The onset of $\Lambda\Lambda$ hypernuclear binding

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(Phys. Lett. B797 (2019) 134893)

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Why all this work ?

What we have ?

- experimentaly observed more than 30 Λ-hypernuclei and three well-established $\Lambda\Lambda$-hypernuclei (emulsion experiments)
  $\rightarrow$ available experimental B_Λ separation energies
- rather precise spectroscopic Λ-hypernucear data (for p-shell hypernuclei extremely precise)

on the other hand ...

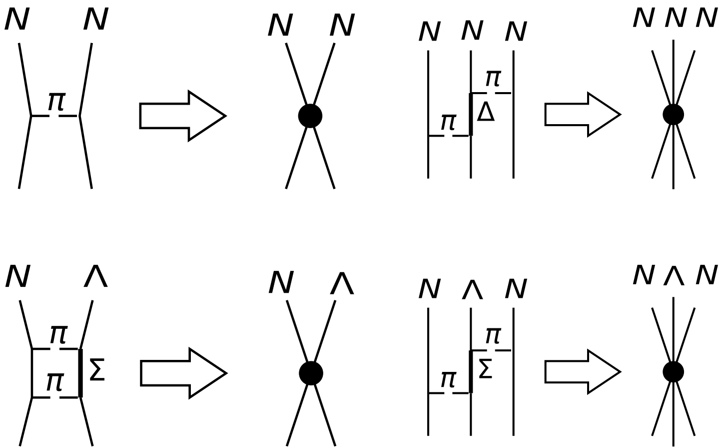
- scarce ΛN scattering data
  $\rightarrow$ large theoretical model dependencies

s-shell hypernuclei

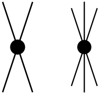
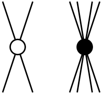
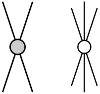
- precise B_Λ separation energies
- few-body character of these systems makes easier to track effects of underlying hypernuclei interaction
- $\Lambda N - \Sigma N$ mixing, Charge Symmetry Breaking
- ${}^{5}_{\Lambda}\mathrm{He}$ overbinding problem
- question of bound ${}^{5}_{\Lambda\Lambda}\mathrm{He}$ (J-PARC P75 proposal) and ${}^{4}_{\Lambda\Lambda}\mathrm{H}(1^+)$
- so far theoretically debated $nn\Lambda$, $n\Lambda\Lambda$, $nn\Lambda\Lambda$

Single Λ pionless EFT - basic idea

(L. Contessi, N. Barnea, and A. Gal, Phys. Rev. Lett. 121 (2018) 102502)

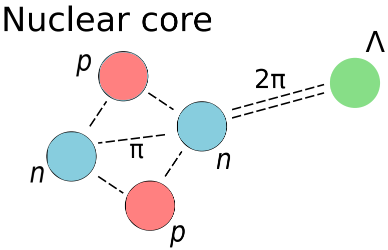


Pionless EFT - Expansion and derivatives

LO			$\delta(\mathbf{r}_{12})$, $\delta(\mathbf{r}_{12})\delta(\mathbf{r}_{23})$
NLO		. . .	$\nabla^2_{\mathbf{r}_{12}}\delta(\mathbf{r}_{12})$, $\delta(\mathbf{r}_{12})\delta(\mathbf{r}_{23})\delta(\mathbf{r}_{34})$
N^2LO		. . .	$(\nabla_{\mathbf{r}_1} \cdot \nabla_{\mathbf{r}_2})\delta(\mathbf{r}_{12})$, tensor, LS, more 3-body ?
$N^{>2}LO$	. . .		

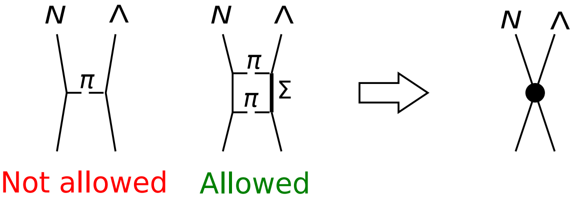
Single-Λ pionless EFT - scales

(L. Contessi, N. Barnea, and A. Gal, Phys. Rev. Lett. 121 (2018) 102502)



- Λ is weakly bound to the nuclear core (small typical exchange momentum Q)
- long distance forces → 2π exchange (≈ 280 MeV)

$$\frac{Q}{2m_\pi} << 1$$



Single-Λ pionless EFT

(L. Contessi, N. Barnea, and A. Gal, Phys. Rev. Lett. 121 (2018) 102502)

