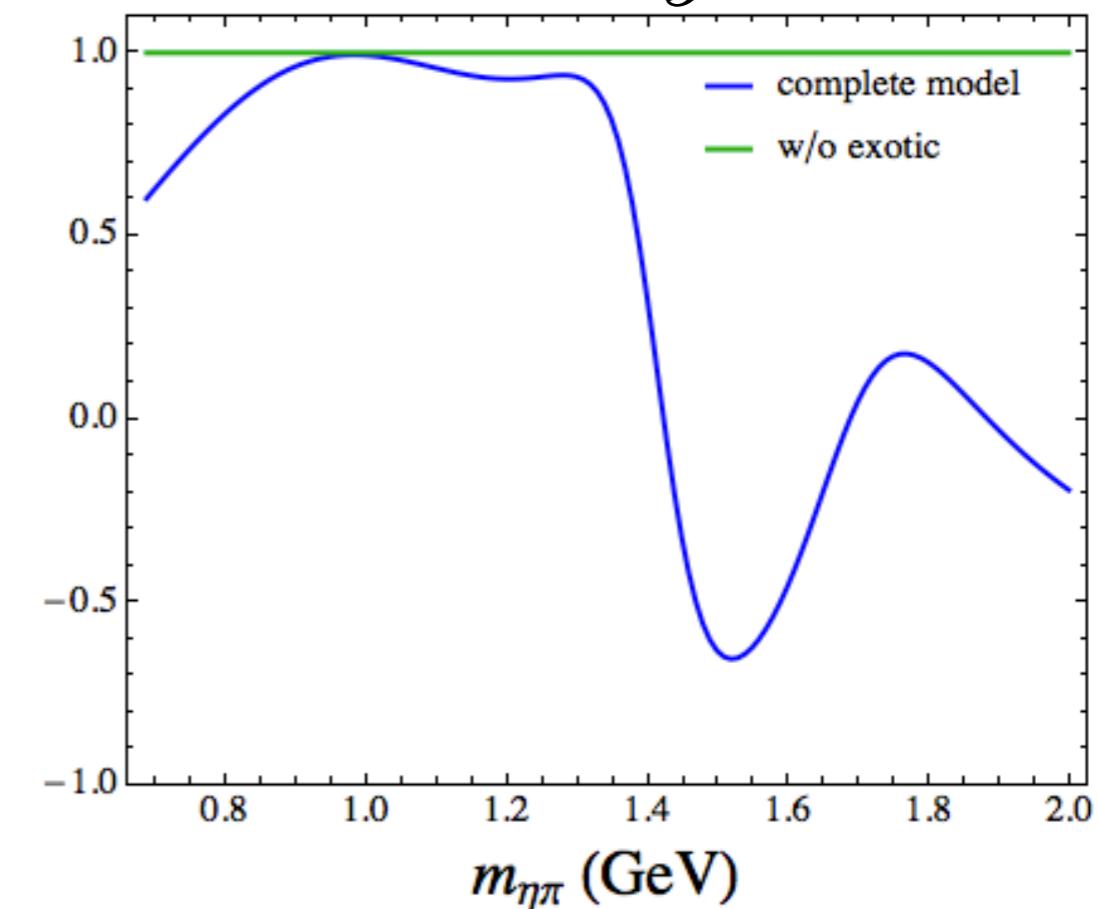


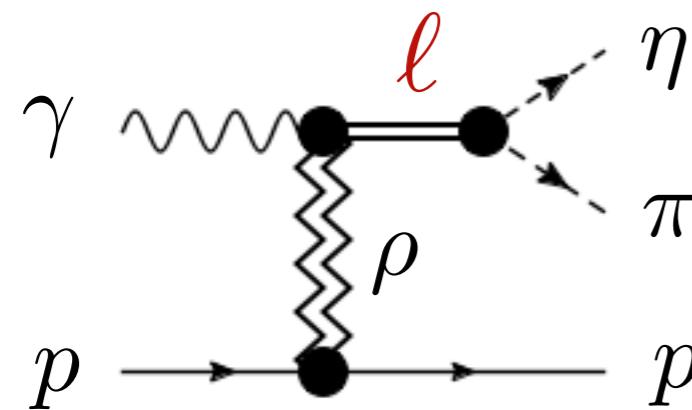
$$\Sigma_y$$



Beam Asymmetries

24

 $\Sigma_{4\pi}$  Σ_y 



$$R = \underbrace{\{a_0(980), \pi_1(1600)\}}_{S_0^{(+)}} \underbrace{\{a_2(1320)\}}_{P_{0,1}^{(+)}} \underbrace{\{a_2(1700)\}}_{D_{0,1,2}^{(+)}}$$

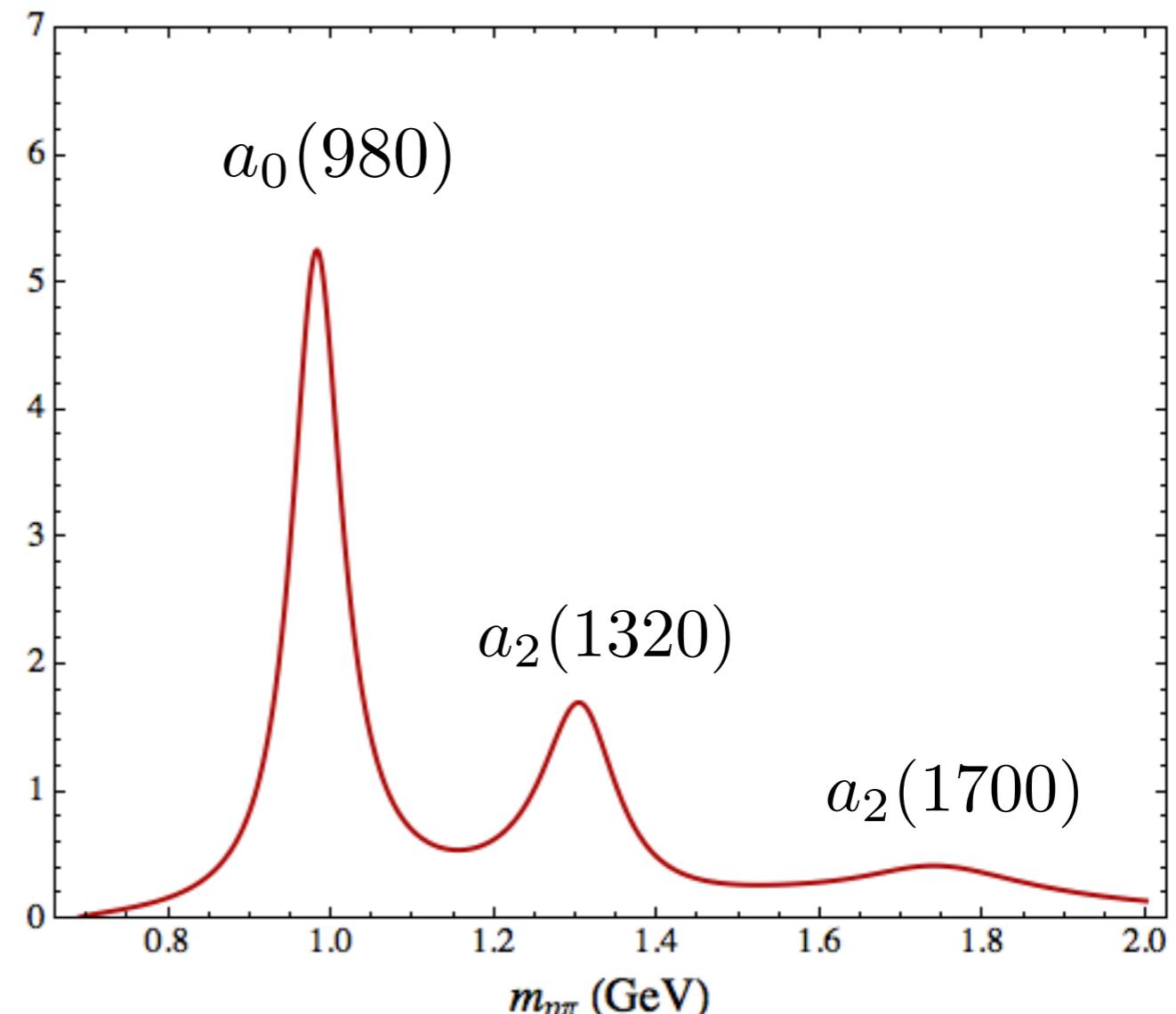
Quark content of the $a_0(980)$ resonance: u, d, g quarks.

production: natural exchanges

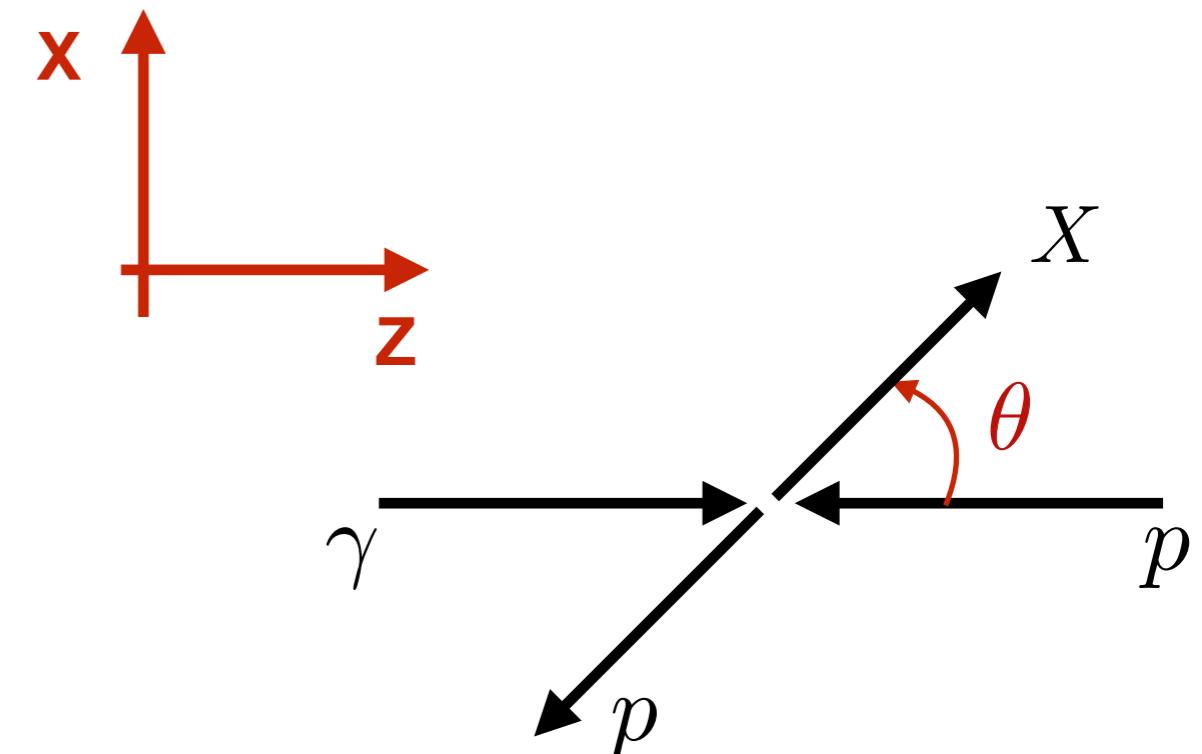
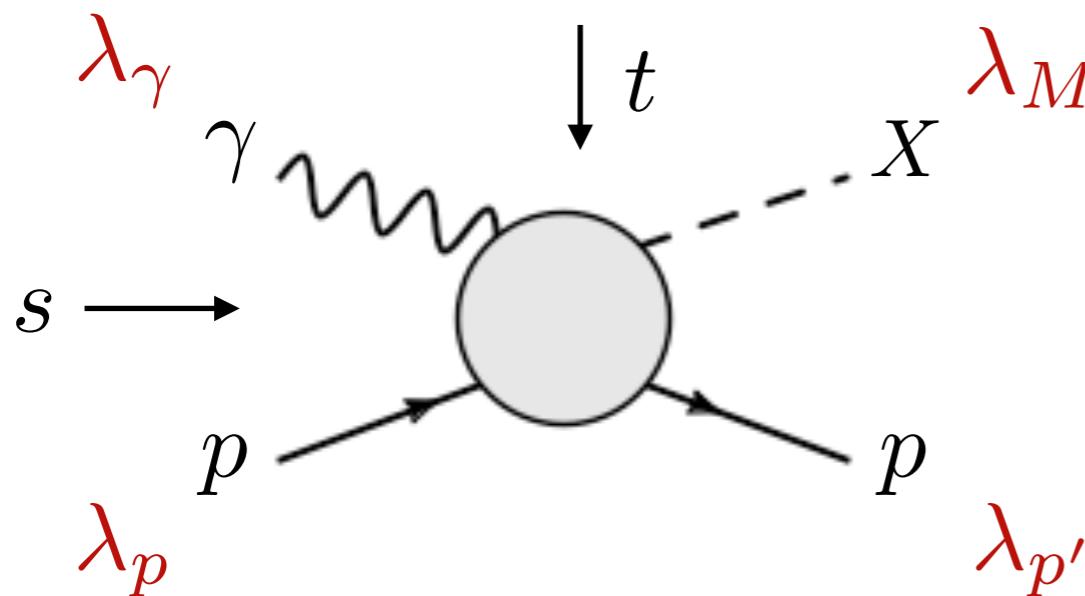
line shape: Breit-Wigner form

parameters: arbitrary

Small exotic wave,
not apparent in the diff. cross. section



Kinematics



$$A_{\lambda_p \lambda_{p'}}^{\lambda_\gamma \lambda_M}(s, t)$$

λ_i = s-channel helicity of particle i

t = momentum transferred squared

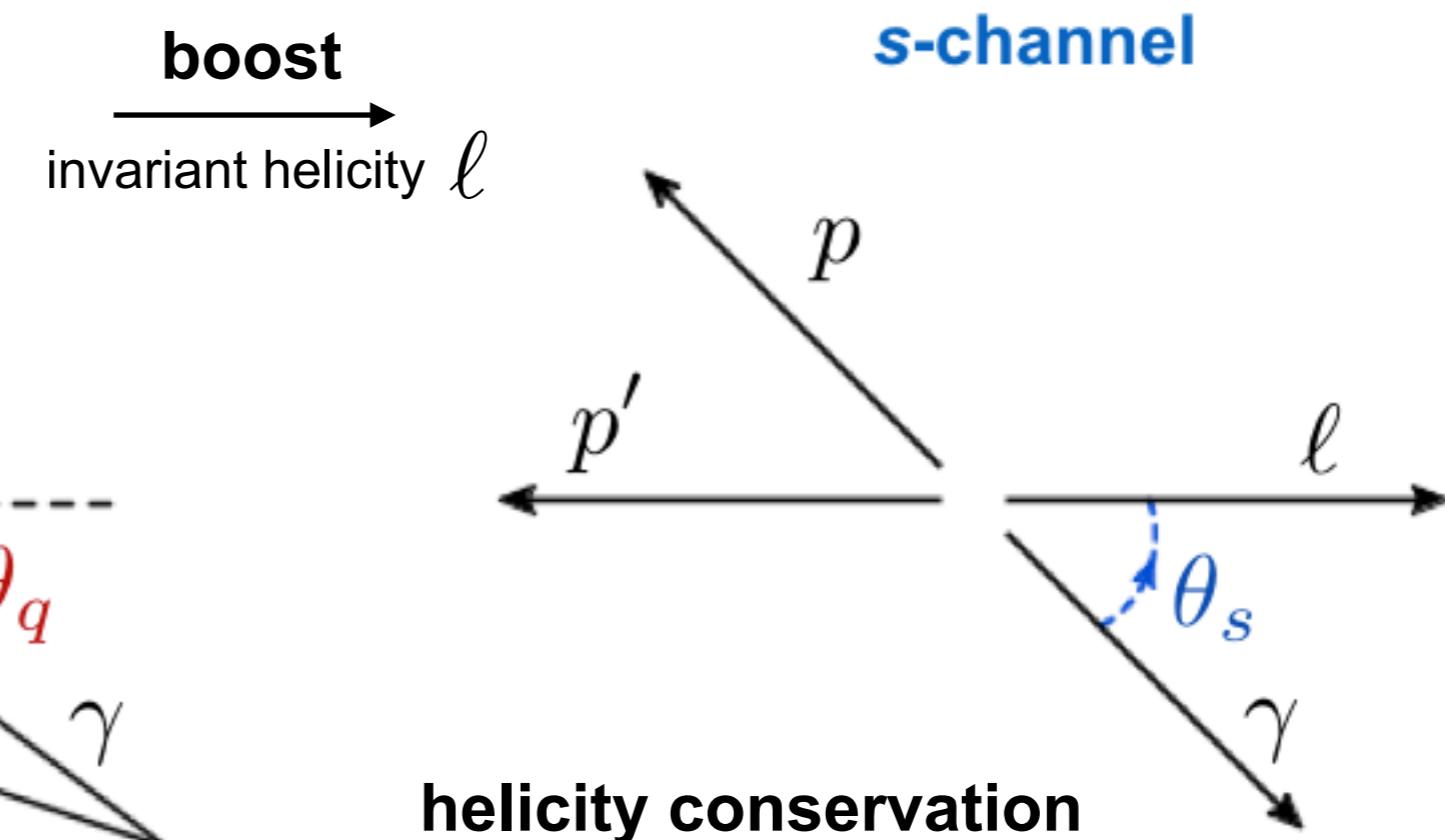
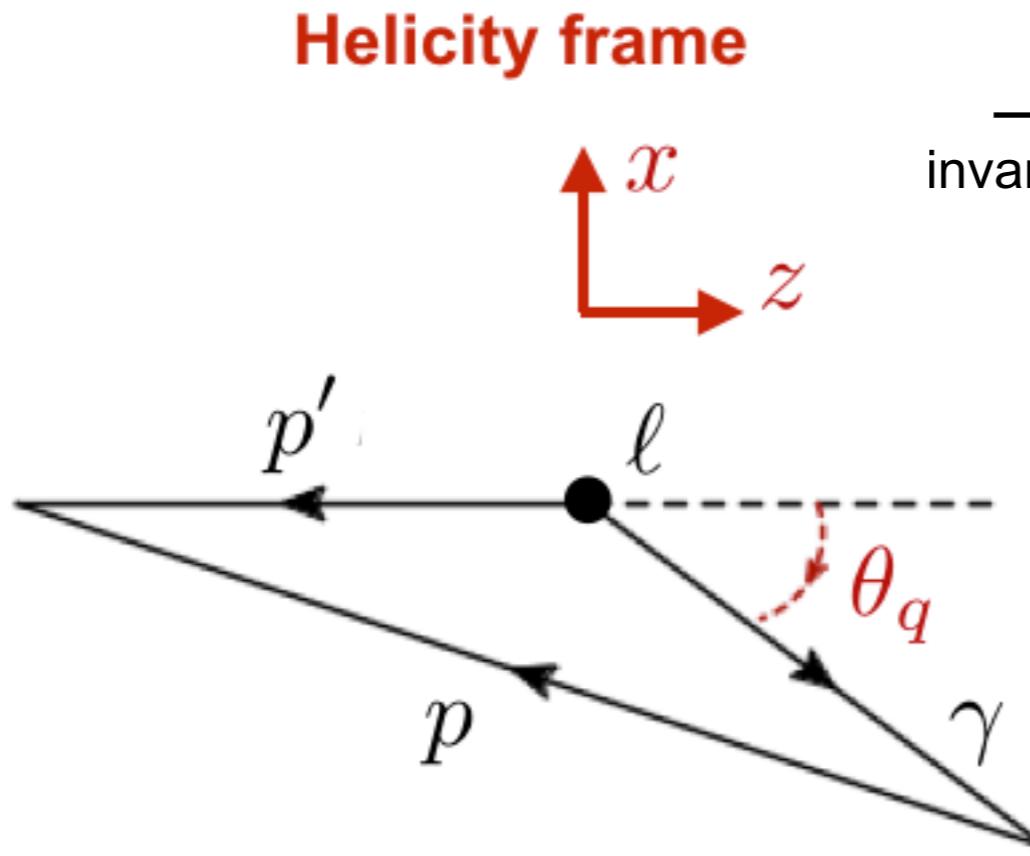
s = center of mass energy squared

High energy approximation

$$\cos \theta \rightarrow 1 + \frac{2t}{s}$$

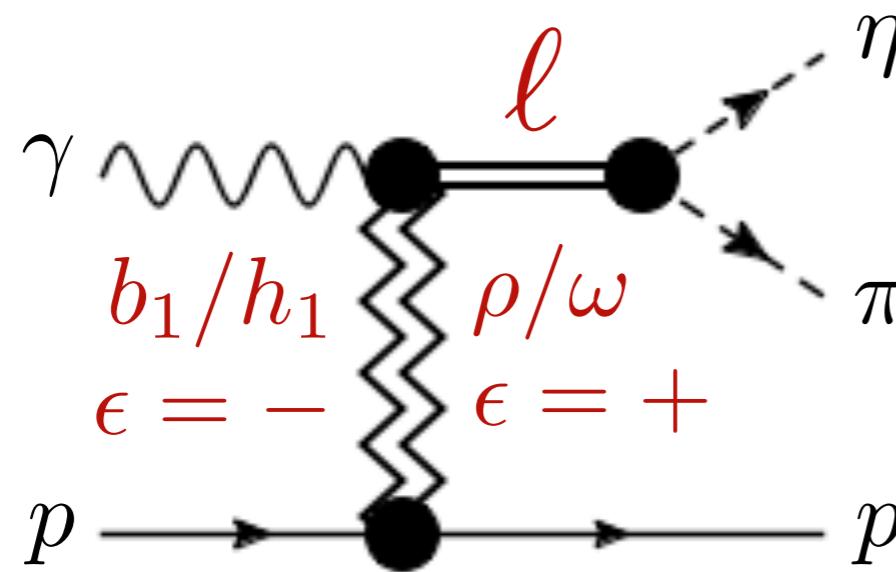
$$\sin \theta \rightarrow 2\sqrt{-t/s}$$

$$\sin \theta/2 \rightarrow \sqrt{-t/s}$$



between γ and ℓ

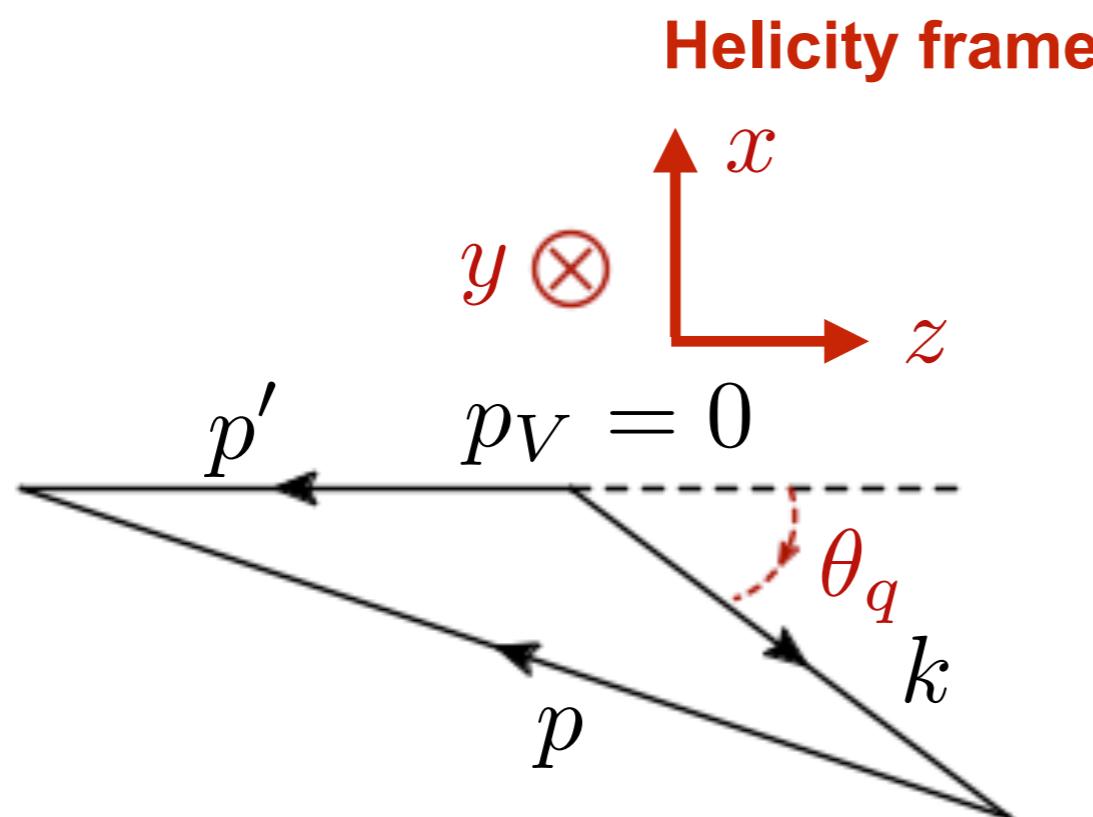
$$T_{\lambda_\gamma m} \simeq \delta_{\lambda_\gamma, m} T_{\lambda_\gamma m} + \dots$$



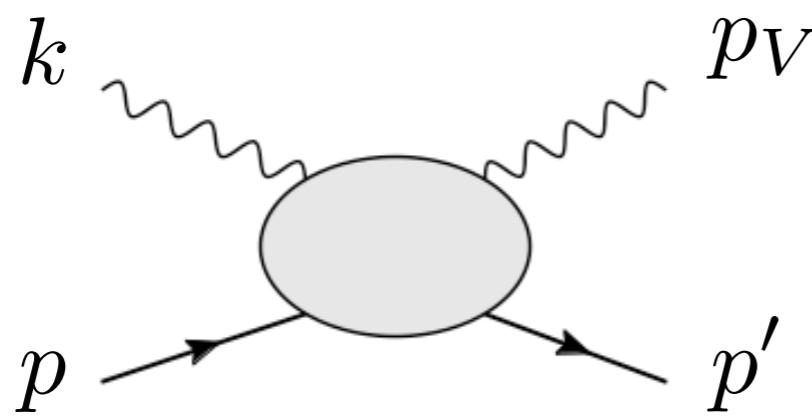
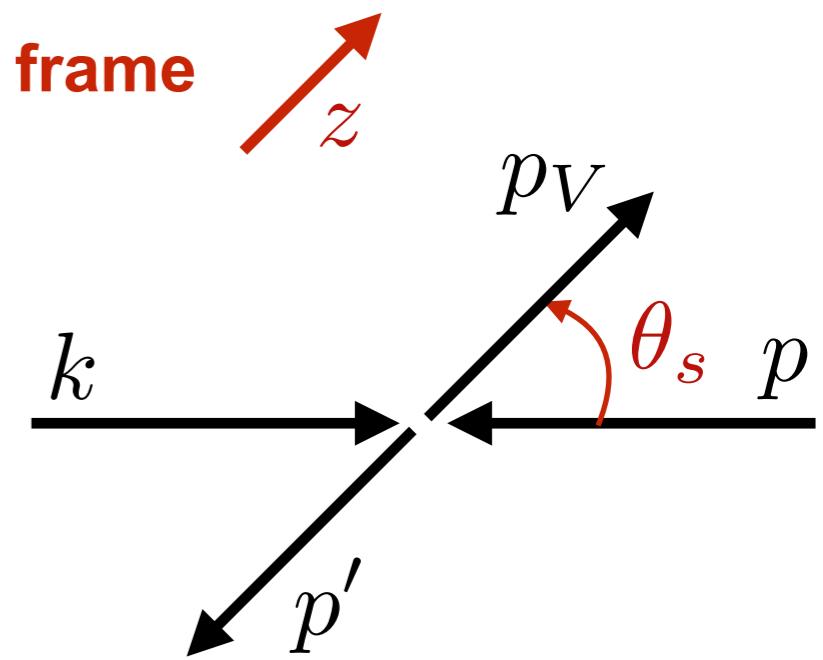
Reflectivity basis:

$$[\ell]_m^{(\epsilon)} = T_{1m} - \epsilon T_{-1-m}$$

Dominant: $(\epsilon = +, m = 1)$

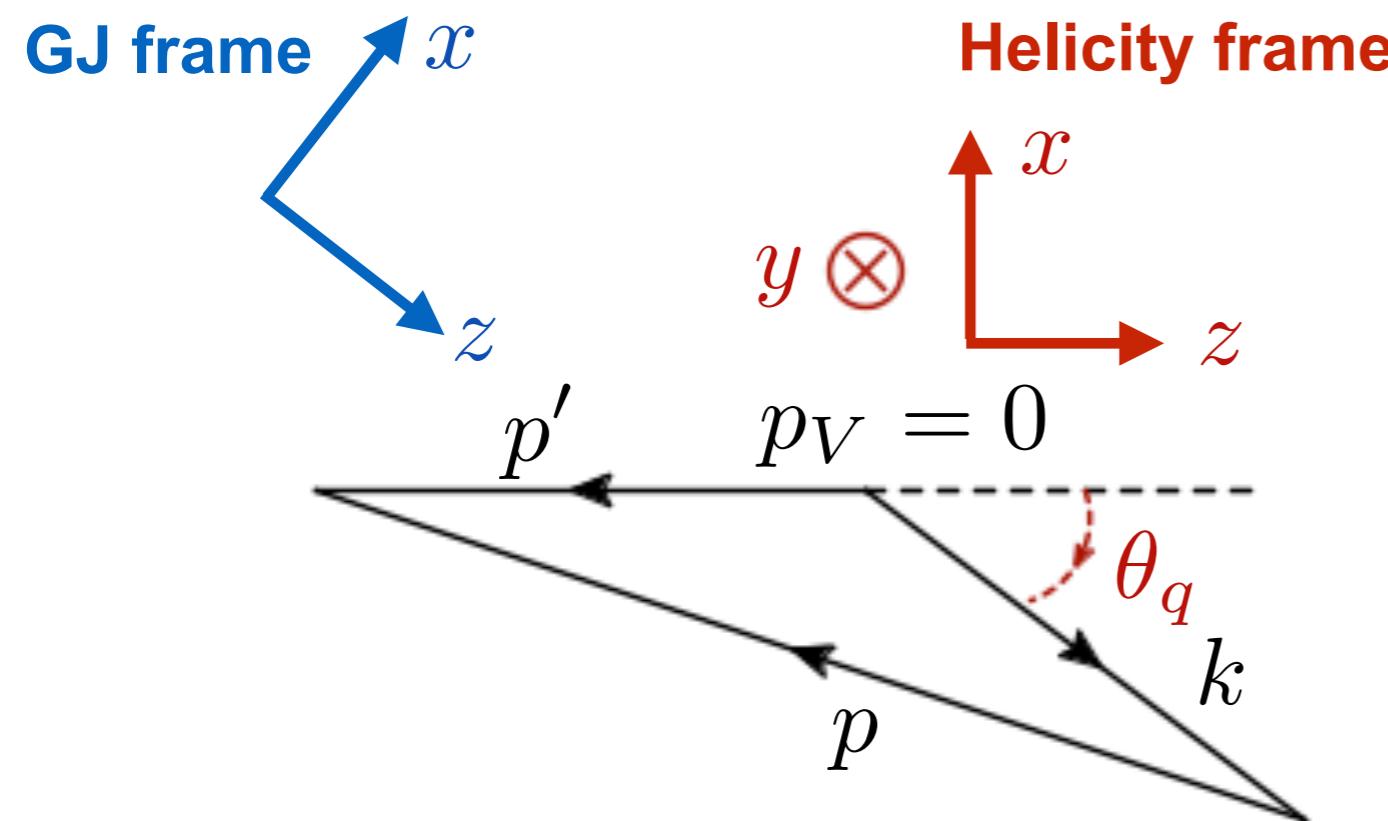


s-channel frame

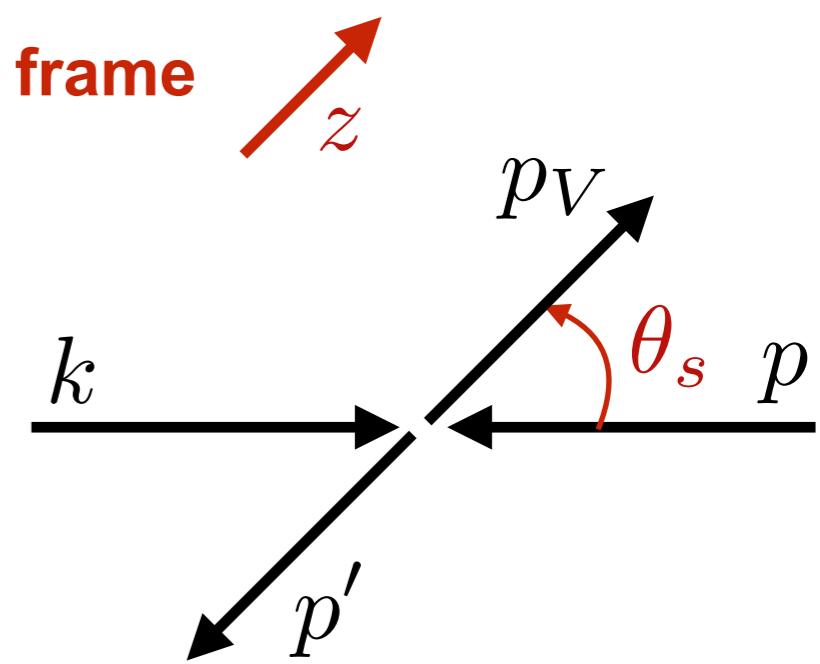


Frames

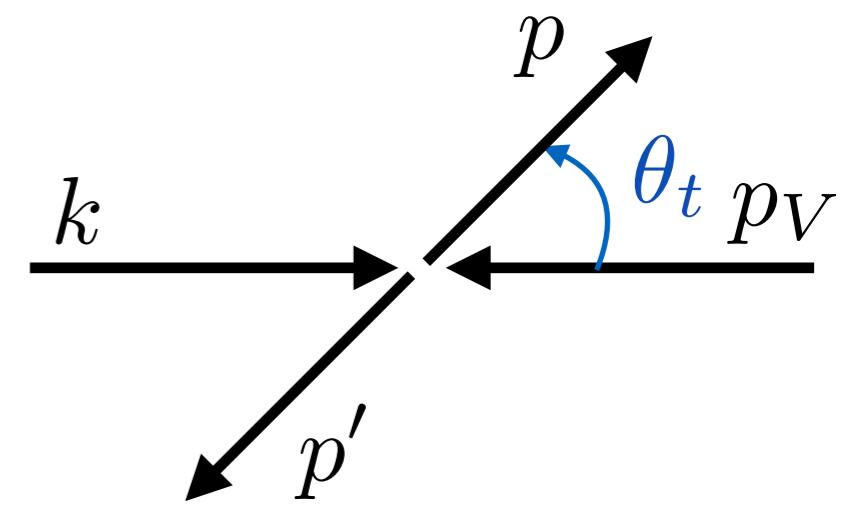
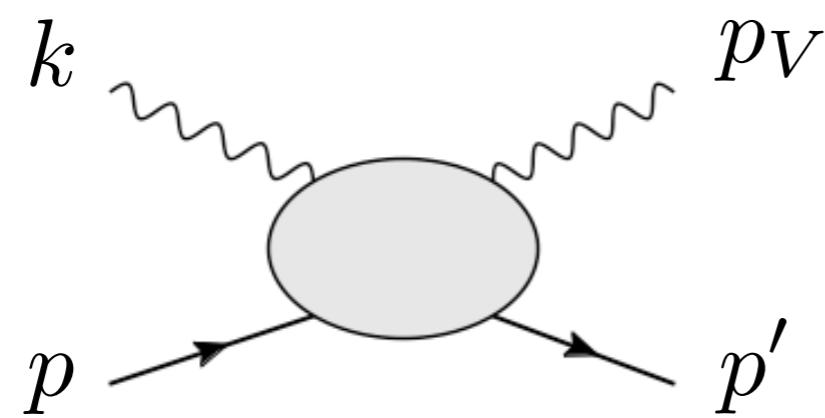
28



