# Coherent and chaotic dynamics of open quantum systems

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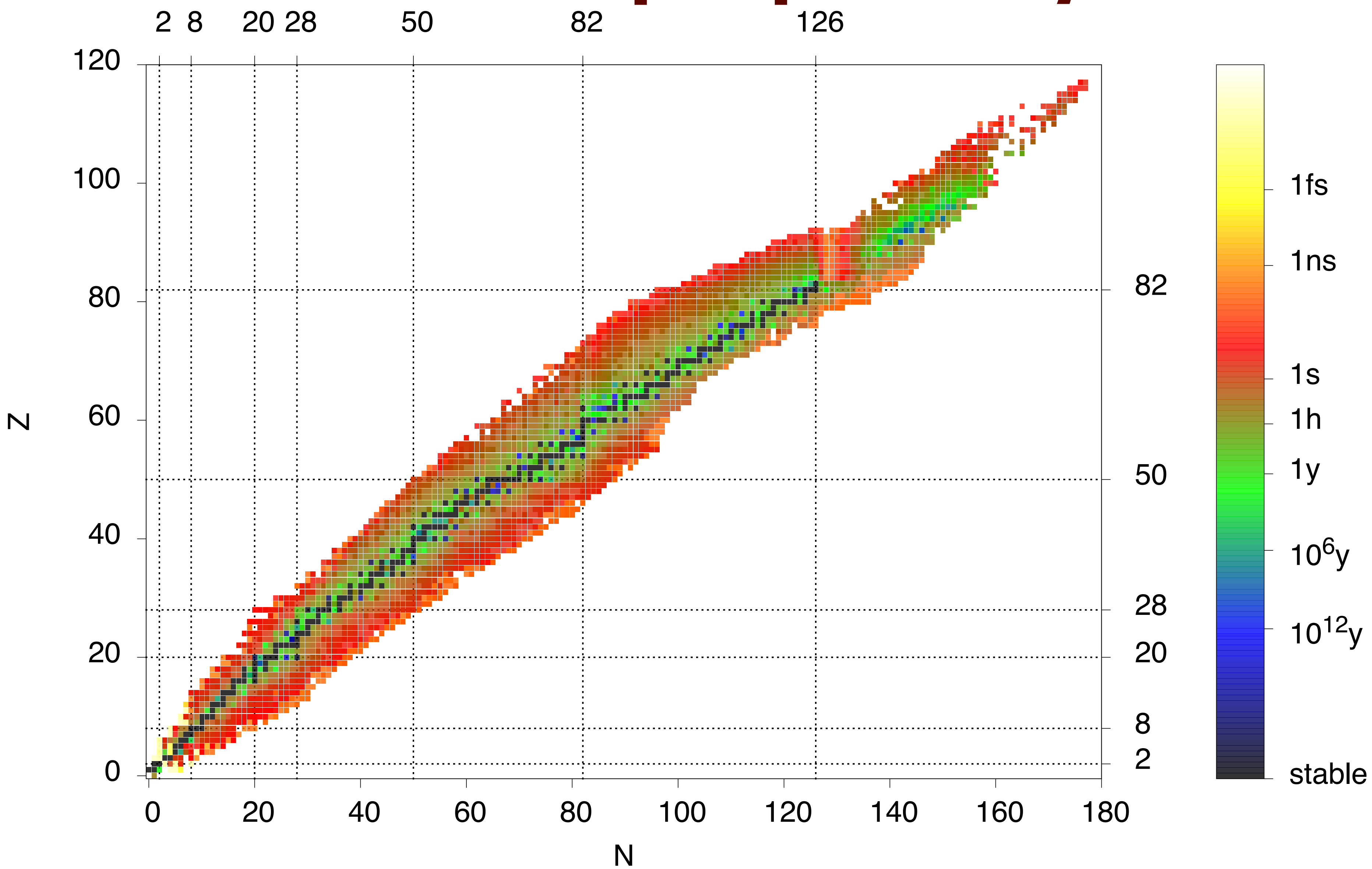
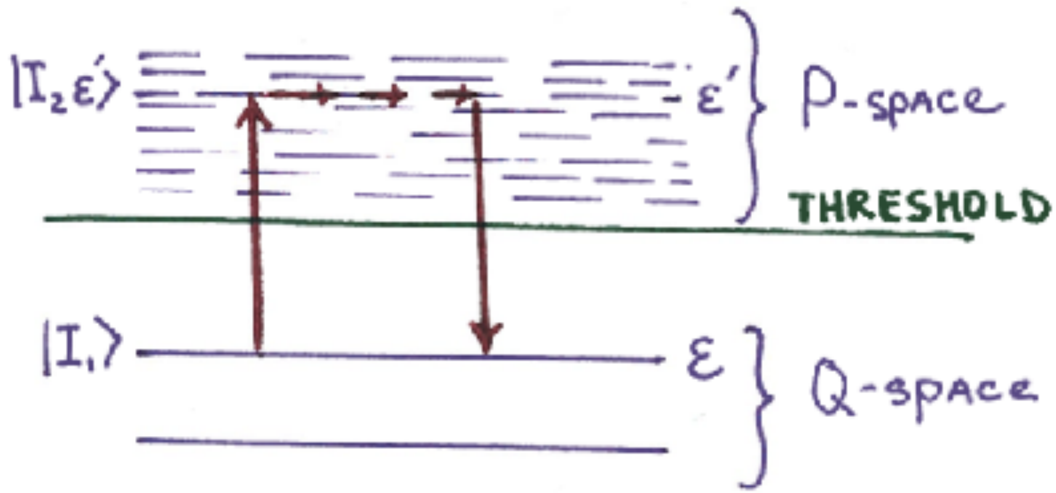
# Atomic nuclei are open quantum systems



# Topics for discussion

- Physics of coupling to continuum
  - Effective Non-Hermitian Hamiltonian formalism
  - Time dependent approach
- Features of open systems
  - Virtual excitations into continuum
  - Resonances and direct decay
  - Superradiance, alignment of structure
- Decay collectivity and intrinsic collectivities.
- Related questions

# Physics of coupling to continuum



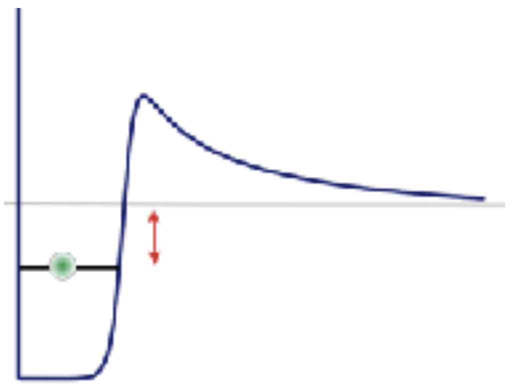

**The role of continuum-coupling**

$$H'(\epsilon) = \int_0^{\infty} d\epsilon' \, A^*(\epsilon') \frac{1}{\epsilon - \epsilon' + i0} A(\epsilon') \qquad A(\epsilon') \equiv \langle I_2, \epsilon' | H_{PQ} | I_1 \rangle$$

[1] C. Mahaux and H. Weidenmüller, *Shell-model approach to nuclear reactions*, North-Holland Publishing, Amsterdam 1969

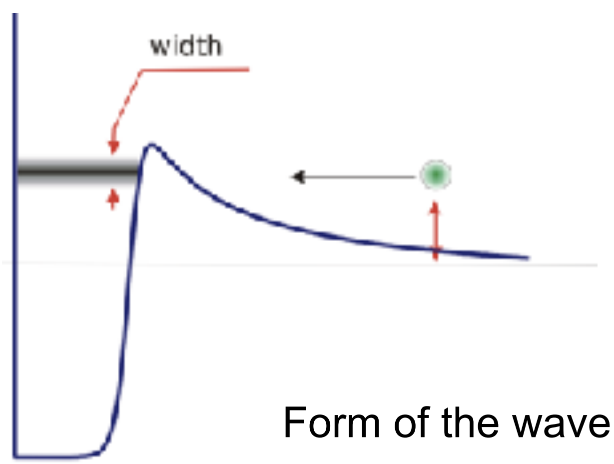
# Physics of coupling to continuum

$$H'(\epsilon) = \int_0^\infty d\epsilon' \frac{|A(\epsilon')|^2}{\epsilon - \epsilon' + i0},$$



**Integration region involves no poles**

$$H'(\epsilon) = \Delta(\epsilon) \qquad \Delta(\epsilon) = \fint d\epsilon' \frac{|A(\epsilon')|^2}{\epsilon - \epsilon' + i0}$$




**State embedded in the continuum**

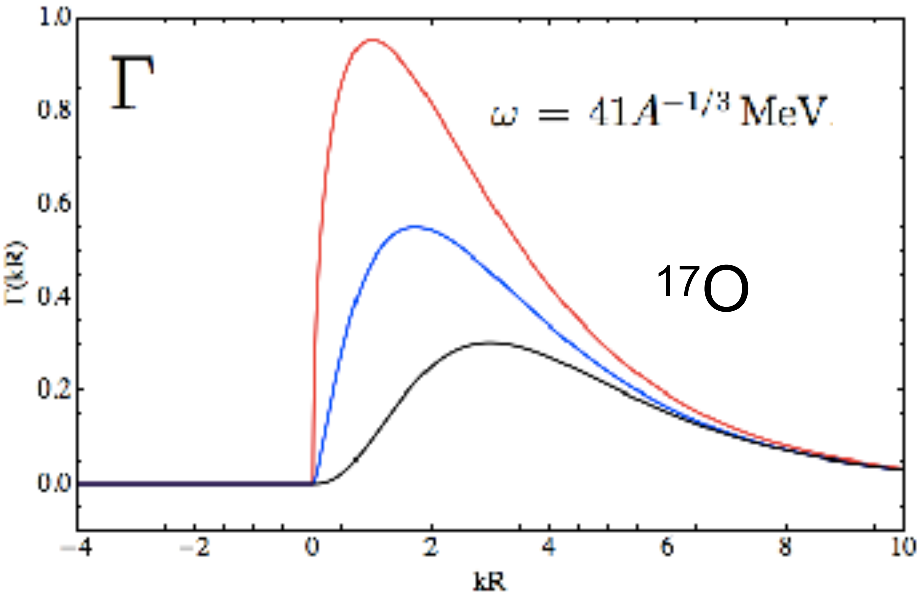
$$\frac{1}{x \pm i0} = \text{p.v.}\frac{1}{x} \mp i\pi\delta(x)$$

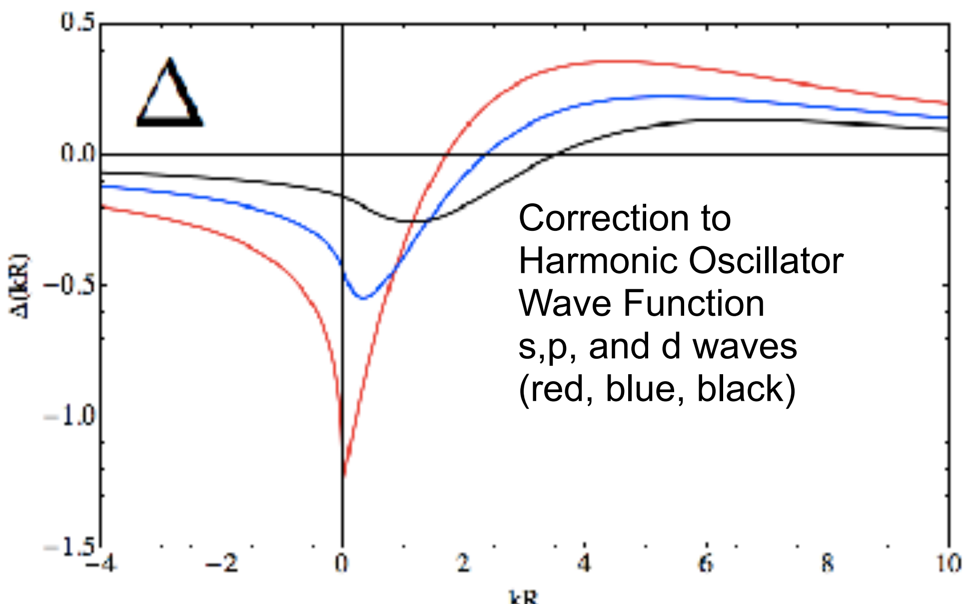
$$H'(\epsilon) = \Delta(\epsilon) - \frac{i}{2}\Gamma(\epsilon) \quad \Gamma(\epsilon) = 2\pi\,|A(\epsilon)|^2$$

Form of the wave function and probability

$$|\exp(-iEt)|^2 = 1 \to |\exp(-iEt - \Gamma t/2)|^2 = \exp(-\Gamma t)$$

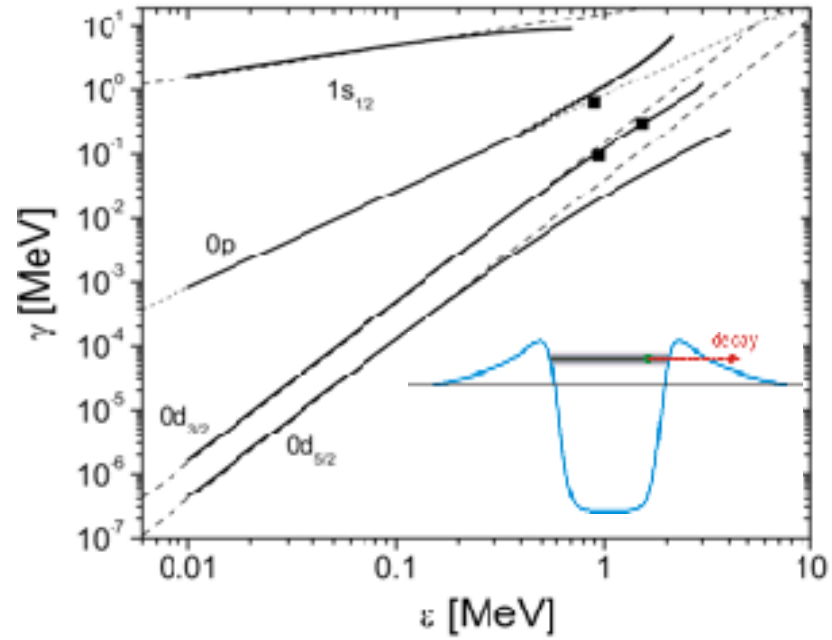
# Self energy, interaction with continuum



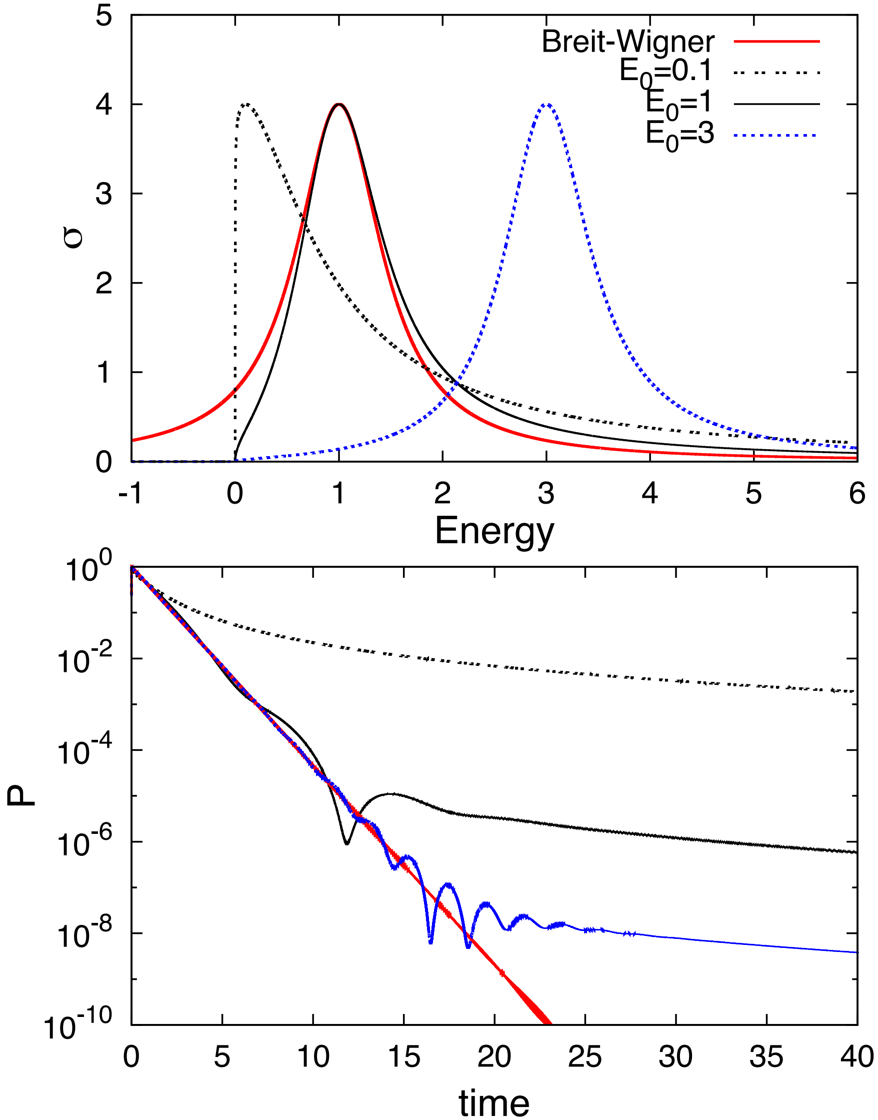

momentum

**Width as a function of energy**




D. Abrahamsen, A. Volya, and I. Wiedenhoever, *Effective R-matrix parameters of the Woods-Saxon nuclear potential,* APS Volume 57, Number 16, section KA 26 (2012).

