Nucleon Form Factors at High Momentum Transfer

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Outline

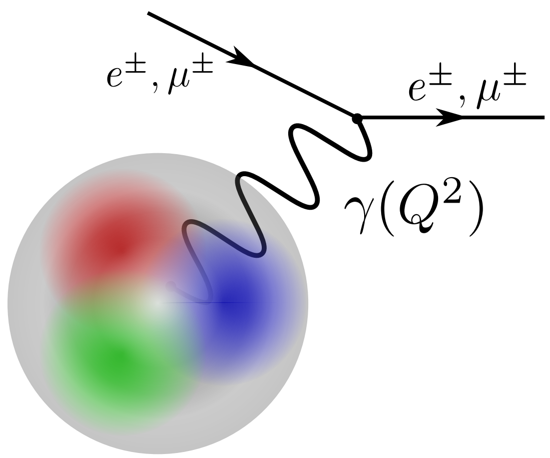
- Phenomenological motivation
- Challenges for high-momentum nucleon structure
- Details of calculation
- Results and comparison to phenomenology
- Summary and Outlook

Nucleon Vector Form Factors and G_{Ep}/G_{Mp}

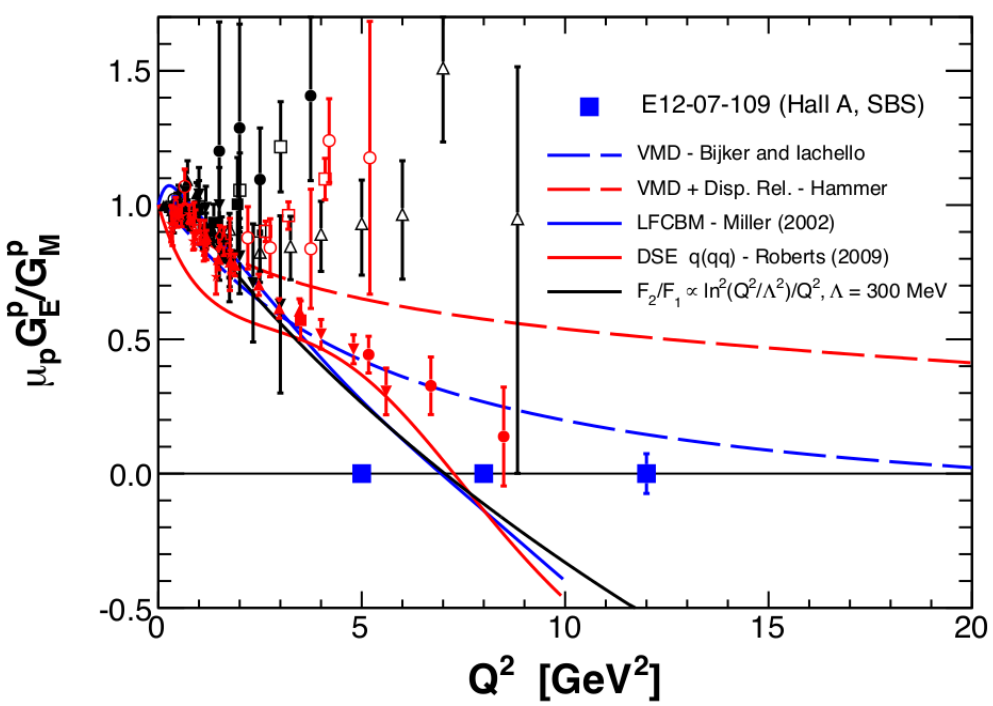
$$\langle P+q|\,\bar{q}\gamma^\mu q\,|P\rangle = \bar{U}_{P+q}\left[F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\right]U_P$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)$$

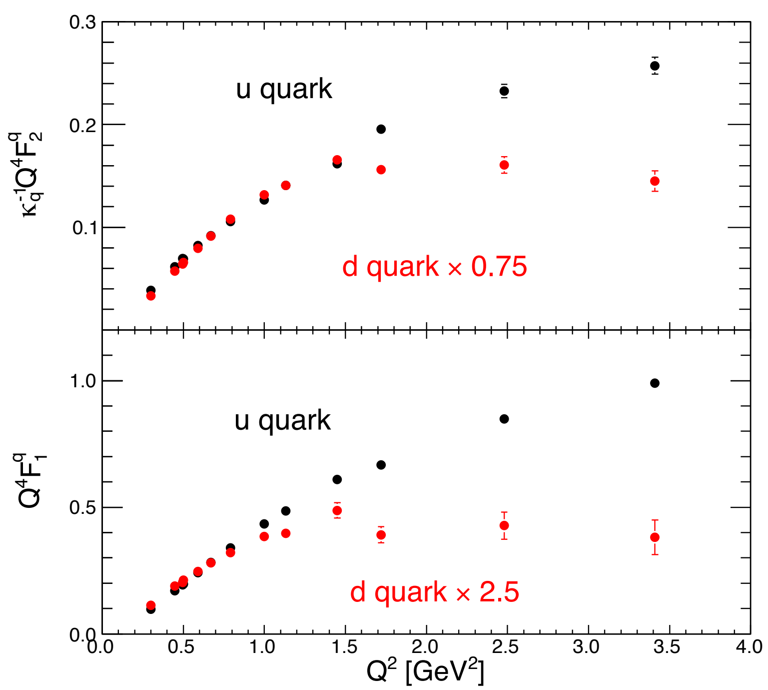
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