s-channel frame

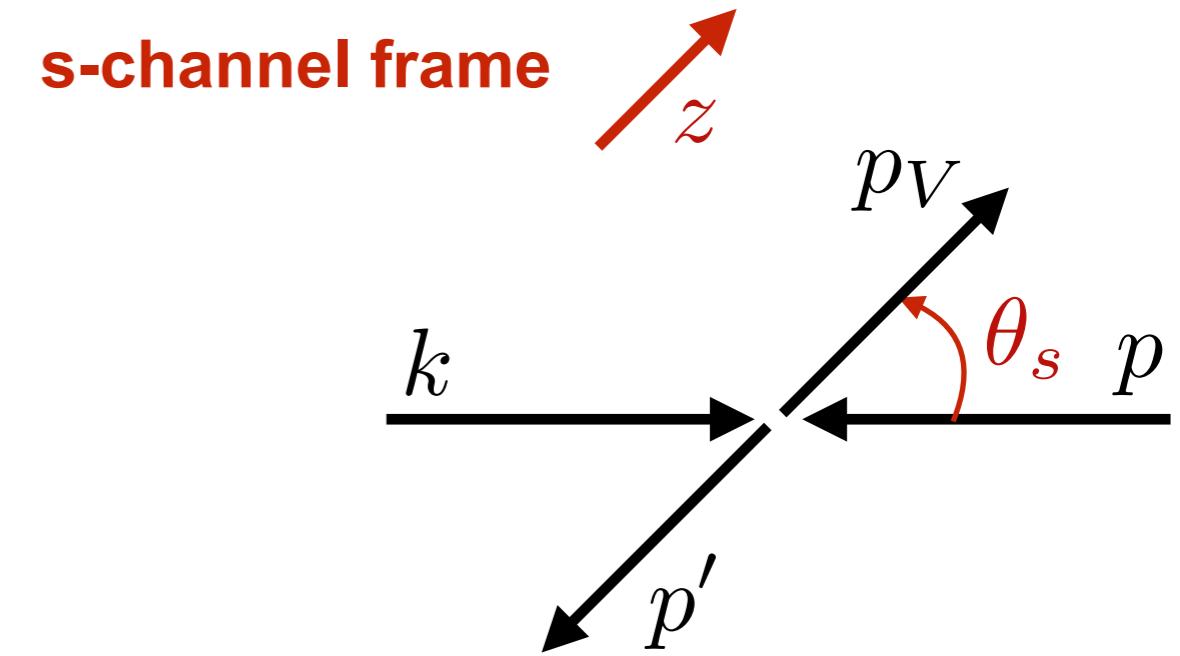
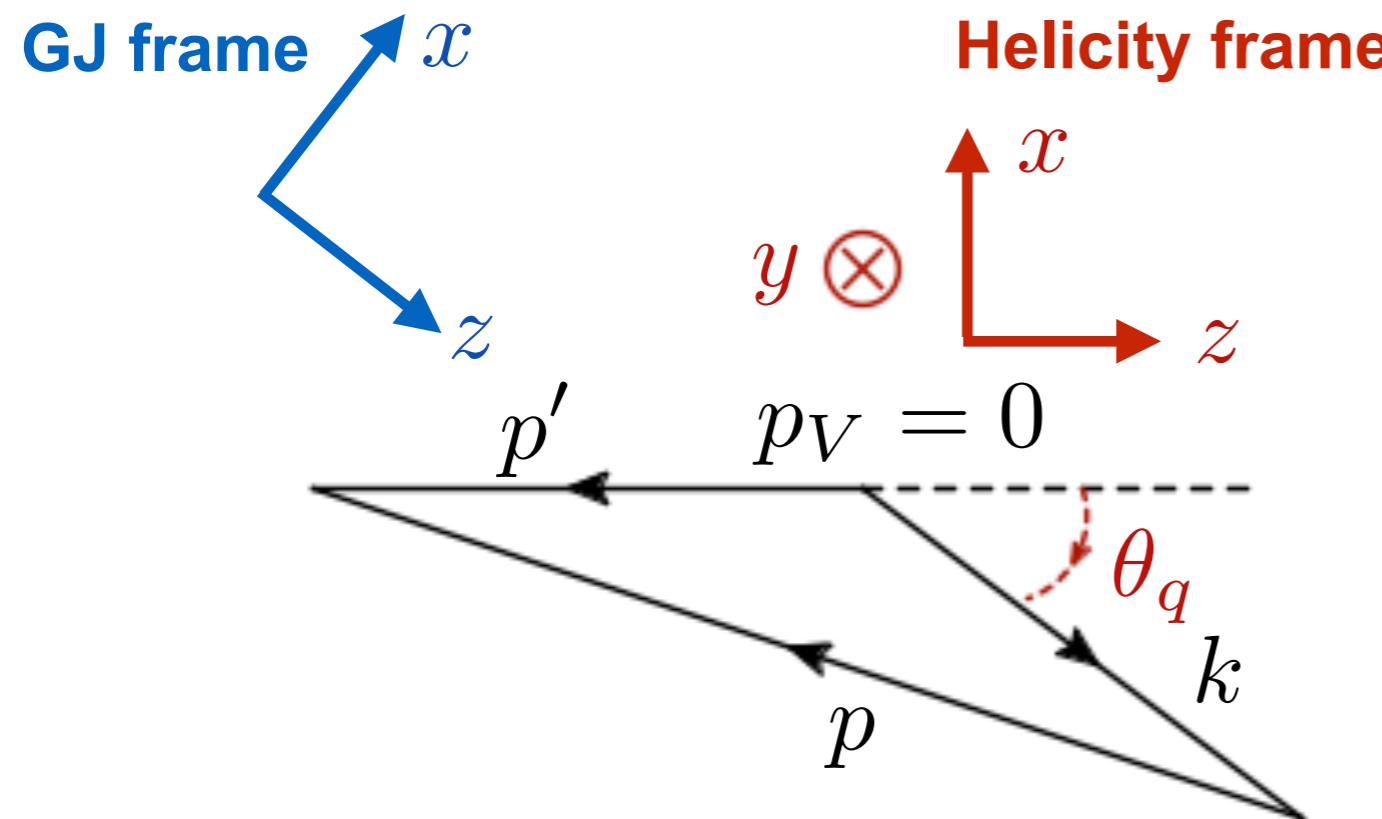


t-channel frame



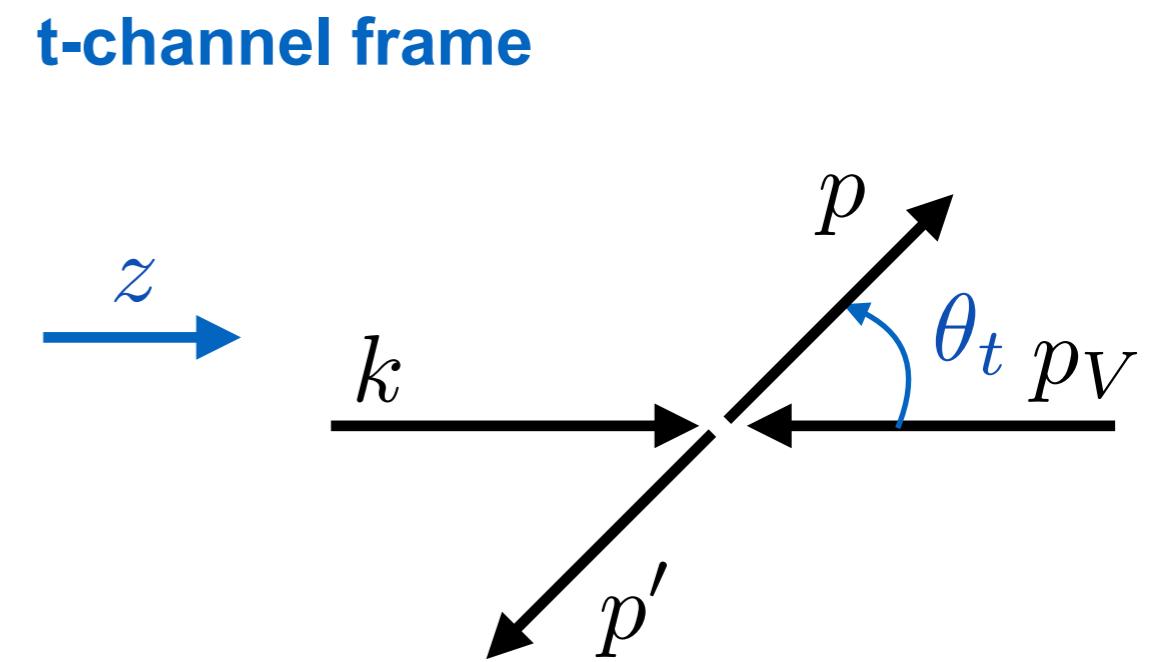
Frames

28



rotation

$$\rho_{MM'}|_H = \rho_{MM'}|_{s\text{-chan}}$$
$$\rho_{MM'}|_{GJ} = \rho_{MM'}|_{t\text{-chan}}$$

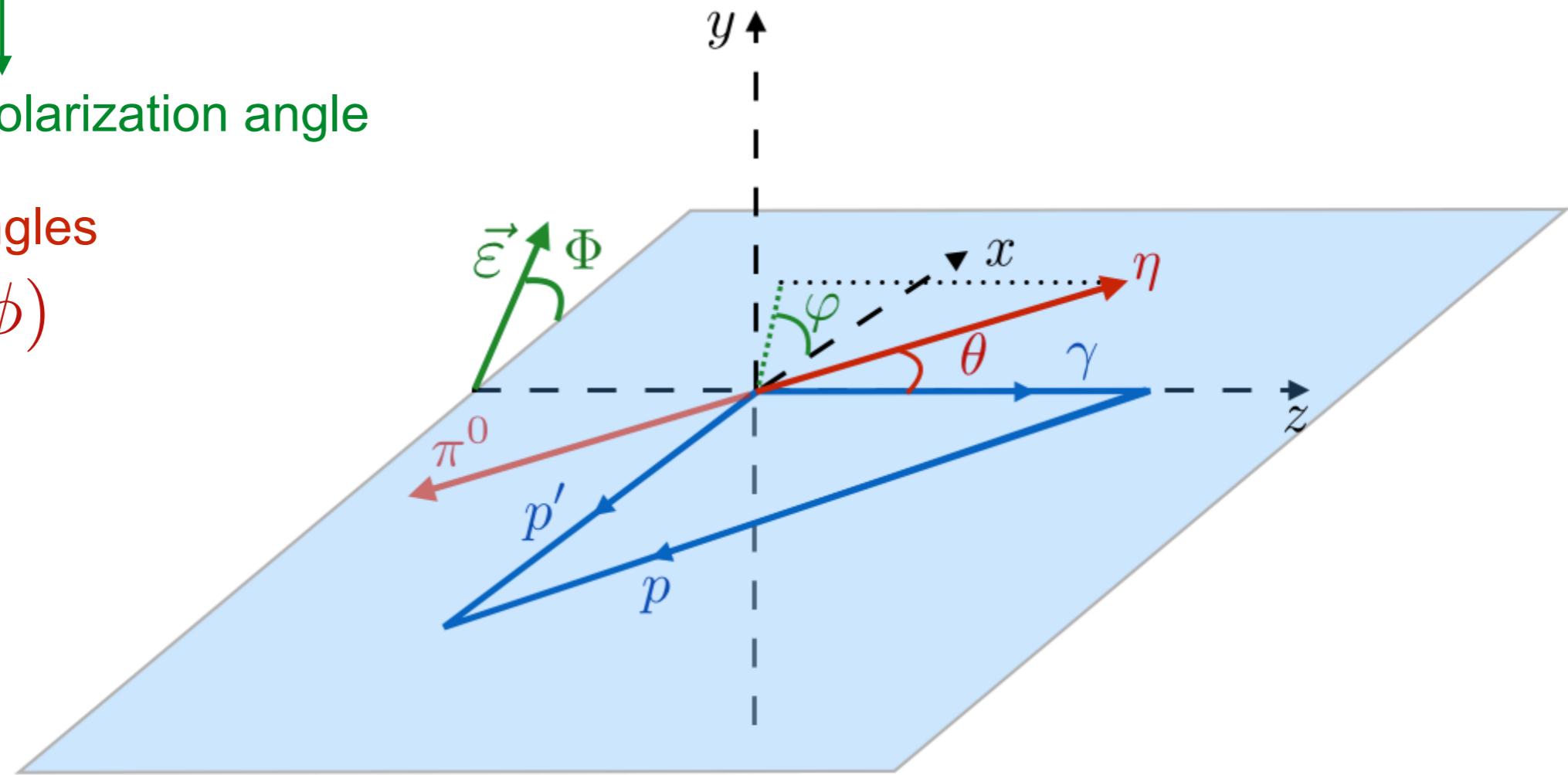


Measured Intensities

$$I(\Omega, \Phi) = I^0(\Omega) - P_\gamma I^1(\Omega) \cos 2\Phi - P_\gamma I^2(\Omega) \sin 2\Phi$$

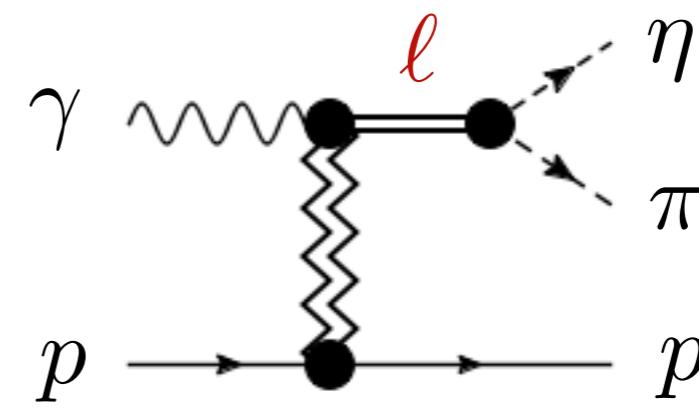

 polarization angle

η decay angles
 $\Omega = (\theta, \phi)$



Implicit variables

- Beam energy (fixed)
- momentum transfer (integrated)
- $\eta\pi$ invariant mass (binned)



Spin Density Matrix Elements

30

$$\rho_{00}^0 = \frac{2}{N} \sum_{\lambda, \lambda'} \left| T_{\lambda, \lambda'}^{1,0} \right|^2$$

$$N = \sum_{\lambda, \lambda', \lambda_\gamma, \lambda_V} \left| T_{\lambda_\gamma, \lambda_V}^{\lambda, \lambda'} \right|^2$$

$$\text{Re } \rho_{10}^0 = \frac{1}{N} \text{Re} \sum_{\lambda, \lambda'} \left(T_{\lambda, \lambda'}^{1,1} - T_{\lambda, \lambda'}^{-1,-1} \right) T_{\lambda, \lambda'}^{1,0}$$

$$\rho_{11}^1 = \frac{2}{N} \text{Re} \sum_{\lambda, \lambda'} T_{\lambda, \lambda'}^{-1,1} T_{\lambda, \lambda'}^{1,1}$$

$$\rho_{1-1}^0 = \frac{2}{N} \text{Re} \sum_{\lambda, \lambda'} T_{\lambda, \lambda'}^{1,1} T_{\lambda, \lambda'}^{1,-1}$$

$$\rho_{00}^1 = \frac{2}{N} \text{Re} \sum_{\lambda, \lambda'} T_{\lambda, \lambda'}^{-1,0} T_{\lambda, \lambda'}^{1,0}$$

$$\rho_{1-1}^1 + \text{Im } \rho_{1-1}^2 = \frac{2}{N} \sum_{\lambda, \lambda'} T_{\lambda, \lambda'}^{-1,1} T_{\lambda, \lambda'}^{1,-1}$$

$$\text{Re } \rho_{10}^1 + \text{Im } \rho_{10}^2 = \frac{1}{N} \text{Re} \sum_{\lambda, \lambda'} \mathcal{M}_{\lambda, \lambda'}^{-1,1} \mathcal{M}_{\lambda, \lambda'}^{1,0}$$

$$\rho_{1-1}^1 - \text{Im } \rho_{1-1}^2 = \frac{2}{N} \sum_{\lambda, \lambda'} T_{\lambda, \lambda'}^{1,1} T_{\lambda, \lambda'}^{-1,-1}$$

$$\text{Re } \rho_{10}^1 - \text{Im } \rho_{10}^2 = \frac{1}{N} \text{Re} \sum_{\lambda_\gamma, \lambda, \lambda'} \mathcal{M}_{\lambda, \lambda'}^{1,1} \mathcal{M}_{\lambda, \lambda'}^{-1,0}$$

Observables: Moments of Angular distribution

$$H^0(LM) = \frac{1}{2\pi} \int I(\Omega, \Phi) d_{M0}^L(\theta) \cos M\phi \, d\Omega d\Phi$$

$$H^1(LM) = \frac{-1}{\pi P_\gamma} \int I(\Omega, \Phi) \cos 2\Phi d_{M0}^L(\theta) \cos M\phi \, d\Omega d\Phi$$

$$\text{Im } H^2(LM) = \frac{1}{\pi P_\gamma} \int I(\Omega, \Phi) \sin 2\Phi d_{M0}^L(\theta) \sin M\phi \, d\Omega d\Phi$$

$$H^1(LM) + \text{Im } H^2(LM) \propto \sum_{\epsilon, \ell\ell', mm'} \left(\frac{2\ell' + 1}{2\ell + 1} \right)^{1/2} \epsilon (-1)^m C_{\ell' 0 L 0}^{\ell 0} C_{\ell' m' L M}^{\ell m} [\ell]_{-m}^{(\epsilon)} [\ell']_{m'}^{(\epsilon)*}$$

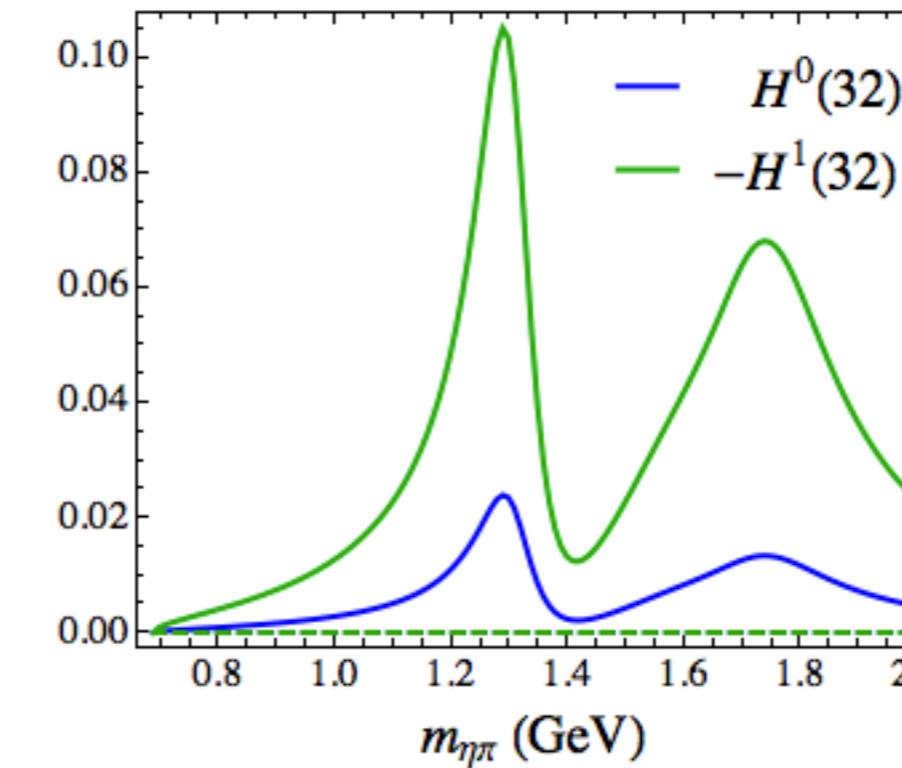
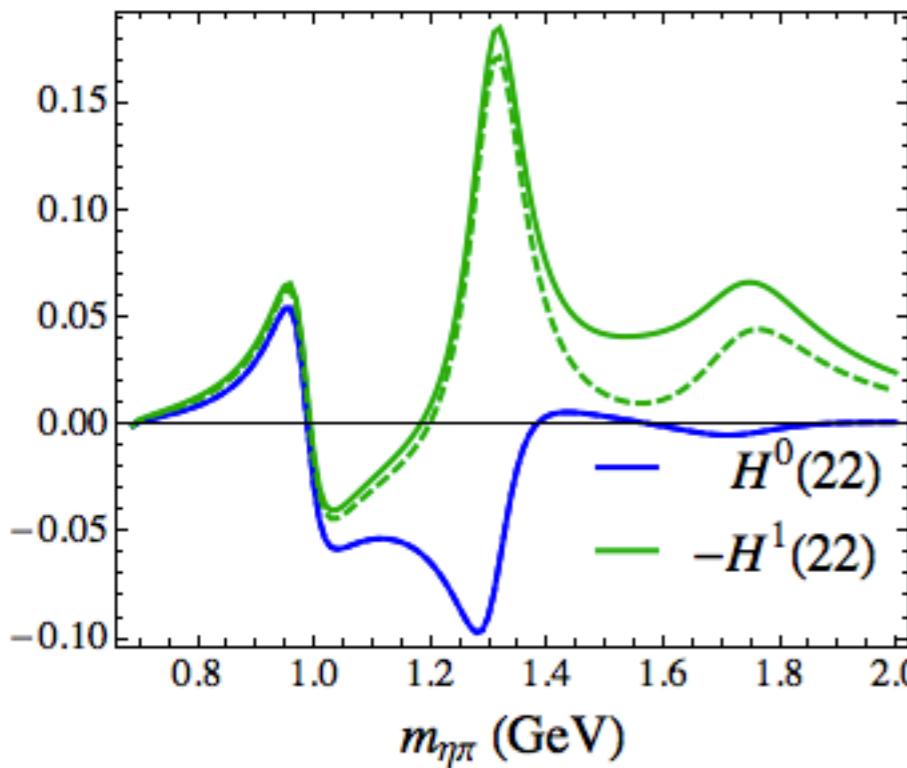
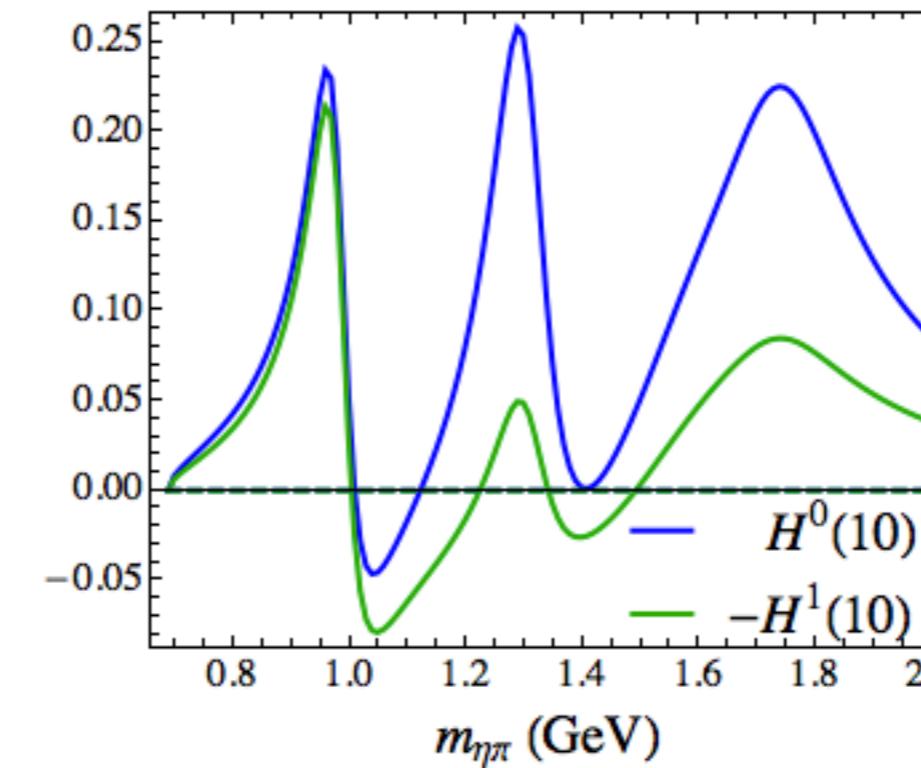
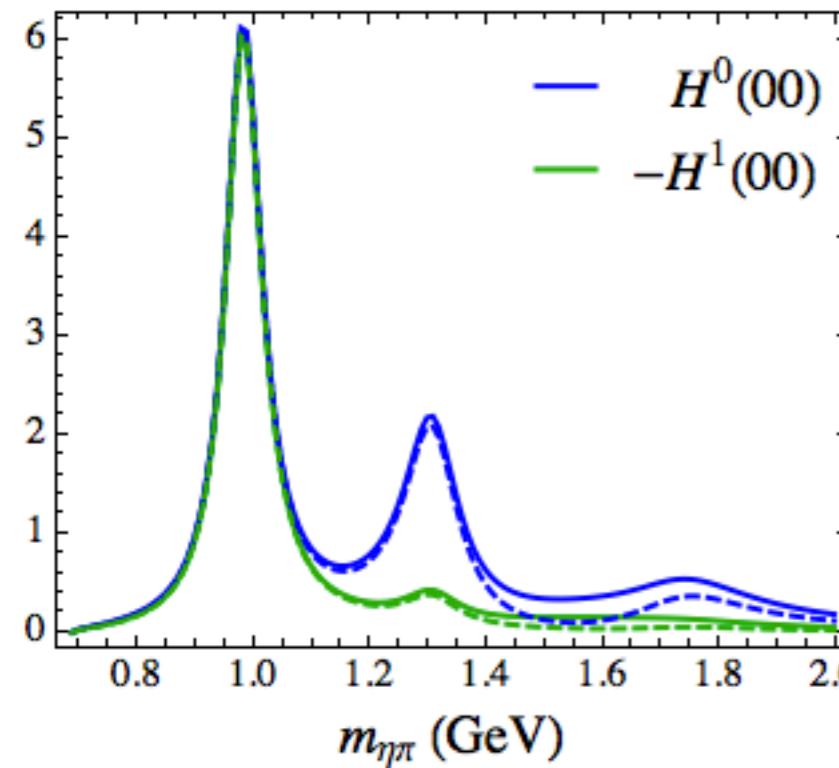
$m' = m - M$

 $0 \leq -m ; 0 \leq m'$

The model features
only positive projections

$$H^1(LM) + \text{Im } H^2(LM) = 0 \quad M \geq 1$$

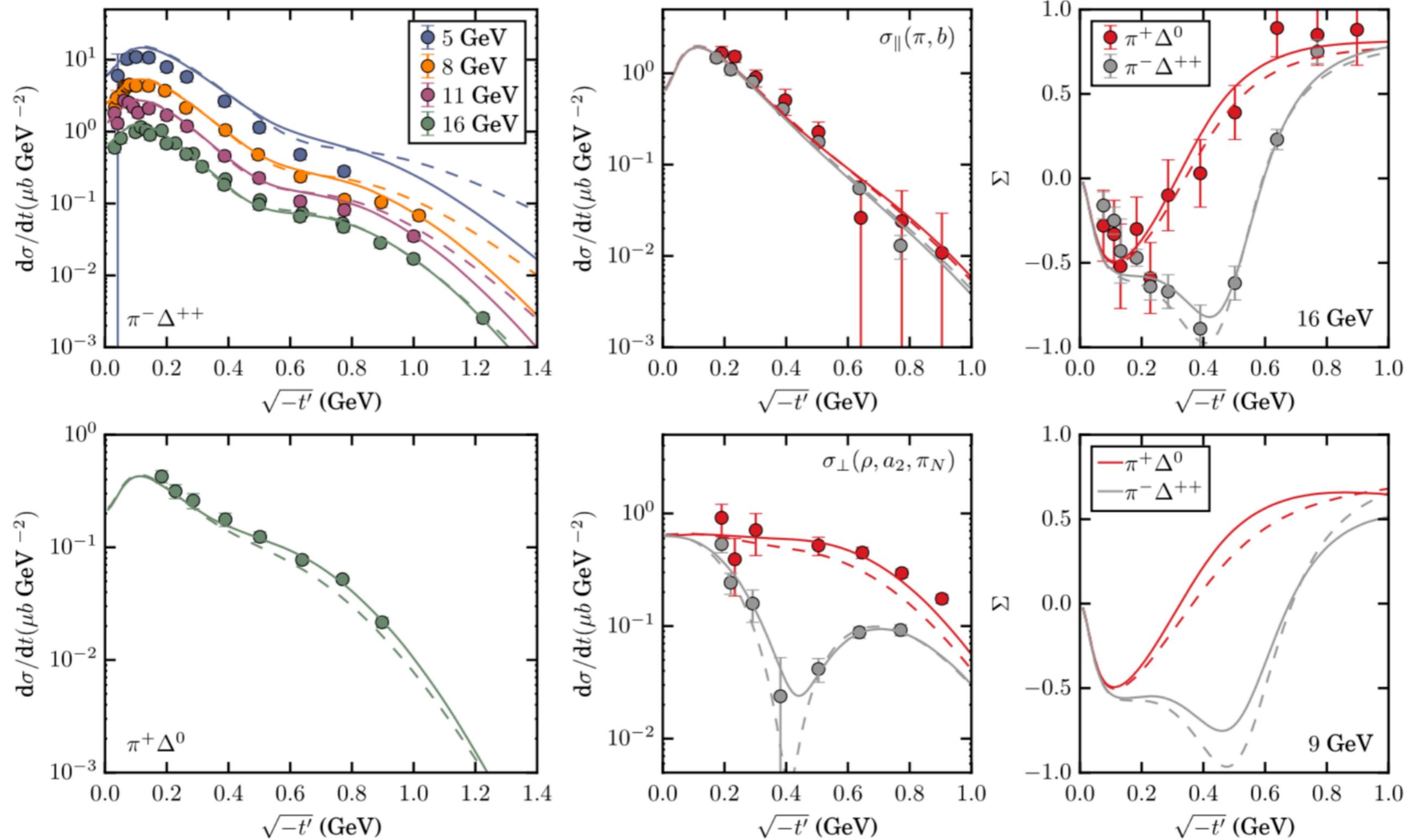
Moments



**P- wave apparent
in odd moments but
not in even moments**

$a_2(1700)$ more apparent
in odd moments than
in even moments

solid lines: $S + P + D$ waves
dashed lines: $S + D$ waves

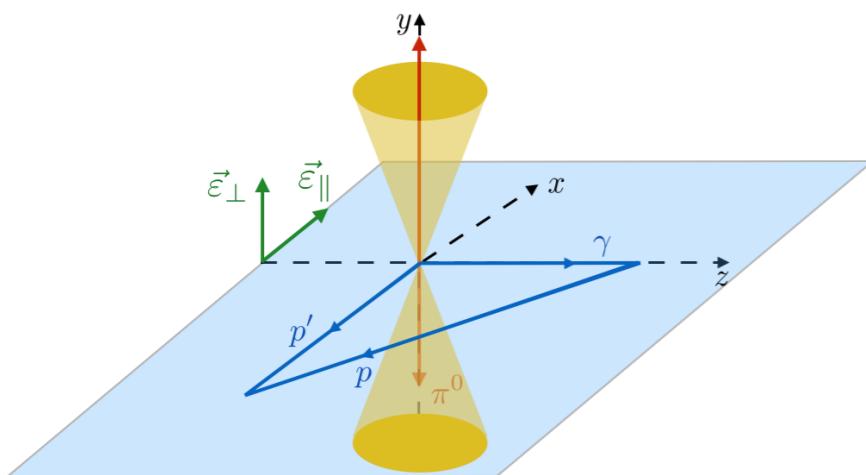
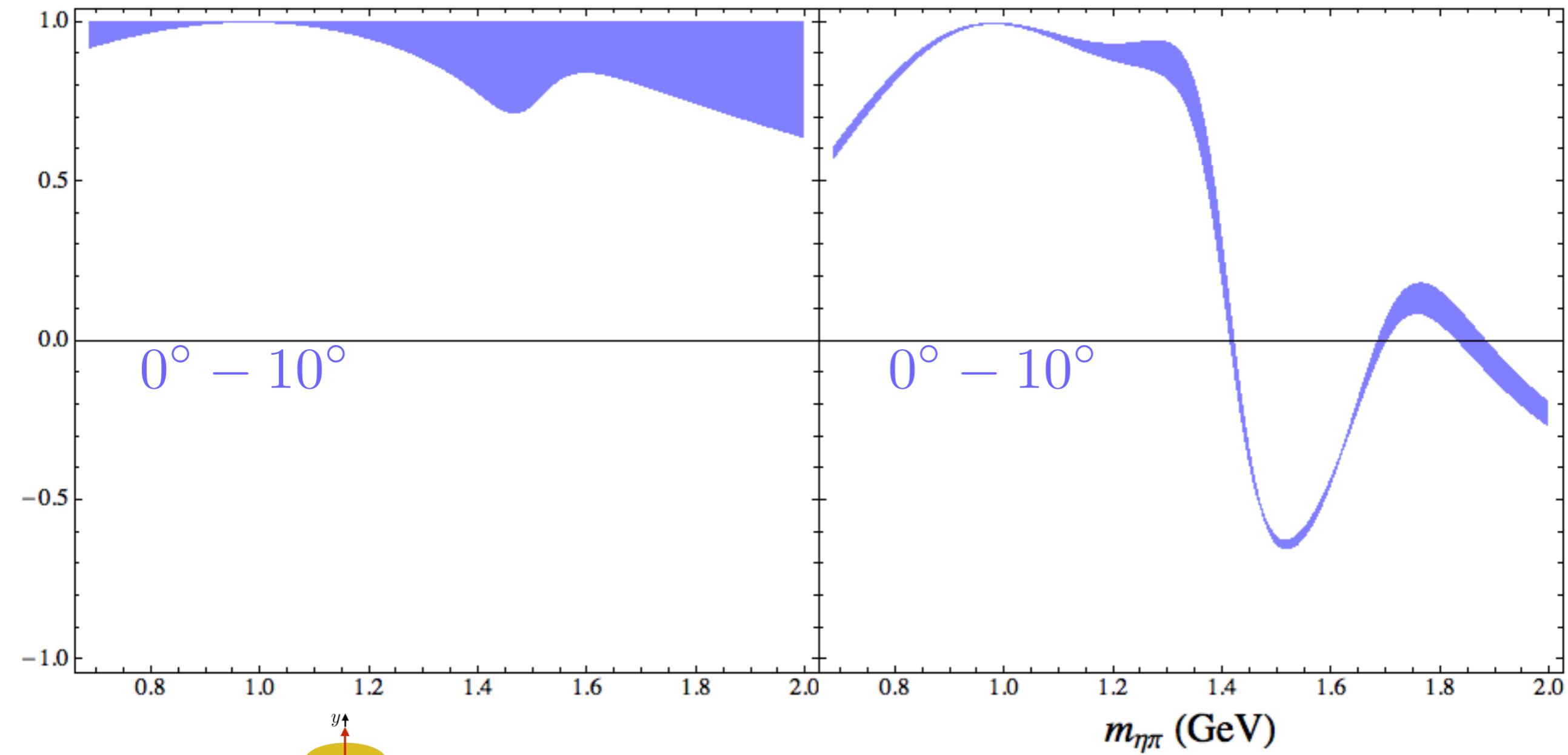