# Time-dependent picture



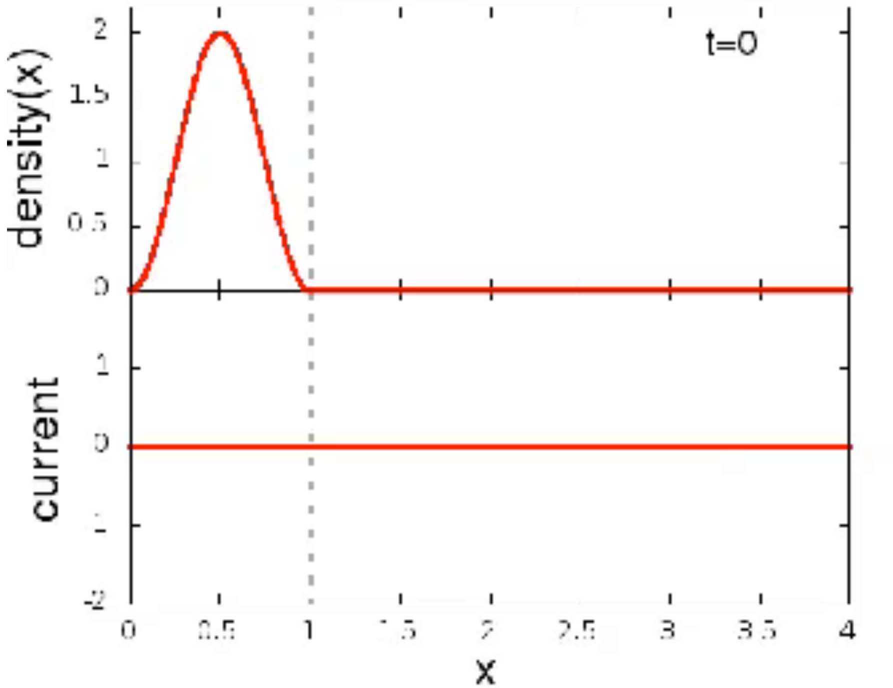

$$\mathcal{G} = \frac{1}{E - E_o + i/2\,\Gamma(E)}$$

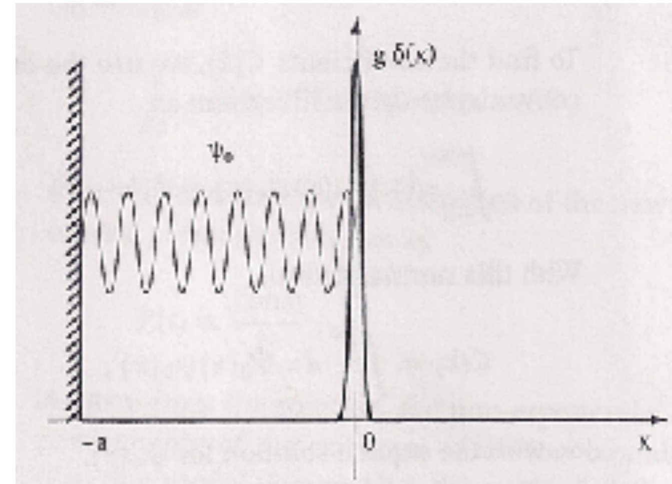
$$\Gamma(E) \propto \sqrt{E}$$

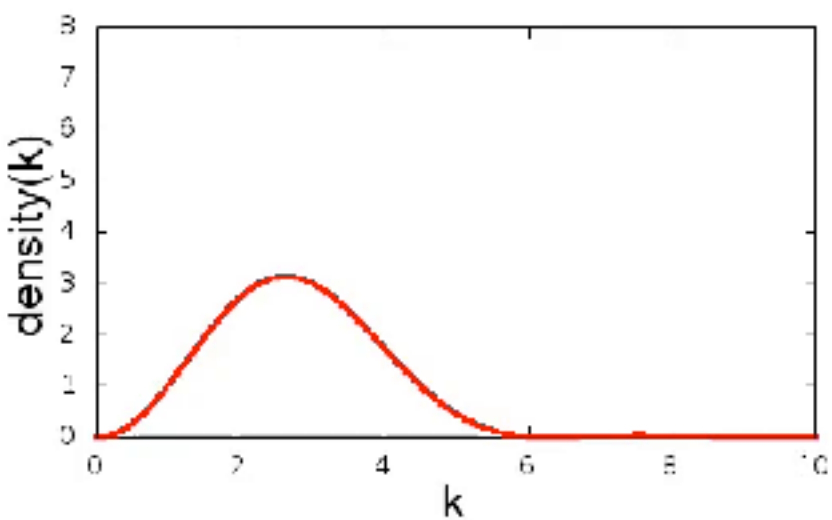
Power-law remote decay rate!

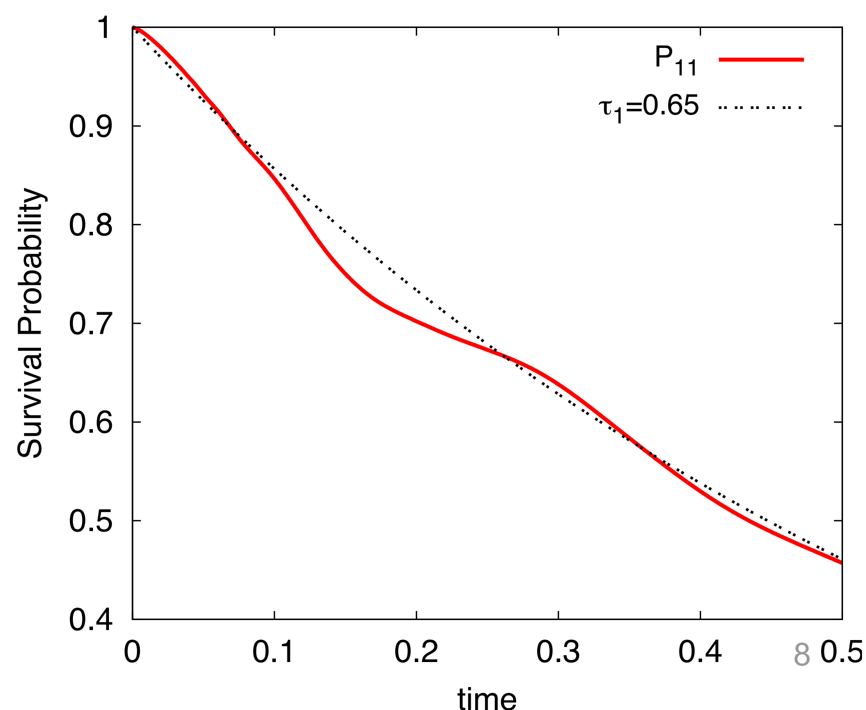
# Time dependence of decay, Winter's model

