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# Gravity, entanglement, and $\mathcal{CPT}$ violation in particle mixing

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# State of the art

- **particle mixing and oscillations: neutral kaons, B-mesons, neutrinos, neutrons, ...**
- physical fields are superpositions of free fields with different masses,
- small  $\Delta m$  → weak perturbations are also relevant,
- idea: a system where the intensity of g. interaction depends on internal degrees of freedom,
- gravity is considered as one of the possible sources of decoherence in flavor oscillations that leads to many interesting effects, in particular,  $CPT$  violation.

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# One flavored particle: Evolution

Mixing relations:

$$\begin{aligned} |\mathbf{n}_A\rangle &= \cos\theta|\mathbf{m}_1\rangle + \mathbf{e}^{i\phi}\sin\theta|\mathbf{m}_2\rangle, \\ |\mathbf{n}_B\rangle &= -\mathbf{e}^{-i\phi}\sin\theta|\mathbf{m}_1\rangle + \cos\theta|\mathbf{m}_2\rangle. \end{aligned}$$

Hamiltonian:

$$\hat{H}^{(1)} = E + \frac{c^2}{2E} \sum_{i=1,2} m_i^2 |\mathbf{m}_i\rangle\langle\mathbf{m}_i|. \quad (1)$$

Modified Hamiltonian:

$$\begin{aligned} \hat{H}^{(1)} &= \omega_0 \hat{\sigma}^z, \\ \omega_0 &= \frac{c^2}{2E} (m_1^2 - m_2^2), \\ \hat{\sigma}^z &= |\mathbf{m}_1\rangle\langle\mathbf{m}_1| - |\mathbf{m}_2\rangle\langle\mathbf{m}_2|. \end{aligned} \quad (2)$$

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Pontecorvo formula:

$$P_{n_A \rightarrow n_B}(t) = \sin^2(2\theta) \sin^2(\omega_0 t), \quad (3)$$

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$\mathcal{T}$ -symmetry is conserved!

# Two flavored particles: Evolution

$$\hat{H}^{(2)} = \hat{H}_a^{(1)} + \hat{H}_b^{(1)} - \sum_{i,j=1,2} \frac{G m_i m_j}{d} |\mathbf{m}_i, \mathbf{m}_j\rangle\langle\mathbf{m}_i, \mathbf{m}_j|. \quad (5)$$

Modified Hamiltonian:

$$\begin{aligned}\hat{H}^{(2)} &= \omega(\hat{\sigma}_a^z + \hat{\sigma}_b^z) + \Omega \hat{\sigma}_a^z \hat{\sigma}_b^z, \\ \omega &= \omega_0 + \mathbf{g}(\mathbf{m}_1^2 - \mathbf{m}_2^2), \\ \Omega &= \mathbf{g}(\mathbf{m}_1 - \mathbf{m}_2)^2, \\ \mathbf{g} &= -\frac{G}{4d}.\end{aligned}$$

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- 1) Newtonian potential;
- 2)  $m_G = m_I$

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Interaction

# Two flavored particles: Entanglement

Initial state:

$$|\psi(0)\rangle = |\mathbf{n}_\eta\rangle \otimes |\mathbf{n}_\chi\rangle. \quad (6)$$

We can measure the entanglement between the particles through:

$$\underbrace{\mathcal{P}(\rho_i(\mathbf{t})) = \text{Tr}(\rho_i^2(\mathbf{t}))}_{\text{purity of } \rho_i(\mathbf{t}) = \text{Tr}_j(|\psi(\mathbf{t})\rangle\langle\psi(\mathbf{t})|)} \implies \underbrace{S_2 = -\ln(\mathcal{P}(\rho_a(\mathbf{t})))}_{\text{2-Renyi entropy}} \quad (7)$$

Purity:

$$\mathcal{P}(\rho_i(\mathbf{t})) = 1 - \frac{1}{2} \sin^4(2\theta) \sin^2(2\Omega\mathbf{t}). \quad (8)$$

