Dalitz Plot Decomposition

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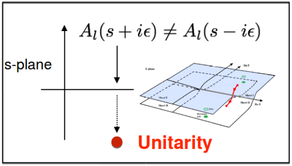
Non-perturbative hadron physics

obeys general principles of the scattering theory

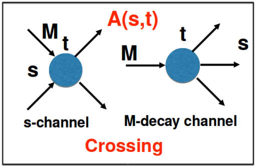
- Lorentz invariance = independence of the reference frame, known behavior under boosts and rotations
- Unitarity = constraint to imaginary part of scattering amplitude
- Analyticity = implementation of relevant, closest singularities
- Crossing = decay and scattering regions are analytically connected



$$A(s,t) = \sum_l A_l(s) P_l(z_s)$$

Analyticity

$$A_l(s) = \lim_{\epsilon \to 0} A_l(s+i\epsilon)$$

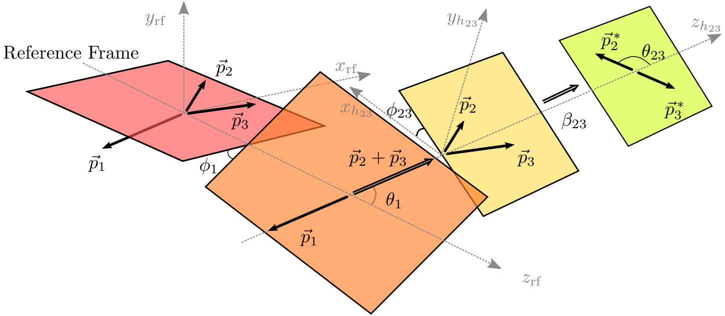


Introduction

Decay variables and decay chain

Three-body kinematics

- depends on 5 variables
- a decay chain - a sequence of decays, (here is two)



$$D^J(\phi_1, \theta_1, 0)\ X(m_{23}^2)\ D^S(\phi_{23}, \theta_{23}, 0)$$

- the full amplitude - sum of many decay chains
- the intensity - amplitude squared, summed over spin orientation

Conventional helicity approach

Complicated cases: particles with spin in isobar model [Herndon(1975)], [Hansen (1974)]

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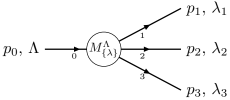
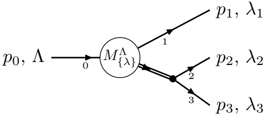
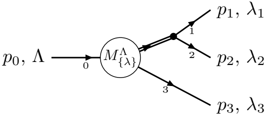
$$M^{\Lambda}_{\{\lambda\}} = M^{\Lambda}_{1,\{\lambda\}} + M^{\Lambda}_{2,\{\lambda\}} + M^{\Lambda}_{3,\{\lambda\}}$$

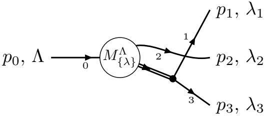
$$\underbrace{M^{\Lambda}_{\{\lambda\}}} \qquad = \underbrace{H_1 D(\phi_1, \theta_1, 0)\, D(\phi_{23}, \theta_{23}, 0) W_1(\dots)} + \underbrace{H_3 D(\phi_3, \theta_3, 0)\, D(\phi_{12}, \theta_{12}, 0) W_3(\dots)}$$







$$+ \underbrace{H_2 D(\phi_2, \theta_2, 0)\, D(\phi_{31}, \theta_{31}, 0) W_2(\dots)}$$



- A special set of angles for every decay chain

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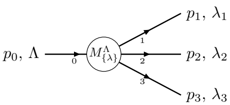

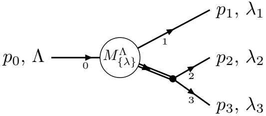

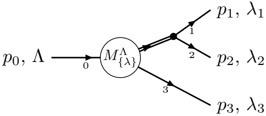

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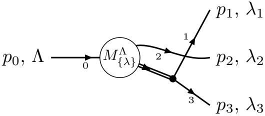
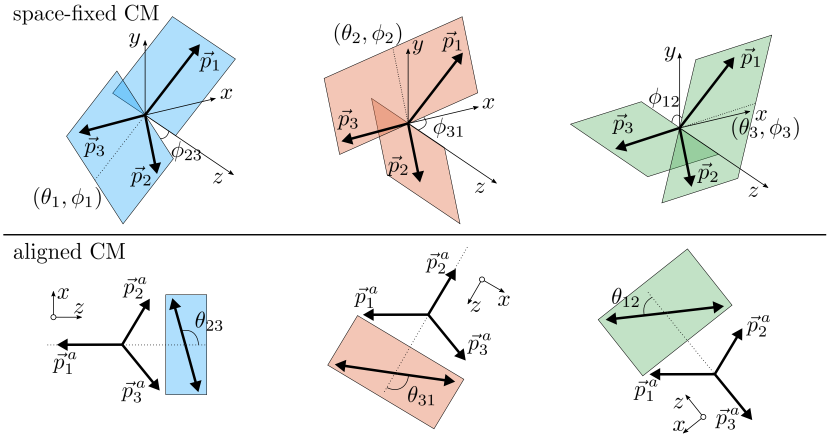