*Winter, Phys. Rev., **123**,1503 1961.*

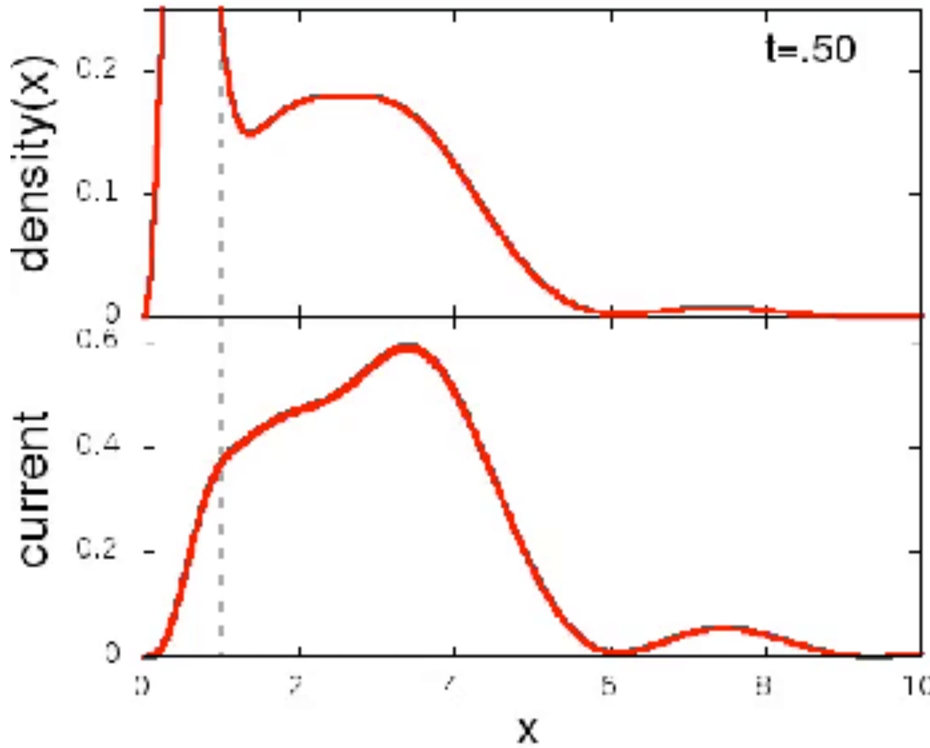


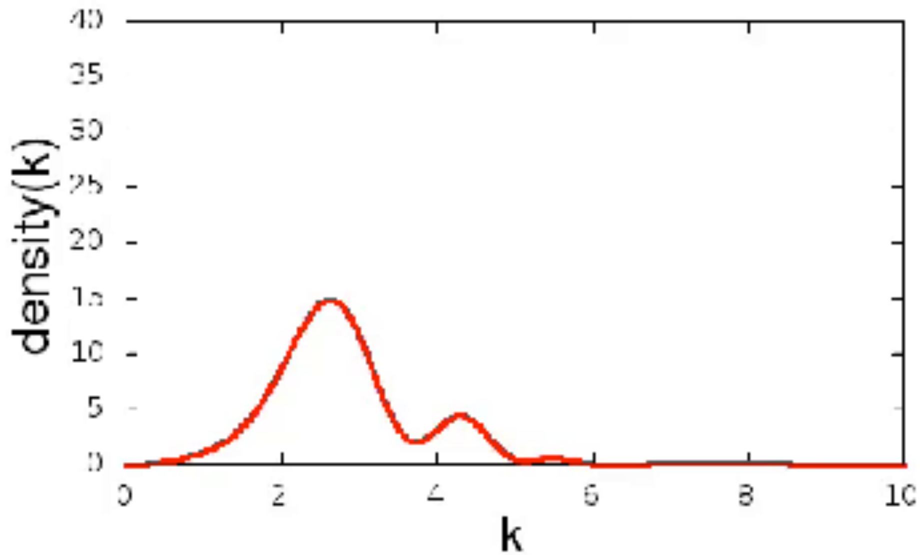



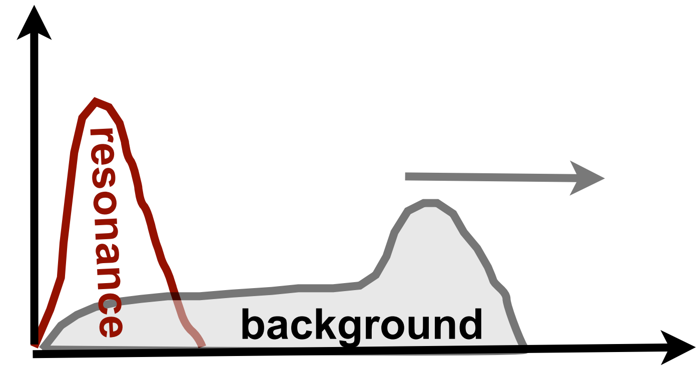



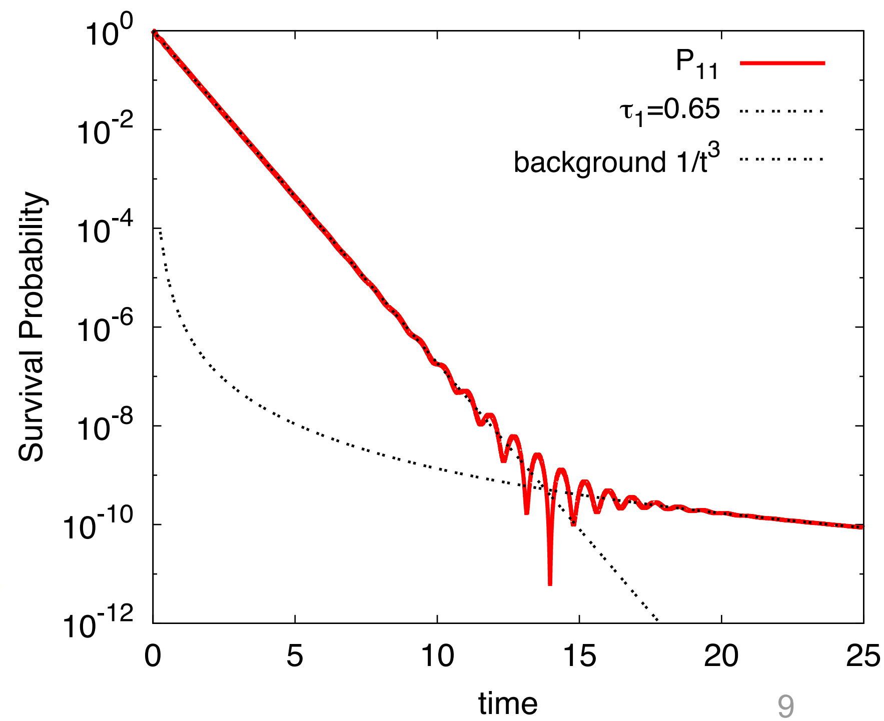



## Winter's model:
## Dynamics at remote times

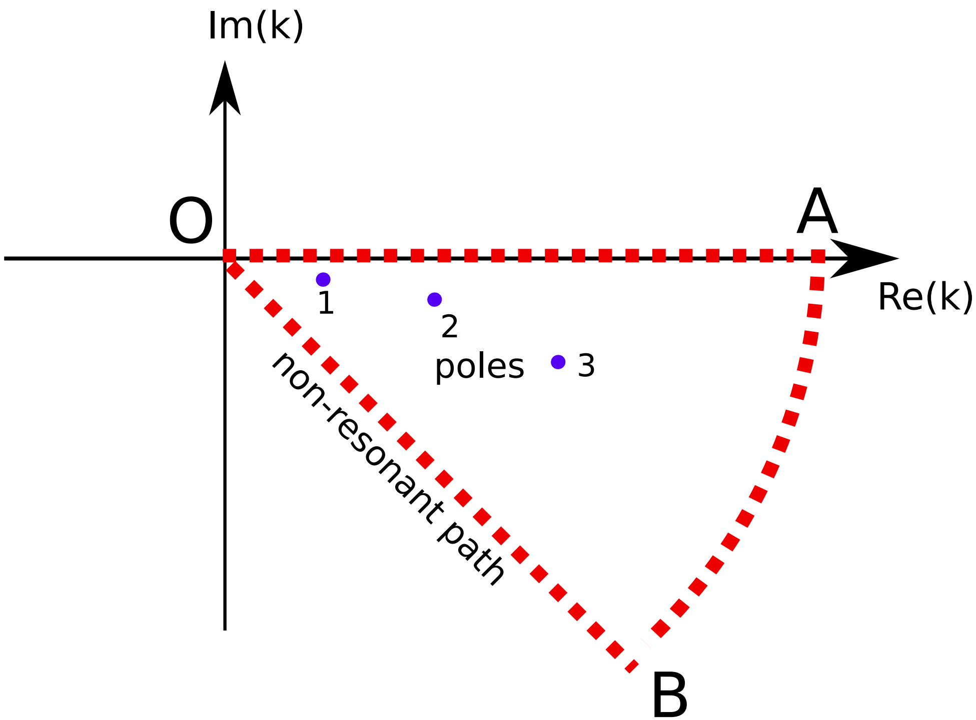


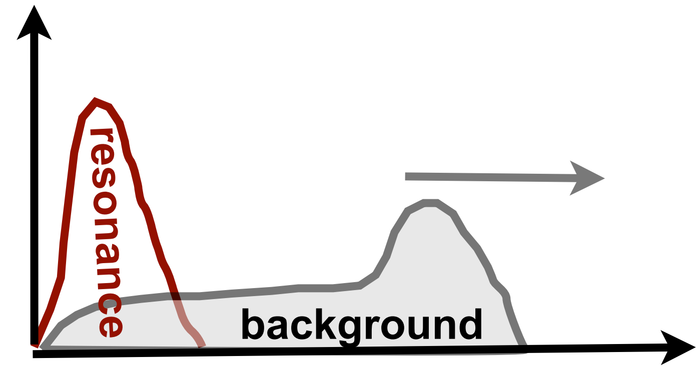



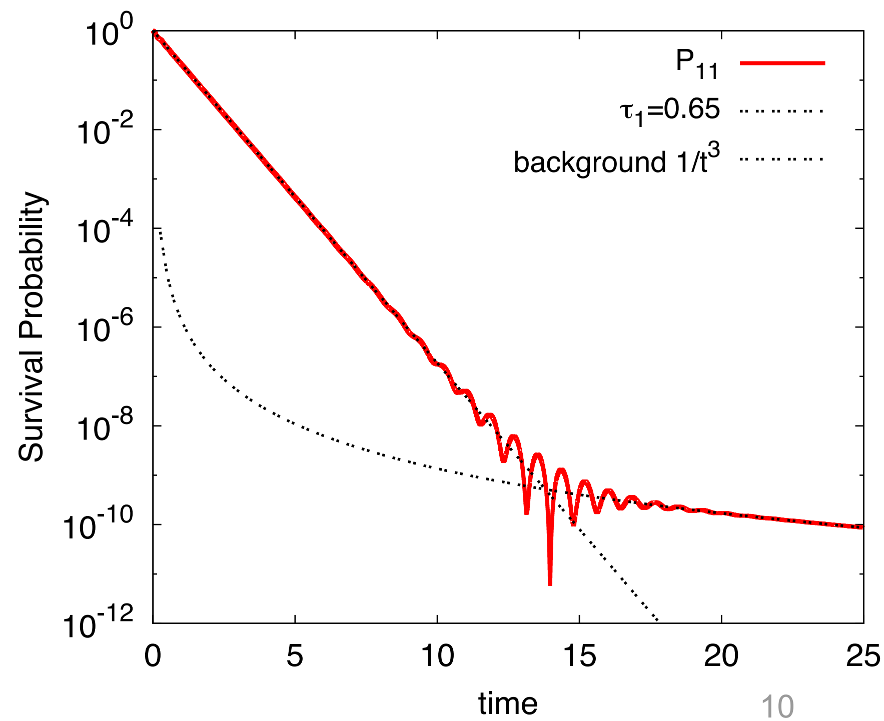



## Winter's model: Dynamics at remote times

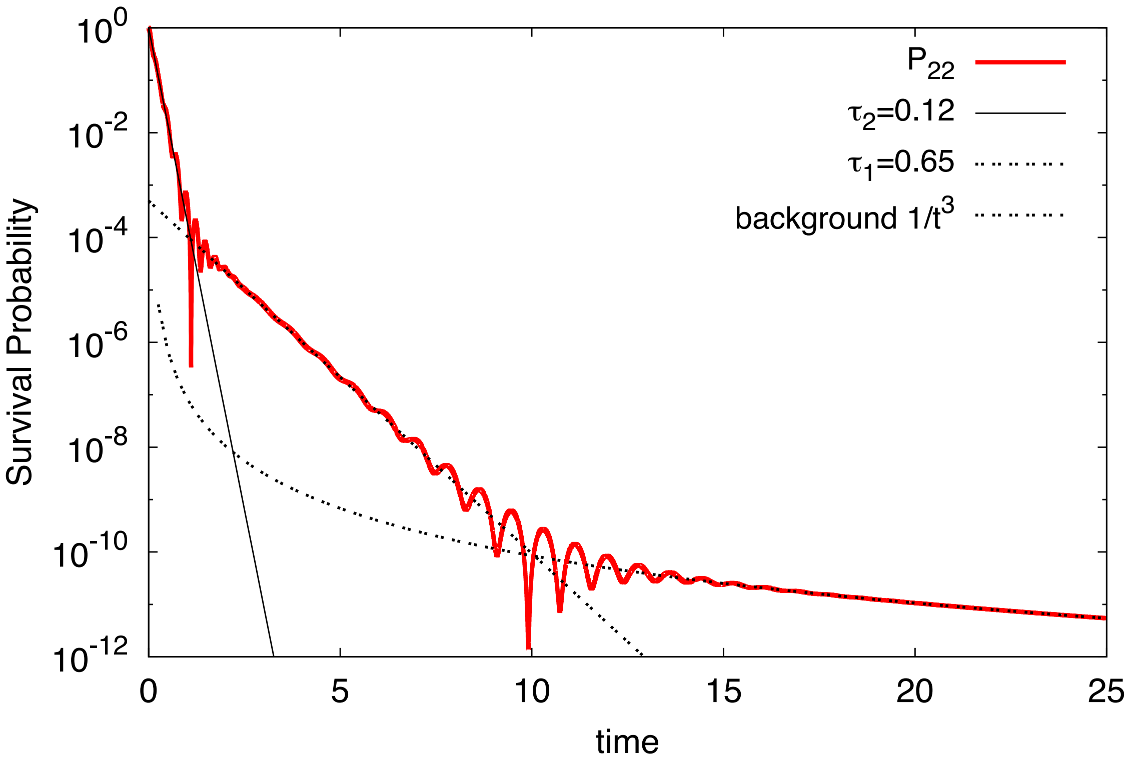


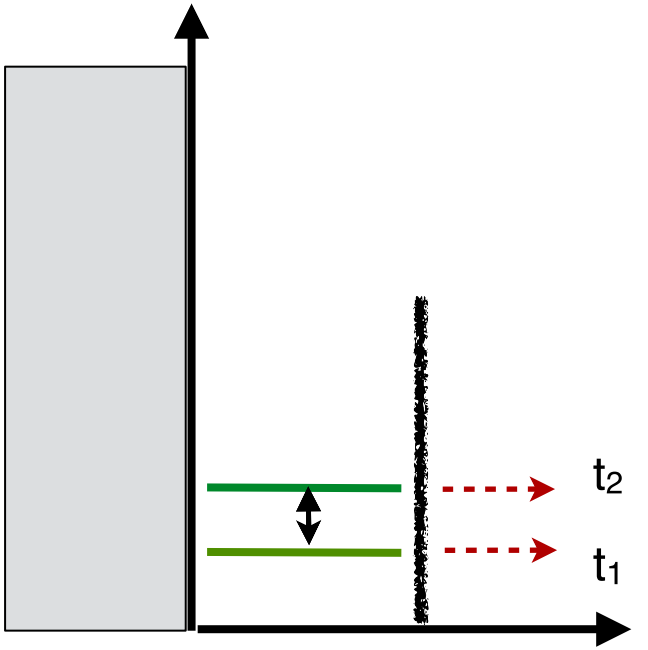




M. Peskin, AV, V. Zelevinsky, EPL, 107(4), 40001 (2014).

# Internal dynamics in decaying system
# Winter's model



# Effective Hamiltonian Formulation

The Hamiltonian in P is: $$\mathcal{H}(E) = H + \Delta(E) - \frac{i}{2}W(E)$$

Channel-vector: $$|A^c(E)\rangle = H_{QP}|c; E\rangle$$

Self-energy: $$\Delta(E) = \frac{1}{2\pi} \int dE' \sum_c \frac{|A^c(E')\rangle\langle A^c(E')|}{E - E'}$$

Irreversible decay to the excluded space: $$W(E) = \sum_{c(\text{open})} |A^c(E)\rangle\langle A^c(E)|$$

[1] C. Mahaux and H. Weidenmüller, *Shell-model approach to nuclear reactions*, Amsterdam 1969
[2] A. Volya and V. Zelevinsky, Phys. Rev. Lett. **94**, 052501 (2005).
[3] A. Volya, Phys. Rev. C **79**, 044308 (2009).

# Scattering matrix and reactions

$$\mathbf{T}_{cc'}(E) = \langle A^c(E)| \left( \frac{1}{E - \mathcal{H}(E)} \right) |A^{c'}(E)\rangle$$

$$\mathbf{S}_{cc'}(E) = \exp(i\xi_c)\ \{\delta_{cc'} - i\,\mathbf{T}_{cc'}(E)\} \exp(i\xi_{c'})$$

Cross section:

$$\sigma = \frac{\pi}{k'^2} \sum_{cc'} \frac{(2J+1)}{(2s'+1)(2I'+1)} |\mathbf{T}_{cc'}|^2$$

**Additional topics:**

- Angular (Blatt-Biedenharn) decomposition
- Coulomb cross sections, Coulomb phase shifts, and interference
- Phase shifts from remote resonances.

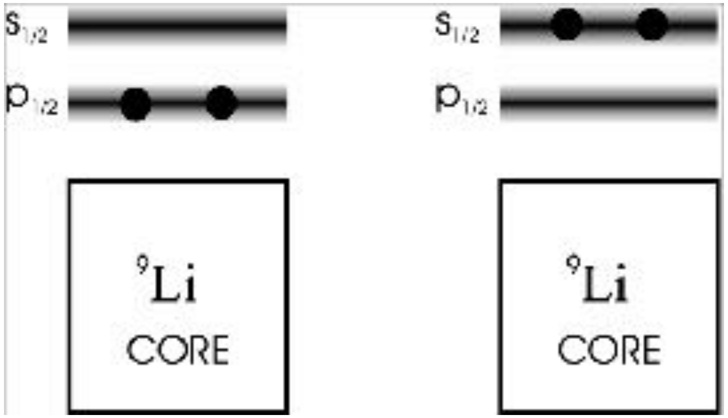
# Interference between resonances

# $^{11}$LI model

## Dynamics of two states coupled to a common decay channel

- Model $\mathcal{H}$

$$\mathcal{H}(E) = \begin{pmatrix} \epsilon_1 - \frac{i}{2}\gamma_1 & v - \frac{i}{2}A_1 A_2 \\ v - \frac{i}{2}A_1 A_2 & \epsilon_2 - \frac{i}{2}\gamma_2 \end{pmatrix}$$



# $^{11}$Li model

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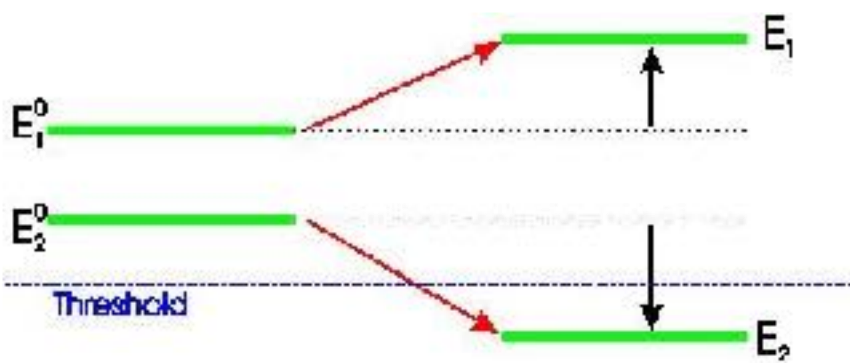


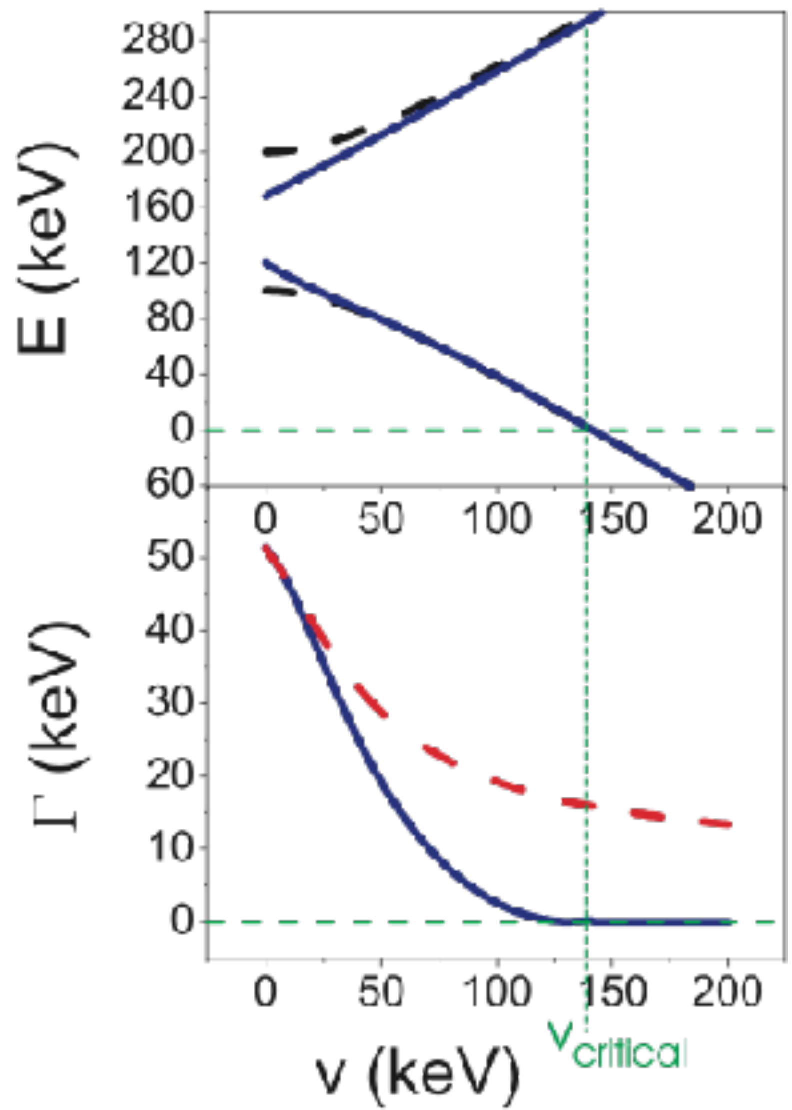

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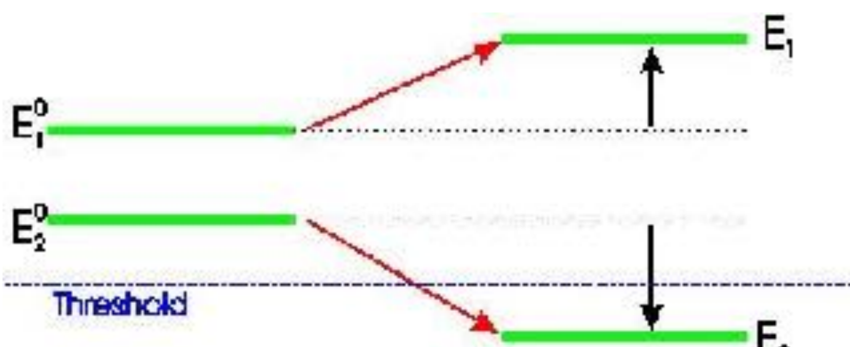
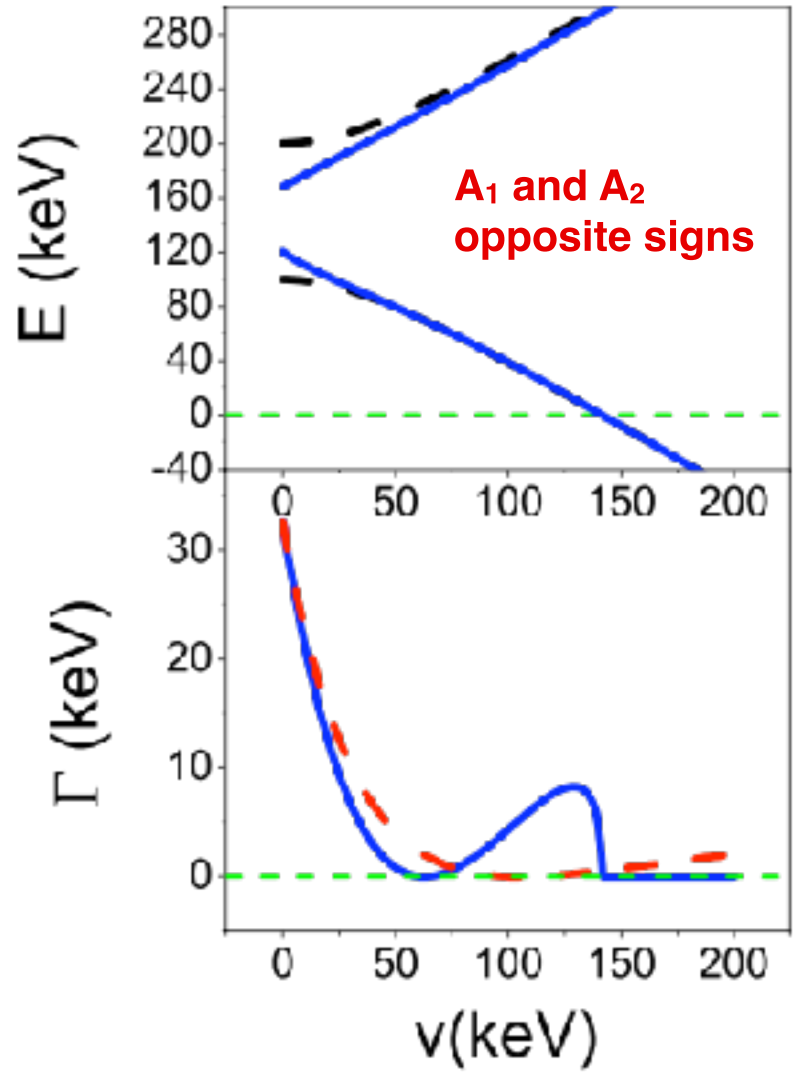
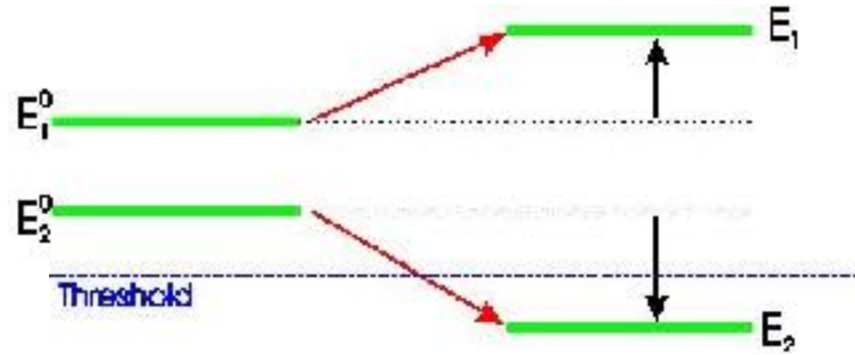
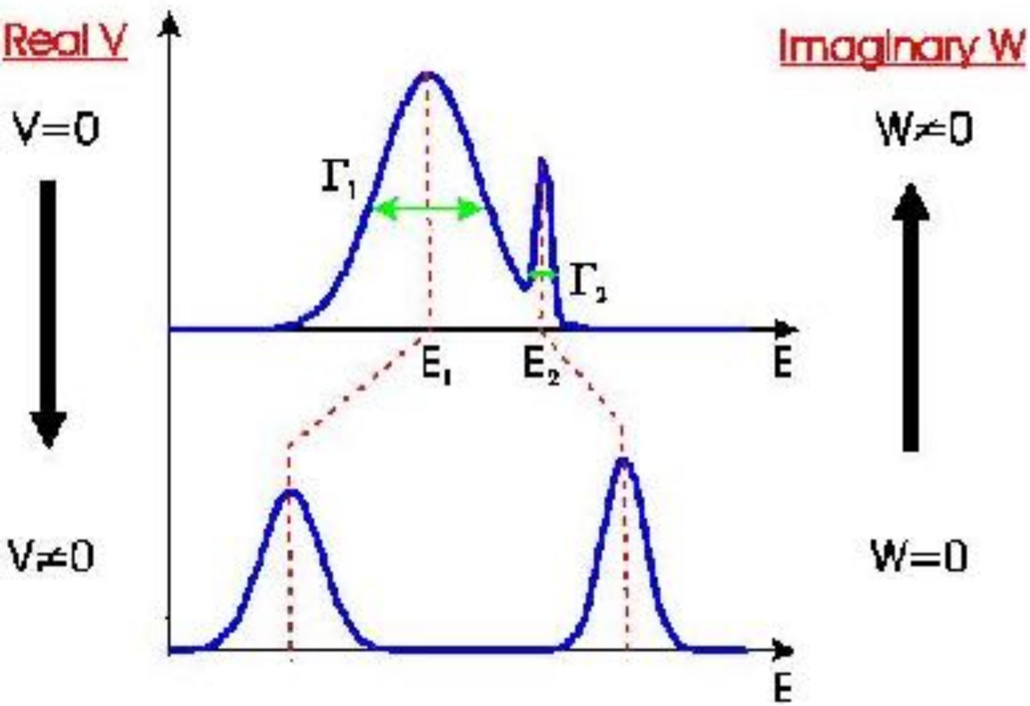