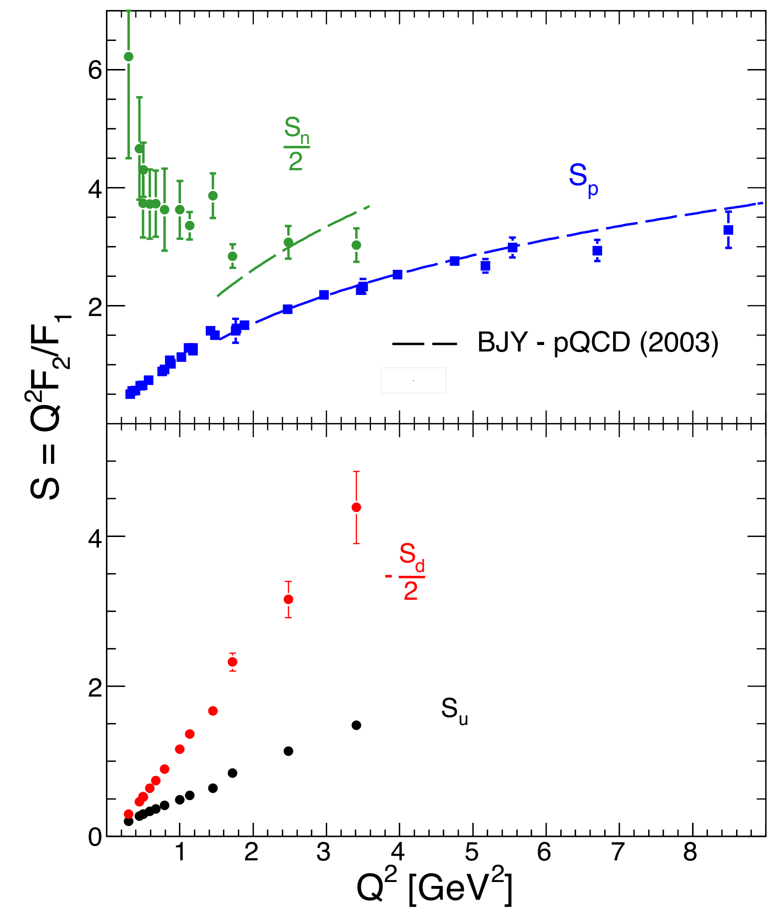


- (G_E/G_M) dependence
- (F_1/F_2) scaling at $Q^2 \to \infty$
- *u-*, *d*-flavor contributions to form factors



[Research Mgmt. Plan for SBS(JLab Hall A)]





