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Open Effective Field Theories and Universality

H.-W. Hammer

Institut für Kernphysik, TU Darmstadt and Extreme Matter Institute EMMI

EMMI

Bundesministerium für Bildung und Forschung

Deutsche Forschungsgemeinschaft DFG

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Outline

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- Introduction
  - Low-energy universality
  - Resonant interactions and the unitary limit
- Universality in Few-Body Systems
  - Efimov effect and few-body losses
- Inelastic Processes in a Many-Body System and Open EFT
  - Inelastic 2- and 3-atom losses
- Summary and Outlook

E. Braaten, HWH, G.P. Lepage, Phys. Rev. D **94** (2016) 056006

E. Braaten, HWH, G.P. Lepage, Phys. Rev. A **95** (2017) 012708

M. Schmidt, L. Platter, HWH, in preparation

Low-Energy Universality

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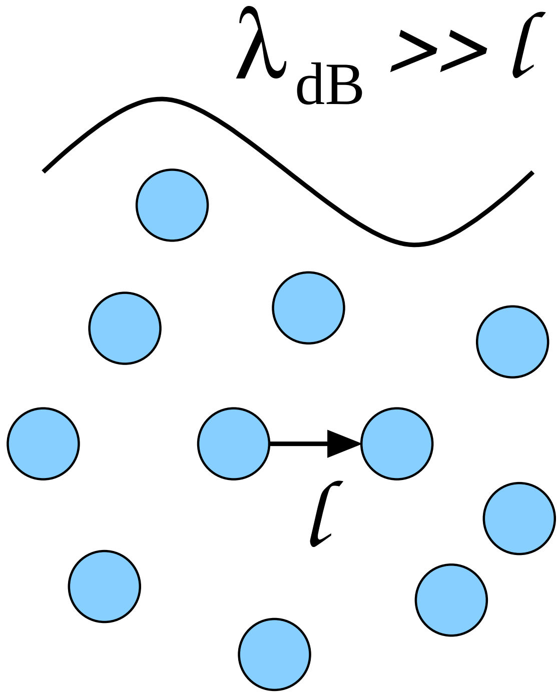
- Ultracold Atoms: small kinetic energy
- Separation of scales:

  $1/k = \lambda_{dB} \gg \ell$
- Limited resolution at low energy:

  $\longrightarrow$ expand in powers of $k\ell$
- Generic/natural case: $|a| \sim \ell$



Low-Energy Universality

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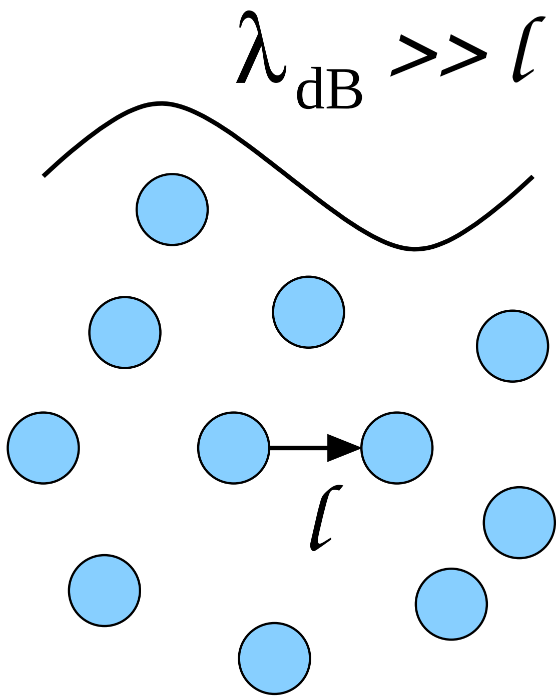

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  $1/k = \lambda_{dB} \gg \ell$
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  $\longrightarrow$ expand in powers of $k\ell$
- Generic/natural case: $|a| \sim \ell$

- Resonant case: $|a| \gg \ell$

$\Longrightarrow$ non-perturbative resummation required for $k \sim |a|$

$\Longrightarrow$ expansion around unitary limit $1/a = 0$

Physics Near the Unitary Limit

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- Consider system with short-ranged, resonant interactions
- Unitary limit: $a \to \infty$, $\ell \to 0$ (cf. Bertsch problem, 2000)

$$\mathcal{T}_2(k,k) \propto \Big[\underbrace{k \cot \delta}_{-1/a + r_e k^2/2 + \ldots} - ik \Big]^{-1} \Longrightarrow i/k$$

