Pion parton distribution function

base on arxiv:1905.05208 [nucl-th]

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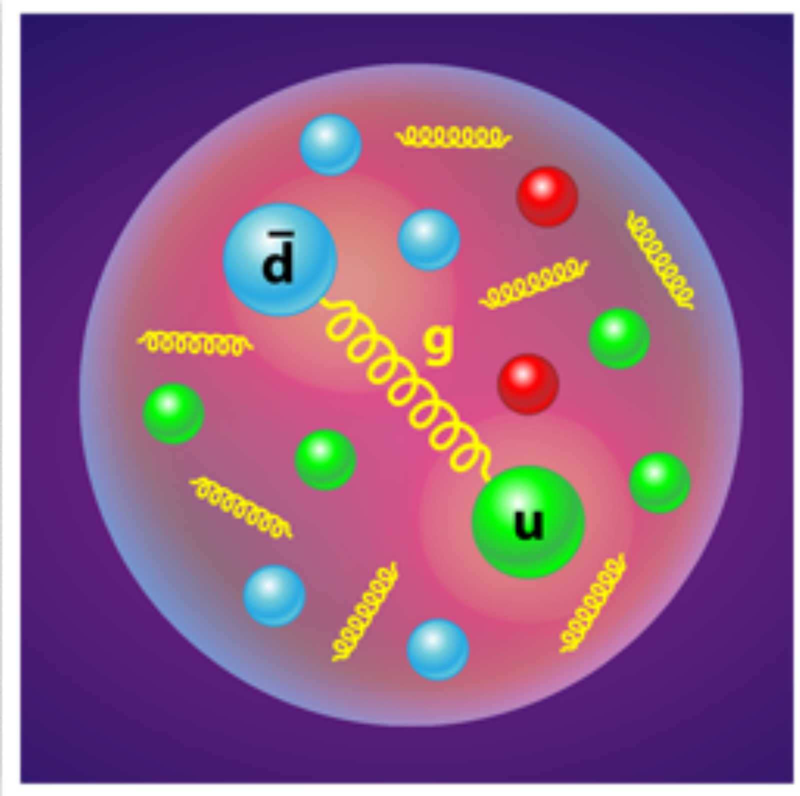
Diquark Correlations in Hadron Physics: Origin, Impact and Evidence
Trento, Italy, Sept. 23-29, 2019.

The pion in QCD

- Pion is made of
  - 1 up quark + 1 down antiquark ⇒ valence quarks
  - + any number of quark- antiquark pairs ⇒ sea quarks
  - + any number of gluons



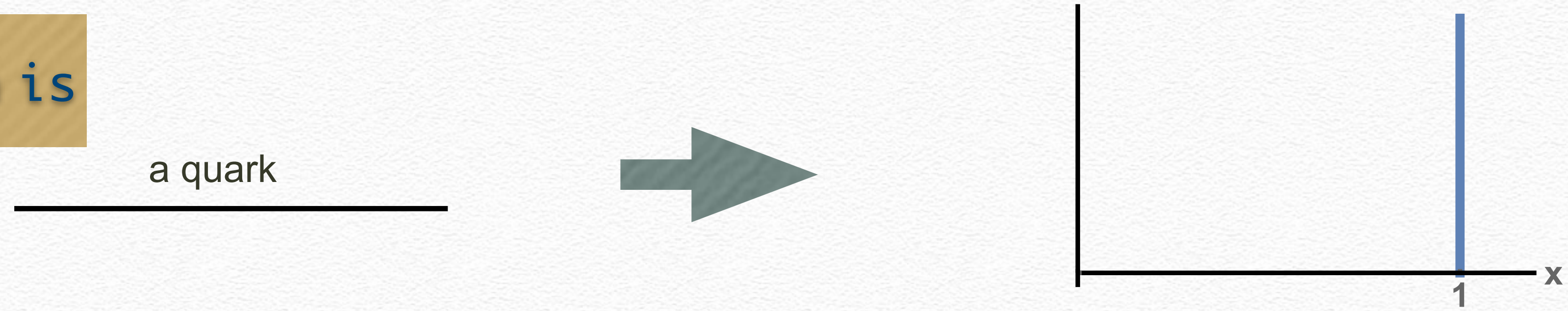
APS/Alan Stonebraker

- Infinite many body dynamic system of quarks and gluons

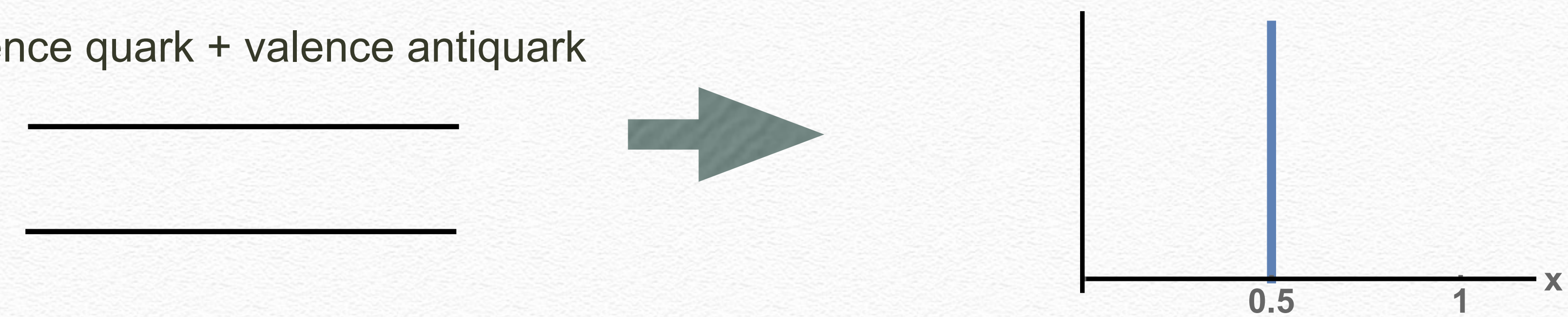
Pion parton distribution function

If the pion is → **then $q(x)$ is**

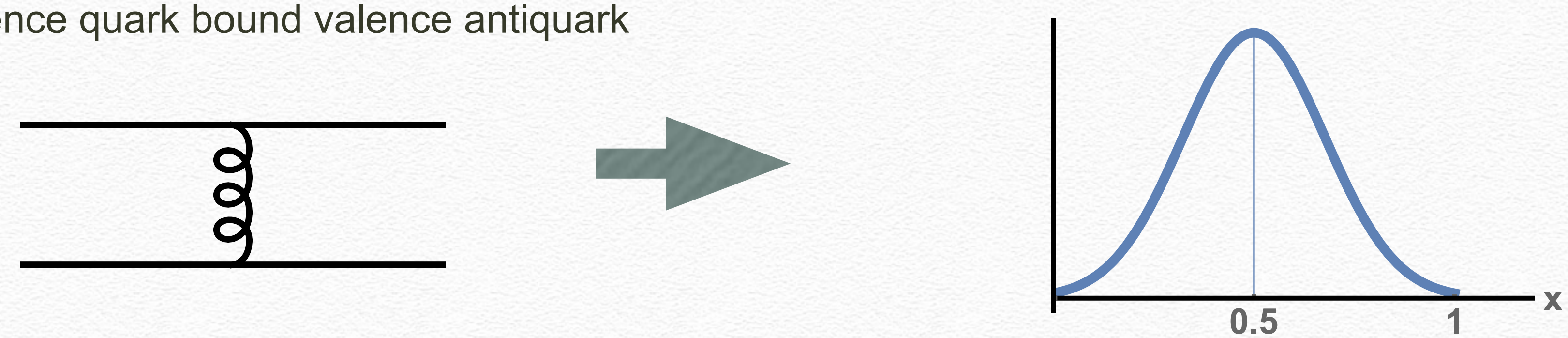
Halzen and Martin (1984)



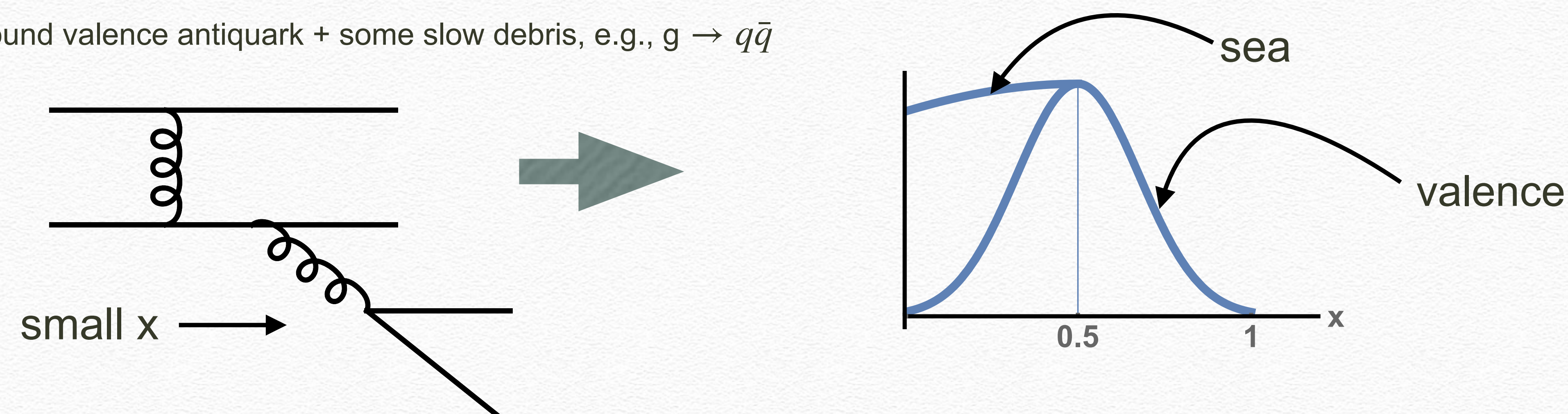
valence quark + valence antiquark



valence quark bound valence antiquark



valence quark bound valence antiquark + some slow debris, e.g., g $\rightarrow q\bar{q}$