Beam Asymmetries: $\Sigma_{y \pm \delta^\circ}$

34

only S and D waves

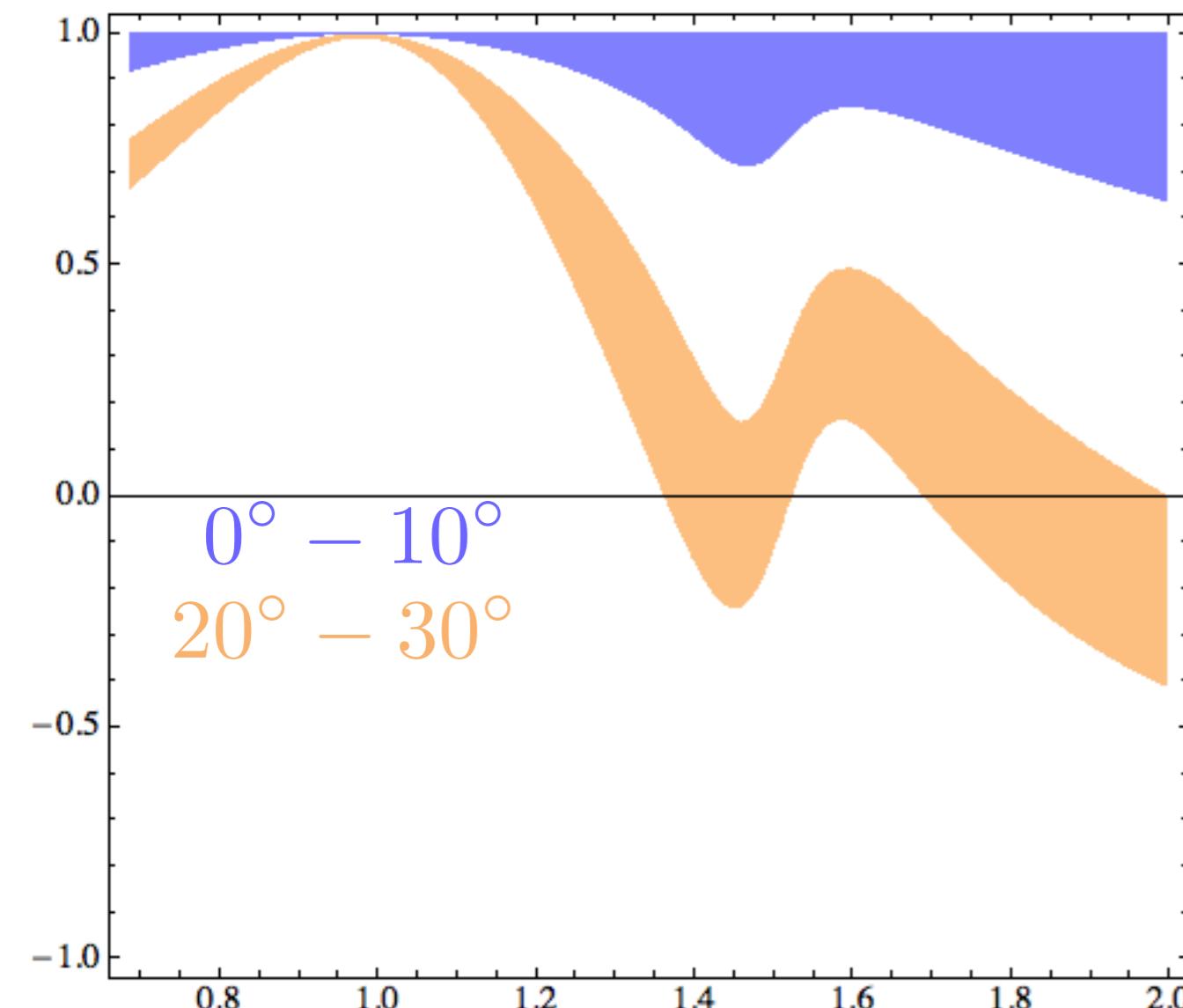
S, P and D waves



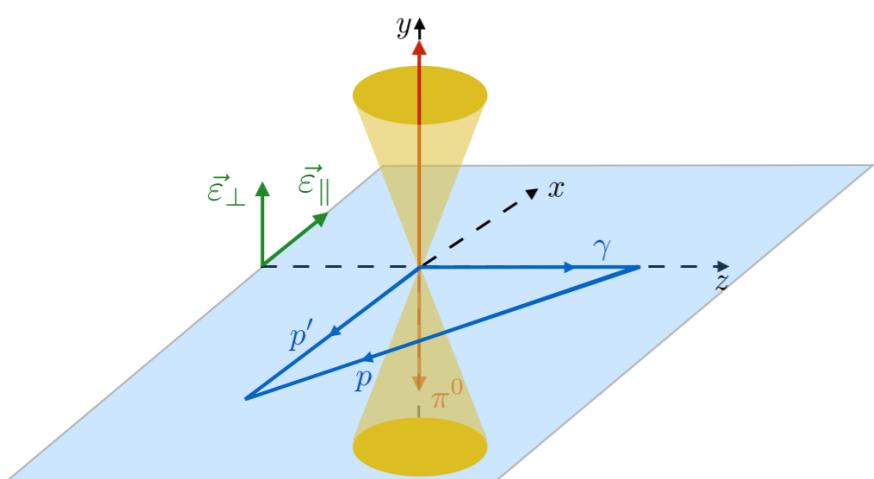
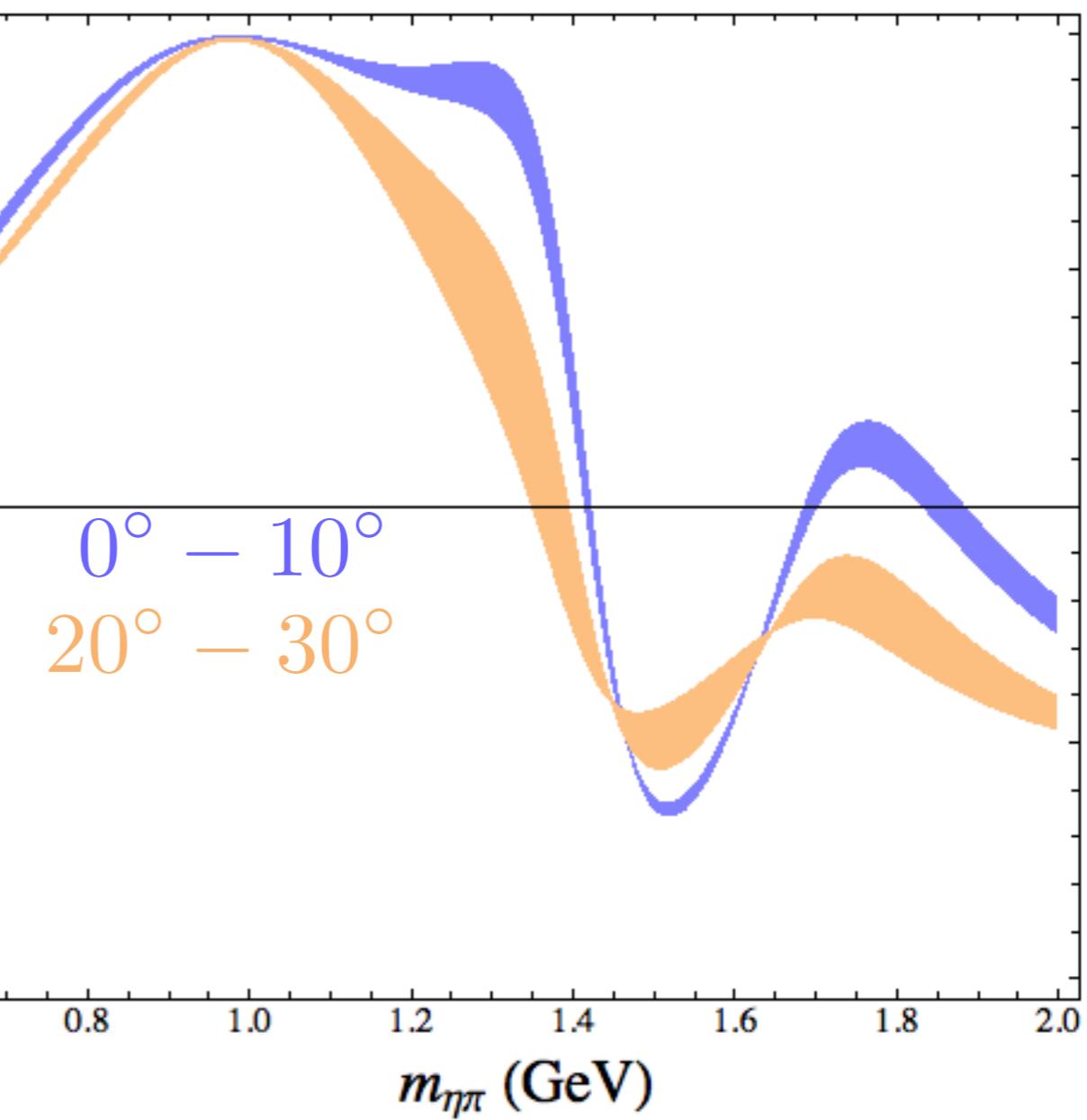
Beam Asymmetries: $\Sigma_{y \pm \delta^\circ}$

34

only S and D waves

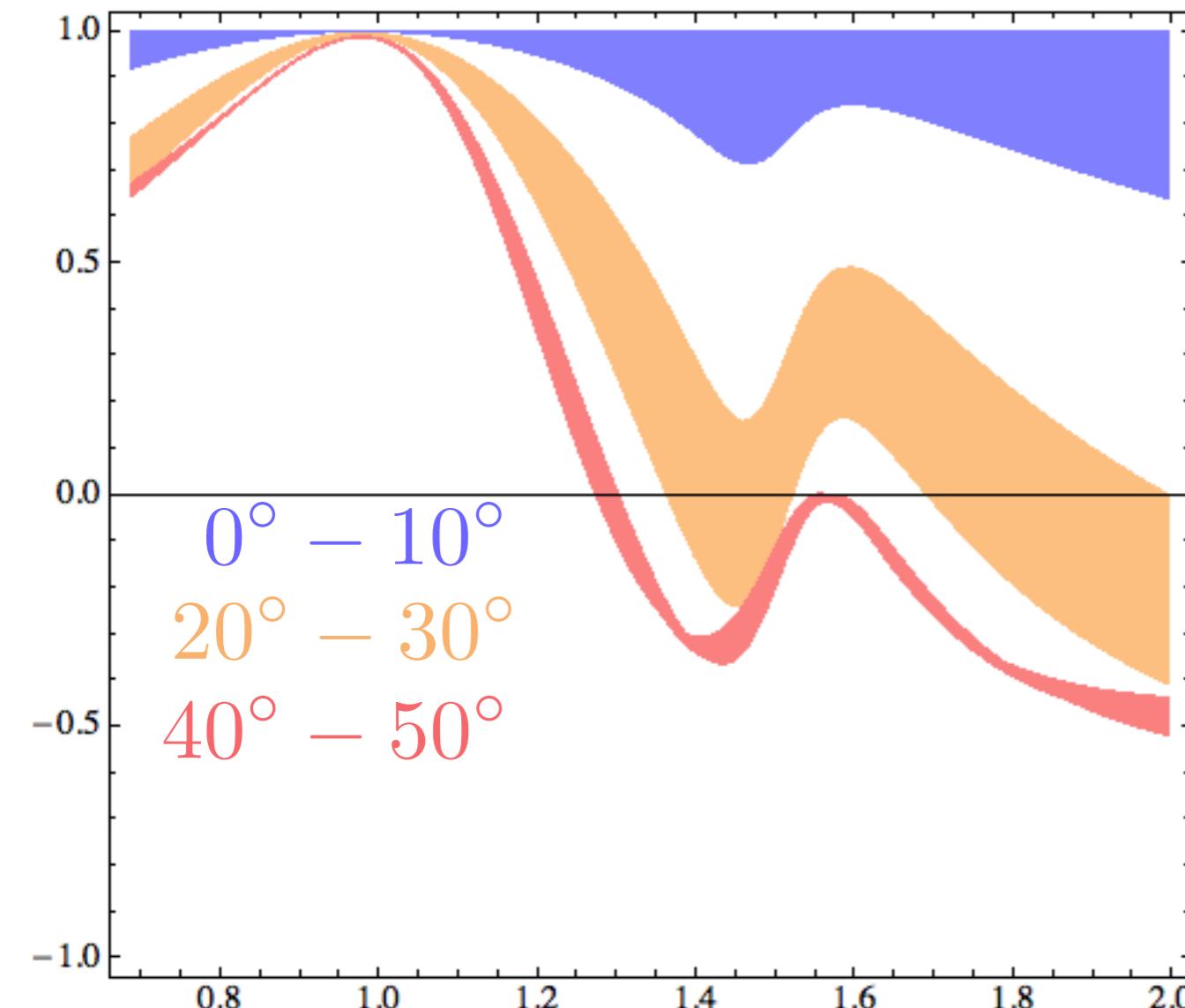


S, P and D waves

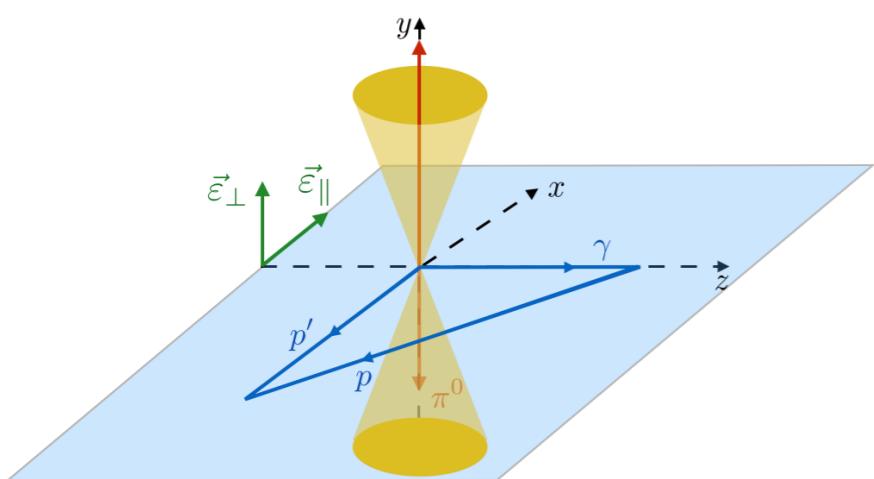
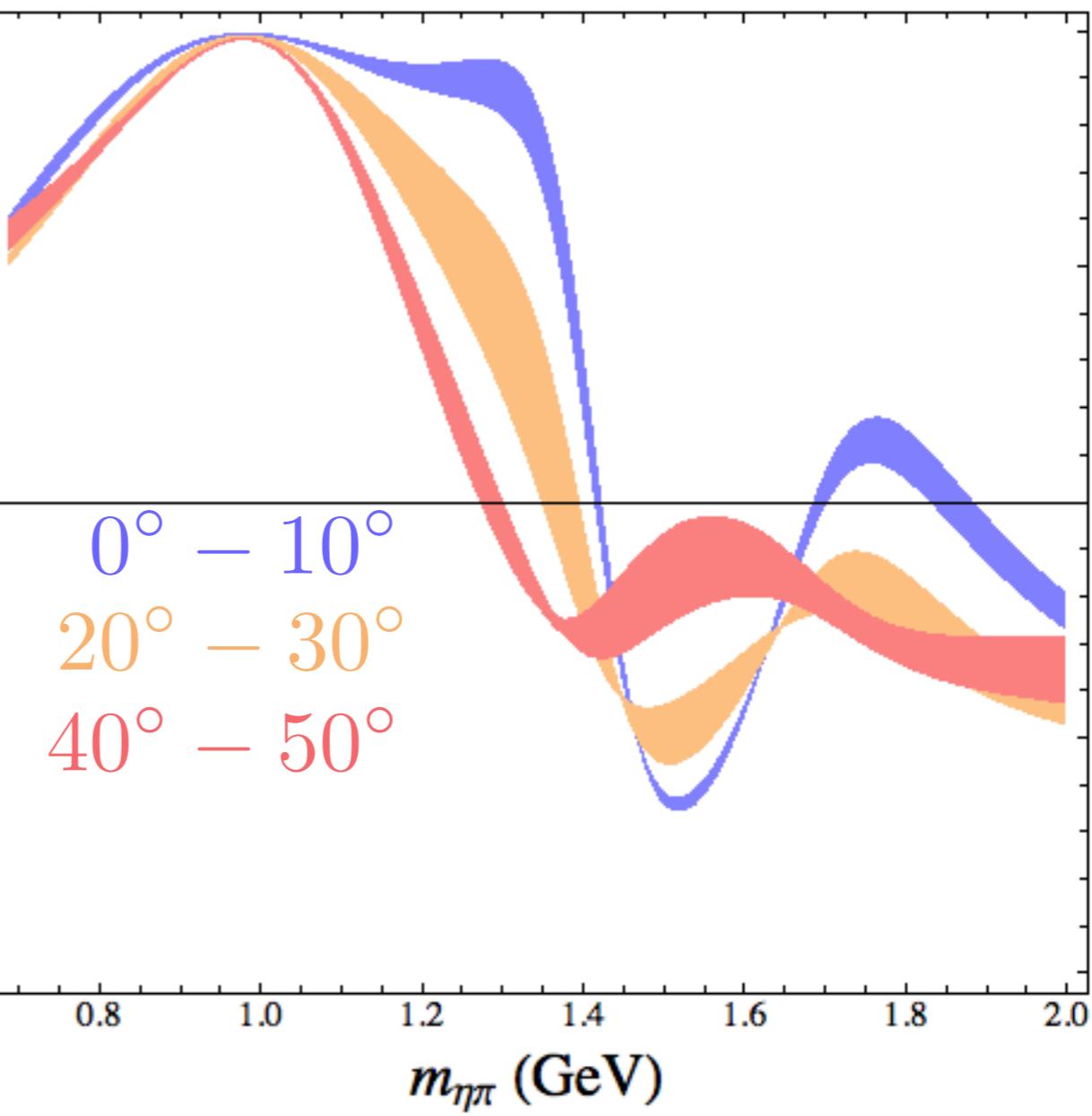


Beam Asymmetries: $\Sigma_{y \pm \delta^\circ}$

only S and D waves



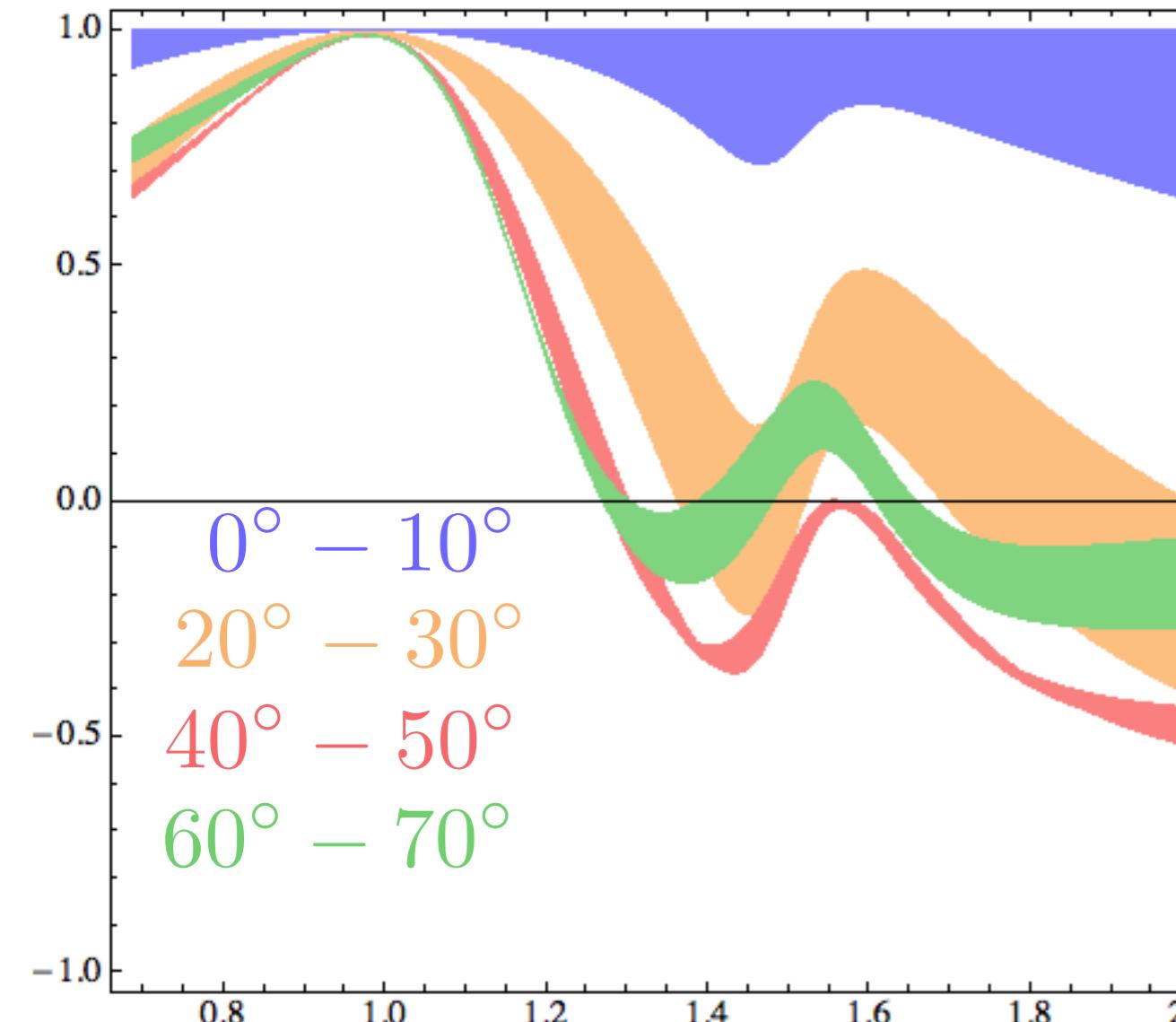
S, P and D waves



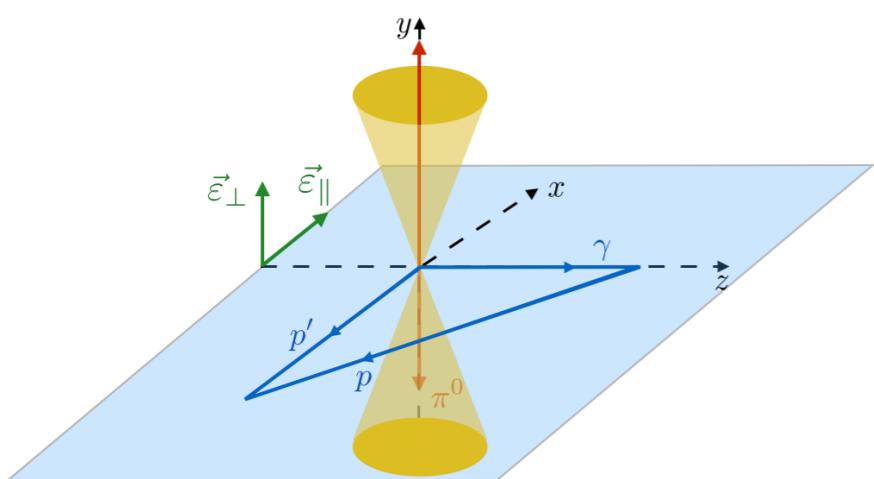
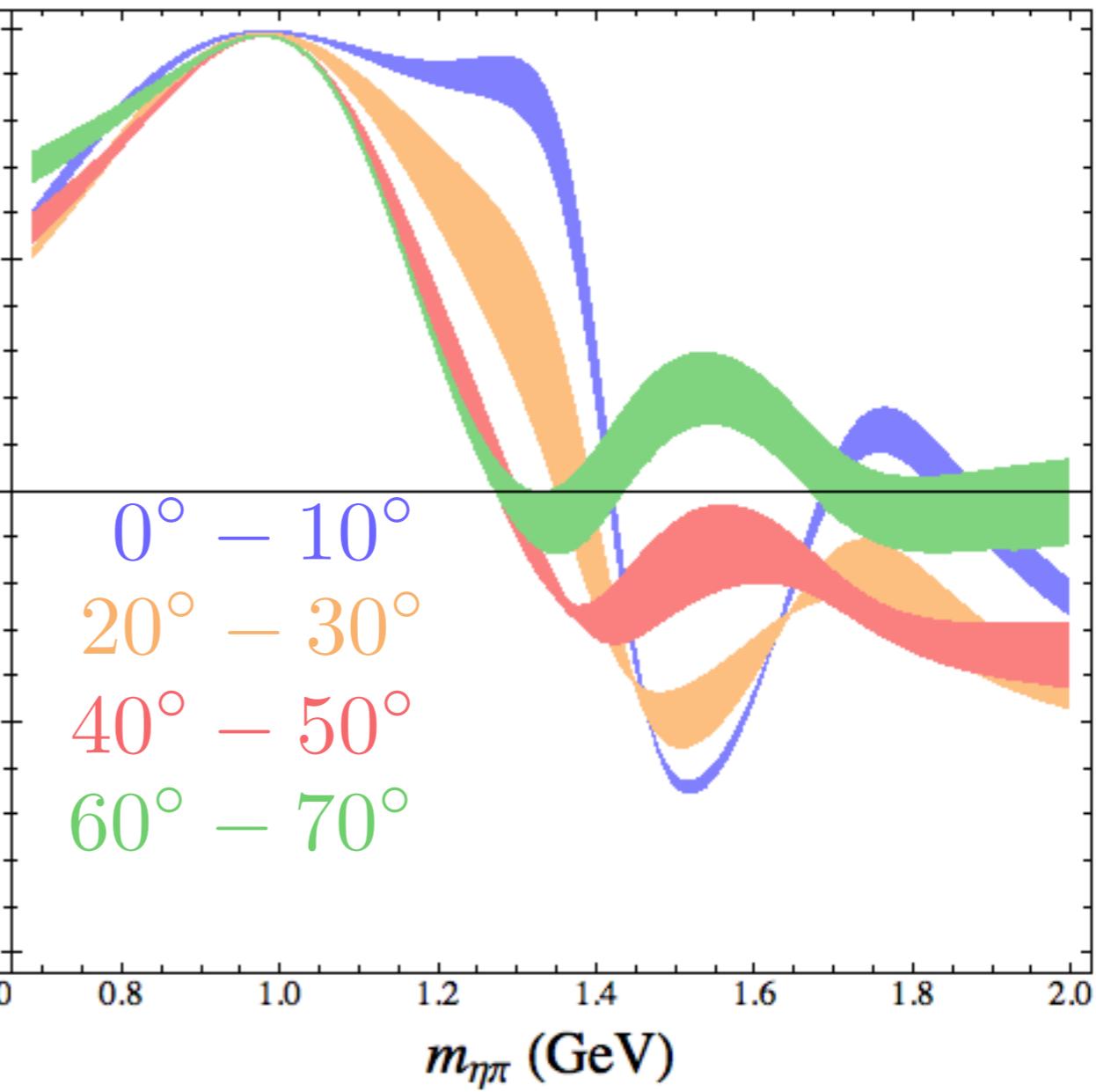
**with an opening angle greater than 30°
the observables is not sensitive to the P-wave
(with our model)**

Beam Asymmetries: $\Sigma_{y \pm \delta^\circ}$

only S and D waves



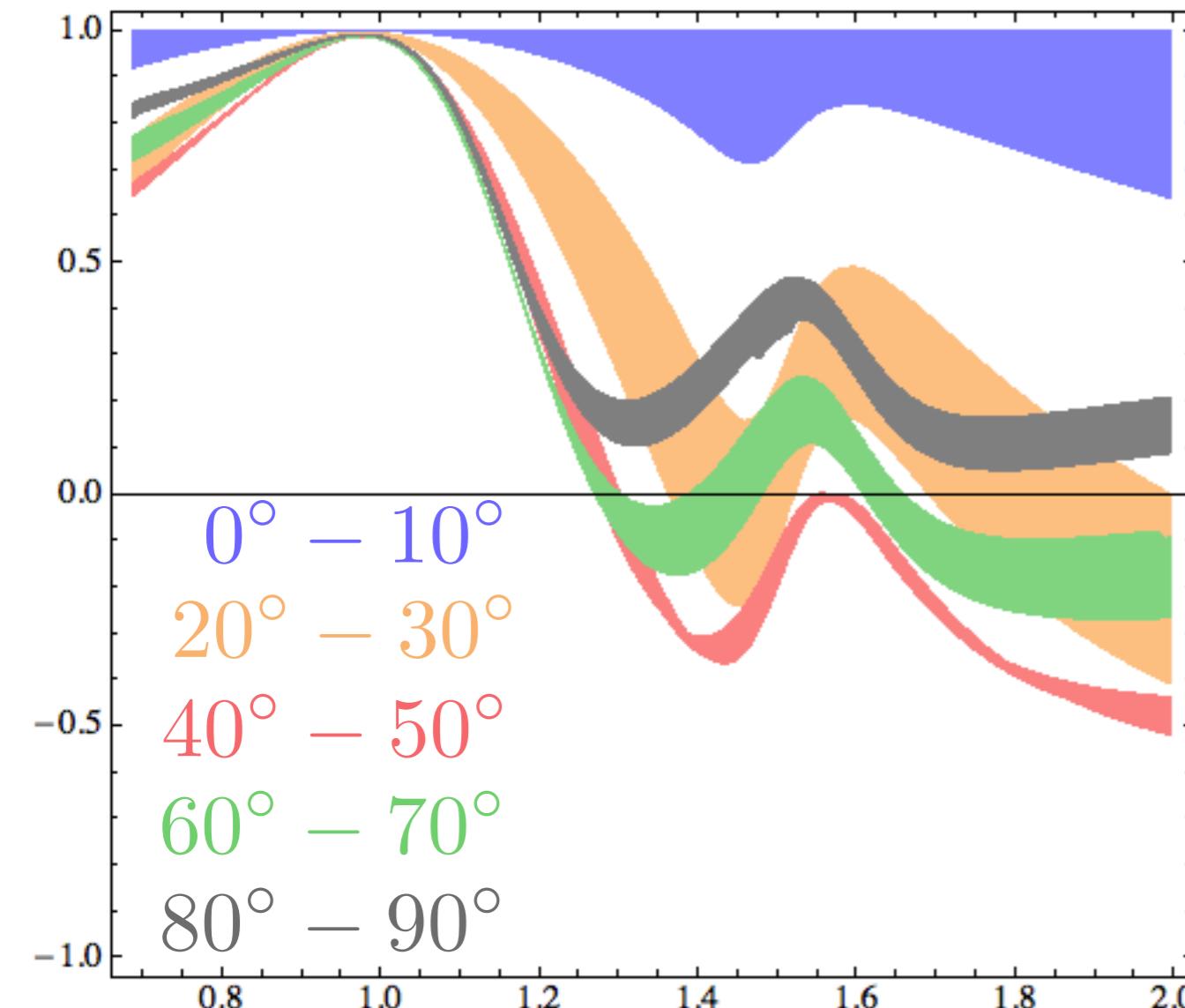
S, P and D waves



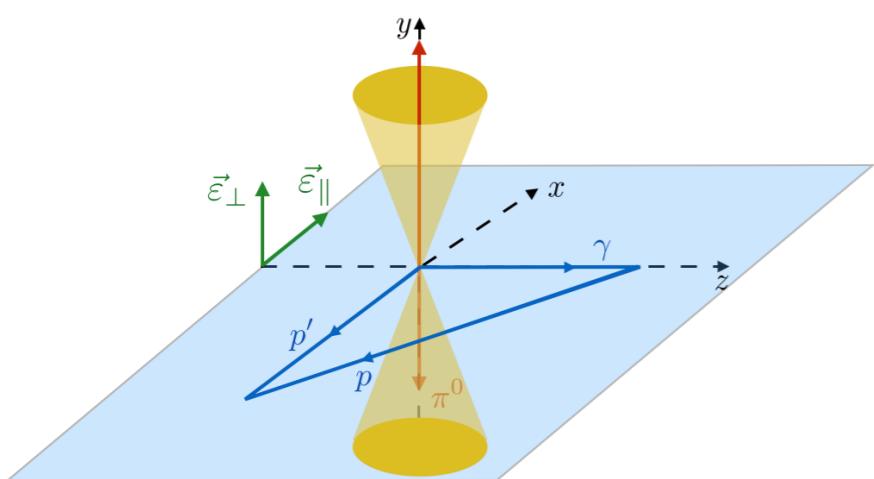
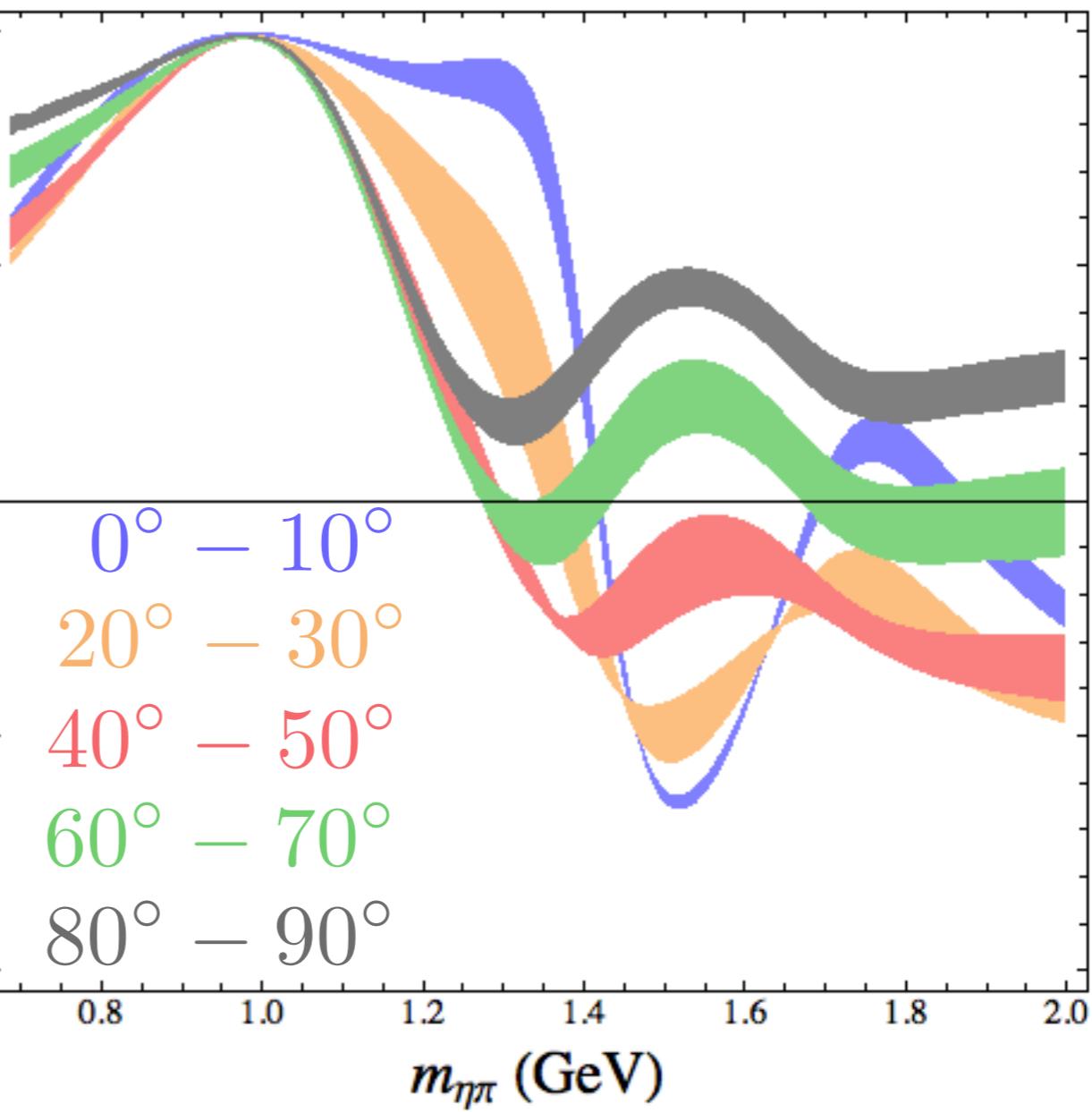
**with an opening angle greater than 30°
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Beam Asymmetries: $\Sigma_{y \pm \delta^\circ}$

only S and D waves



S, P and D waves



**with an opening angle greater than 30°
the observables is not sensitive to the P-wave
(with our model)**

Observables: Moments of Angular distribution

$$H^0(LM) = \frac{1}{2\pi} \int I(\Omega, \Phi) d_{M0}^L(\theta) \cos M\phi \, d\Omega d\Phi$$

$$H^1(LM) = \frac{-1}{\pi P_\gamma} \int I(\Omega, \Phi) \cos 2\Phi d_{M0}^L(\theta) \cos M\phi \, d\Omega d\Phi$$