  - Scattering amplitude scale invariant, saturates unitarity bound

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- Use as starting point for description of few-body properties
  - Large scattering length: $|a| \gg \ell \sim r_e, l_{vdW}, \ldots$
  - Natural expansion parameter: $\ell/|a|$, $k\ell$,...
  - Universal dimer with energy $E_d = -\hbar^2/(ma^2)$ $(a > 0)$

    size $\langle r^2 \rangle^{1/2} = a/2$

Broken Scale Invariance

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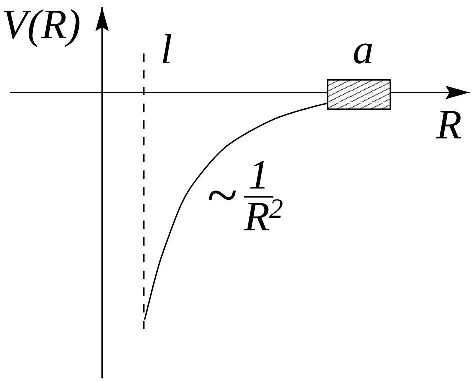
- Three-boson system near the unitary limit (Efimov, 1970)
- Hyperspherical coordinates: $R^2 = (r_{12}^2 + r_{13}^2 + r_{23}^2)/3$
- Schrödinger equation simplifies for $|a| \gg R \gg l$:

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial R^2} + \frac{s_0^2 + 1/4}{R^2}\right] f(R) = \underbrace{-\frac{\hbar^2\kappa^2}{m}}_{E} f(R)$$



- Singular Potential: renormalization required
- Boundary condition at small R: breaks scale invariance

  $\Longrightarrow$ scale invariance is anomalous
  $\Longrightarrow$ observables depend on boundary condition and a
- EFT formulation $\Rightarrow$ 3-body interaction

Limit Cycle

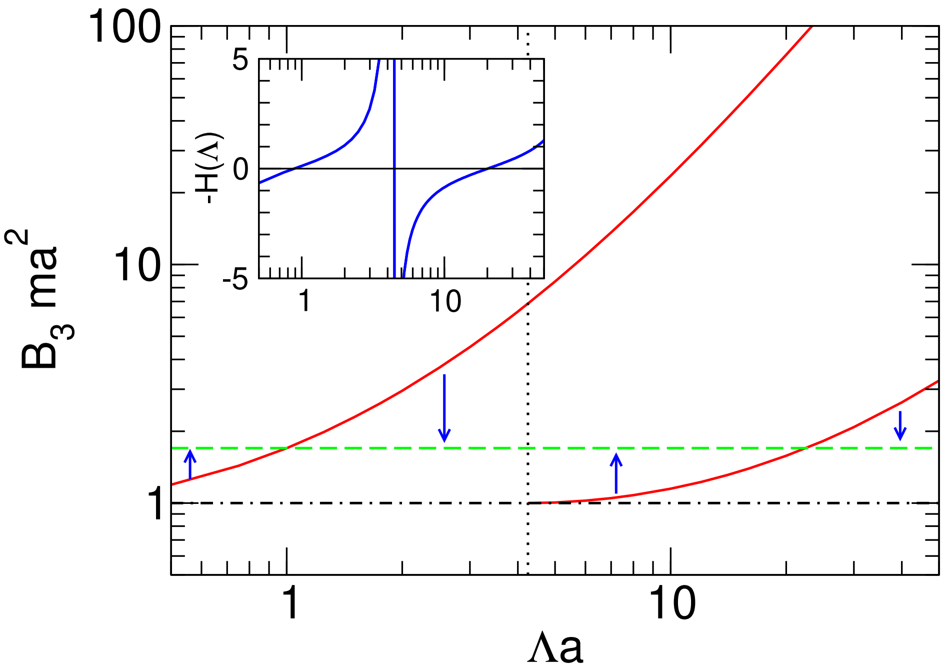
- EFT framework $\Longrightarrow$ running coupling $H(\Lambda)$ $(\Lambda \sim 1/R)$

- $H(\Lambda)$ periodic: limit cycle

$$\Lambda \to \Lambda\, e^{n\pi/s_0} \approx \Lambda(22.7)^n$$

(cf. Wilson, 1971)

- Anomaly: scale invariance broken to discrete subgroup



$$H(\Lambda) \approx \frac{\cos(s_0 \ln(\Lambda/\Lambda_*) + \arctan(s_0))}{\cos(s_0 \ln(\Lambda/\Lambda_*) - \arctan(s_0))}\,, \qquad s_0 \approx 1.00624$$

(Bedaque, HWH, van Kolck, 1999)

- Three-body parameter: $\Lambda_*, \ldots$

- Limit cycle $\Longleftrightarrow$ Discrete scale invariance $\Longleftrightarrow$ Efimov physics