# Example of interacting resonances




$$\mathcal{H} = H^0 + V - iW/2$$

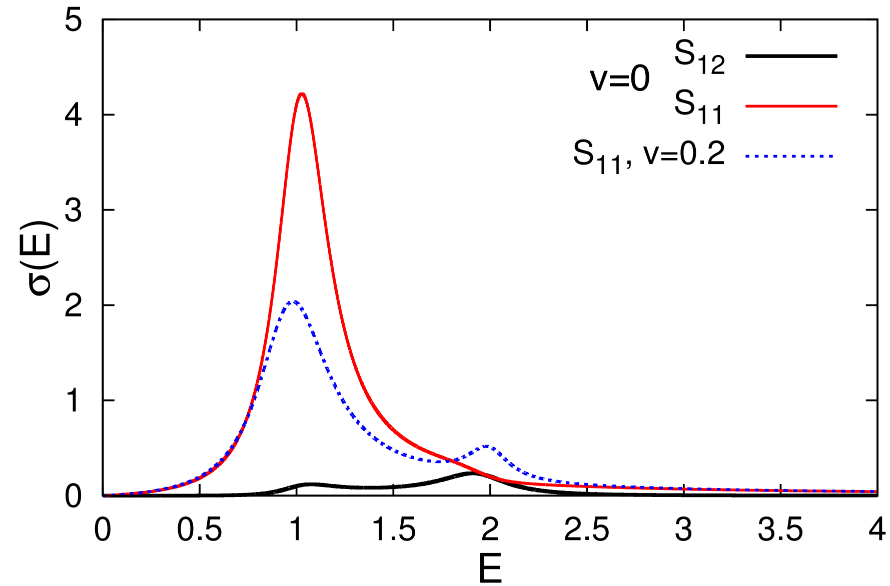


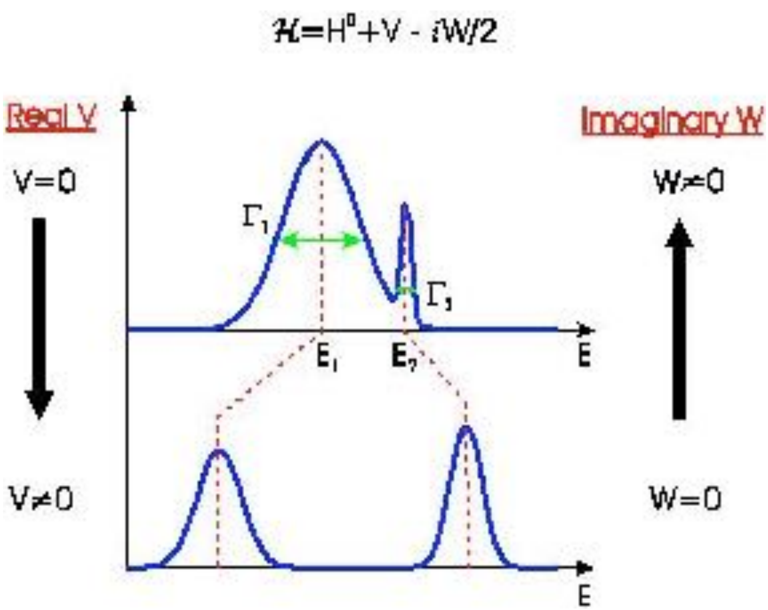
# Two-level system

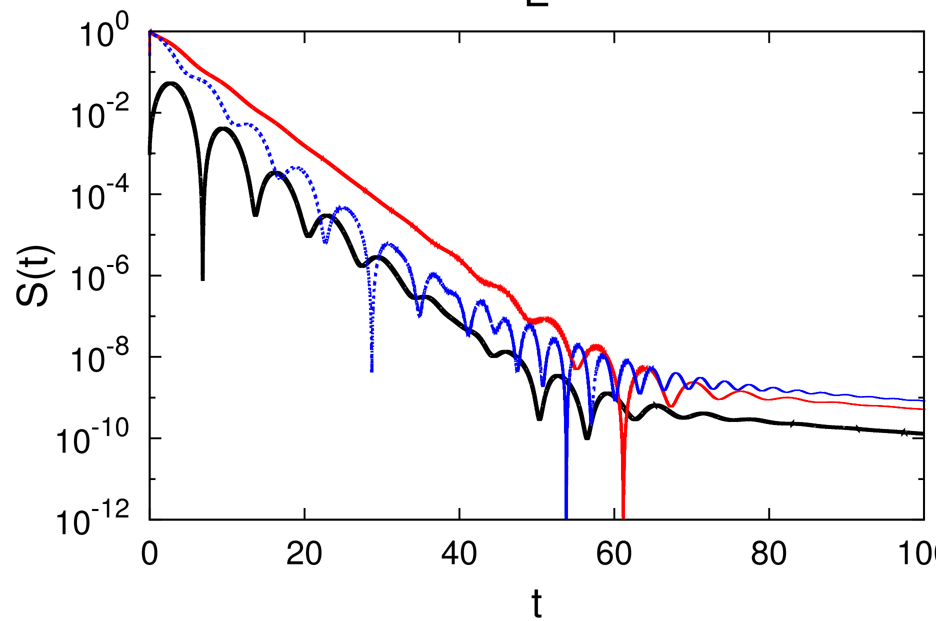
$$\mathcal{H} = \begin{pmatrix} \epsilon_1 - (i/2)\Gamma_1 & v - (i/2)A_1A_2 \\ v - (i/2)A_1A_2 & \epsilon_2 - (i/2)\Gamma_2 \end{pmatrix}$$

$$\Gamma_1 = A_1^2, \quad \Gamma_2 = A_2^2,$$

$$S(t) = |\langle \Psi(0)|\Psi(t)\rangle|^2$$



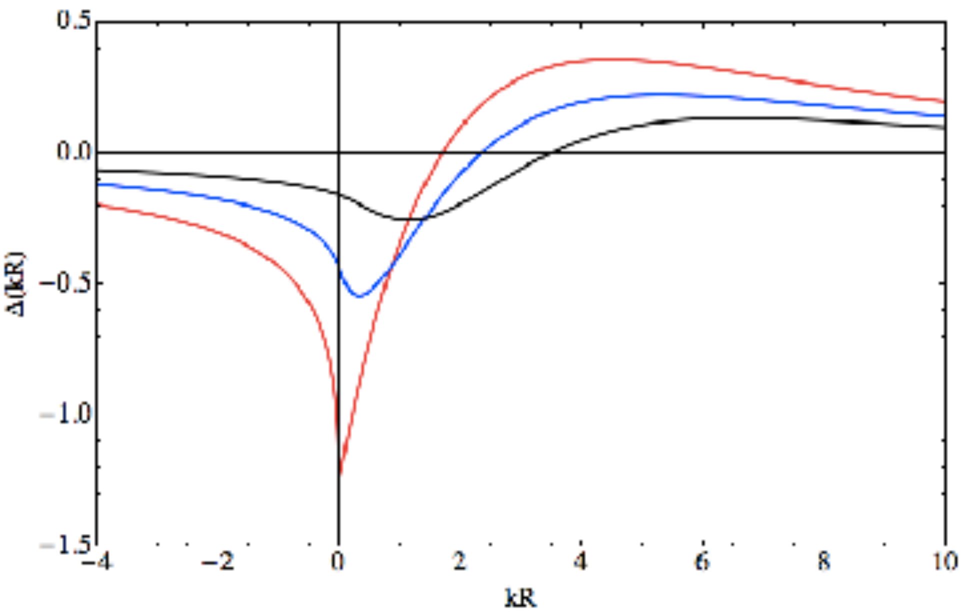



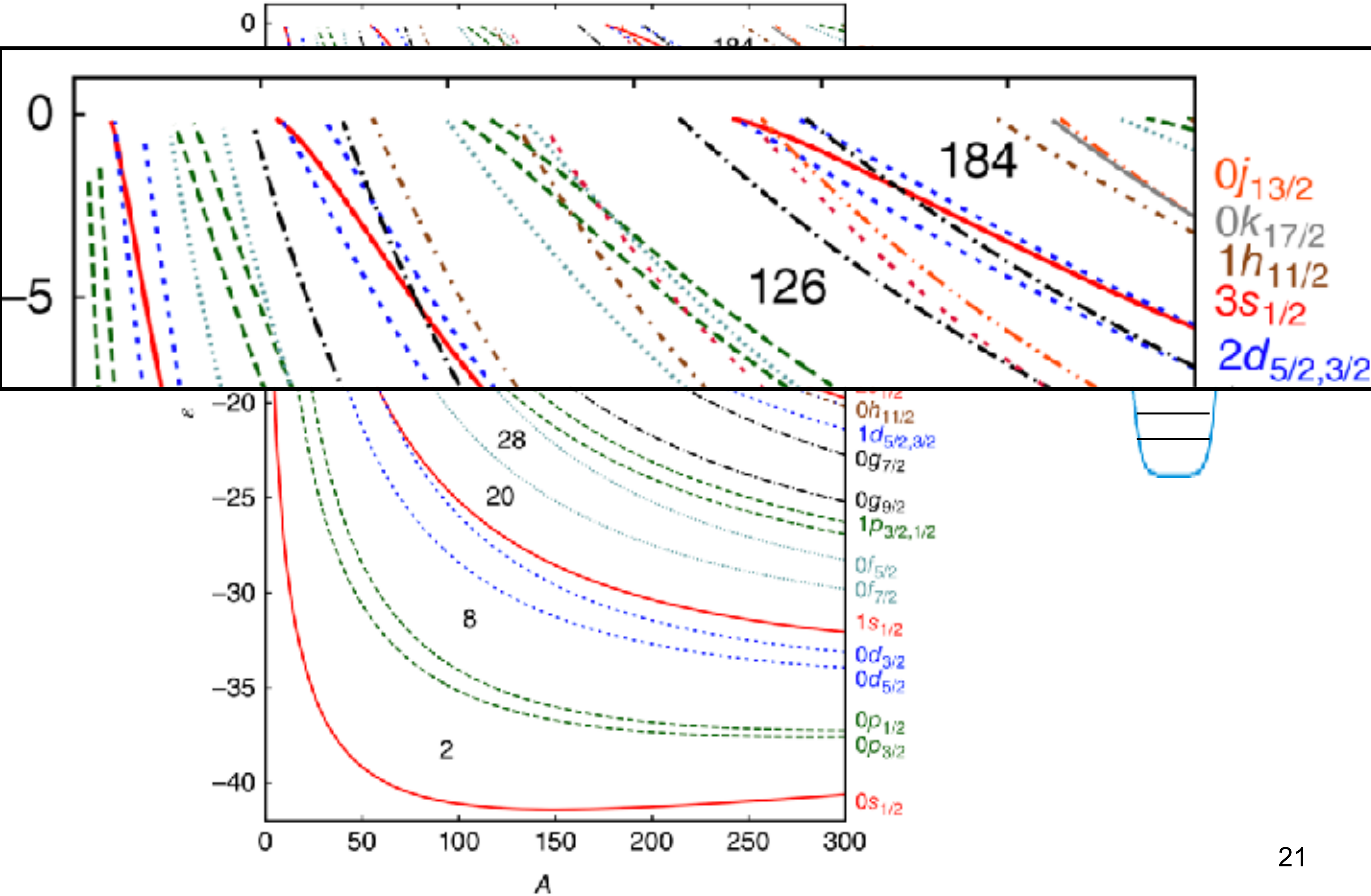
# Virtual excitations

$$H'(\epsilon) = \Delta(\epsilon) - \frac{i}{2}\Gamma(\epsilon)$$

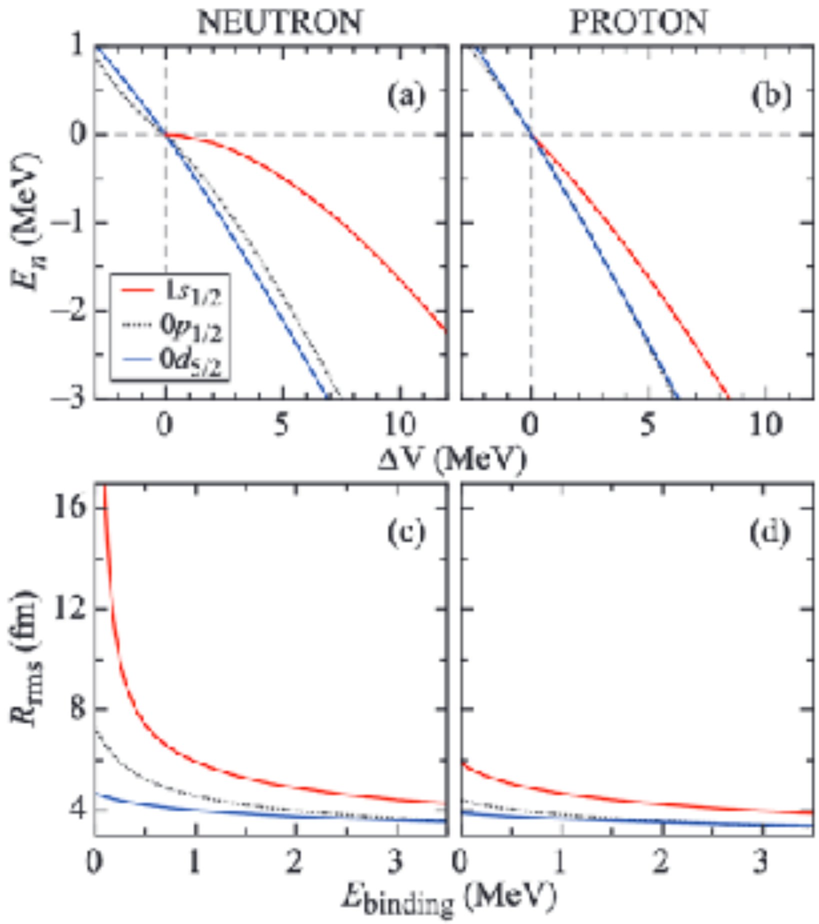
$$\Delta(E) = \frac{1}{2\pi} \int dE' \sum_c \frac{|A^c(E')\rangle\langle A^c(E')|}{E - E'}$$