- nuclear pionless EFT has large truncation error at LO ($Q < M$)
  $\rightarrow$ however, it works well in few body physics
- hypernuclear pionless EFT has good accuracy at LO ($Q << M$)

$\rightarrow$ Truncation error at LO : $\delta_{LO} = \left(\frac{Q}{M}\right)^2 \approx \left(\frac{\sqrt{2M_\Lambda B_\Lambda}}{2m_\pi}\right)^2 = 9\%$

Regularization/Renormalization

$$C\ \delta(\mathbf{r}_{ij}) \rightarrow C(\lambda)\left(\frac{\lambda}{2\sqrt{\pi}}\right)^3 \mathrm{e}^{\frac{-\lambda^2 r_{ij}^2}{4}}$$

$$D\ \delta(\mathbf{r}_{ij})\delta(\mathbf{r}_{jk}) \rightarrow D(\lambda)\left(\frac{\lambda}{2\sqrt{\pi}}\right)^6 \mathrm{e}^{\frac{-\lambda^2 (r_{ij}^2 + r_{jk}^2)}{4}}$$

- $C(\Lambda), D(\lambda)$ are low energy constants (LECs) tuned to reproduce two-body resp. three-body observables for each λ
- required (RG invariance for $\lambda >> M$)
  $\rightarrow$ all observable will become λ independent when $\lambda \rightarrow \infty$

Single-Λ pionless EFT - fitting low energy constants

(L. Contessi, N. Barnea, and A. Gal, Phys. Rev. Lett. 121 (2018) 102502)

- 8 LEC constants (3 nuclear and 5 hypernuclear) for each value of cut-off λ which we fit using available experimental data

Nuclear LECs :

C_1	NN	$S = 0$ $I = 1$	$a_{NN}(^1S_0)$	= -18.63 fm
C_2	NN	$S = 1$ $I = 0$	$B(^2\mathrm{H})$	= 2.22452 MeV
D_1	NNN	$S = \frac{1}{2}$ $I = \frac{1}{2}$	$B(^3\mathrm{H})$	= 8.482 MeV

Hypernuclear LECs :

C_3	ΛN	$S = 0$ $I = \frac{1}{2}$	$a_{\Lambda N}(^1S_0)$	
C_4	ΛN	$S = 1$ $I = \frac{1}{2}$	$a_{\Lambda N}(^3S_1)$	
D_2	ΛNN	$S = \frac{1}{2}$ $I = 0$	$B_\Lambda(^3_\Lambda\mathrm{H})$	= 0.13(5) MeV
D_3	ΛNN	$S = \frac{3}{2}$ $I = 0$	$B_\Lambda(^4_\Lambda\mathrm{H}, 0^+)$	= 2.16(8) MeV
D_4	ΛNN	$S = \frac{1}{2}$ $I = 1$	$E_{ex}(^4_\Lambda\mathrm{H}, 1^+)$	= 1.09(2) MeV

ΛΛ hypernuclei

extreme lack of experimental data
→ just 3 experimental data vs. dozens for single-Λ hypernuclei
($^{6}_{\Lambda\Lambda}\mathrm{He}$, $^{10}_{\Lambda\Lambda}\mathrm{Be}$, $^{13}_{\Lambda\Lambda}\mathrm{B}$)

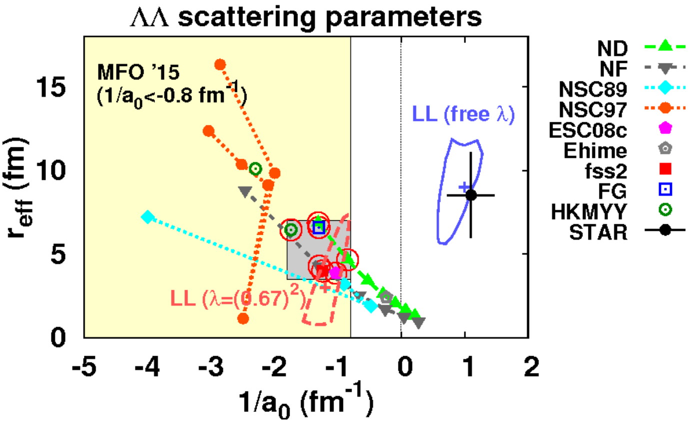
$^{6}_{\Lambda\Lambda}\mathrm{He}$ **Nagara event**

- $\Delta B_{\Lambda\Lambda} = 0.67 \pm 0.17$ MeV, KEK-E373, (PRL87, 212505, 2001)

