



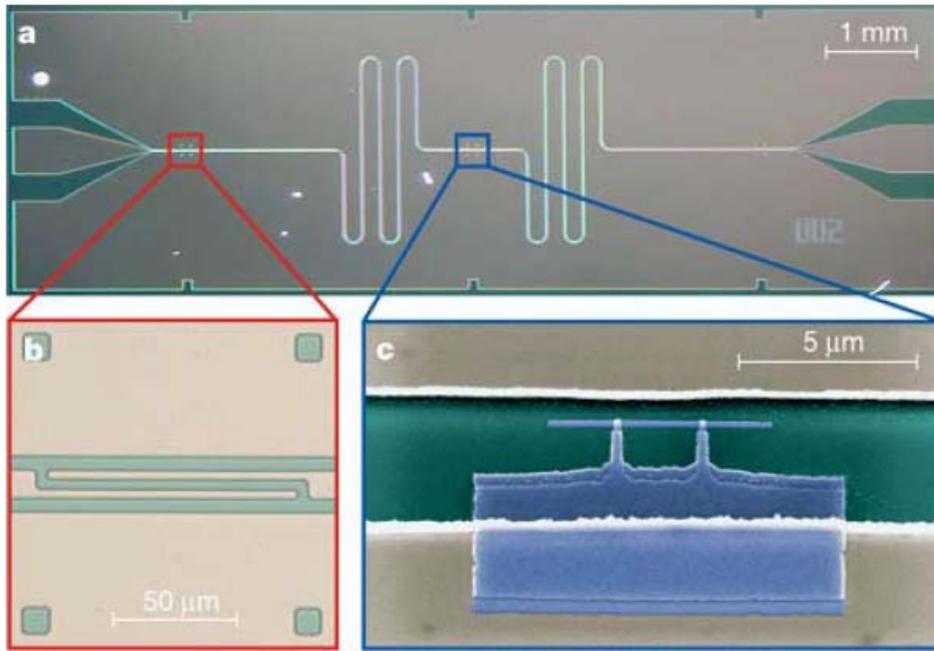
Superconducting Circuits as Testbeds for Quantum Optics and Atomic Physics

Joachim Ankerhold

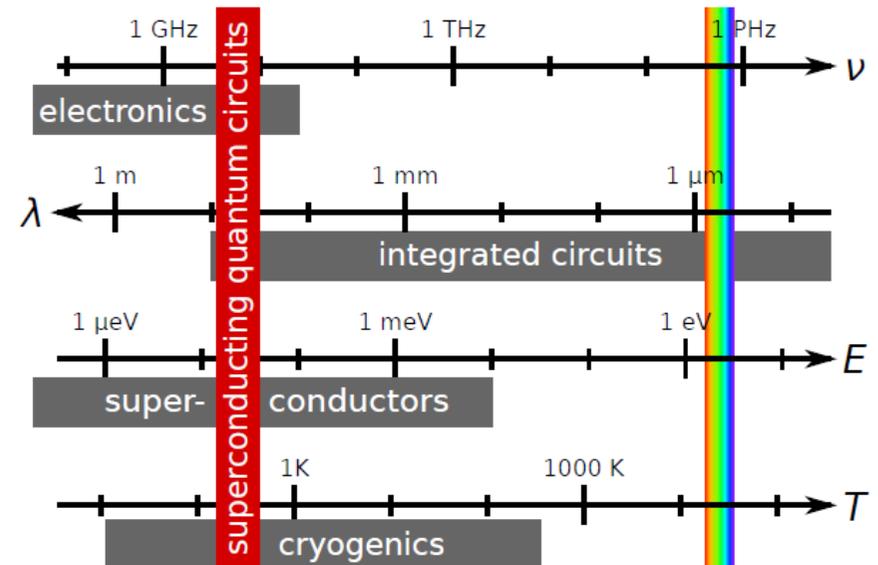
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S. Dambach, V. Gramich, B. Kubala (Ulm)
F. Portier, D. Esteve (Saclay)
A. Armour (Nottingham)
M. Dykman (Michigan)
M. Hofheinz (Grenoble)
A. Rimberg, M. Blencowe (Dartmouth)
C. Ast, K. Kern (Stuttgart)

Circuit QED – quantum optics/atomic physics on macro scales



Schoelkopf et al, Nature 431, 162 (04)



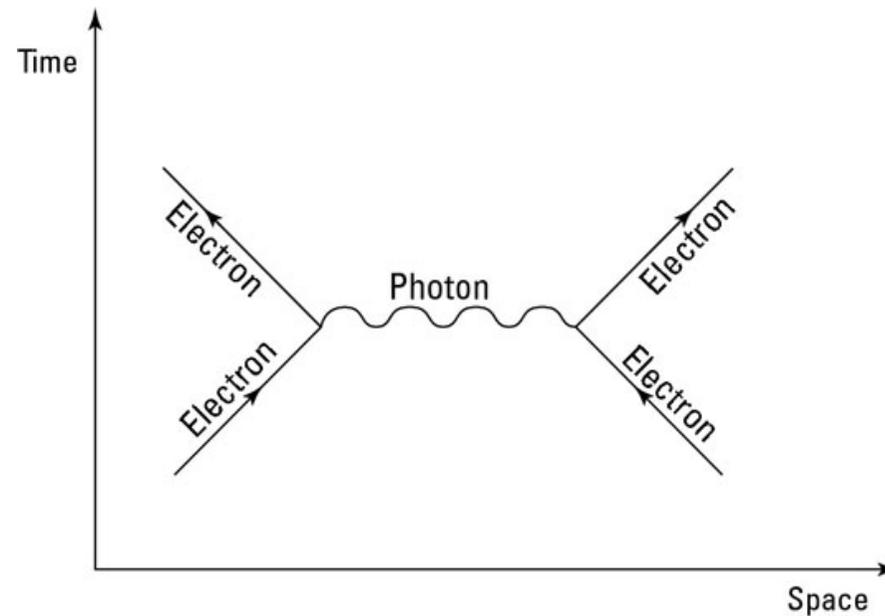
Quantum Electrodynamics (QED)



Dirac

Bethe
Schwinger
Feynman
Lamb

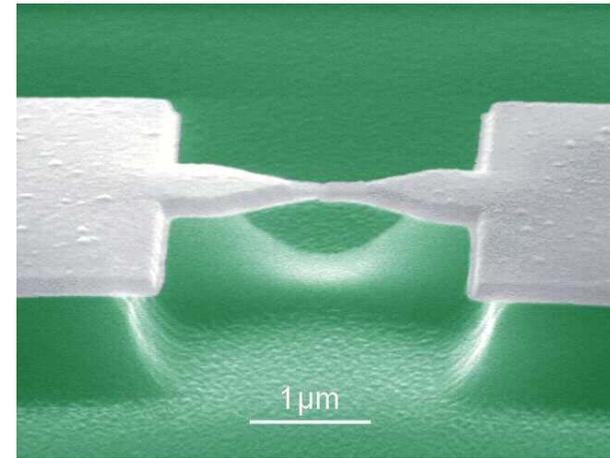
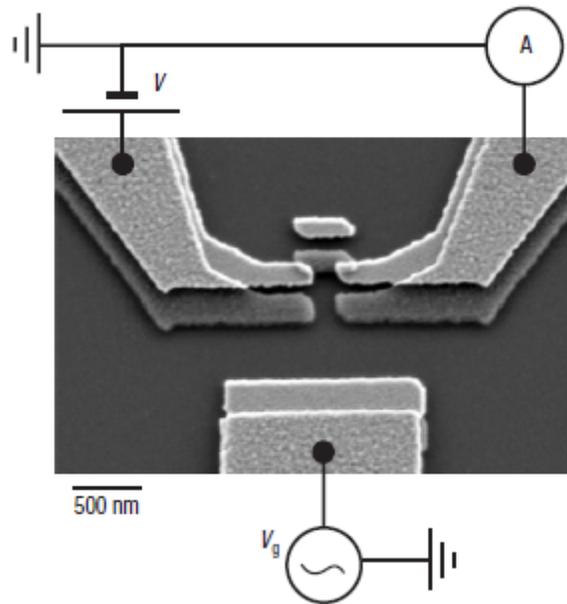
.....



$$\begin{aligned} \text{Fine structure constant } \alpha_{\text{QED}} &= \frac{e^2}{4\pi\epsilon_0 \hbar c} \approx \frac{1}{137} \\ &= \frac{Z_0}{2R_K} \end{aligned} \quad Z_0 = \mu_0 c, \quad R_K = \frac{h}{e^2}$$

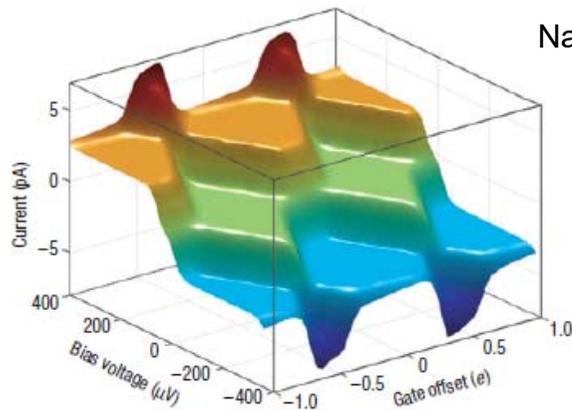
Quantum electronics

Controlled transfer of single charge quanta (Coulomb blockade)