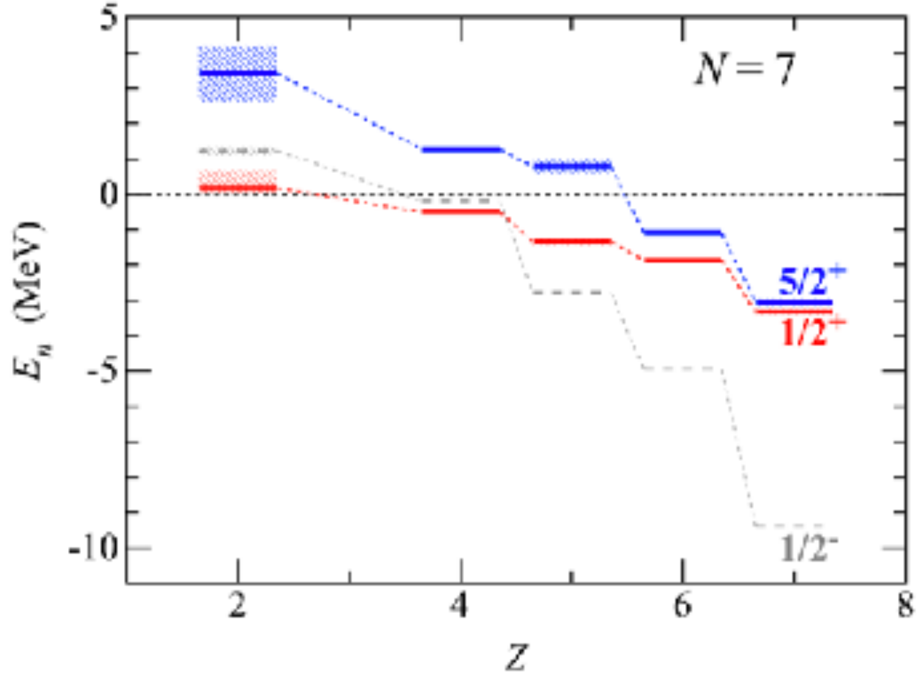


# Evolution of single particle energies



# Effect of weak binding




C. R. Hoffman, B. P. Kay, and J. P. Schiffer Phys. Rev. C 89, 061305(R)
B. P. Kay, C. R. Hoffman, and A. O. Macchiavelli Phys. Rev. Lett. 119, 182502

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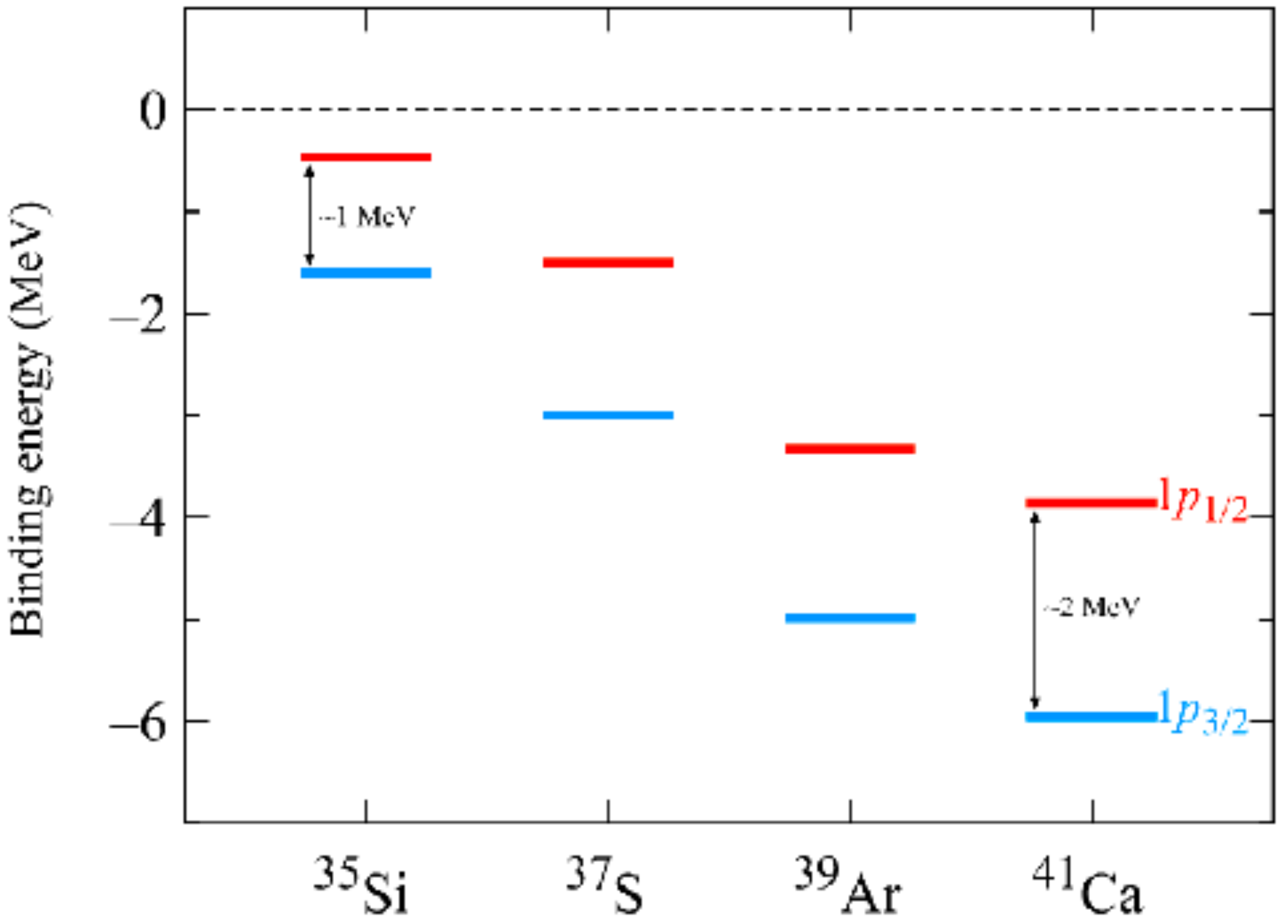
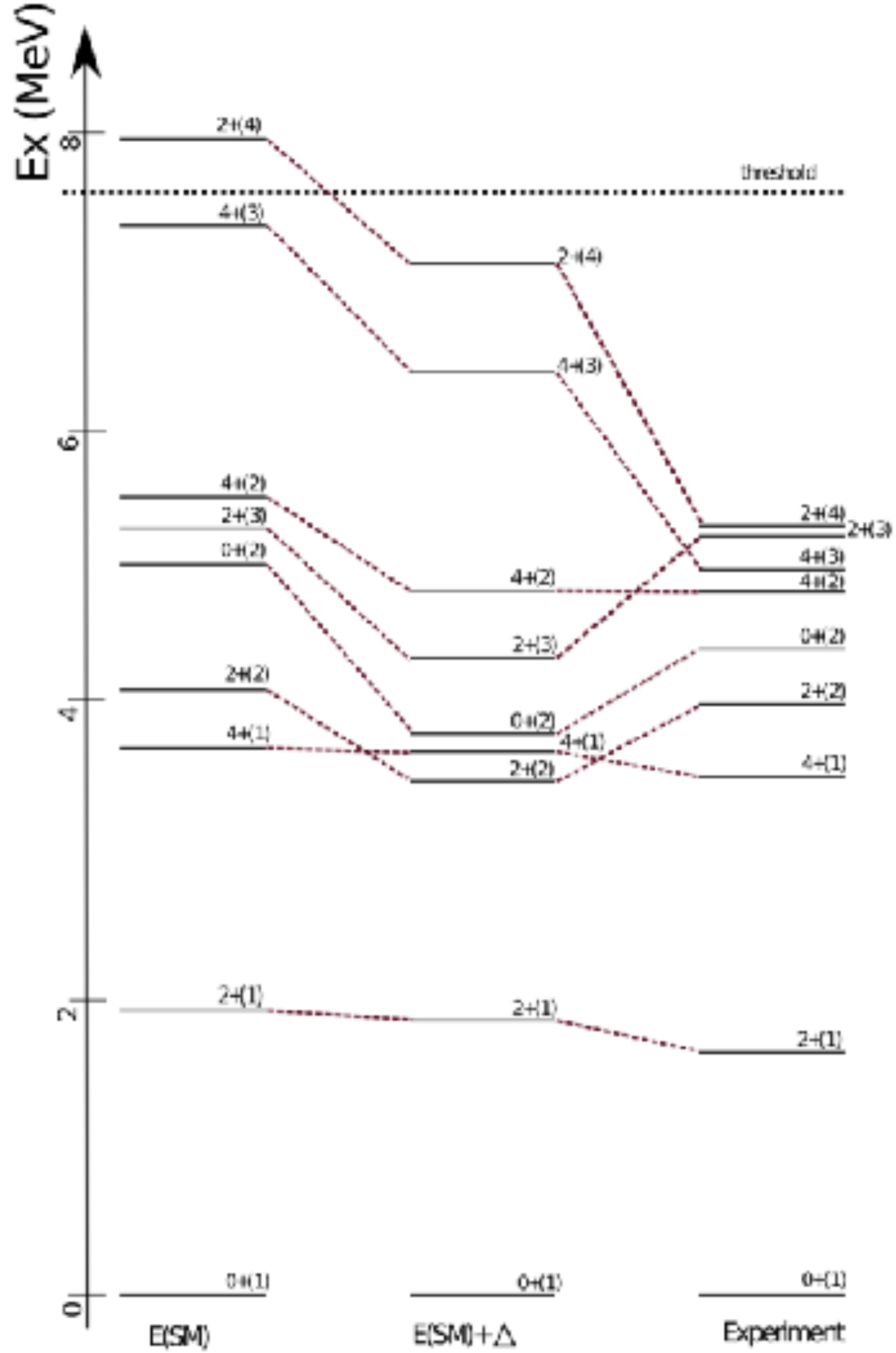



C. R. Hoffman, B. P. Kay, and J. P. Schiffer Phys. Rev. C 89, 061305(R)
B. P. Kay, C. R. Hoffman, and A. O. Macchiavelli Phys. Rev. Lett. 119, 182502

# Role of virtual excitations

Spectrum of $^{20}O$




A. Volya, JPS Conf. Proc. 6, 030059 (2015)

# Superradiance

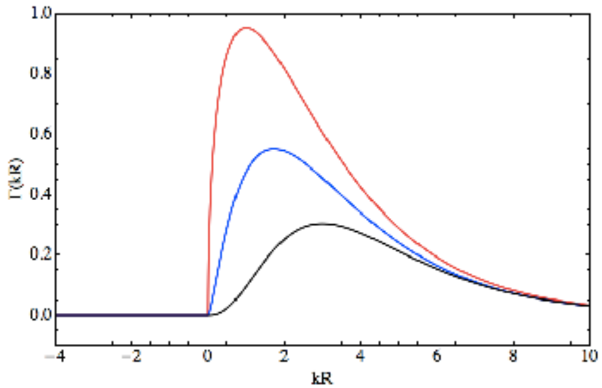
$$\mathcal{H}(E) = H + \Delta(E) - \frac{i}{2}W(E)$$

$$W_{12}(E) = 2\pi \sum_{c(\text{open})} A_1^c(E) A_2^{c*}(E)$$

**Factorized operator**

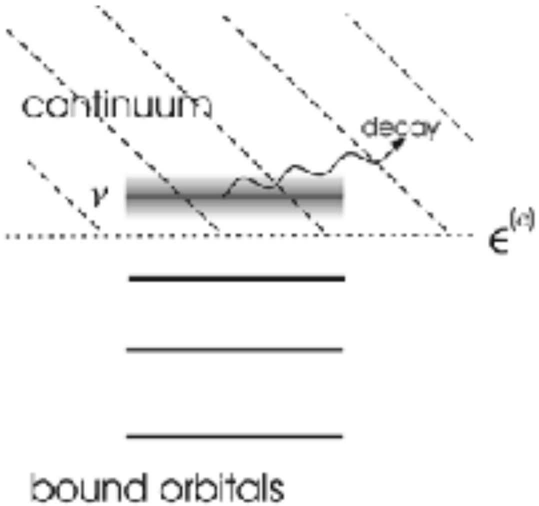
**Factorized form leads to unitarity of scattering matrix**

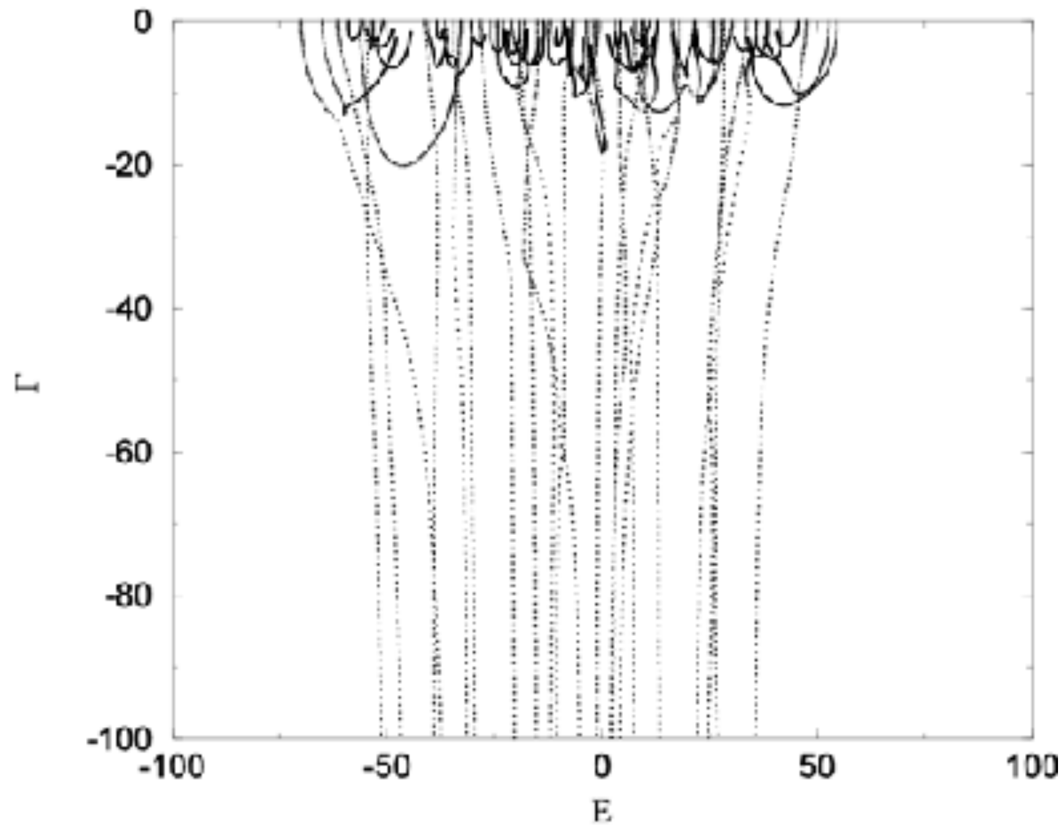
$$\mathbf{T}_{cc'}(E) = \langle A^c(E)| \left( \frac{1}{E - \mathcal{H}(E)} \right) |A^{c'}(E)\rangle$$



# Single-particle decay in many-body system

Evolution of complex energies E=E-i $\Gamma/2$ as a function of $\gamma$



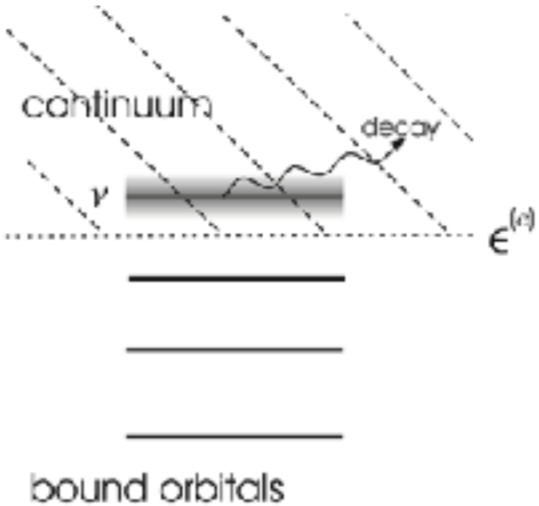




- Assume energy independent W
- Assume one channel $\gamma$=A²
- System 8 s.p. levels, 3 particles
- One s.p. level in continuum e=$\varepsilon$ –i$\gamma$/2