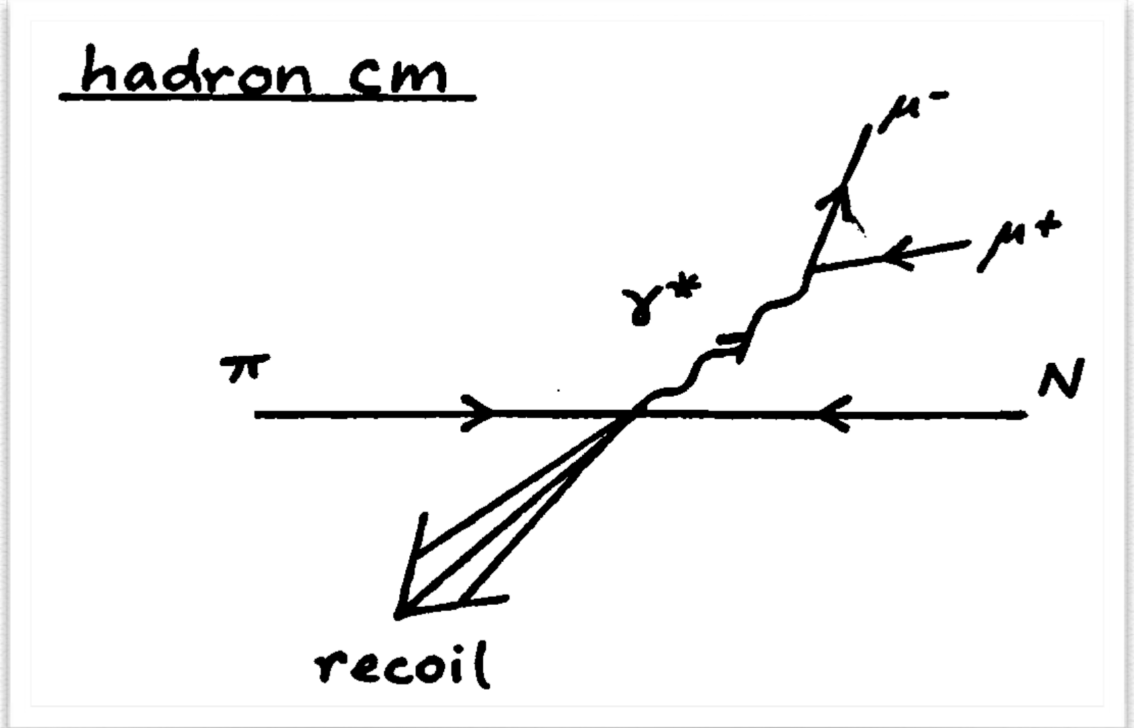
Pion parton distribution function

Four possible scenarios of the quark PDFs within a pion

- A single point-like particle
- Static quark and antiquark, each sharing 1/2 of the momentum
- Interacting quark and antiquark which can exchange momentum
- Interacting quark and antiquark including higher-order diagrams

PDFs depend on the detailed dynamics of the pion, not a priori known and obtained from experiment.

Experiment

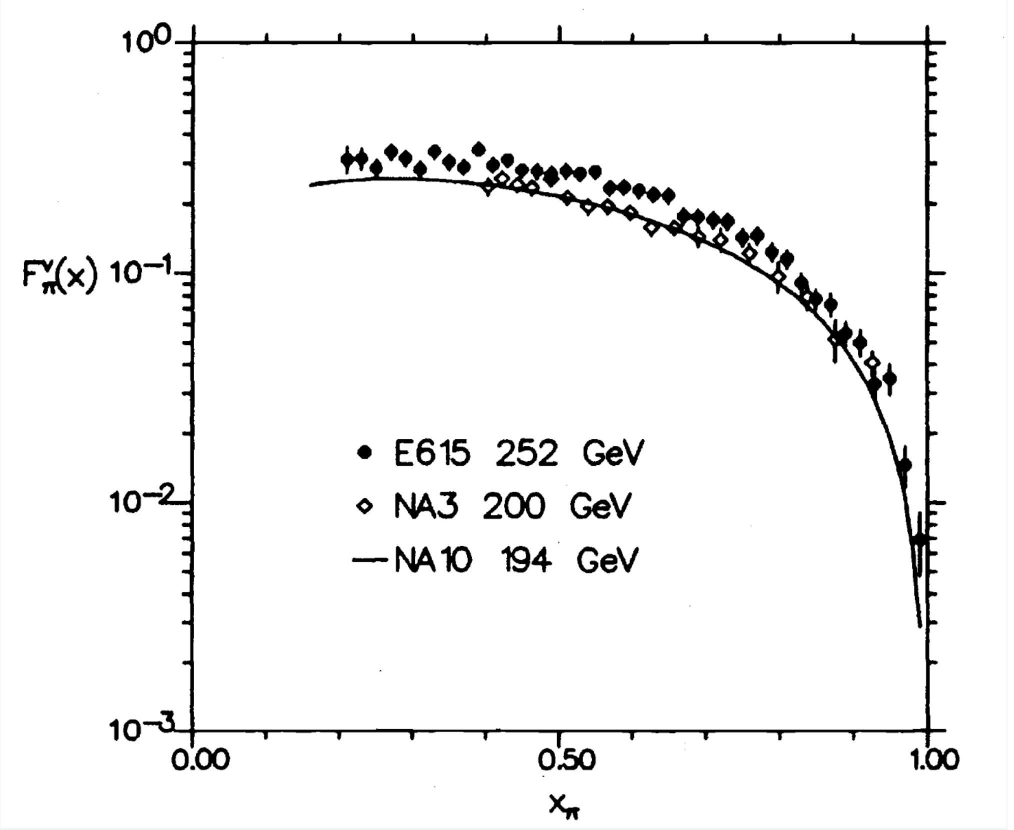


J.S.Conway et al..Phys.Rev. D39 (1989) 92-122

$$\pi^- N \rightarrow \mu^+ \mu^- X$$

- E615 Collaboration at Fermilab
- Muon pair production
- Pion-nucleon collisions
- High x_F
- Hadronic structure functions

Experiment



$$F_\pi^\nu(x) = A^\nu[x^\alpha(1-x)^\beta + \gamma\frac{2x^2}{9m_{\mu\mu}^2}]$$

J.S.Conway et al..Phys.Rev. D39 (1989) 92-122

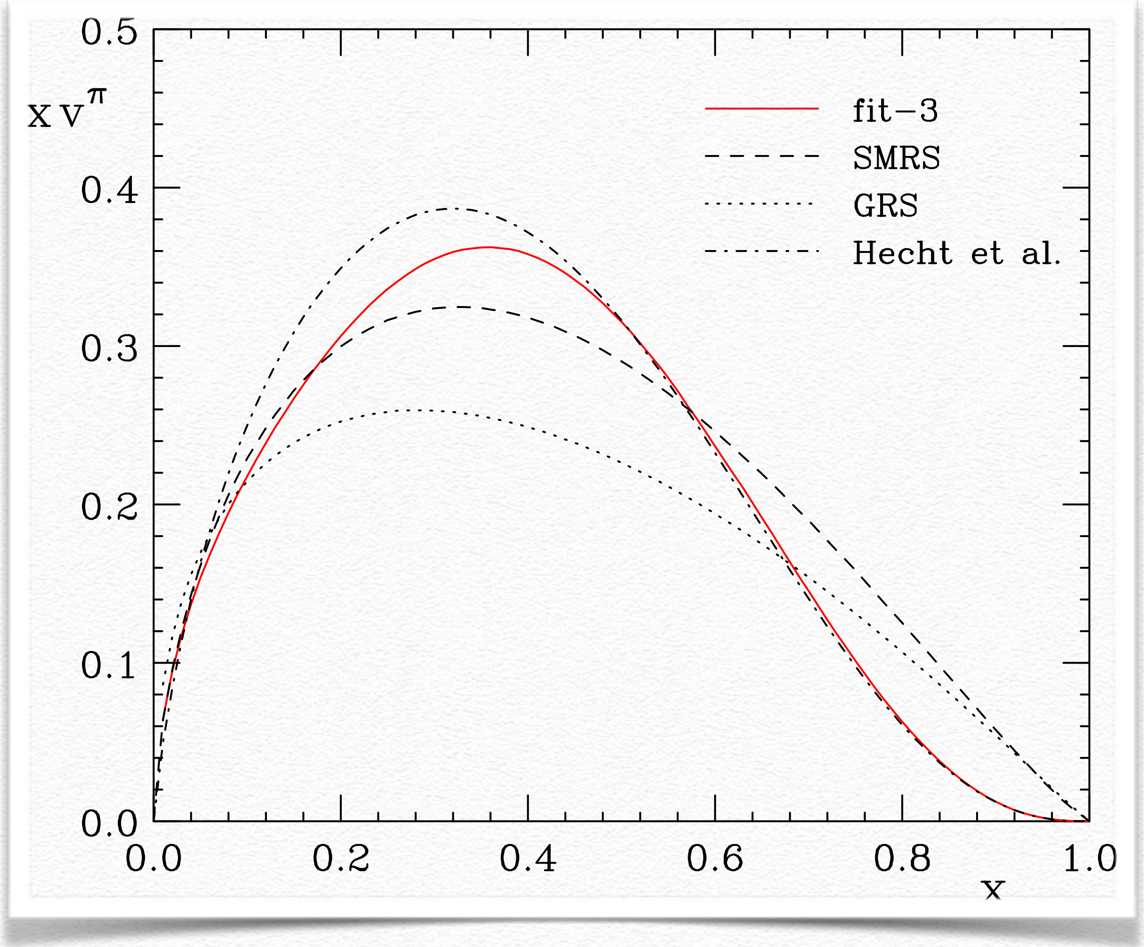
NA3(1983):
$\alpha = 0.45 \pm 0.03$
$\beta = 1.17 \pm 0.02$