Limit Cycle: Efimov Effect

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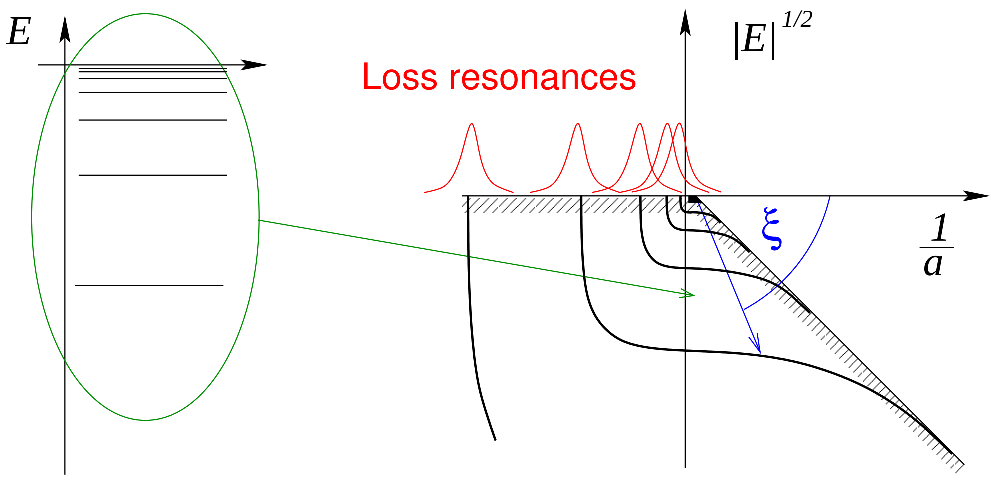
- Universal spectrum of three-body states (Efimov, 1970)



  - Discrete scale invariance for fixed angle ξ
  - Geometrical spectrum for $1/a \to 0$

$$B_3^{(n)}/B_3^{(n+1)} \xrightarrow{1/a\to 0} \left(e^{\pi/s_0}\right)^2 = 515.035...$$

- Universal four- and higher-body states
- Ultracold atoms $\Longrightarrow$ variable scattering length $\Longrightarrow$ loss resonances

Three-Body Recombination

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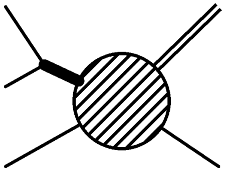

- Three-body recombination:

  3 atoms $\rightarrow$ dimer + atom $\Rightarrow$ loss of atoms
- Recombination constant: $\dot{n}_A = -K_3\, n_A^3$
- K_3 has log-periodic dependence on scattering length

  (Nielsen, Macek, 1999; Esry, Greene, Burke, 1999; Bedaque, Braaten, HWH, 2000)
- Deep dimers: Efimov trimers aquire width $\Rightarrow$ resonances
- Loss term in short distance b.c.: $\Lambda_* \longrightarrow \Lambda_* \exp^{i\eta_*/s_0}$

  $\Longrightarrow$ non-hermitian Hamiltonian
- Universal line shape of recombination resonance ($a < 0$)

$$K_3^{deep} = \frac{64\pi^2(4\pi - 3\sqrt{3})\ \coth(\pi s_0)\sinh(2\eta_*)}{\sin^2\left[s_0 \ln(a/a_-)\right] + \sinh^2 \eta_*} \frac{\hbar\, a^4}{m}\,, \qquad s_0 \approx 1.00624..$$

and other observables . . .

Efimov Physics in Ultracold Atoms

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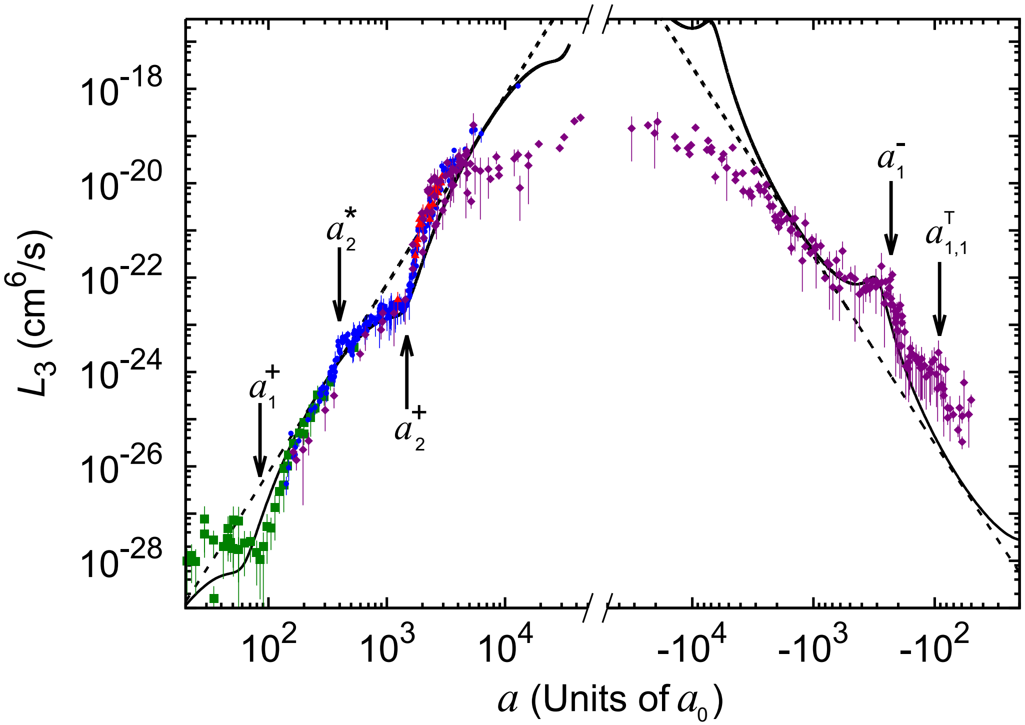
- First experimental evidence in ^{133}Cs (Kraemer et al. (Innsbruck), 2006)
  now also ^{6}Li, ^{7}Li, ^{39}K, ^{41}K/^{87}Rb, ^{6}Li/^{133}Cs
- Example: Efimov spectrum in ^{7}Li