Other measuremets

- no conclusive evidence of bound $^{4}_{\Lambda\Lambda}\mathrm{H}(1^{+})$ system; AGS-E906 counter experiment (PRL87 132504, 2001; PRC76, 064308, 2007)
- neutral $^{4}_{\Lambda\Lambda}\mathrm{n}$ system assigned to the main yet unexplained signal; AGS-E906 counter experiment (PLB790, 502, 2019)
- search for $^{5}_{\Lambda\Lambda}\mathrm{H}$ system included in a recent J-PARC proposal (submitted 18 December 2018)

ΛΛ interaction



- invariant mass spectrum, (PRC85, 015204, 2012)
  $^{12}C(K^-,K^+)\Lambda\Lambda X$, $a_{\Lambda\Lambda} = -1.2 \pm 0.6$ fm
- ΛΛ correlations, STAR experiment (Morita)
  $a_{\Lambda\Lambda} = -0.79^{(+0.29)}_{(-1.13)}$ fm, $r_{eff} = 1.76 \pm 11.62$ fm
  (PRC91, 024916,2016; PRL114,022301, 2015)
- χEFT, (NPA954, 273, 2016)
  (LO) $a_{\Lambda\Lambda} = -1.52$ fm, $r_{eff} = 0.59$ fm
  (NLO) $a_{\Lambda\Lambda} = -0.66$ fm, $r_{eff} = 5.05$ fm

Previous works - ΛΛ few-body systems

- **Filikhin and Gal** (PRL89,172502, 2002)
  ${}^{4}_{\Lambda\Lambda}\mathrm{H}(1^{+})$, Faddeev-Yakubovski calculations, ΛΛ fitted to $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He})$, no bound state
- **Nemura at al.** (PRC67, 051001(R), 2003)
  ${}^{4}_{\Lambda\Lambda}\mathrm{H}(1^{+})$, Stochastic Variational calculations, interactions same as above, bound state
- **Nemura et al.** (PRL94, 202502, 2005)
  all s-shell ΛΛ hypernuclei, Stochastic Variational full coupled-channel calculations, simulated form of *YN* and *YY* Nijmegen potentials, onset at $B_{\Lambda}({}^{4}_{\Lambda\Lambda}\mathrm{H}(1^{+}))$=2 keV
- **J.-M. Richard et al.** (PRC91, 014003, 2015)
  ${}^{4}_{\Lambda\Lambda}\mathrm{n}(0^{+})$, claim that a stability of this system is within uncertainties of the baryon-baryon interaction
- **H. Garcilazo et al.** (Chin. Phys. C41, 0741102, 2017)
  ${}^{4}_{\Lambda\Lambda}\mathrm{n}(0^{+})$, *NN* Yukawa type Malfliet-Tjon interaction and *N*Λ, ΛΛ ESC08c Nijmegen potential, unbound - just above the threshold

Double-Λ pionless EFT

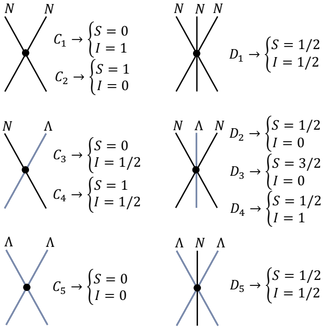
LO pionless EFT interaction (s-wave only):

$$\mathcal{L}^{(\mathrm{LO})} = \sum_B B^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M_B}\right) B - \mathcal{V}_2 - \mathcal{V}_3$$

$B \ldots N, \Lambda$ baryonic fields

$$V_2 = \sum_{IS} C_\lambda^{IS} \sum_{i<j} \mathcal{P}_{IS}(ij)\delta_\lambda(\vec{r}_{ij})$$

$$V_3 = \sum_{\alpha IS} D_{\alpha\lambda}^{IS} \sum_{i<j<k} \mathcal{Q}_{IS}(ijk) \left(\sum_{\mathrm{cyc}} \delta_\lambda(\vec{r}_{ij})\delta_\lambda(\vec{r}_{jk})\right)$$



$\rightarrow$ two new ΛΛ LEC constants C_5 and D_5

$C_5 \ldots$ fitted to $a_{\Lambda\Lambda}$ scattering length

$\ldots\ a_{\Lambda\Lambda} \in \langle -1.9; -0.5\rangle$ fm

$D_5 \ldots$ fitted to $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) = 0.67 \pm 0.17$ MeV

$\ldots\ \Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) = B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) - 2B_\Lambda({}^5_\Lambda\mathrm{He})$

Double-Λ pionless EFT

What is the breakup scale of our theory ?