NA10(1985):
$\alpha = 0.40 \pm 0.02$
$\beta = 1.17 \pm 0.03$

E615(1989):
$\alpha = 0.6 \pm 0.03$
$\beta = 1.26 \pm 0.04$

$\gamma\frac{2x^2}{9m_{\mu\mu}^2}$: higher twist
$\gamma = 0.83 \pm 0.26$

Experiment



Soft-Gluon Resummation and the Valence Parton Distribution Function of the Pion

Matthias Aicher, Andreas Schafer and Werner Vogelsang, PRL 105, 252003 (2010)

Hecht et al. DSE in 2001 Phys. Rev. C 63, 025213 (2001)

E615(2010):

$\alpha = 0.7 \pm 0.07$

$\beta = 2.03 \pm 0.06$

Evolve to $Q = 4$ GeV

$\beta = 2.34$

Perturbative QCD

$\beta = 2$

E. L. Berger and S. J. Brodsky, Phys. Rev. Lett. 42, 940 (1979)

Theory

- Nambu-Jona-Lasinio models — Phys.Lett. B308 (1993) 383-388 et al..
- AdS/QCD models — J.Phys. G42 (2015) no.9, 095005 et al..
- Light Front Hamiltonian — Phys.Rev.Lett. 122 (2019) no.17, 172001 et al..
- Lattice QCD — Phys.Rev. D99 (2019) no.1, 014508 et al..
- Relativistic constituent-quark models — Phys.Rev. D94 (2016) no.11, 114008 et al..
- Instanton-based models — Phys.Rev. D62 (2000) 014016 et al..
- Dyson-Schwinger equations
  - Phys.Rev.C 63 (2001) 025213
  - Rev.Mod.Phys. 82 (2010) 2991-3044
  - Phys.Rev. C 83 (2011) 062201
  - Phys.Lett.B 737 (2014) 23-29
  - Phys.Lett. B 749 (2015) 547-550
  - et al..

Dyson-Schwinger equations: PDF Definition

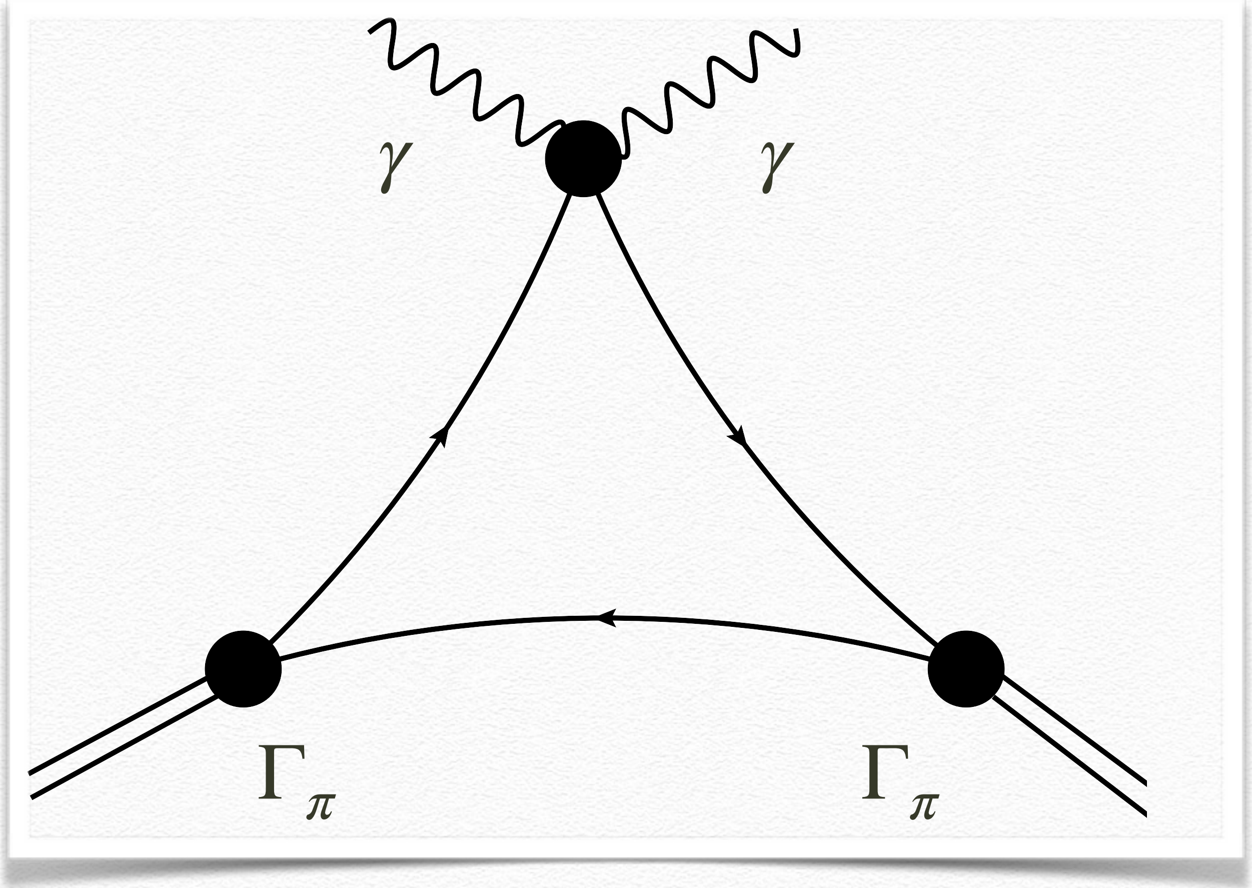
Twist-2 valence quark PDF

$$q(x) = \int d\lambda e^{-ixP\cdot n\lambda} \langle \pi(P) | \bar{\psi}(\lambda n)\gamma \cdot n \psi(0) | \pi(P) \rangle$$

Forward virtual quark-target scattering amplitude

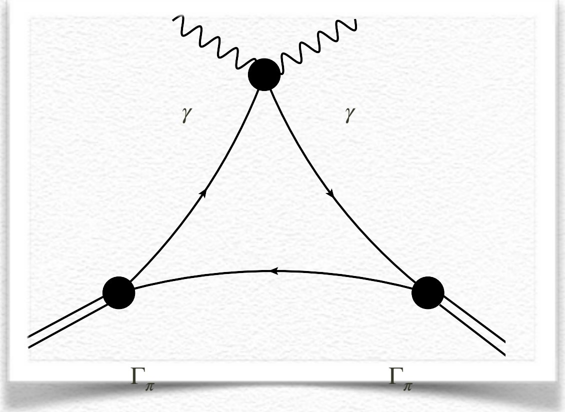
$$q(x) = \int dk \delta(n \cdot k - xn \cdot P)$$

$$Tr[i\gamma \cdot n G(k, P)]$$



"Handbag diagram"