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$$\left. \begin{array}{l} \theta \neq \frac{\pi n}{2} \text{ (flavor mixing)} \\ \mathbf{t} \neq \frac{\pi n}{2\Omega} \\ \mathbf{n} \in \mathbb{Z} \end{array} \right\} \implies |\psi(\mathbf{t})\rangle \text{ is entangled}$$

# Two flavored particles: Oscillations

$$P_{n_A \rightarrow n_B}(\mathbf{t}) = \frac{1}{2} \sin^2(2\theta) [1 - \cos(2\omega\mathbf{t}) \cos(2\Omega\mathbf{t}) + \cos(2\theta) \sin(2\omega\mathbf{t}) \sin(2\Omega\mathbf{t})], \quad (9)$$

$$P_{n_B \rightarrow n_A}(\mathbf{t}) = \frac{1}{2} \sin^2(2\theta) [1 - \cos(2\omega\mathbf{t}) \cos(2\Omega\mathbf{t}) - \cos(2\theta) \sin(2\omega\mathbf{t}) \sin(2\Omega\mathbf{t})]. \quad (10)$$

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$\mathcal{T}$  violation:

$$\Delta_{\mathcal{T}}(\mathbf{t}) = \sin^2(2\theta) \cos(2\theta) \sin(2\omega\mathbf{t}) \sin(2\Omega\mathbf{t})$$

# Two flavored particles: Oscillations

**TOO WEAK!**

**Solution:** considering more particles

$$P_{n_A \rightarrow n_B}(\mathbf{t}) = \frac{1}{2} \sin^2(2\theta) [1 - \cos(2\omega\mathbf{t}) \cos(2\Omega\mathbf{t}) + \cos(2\theta) \sin(2\omega\mathbf{t}) \sin(2\Omega\mathbf{t})], \quad (9)$$

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# $N$ flavored particles: Evolution

**Hamiltonian:**

$$\hat{\mathcal{H}}^{(2)} = \sum_i \omega_i \hat{\sigma}_i^z + \sum_{i,j} \Omega_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z,$$

$$\omega_i = \omega_0 + \sum_j \mathbf{g}_{ij} (\mathbf{m}_1^2 - \mathbf{m}_2^2),$$

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**Initial state:**

$$|\psi(0)\rangle = \otimes_{i=1}^M |\mathbf{n}_A\rangle_i \otimes_{j=M+1}^N |\mathbf{n}_B\rangle_j. \quad (11)$$

Reduced density matrix:

$$\rho_i(\mathbf{t}) = \frac{1}{2} \begin{pmatrix} 1 + \zeta_i \cos(2\theta) & \zeta_i e^{-i\phi} \sin(2\theta) \mathbf{a}_i^*(\mathbf{t}) \\ \zeta_i e^{i\phi} \sin(2\theta) \mathbf{a}_i(\mathbf{t}) & 1 - \zeta_i \cos(2\theta) \end{pmatrix}, \quad (12)$$

$$\mathbf{a}_i(\mathbf{t}) = e^{i\omega_i t} \prod_{j=1}^N (\cos(2\Omega_{ij}\mathbf{t}) + i\zeta_j \cos(2\theta) \sin(2\Omega_{ij}\mathbf{t})),$$

$$\zeta_i = \text{sgn}(M - i).$$

Purity:

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# $N$ flavored particles: Oscillations

For  $|\psi(0)\rangle = \otimes_{i=1}^N |\mathbf{n}_{A/B}\rangle_i$ :

$$\begin{aligned} P_{\mathbf{n}_A \rightarrow \mathbf{n}_B}(\mathbf{t}) &= \frac{1}{2} \sin^2(2\theta) [1 - \frac{1}{N} \sum_i \text{Re}(\mathbf{a}_i^{(A)}(\mathbf{t}))], \\ P_{\mathbf{n}_B \rightarrow \mathbf{n}_A}(\mathbf{t}) &= \frac{1}{2} \sin^2(2\theta) [1 - \frac{1}{N} \sum_i \text{Re}(\mathbf{a}_i^{(B)}(\mathbf{t}))]. \end{aligned}$$