- ΛΛ one-pion-exchange (OPE) forbidden by isospin invariance
- the lowest mass pseudoscalar mesone exchange provided by η (0.4 fm) (short range, rather weak)
- excitation from ΛΛ into intermediate ΞN through short-range K meson exchange accounted for implicitely by the chosen pionless EFT interaction (together with other short range exchanges)
- pions appear through excitation to fairly high-lying ΣΣ intermediate states

$\rightarrow$ **our breakup scale is** $2m_\pi$

What is the LO accuracy ?

- $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) < 1$ MeV

$\rightarrow$ Λ momentum scale Q in ${}^{6}_{\Lambda\Lambda}\mathrm{He}$ can be roughly approximated by the one in ${}^{5}_{\Lambda}\mathrm{He}$
$p_\Lambda \approx \sqrt{2M_\Lambda B_\Lambda}$=83 MeV/c
$\rightarrow$ expansion parameter $(Q/2m_\pi)$ ≈0.3
$\rightarrow$ leading order accuracy $(Q/2m_\pi)^2$ ≈0.09

Double-Λ pionless EFT

What is the role of $\Lambda\Lambda - \Xi N$?

- included implicitely in $a_{\Lambda\Lambda}$
- for ${}^{6}_{\Lambda\Lambda}\mathrm{He}$ $\Lambda\Lambda - \Xi N$ channel is partially Pauli blocked (newly created neutron has to go into p-shell)

$\rightarrow$ it can be argued that the value of $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He})$ used to fix $\Lambda\Lambda N$ LEC constant should be somewhat increased in order to implicitely account for the ΞN channel in lighter systems

- full coupled-channel calculations suggest increase of $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He})$ by ≈ 0.25 MeV (PRC70, 024306, 2004)

s-shell systems

A=2	A=3	A=4	A=5	A=6
$a_{NN}(^1S_0)$ $^2\mathrm{H}(1^+)$	$^3\mathrm{H}(\frac{1}{2}^+)$	$^4\mathrm{He}(0^+)$		
$a_{N\Lambda}(^1S_0)$ $a_{N\Lambda}(^3S_1)$	$^3_\Lambda\mathrm{H}(\frac{1}{2}^+)$ $^3_\Lambda\mathrm{H}(\frac{3}{2}^+)$ $^3_\Lambda\mathrm{n}(\frac{1}{2}^+)$	$^4_\Lambda\mathrm{H}(0^+)$ $^4_\Lambda\mathrm{H}(1^+)$	$^5_\Lambda\mathrm{He}(\frac{1}{2}^+)$	
$a_{\Lambda\Lambda}(^1S_0)$	$^3_{\Lambda\Lambda}\mathrm{n}(\frac{1}{2}^+)$	$^4_{\Lambda\Lambda}\mathrm{H}(1^+)$ $^4_{\Lambda\Lambda}\mathrm{n}(0^+)$	$^5_{\Lambda\Lambda}\mathrm{H}(\frac{1}{2}^+)$	$^6_{\Lambda\Lambda}\mathrm{He}(0^+)$

. . . fitted (scattering lengths, bound state energies)

. . . prediction (bound states, resonances)

Stochastic Variational Method

(K. Varga et al., Nucl. Phys. **A 571** (1994) 447)
(K. Varga, Y. Suzuki, Phys. Rev. **C 52** (1995) 2885)

optimizes variational basis in a **random trial and error procedure**

Variational basis states

- antisymmetrized correlated Gaussians (assuming L=0)

$$\psi_{SM_S TM_T}(\mathbf{x}, A) = \mathcal{A}\{G_A(\mathbf{x})\chi_{SM_S}\eta_{TM_T}\}, \quad G_A(\mathbf{x}) = e^{-\frac{1}{2}\mathbf{x}A\mathbf{x}}$$

- Jacobi coordinates $\mathbf{x}$, symmetric positive definite matrix of variational parameters A, spin χ_{SM_S} and isospin η_{TM_T} parts
- $\frac{N(N-1)}{2}$ real parameters for one basis state
- explicit antisymmetrization → computational complexity grows with N!