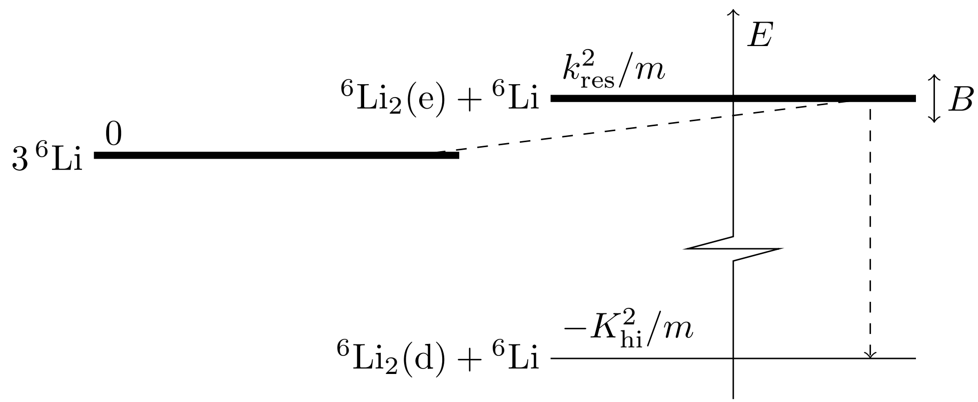
Pollack et al. (Rice), Science **326** (2009) 1683; Phys. Rev. A **88** (2013) 023625

- vdW tail determines resonance position: $a_-/l_{vdW} \approx -10\ (\pm 15\%)$
  but not width (Wang et al., 2012; Naidon et al. 2012, 2014; ...)

Polarized P-Wave Fermi Gas

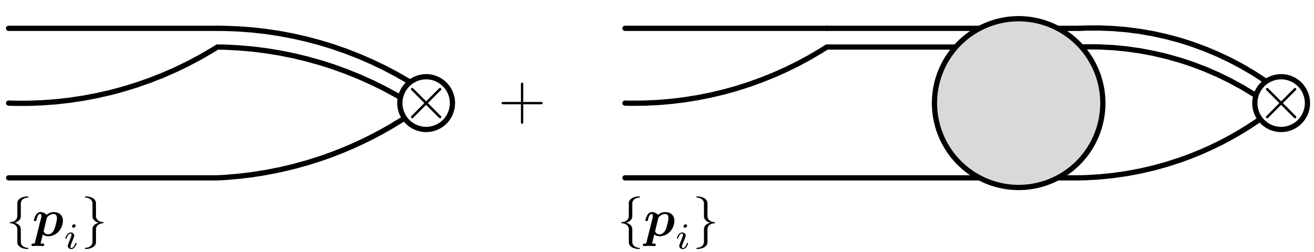
- Three-body recombination in spin-polarized ^{6}Li gas with P-wave Feshbach resonance ($|F = 1/2, m_F = 1/2\rangle$)

  Waseem et al., Phys. Rev. A **98** (2018) 020702



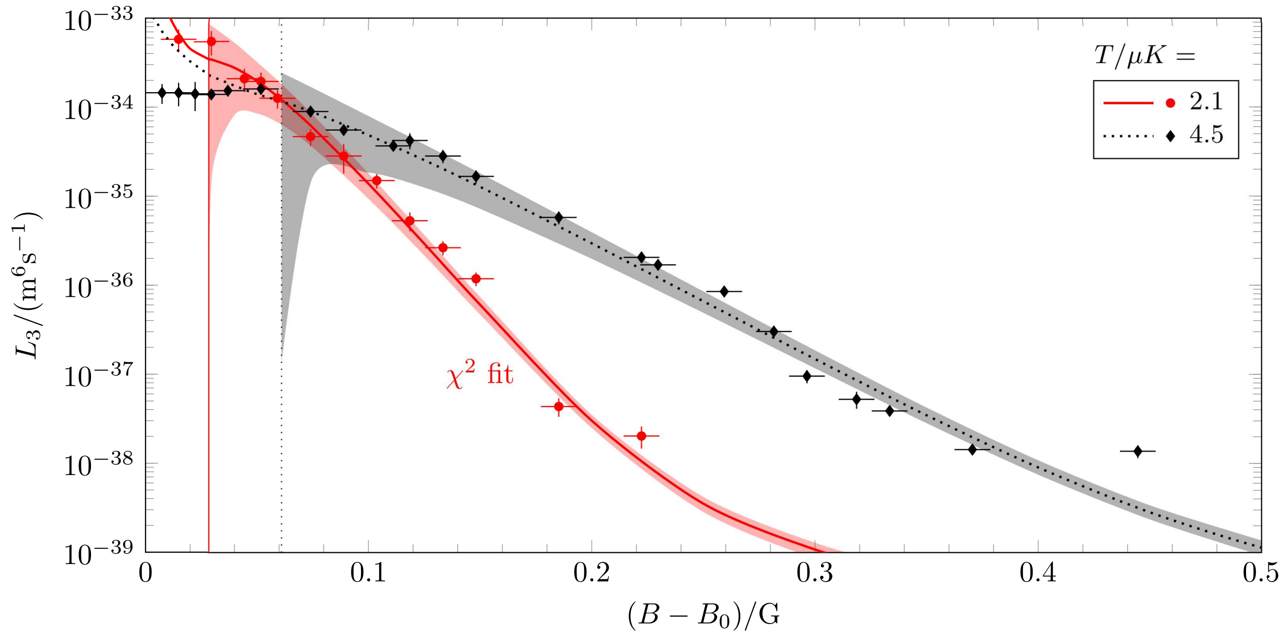
- Describe with complex three-body force