- A special set of angles for every decay chain
- Consistently of quantization direction – **Wigner rotations**

$$W_i(\dots) = D^{j_1}(\tilde{\phi}^i_1, \tilde{\theta}^i_1, 0)\, D^{j_2}(\tilde{\phi}^i_2, \tilde{\theta}^i_2, 0)\, D^{j_3}(\tilde{\phi}^i_3, \tilde{\theta}^i_3, 0)$$

Different angles for decay chains



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The Dalitz-Plot decomposition

[MM et al.(JPAC), arXiv:1910.04566]

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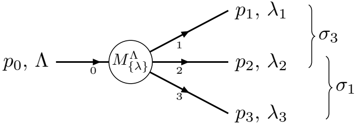
Reformulation of the helicity approach



$$= \sum_{\nu} \underbrace{D^{J*}_{\Lambda\nu}(\phi_1, \theta_1, \phi_{23})}_{\text{Decay-plane orientation}} \times \underbrace{O^{\nu}_{\{\lambda\}}(\{\sigma\})}_{\text{Dalitz-plot function}}$$

Model-independent factorization of the overall rotation:

- Exploits properties of the Lorentz group (orientation – just three Euler angles)
- Dalitz-plot function depends entirely on 2 variables, $\{\sigma\} \equiv \{\sigma_1, \sigma_2, \sigma_3\}$
- No azimuthal phase factors in $O^{\nu}_{\{\lambda\}}$.

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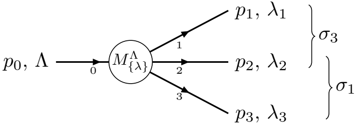

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Gives significant benefits to

- Pentaquark analysis, Λ_b/Λ_c polarionation measurements, Baryonic decay chains,. . .

Dalitz-Plot function

[MM et al.(JPAC), arXiv:1910.04566]

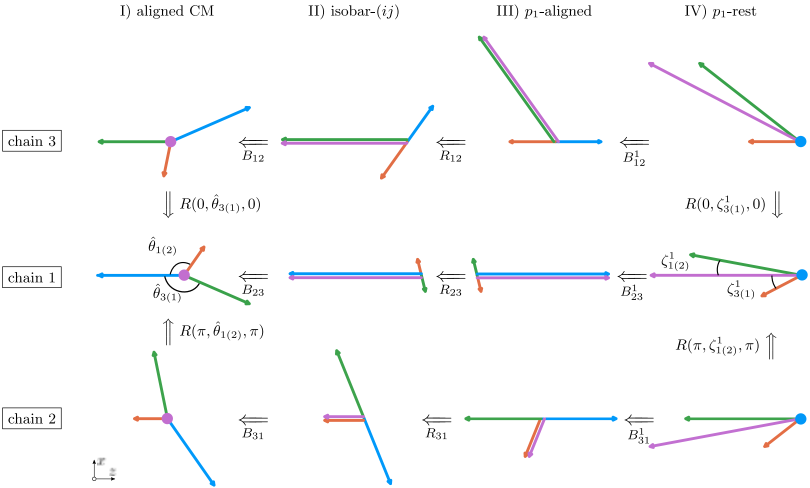
Master formula $0 \to 1\,2\,3$ decay with arbitrary spins

$$O^{\nu}_{\{\lambda\}}(\{\sigma\}) = \sum_{(ij)k} \sum_{s}^{(ij)\to i,j} \sum_{\tau} \sum_{\{\lambda'\}} n_J n_s \, d^{J}_{\nu,\tau-\lambda'_k}(\hat{\theta}_{k(1)}) X_s^{\tau,\lambda'_k;\lambda'_i,\lambda'_j}(\sigma_k) \, d^{s}_{\tau,\lambda'_i-\lambda'_j}(\theta_{ij})$$
$$\times \, d^{j_1}_{\lambda'_1,\lambda_1}(\zeta^{1}_{k(0)}) \, d^{j_2}_{\lambda'_2,\lambda_2}(\zeta^{2}_{k(0)}) \, d^{j_3}_{\lambda'_3,\lambda_3}(\zeta^{3}_{k(0)}),$$