Nature Physics 4, 120 (2008)

Nature 394, 154 (1998)
Science 349, 1199 (2015)

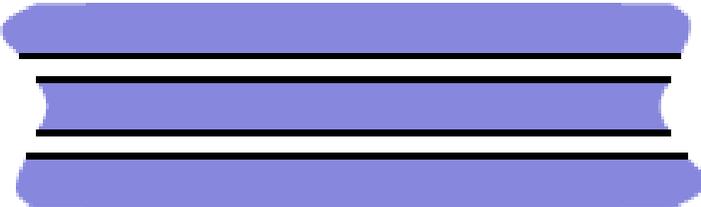


$$I = n e f$$

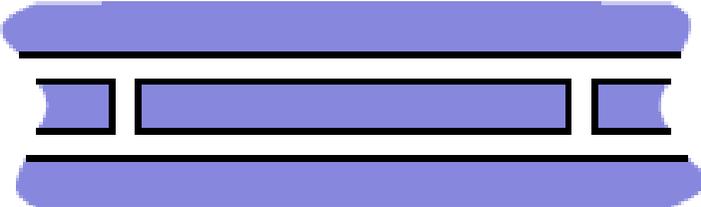
$$G_0 = \frac{e^2}{h}$$

Quantum circuits

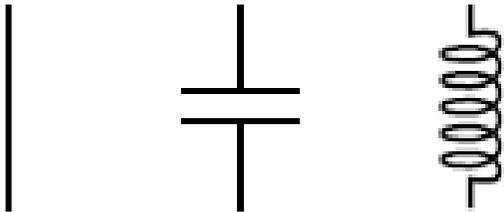
- Transmission lines



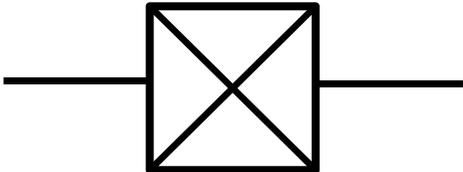
- Resonators



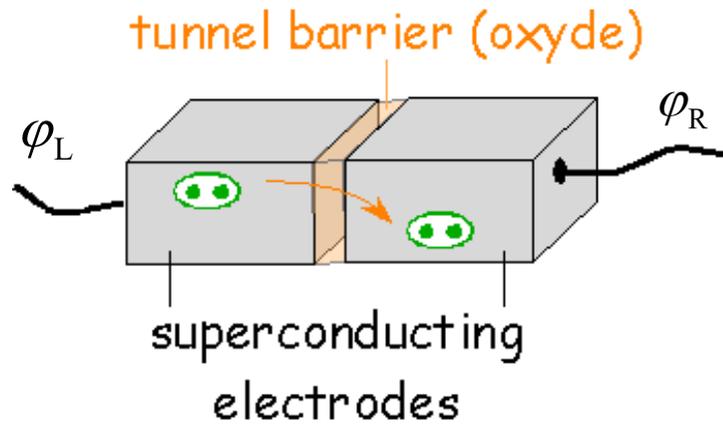
- Linear elements



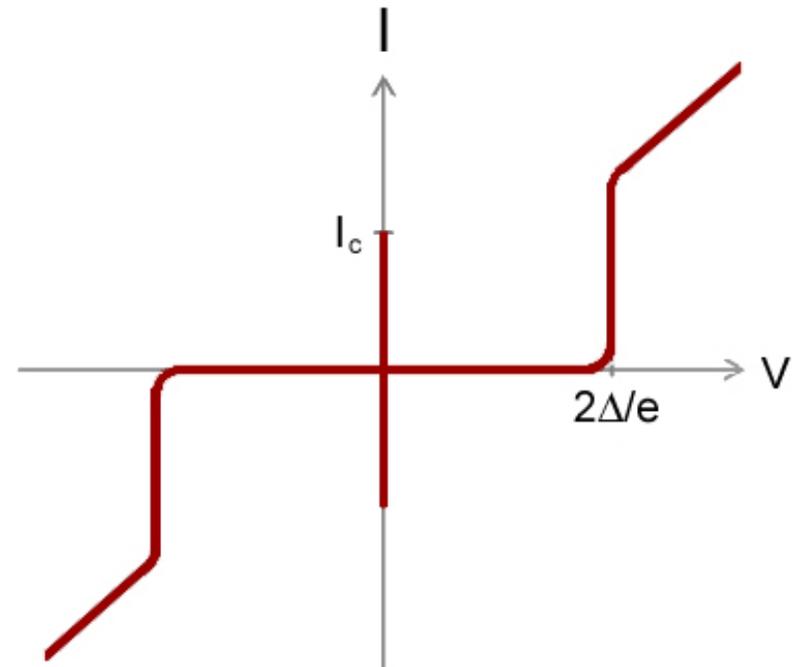
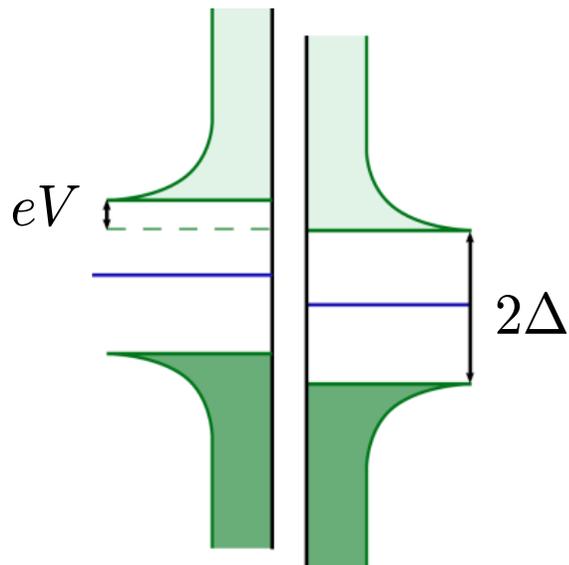
- Nonlinear element:
Josephson junction



Josephson junction



$$I_J = I_c \sin(\varphi), \quad \varphi = \varphi_L - \varphi_R$$
$$V_J = \frac{\hbar}{2e} \dot{\varphi} \quad (\text{ac-Josephson})$$



Quantum: embedded in circuit

Phase is a collective dof, embedded in electromagnetic environment

$$H = H_J + H_Z + H_{\text{int}}$$

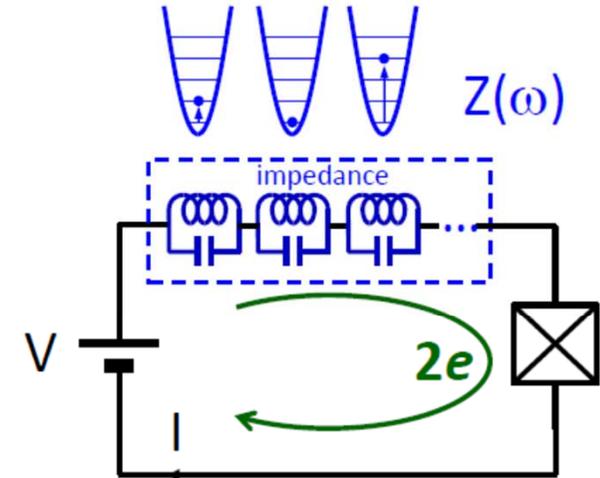
$$V = 0 : H = E_C N^2 - E_J \cos(\varphi) + \sum_n \frac{4e^2 N_n^2}{2C_n} + \frac{\hbar^2 (\varphi_n - \varphi)^2}{4e^2 2L_n}$$

$$E_C = \frac{2e^2}{C}$$

$$[N, \varphi] = -i, [N_n, \varphi_n] = -i$$

Number of transmitted Cooper pairs

$$H_J = \frac{E_J}{2} \left(\overrightarrow{T}_{2e} + \overleftarrow{T}_{2e} \right)$$