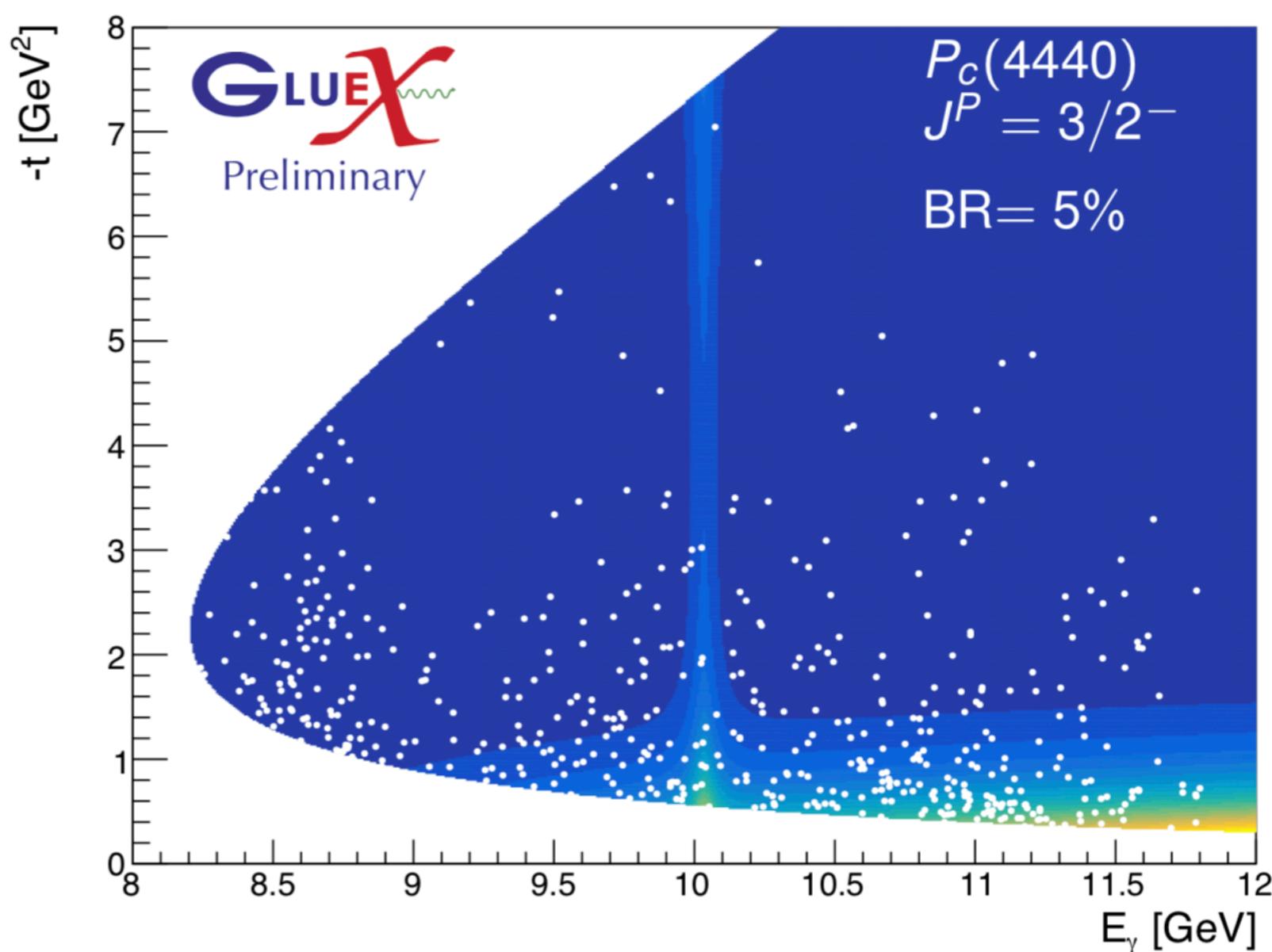
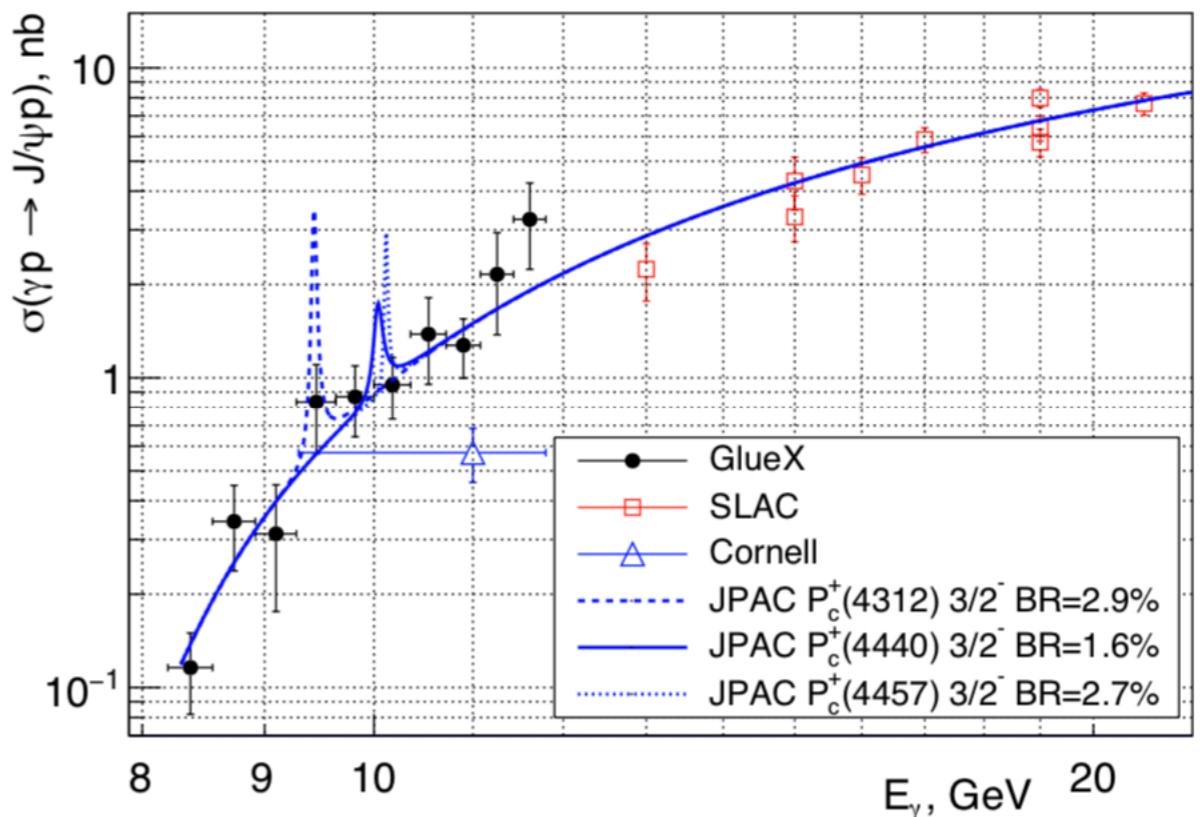
$$\text{Im } H^2(LM) = \frac{1}{\pi P_\gamma} \int I(\Omega, \Phi) \sin 2\Phi d_{M0}^L(\theta) \sin M\phi \, d\Omega d\Phi$$

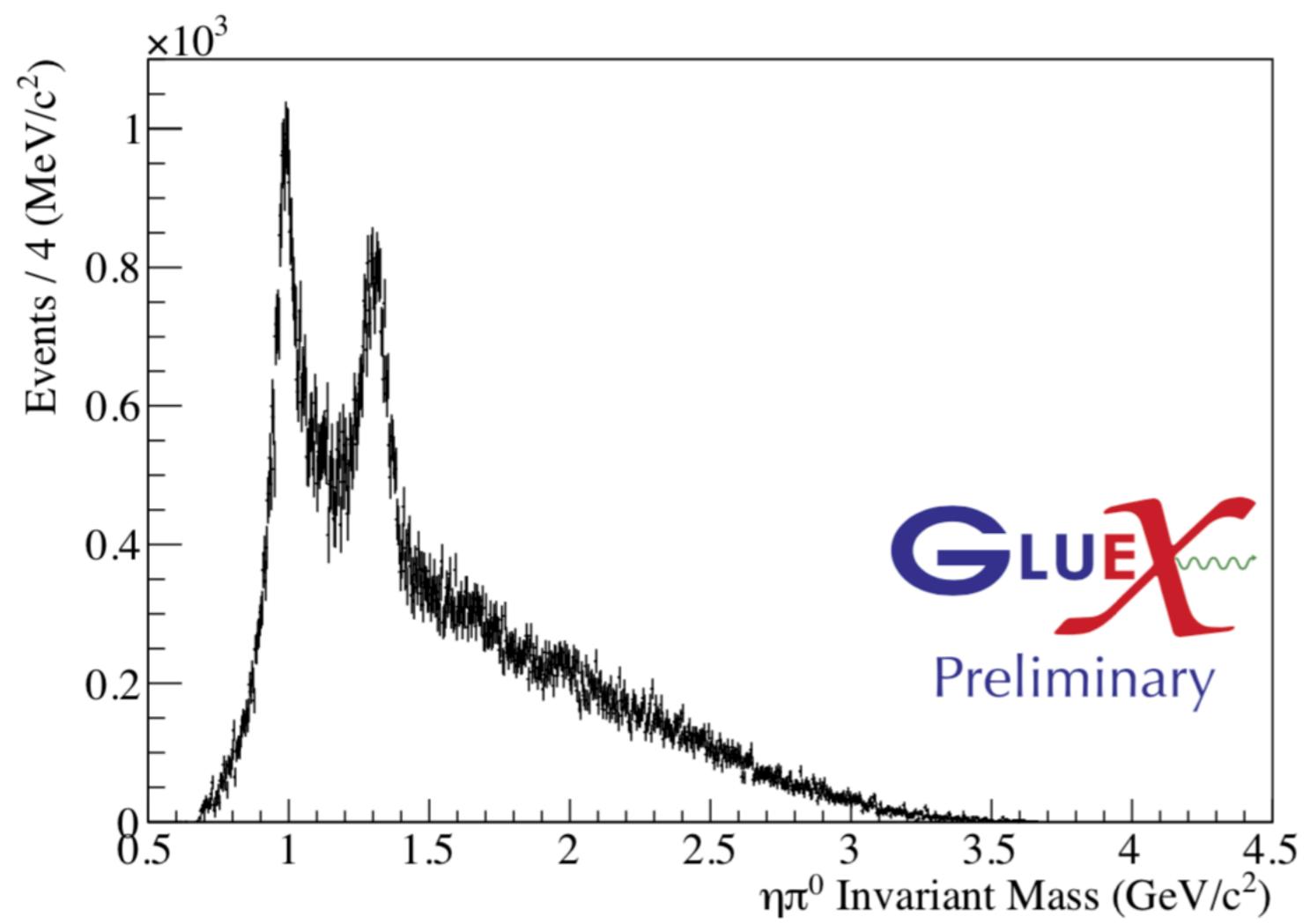
Moments are unambiguously extracted and are related to partial waves (interferences)

$$\begin{aligned} {}^{(+)}H^0(21) &= 2\text{Re} \left[\left(\frac{S_0^{(+)}}{\sqrt{5}} + \frac{D_0^{(+)}}{7} \right) (D_1^{(+)})^* - D_{-1}^{(+)})^* \right] + 2\frac{\sqrt{3}}{5} \text{Re} \left[P_0^{(+)} (P_1^{(+)})^* - P_{-1}^{(+)})^* \right] \\ &\quad + 2\frac{\sqrt{6}}{7} \text{Re} \left[D_1^{(+)} D_2^{(+)})^* - D_{-1}^{(+)} D_{-2}^{(+)})^* \right] \end{aligned}$$

$$\begin{aligned} {}^{(+)}H^1(21) &= 2\text{Re} \left[\left(\frac{S_0^{(+)}}{\sqrt{5}} + \frac{D_0^{(+)}}{7} \right) (D_1^{(+)})^* - D_{-1}^{(+)})^* \right] + 2\frac{\sqrt{3}}{5} \text{Re} \left[P_0^{(+)} (P_1^{(+)})^* - P_{-1}^{(+)})^* \right] \\ &\quad + 2\frac{\sqrt{6}}{7} \text{Re} \left[D_1^{(+)} D_{-2}^{(+)})^* + D_{-1}^{(+)} D_2^{(+)})^* \right] \end{aligned}$$

$$\begin{aligned} {}^{(+)}H^2(21) &= -2\text{Re} \left[\left(\frac{S_0^{(+)}}{\sqrt{5}} + \frac{D_0^{(+)}}{7} \right) (D_1^{(+)})^* + D_{-1}^{(+)})^* \right] - 2\frac{\sqrt{3}}{5} \text{Re} \left[P_0^{(+)} (P_1^{(+)})^* + P_{-1}^{(+)})^* \right] \\ &\quad + 2\frac{\sqrt{6}}{7} \text{Re} \left[D_1^{(+)} D_{-2}^{(+)})^* + D_{-1}^{(+)} D_2^{(+)})^* \right] \end{aligned}$$

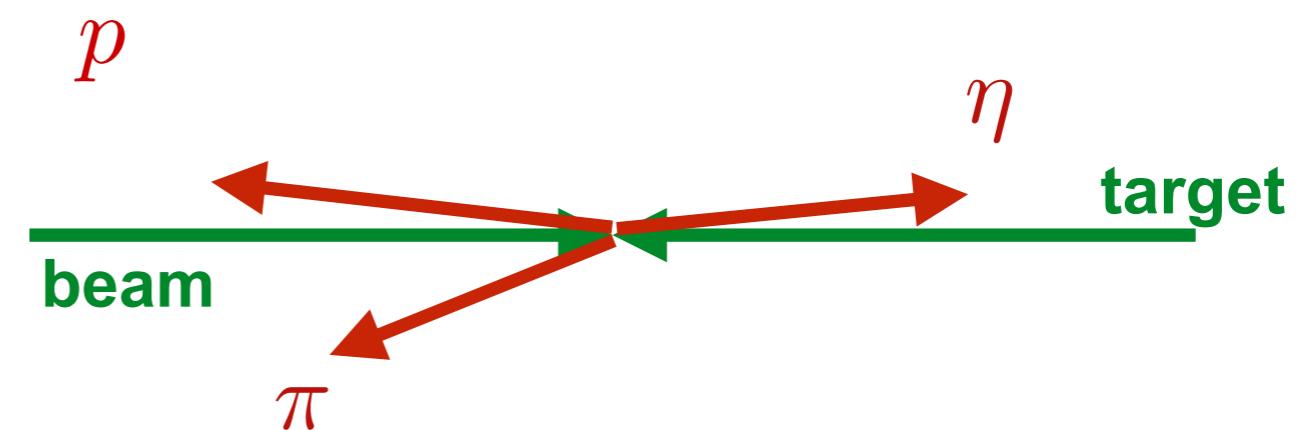
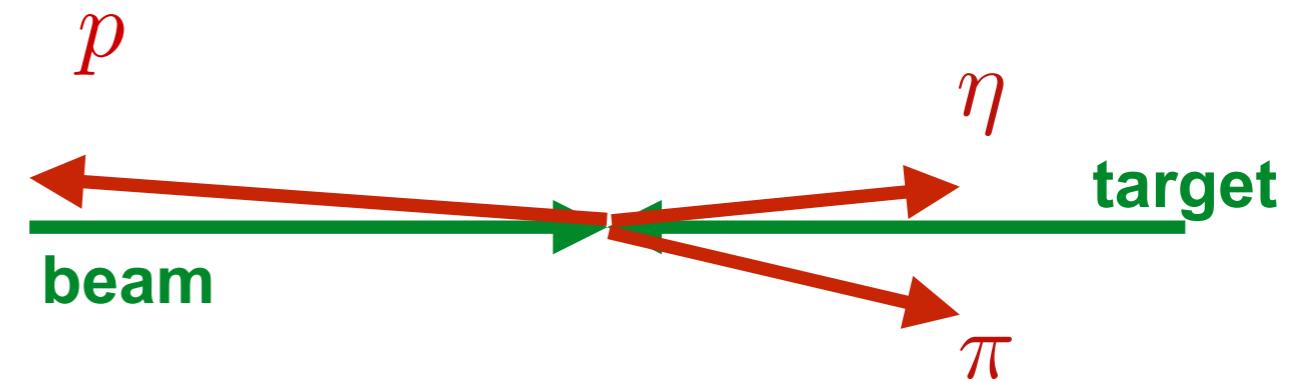
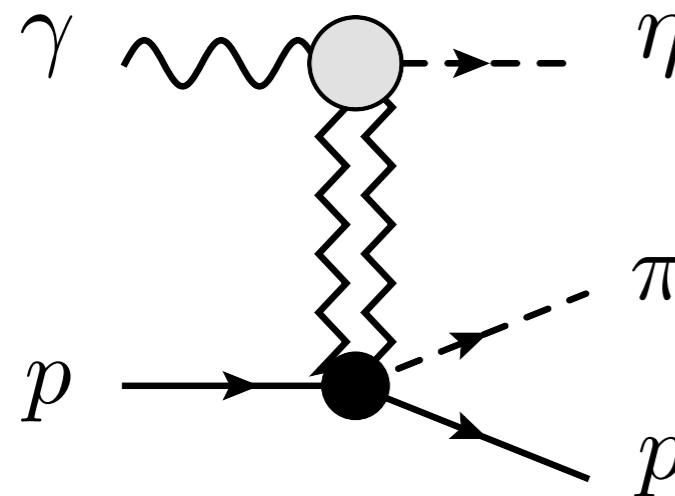
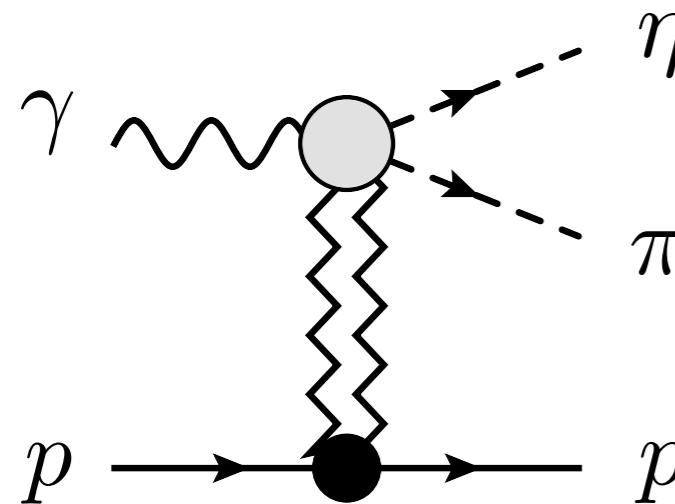




Longitudinal Plot

How do we select beam fragmentation ?

→ Boost in the rest frame



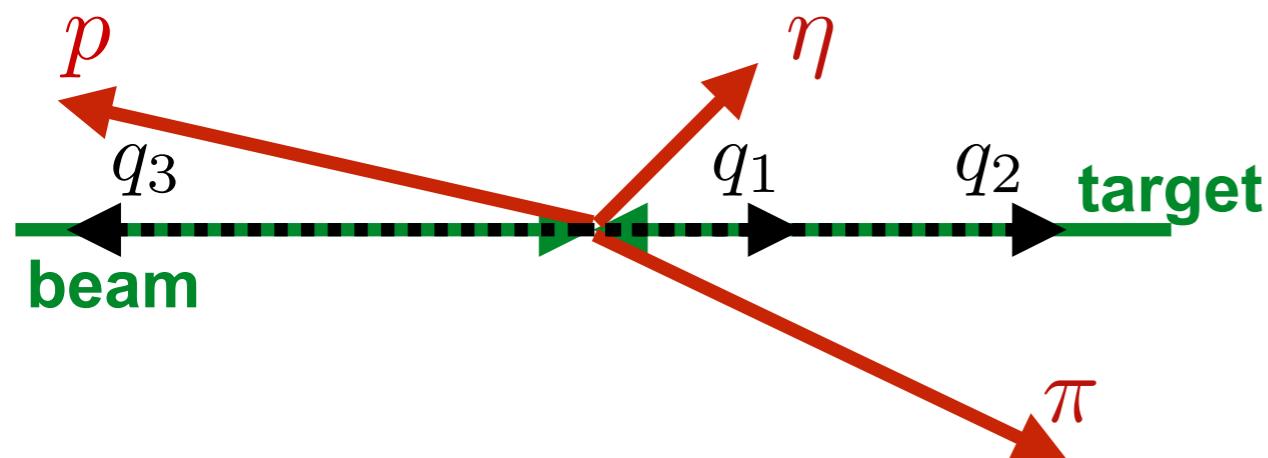
Van Hove NPB9 (1969) 331

Shi et al (JPAC) PRD91 (2015) 034007

Pauli et al PRD98 (2018) 065201

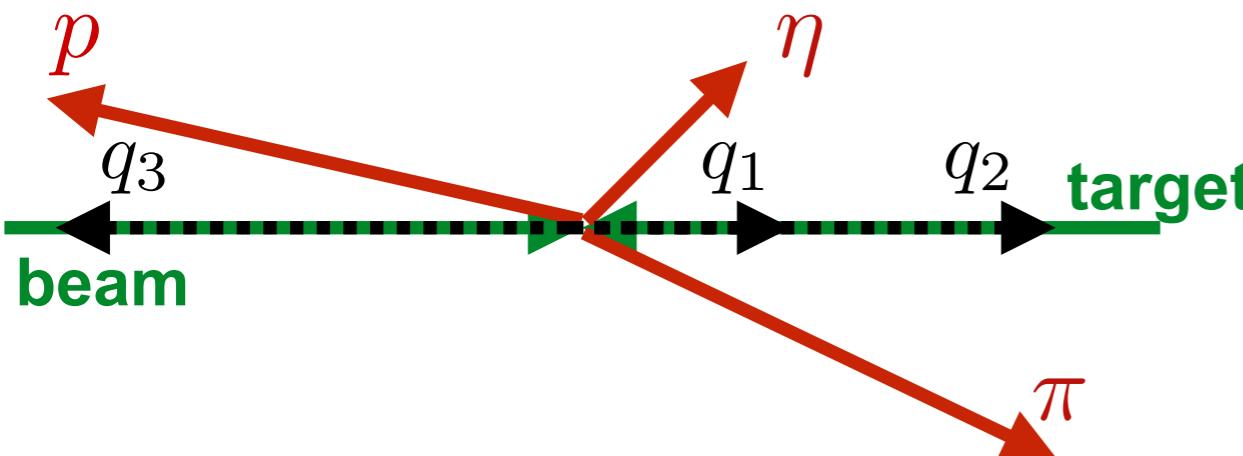
Longitudinal Plot

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only 2 variables since $q_1 + q_2 + q_3 = 0$

Longitudinal Plot



radius: $q^2 = q_1^2 + q_2^2 + q_3^2$

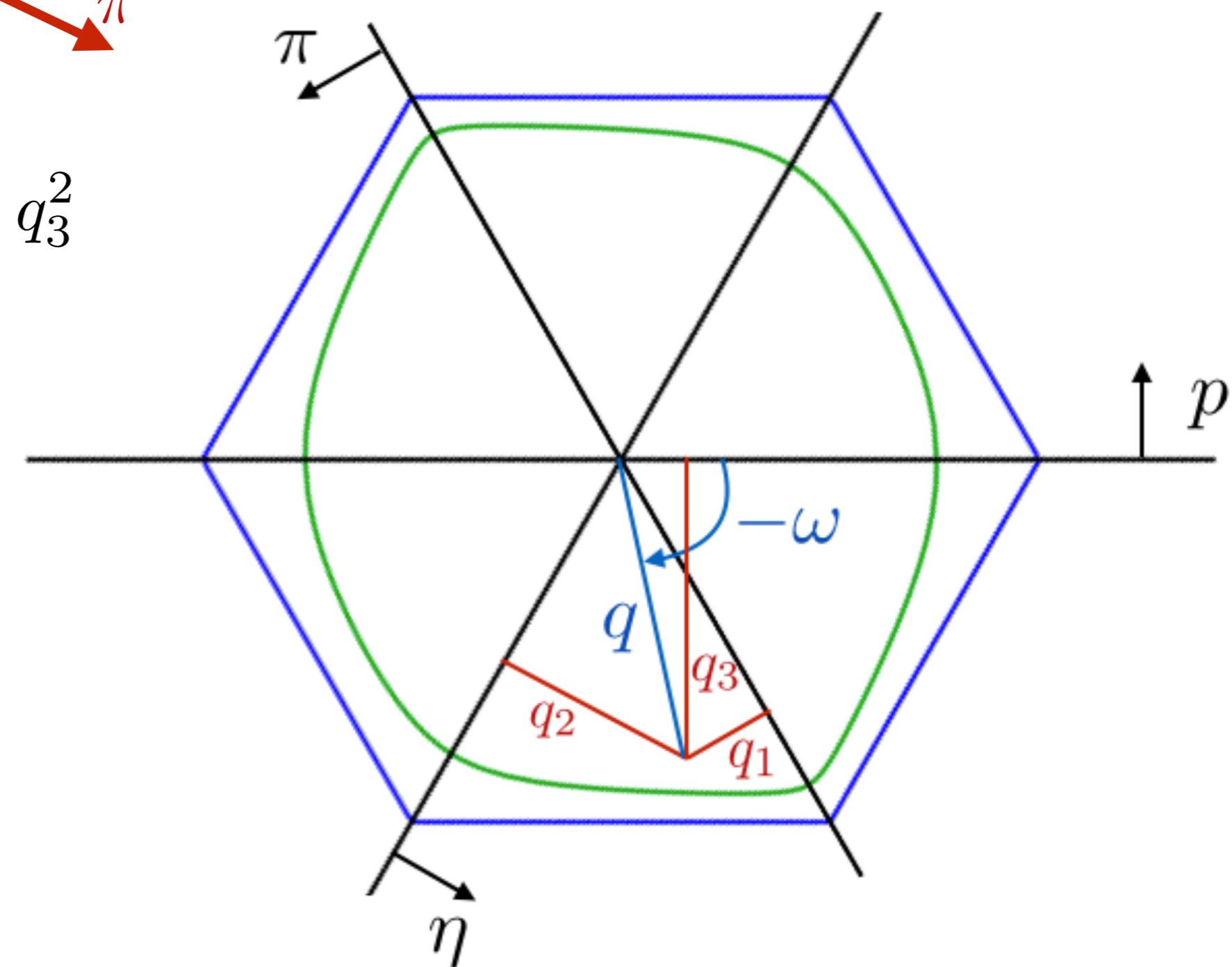
longitudinal angle: ω

$$q_3 = \sqrt{\frac{2}{3}}q \sin \omega$$

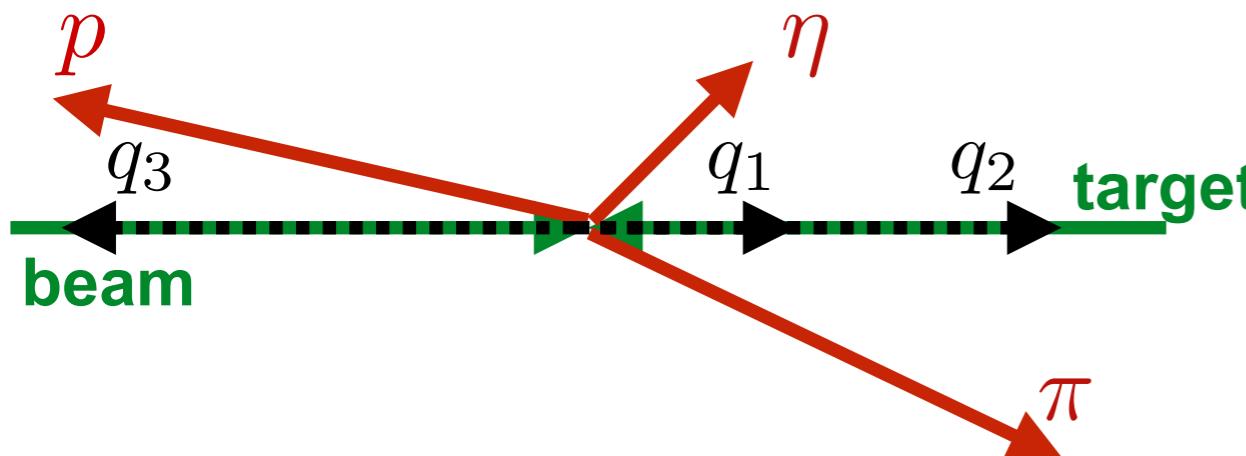
$$q_2 = \sqrt{\frac{2}{3}}q \sin \left(\omega + \frac{2\pi}{3} \right)$$

