# Impact of chiral symmetry constraints on the pole content of the antikaon-nucleon scattering amplitude

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# Outline

# $S=-1$ meson-baryon scattering and the $\Lambda(1405)$

- $I=0$ s-wave resonance $\Lambda(1405)$ just below the $K^- p$ threshold.
- Important e.g. for the formation of deeply bound kaon-nuclear states.
- State-of-the-art description: LSE or BSE for meson-baryon scattering, with kernels derived from a chiral Lagrangian.
- Famous result: "Two-pole structure" of $\Lambda(1405)$
  D. Jido, J. A. Oller, E. Oset, A. Ramos and U.-G. Meißner, Nucl. Phys. A **725** (2003) 181 .

# BChPT and LSE (BSE)

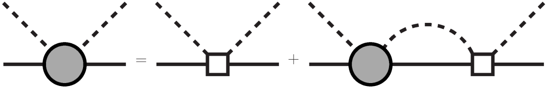
- Due to the presence of the $\Lambda(1405)$ in the threshold region, the perturbative low-energy expansion of BChPT is not effective for $S = -1$ meson-baryon scattering.
- Approach here: Chiral expansion of the kernel (derived from BChPT), infinite iteration of this truncated kernel via LSE/BSE.
- Additional approximations are necessary. E.g., crossing symmetry is sacrificed for exact (coupled-channel) unitarity.

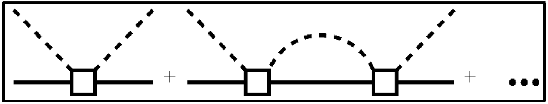
# BChPT and LSE (BSE)

Iteration corresponds (more or less) to a resummation of a class of loop graphs in BChPT.

$$T(q',q;P) = V(q',q;P) + \int \frac{d^4 l}{(2\pi)^4} T(q',l;P) \frac{(-i)}{(\not{P} - \not{l} - M)(l^2 - m^2)} V(l,q;P)\,.$$

Here $P =$ overall c.m. four-momentum, $P^2 =: s$ (Mandelstam variable).
Note the dependence of $T$ and kernel $V$ on the loop momentum $l$ in the integral.





# BChPT and LSE (BSE)

- Very often, the so-called "on-shell factorization" is employed: the vertex $V$ is treated as an on-shell amplitude ($l^2 \to m^2$, baryon four-momenta $\not{p}_B \to M$, etc.), and a partial-wave expansion is performed **inside** the loop integrals.
- The LSE (or BSE) then reduces to a simple geometric series.
- We will shortly see how the effect of this procedure can be quantified.
- Recently, this procedure (and the conclusions drawn from its results) has been criticised by J. Révai,
  "Are the chiral based $\bar{K}N$ potentials really energy dependent?",
  Few Body Syst. **59** (2018) no.4, 49,
  - "Energy dependence of the $\bar{K}N$ interaction and the two-pole structure of the $\Lambda(1405)$ – are they real?," arXiv:1811.09039 [nucl-th].

# Révai's approach

In his work he uses non-relativistic kinematics, e.g. for the c.m. momentum

$$\bar{q}_{\rm cm} \to k = \sqrt{2\mu(\sqrt{s} - m - M)}\,,$$

$\mu := mM/(M+m)$, and writes the leading (Weinberg-Tomozawa) kernel as

$$u(q)\,(\gamma(q)\lambda + \lambda\gamma(q))\,u(q)\,,$$

with a coupling matrix $\lambda$ in the coupled-channel space, $\gamma(q) := \frac{q^2}{2\mu} + m$ and form factors $u(q) := \frac{\beta^4}{(\beta^2+q^2)^2}$, with inverse ranges $\beta$ to be fitted to data.
Note that on shell, the meson momentum $q \to k$. Révai obtains the solution to the LSE without employing the "on-shell approximation".
His solution "…supports only **one** pole in the region of the $\Lambda(1405)$ resonance. Thus the almost overall accepted view, that chiral-based interactions lead to a two-pole structure of the $\Lambda(1405)$, becomes questionable."