Quasi-continuum of modes:
open quantum system

Linear impedances (Gaussian
fluctuations) characterized by $Z(\omega)$

Non-Markovian (memory),
weak to strong coupling

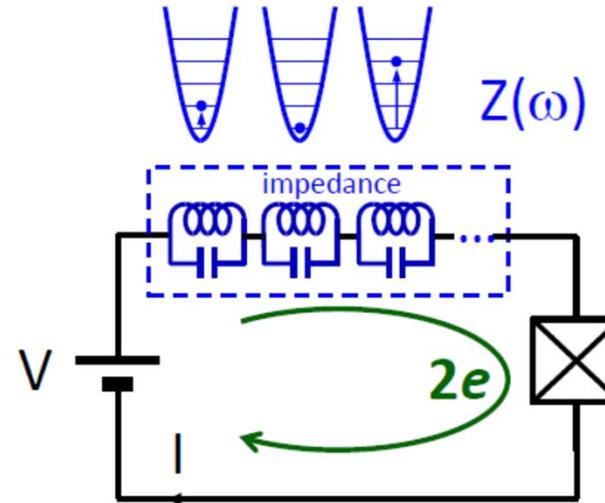
Low transmitting junction

$$E_J \ll E_C, k_B T$$

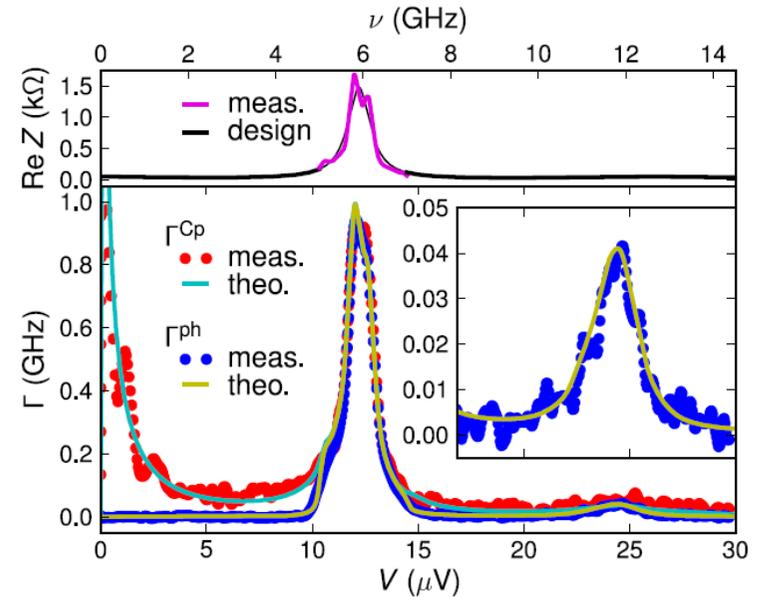
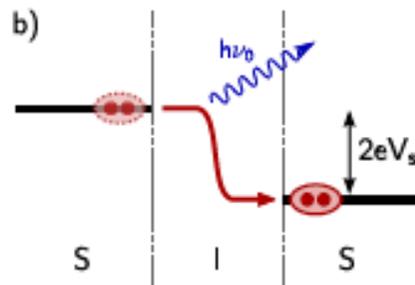
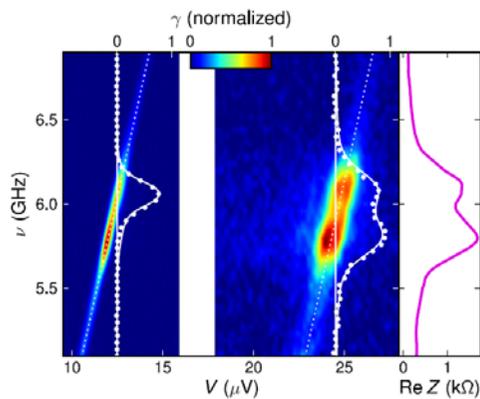
Fermi's golden rule (sequential charge transfer)

$$I_J = \frac{\pi e E_J^2}{\hbar} P(2eV) \quad , \quad T = 0$$

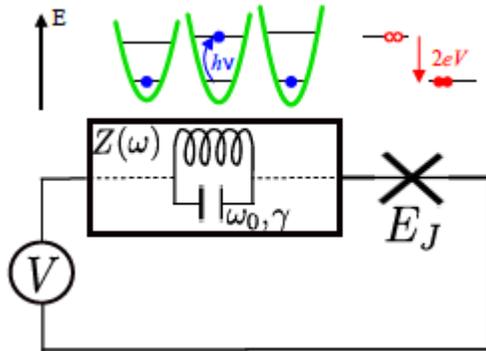
↑
Probability of the environment
to absorb energy: phase-phase correlation



Limit: Single mode environment



Beyond low transmission: Josephson photonics



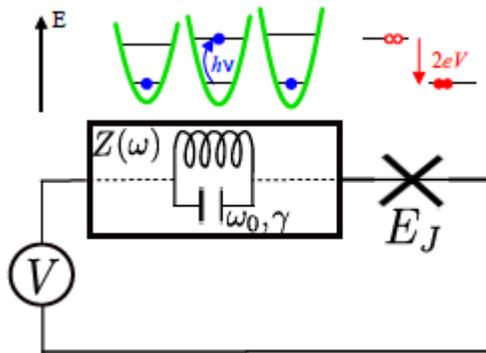
$$\omega_J = \frac{2eV}{\hbar}$$

$$H = \hbar\omega_0 \left(a^\dagger a + \frac{1}{2} \right) - E_J \cos(\varphi)$$

$$\varphi = \omega_J t + \sqrt{\alpha}(a^\dagger + a)$$

Effective fine structure constant $\alpha = \frac{Z_{LC}}{2R_K}$

Josephson photonics



p-photon resonance $\omega_J \approx p\omega_0$

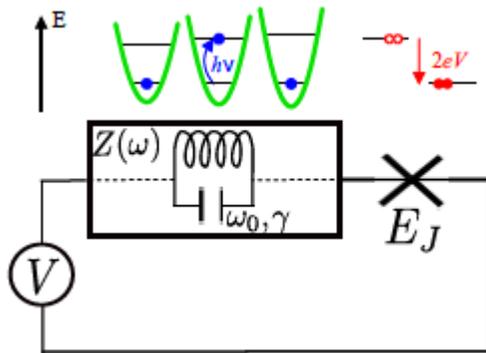
$$H_{\text{RWA}}^{(p)} = \hbar\Delta a^\dagger a - (-i)^p \frac{E_J^*}{2} : [(a^\dagger)^p + (-1)^p a^p] \frac{J_p(2\sqrt{\alpha} a^\dagger a)}{(a^\dagger a)^{p/2}} :$$

$$\alpha = \frac{Z_{LC}}{2R_K} (\approx 0.07 \dots 1.5 \dots) \quad Z_{LC} = \sqrt{\frac{L}{C}}$$

Renormalized
JJ-coupling

$$E_J^* = E_J e^{-\alpha/2}$$

Josephson photonics



p-photon resonance $\omega_J \approx p\omega_0$

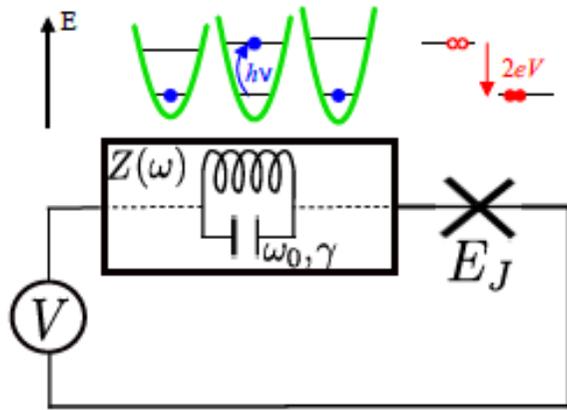
$$H_{\text{RWA}}^{(p)} = \hbar\Delta a^\dagger a - (-i)^p \frac{E_J^*}{2} : [(a^\dagger)^p + (-1)^p a^p] \frac{J_p(2\sqrt{\alpha} a^\dagger a)}{(a^\dagger a)^{p/2}} :$$

+ cavity damping

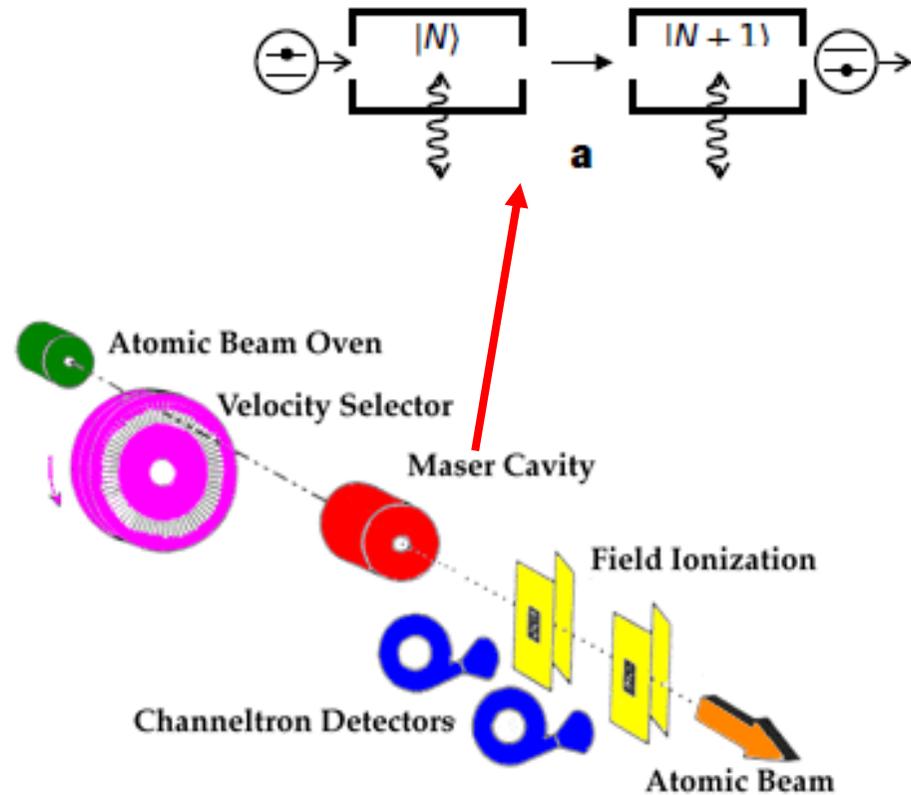
(+ low frequency voltage noise)

$$\dot{\rho} = \frac{1}{i\hbar} [H_{\text{RWA}}^{(p)}, \rho] + \gamma \mathcal{L}_{\text{res}}[\rho]$$