$$q_1 = \sqrt{\frac{2}{3}}q \sin \left(\omega + \frac{4\pi}{3} \right)$$

only 2 variables since $q_1 + q_2 + q_3 = 0$



Longitudinal Plot



only 2 variables since $q_1 + q_2 + q_3 = 0$

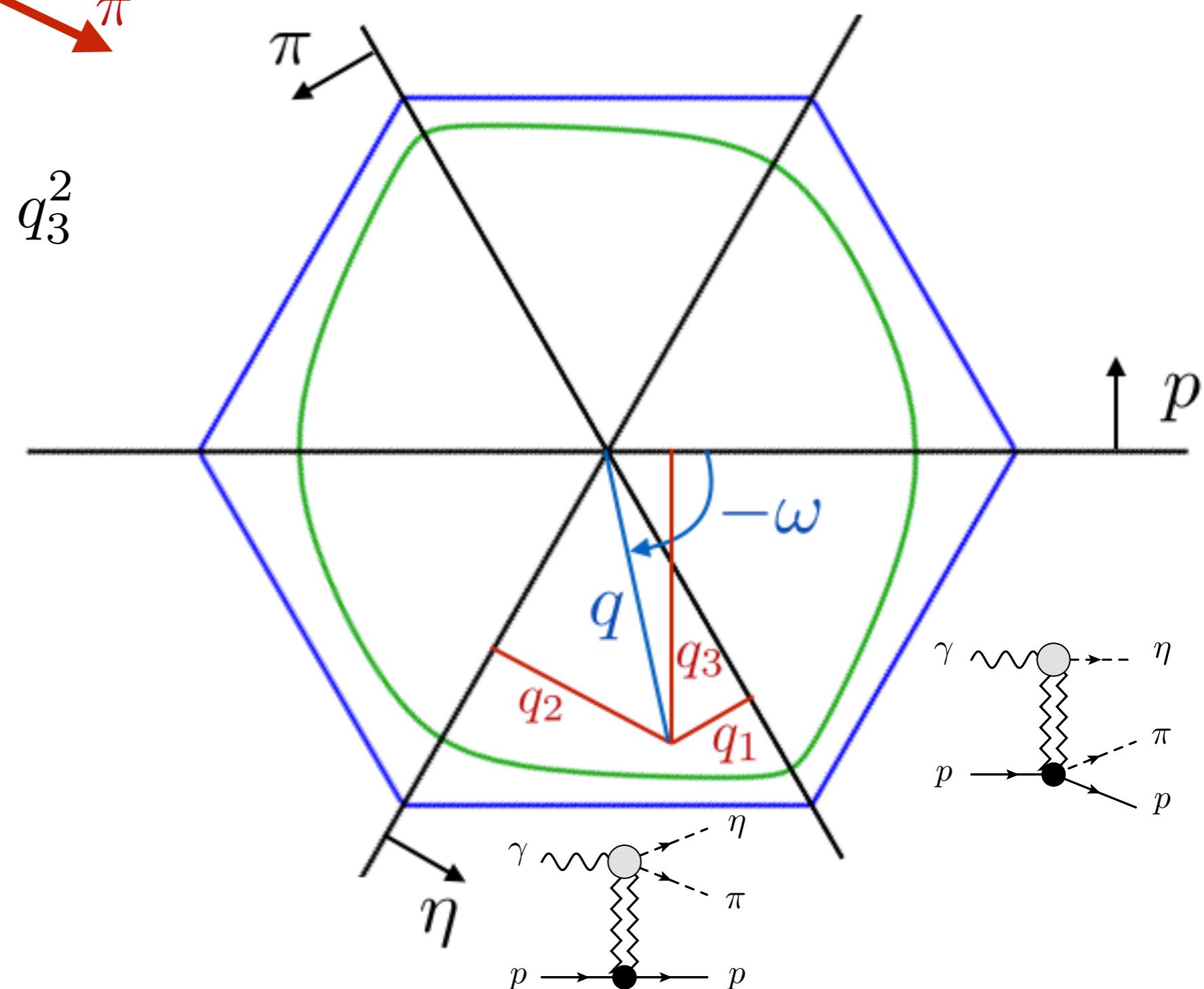
$$\text{radius: } q^2 = q_1^2 + q_2^2 + q_3^2$$

longitudinal angle: ω

$$q_3 = \sqrt{\frac{2}{3}}q \sin \omega$$

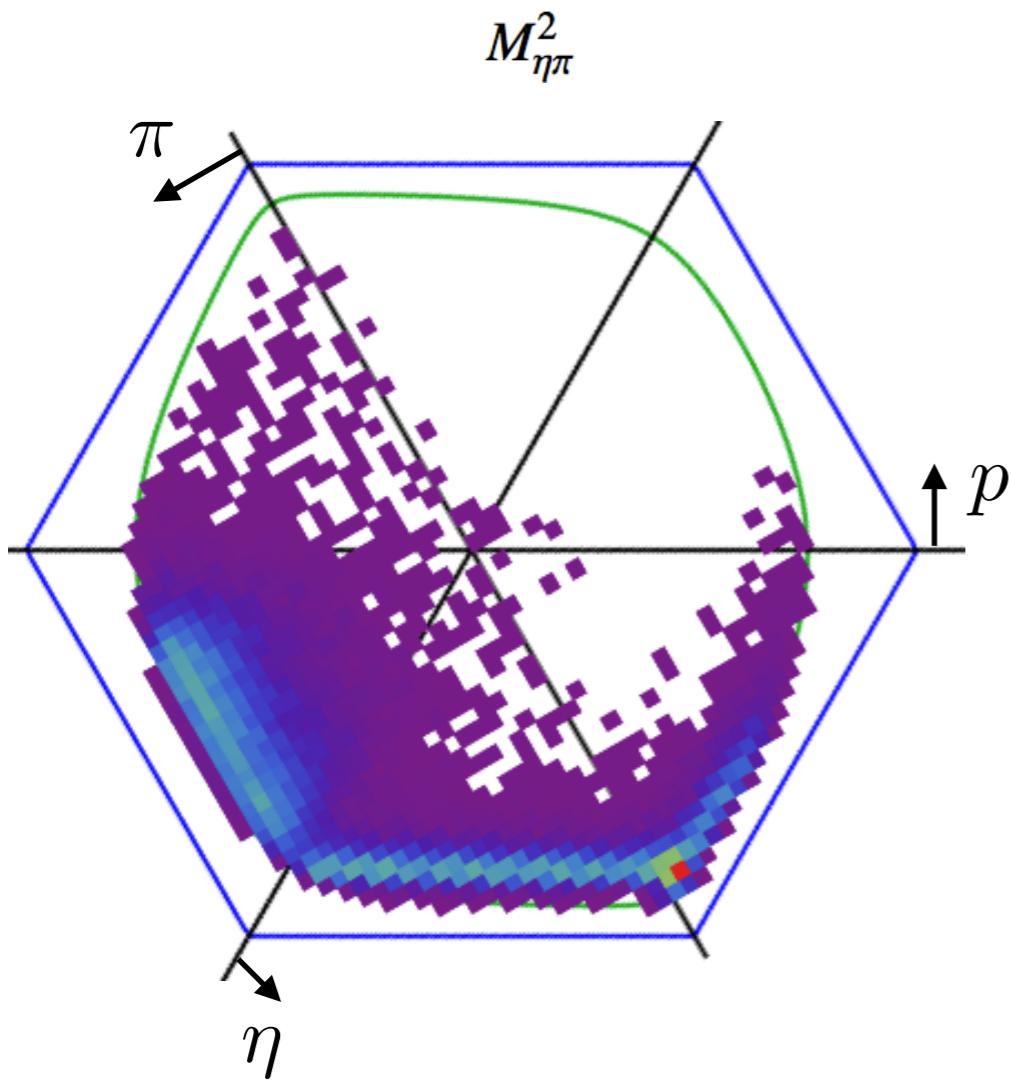
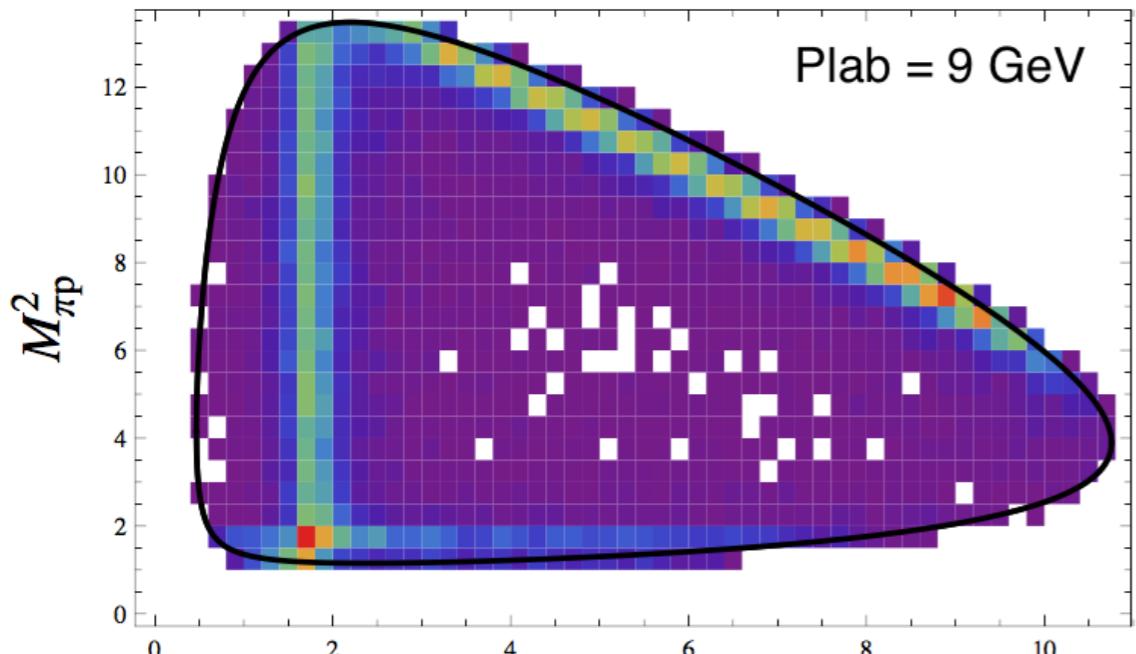
$$q_2 = \sqrt{\frac{2}{3}}q \sin \left(\omega + \frac{2\pi}{3} \right)$$

$$q_1 = \sqrt{\frac{2}{3}}q \sin \left(\omega + \frac{4\pi}{3} \right)$$



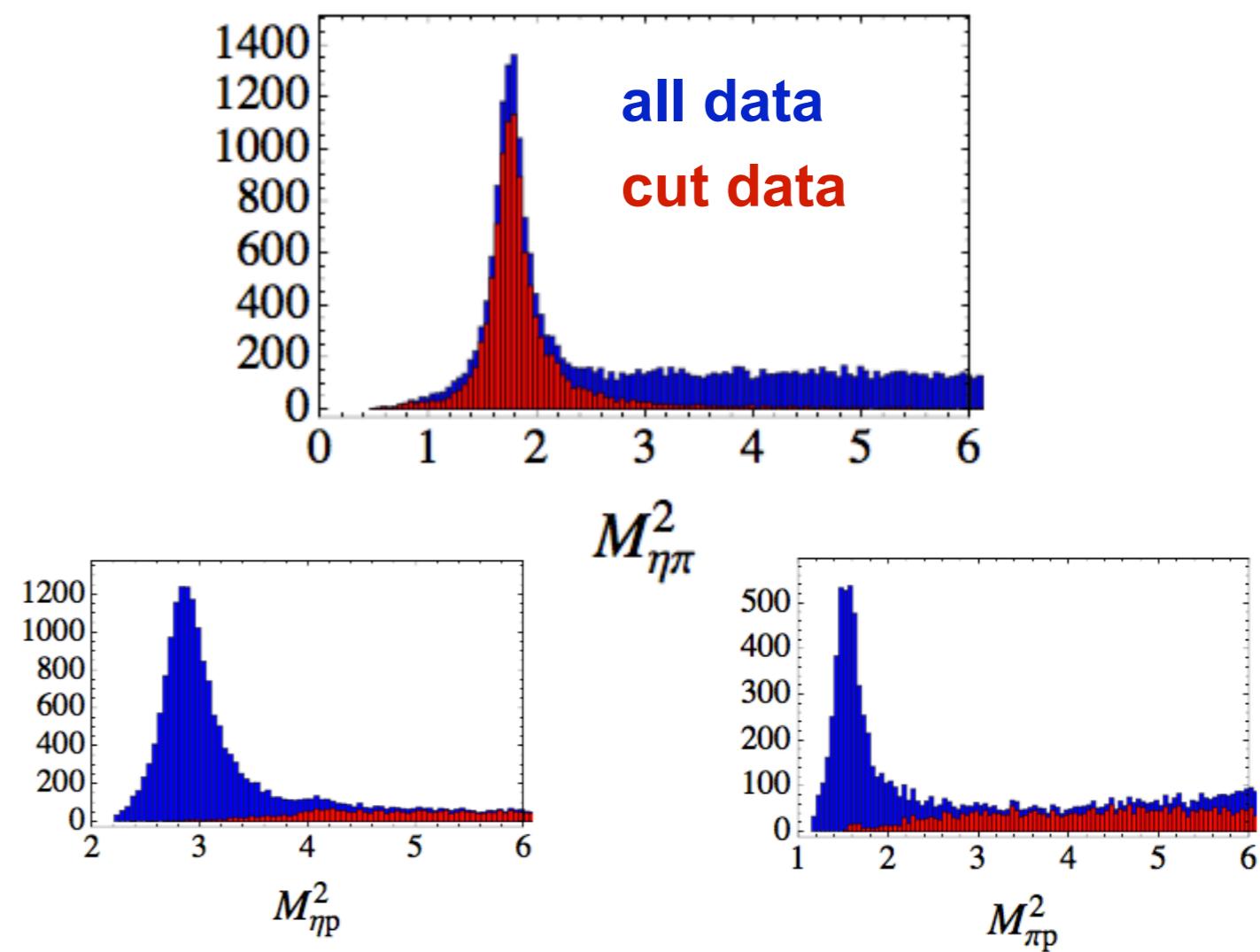
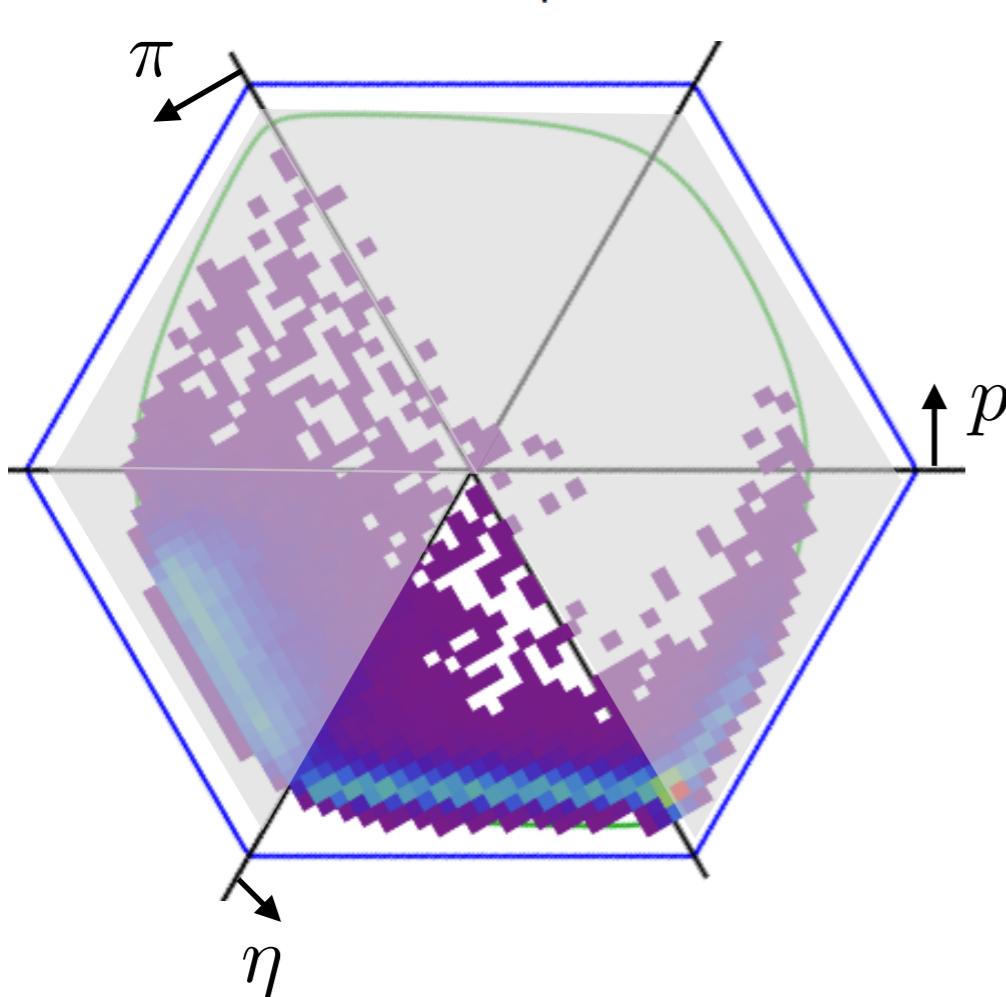
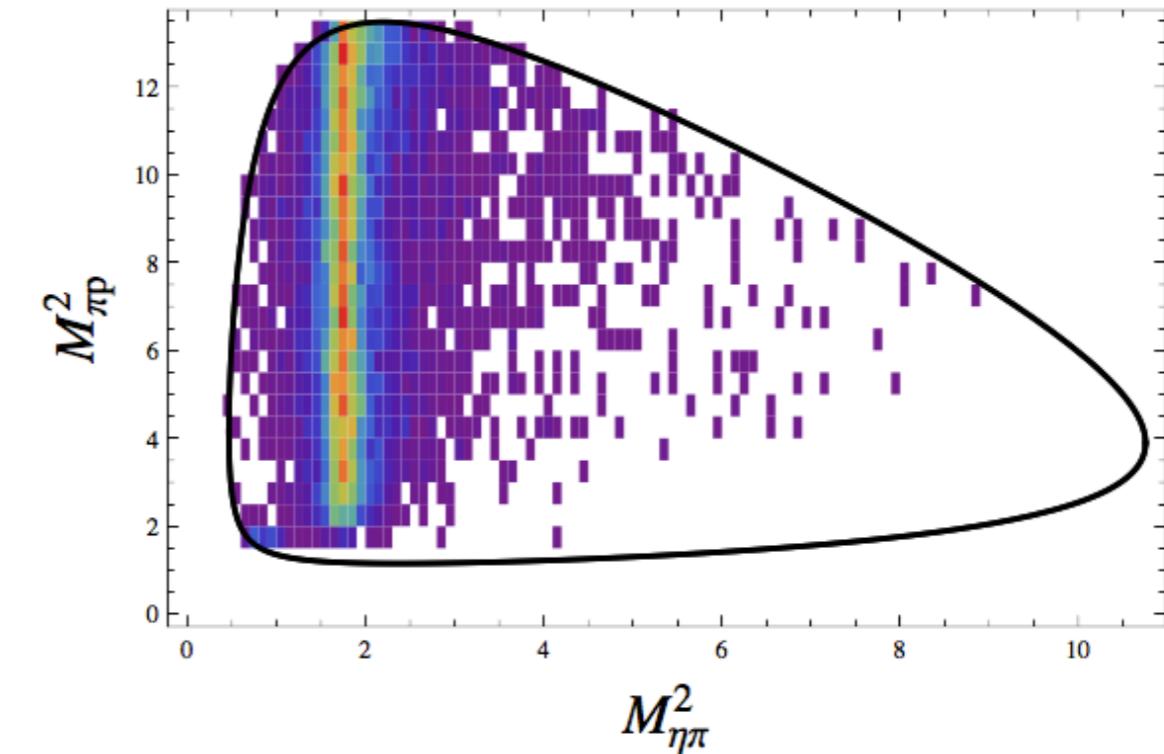
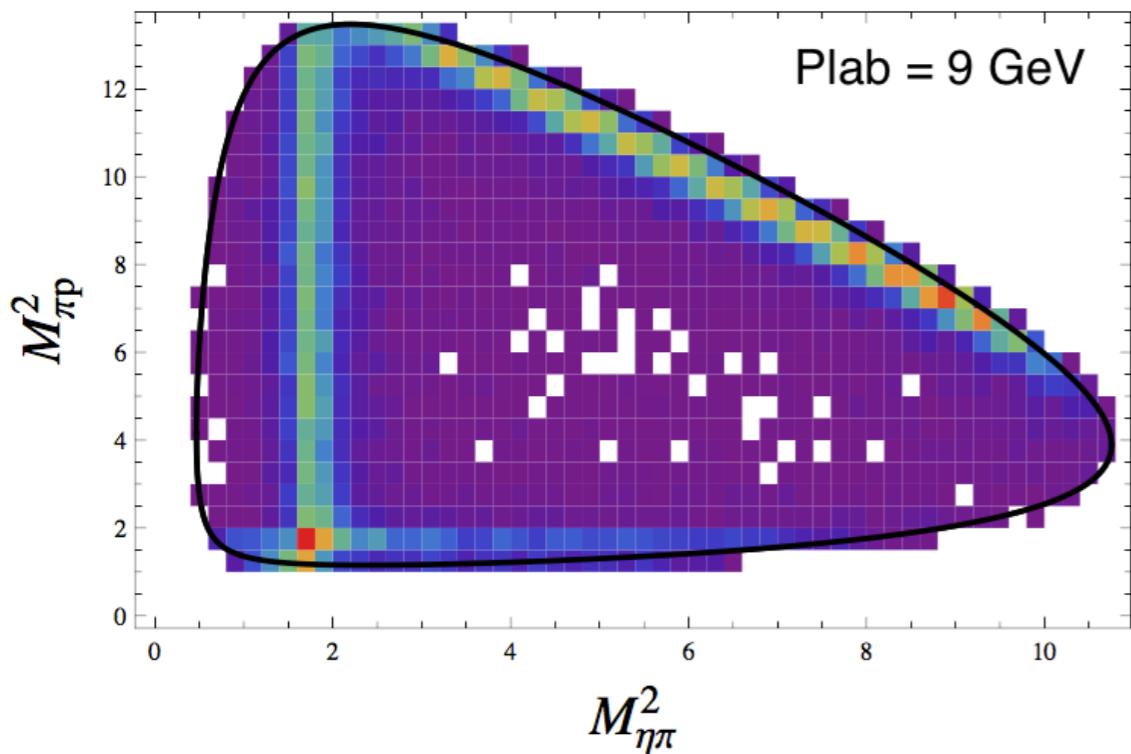
Cut in Longitudinal Angle

40



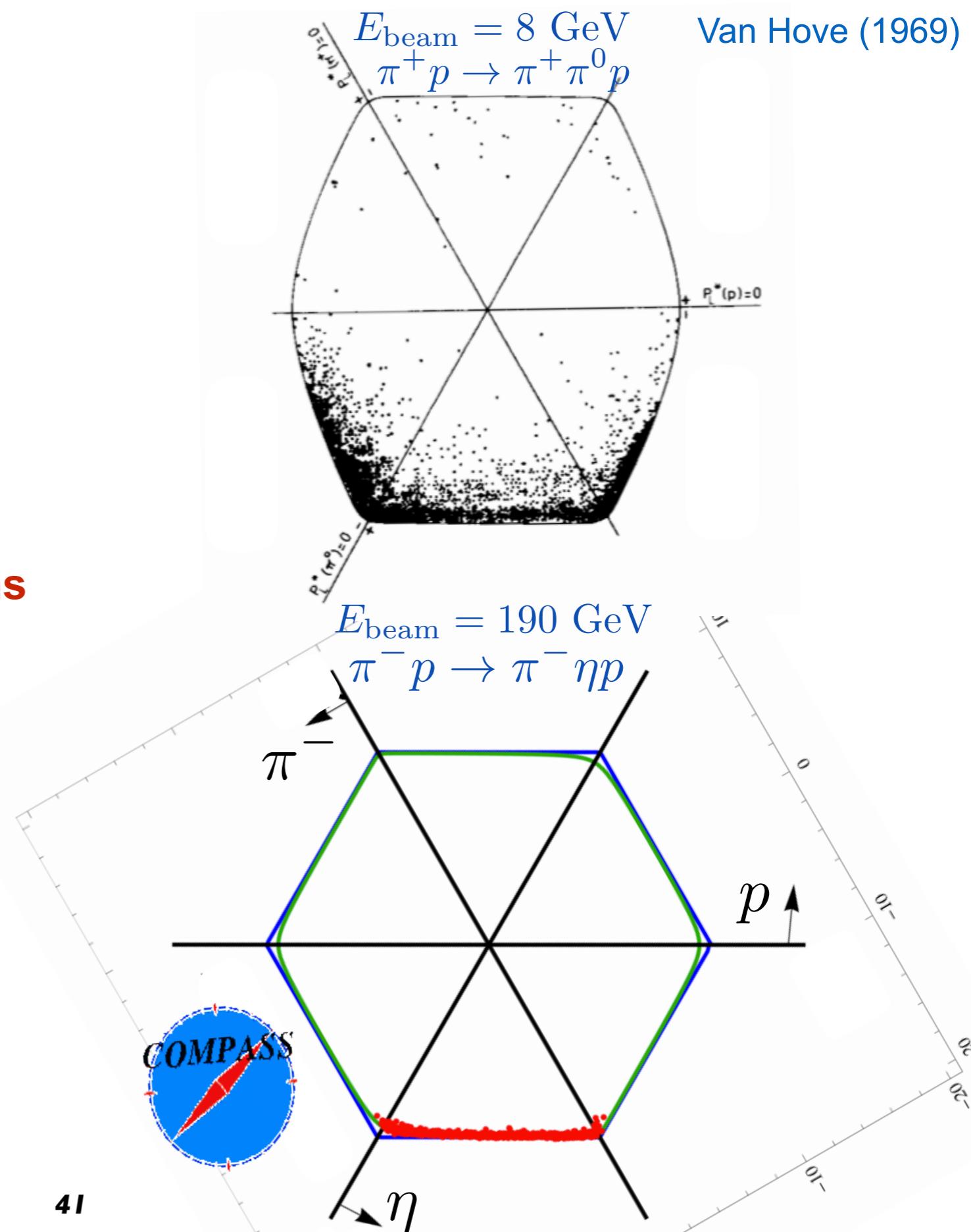
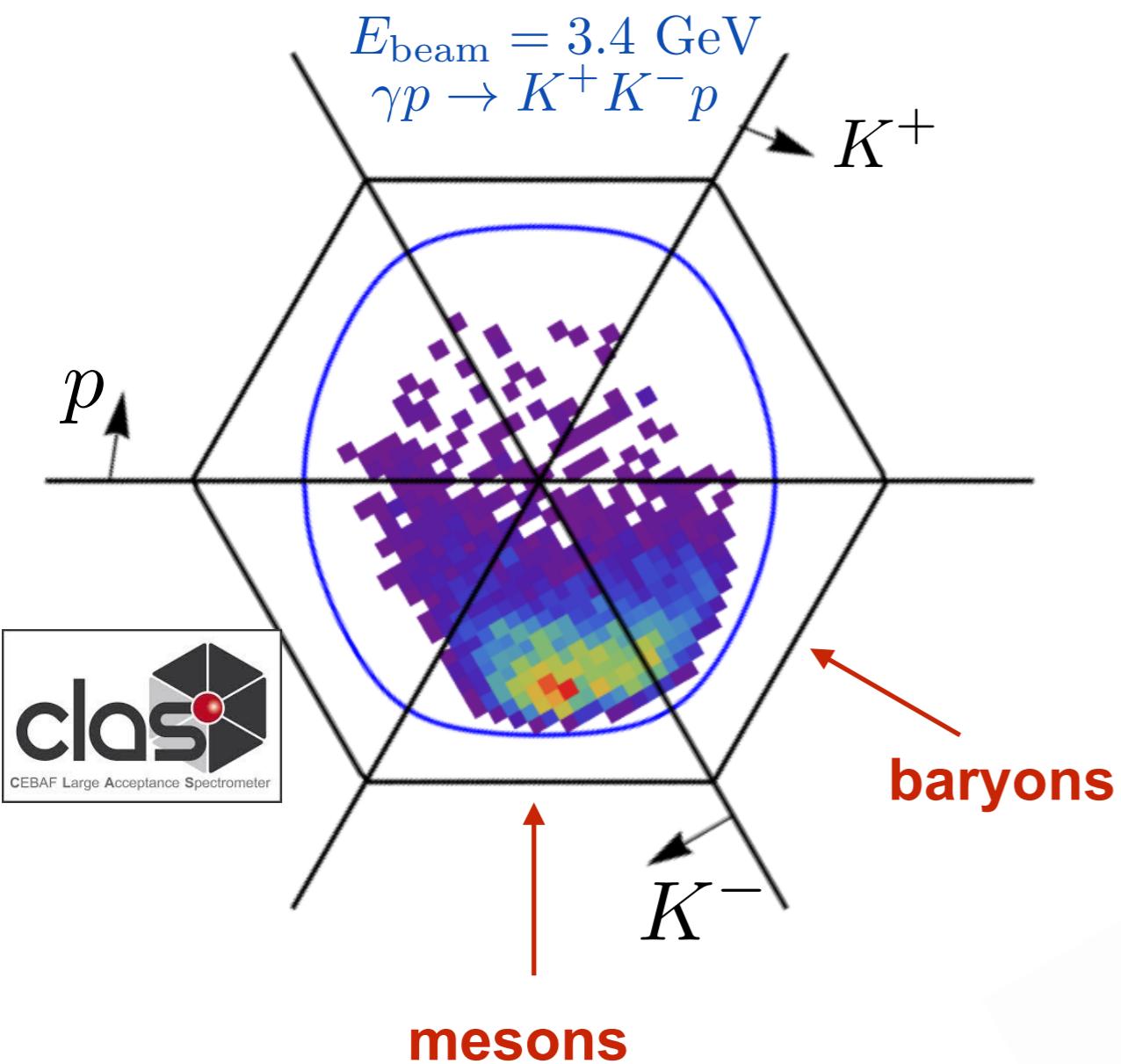
Cut in Longitudinal Angle

40



Longitudinal Plot: Energy Evolution

41

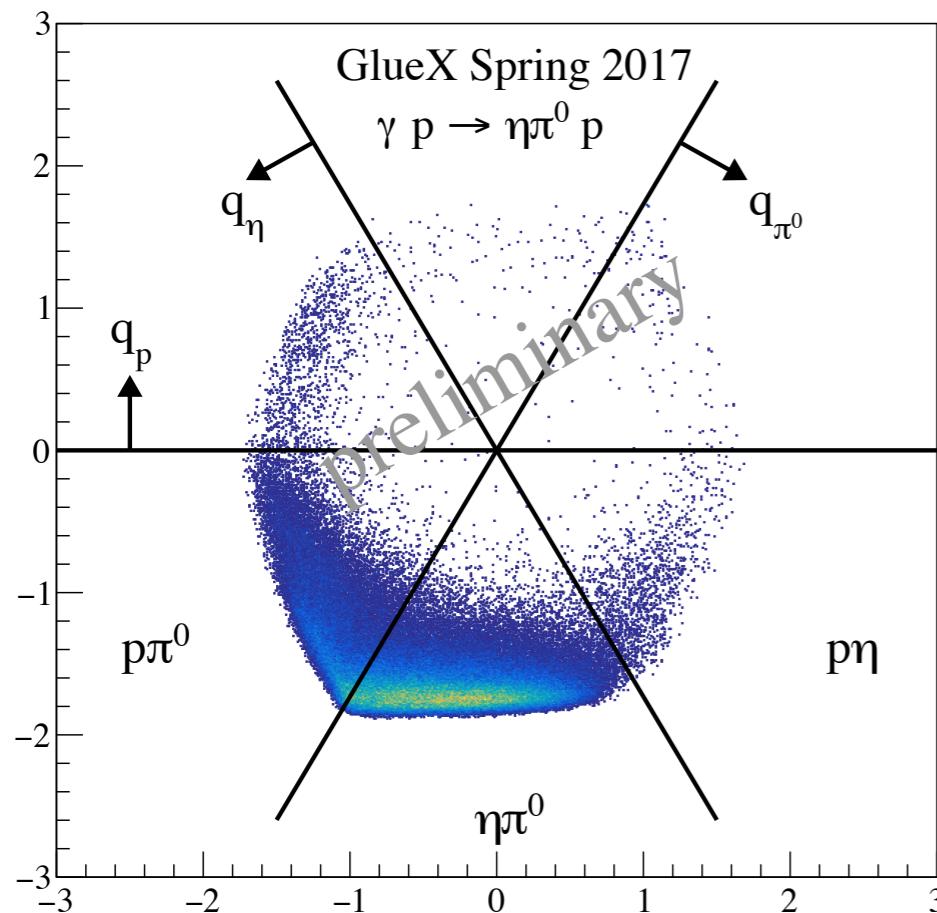


GlueX Preliminary Results

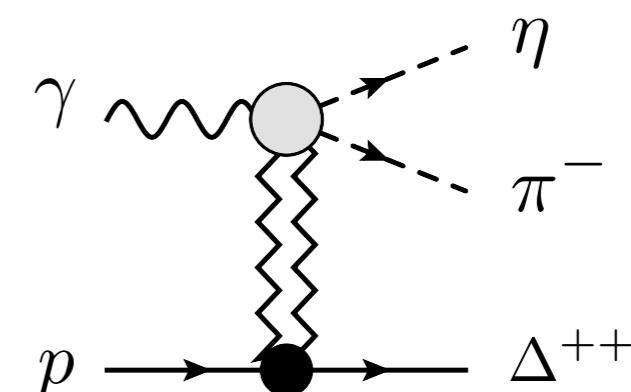
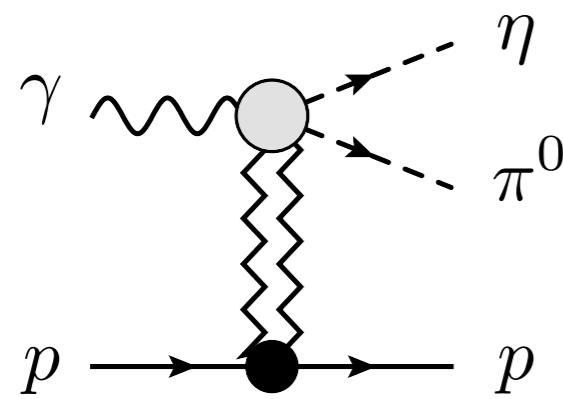
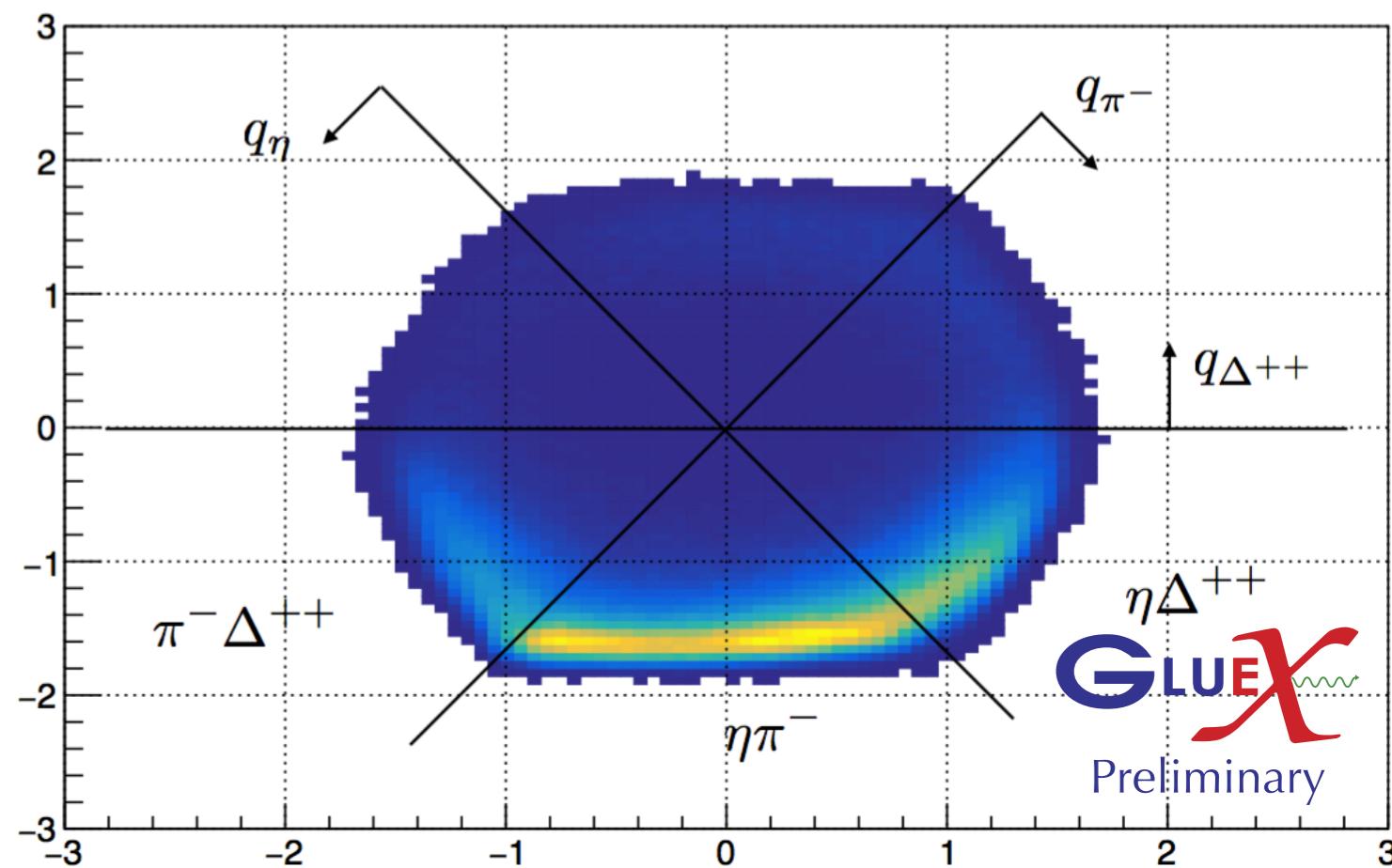
42