  Schmidt, Platter, HWH, in preparation



Polarized P-Wave Fermi Gas

- Good description for $k_{res} < k_{thermal} = \sqrt{5mk_BT/2}$ (non-unitary regime)



Schmidt, Platter, HWH, in preparation

- Prediction of shallow three-body bound state
- Open questions in unitary regime

Inelastic Processes

- Loss coefficients used in few-body rate equations
- Complete many-body description requires density matrix
- Effective density matrix from tracing over high-energy states?
- Naive evolution equation for $H_{eff} = H - iK$

$$i\hbar\, \partial_t \rho = H_{\text{eff}}\, \rho - \rho H_{\text{eff}}^{\dagger} = [H, \rho] - i\{K, \rho\}$$

- Implies $\partial_t \mathrm{Tr}\,(\rho) = -\mathrm{Tr}(2K\rho)/\hbar$

$\Longrightarrow$ probability not conserved

- Need evolution equation for open system

$\Longrightarrow$ Lindblad equation (Lindblad; Gorini, Kossakowski, Sudarshan, 1976)

- Derive from Quantum Field Theory

Inelastic Processes

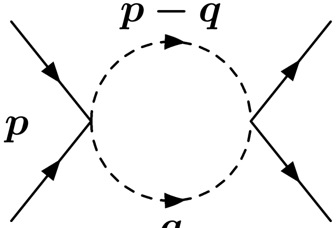
- Consider model with two fields ψ and ϕ: $\quad H_{\text{full}} = H^{\psi} + H^{\phi} + H_{\text{int}}$

$$H_{\text{int}} = \frac{1}{4} g \int_{\boldsymbol{r}} \left(\psi^{\dagger 2}(\boldsymbol{r}) \phi^2(\boldsymbol{r}) + \psi^2(\boldsymbol{r}) \phi^{\dagger 2}(\boldsymbol{r}) \right)$$

- Reaction $\psi\psi \to \phi\phi$ has large energy release E_{deep}

  $\Longrightarrow$ process is effectively local and instantaneous

- Leading contribution in g to imaginary part of $\psi\psi \to \psi\psi$



$$\text{Im}\, T(E, \boldsymbol{p}) = \text{Im}\left(-\frac{g^2}{2} \int_{\boldsymbol{q}} \frac{1}{E - \omega_{\boldsymbol{q}} - \omega_{\boldsymbol{p}-\boldsymbol{q}} + i\epsilon}\right)$$

  $\Longrightarrow$ expand in powers of $\boldsymbol{p}^2 / m E_{deep}$

- Effect of high-energy ϕ particles on low-energy ψ particles is local

Inelastic Processes

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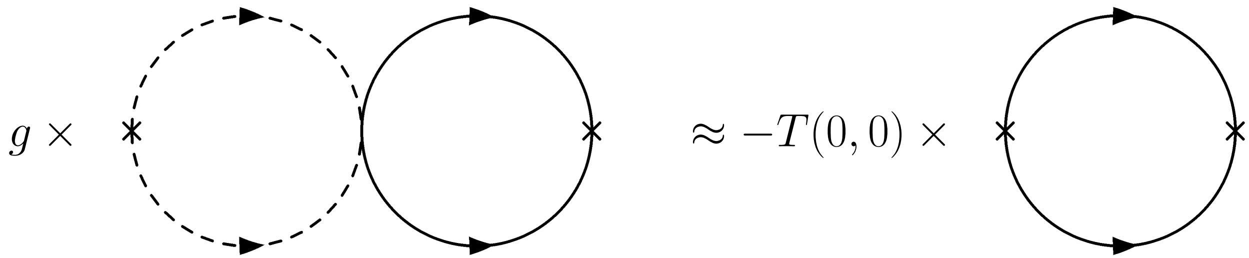
- Effective Field Theory without explicit ϕ dof

$$H - iK = H^{\psi} - \frac{1}{4}T(0,0)\int_{\boldsymbol{r}}(\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r}))^2$$