Josephson + cavity – micromaser (single atom maser)



$$\dot{\rho} = \frac{1}{i\hbar} [H_{\text{RWA}}^{(p)}, \rho] + \gamma \mathcal{L}_{\text{res}}[\rho]$$

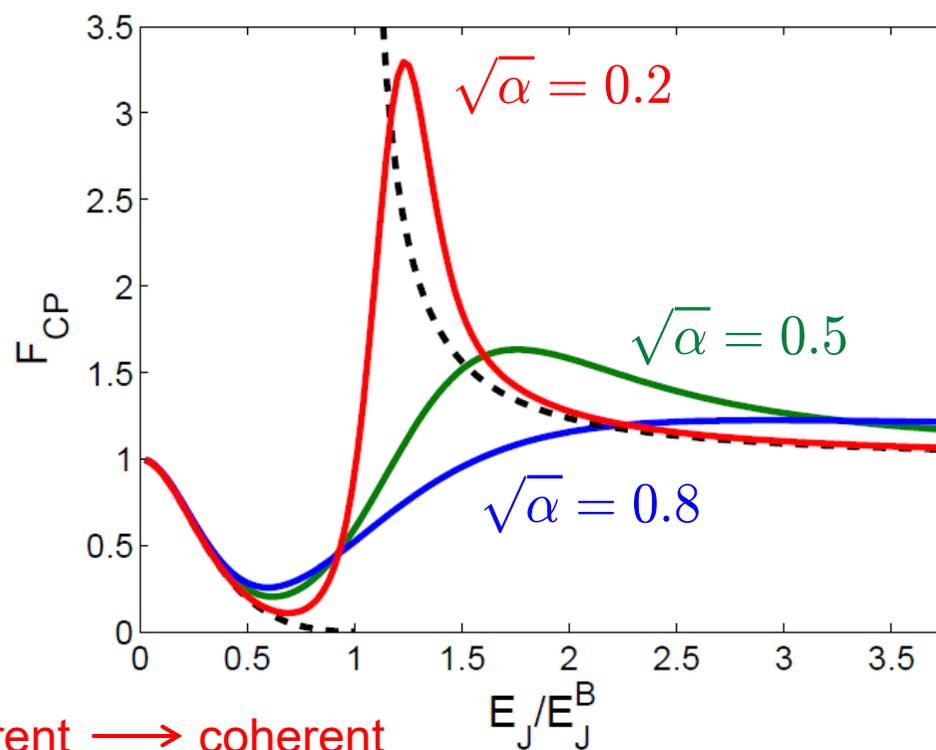


$$\dot{\rho} = \mathcal{L}_{\text{atom-field}}[\rho] + \gamma \mathcal{L}_{\text{res}}[\rho]$$

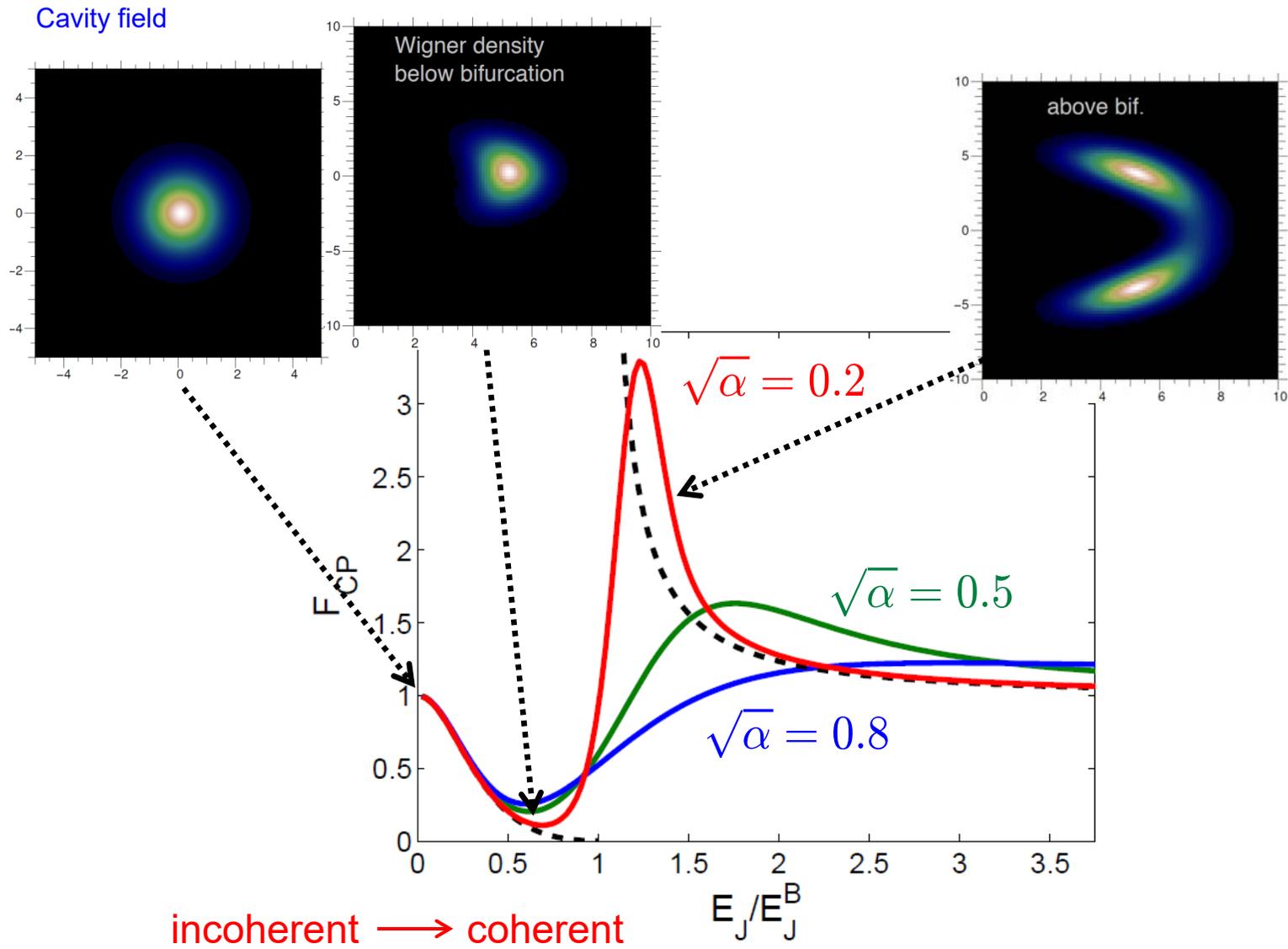
Sequential to coherent charge flow: current noise

$$\delta I_{CP} = I_{CP} - \langle I_{CP} \rangle$$

Fano factor
$$F_{CP} = \frac{1}{2eI_{CP}} \int d\tau \langle \delta I_{CP}(t + \tau) \delta I_{CP}(t) \rangle_{st}$$