not corrected for acceptance

$$\gamma p \rightarrow \eta \pi^0 p$$



$$\gamma p \rightarrow \eta \pi^- \Delta^{++}$$



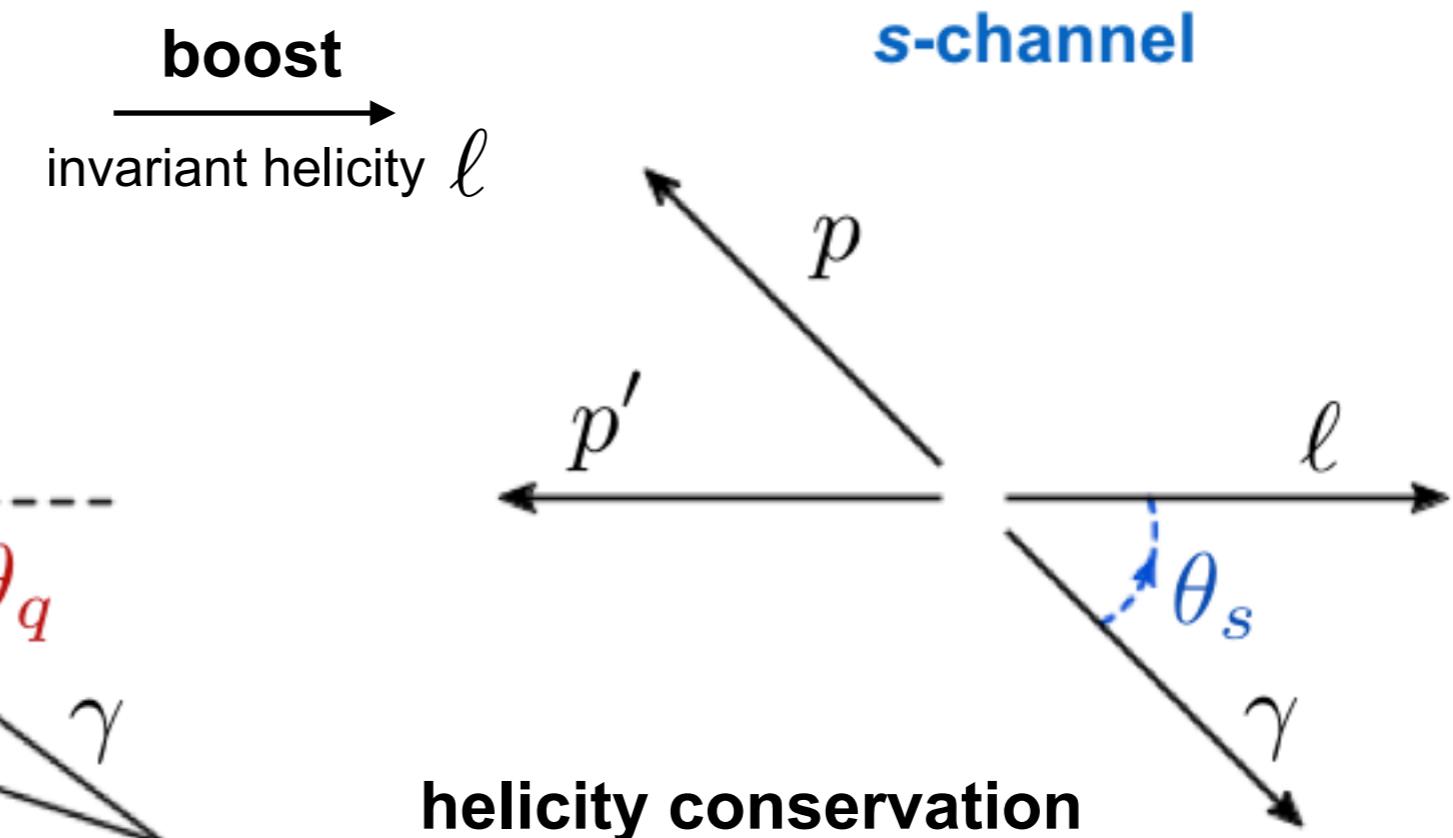
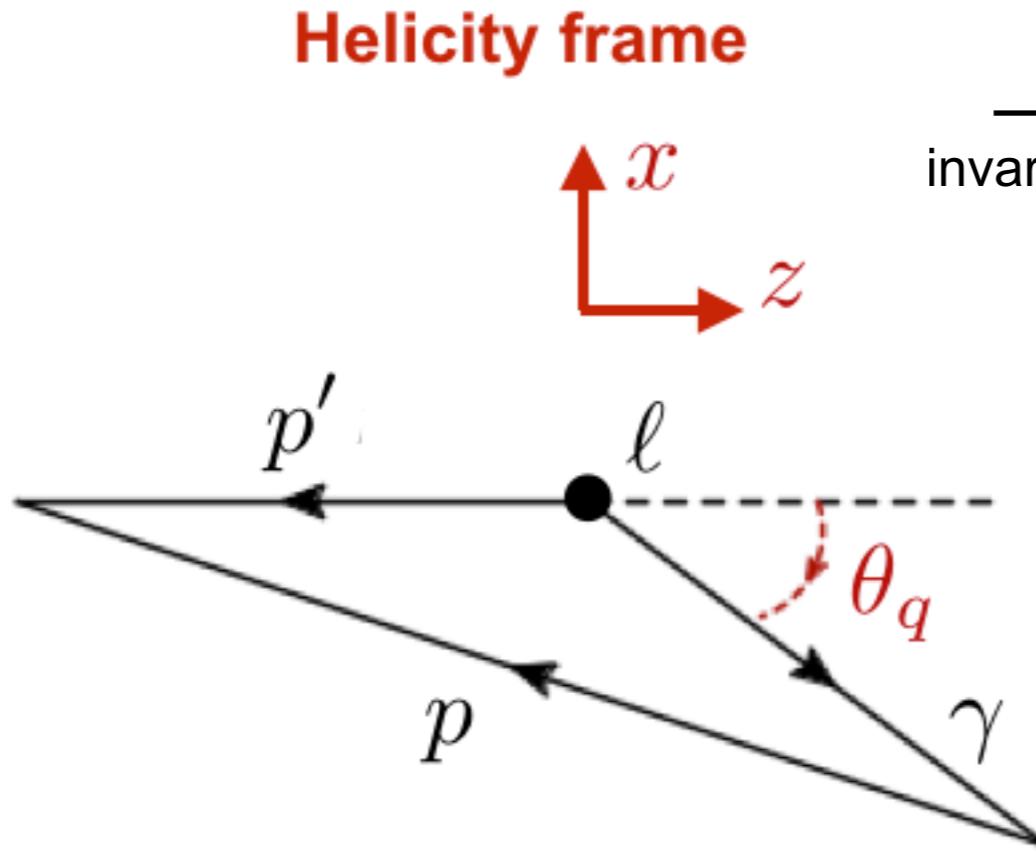
Courtesy of A. Austregesilo and C. Gleason

The resulting photon spin density matrix reads

$$L_{\lambda\lambda'} = \frac{Q^2}{2(1-\epsilon)} \begin{pmatrix} 1 & \sqrt{\epsilon(1+\epsilon+2\delta)}e^{-i\Phi} & -\epsilon e^{-2i\Phi} \\ \sqrt{\epsilon(1+\epsilon+2\delta)}e^{i\Phi} & 2(\epsilon+\delta) & -\sqrt{\epsilon(1+\epsilon+2\delta)}e^{-i\Phi} \\ -\epsilon e^{2i\Phi} & -\sqrt{\epsilon(1+\epsilon+2\delta)}e^{i\Phi} & 1 \end{pmatrix} \quad (44)$$

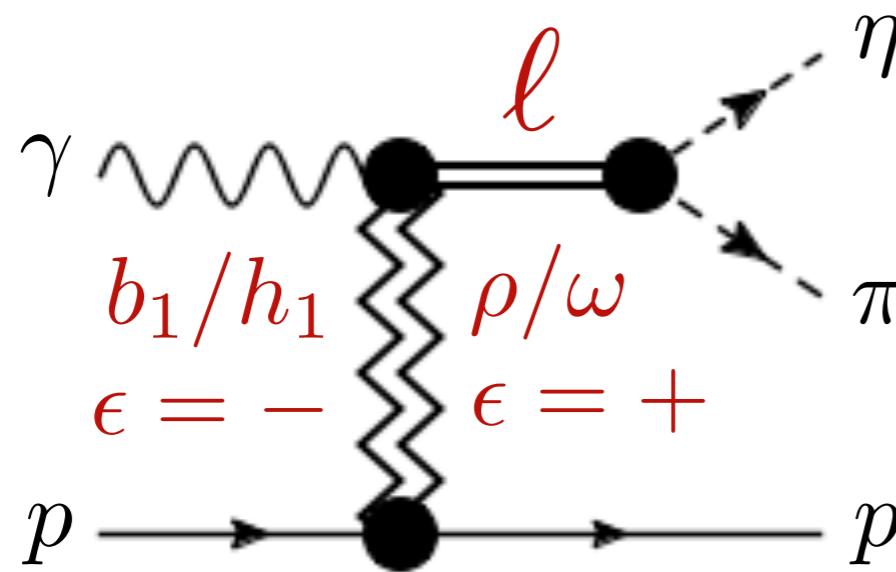
$$\frac{Q^2 \epsilon}{2(1-\epsilon)} = l_x^2$$

Frame



between γ and ℓ

$$T_{\lambda_\gamma m} \simeq \delta_{\lambda_\gamma, m} T_{\lambda_\gamma m} + \dots$$



Reflectivity basis:

$$[\ell]_m^{(\epsilon)} = T_{1m} - \epsilon T_{-1-m}$$

Dominant: $(\epsilon = +, m = 1)$

Observables: Moments of Angular distribution

$$H^0(LM) = \frac{1}{2\pi} \int I(\Omega, \Phi) d_{M0}^L(\theta) \cos M\phi \, d\Omega d\Phi$$

$$H^1(LM) = \frac{-1}{\pi P_\gamma} \int I(\Omega, \Phi) \cos 2\Phi d_{M0}^L(\theta) \cos M\phi \, d\Omega d\Phi$$

$$\text{Im } H^2(LM) = \frac{1}{\pi P_\gamma} \int I(\Omega, \Phi) \sin 2\Phi d_{M0}^L(\theta) \sin M\phi \, d\Omega d\Phi$$

$$H^1(LM) + \text{Im } H^2(LM) \propto \sum_{\epsilon, \ell\ell', mm'} \left(\frac{2\ell' + 1}{2\ell + 1} \right)^{1/2} \epsilon (-1)^m C_{\ell' 0 L 0}^{\ell 0} C_{\ell' m' L M}^{\ell m} [\ell]_{-m}^{(\epsilon)} [\ell']_{m'}^{(\epsilon)*}$$

$m' = m - M$

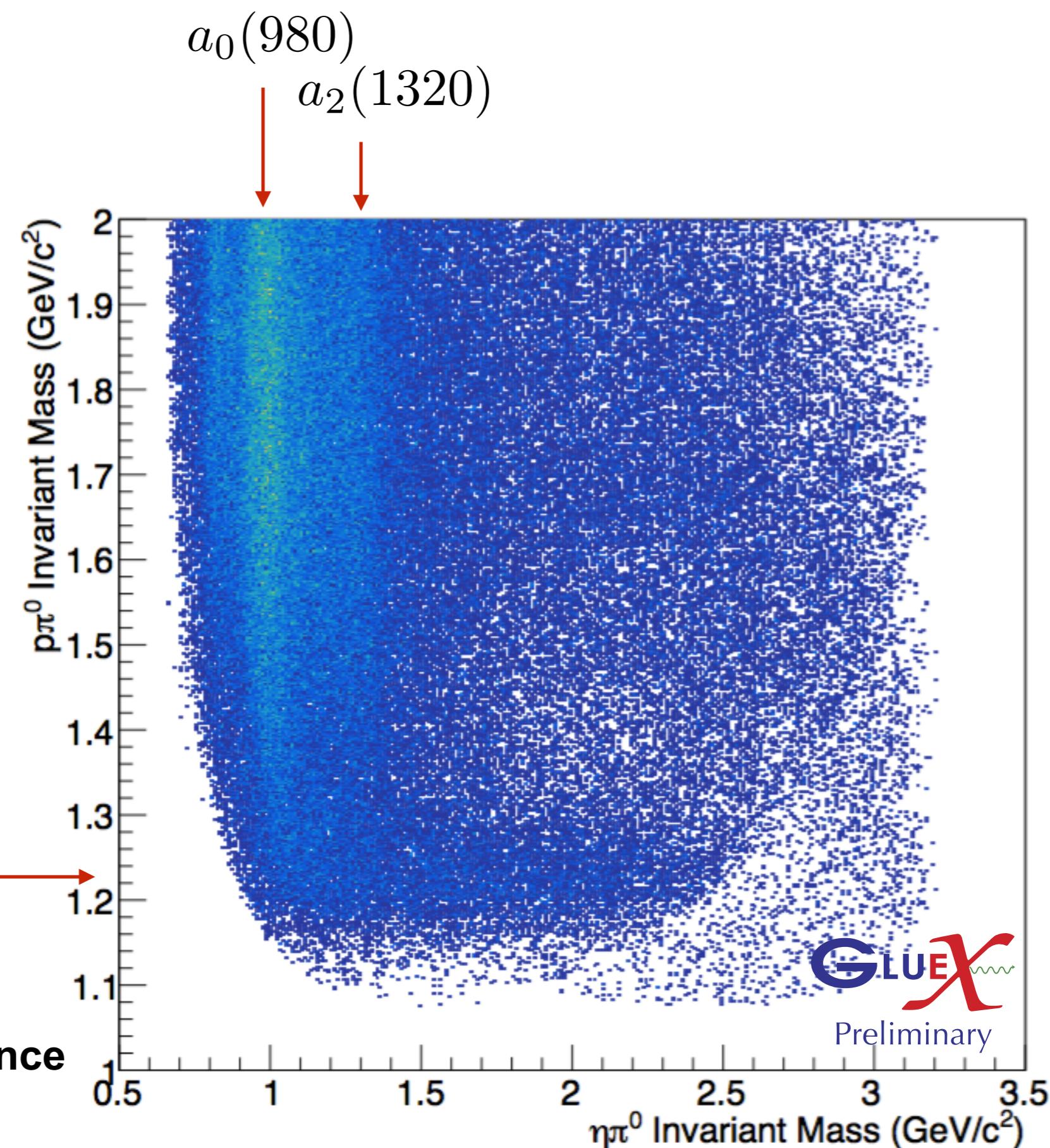
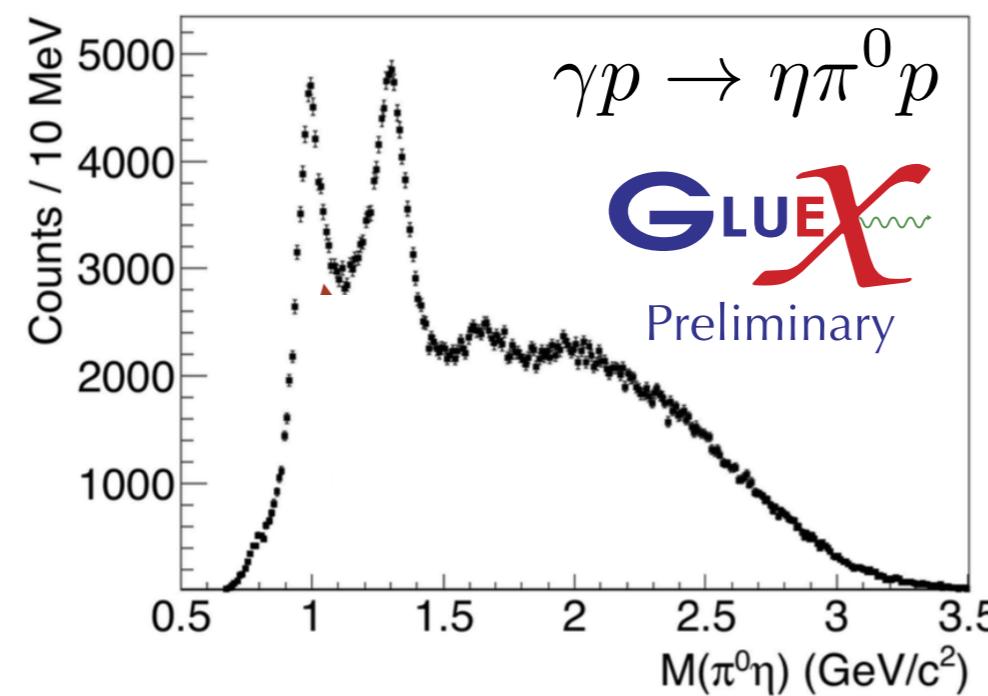
 $0 \leq -m ; 0 \leq m'$

The model features
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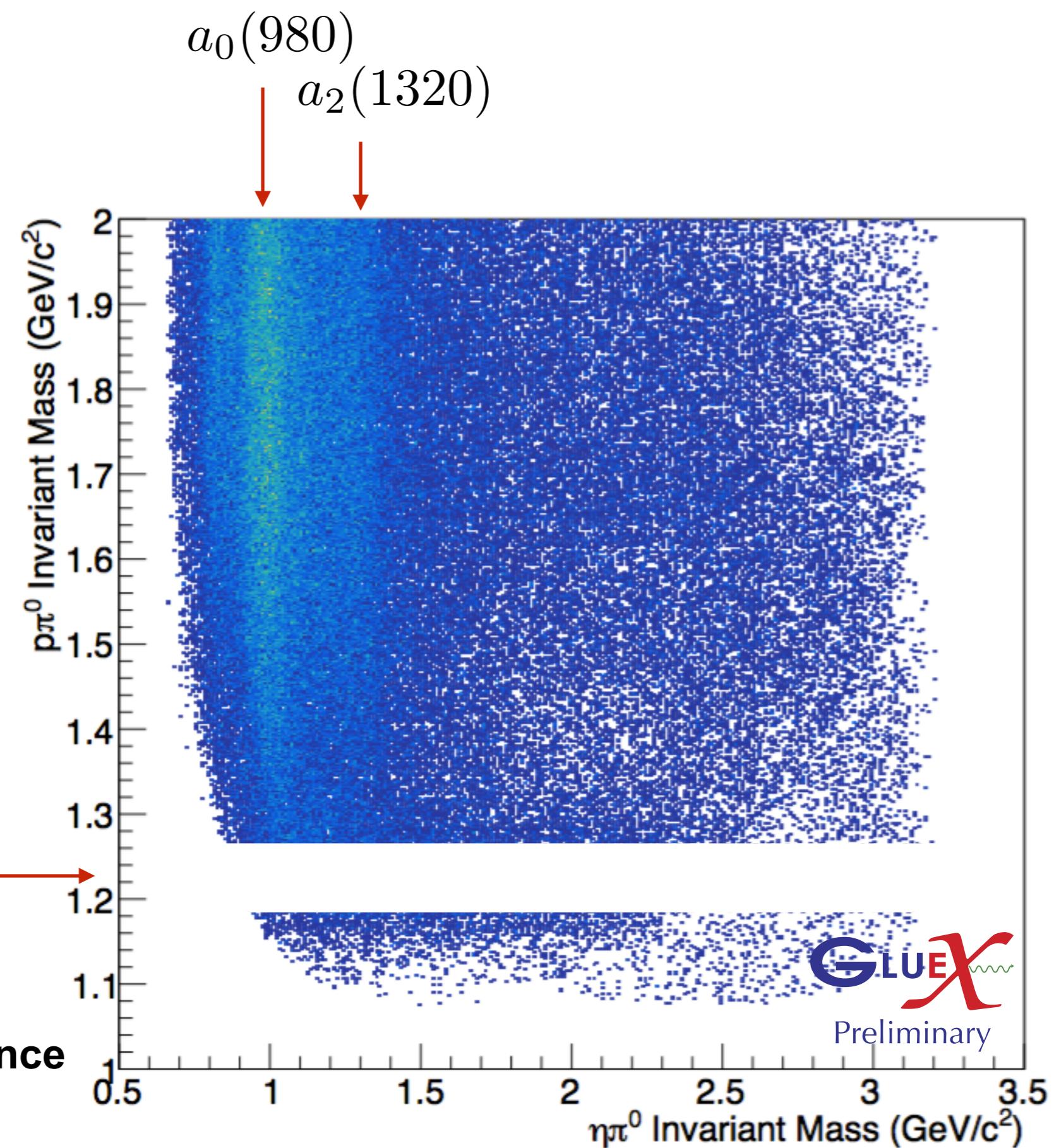
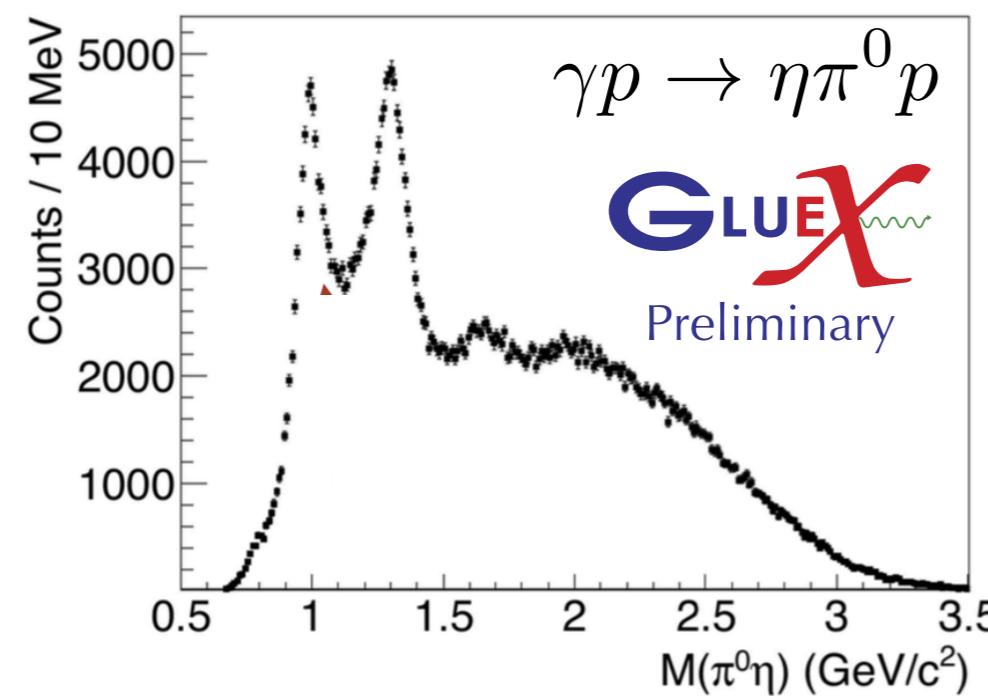
Eta-Pi Production@GlueX

46

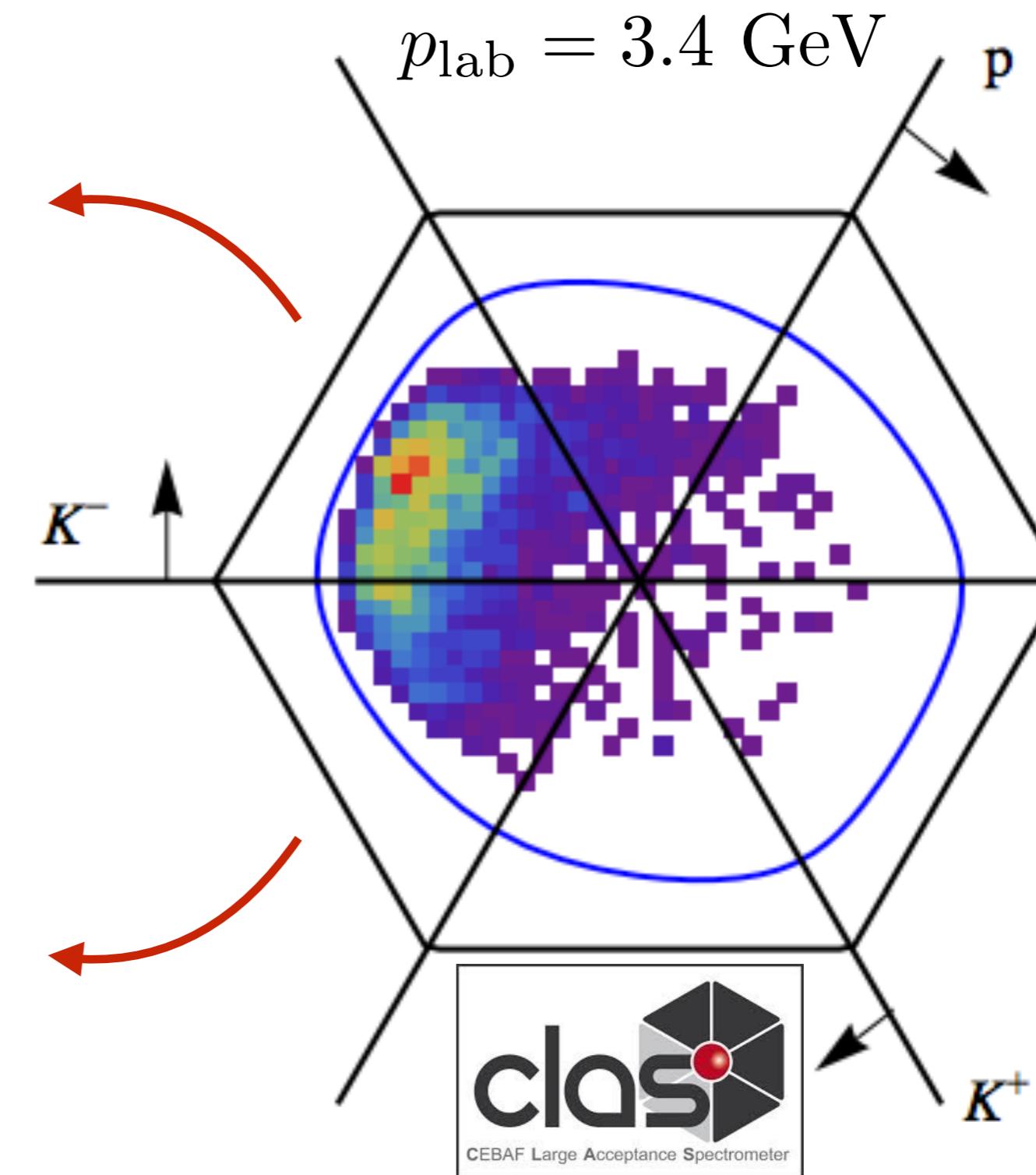
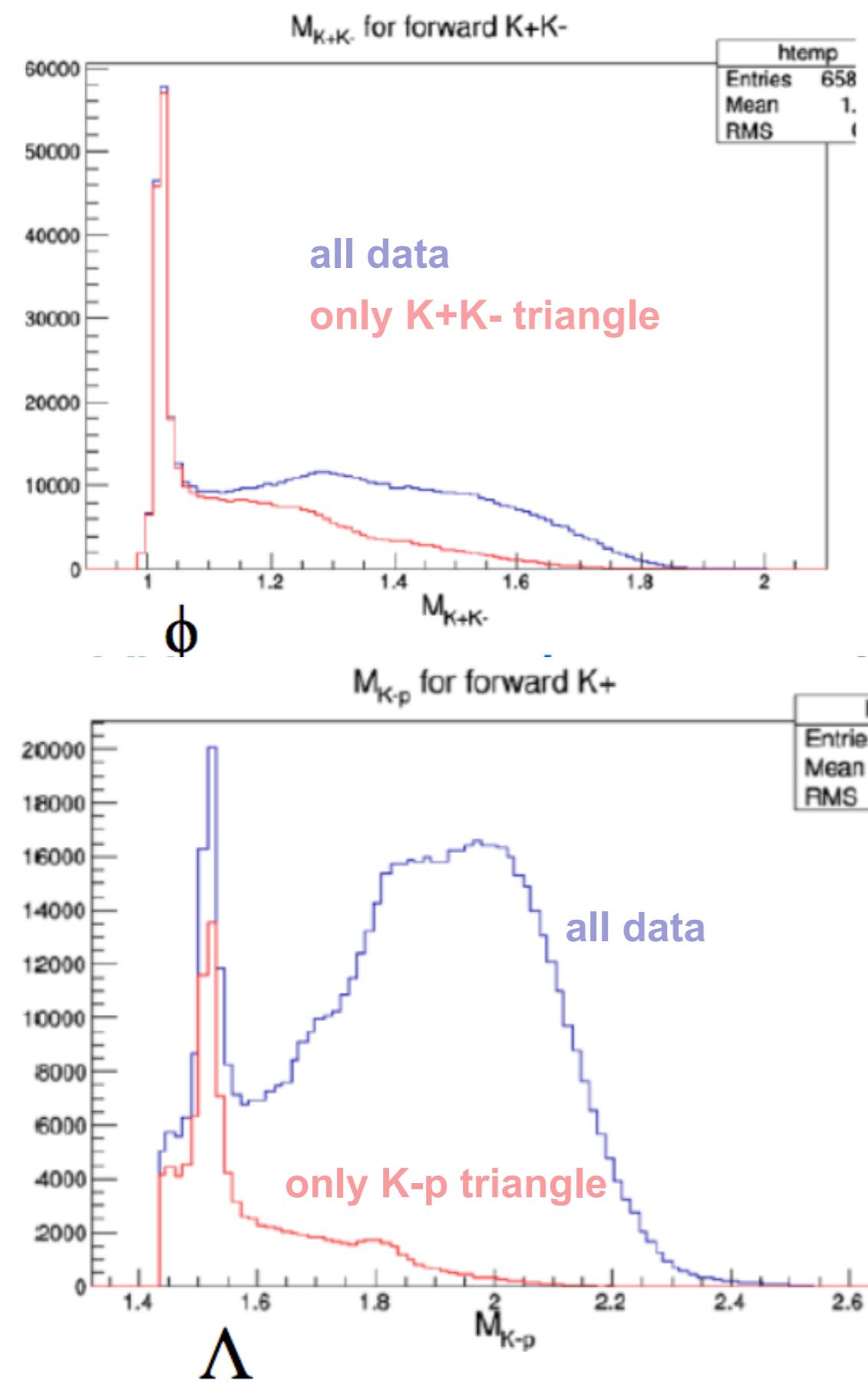


Eta-Pi Production@GlueX

46

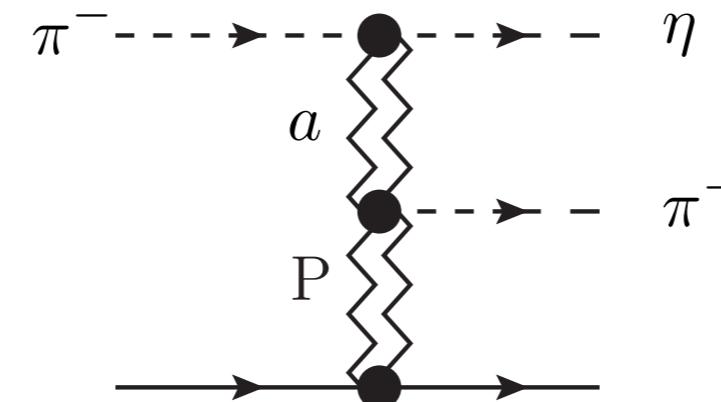


Longitudinal Plot



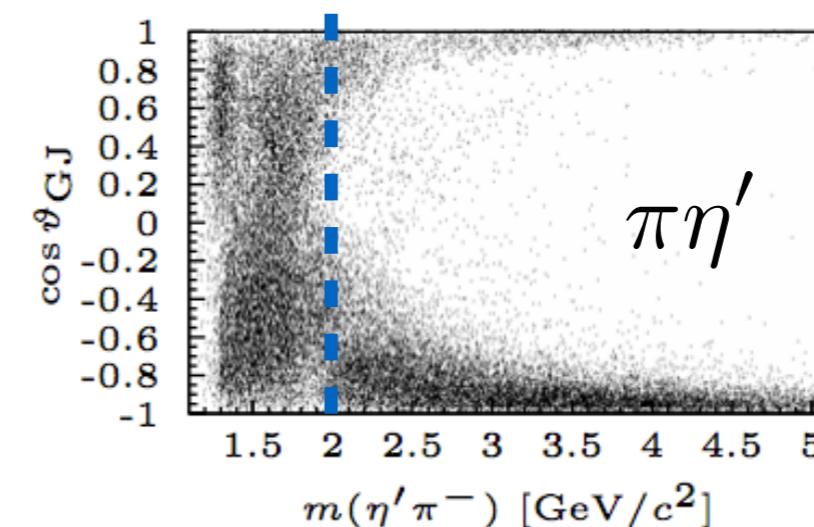
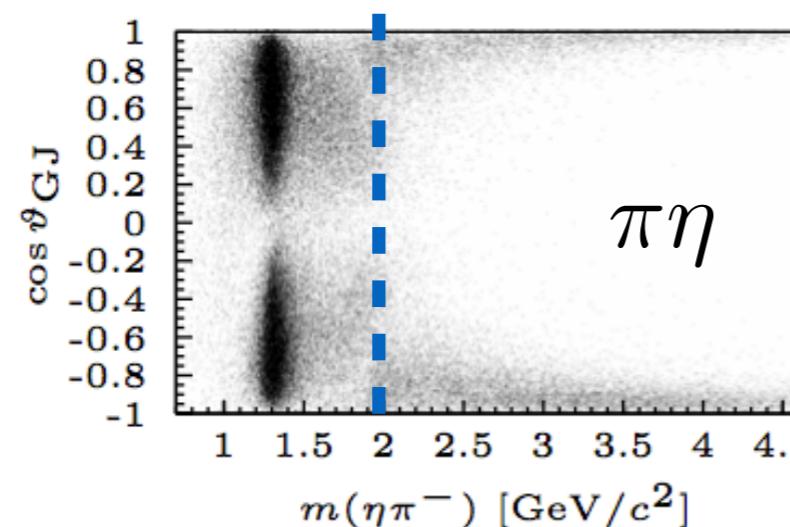
High Mass Region