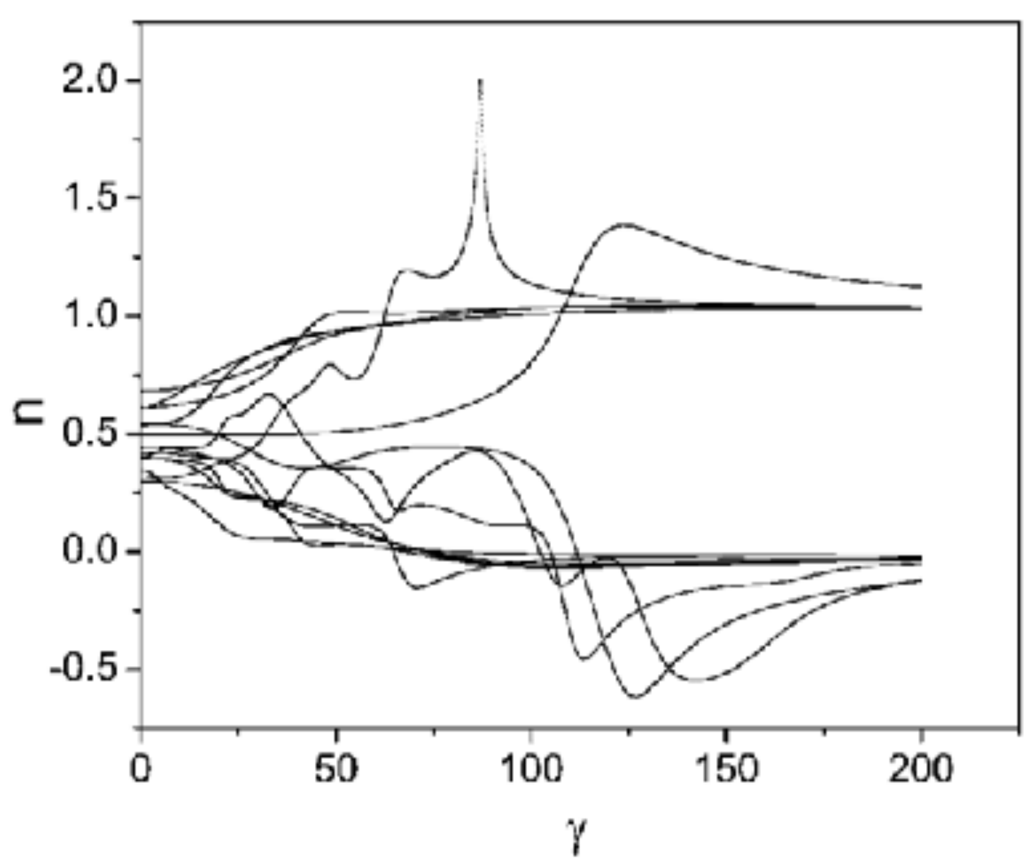
Total states 8!/(3! 5!)=56; states that decay fast 7!/(2! 5!)=21

# Single-particle decay in many-body system

**Evolution of occupancies as a function of $\gamma$**




$$n_\nu(j;\gamma) = \frac{\partial \Gamma_j(\gamma)}{\partial \gamma}$$

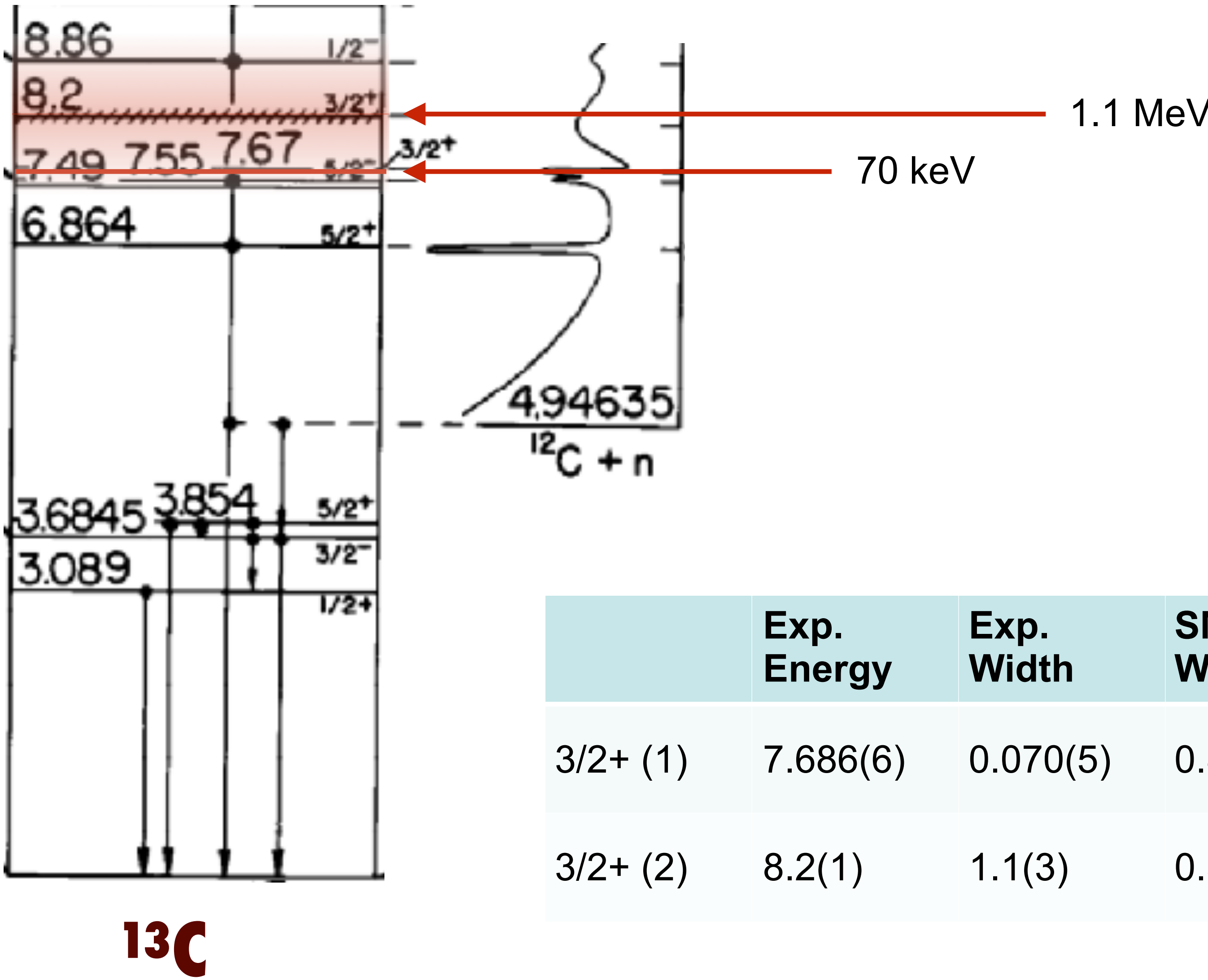



A. Volya and V. Zelevinsky, J. Opt. B (2003) S450

# Superradiance

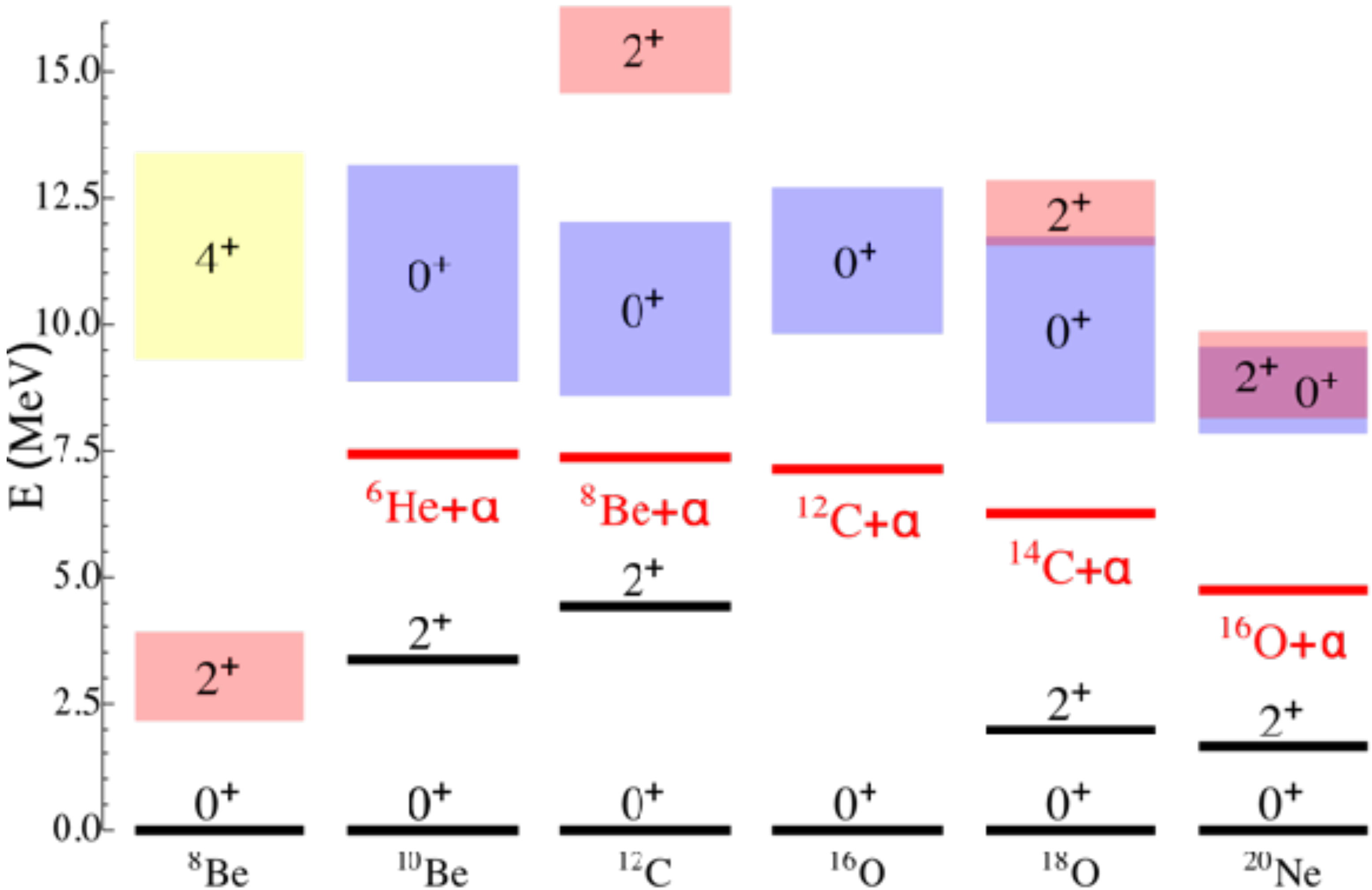
- Factorized form of non-herminitan component consistent with unitarity
- Low operator rank, number of channels versus number of many-body states
- When imaginary part dominates states separate into
    - Supperradiant (strongly coupled to continuum)
    - Decoupled from decay
- Internal wave functions are "reoriented" either along or away from decay
- Coupling to decay is a collective phenomenon
- There is a phase transition in many-body dynamics associated with superradiance

# Superradiance in $^{13}C$




| | Exp. Energy | Exp. Width | SM. Width | CSM Width |
|---|---|---|---|---|
| 3/2+ (1) | 7.686(6) | 0.070(5) | 0.858 | 0.098 |
| 3/2+ (2) | 8.2(1) | 1.1(3) | 0.342 | 1.031 |

# Searching for clustering states



# Interplay of collectivities

**Definitions**

**n - labels particle-hole state**

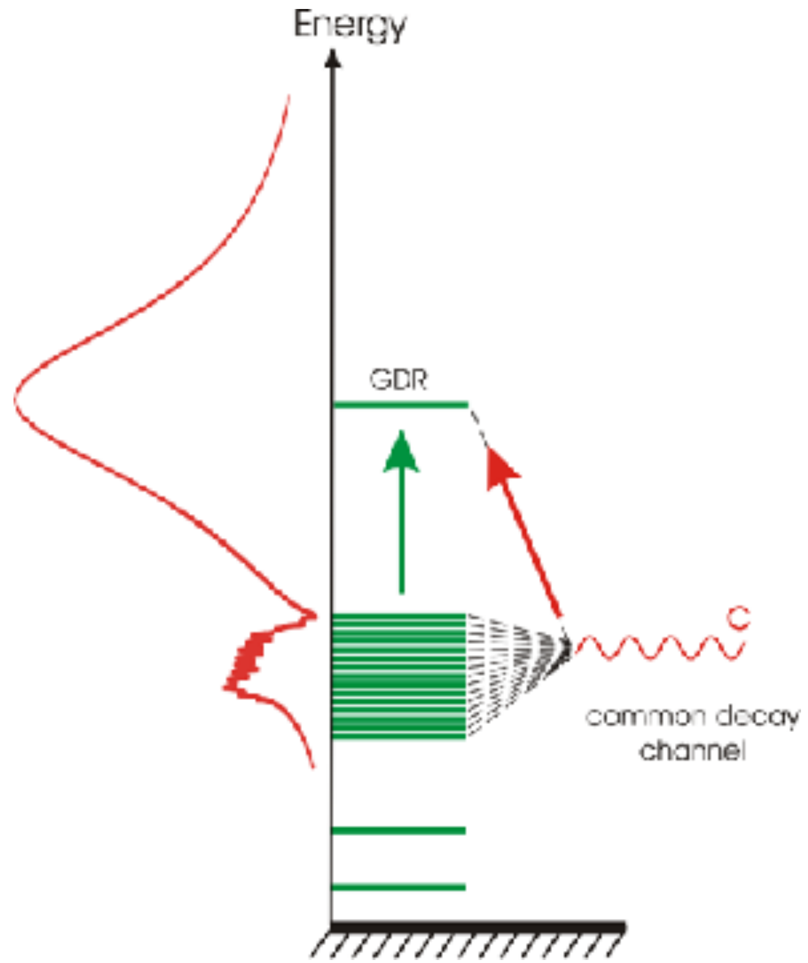
**$\varepsilon_n$ – excitation energy of state n**

**$d_n$ - dipole operator**

**$A_n$ – decay amplitude of n**

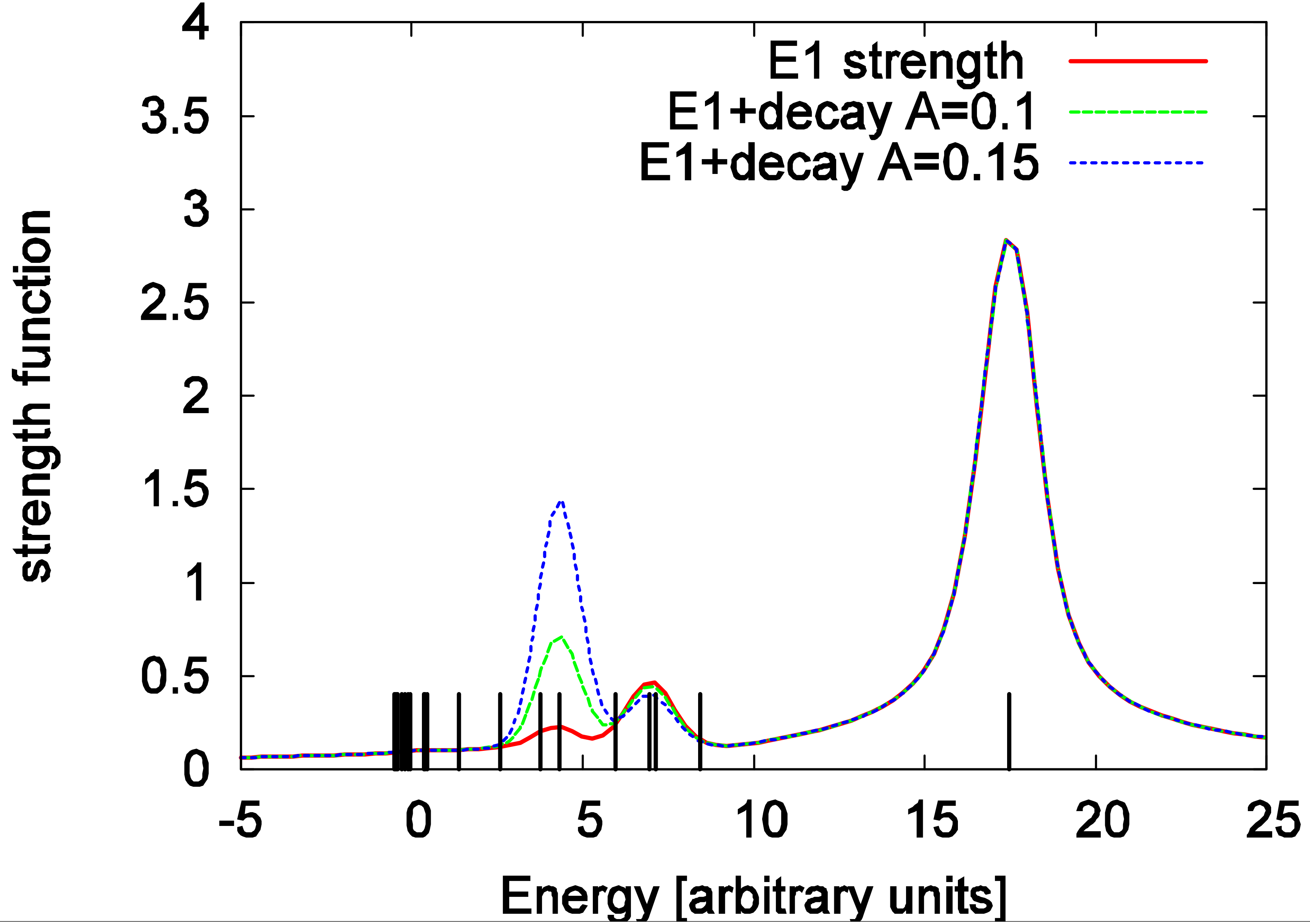
**Model Hamiltonian**

$$\mathcal{H}_{nn'} = \epsilon_n \delta_{nn'} + \lambda d_n d_{n'} - \frac{i}{2} A_n A_{n'},$$