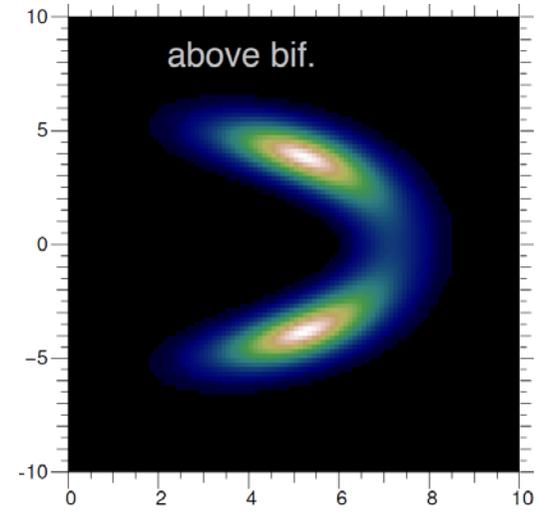
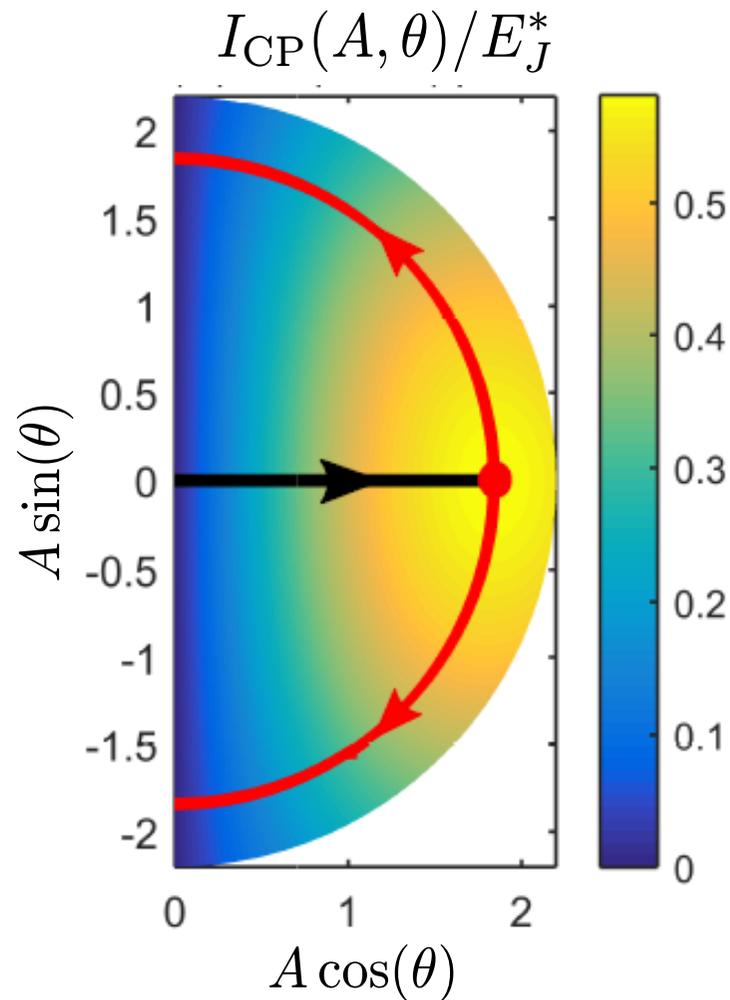
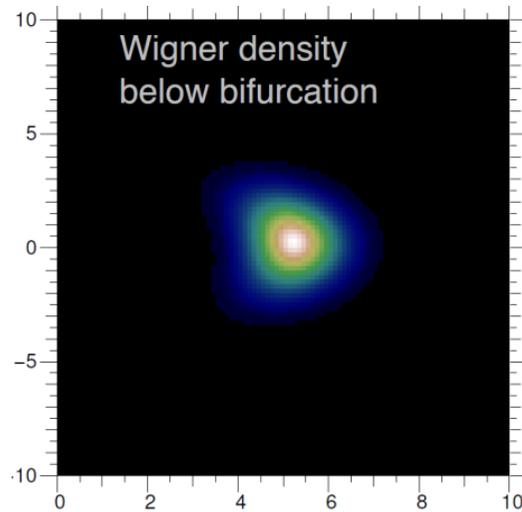


„Phase transition“-like behavior



Semiclassics: critical behavior

$$\langle a \rangle = A e^{i\theta}$$



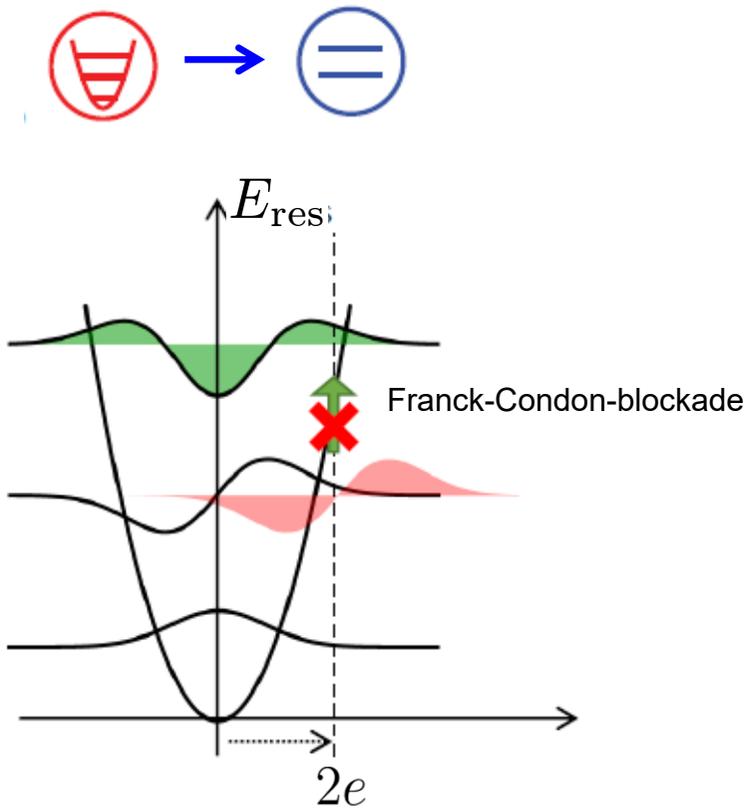
From phase locking to amplitude locking

Towards single photon sources for microwaves

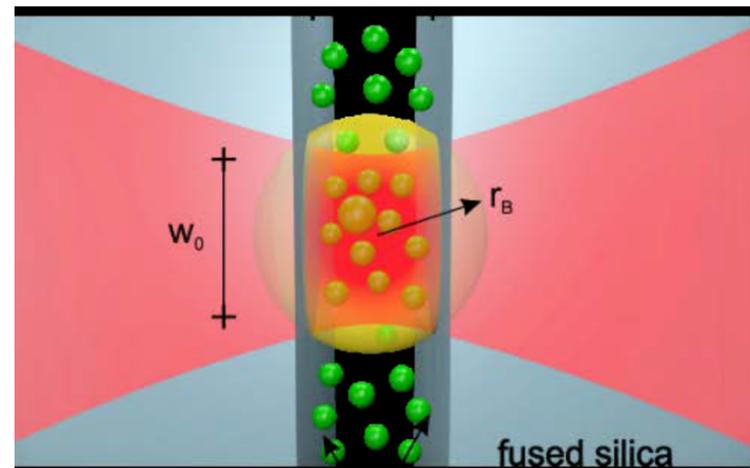
Weak driving $g^{(2)}(0) \approx \left(1 - \frac{\alpha}{2}\right)^2$

$$g^{(2)}(\tau) = \frac{\langle a^\dagger(\tau) a^\dagger a a(\tau) \rangle_{st}}{\langle n \rangle_{st}^2}$$

$$T_{12}(\alpha = 2) = \langle 2 | H_{RWA}^{(1)} | 1 \rangle = 0$$



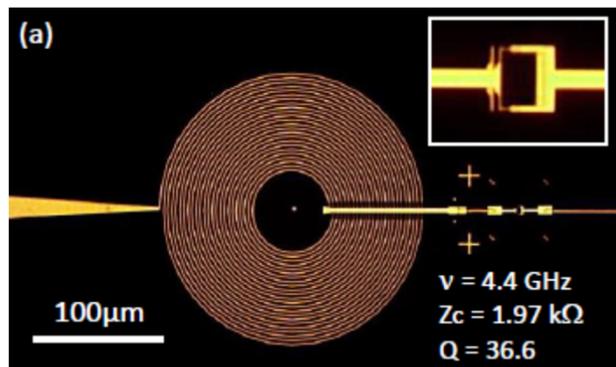
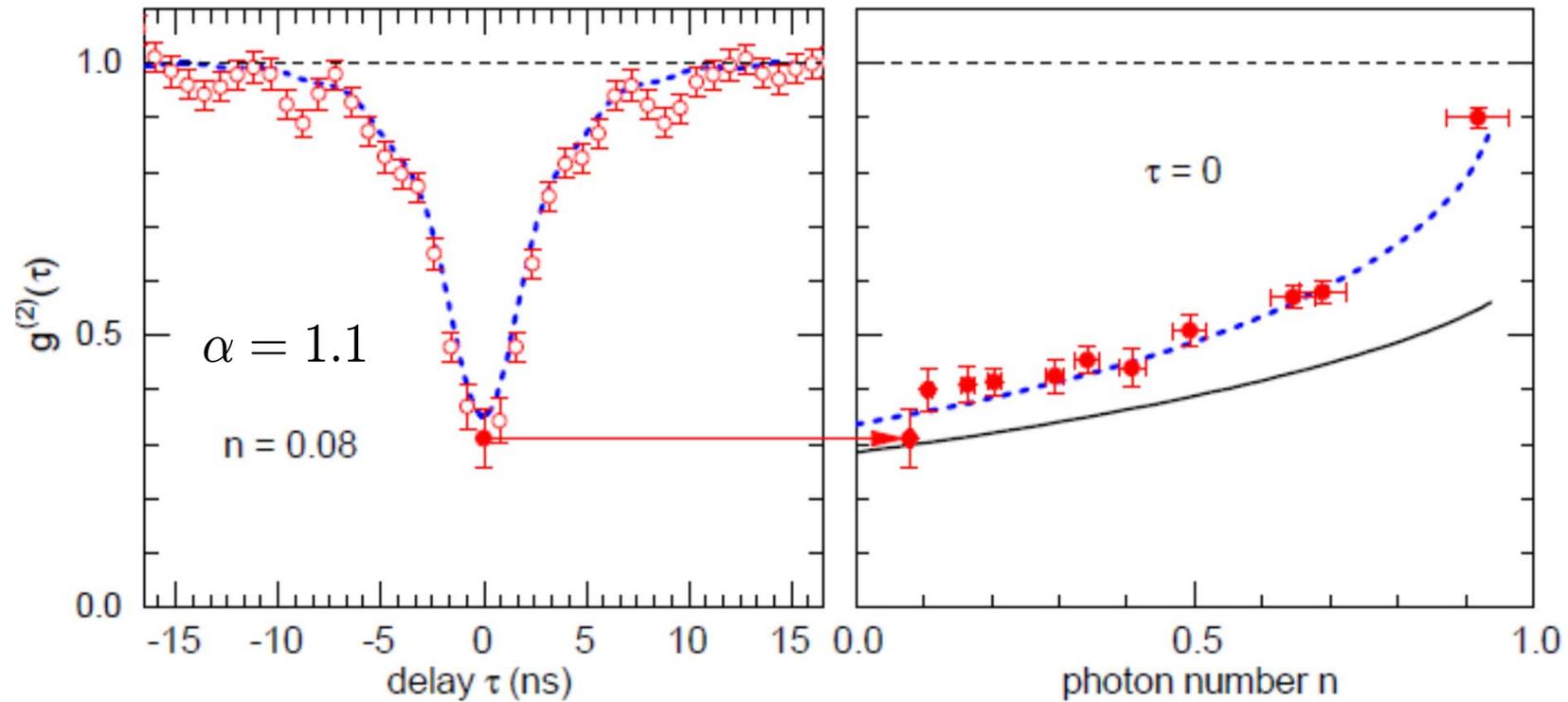
Atom physics: Rydberg blockade