## Révai's approach

In the course of the solution of the LSE, integrals like

$$
\begin{aligned}
G_{AA} &:= 8\pi\mu\int_0^\infty \frac{q^2(u(q))^2dq}{k^2-q^2+i\epsilon} \\
&= -4\pi^2\mu(u(k))^2\left(\frac{\beta}{16}\left(5-15\,(k/\beta)^2-5\,(k/\beta)^4-(k/\beta)^6\right)+ik\right)\,, \qquad (1) \\
G_{AB} &:= 8\pi\mu\int_0^\infty \frac{q^2(u(q))^2\gamma(q)dq}{k^2-q^2+i\epsilon} = \bar{\gamma}G_{AA}-I_0\,, \qquad (2) \\
I_n &:= \frac{4\pi}{(2\mu)^n}\int_0^\infty q^2(u(q))^2(q^2-k^2)^n dq\,, \qquad (3)
\end{aligned}
$$

occur. Note that setting $\gamma(q)\to\gamma(k)\equiv\bar{\gamma}$ in the numerator of the integrand of $G_{AB}$ ("on-shell approximation") corresponds to dropping the "tadpole" integral $I_0$.
(These are real polynomials in the energy, e.g. $I_0=\pi^2\beta^3/8\,,\quad I_1=\frac{\pi^2\beta^3}{16\mu}(\beta^2-k^2)\,.$)

# Conflict with chiral symmetry?

We found that Révai's solution can be cast in a transparent form,

$$T_{\mathrm{Rev}}(k) = u(k)\left[\tilde{W}^{-1} - G_{AA}\right]^{-1} u(k)\,,$$

$$\tilde{W} = [\mathbb{1} + \lambda I_0]^{-1}\left(\bar{\gamma}\lambda + \lambda\bar{\gamma} - \lambda I_1 \lambda\right)[\mathbb{1} + I_0\lambda]^{-1}\ .$$

- The scattering lengths following from this amplitude do **NOT** vanish in the chiral limit (where $m_K,\ m_\pi,\ m_\eta \to 0$).
- However, chiral symmetry generally demands that this must happen. E.g.,

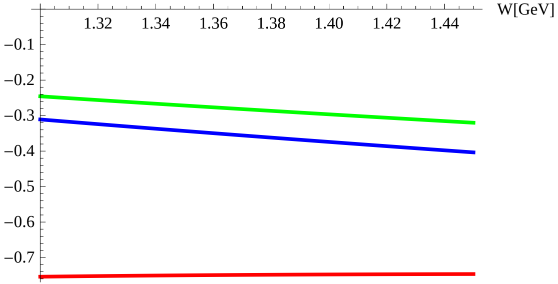
$$a_{0+}^{\bar{K}N,I=0} = \frac{M_N}{4\pi(M_N + m_K)}\left(\frac{3m_K}{2F_K^2} + \mathcal{O}(m^2)\right)\ .$$

- Spoilt by tadpole terms $I_{0,1} \neq 0$ included to **improve** the amplitude!

## BBP amplitude

- We have constructed a generalization of Révai's solution, which can be written in a similar way:
$$T_{\mathrm{BBP}}(s) = u(\bar{q})\left[\tilde{W}^{-1}_{\mathrm{BBP}}(s) - G^{\mathrm{rel}}_{AA}(s)\right]^{-1} u(\bar{q})\,.$$
- It corresponds directly to the iterated set of Feynman graphs, without any further approximation (but with the form factors $u(q)$ of Révai's original model).
- **Relativistic** propagators and kinematics, $G_{AA} \to G^{\mathrm{rel}}_{AA}$ etc.
- BBP model is "chirally improved": The scattering lengths now vanish in the chiral limit, as they should!
- The "effective potential" $\tilde{W}_{\mathrm{BBP}}$ is closer to the original chiral Weinberg-Tomozawa kernel than the former $\tilde{W}$.