$$\mathcal{A} = \sum_{i=1}^{N!} p_i \mathcal{P}_i$$

${}^{4}_{\Lambda\Lambda}\mathrm{H}$ ($I = 0$, $J^{\pi} = 1^{+}$) hypernucleus

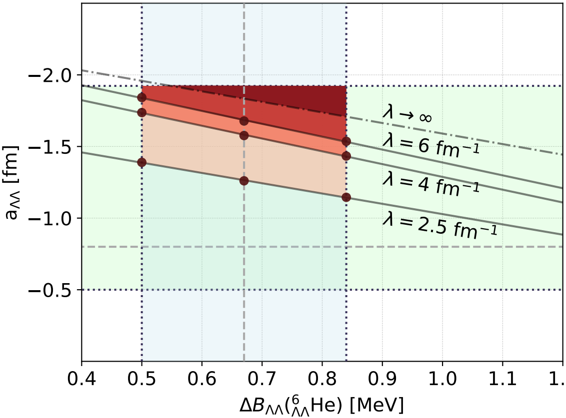


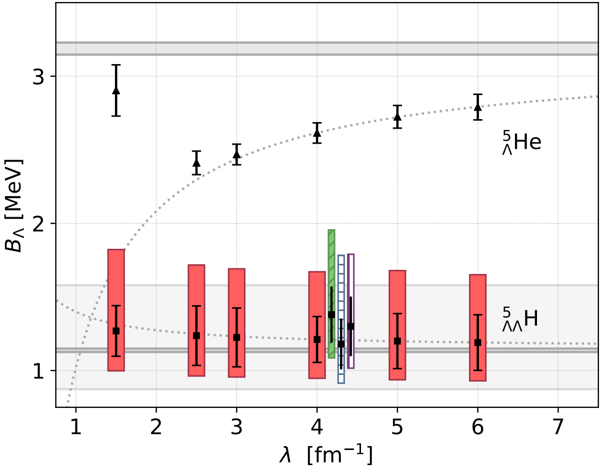
Figure: Minimum values of $|a_{\Lambda\Lambda}|$ for which ${}_{\Lambda\Lambda}{}^{4}\mathrm{H}$ becomes bound are plotted, for given values of cutoff λ, as a function of $\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}{}^{6}\mathrm{He})$. The vertical dotted lines mark the experimental uncertainty of $\Delta B_{\Lambda\Lambda}$. The horizontal dotted lines mark the range of $a_{\Lambda\Lambda}$ values $[-0.5, -1.9]$ fm suggested by studies of ΛΛ correlations. The $\lambda \to \infty$ limit is reached assuming a Q/λ asymptotic behavior.

Double-Λ neutral systems

Table: Λ separation energies $B_\Lambda({}_{\Lambda\Lambda}^{A}\mathrm{Z})$ for A=3–6, calculated using $a_{\Lambda\Lambda}=-0.8$ fm, cutoff $\lambda=4\ \mathrm{fm}^{-1}$ and the Alexander[B] ΛN interaction model. In each row a $\Lambda\Lambda N$ LEC was fitted to the underlined binding energy constraint.

Constraint (MeV)	${}_{\Lambda\Lambda}^{3}\mathrm{n}$	${}_{\Lambda\Lambda}^{4}\mathrm{n}$	${}_{\Lambda\Lambda}^{4}\mathrm{H}$	${}_{\Lambda\Lambda}^{5}\mathrm{H}$	${}_{\Lambda\Lambda}^{6}\mathrm{He}$
$\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{6}\mathrm{He})=\underline{0.67}$	–	–	–	1.21	3.28
$B_\Lambda({}_{\Lambda\Lambda}^{4}\mathrm{H})=\underline{0.05}$	–	–	0.05	2.28	4.76
$B({}_{\Lambda\Lambda}^{4}\mathrm{n})=\underline{0.10}$	–	0.10	0.86	4.89	7.89
$B({}_{\Lambda\Lambda}^{3}\mathrm{n})=\underline{0.10}$	0.10	15.15	18.40	22.13	25.66

${}^{5}_{\Lambda\Lambda}\mathrm{H}$ ($I = 1/2$, $J^{\pi} = 1/2^{+}$) hypernucleus



$B_{\Lambda}({}^{5}_{\Lambda\Lambda}\mathrm{H};\infty) = 1.14 \pm 0.01^{+0.44}_{-0.26}$ MeV

${}^{5}_{\Lambda\Lambda}\mathrm{H}$ ($I = 1/2$, $J^{\pi} = 1/2^{+}$) hypernucleus