48

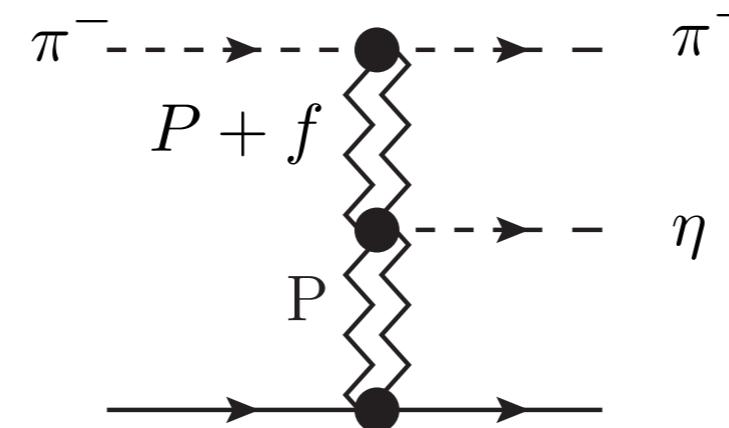


$$a : I^G J^{PC} = 1^-(0, 2, 4, 6, \dots)^{++}$$

$\cos \theta_{GF} \sim 1 \rightarrow \eta$ forward



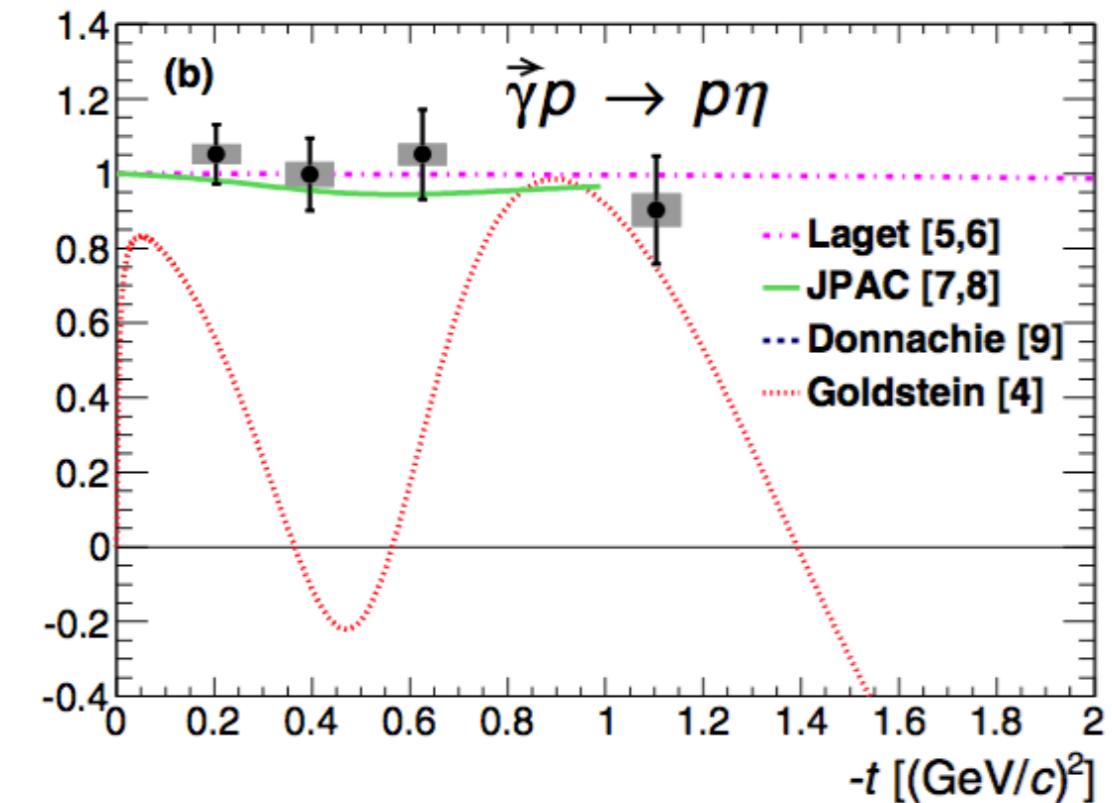
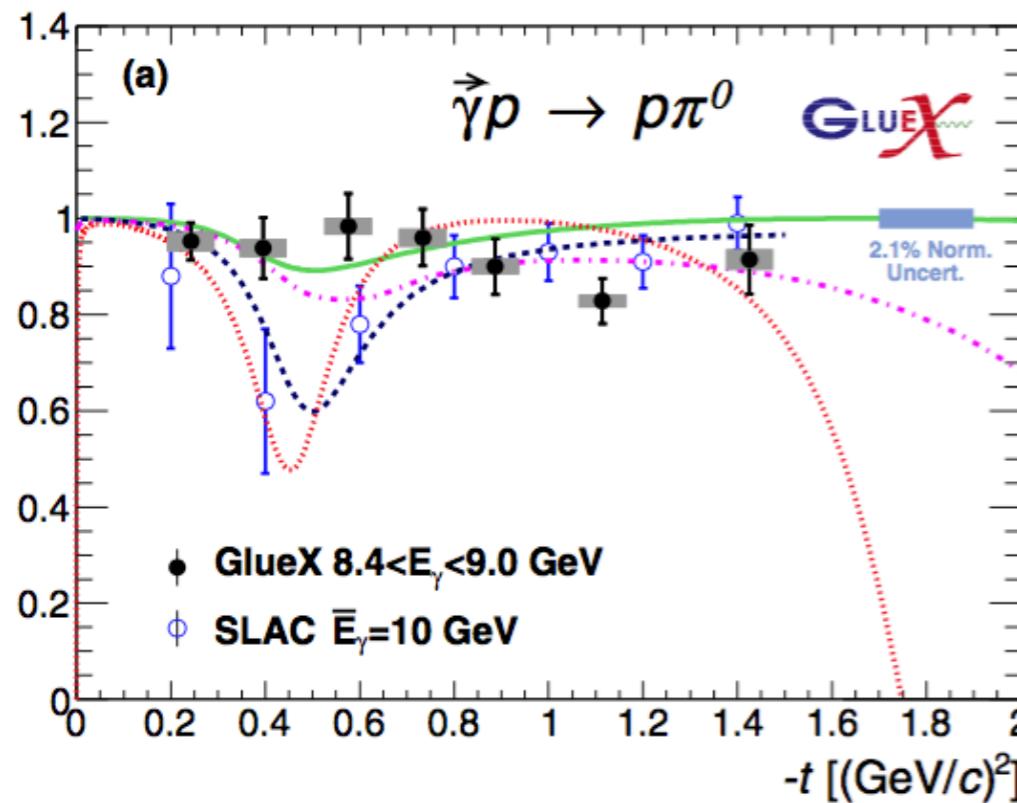
$\cos \theta_{GF} \sim -1 \rightarrow \eta$ backward



$$f : I^G J^{PC} = 0^+(0, 2, 4, 6, \dots)^{++}$$

GlueX Results

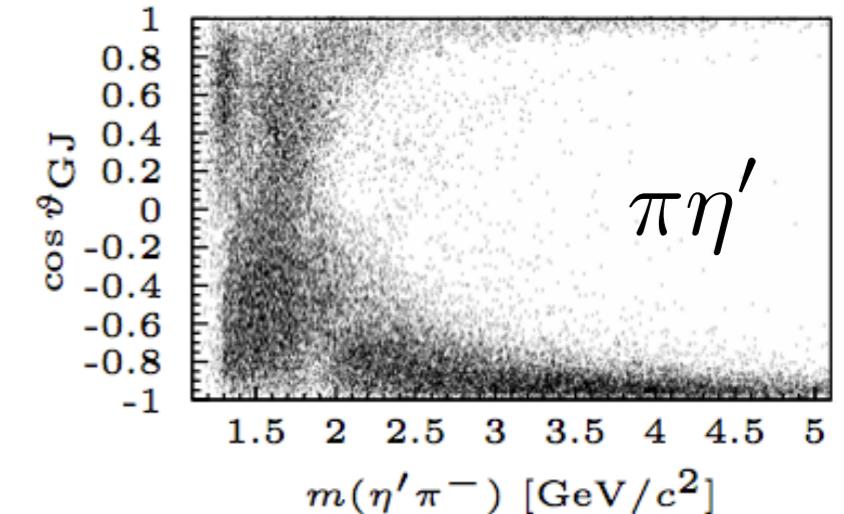
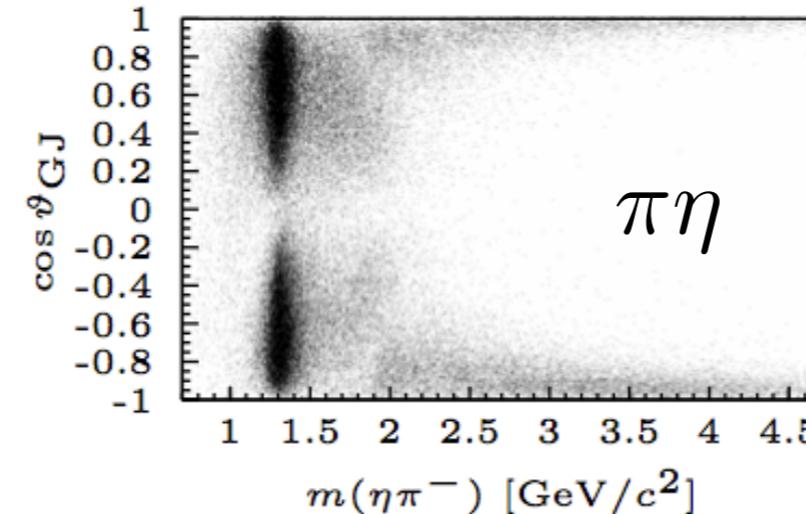
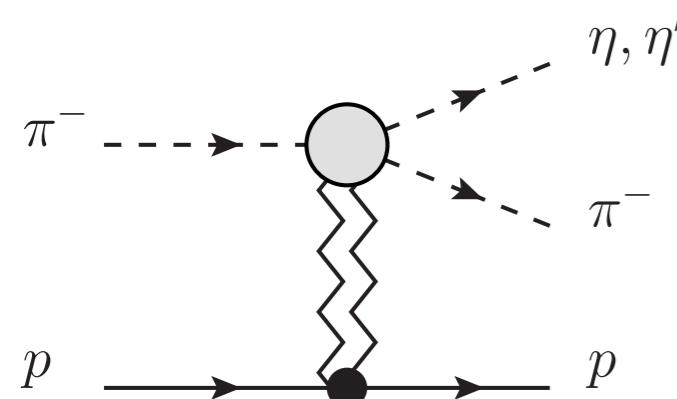
49



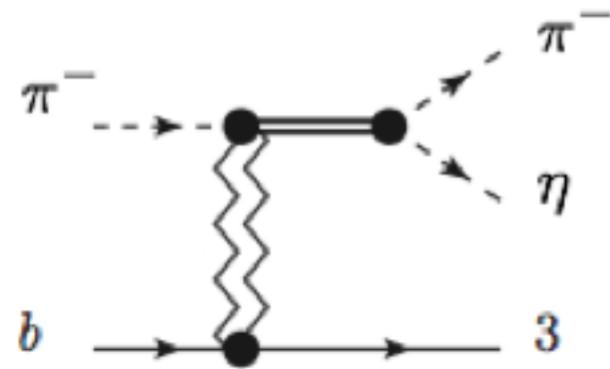
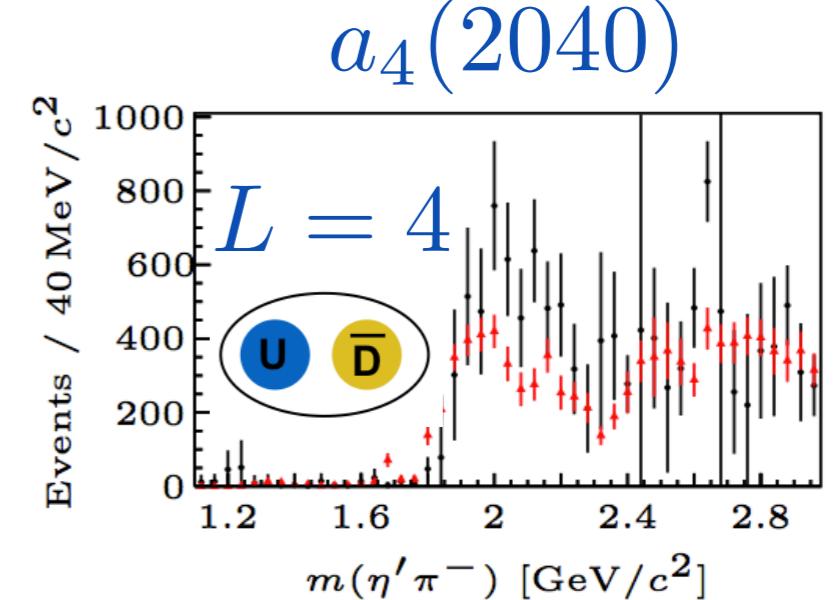
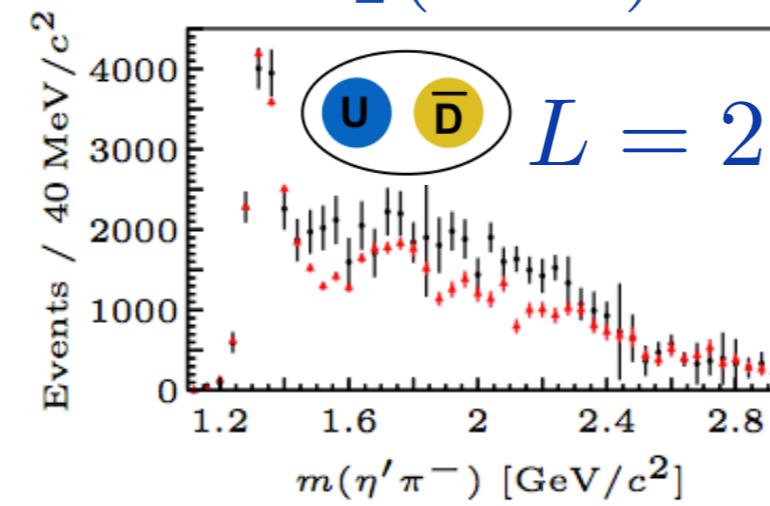
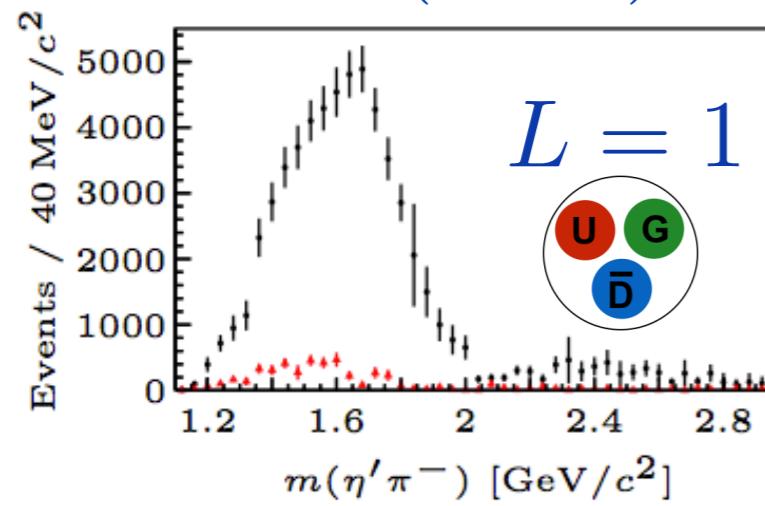
GlueX, VM and J. Nys PRC95 (2017)

Eta-Pi @COMPASS

COMPASS Phys. Lett. B740 (2015)

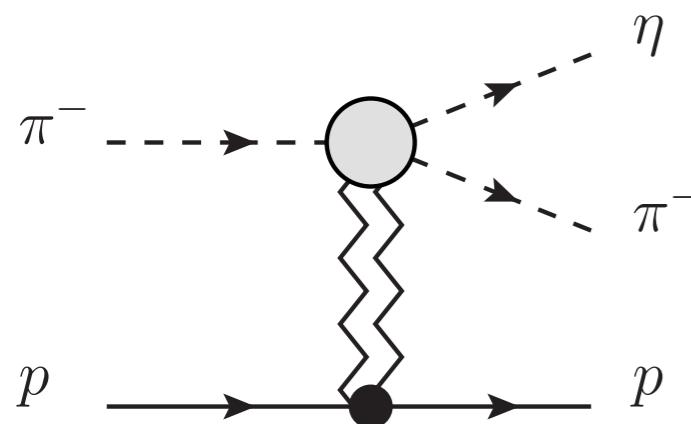


$\pi_1(1600)?$

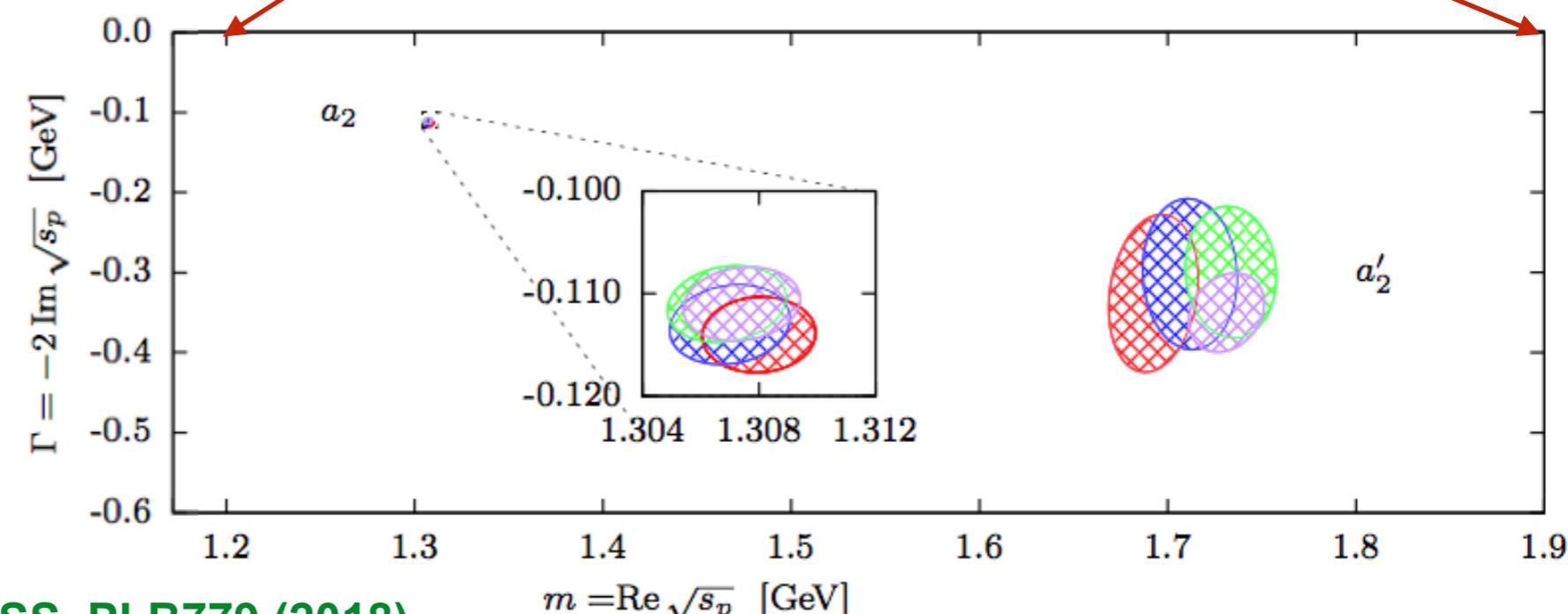
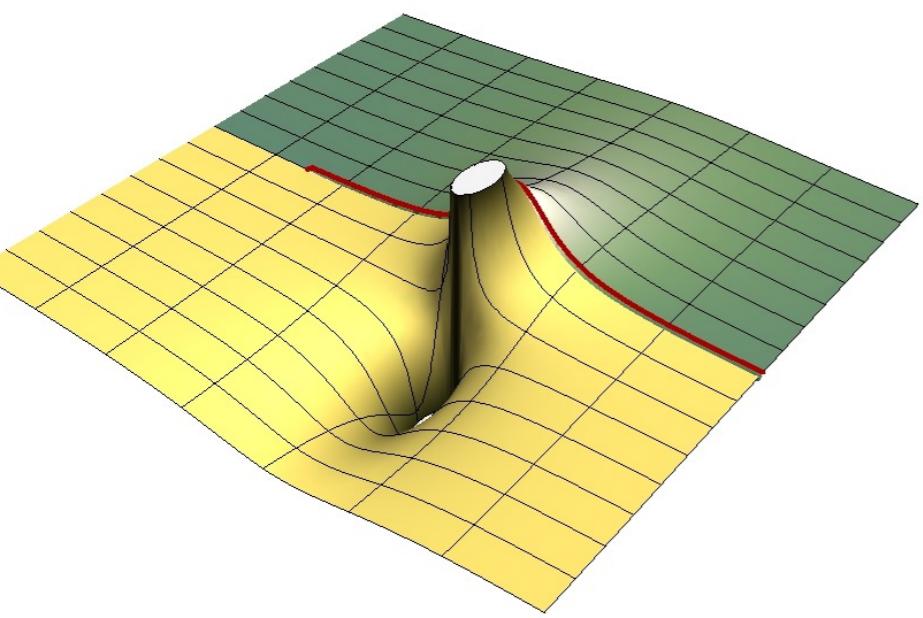
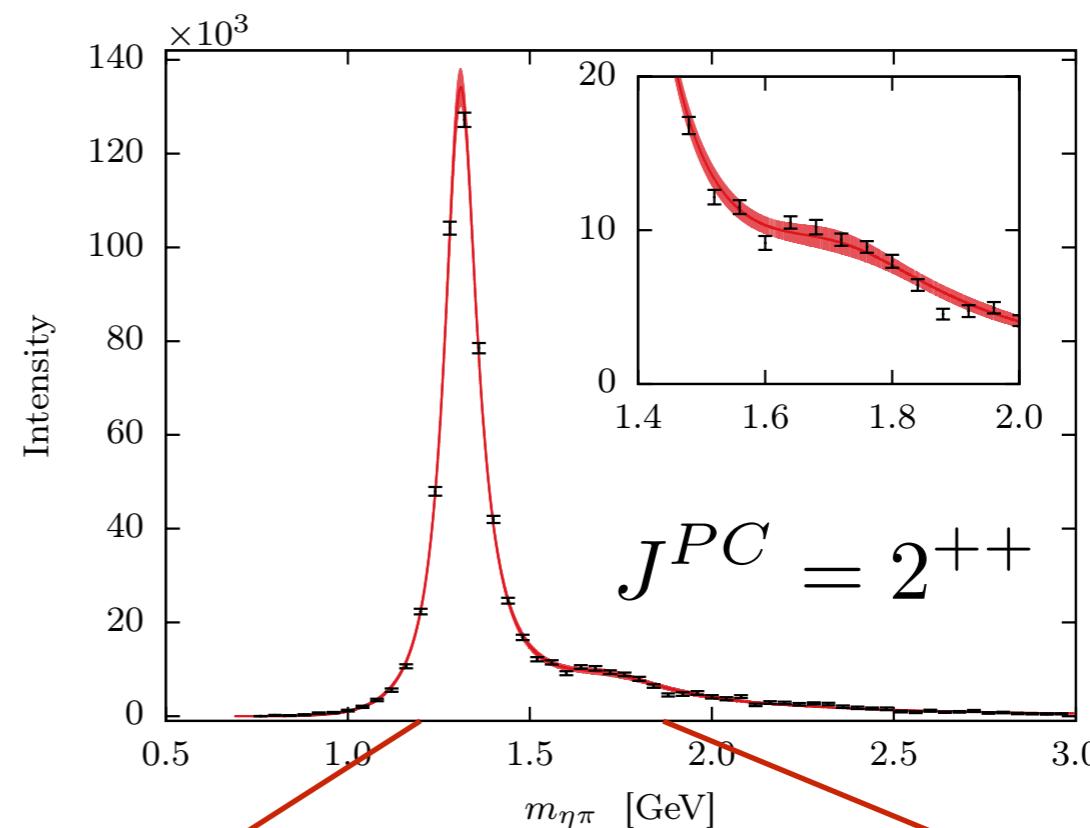


black: $\pi\eta'$
red: $\pi\eta$ (scaled)

Resonance in angular mom. $L = 1$?

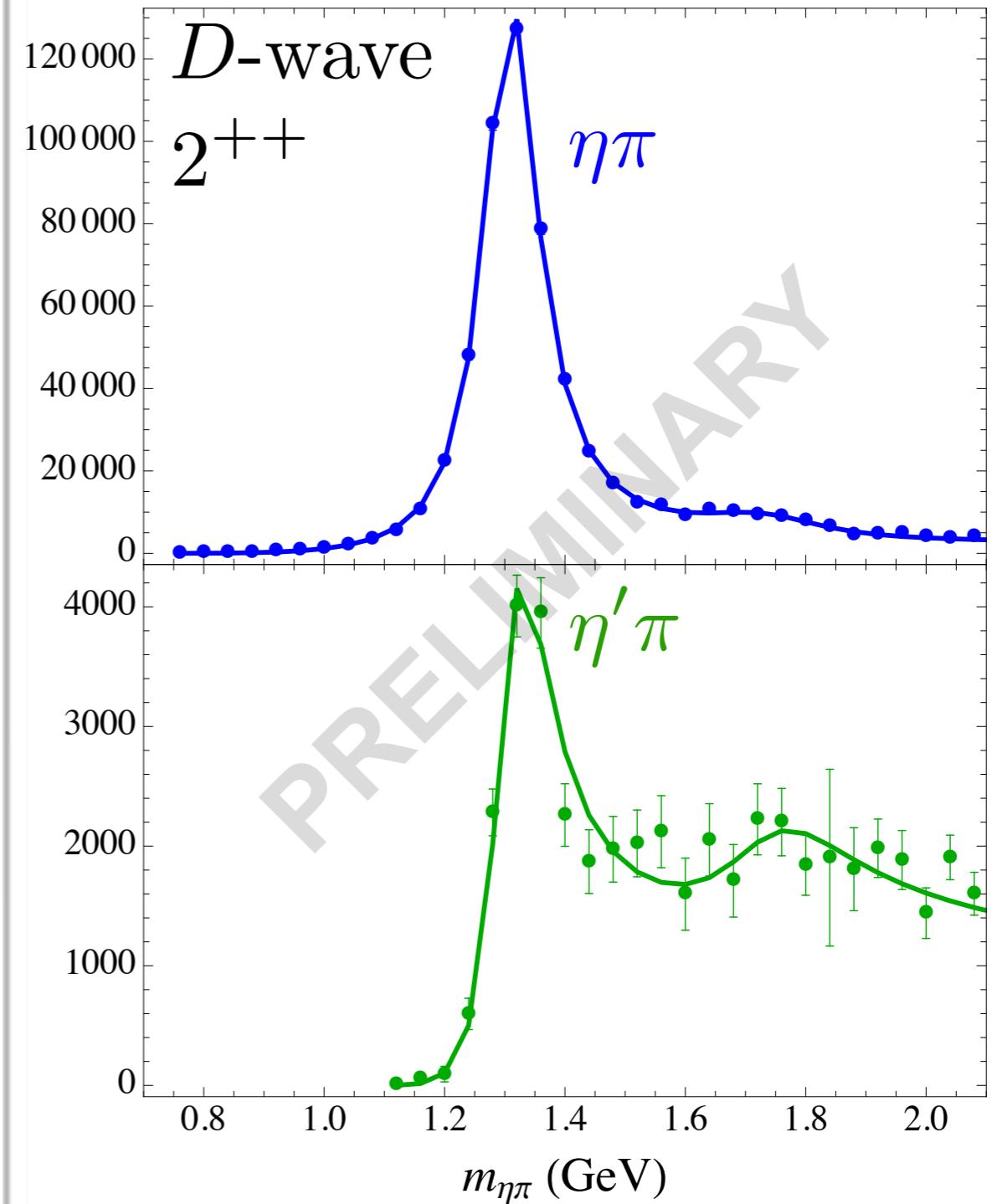
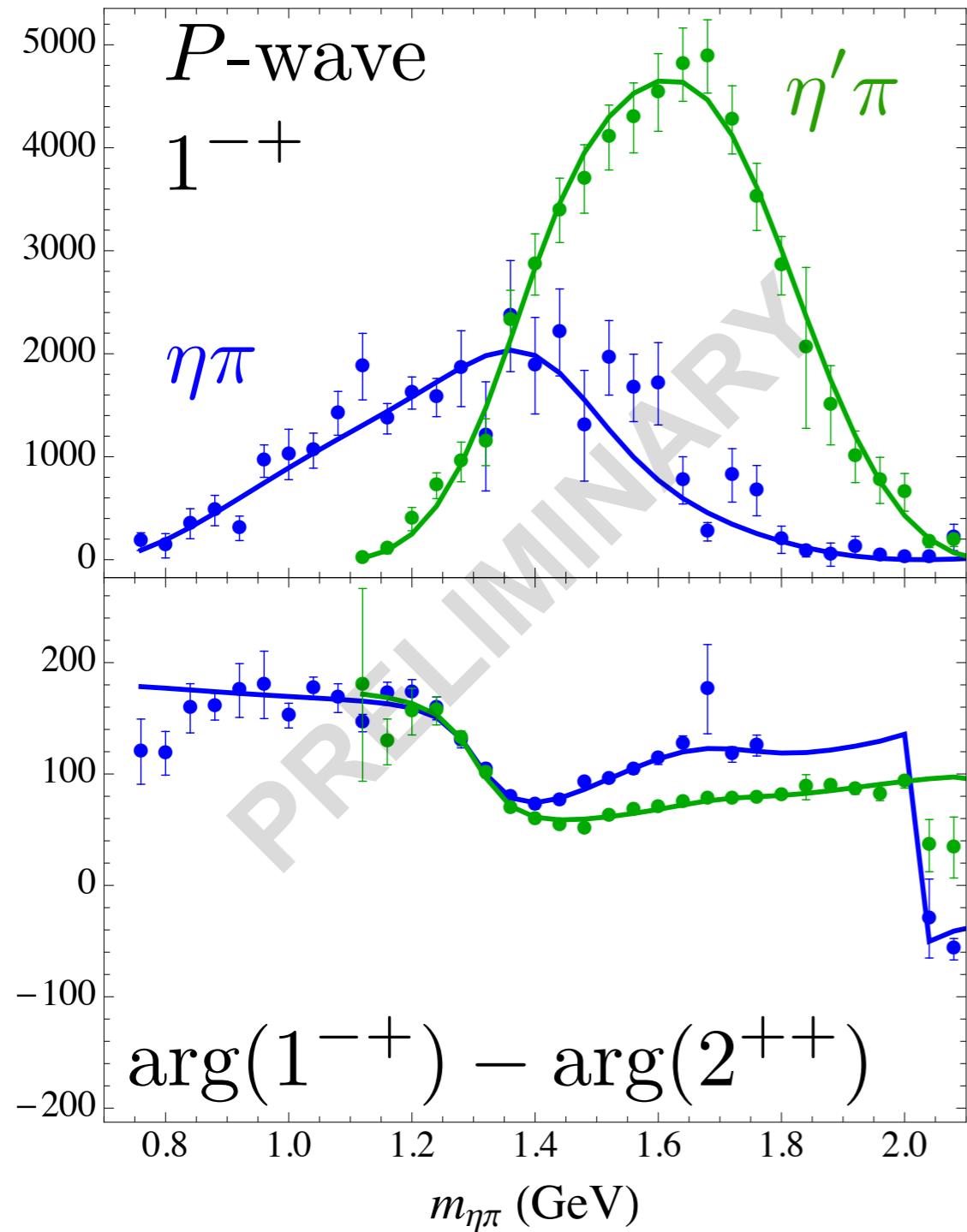