# BBP amplitude




$\bar{K}N(I=0)$ effective potentials over the c.m. energy,
for the parameters from Révai's published fit.

Green: “pure” Weinberg-Tomozawa potential (no tadpoles).
Blue: $\tilde{W}_{\mathrm{BBP}}$, Red: Révai's $\tilde{W}$.

# Fits to data

Our own fits yield

- $\chi^2$/dof $\sim 2.4\ldots 2.9$,
- fit parameters of expected (natural) size,
- 1s level shift in kaonic hydrogen: $(285 - i333)\,\mathrm{MeV}$ (best fit) (exp.: $((283 \pm 36) - i(271 \pm 46))\,\mathrm{MeV}$) via improved Deser formula.
- **two** resonance poles in the BBP amplitude in the relevant energy region!

Our best fit has $I = 0$ poles at $z_1 = (1440 - i23)\,\mathrm{MeV}$, $z_2 = (1316 - i7)\,\mathrm{MeV}$ .

The second pole is a bit off the positions usually obtained in more sophisticated approaches, but it is known to be not well determined by the data, and scatters widely in the various models.

For comparison: Révai obtained **one** pole at $z_R = (1422 - i26)\,\mathrm{MeV}$.

# Fits to data

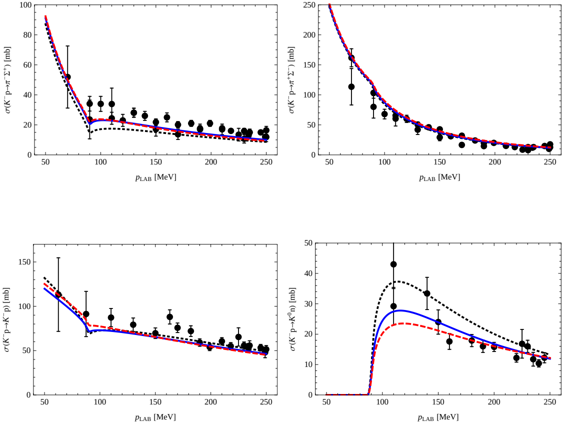



Figure: Blue and red: Fits with BBP, black: curves from Cieply&Smejkal (2011)

# Fits to data

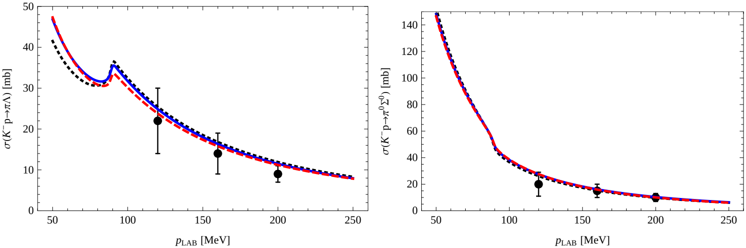



Figure: Blue and red: Fits with BBP, black: curves from Cieply&Smejkal (2011)

# Conclusions

It is always good to scrutinize procedures like the “on-shell approximation” often used in chiral coupled-channel calculations, as Révai did. However,

- the unfortunate combination of the off-shell extrapolation, his chosen regularization scheme and non-relativistic approximations leads to a strong departure from the leading chiral-symmetric kernel, and strongly violates chiral symmetry.
- It is possible to devise an improved version of his approach, which is more in line with chiral symmetry **and** features **two** poles in the $S = -1$ meson-baryon threshold region.
- Fits to data employing the improved model work reasonably well.
- BBP model could be further improved by including higher-order interaction kernels.
- The models can be further tested in an analysis of CLAS data for two-meson photoproduction.