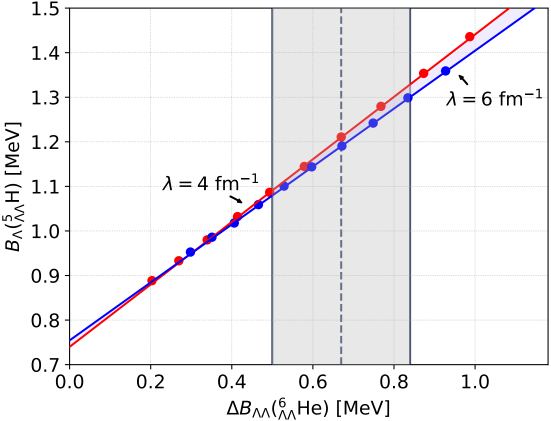


Figure: Calculated Λ separation energies $B_{\Lambda}({}_{\Lambda\Lambda}{}^{5}\mathrm{H})$ are plotted as a function of the constrained value assumed for $\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}{}^{6}\mathrm{He})$ for two cutoff values, using $a_{\Lambda\Lambda}$=−0.8 fm. The shaded vertical area marks the observed value $\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}{}^{6}\mathrm{He})$=0.67±0.17 MeV.

s-shell ΛΛ hypernuclei from pionless EFT - Conclusions

- successful extension of LO pionless EFT to $S = -2$ sector (covered the whole range of s-shell ΛΛ hypernuclei)
- comprehensive study of the onset of ΛΛ hypernuclear binding
  $\rightarrow a_{\Lambda\Lambda} \in \langle -1.9; -0.5 \rangle$ fm (ΛΛ correlations, multiple interaction models)
  $\rightarrow \Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He})$=0.67 ± 0.17 MeV (Nagara event)

${}^{5}_{\Lambda\Lambda}\mathrm{H}$

- particle stable taking into account both theoretical and experimental uncertainties
- $B_{\Lambda}({}^{5}_{\Lambda\Lambda}\mathrm{H}; \infty)$=$1.14 \pm 0.01^{+0.44}_{-0.26}$ MeV

${}^{4}_{\Lambda\Lambda}\mathrm{H}(1^{+})$

- particle stability requires value $|a_{\Lambda\Lambda}| \geq$1.5 fm
- for bound system ΛΛ intreaction would acquire almost the same strength as ΛN interaction → rather unlikely

neutral systems ${}^{4}_{\Lambda\Lambda}\mathrm{n}$, ${}^{3}_{\Lambda\Lambda}\mathrm{n}$

- rather far from being bound
- observed experimentally would be with serious disagreement with experimentally established $\Delta B_{\Lambda\Lambda}$ of Nagara event

Current work - few-body hypernuclear resonances

s-shell Λ hypernuclear candidates :

- ${}^{3}_{\Lambda}\mathrm{H}$ ($J^{\pi} = \frac{3}{2}^{+}$), $\mathrm{nn}\Lambda$ ($J^{\pi} = \frac{1}{2}^{+}$)

s-shell ΛΛ hypernuclear candidates :