- Only imaginary part of T physically relevant, real part renormalized away
- Consider correlation function $\quad g\langle 0|\phi^2(\boldsymbol{r},t)\psi^{\dagger 2}(\boldsymbol{r}',0)|0\rangle$



- Replacement for internal ϕ particles in correlation functions

$$g\,\phi^2(\boldsymbol{r},t) \to -T(0,0)\,\psi^2(\boldsymbol{r},t), \qquad g\,\phi^{\dagger 2}(\boldsymbol{r},t) \to -T^*(0,0)\,\psi^{\dagger 2}(\boldsymbol{r},t)$$

Inelastic Processes

- Derive effective density matrix for low-energy particles

$$\rho(t) \equiv \mathrm{Tr}_\phi\left(\rho_{\text{full}}(t)\right) = \sum_{m=0}^{\infty} \int_{\boldsymbol{y}_1 \ldots \boldsymbol{y}_m} {}_\phi\langle \boldsymbol{y}_1 \ldots \boldsymbol{y}_m | \rho_{\text{full}}(t) | \boldsymbol{y}_1 \ldots \boldsymbol{y}_m \rangle_\phi$$

- Evolution of effective density matrix

$$i\hbar\partial_t \rho = \mathrm{Tr}_\phi\big(H_{\text{full}}\,\rho_{\text{full}} - \rho_{\text{full}}\,H_{\text{full}}\big)$$

- Four different contributions from interaction term

$$\mathrm{Tr}_\phi\left[\left(g\int_{\boldsymbol{r}} \psi^{\dagger 2}(\boldsymbol{r})\phi^2(\boldsymbol{r})\right)\rho_{\text{full}}\right] \longrightarrow -T(0,0)\int_{\boldsymbol{r}} (\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r}))^2 \rho$$

- Analog for other three contributions

$$\mathrm{Tr}_\phi[\rho_{\text{full}}(g\textstyle\int_{\boldsymbol{r}} \phi^{\dagger 2}(\boldsymbol{r})\psi^2(\boldsymbol{r}))],\ \mathrm{Tr}_\phi[(g\textstyle\int_{\boldsymbol{r}} \phi^{\dagger 2}(\boldsymbol{r})\psi^2(\boldsymbol{r}))\rho_{\text{full}}],\ \ldots$$

Open EFT

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- Evolution equation for effective density matrix

$$i\hbar\partial_t\rho = \left[H,\rho\right] - \frac{i}{4}\,\mathrm{Im}\,T\int_{\boldsymbol{r}}\left[\left(\psi^\dagger\psi(\boldsymbol{r})\right)^2\rho + \rho\left(\psi^\dagger\psi(\boldsymbol{r})\right)^2 - 2\,\psi(\boldsymbol{r})^2\,\rho\,\psi^{\dagger 2}(\boldsymbol{r})\right]$$

$\Longrightarrow$ Lindblad form

- General Hamiltonian with a loss term

$$H_{\mathrm{eff}} = H - iK, \qquad K = \sum_i \gamma_i \int d^3r\,\Phi_i^\dagger\Phi_i$$

- Lindblad equation

$$i\hbar\partial_t\rho = [H,\rho] - i\sum_i\gamma_i\int d^3r\left(\Phi_i^\dagger\Phi_i\rho + \rho\Phi_i^\dagger\Phi_i - 2\Phi_i\rho\Phi_i^\dagger\right)$$

$\Longrightarrow$ Open EFT (Burgess et al., 2015)

Inelastic 2-Body Losses

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- Application to inelastic 2-body losses
- Fermionic atoms with a loss channel $\Rightarrow$ a complex

$$K = (4\pi\hbar^2/m)\,\mathrm{Im}(1/a)\int d^3r\,\Phi^\dagger\Phi, \qquad \Phi = 4\pi a\,\psi_2\psi_1$$

- Particle losses: $\langle N\rangle = \mathrm{Tr}(\rho N)$

$$\frac{d}{dt}\langle N_1\rangle = \frac{d}{dt}\langle N_2\rangle = -\frac{\hbar}{2\pi m}\mathrm{Im}(1/a)\int d^3r\,\left\langle\Phi^\dagger\Phi\right\rangle$$

where $\mathcal{C} = \left\langle\Phi^\dagger\Phi\right\rangle$ contact operator

2-Atom Losses and the Contact

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- Universal relations involving the contact: $C = \int d^3r\, \mathcal{C}(\boldsymbol{r})$

  measures number of pairs at short distances (Tan, 2005-2008)

  e.g. adiabatic relation $$\frac{d}{da^{-1}} E = -\frac{\hbar^2}{4\pi m}\, C$$

  also RF spectroscopy, photoassociation, ...
- Here: inelastic loss rate for mixture of atom species $\sigma = 1, 2$
- Inelastic short-distance processes parameterized by complex scattering length

$$\frac{d}{dt} N_\sigma = -\frac{\hbar}{2\pi m}\, \mathrm{Im}(1/a)\, C$$

(Tan, 2008; Braaten, Platter, 2008)