**first precise determination of
 $a_2(1700)$ pole location**



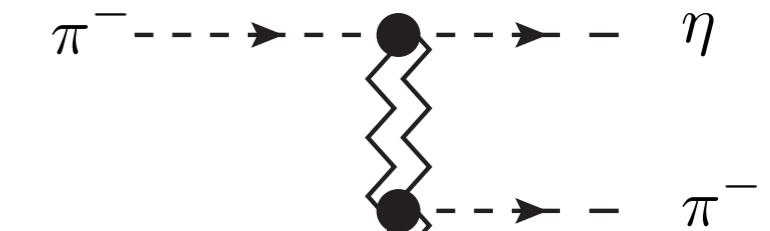
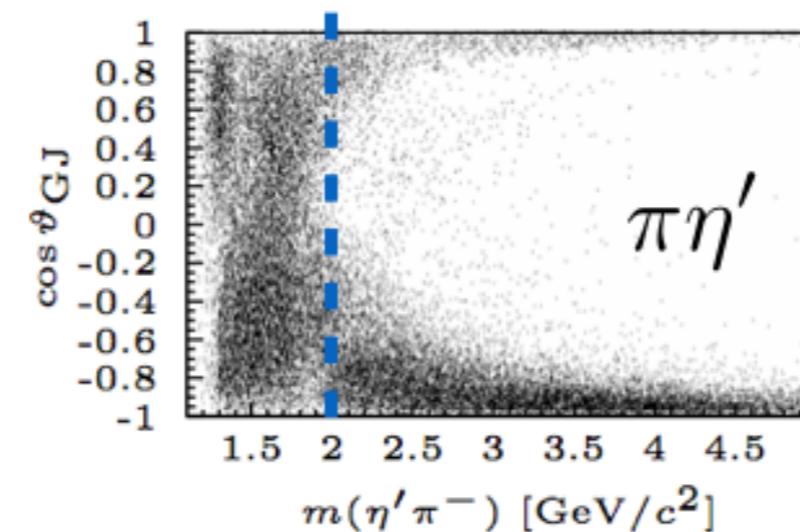
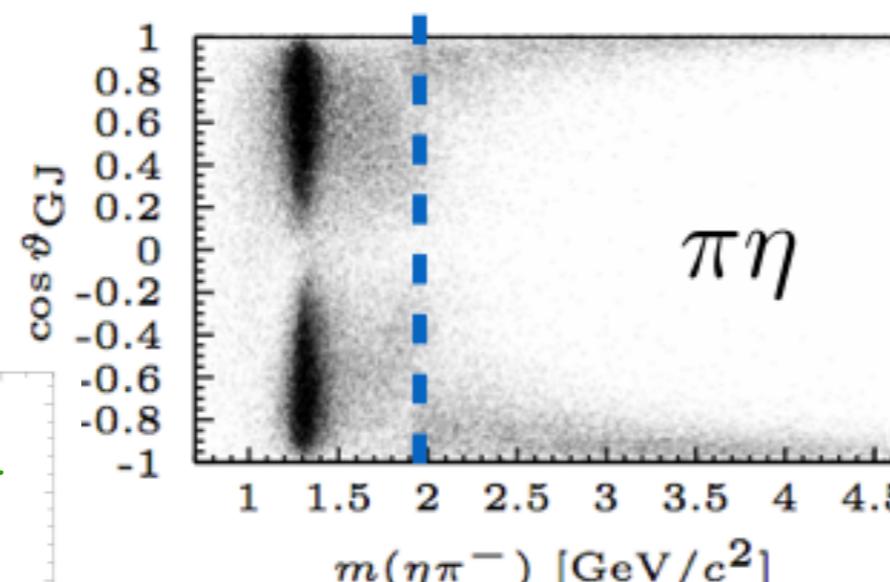
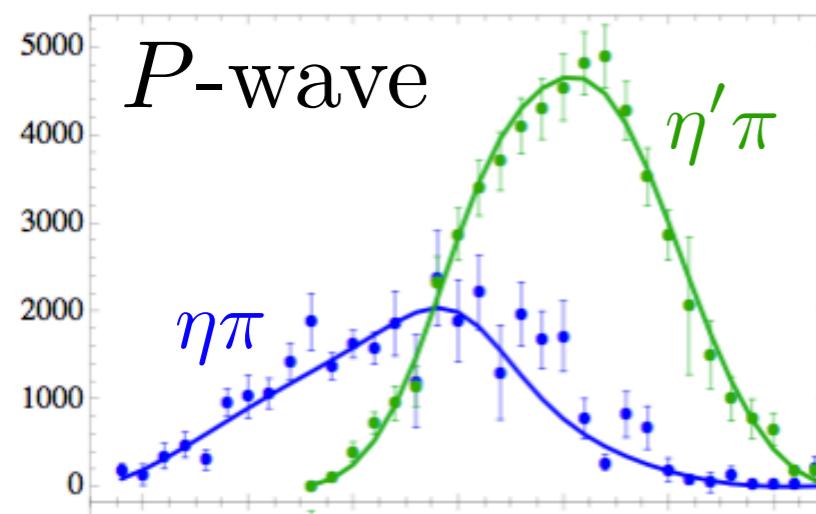
Exotic wave @COMPASS

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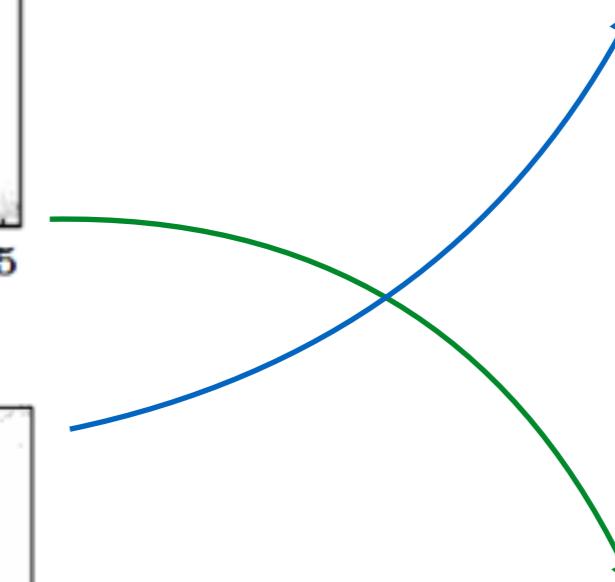


On-going analysis: Systematic studies and exploration of the complex plane

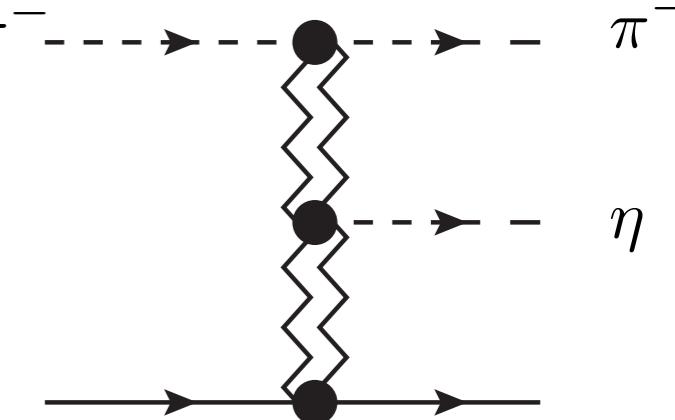
Dispersion relation relates the high (exchanges) and the low (resonances) regions

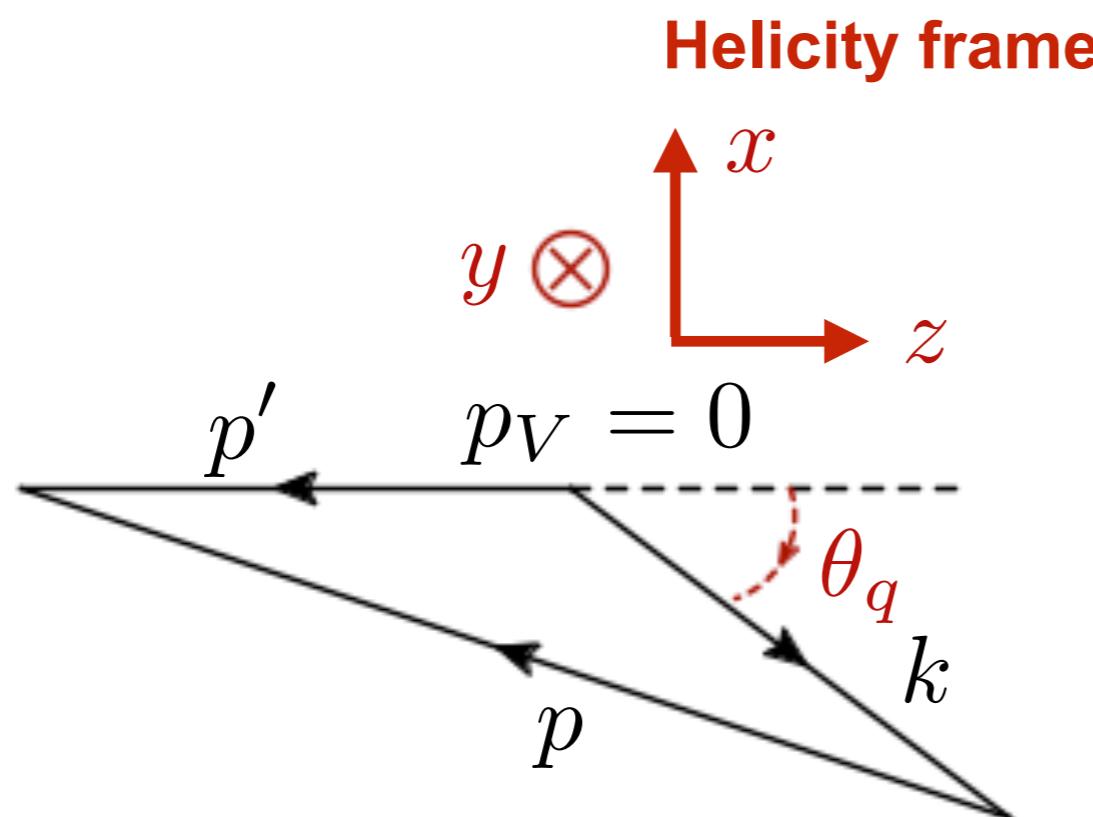


$\cos \theta_{GF} \sim 1 \rightarrow \eta$ forward

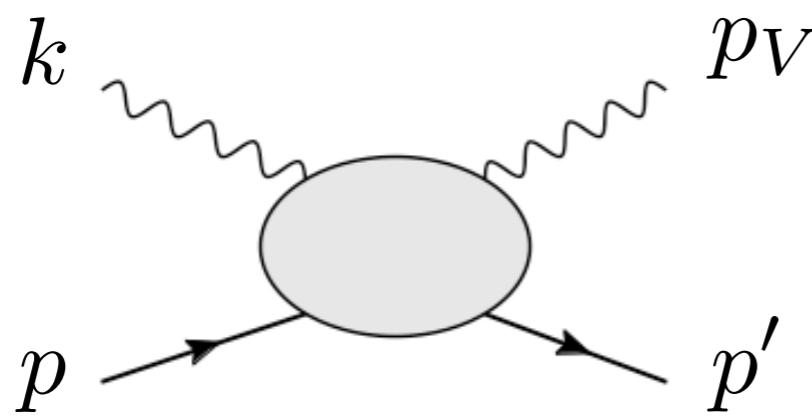
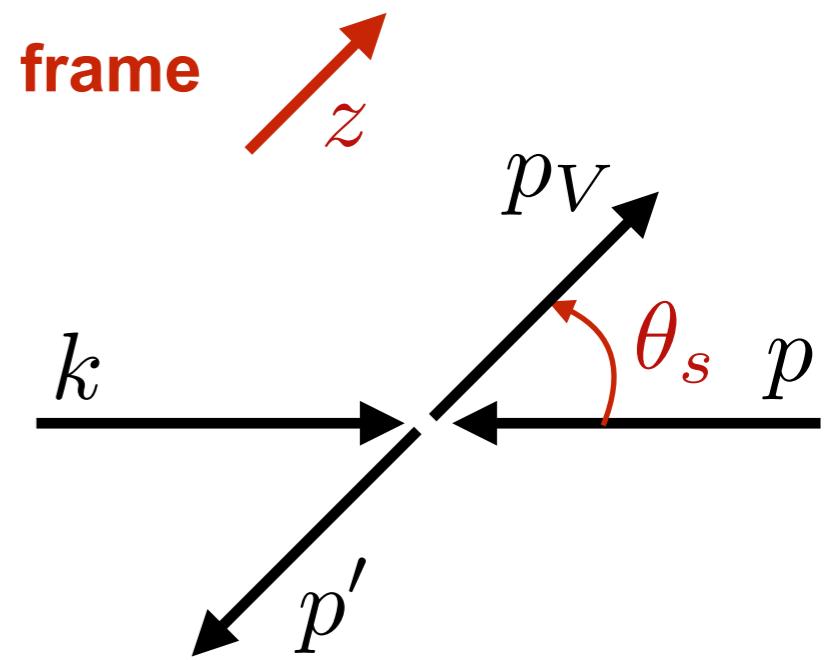


$\cos \theta_{GF} \sim -1 \rightarrow \eta$ backward



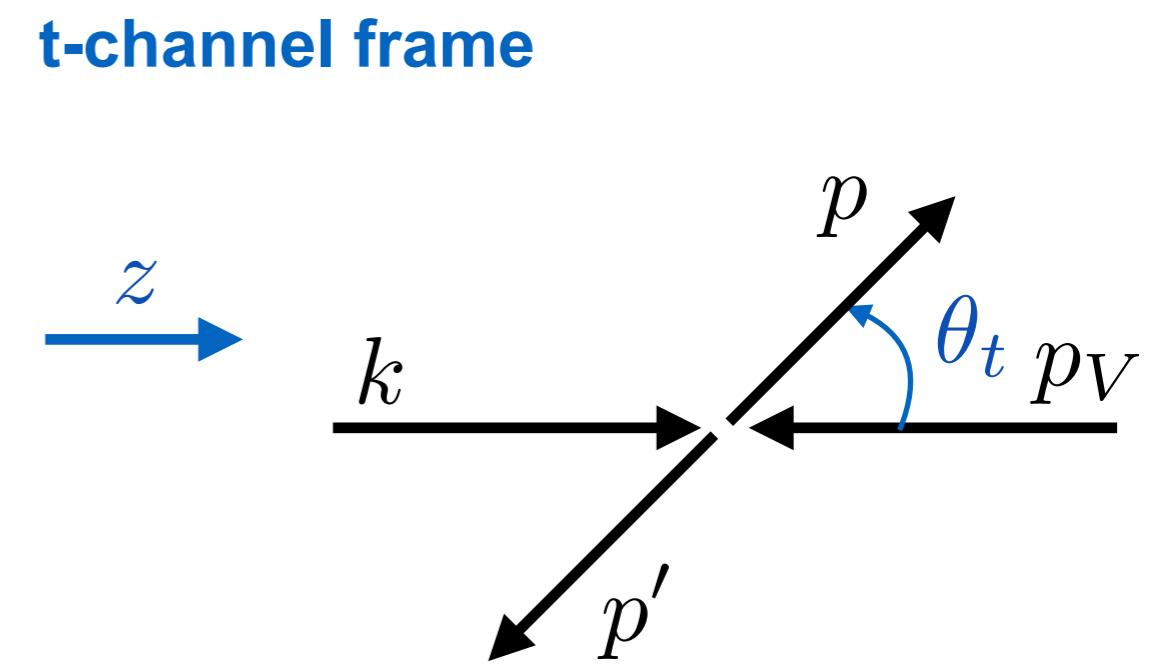
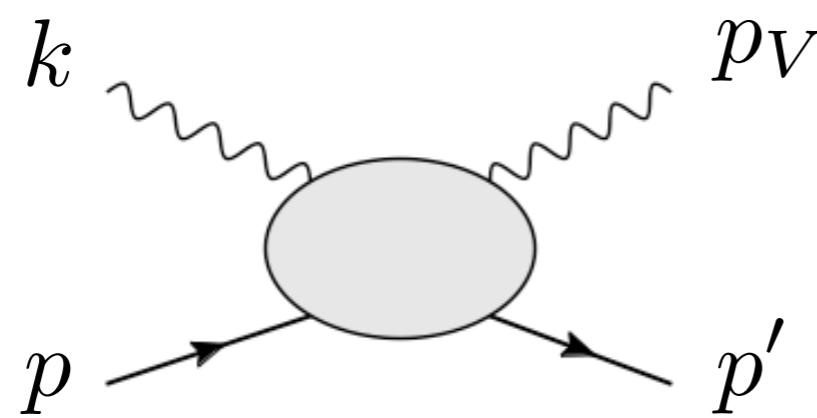
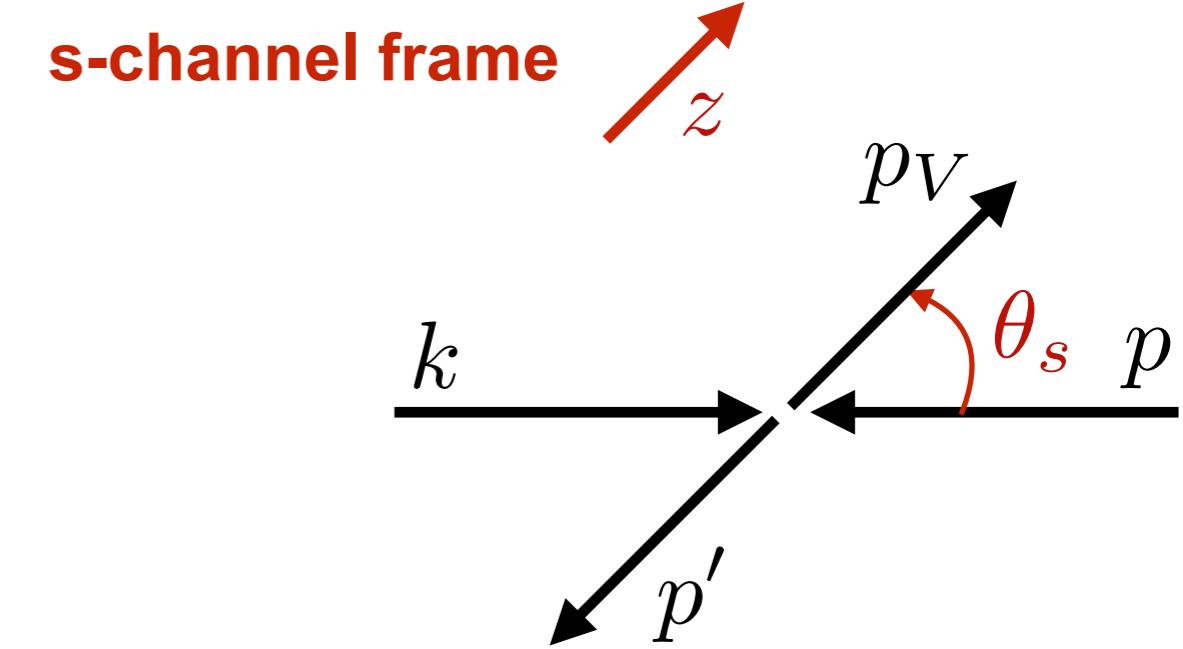
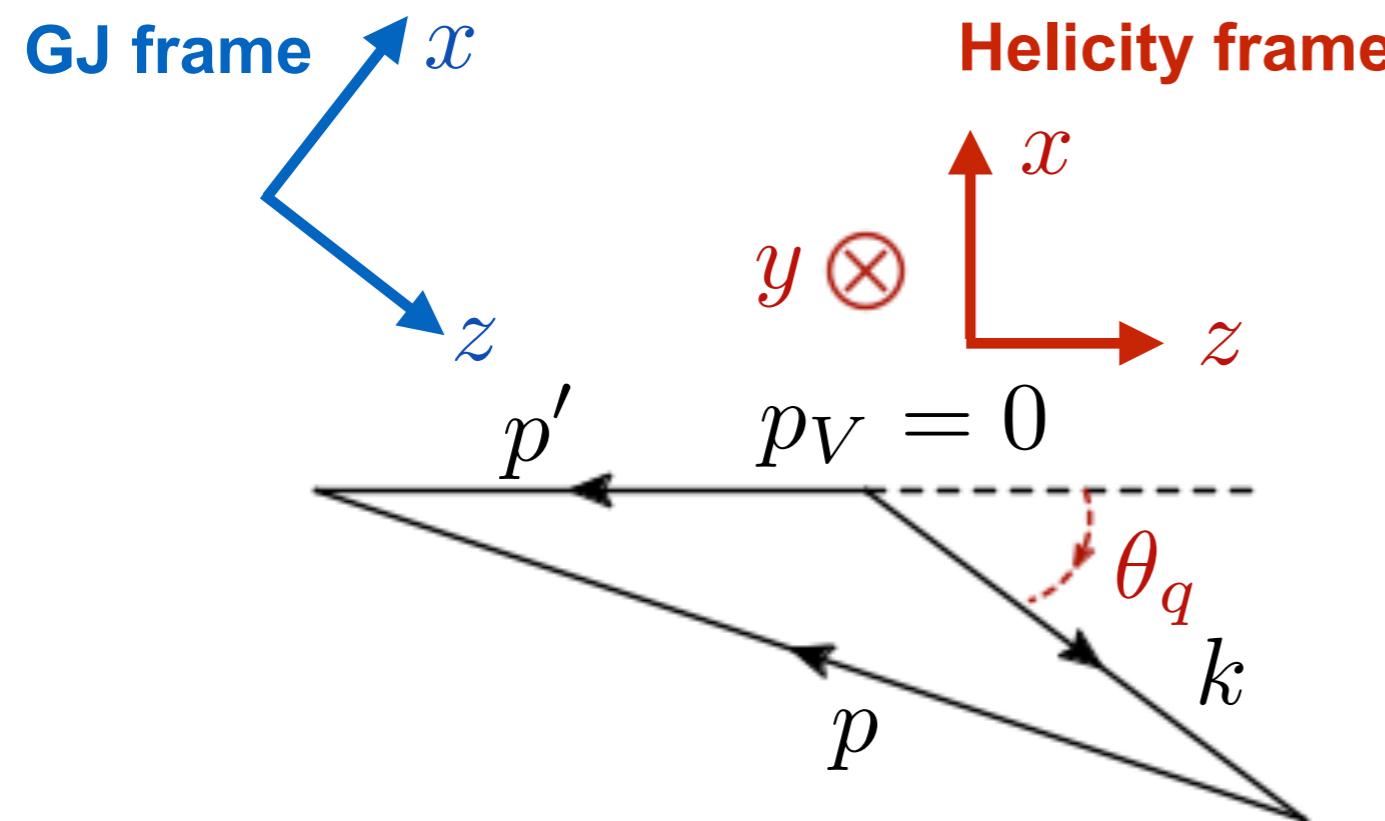


s-channel frame



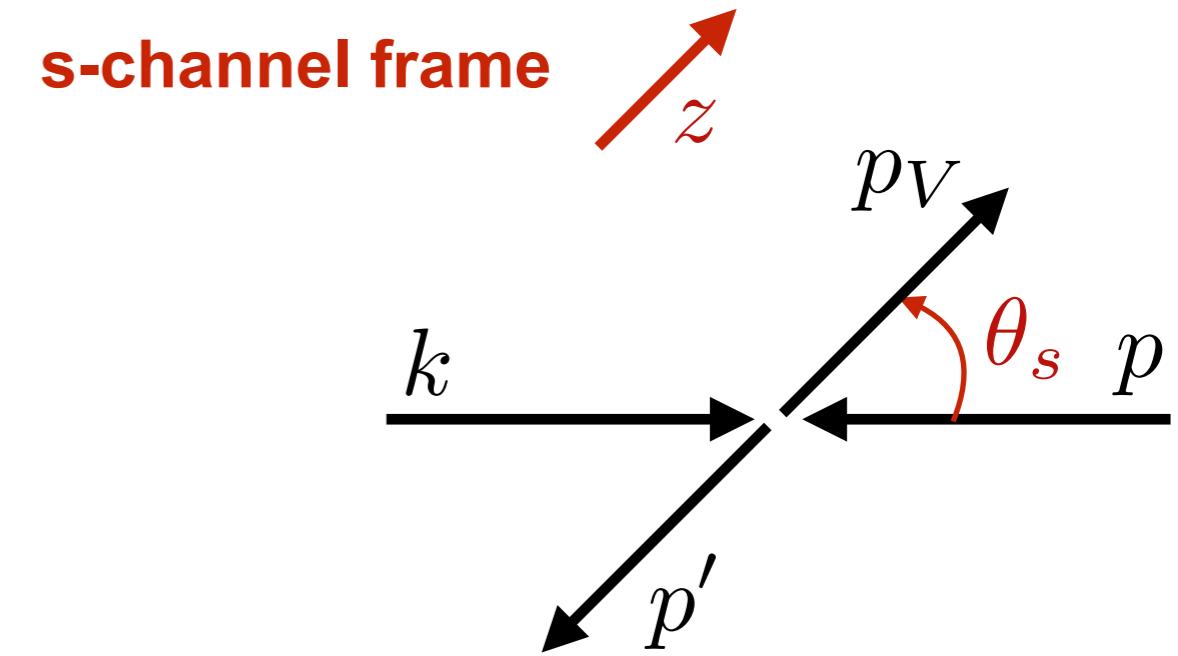
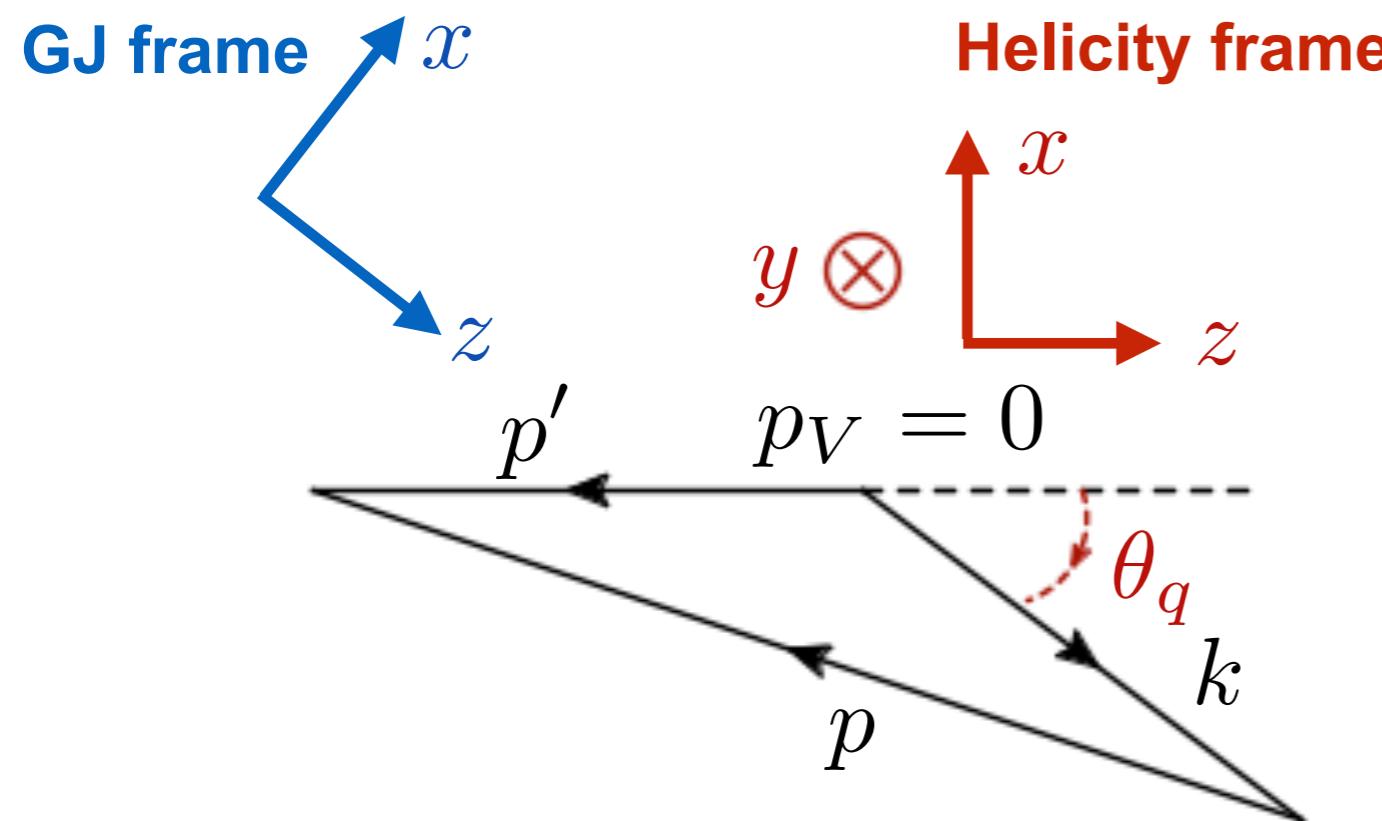
Frames

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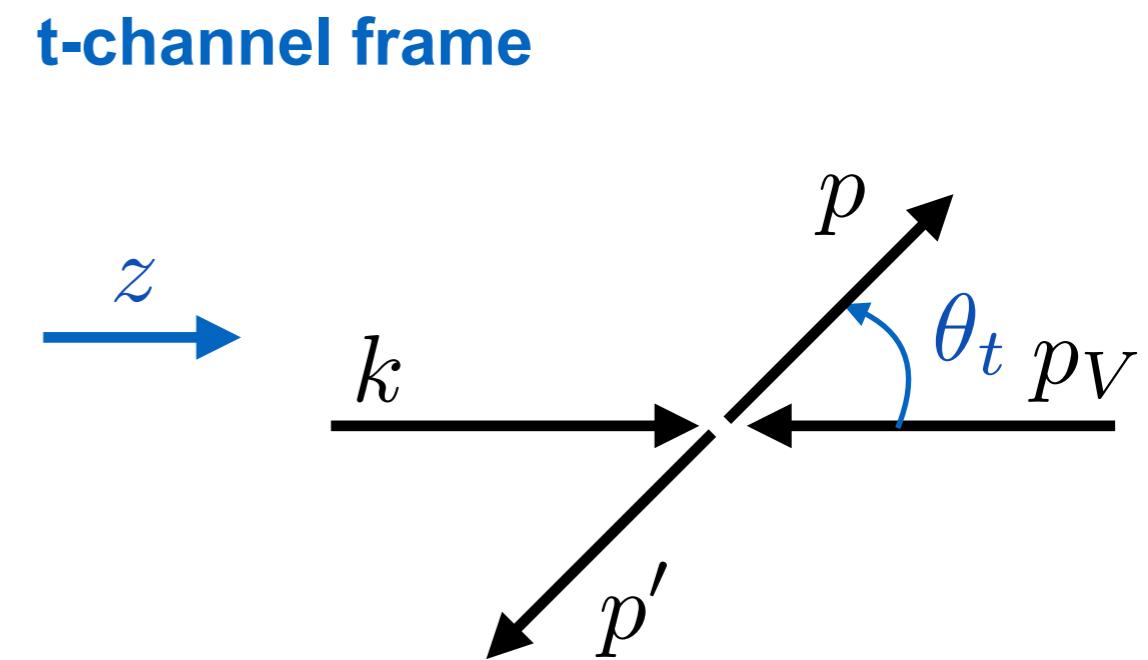
Frames

54



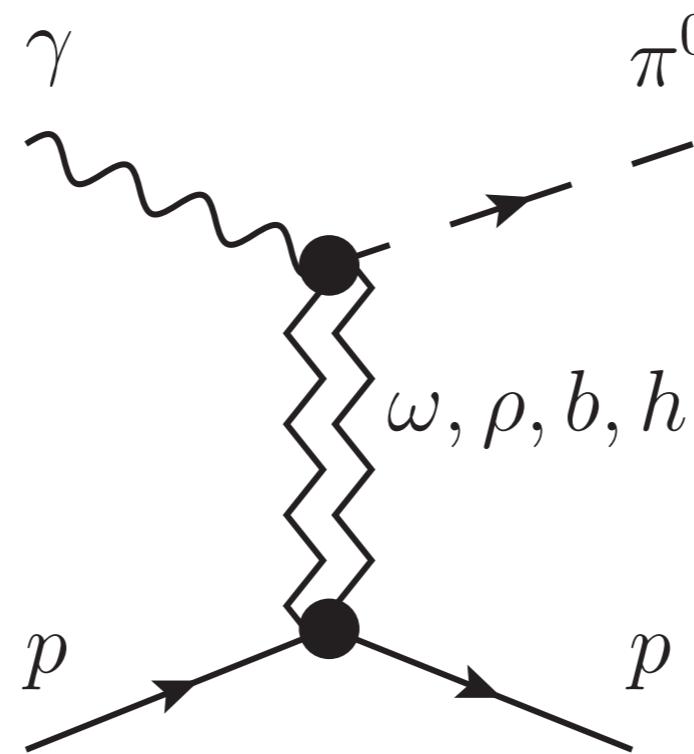
rotation

$$\rho_{MM'}|_H = \rho_{MM'}|_{s\text{-chan}}$$
$$\rho_{MM'}|_{GJ} = \rho_{MM'}|_{t\text{-chan}}$$



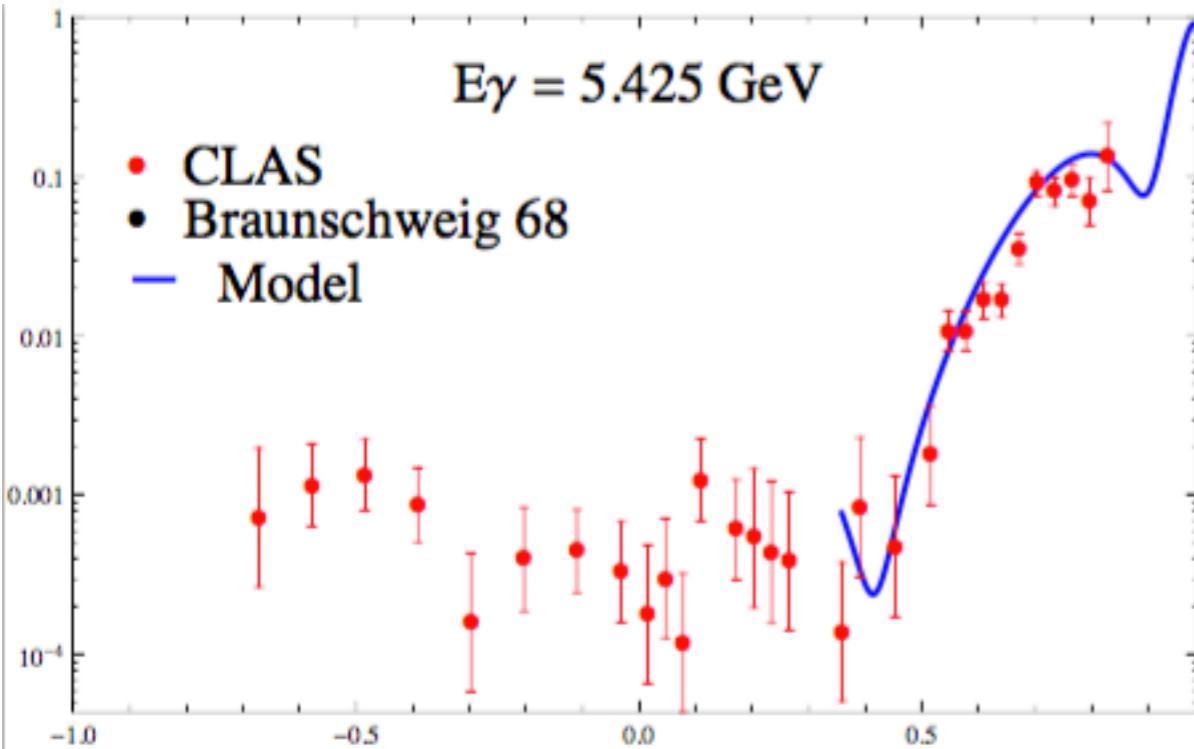
4. Parametrize Reactions: $\gamma p \rightarrow \pi^0 p$

At high energy, particles are produced via the exchange of a force



4. Parametrize Reactions: $\gamma p \rightarrow \pi^0 p$

Blue line: Model from VM et al arXiv:1505.02321



Red points: Data from CLAS (in preparation)