Gramich et al, PRL 113, 027001 ('14)
Dambach et al, PRB 92, 054508 ('15)

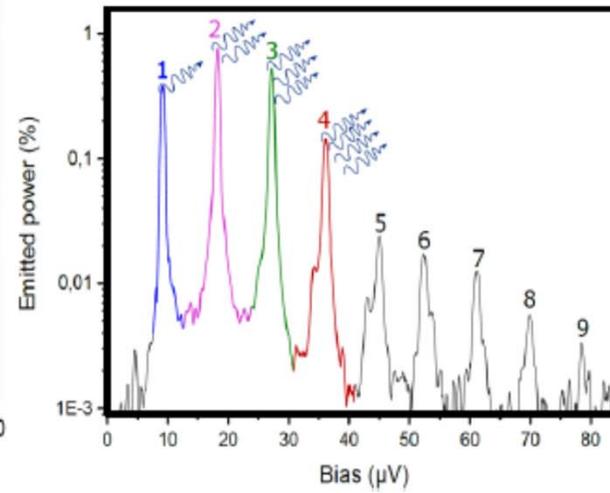
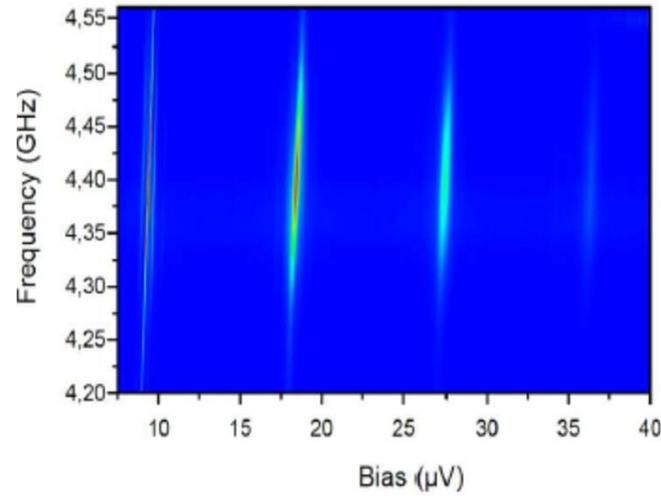
Ripka et al, Science 362, 446 (2018)

Ultra-large coupling regime: single photon sources

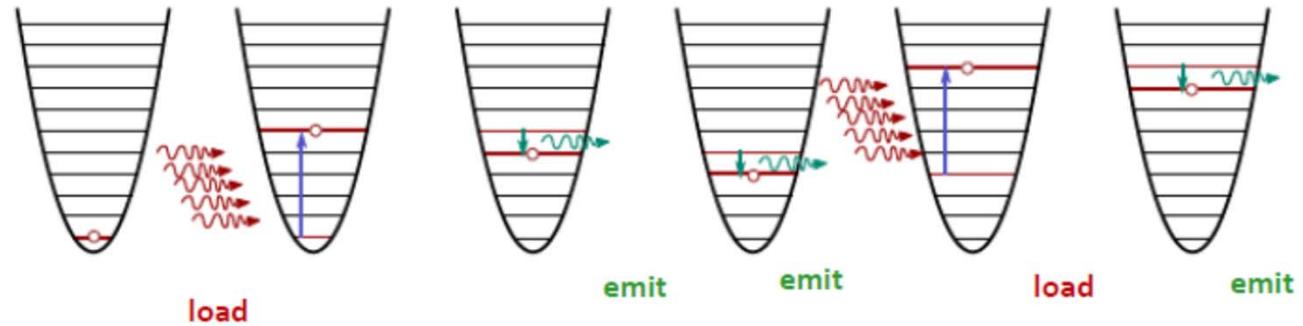


$\sim 10^8$ photons/s

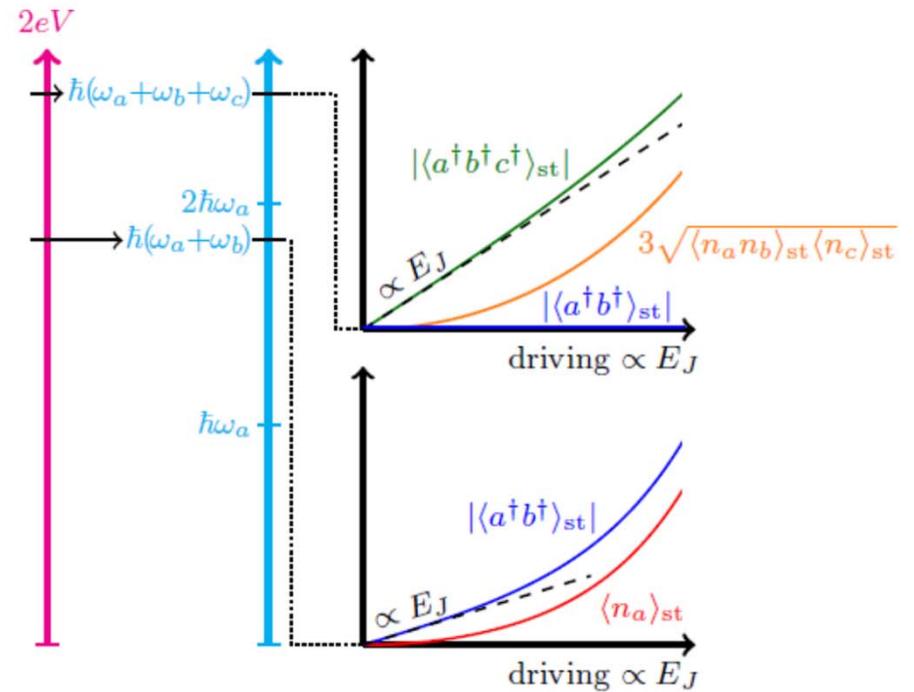
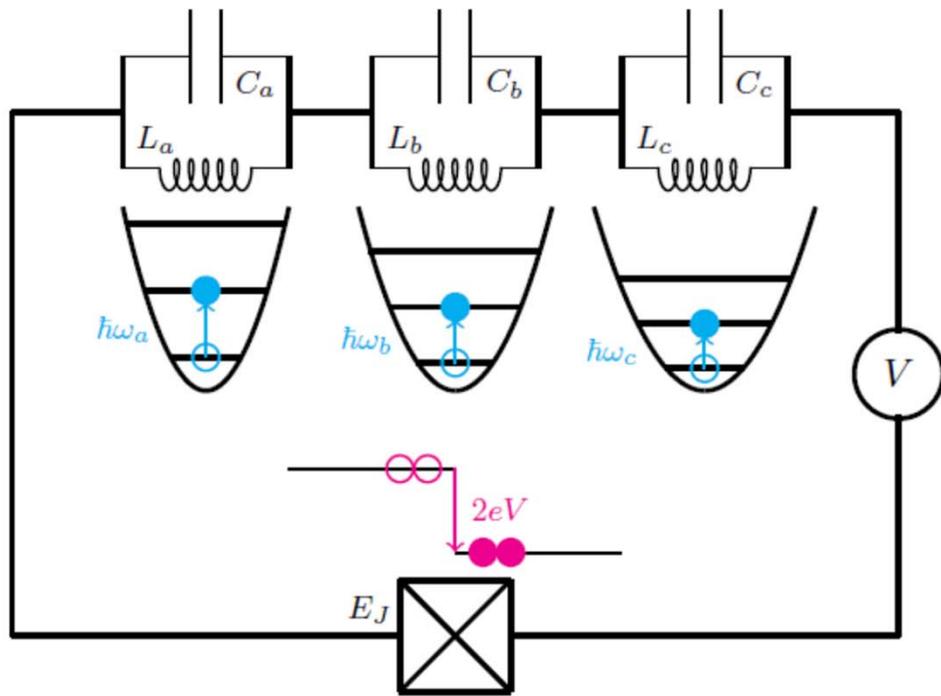
Large coupling regime: multi-photon events



Out of
equilibrium
tunneling



Multi-photon entanglement



$$H_N = \frac{E_J^*}{2} : \left(\prod_{k=1}^N a_k^\dagger + \prod_{k=1}^N a_k \right) \prod_{k=1}^N \frac{J_1(\sqrt{\alpha a_k^\dagger a})}{\sqrt{a_k^\dagger a_k}} :$$