Dyson-Schwinger equations: PDF Definition

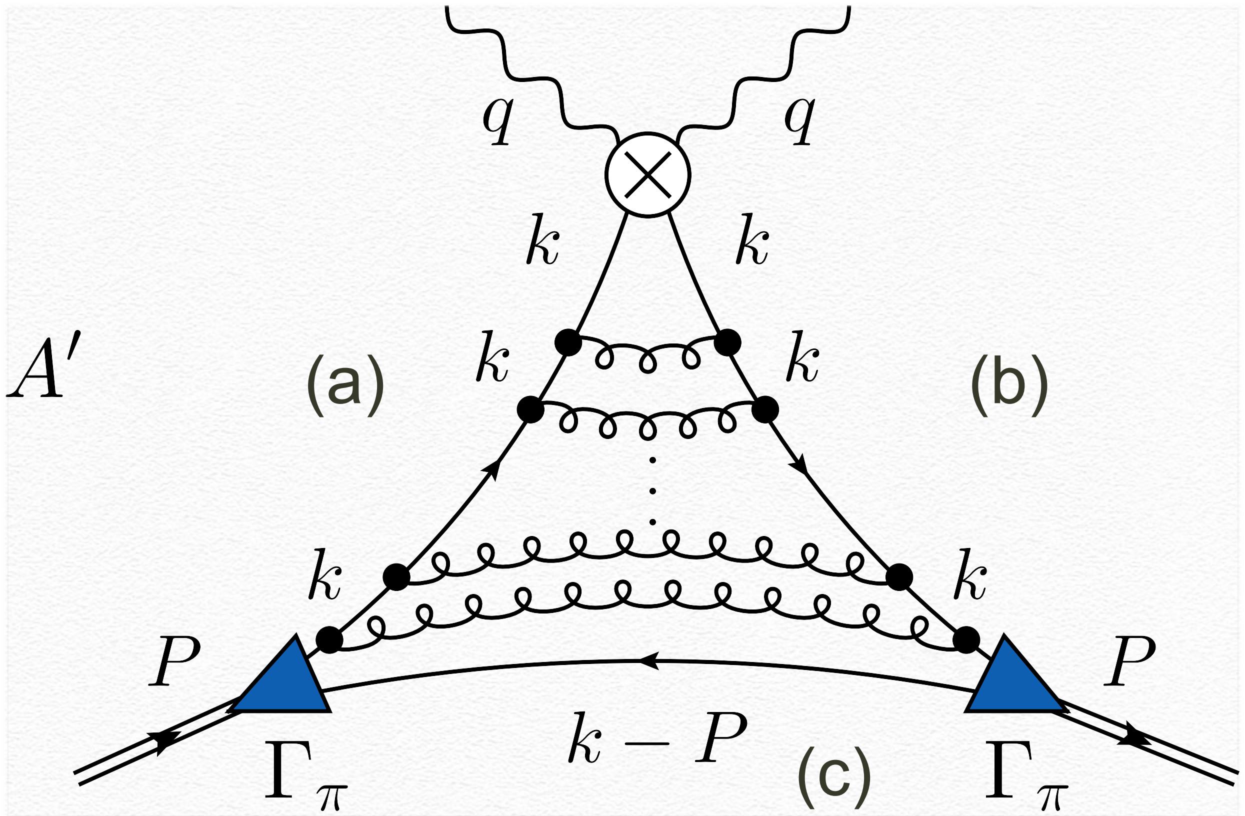


Rainbow Ladder truncation

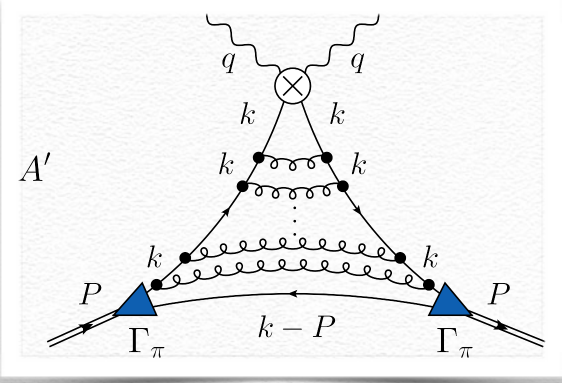
$$q_A^\pi(x;\zeta_H) = N_c \mathrm{tr} \int_{dk} \delta_n^x(k_\eta)\, i\Gamma_\pi(k_\eta, -P)\, S(k_\eta)$$
$$\times\, i\Gamma^n(k;x;\zeta_H)\, S(k_\eta)\, i\Gamma_\pi(k_{\bar\eta}, P)\, S(k_{\bar\eta})$$

Any gluon line can be absorbed into $S(k)$, $\Gamma(k,P)$ or $\Gamma^n(k;x;\zeta_H)$

- $S(k)$ quark propagator
- $\Gamma(k,P)$ Bethe-Salpeter amplitude
- $\Gamma^n(k;x;\zeta_H)$ quark photon vertex



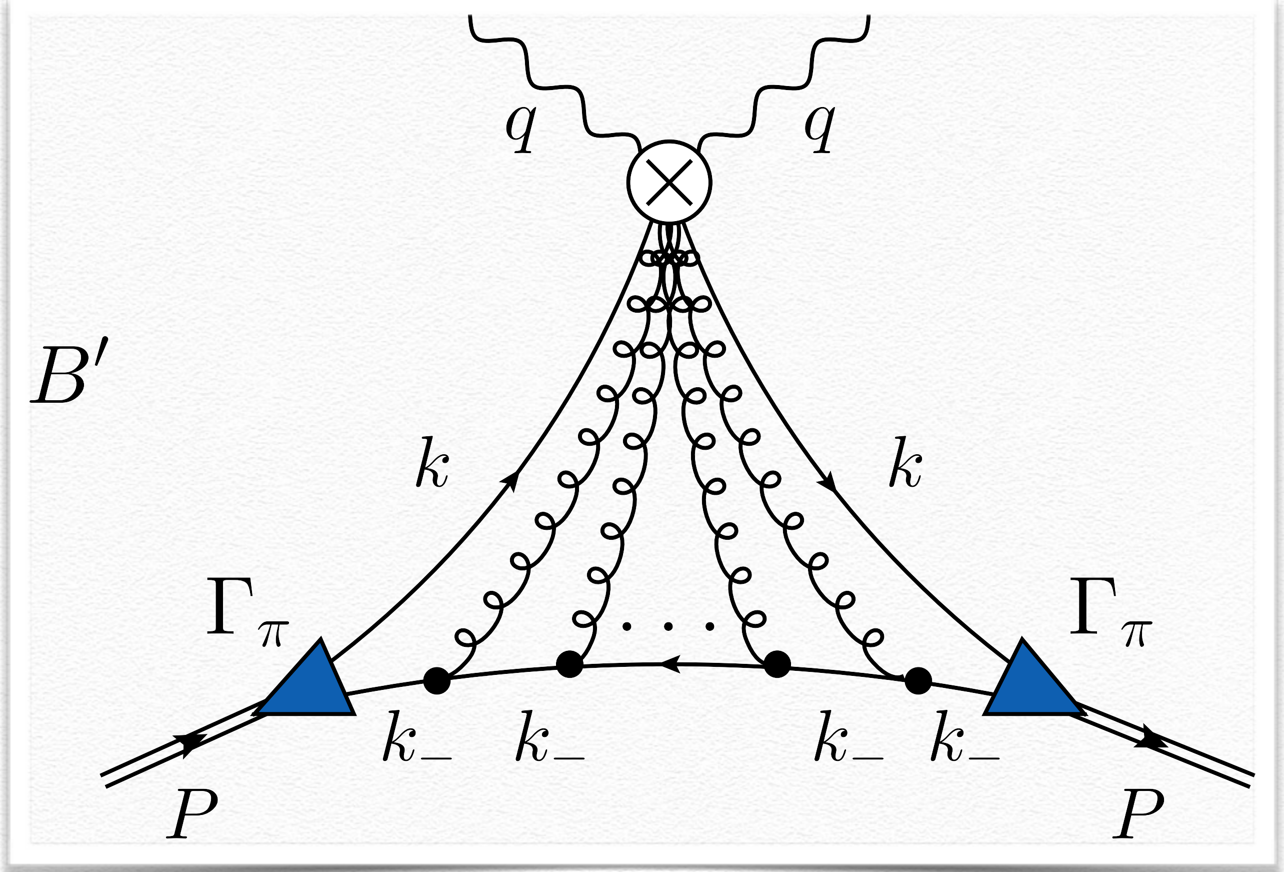
Dyson-Schwinger equations: A & BC term



Failure of impulse approximation

$$q^{\pi}_{BC}(x;\zeta_H) = N_c \mathrm{tr}\int_{dk} \delta^x_n(k_\eta)\,\Gamma^n_\pi(k_\eta, -P;\zeta_H)$$
$$\times\, S(k_\eta)\Gamma_\pi(k_{\bar\eta}, P)\, S(k_{\bar\eta})$$

- Gluon exchanges are soft
- Twist-2 contribution
- Same order as Fig.A



Phys.Lett. B737 (2014) 23-29

Dyson-Schwinger equations: A & BC term

Pion PDF

$$q^\pi(x;\zeta_H) = q^\pi_{\rm A}(x;\zeta_H) + q^\pi_{\rm BC}(x;\zeta_H)$$

$q^\pi_{\rm A}$ & $q^\pi_{\rm BC}$ properties

$$\int_0^1 dx\, q^\pi_{\rm A}(x;\zeta_H) = 1\,, \quad \int_0^1 dx\, q^\pi_{BC}(x;\zeta_H) = 0$$

Valence quark PDF Mellin moments →

$$\langle x^0_\pi \rangle = 1\,, \quad \langle x^1_\pi \rangle = \frac{1}{2}$$

- Pion is purely a bound-state of a dressed-quark and dressed-antiquark at the hadronic scale ζ_H.
- A novel term $q^\pi_{\rm BC}(x;\zeta_H)$ is necessary to keep $q^\pi(x;\zeta_H) = q^\pi(1-x;\zeta_H)$ and then $\langle x^1_\pi \rangle = 1/2$.