Inelastic 3-Atom Losses

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- Inelastic three-atom loss rate

$$\frac{d}{dt}\langle N\rangle = -\frac{6\hbar}{ms_0}\sinh(2\eta_*)C_3$$

(linear term in η_*: Werner, Castin, 2012; Smith, Braaten, Kang, Platter, 2014)

- Three-body contact: $C_3 = f(\Lambda)\int d^3r\,\langle(\psi^3)^\dagger\psi^3\rangle$

where $f(\Lambda)$ is scheme-dependent

- Equivalent definition (Braaten, Kang, Platter, 2011)

$$\text{with}\qquad \Lambda_*\frac{\partial\langle H\rangle}{\partial\Lambda_*}\bigg|_a = -\frac{2\hbar^2}{m}C_3$$

- Tail of momentum distribution (Braaten, Kang, Platter, 2011)

$$k^4 n(k) \longrightarrow C_2 + A\sin[2s_0\ln(k/\kappa*) + \phi]C_3/k$$

Three-body contact

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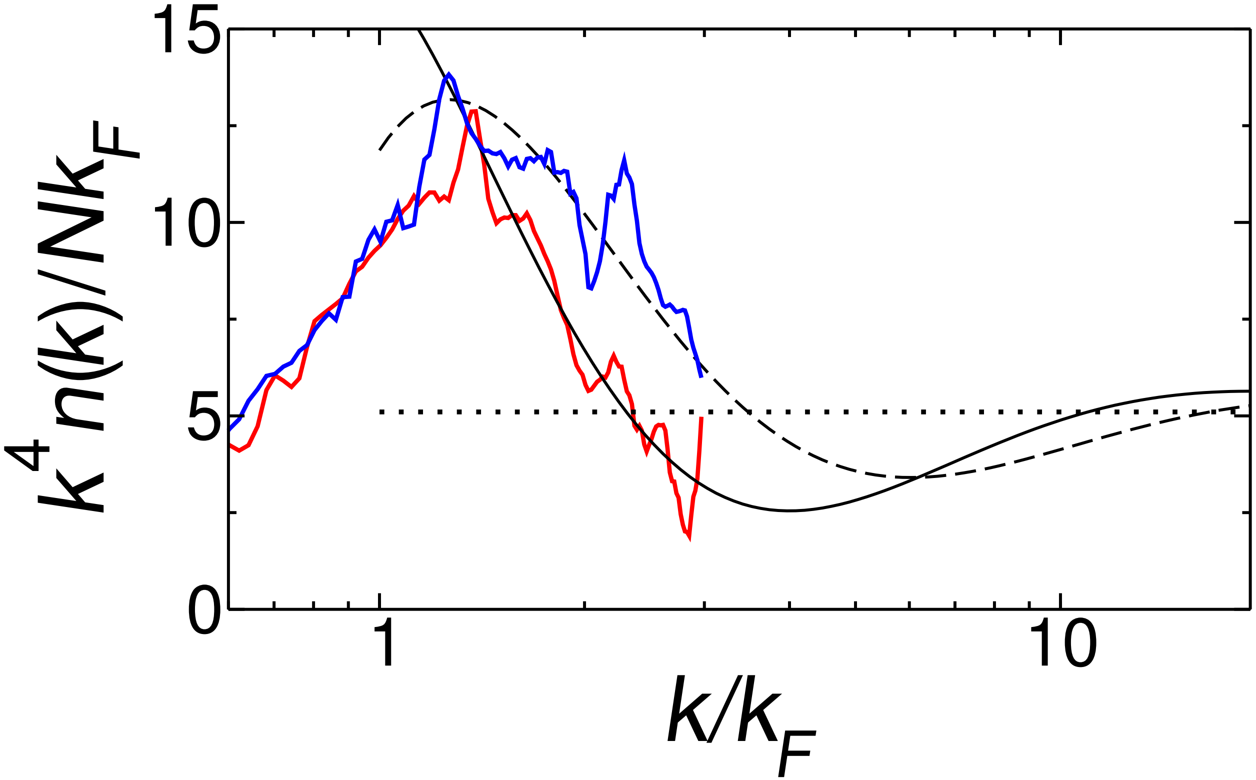
- Consistent with experiment ($\langle n_1 \rangle < \langle n_2 \rangle$)

$$k^4 n(k) \longrightarrow C_2 + A \sin[2 s_0 \ln(k/\kappa *) + \phi] C_3/k$$



Exp.: Makotyn, Klauss, Goldberger Cornell, Jin, Nature Phys. **88**, 116 (2014)