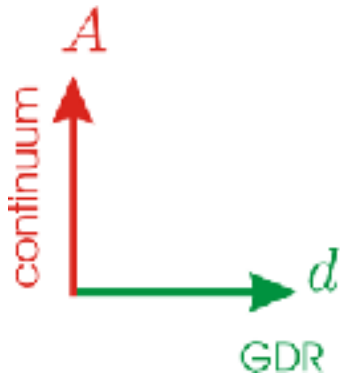



**Everything depends on angle between multi dimensional vectors A and d**

# Model Example



Pigmy resonance

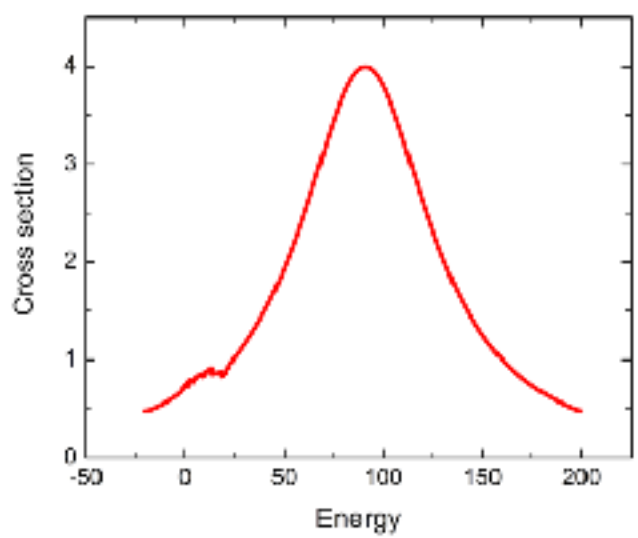


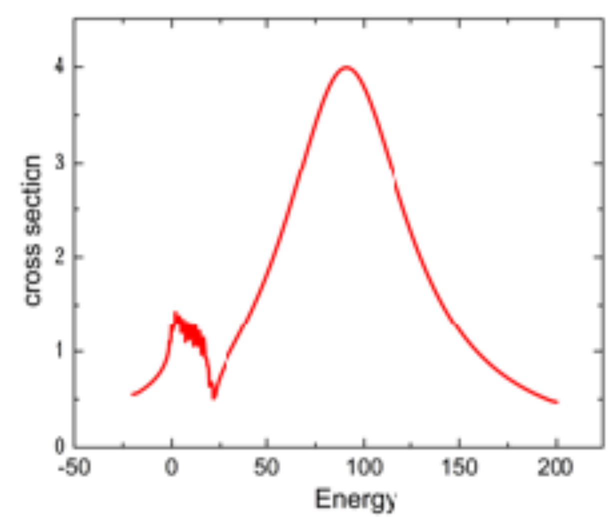



**Orthogonal:**
GDR not seen

**Parallel:**
Most effective excitation of GDR from continuum

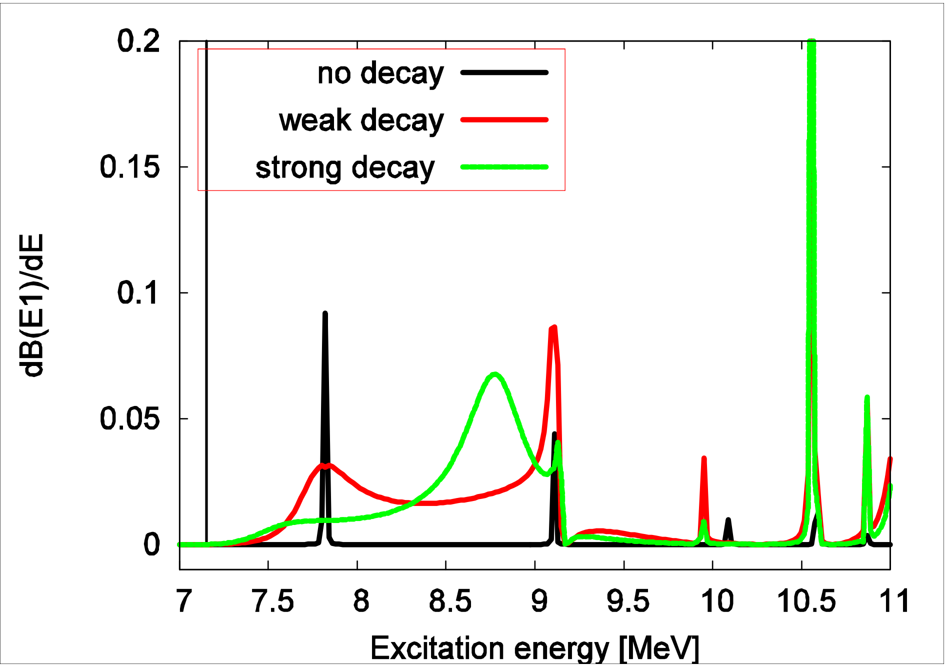
**At angle:**
excitation of GDR and pigmy

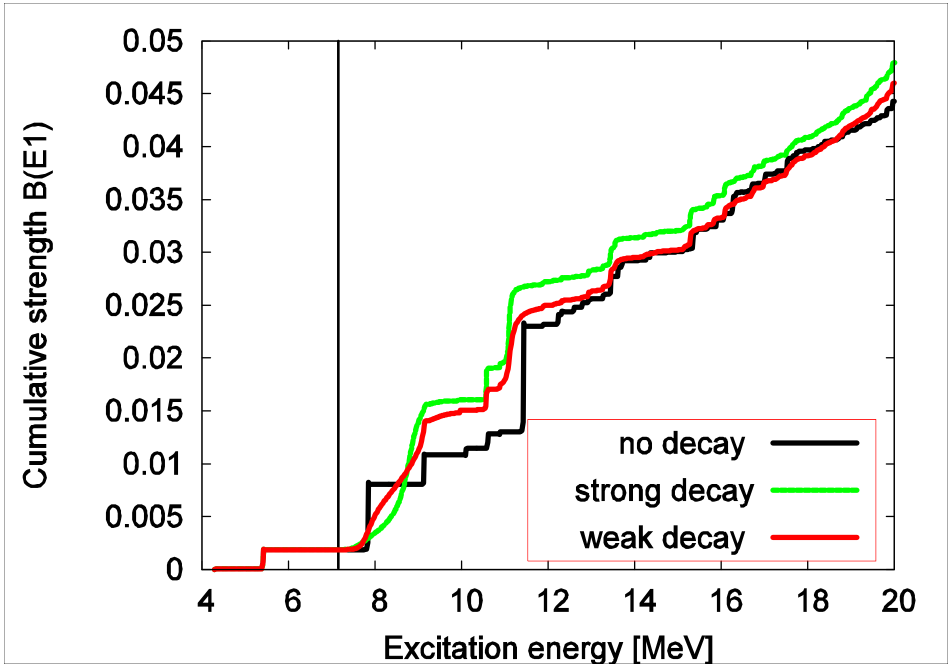



A model of 20 equally distant levels is used

# B(E1) strength in $^{22}O$

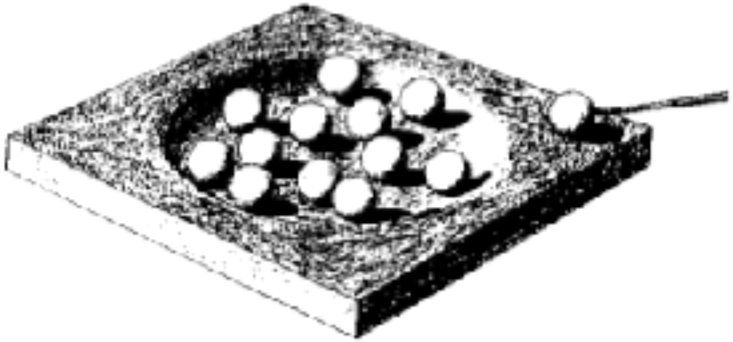


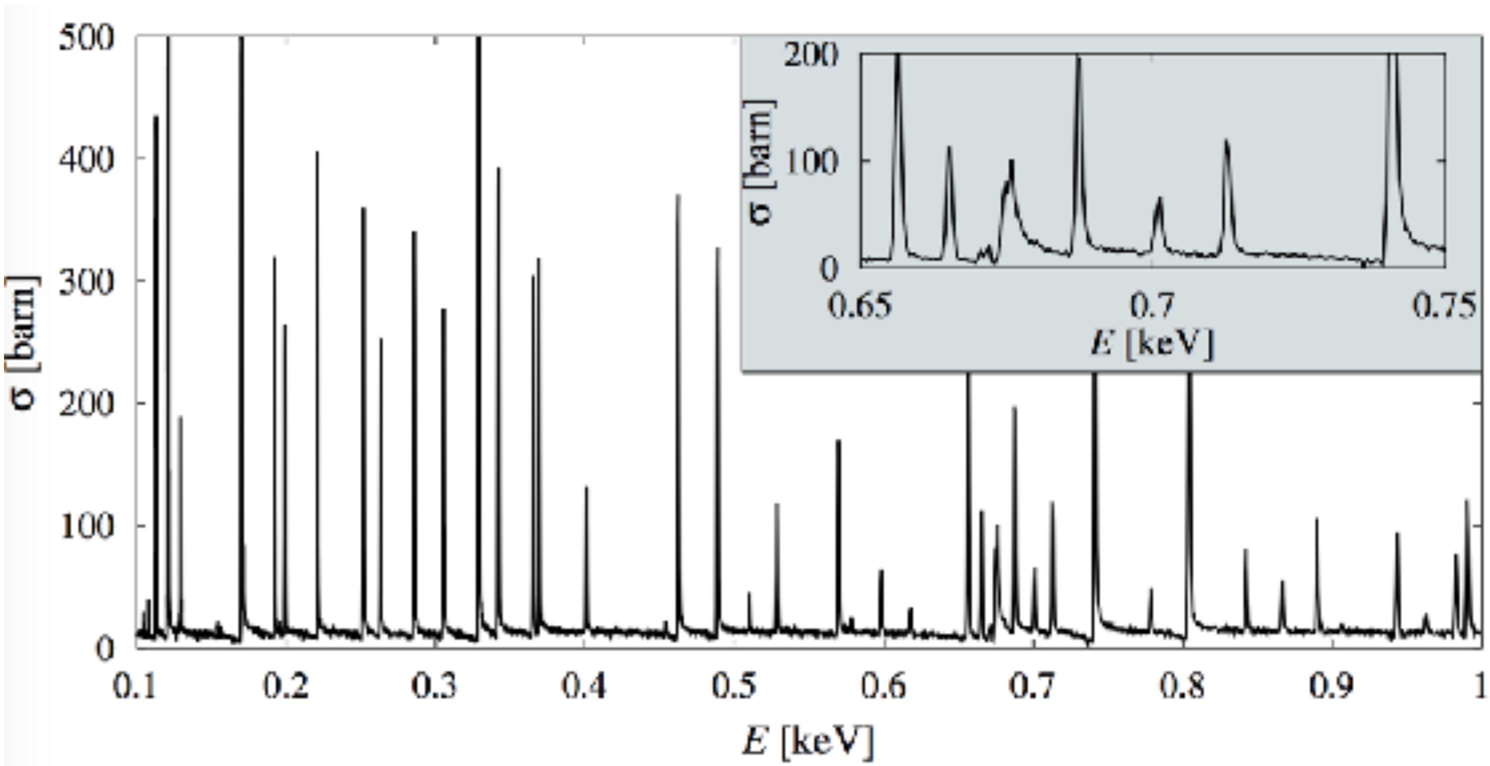

**Super-radiance phenomenon, strong decay makes leads to width distribution**

**Cumulative strength shows that super-radiance increases low-lying**
**dipole strength**

# Distribution of decay widths in a chaotic system



Wooden toy model illustrating Bohr's compound nucleus, from Nature **137**, 351 (1936)



# Many-body complexity and reduced widths

$|c\rangle$ Channel-vector (normalized)

$|I\rangle$ Eigenstate

Reduced width

$$\gamma_I^c = |\langle I|c\rangle|^2$$

What is the distribution of the reduced width?

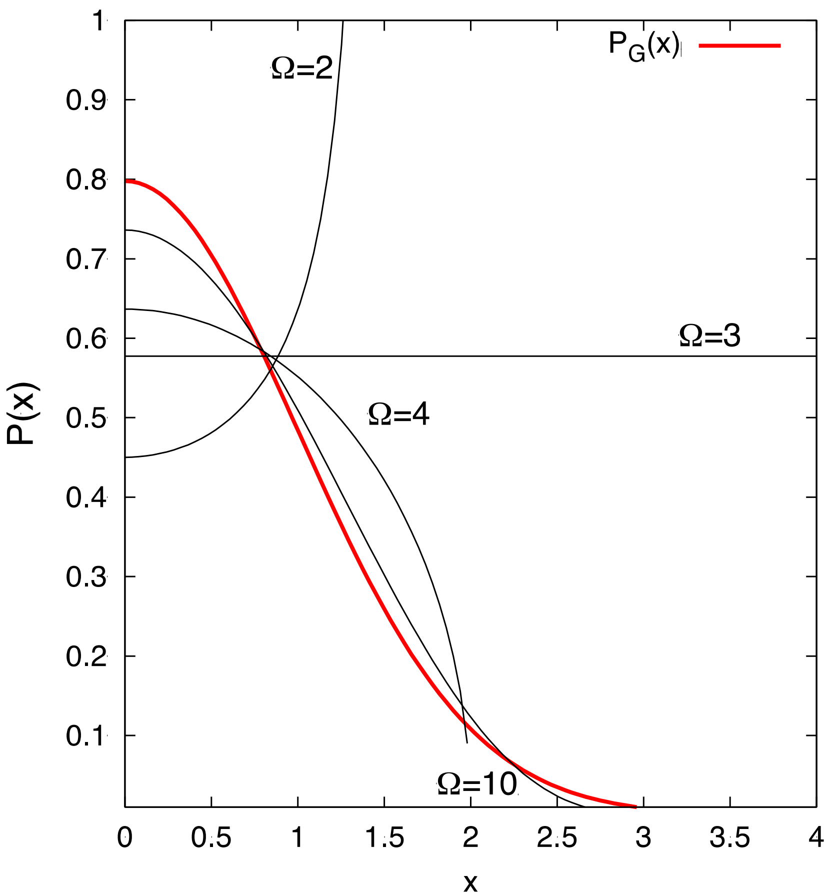
Average width $$\overline{\gamma} = \frac{1}{\Omega}\sum_I \gamma_I^c = \frac{\langle c|c\rangle}{\Omega}$$

Amplitude $$x_I = \sqrt{\gamma_I/\overline{\gamma}}$$

If any direction in the Ω-dimensional Hilbert space is equivalent

$$P(x_{I_1}, \ldots x_{I_\Omega}) \sim \delta\left(\Omega - \sum_I x_I^2\right)$$

# Why Porter-Thomas Distribution?




Projection of a randomly oriented vector in Ω-dimensional space

$$P(x) = \frac{V_{\Omega-1}}{\sqrt{\Omega}V_\Omega}\left(1 - x^2/\Omega\right)^{(\Omega-3)/2}$$

$$V_\Omega = \frac{\Omega \pi^{\Omega/2}}{\Gamma(\Omega/2+1)}$$

For large Ω this leads to Gaussian

$$P_G(x) = \sqrt{\frac{2}{\pi}} \exp\left(-x^2/2\right)$$

For large ν channels

$$P_\nu(\gamma) = \frac{1}{\gamma}\left(\frac{\nu\gamma}{2\overline{\gamma}}\right)^{\nu/2} \frac{1}{\Gamma(\nu/2)} \exp\left(-\frac{\nu\gamma}{2\overline{\gamma}}\right)$$