Dyson-Schwinger equations: Ward identity

- Ward-Takahashi Identity $iQ_\mu \Gamma_\mu(k;Q) = S^{-1}(k+Q/2) - S^{-1}(k-Q/2)$
  - Differential Ward identity $Q=0$ $\quad i\Gamma^n(k;x;\zeta_H) = n\cdot\partial_{k_\eta} S^{-1}(k_\eta)$
- PDF A term

$$q^\pi_A(x;\zeta_H) = N_c \mathrm{tr}\int_{dk} \delta^x_n(k_\eta)\,\Gamma_\pi(k_\eta,-P)\,S(k_\eta)\,i\Gamma^n(k;x;\zeta_H)\,S(k_\eta)\,\Gamma_\pi(k_{\bar\eta},P)\,S(k_{\bar\eta})$$

$$= N_c \mathrm{tr}\int_{dk} \delta^x_n(k_\eta)\,\left[\Gamma_\pi(k_\eta,-P)\,n\cdot\partial_{k_\eta} S(k_\eta)\right]\,\Gamma_\pi(k_{\bar\eta},P)\,S(k_{\bar\eta})$$

- PDF BC term

$$q^\pi_{BC}(x;\zeta_H) = N_c \mathrm{tr}\int_{dk} \delta^x_n(k_\eta)\,\Gamma^n_\pi(k_\eta,-P;\zeta_H) S(k_\eta)\Gamma_\pi(k_{\bar\eta},P)\,S(k_{\bar\eta})$$

$$\Gamma^n_\pi(k_\eta,-P;\zeta_H)$$
$$= n\cdot\partial_{k_\eta}\Gamma_\pi(k_\eta,-P;\zeta_H)$$

$$= N_c \mathrm{tr}\int_{dk} \delta^x_n(k_\eta)\,\left[n\cdot\partial_{k_\eta}\Gamma_\pi(k_\eta,-P;\zeta_H) S(k_\eta)\right]\,\Gamma_\pi(k_{\bar\eta},P)\,S(k_{\bar\eta})$$

- Pion PDF $\quad q^\pi(x;\zeta_H) = q^\pi_A(x;\zeta_H) + q^\pi_{BC}(x;\zeta_H)$

$$q^\pi(x;\zeta_H) = N_c \mathrm{tr}\int_{dk} \delta^x_n(k_\eta)\; n\cdot\partial_{k_\eta}\left[\Gamma_\pi(k_\eta,-P;\zeta_H) S(k_\eta)\right]\,\Gamma_\pi(k_{\bar\eta},P)\,S(k_{\bar\eta})$$

Computing inputs

- $q^\pi(x;\zeta_H)$ is completely determined once an interaction kernel is specified

$$K_{\alpha_1\alpha_1',\alpha_2\alpha_2'} = G_{\mu\nu}(k)[i\gamma_\mu]_{\alpha_1\alpha_1'}[i\gamma_\nu]_{\alpha_2\alpha_2'}\,, \quad G_{\mu\nu}(k) = \tilde{G}(k^2)T_{\mu\nu}(k)$$

$$\frac{1}{Z_2^2}\tilde{G}(s) = \frac{8\pi^2 D}{\omega^4}e^{-s/\omega^2} + \frac{8\pi^2\gamma_m \mathcal{F}(s)}{\ln\left[\tau + (1+s/\Lambda_{\rm QCD}^2)^2\right]}\,, \quad k^2 T_{\mu\nu}(k) = k^2\delta_{\mu\nu} - k_\mu k_\nu$$

- Mass-independent renormalisation scheme
  - Z_2 & Z_4 defined in chiral limit and invariant for any current quark mass
- Renormalisation scale ζ
  - A scale where dressed quasiparticles are the correct degrees-of-freedom
  - Meson's Poincare covariant wave function must evolve with ζ

Computing inputs: renormalisation scale

- Process-independent effective charge $\alpha_{\rm PI}(k^2)$
  - Saturates in the infrared: $\alpha_{\rm PI}(0)/\pi \approx 1$
  - Dynamical generation of a gluon mass-scale

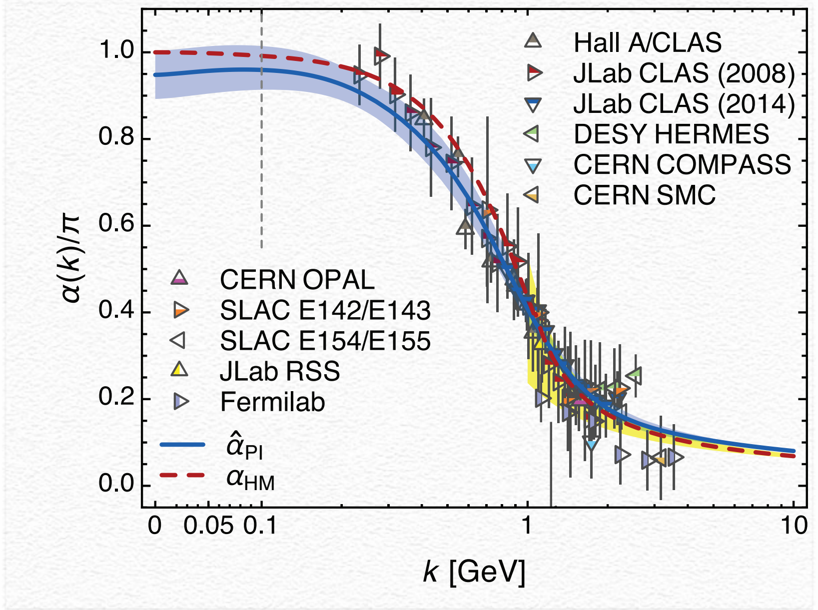
$$\alpha_{PI}(k) = \frac{4\pi}{\beta_0 \ln[({m_\alpha}^2 + k^2)/\Lambda_{\rm QCD}^2]}$$

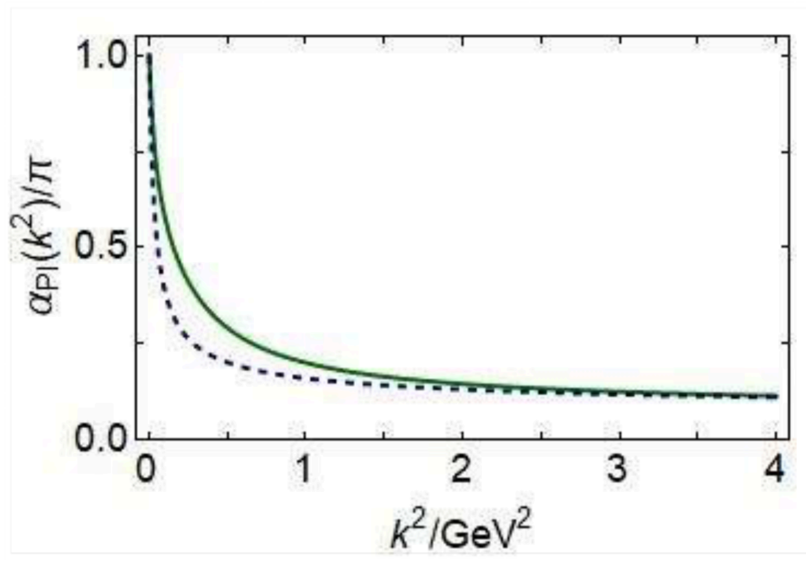
- $m_\alpha = 0.30\, GeV \gtrsim \Lambda_{\rm QCD}$ is an nonperturbative scale ensures modes with $k^2 \lesssim m_\alpha^2$ are screened from interactions.

- Renormalisation scale $\zeta_H = m_\alpha$

- m_α therefore serves to define the natural boundary between soft and hard physics

Phys. Rev. D 96, 054026 (2017)





Solid Green = original

Dashed Blue = simplified expression

Mellin moments

- $q^{\pi}(x;\zeta_H)$ is reconstructed from Mellin moments

$$\langle x^m\rangle^{\pi}_{\zeta_H} = \int_0^1 dx\, x^m q^{\pi}(x;\zeta_H) = \frac{N_c}{n\cdot P} tr \int_{dk} \left[\frac{n\cdot k_\eta}{n\cdot P}\right]^m \Gamma_\pi(k_{\bar\eta},P)\, S(k_{\bar\eta})\; n\cdot\partial_{k_\eta}\left[\Gamma_\pi(k_\eta,-P)S(k_\eta)\right]$$

- Direct calculation oscillations produced by the factor $[n\cdot k_\eta]^m$, introduce a convergence-factor $C_m(k^2r^2) = 1/[1+k^2r^2]^{m/2}$

- Odd moments are not independent

- Schlessinger point method (SPM)
  - Direct can compute m=0~5
  - SPM Extrapolate m=6~10

$$\langle x\rangle^{\pi}_{\zeta_H} = \frac{1}{2}\langle x^0\rangle^{\pi}_{\zeta_H} = \frac{1}{2},$$

$$\langle x^3\rangle^{\pi}_{\zeta_H} = -\frac{1}{4}\langle x^0\rangle^{\pi}_{\zeta_H} + \frac{3}{2}\langle x^2\rangle^{\pi}_{\zeta_H},$$

$$\langle x^5\rangle^{\pi}_{\zeta_H} = \frac{1}{2}\langle x^0\rangle^{\pi}_{\zeta_H} - \frac{5}{2}\langle x^2\rangle^{\pi}_{\zeta_H} + \frac{5}{2}\langle x^4\rangle^{\pi}_{\zeta_H},$$

$$\langle x^7\rangle^{\pi}_{\zeta_H} = -\frac{17}{8}\langle x^0\rangle^{\pi}_{\zeta_H} + \frac{21}{2}\langle x^2\rangle^{\pi}_{\zeta_H} - \frac{35}{4}\langle x^4\rangle^{\pi}_{\zeta_H} + \frac{7}{2}\langle x^6\rangle^{\pi}_{\zeta_H}.$$