Classical: Collective modes

REPORT

OPTOELECTRONICS

Kouwenhoven et al, *Science* 355, 939 ('17)

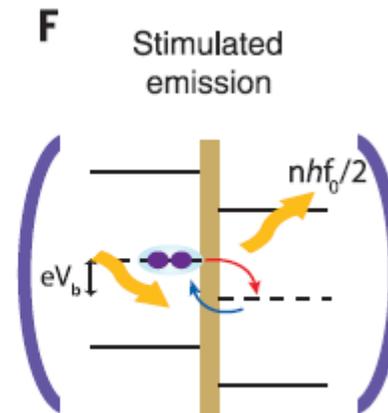
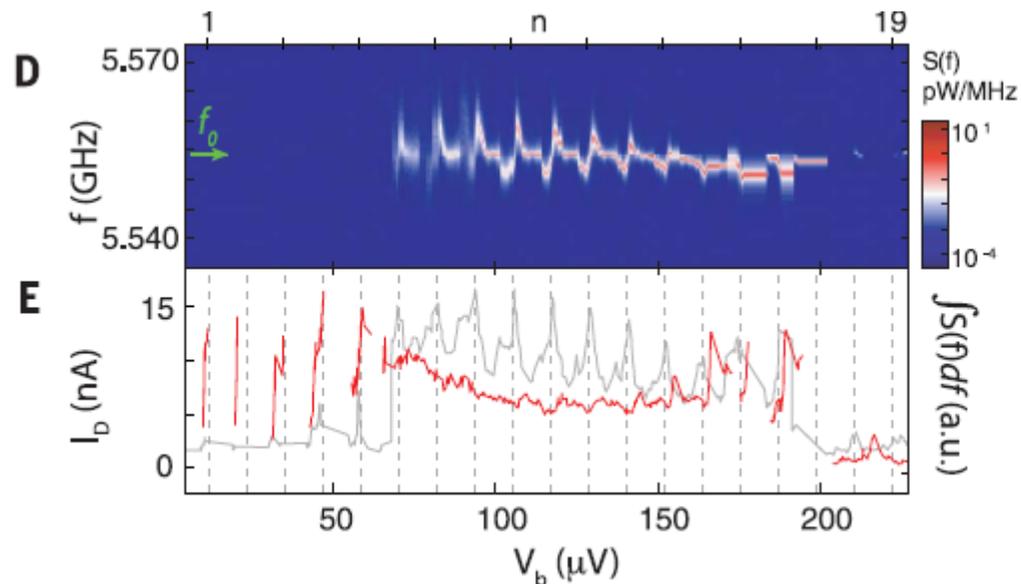
Demonstration of an ac Josephson junction laser

M. C. Cassidy,¹ A. Bruno,¹ S. Rubbert,² M. Irfan,² J. Kammhuber,¹ R. N. Schouten,^{1,2}
A. R. Akhmerov,² L. P. Kouwenhoven^{1,2*}

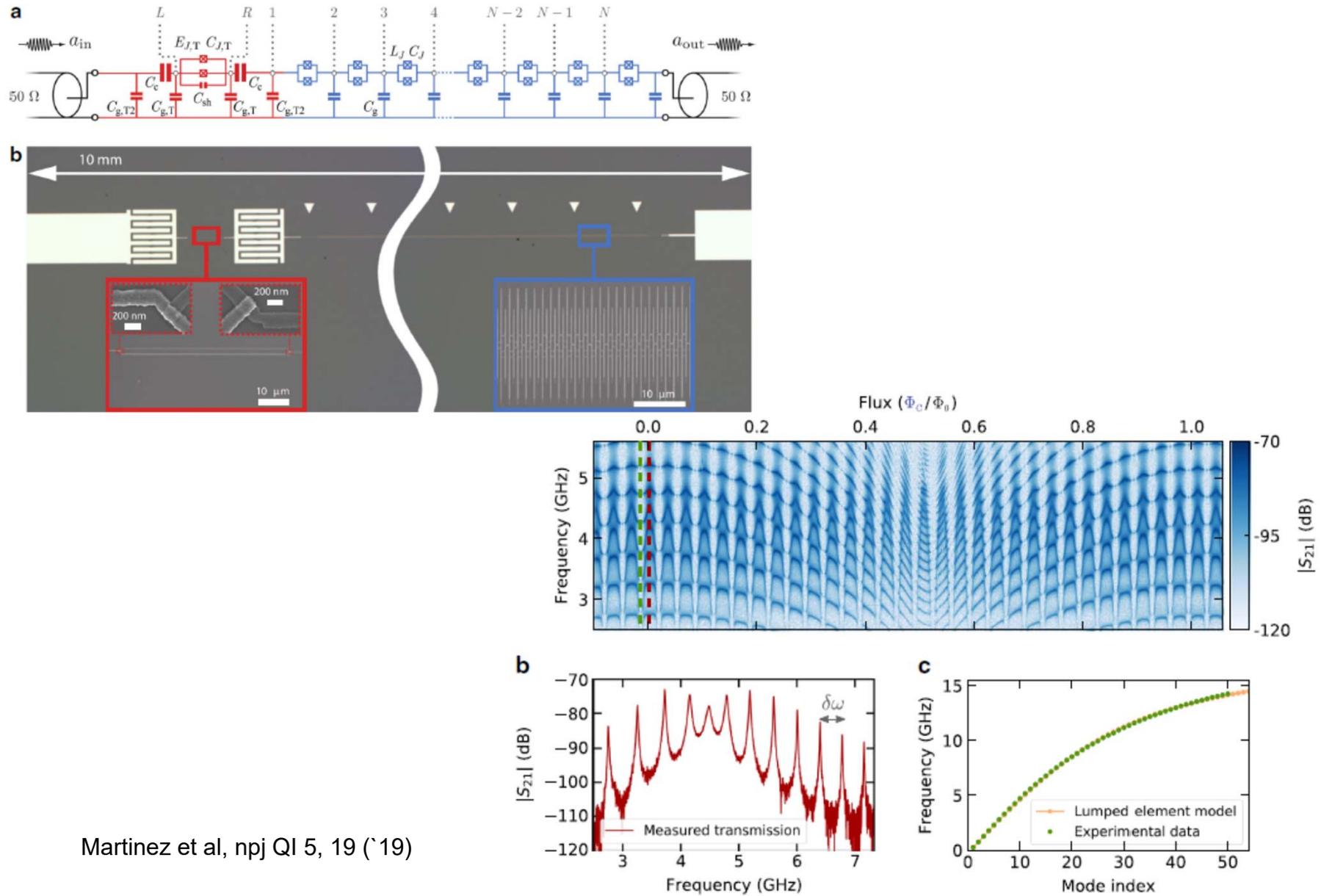
Collective excitations

$$\psi = \sum_n a_n \varphi_n$$

Classical: Simon et al, PRL 121, 027004 ('18)



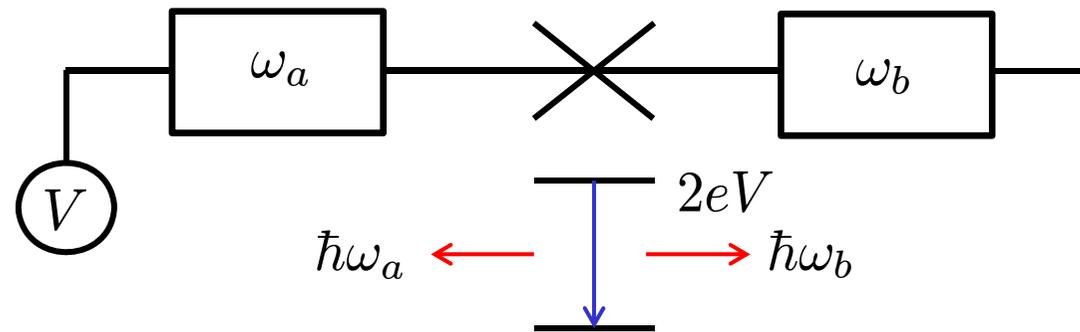
Tunable N-mode environment



Martinez et al, npj QI 5, 19 (2019)