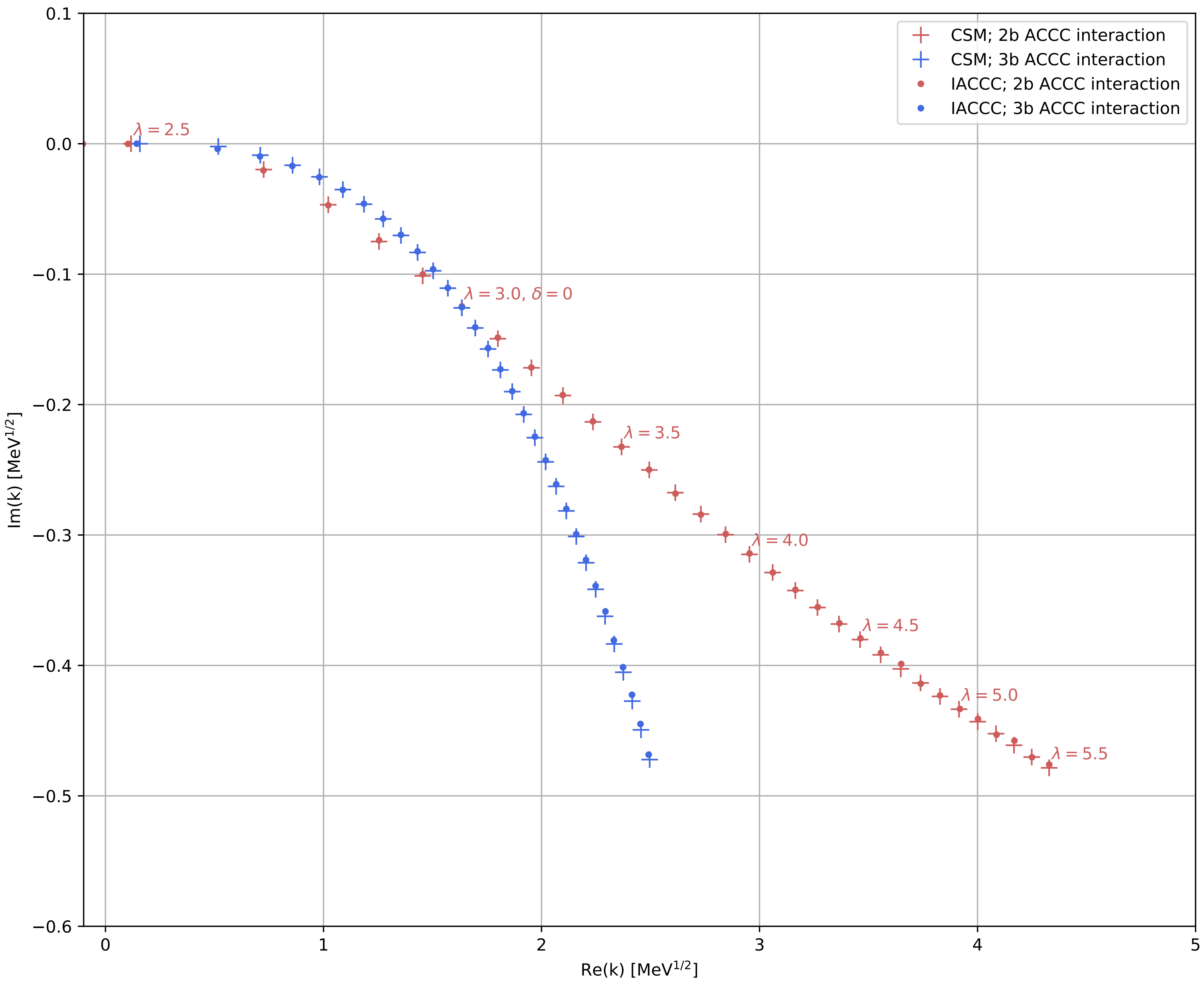
- $\mathrm{n}\Lambda\Lambda$ ($J^{\pi} = \frac{1}{2}^{+}$), ${}^{4}_{\Lambda\Lambda}\mathrm{n}$ ($J^{\pi} = 0^{+}$), ${}^{4}_{\Lambda\Lambda}\mathrm{H}$ ($J^{\pi} = 1^{+}$)

$\rightarrow$ **developed SVM-CSM-IACCC code**

- Complex Scaling Method (CSM)
  Inverse Analytical Continuation in Coupling Constant (IACCC)
- stability, systematic increase of an accuracy (benchmark calculations)
- maximal number of particles given by the SVM (computational)

$\rightarrow$ ${}^{3}_{\Lambda}\mathrm{H}$ ($J^{\pi} = \frac{3}{2}^{+}$), $\mathrm{nn}\Lambda$ ($J^{\pi} = \frac{1}{2}^{+}$)

3-body bosonic system with 1+1+1 resonance; Phys. Rev. C 56 (1997) 56



→ borromean system (no 2-body or 3-body bound states)

$H = T_k + V^{phys} + V^{accc}$

ACCC using 2-body repulsive interaction :

$V^{phys} = -120e^{-r^2}$

$V^{accc} = 3\lambda e^{-\frac{r^2}{9}}$

ACCC using 3-body attractive interaction :