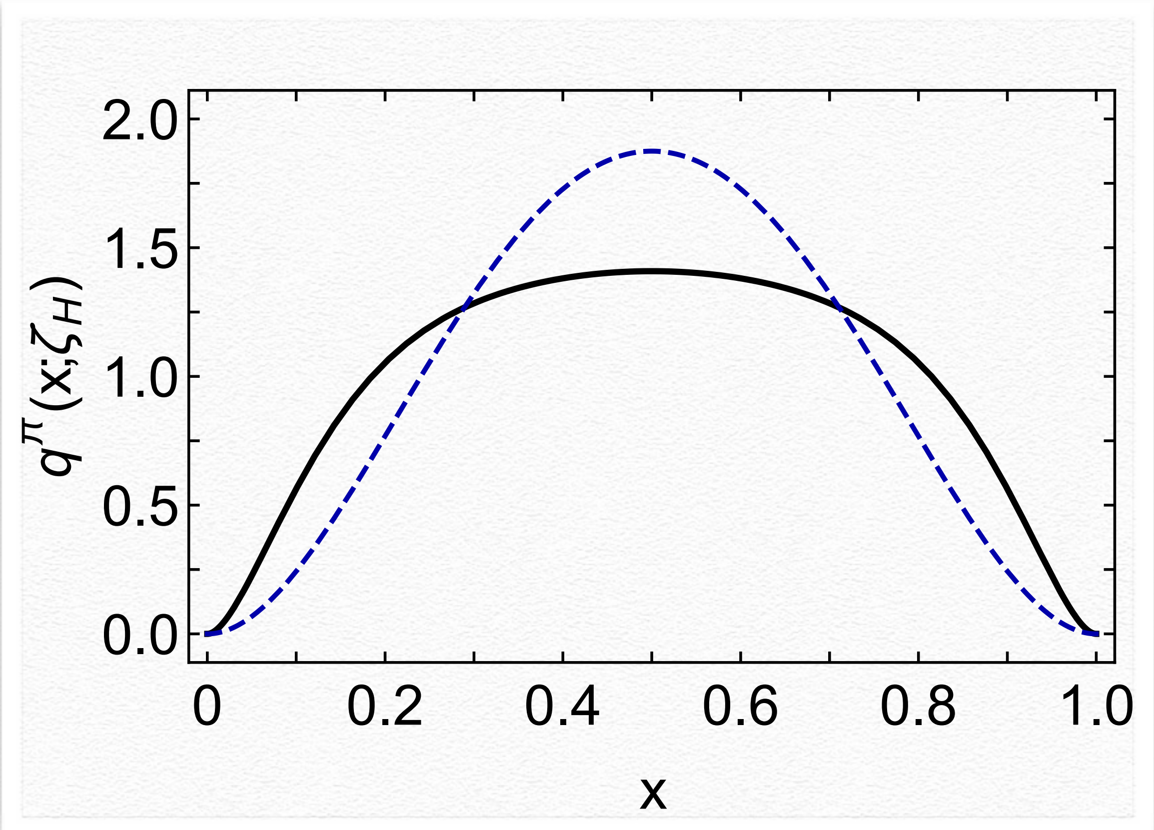
L. Schlessinger and C. Schwartz, Phys. Rev. Lett. 16, 1173 (1966)

L. Schlessinger, Phys. Rev. 167, 1411 (1968)

Valence quark PDF at ζ_H

- $q^\pi(x;\zeta_H)$ is reconstructed from Mellin moments

$$q^\pi(x;\zeta_H) = 213.32\,x^2(1-x)^2[1 - 2.9342\sqrt{x(1-x)} + 2.2911\,x(1-x)]$$



Dashed Blue = scale free distribution

$$q_{\rm sf}(x) \approx 30\,x^2(1-x)^2$$

Solid Black = $q^\pi(x;\zeta_H)$

- Broad concave function
- Dynamical chiral symmetry breaking
- PDA, FFs

Evolution of pion distribution functions

- Existing lQCD calculations of low-order moments and phenomenological fits to pion parton distributions are typically quoted at $\zeta_2 = 2$ GeV.

$$\zeta_H = m_\alpha \longrightarrow \zeta_2 = 2 \text{ GeV}$$

- E615 experiment take the scale $\zeta_5 = 5.2$ GeV

$$\zeta_H = m_\alpha \longrightarrow \zeta_5 = 5.2 \text{ GeV}$$

- Process-independent running coupling $\alpha_{\mathrm{PI}}(\zeta_H)/(2\pi) = 0.20\,, [\alpha_{\mathrm{PI}}(\zeta_H)/(2\pi)]^2 = 0.04$

  Leading order evolution should serve as a good approximation.

- Results report with $\zeta \to (1 \pm 0.1)\zeta$

Evolution of pion distribution functions

DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi) equation

$$\frac{d}{dt}q(x,t) = -\frac{\alpha(t)}{4\pi}\int_x^1 \frac{dy}{y} q(y,t) P(\frac{x}{y}) + \dots$$

- Moments evolution equation

$$M_n(t) = \int_0^1 dx x^n q(x,t) \qquad t = \ln\Big(\frac{\zeta^2}{\zeta_0^2}\Big)$$

$$\frac{d}{dt}M_n(t) = -\frac{\alpha(t)}{4\pi}\gamma_0^n M_n(t) + \dots$$

$$M_n(t) = M_0(t_0) exp\Big(-\frac{\gamma_0^n}{4\pi}\int_{t_0}^t dz \alpha(z)\Big)$$

- Splitting function

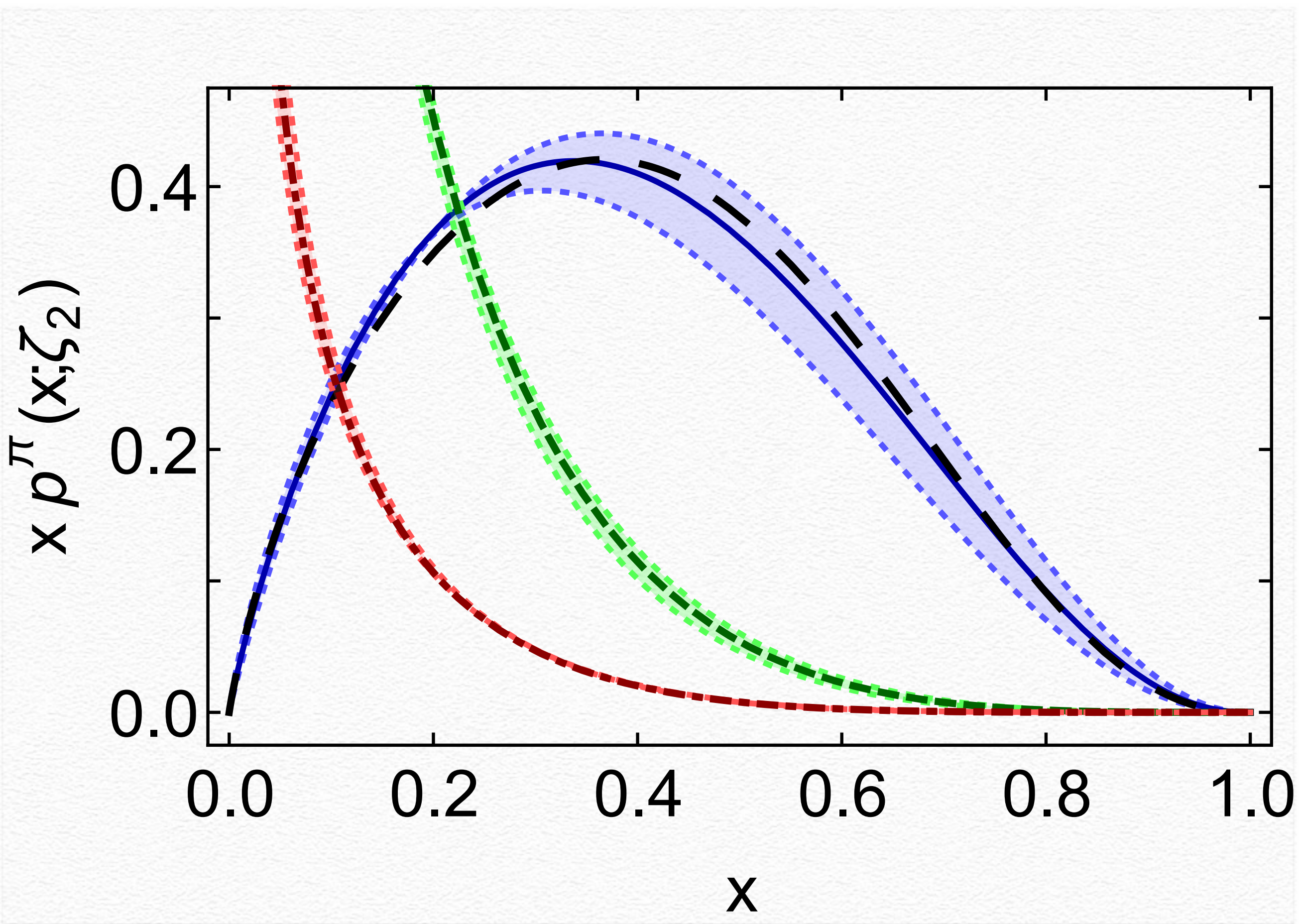
$$\int_0^1 dx P(x) = \gamma_0^n \qquad \gamma_0^n = -\frac{4}{3}\Big(3 + \frac{2}{(n+1)(n+2)} - 4\sum_{i=1}^{n+1}\frac{1}{i}\Big)$$

$$P(x) = \frac{8}{3}\Big(\frac{1+z^2}{(1-x)_+} + \frac{3}{2}\delta(x-1)\Big)$$

- Running coupling

$$\alpha(\zeta) = \frac{4\pi}{\beta_0 \ln[({m_\alpha}^2 + \zeta^2)/\Lambda_{QCD}^2]}$$