- Three decay chains, $(ij)k \in \{(12)3, (23)1, (31)2\}$.
- $\theta_{ij} = \theta_{ij}(\{\sigma\})$ is an isobar decay angle
- $\hat{\theta}_{k(1)} = \hat{\theta}_{k(1)}(\{\sigma\})$ is the particle-0 Wigner angle
- $\zeta^{i}_{k(0)} = \zeta^{i}_{k(0)}(\{\sigma\})$ is the particle-i Wigner angle
- $X_s^{\tau,\lambda'_k;\lambda'_i,\lambda'_j}(\sigma_k) \Rightarrow X_s^{LS;l's'}(\sigma_k)$ is the only model-dependent input (lineshape functions)

Origin of the Wigner angle

[MM et al.(JPAC), arXiv:1910.04566]



$\Lambda_c \to pK\pi$ polarization studies

- proposal for the electromagnetic dipole moments of charmed baryons
  [EPJC 77 (2017) 181, arXiv:1708.08483]
- polarization information for complex decay chains

$$\boxed{M_{\lambda}^{\Lambda} = \sum_{\nu} D_{\Lambda\nu}^{1/2*}(\phi_1, \theta_1, \phi_{23}) O_{\lambda}^{\nu}(\sigma_1, \sigma_3),}$$

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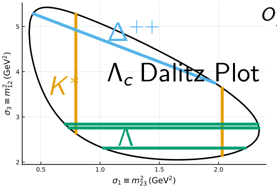
$$M_\lambda^\Lambda = \sum_\nu D_{\Lambda\nu}^{1/2*}(\phi_1, \theta_1, \phi_{23}) O_\lambda^\nu(\sigma_1, \sigma_3),$$

- Resonances in all channels, Λ^0, K^{*0}, Δ^{++}
- Possible Triangle Singularity near $\Lambda\eta$ threshold [Liu, Xiao-Hai et al., PRD100 (2019)]



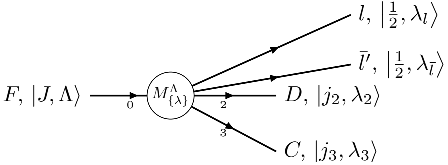
$$\begin{aligned} O_\lambda^\nu(\sigma_1, \sigma_3) = &\sum_s^{K^*\to K\pi} \sum_\tau d_{\nu,\tau-\lambda}^{1/2}(0)\, X_s^{\tau,\lambda}(\sigma_1)\, d_{\tau,0}^s(\theta_{23}) \\ &+ \sum_s^{\Delta\to\pi p} \sum_{\tau,\lambda'} d_{\nu,\tau}^{1/2}(\hat\theta_{2(1)})\, X_s^{\tau,\lambda'}(\sigma_2)\, d_{\tau,-\lambda'}^s(\theta_{31})\, d_{\lambda',\lambda}^{1/2}(\tilde\theta_{2(1)}^1) \\ &+ \sum_s^{\Lambda\to pK} \sum_{\tau,\lambda'} d_{\nu,\tau}^{1/2}(\hat\theta_{3(1)})\, X_s^{\tau,\lambda'}(\sigma_3)\, d_{\tau,\lambda'}^s(\theta_{12})\, d_{\lambda',\lambda}^{1/2}(\tilde\theta_{3(1)}^1). \end{aligned}$$

Four-body decay with leptons

Four-body decay - 8 degrees of freedom,

$$M^{\Lambda}_{\lambda_2,\lambda_3,\lambda_l,\lambda_{\bar{l}}}(\underbrace{\phi_1,\theta_1,\phi_{23}}_{\text{polarization}},\ \overbrace{q^2, m^2_{XD}, m^2_{XC}}^{\text{Dalitz-Plot dynamics}},\ \underbrace{\phi_l,\theta_l}_{\text{decay to lepton}}) =$$

horribly complicated ?



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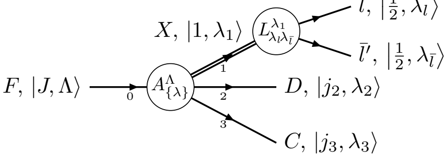

Model-independent factorization:

$$= \sum_{\nu,\lambda_1} D^{J*}_{\lambda,\nu}(\phi_1,\theta_1,\phi_{23})\, O^{\nu}_{\lambda_1,\lambda_2,\lambda_3}(q^2, m_{XD}^2, m_{XC}^2)\, D^{1*}_{\lambda_1,\lambda_l-\lambda_{\bar{l}}}(\phi_l,\theta_l,0).$$

Conclusion

Dalitz-Plot decomposition

General S-**matrix principles** and **symmetries** are the main reliable tools for dealing with non-perturbative theory

- Requirement of Lorentz Invariance
- Explicit separation of degrees of freedom

$\Rightarrow$ significant simplification of amplitude construction

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Practicalities:

- `Julia` framework ThreeBodyDecay.jl
- c++ code (thanks Vincent)

Thank you for attention

JPAC group:

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