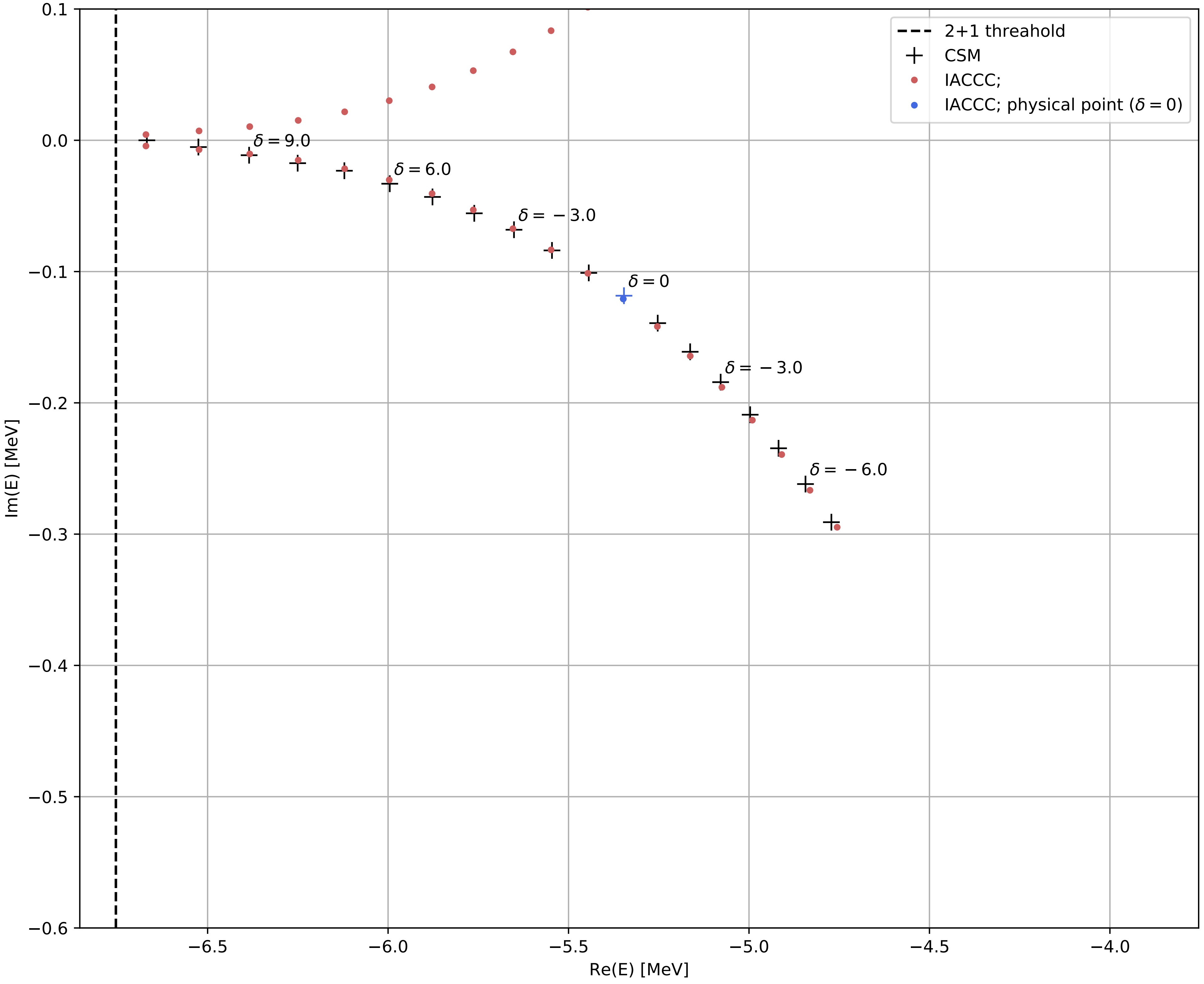
$V^{phys} = -120e^{-r^2} + 9e^{-\frac{r^2}{9}}$

$V^{accc} = -\delta \sum_{i<j<k} \sum_{cyc} e^{-0.25(\vec{r}_{ij} + \vec{r}_{jk})}$

→ both calculations must coincide for $\lambda = 3$ and $\delta = 0$

k_r(CSM)= 1.6347 - 0.1259i MeV$^{1/2}$
k_r(IACCC2b)= 1.6350 - 0.1247i MeV$^{1/2}$
k_r(IACCC3b)= 1.6347 - 0.1253i MeV$^{1/2}$

3-body bosonic system with 2+1 resonance



→ one 2-body and one 3-body bound state

E_{2b} = -6.75414 MeV
E_{3b} = -37.24517 MeV

ACCC using 3-body attractive interaction :

$H = T_k + V^{phys} + V^{accc}$

→ benefit of not affecting 2+1 threshold

$$V^{phys} = -55e^{-0.2r^2} + 1.5e^{-0.01(r-5)^2}$$
$$V^{accc} = -\delta \sum_{i<j<k} \sum_{cyc} e^{-0.25(\vec{r}_{ij} + \vec{r}_{jk})}$$

comparison with other available results :

E_r(CSM)= -5.3464 - 0.1184i MeV
E_r(IACCC)= -5.3484 - 0.1208i MeV
E_r [1]= -5.310 - 0.117i MeV
E_r [2]= -5.96 - 0.4i MeV
E_r [3]= -5.32(1) MeV (width not given)

[1] Phys. Rev. A75 (2007) 042508
[2] Phys. Rev. C98 (2018) 034004
[3] Few. Body Syst. 53 (2007) 153