Valence quark PDF at $\zeta_2 = 2$ GeV



- Solid (blue) curve embedded in shaded band $q^{\pi}(x;\zeta_2)$

$$q^{\pi}(x) = n_{q^{\pi}}\, x^{\alpha}(1-x)^{\beta}$$
$$\times\, [1 + \rho\, x^{\alpha/4}(1-x)^{\beta/4} + \gamma\, x^{\alpha/2}(1-x)^{\beta/2}]$$

- Long-dashed (black), ζ_2 result from DSE in 2001

Phys. Rev. C 63, 025213 (2001)

	$n_{q^{\pi}}$	α	β	ρ	γ
ζ_2	9.83	−0.080	2.29	−1.27	0.511
	8.31	−0.127	2.37	−1.19	0.469
	7.01	−0.162	2.47	−1.12	0.453

$$\beta(\zeta_2) = 2.38(9)$$

Valence quark PDF at $\zeta_2 = 2$ GeV: moment $\langle x^1 \rangle_u^\pi$

- Low-order moments in comparison with recent lQCD simulations

- Both continuum and lQCD results agree

$$\langle 2x \rangle_q^\pi = 0.48(3)$$

- Roughly one-half of the light front momentum fraction is carried by the valence quarks

ζ_2	$\langle x \rangle_u^\pi$	$\langle x^2 \rangle_u^\pi$	$\langle x^3 \rangle_u^\pi$
Ref. [33]	0.24(2)	0.09(3)	0.053(15)
Ref. [34]	0.27(1)	0.13(1)	0.074(10)
Ref. [35]	0.21(1)	0.16(3)	
average	0.24(2)	0.13(4)	0.064(18)
Herein	0.24(2)	0.098(10)	0.049(07)

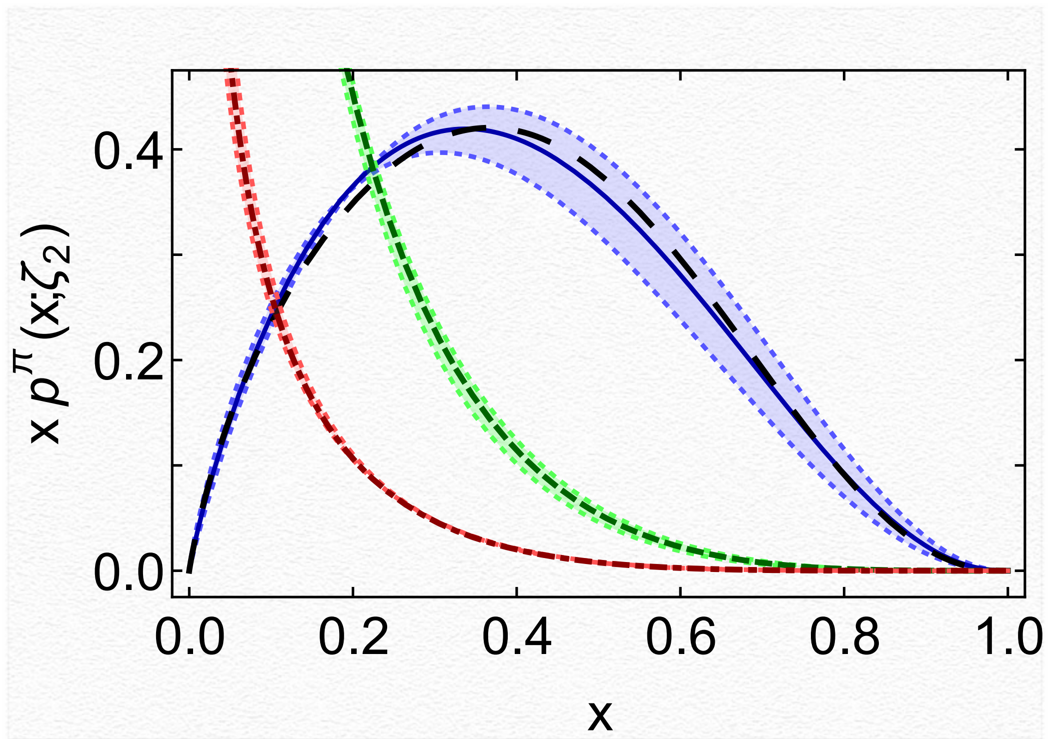
- Phenomenological analysis π-nucleus Drell-Yan and leading neutron electroproduction data.

Phys. Rev. Lett. 121, 152001 (2018)

$$\langle 2x \rangle_q^\pi = 0.48(1)\,, \zeta = 2.24 \text{ GeV}$$

Gluon and sea quark PDF at $\zeta_2 = 2$ GeV

- Pion is purely a bound-state of a dressed-quark and dressed-antiquark at the hadronic scale ζ_H, sea and glue distributions are zero at ζ_H.



	p	$\mathcal{A}$	α	β
ζ_2	g	0.40 ∓ 0.03	-0.55 ∓ 0.03	3.47 ± 0.13
	S	0.13 ∓ 0.01	-0.53 ∓ 0.05	4.51 ± 0.03

$p = g =$ *glue*, $p = S =$ *sea*

- Dashed (green), $xg^\pi(x;\zeta_2)$, gluon.
- Dot-dashed (red), $xS^\pi(x;\zeta_2)$, sea-quark.

$$xp^\pi(x;\zeta) = A\, x^\alpha (1-x)^\beta$$

- First Moment

$$\langle x\rangle_g^\pi = 0.41(2)\,,\ \langle x\rangle_{\rm sea}^\pi = 0.11(2)\,.$$

- Agree with Phys. Rev. Lett. 121, 152001 (2018)

$$\langle x\rangle_g^\pi = 0.30(2)\,,\ \langle x\rangle_{\rm sea}^\pi = 0.16(2)\,.$$

PDF at $\zeta_2 = 2$ GeV

- Valence quark $q^\pi(x;\zeta_2)$
  - Large x behaviour, $(1-x)^\beta$

$$\beta(\zeta_2) = 2.38(9)$$

  - First moment

$$\langle 2x \rangle_q^\pi = 0.48(3)$$

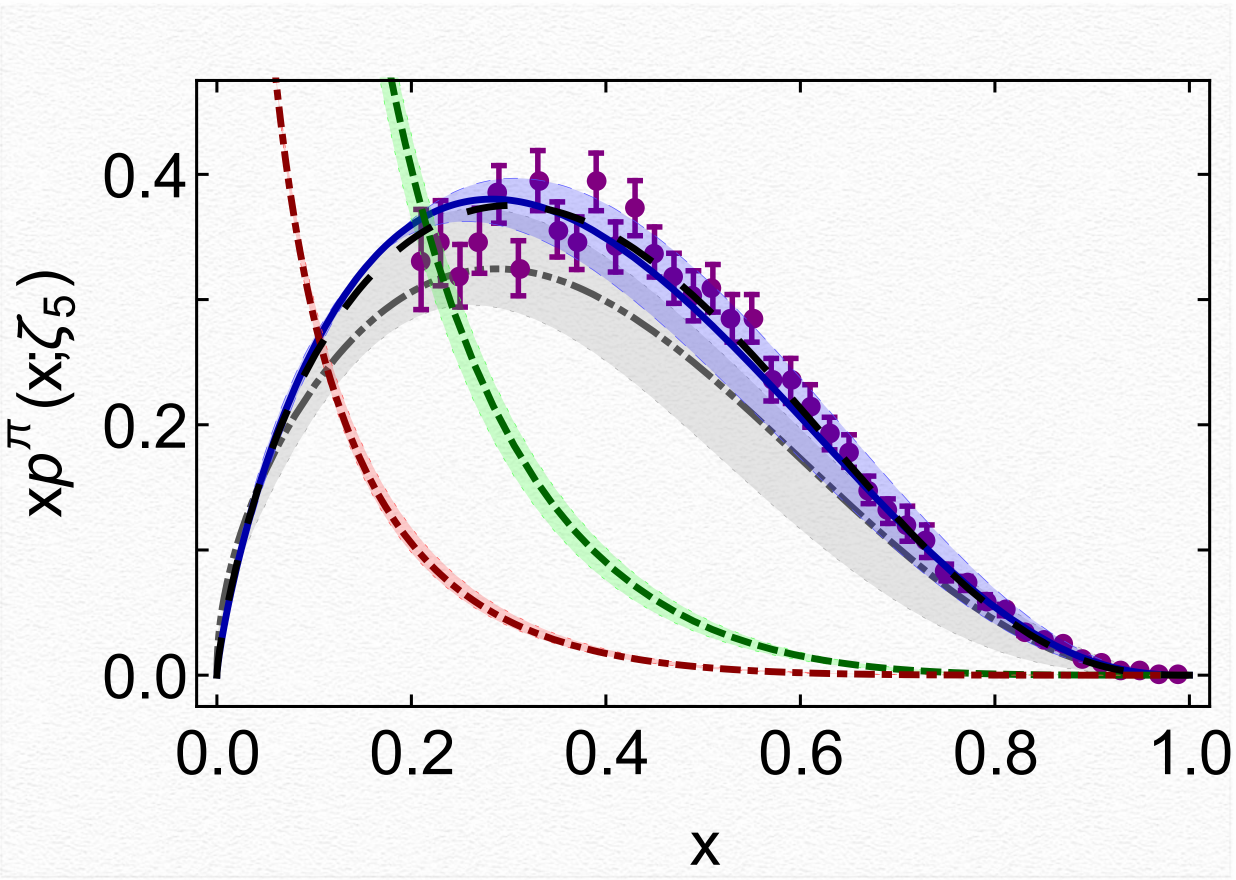
- Gluon and sea quark PDF
  - First moment

$$\langle x \rangle_g^\pi = 0.41(2)$$

$$\langle x \rangle_{\rm sea}^\pi = 0.11(2)$$

- Roughly one-half of the light front momentum fraction is carried by the valence quarks