# Appendix

# Fit strategies

Models:

- CS Prague TW1 model by Cieply, Smejkal, two parameter fit with the same $\beta$ in all channels and $F_\eta = F_K = F_\pi$.
- JR J.Révai's model published in Few Body Syst. **59** (2018).
- N1 non-relativistic (JR) model with $F_K = F_\pi$.
- N2 non-relativistic (JR) model with $F_K = 1.193\, F_\pi$.
- R1 relativistic (BBP) model with $F_K = F_\pi$.
- R2 relativistic (BBP) model with $F_K = 1.193\, F_\pi$.

# Fit parameters

Table: various fits

| model | $F_\pi$ | $F_K$ | $I=0$ sector $\beta_{\pi\Sigma}$ | $I=0$ sector $\beta_{\bar{K}N}$ | $I=1$ sector $\beta_{\pi\Lambda}$ | $I=1$ sector $\beta_{\pi\Sigma}$ | $I=1$ sector $\beta_{\bar{K}N}$ | $\chi^2/dof$ |
|---|---|---|---|---|---|---|---|---|
| CS | 112.8 | 112.8 | 701.5 | 701.5 | 701.5 | 701.5 | 701.5 | 3.6 |
| JR | 73.2 | 98.3 | 451.8 | 830.2 | 352.4 | 471.2 | 934.6 | — |
| N1 | 116.3 | 116.3 | 553.2 | 860.6 | 656.3 | 553.2 | 860.6 | 2.62 |
| N2 | 95.6 | 114.0 | 493.6 | 870.3 | 536.2 | 493.6 | 870.3 | 2.78 |
| R1 | 105.9 | 105.9 | 876.7 | 1065.0 | 773.8 | 876.7 | 1065.0 | 2.39 |
| R2 | 89.4 | 106.6 | 762.2 | 1125.8 | 637.8 | 762.2 | 1125.8 | 2.93 |

# Pole positions

Table: Pole positions (in MeV) on the [-,+] and [-,-,+] RSs for the $I = 0$ and $I = 1$ sectors.

| model | $z_1$ $(I = 0)$ | $z_2$ $(I = 0)$ | $z_3$ $(I = 1)$ |
|---|---|---|---|
| CS | (1432.8, -24.9) | (1370.8, -54.2) | (1408.9,-199.7) |
| JR | (1422.9, -25.7) | — | (1106.5, -71.6) |
| N1 | (1442.8, -23.3) | — | (1141.1, -80.5) |
| N2 | (1441.0, -22.5) | — | (1266.4, 0.0) |
| R1 | (1439.9, -23.3) | (1316.0, -6.76) | (1361.1, -166.9) |
| R2 | (1437.8, -20.9) | (1251.1, 0.0) | (1337.4, -117.3) |

We also checked the sensitivity of the $z_2$ pole position to the $\beta_{\pi\Sigma}$ value. For the R1 model, when the parameter is reduced by 10%, the pole moves to (1326.6, -39.8) MeV, moving away from the real axis. When the $\beta$ value is increased by 10%, the pole moves to the real axis, to the (1274.9, 0.0) MeV position.

# S-wave $K^- p \to K^- p$ amplitudes

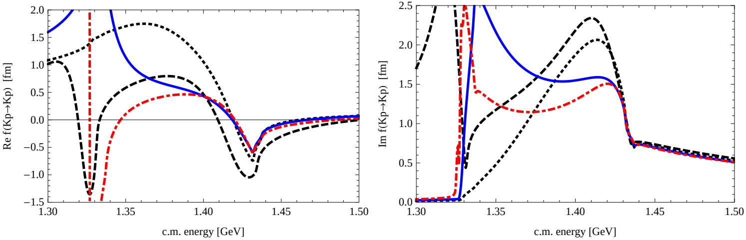



Figure: Black dotted - TW1 model by Cieply and Smejkal; black dashed - original Révai model; red dot-dashed - BPP model, $F_K = 1.193 \cdot F_\pi$; blue continuous - BPP model, $F_K = F_\pi$.