For small  $\Omega_{ij}\mathbf{t}$ :

$$\begin{aligned} \Delta\tau(\mathbf{t}) &= \sin^2(2\theta) \cos(2\theta) \frac{2\mathbf{t}}{N} \sum_{i,j=1}^N \sin(2\omega_i \mathbf{t}) \Omega_{ij} \\ &= \sin^2(2\theta) \cos(2\theta) 2N \langle \sin(2\omega_i \mathbf{t}) \Omega_{ij} \rangle_{ij} \mathbf{t}. \end{aligned} \tag{14}$$

For  $|\psi(0)\rangle = \otimes_{i=1}^{N/2} |\mathbf{n}_A\rangle_i \otimes_{j=N/2+1}^N |\mathbf{n}_B\rangle_j$ :

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$$\begin{aligned} \Delta_{\mathcal{T}}(\mathbf{t}) &= \sin^2(2\theta) \cos(2\theta) \frac{2\mathbf{t}}{N} \sum_{i,j=1}^N \sin(2\omega_i \mathbf{t}) \Omega_{ij} \\ &= \sin^2(2\theta) \cos(2\theta) 2N \langle \sin(2\omega_i \mathbf{t}) \Omega_{ij} \rangle_{ij} \mathbf{t}. \end{aligned} \tag{14}$$

For  $|\psi(0)\rangle = \otimes_{i=1}^{N/2} |\mathbf{n}_A\rangle_i \otimes_{j=N/2+1}^N |\mathbf{n}_B\rangle_j$ :

$$\Delta_{\mathcal{T}} \sim \sqrt{N}. \tag{15}$$

# $N$ flavored particles: Oscillations

For  $|\psi(0)\rangle = \otimes_{i=1}^N |\mathbf{n}_{A/B}\rangle_i$ :

$$P_{n_A \rightarrow n_B}(\mathbf{t}) = \frac{1}{2} \sin^2(2\theta) [1 - \frac{1}{N} \sum_i \text{Re}(\mathbf{a}_i^{(A)}(\mathbf{t}))],$$

$$P_{n_B \rightarrow n_A}(\mathbf{t}) = \frac{1}{2} \sin^2(2\theta) [1 - \frac{1}{N} \sum_i \text{Re}(\mathbf{a}_i^{(B)}(\mathbf{t}))].$$

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many-body effect

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- Gravity in a system of mixed self-interacting particles leads to a  $CPT$  violation.
- The  $CPT$  violation is related to a non-zero entanglement among the particles due to the non-zero mass difference.
- In contrast to the open system approach, the  $CPT$  violation is caused by a  $T$  violation.
- The  $CPT$  violation is a many-body effect being proportional to a number of elements in the system and its density, so it could play a crucial role in galactic and in first stages of the Universe.

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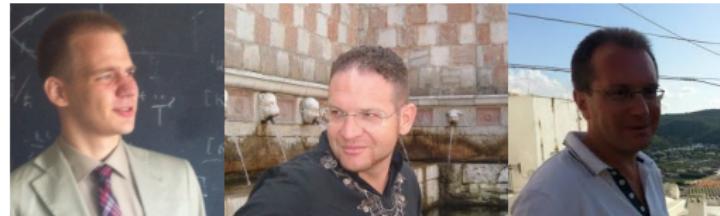
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- The presented result is not limited to the gravity and can be generalized with requirements:
  - ① the presence of neutral particles whose flavor states are superpositions of the eigenstates of a free Hamiltonian,
  - ② the presence of an interaction depending on the eigenstates of the free Hamiltonian.
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# THANK YOU FOR YOUR ATTENTION!

K. Simonov, A. Capolupo, and S. M. Giampaolo, arXiv:1903.10266 (2019).