Valence quark PDF at $\zeta_5 = 5.2$ GeV



	n_{q^π}	α	β	ρ	γ
	7.81	−0.153	2.54	−1.20	0.505
ζ_5	7.28	−0.169	2.66	−1.21	0.531
	6.48	−0.188	2.78	−1.19	0.555

$$\beta(\zeta_5) = 2.66(12)$$

- Agree with Phys. Rev. D 99, 074507 (2019) $\beta_{\rm lQCD}(\zeta_5) = 2.45(58)$

- Solid (blue), $q^\pi(x;\zeta_5)$

$$q^\pi(x) = n_{q^\pi}\, x^\alpha (1-x)^\beta$$
$$\times [1 + \rho\, x^{\alpha/4}(1-x)^{\beta/4} + \gamma\, x^{\alpha/2}(1-x)^{\beta/2}]$$

- Long-dashed (black), ζ_5 result from DSE in 2001

Phys. Rev. C 63, 025213 (2001)

- Dot-dot-dashed (grey), lQCD

Phys. Rev. D 99, 074507 (2019)

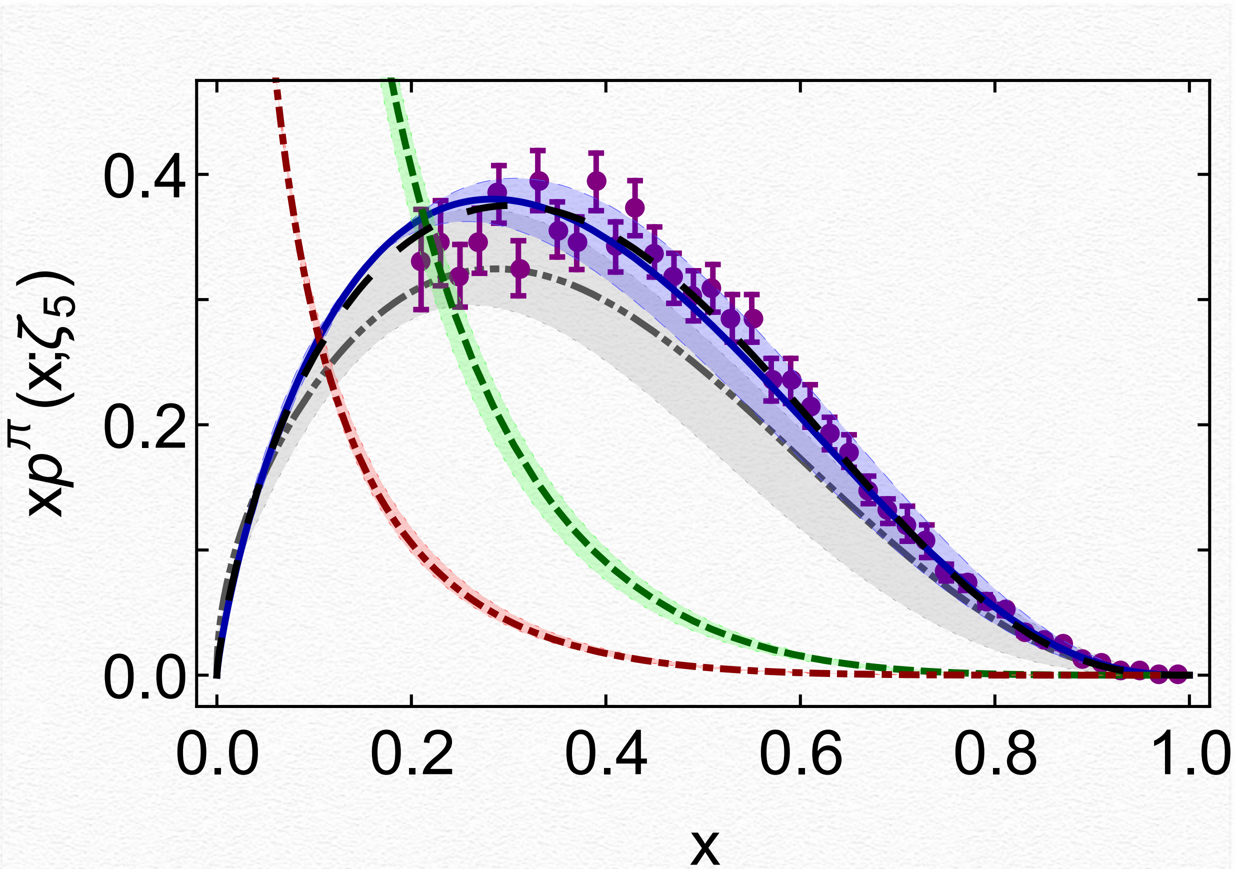
- Data (purple) from

Phys. Rev. D 39, 92 (1989)

- Rescaled analysis

Phys. Rev. Lett. 105, 252003 (2010)

$\zeta_5 = 5.2$ GeV: moment $\langle x^1 \rangle_u^\pi$ & Sea and gluon quark PDF



- Dashed (green), $xg^\pi(x;\zeta_5)$, gluon.
- Dot-dashed (red), $xS^\pi(x;\zeta_5)$, sea-quark.

$$xp^\pi(x;\zeta) = A\, x^\alpha\, (1-x)^\beta$$

	p	$\mathcal{A}$	α	β
ζ_5	g	0.34 ∓ 0.04	-0.62 ∓ 0.04	3.75 ± 0.12
	S	0.12 ± 0.02	-0.61 ∓ 0.07	4.77 ± 0.03

- Low-order moments in comparison with recent lQCD simulations

ζ_5	$\langle x \rangle_u^\pi$	$\langle x^2 \rangle_u^\pi$	$\langle x^3 \rangle_u^\pi$
Ref. [31]	0.17(1)	0.060(9)	0.028(7)
Herein	0.21(2)	0.076(9)	0.036(5)

- First Moment

$$\langle x \rangle_g^\pi = 0.45(1)\,,\ \langle x \rangle_{\rm sea}^\pi = 0.14(2)\,.$$

PDF at $\zeta_5 = 5.2$ GeV

- Valence quark $q^\pi(x;\zeta_5)$
  - Large x behaviour, $(1-x)^\beta$

$$\beta(\zeta_5) = 2.66(12)$$

  - First moment

$$\langle 2x \rangle_q^\pi = 0.42(4)$$

- Gluon and sea quark PDF
  - First moment

$$\langle x \rangle_g^\pi = 0.45(1)$$

$$\langle x \rangle_{\text{sea}}^\pi = 0.14(2)$$

- We agrees with rescaled E615 data and lQCD prediction

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Summary

- Using a continuum approach, presented a symmetry-preserving calculation of the pion's valence-quark PDF; A novel term $q^{\pi}_{\rm BC}(x;\zeta_H)$ is necessary to keep $q^{\pi}(x;\zeta_H) = q^{\pi}(1-x;\zeta_H)$ and then $\langle x_{\pi}^{1}\rangle = 1/2$; $\zeta_H = 0.30\,{\rm GeV}$ is the hadronic scale, and is determined by connecting the one-loop running coupling with QCD's process-independent effective charge.

- $q^{\pi}(x;\zeta_H)$ is a broad concave function and is a consequence of dynamical chiral symmetry breaking (DCSB). Valence quark $q^{\pi}(x;\zeta_2)$ large x behaviour $\beta(\zeta_2) = 2.38(9)$, and first moment $\langle 2x\rangle^{\pi}_{q} = 0.48(3)$. Valence quark $q^{\pi}(x;\zeta_5)$ agrees with rescaled E615 data and lQCD prediction, large x behaviour $\beta(\zeta_5) = 2.66(12)$, and first moment $\langle 2x\rangle^{\pi}_{q} = 0.42(4)$. Gluon and sea quark PDF ζ_2, $\langle x\rangle^{\pi}_{g} = 0.41(2)$, $\langle x\rangle^{\pi}_{\rm sea} = 0.11(2)$, ζ_5, $\langle x\rangle^{\pi}_{g} = 0.45(1)$, $\langle x\rangle^{\pi}_{\rm sea} = 0.14(2)$.

Outlook

- Kaon PDF, $u_K(x)/u_{\pi}(x)$, gluon content, Approved experiment, using tagged DIS at JLab 12, Electron–ion collider (EIC).

- Nucleon PDF, quark-diquark model.