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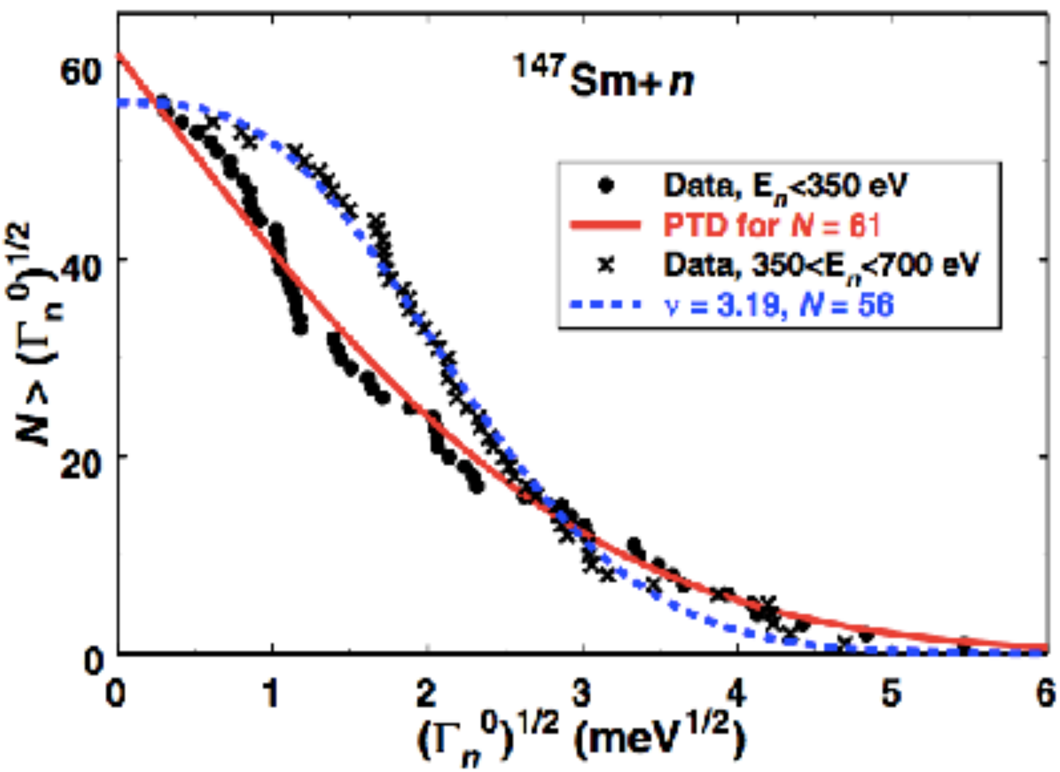
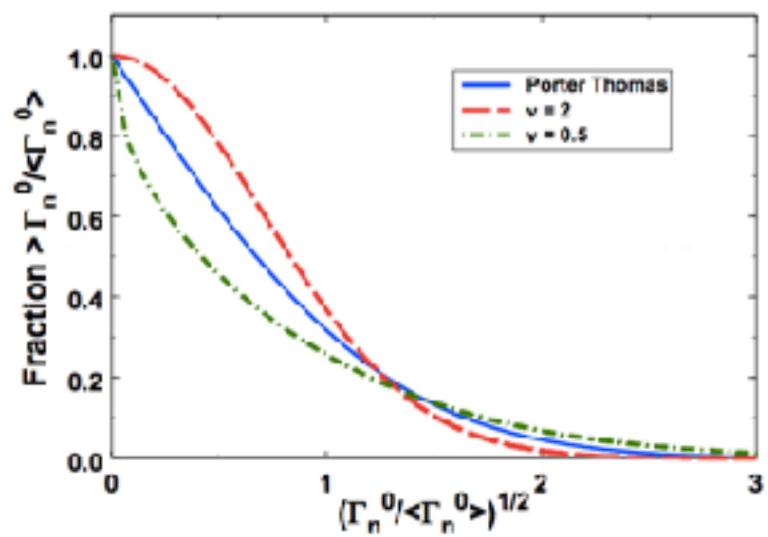
# Nuclear theory nudged? Violation of Porter-Thomas Distribution

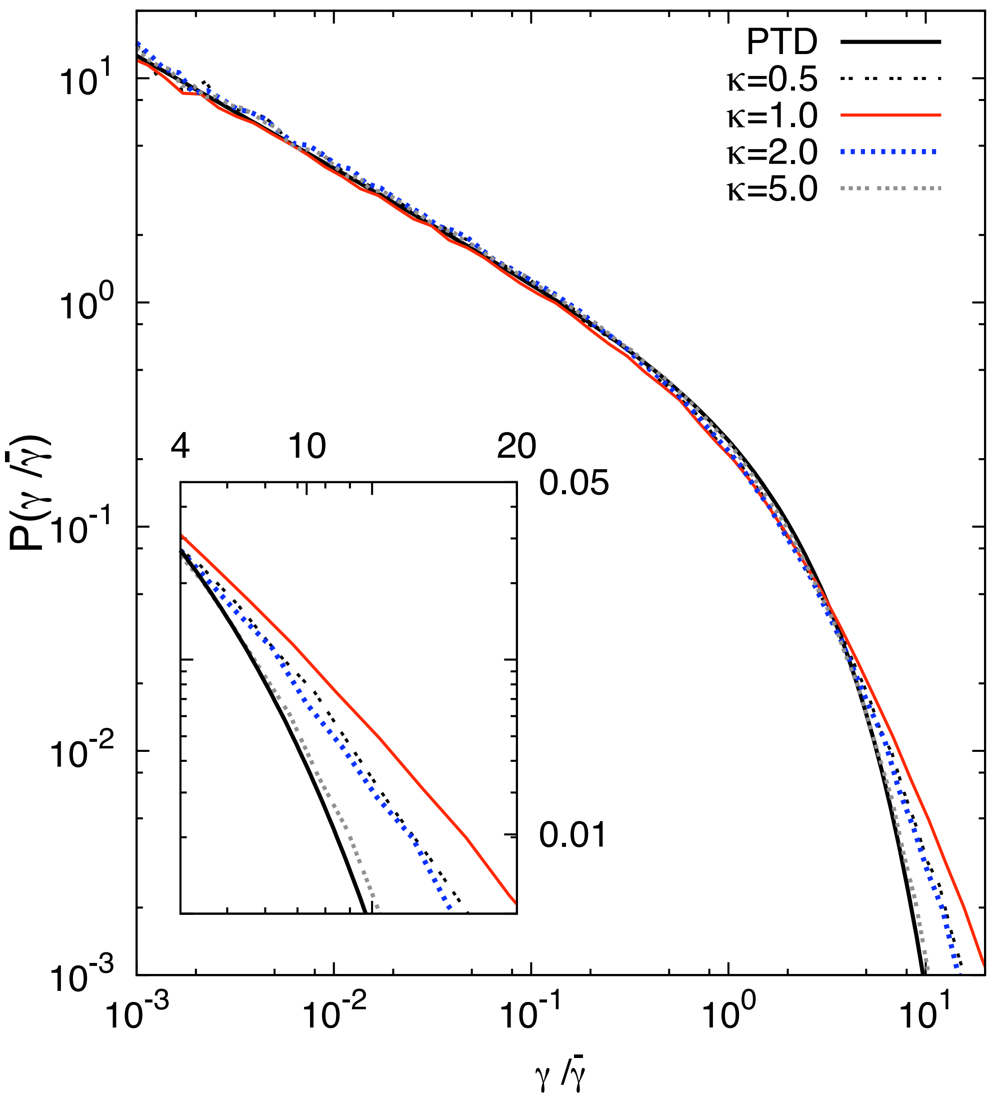
Random matrix theory is rejected with 99.997% probability [Koehler, et. al. Phys. Rev. Lett. 105, 072502 (2010)] In platinum $\nu = 0.5$

**Implications:**
Capture rates, astrophysical reactions, nuclear reactors, critical mass, shielding...



# Superradiance: decay collectivity

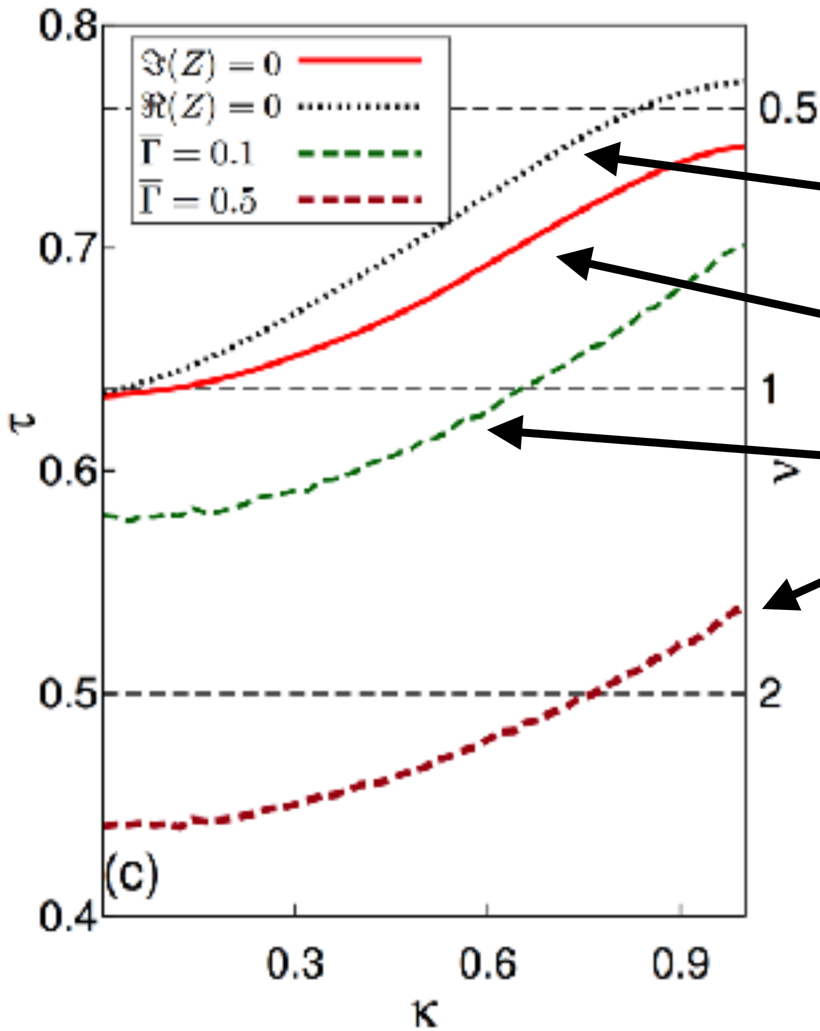



Ω=10000

SR leads to a small number of broad states.

# Virtual excitations as possible explanation

$$\mathcal{H}_{\text{eff}} = H_{\text{GOE}} + \Delta_{\text{n}} + \frac{i}{2}W_{\text{n}} + \frac{i}{2}W_{\gamma}$$




Open neutron channels

Virtual neutron channels $\kappa = \Delta_n / \lambda$

Additional gamma channels

PRL 115, 052501 (2015)

# Open Question:
# Dipole-moment and violation of
# P and T-symmetries

$$\mathrm{d} = \frac{\langle \mathbf{d} \cdot \mathbf{J} \rangle}{J(J+1)} \mathbf{J}$$

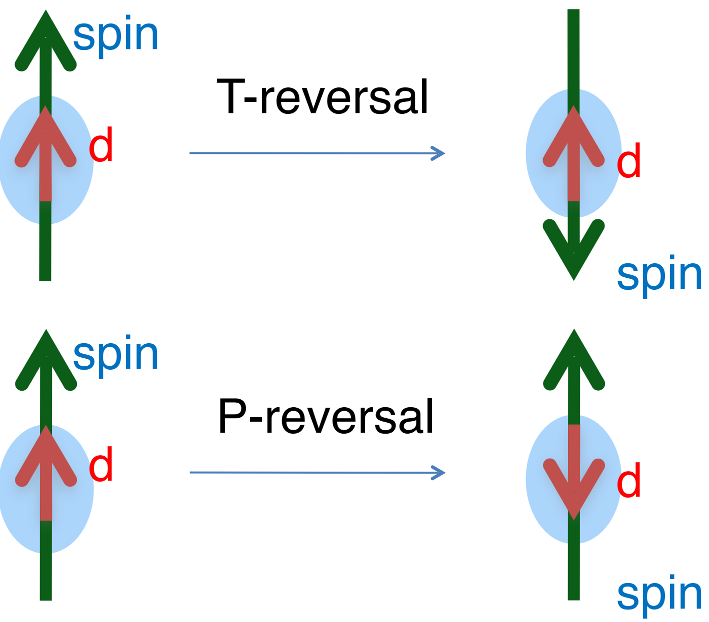
Observation of the dipole moment is an indication of parity and time-reversal violation

**Limit on EDM in electron**

Experiment $10^{-27}$ e cm

Standard model » $10^{-38}$ e cm

Physics beyond SM » $10^{-28}$ e cm



# *Why is this interesting?*

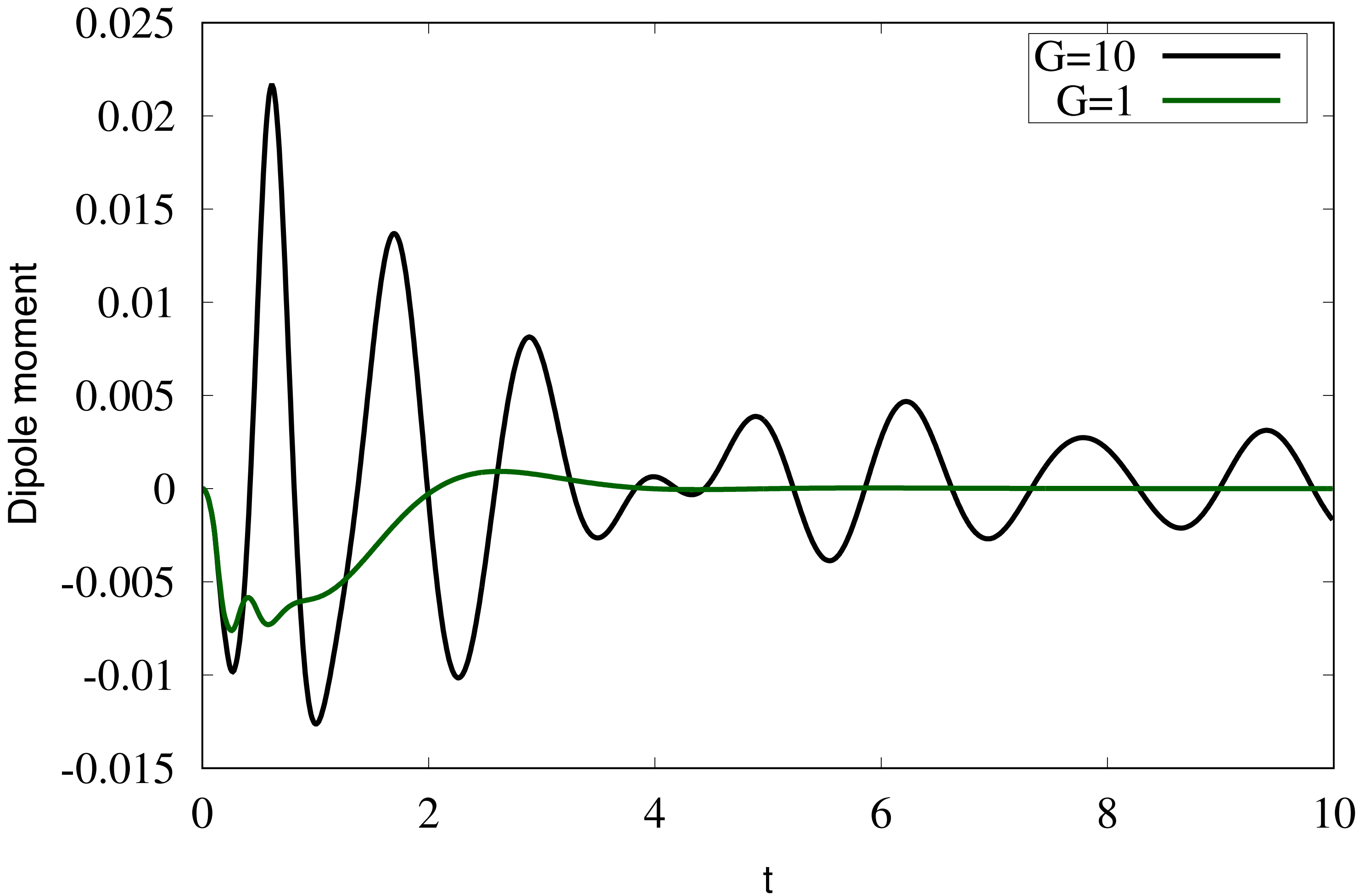
- Sensitive test of CP violation in the standard model
- Baryon asymmetry in the universe.
- Physics beyond standard model.

| system | EDM limit | SM |
|---|---|---|
| e (electron) | $10^{-27}$ | $10^{-40}$ |
| n (neutron) | $3.0 \times 10^{-26}$ | $10^{-32}$ |
| $^{225}Ra$ | $1.4 \times 10^{-23}$ | $10^{-33}$ |
| $^{199}Hg$ | $7.4 \times 10^{-30}$ | $10^{-33}$ |

T. E. Chupp, P. Fierlinger, M. J. Ramsey-Musolf, and J. T. Singh, *Electric dipole moments of atoms, molecules, nuclei, and particles*, Rev. Mod. Phys. **91**, 015001 (2019).

# Dipole moment in decaying system

Symmetric Winter's model




Winter's model, with slightly broken parity by coupling to continuum strength

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