Theo.: Braaten, Kang, Platter, Phys. Rev. Lett **112**, 110402 (2014)

Summary and Outlook

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- Universality: Effective field theory for large scattering length
  - Discrete scale invariance, universal correlations,...
- Applications in atomic, nuclear, and particle physics
  - Ultracold atoms close to Feshbach resonance
  - Few-body nuclei
  - Hadronic molecules: $X(3872)$, $\ldots$

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  - Hadronic molecules: $X(3872), \ldots$
- Open Effective Field Theory and inelastic processes
- Lindblad equation for density matrix
- Universal relation for the inelastic 2-atom loss rate
  - Losses proportional to $\mathrm{Im}(a)$ and Tan contact
- Universal relation for the inelastic 3-atom loss rate
  - Losses proportional to η_* and 3-body contact

Additional Slides

Universal EFT

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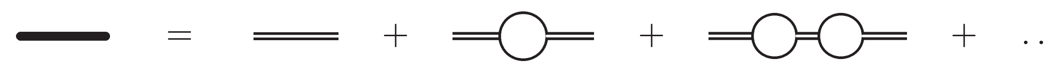
- Effective Lagrangian (Kaplan, 1997; Bedaque, HWH, van Kolck, 1999)

$$\mathcal{L}_d = \psi^\dagger \left(i\partial_t + \frac{\vec{\nabla}^2}{2m} \right) \psi + \frac{g_2}{4} d^\dagger d - \frac{g_2}{4} (d^\dagger \psi^2 + (\psi^\dagger)^2 d) - \frac{g_3}{36} d^\dagger d \psi^\dagger \psi + \ldots$$

- 2-body amplitude: 
- 2-body coupling g_2 near fixed point $(1/a = 0)$

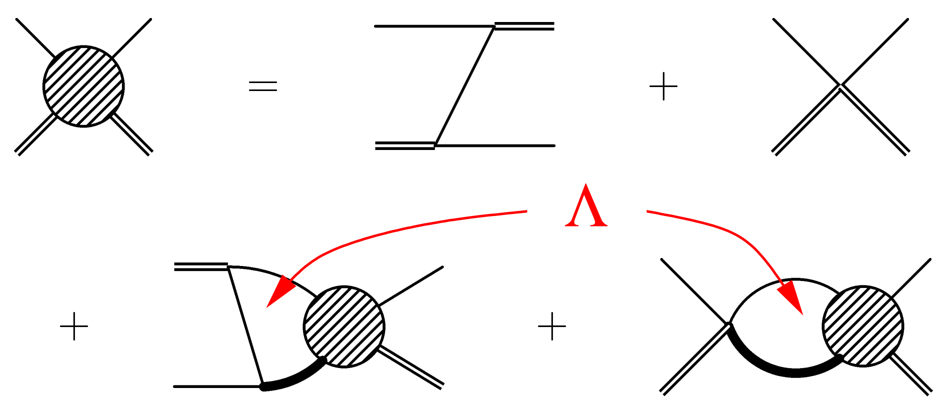
  $\Rightarrow$ scale and conformal invariance $\Longleftrightarrow$ unitary limit

  (Mehen, Stewart, Wise, 2000; Nishida, Son, 2007; ...)

- 3-body amplitude:

  $g_3(\Lambda)$ $\Rightarrow$ limit cycle

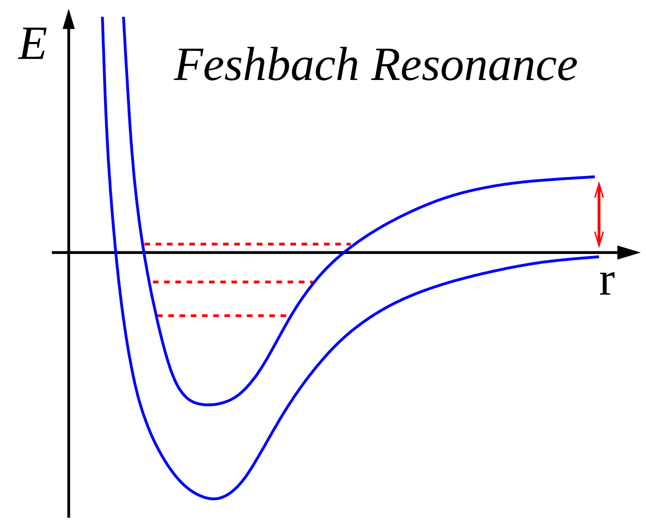
  $\Rightarrow$ discrete scale inv.



Variable Scattering Length

- **Feshbach Resonance:**

  energy of molecular state in closed channel close to energy of scattering state



- **Tune scattering length via external magnetic field**

  (Tiesinga, Verhaar, Stoof, 1993)

- **Observation in a Na BEC**

  (Inouye et al. (MIT), 1998)

$$\frac{a(B)}{a_0} = 1 + \frac{\Delta}{B_0 - B}$$