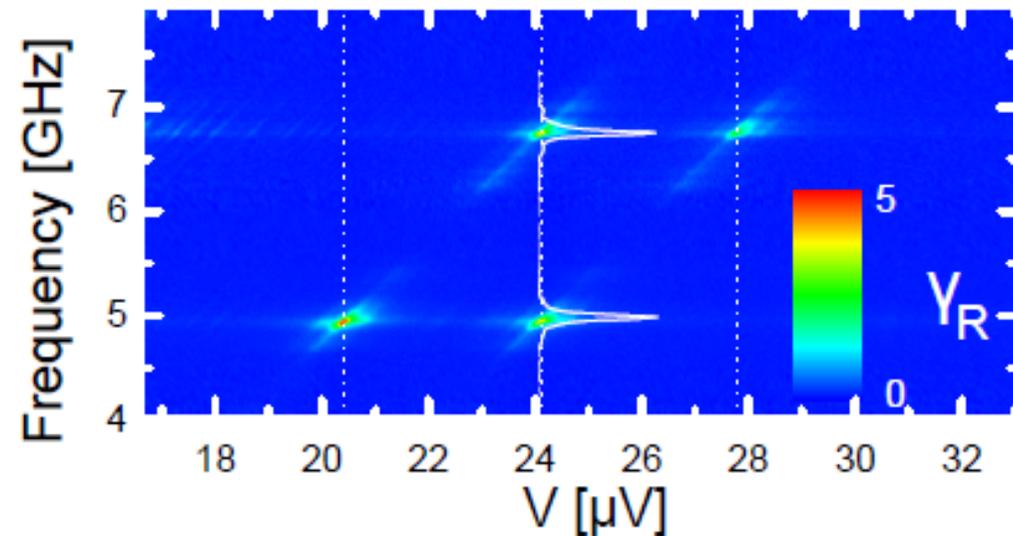
Towards entangled photon sources

$$2eV = \hbar(\omega_a + \omega_b)$$



Non-classical light:

$$g_{ab}^{(2)}(0) \not\leq \sqrt{g_{aa}^{(2)}(0) g_{bb}^{(2)}(0)}$$

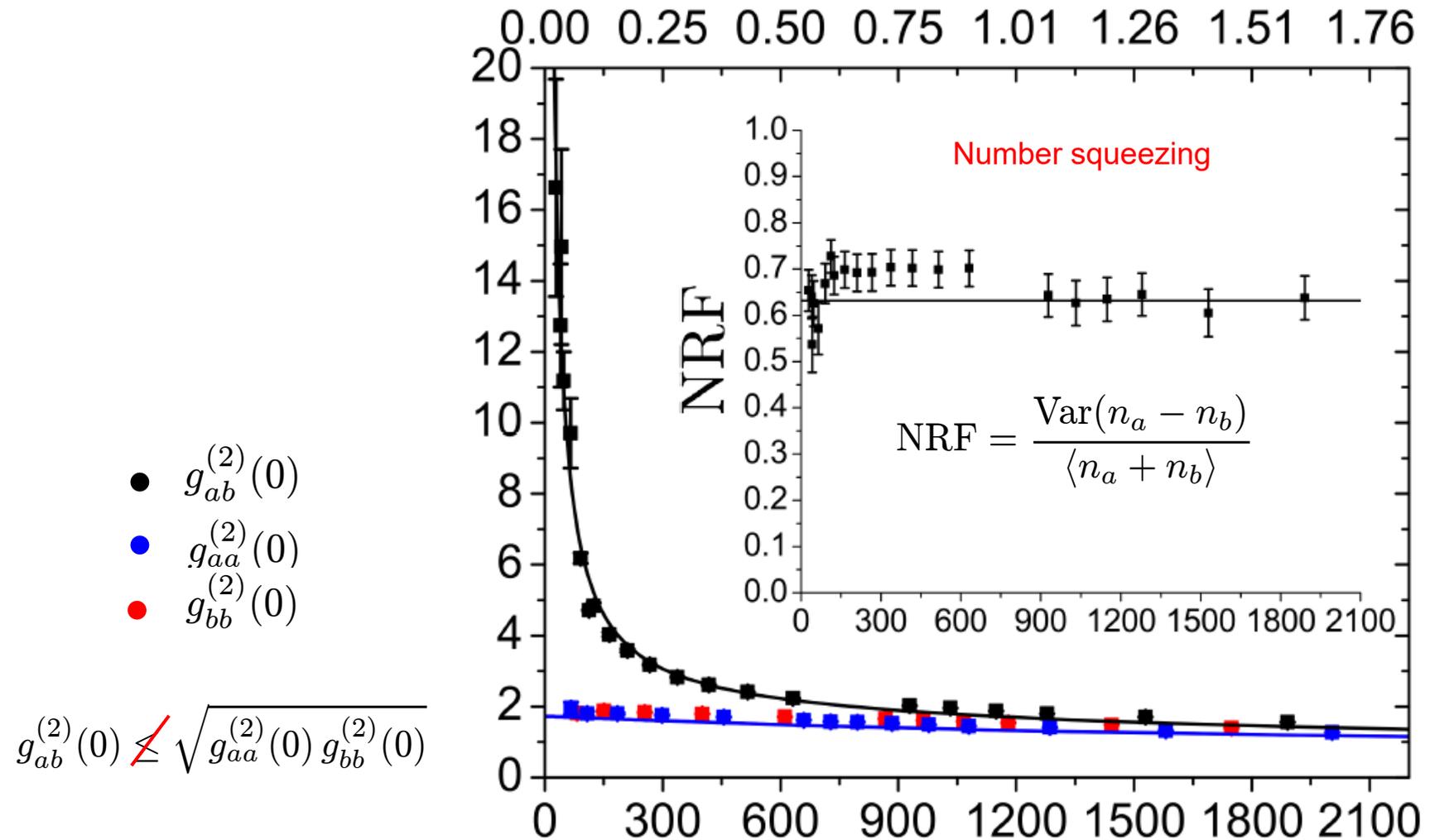


Saclay-Ulm, PRL 119, 137001 (2017)

Leppäkangas et al, NJP 2014

Armour, Kubala, JA, PRB (2015)

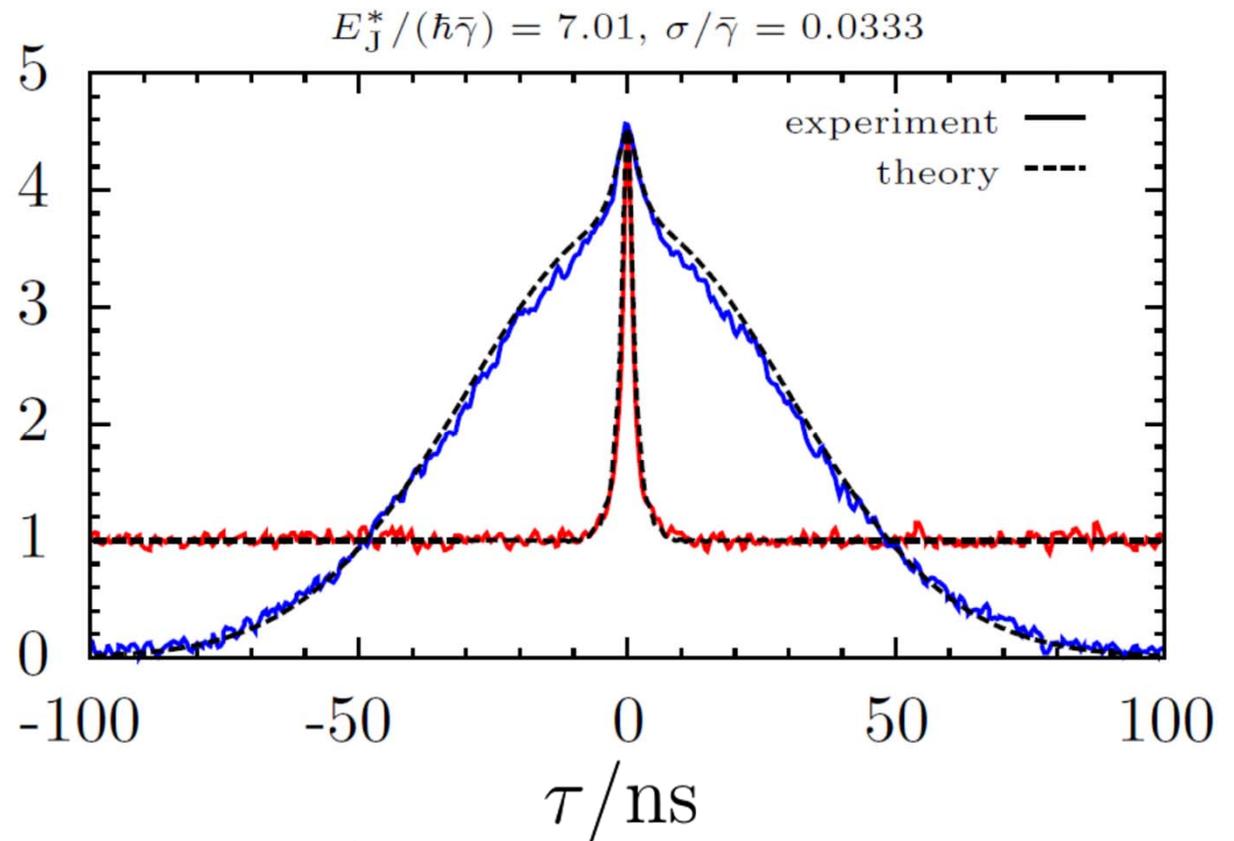
Two cavities: correlated photons



Entangled photon pairs

Entanglement witness

$$g_\phi(\tau) > g_{ab}(\tau)$$



Blue

$$g_\phi(\tau) = \langle a^\dagger(\tau)b^\dagger(\tau)a(0)b(0) \rangle$$

Red

$$g_{ab}(\tau) = \langle b^\dagger(0)a^\dagger(\tau)a(\tau)b(0) \rangle$$

Summary

- Superconducting circuits as open quantum systems
- JJ-cavity devices: Weak to strong coupling
- Single to few-body quantum dynamics far from equilibrium
- Tailored light sources