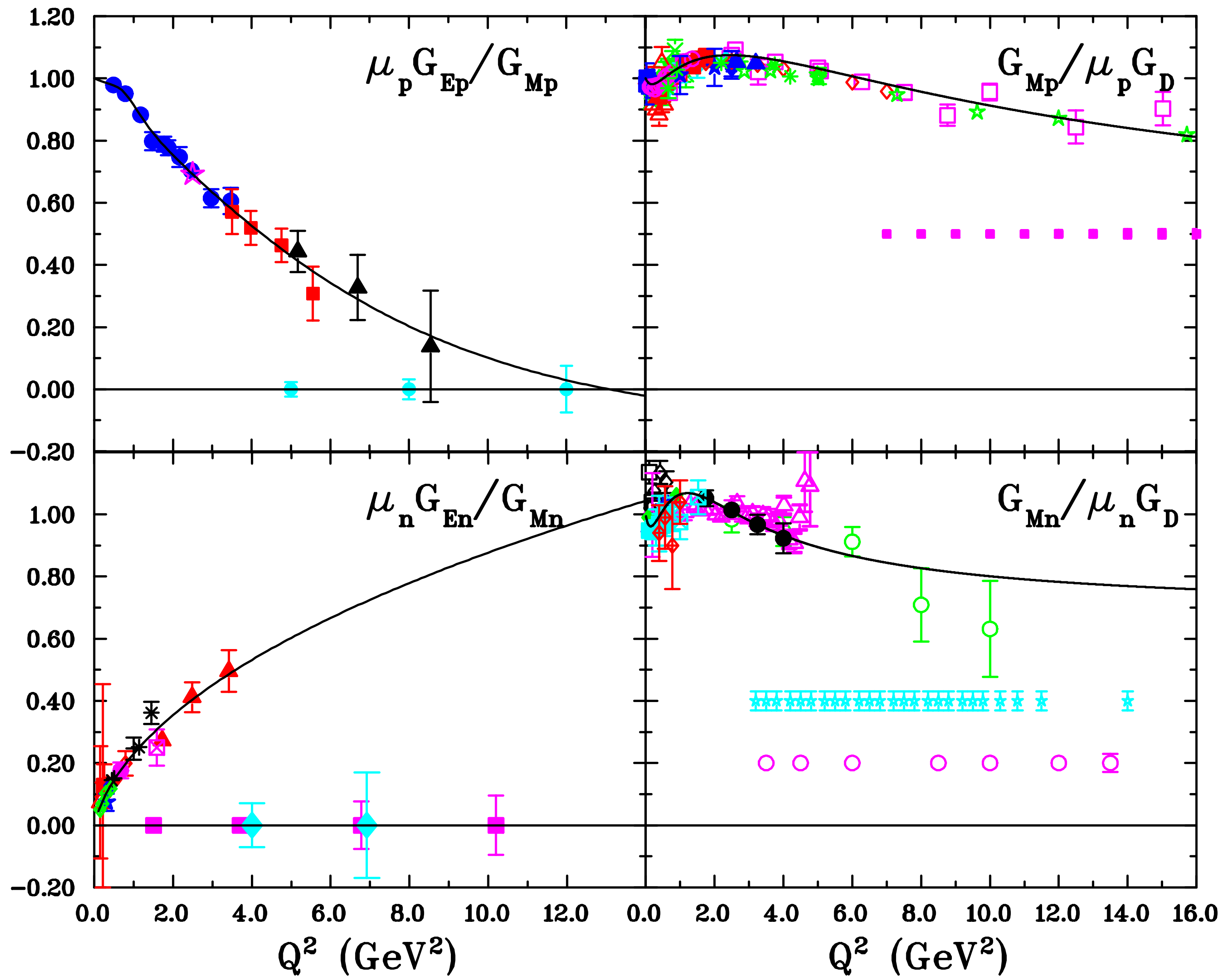
[G.D.Cates, C.W.de Jager, S.Riordan, B.Wojtsekhovski, PRL106:252003, arXiv:1103.1808]

Experimental Prospects

[V. Punjabi et al, EPJ A51: 79 (2015); arXiv: 1503.01452]

JLab:

- Hall A (HRS):
  G_{Mp} @ $Q^2 \lesssim 17.5\ \text{GeV}^2$
- Hall A (SBS):
  G_{Ep}/G_{Mp} @ $Q^2 \lesssim 15\ \text{GeV}^2$;
  G_{En}/G_{Mn} @ $Q^2 \lesssim 10.2\ \text{GeV}^2$;
  G_{Mn} @ $Q^2 \lesssim 18\ \text{GeV}^2$
- Hall B (CLAS12):
  G_{Mn} @ $Q^2 \lesssim 14\ \text{GeV}^2$
- Hall C :
  G_{En}/G_{Mn} @ $Q^2 \lesssim 6.9\ \text{GeV}^2$



Basics of Hadron Structure in Lattice QCD

Lattice Field Theory ⇔ Numerical evaluation of the Path Integral

$$\langle q_x \bar{q}_y \ldots \rangle = \int \mathcal{D}\big(Glue\big) \int \mathcal{D}\big(Quarks\big)\; e^{-S_{Glue} - \bar{q}\left(\not{D}+m\right)q} \left[q_x \bar{q}_y \ldots\right]$$

Grassmann integration

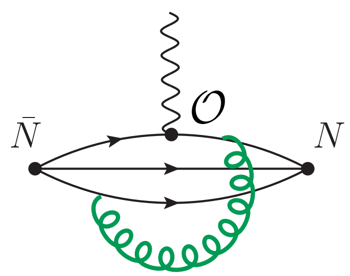
$$= \int \mathcal{D}\big(Glue\big)\; e^{-S_{Glue}}\; \mathrm{Det}\big(\not{D}+m\big) \; \left[\big(\not{D}+m\big)^{-1}_{x,y} \ldots\right]$$

Hybrid Monte Carlo sampling of gluon background

$(\not{D}+m)\cdot q = 0$
quark motion in gluon background

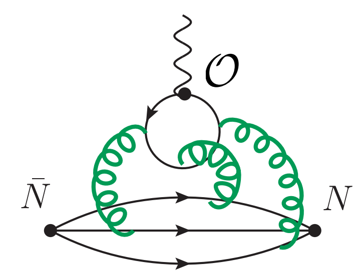
Hadron Matrix Elements:

$$C^{\mathcal{O}}_{3\mathrm{pt}}(T) = \langle N(T)\mathcal{O}(\tau)\bar{N}(0)\rangle =$$

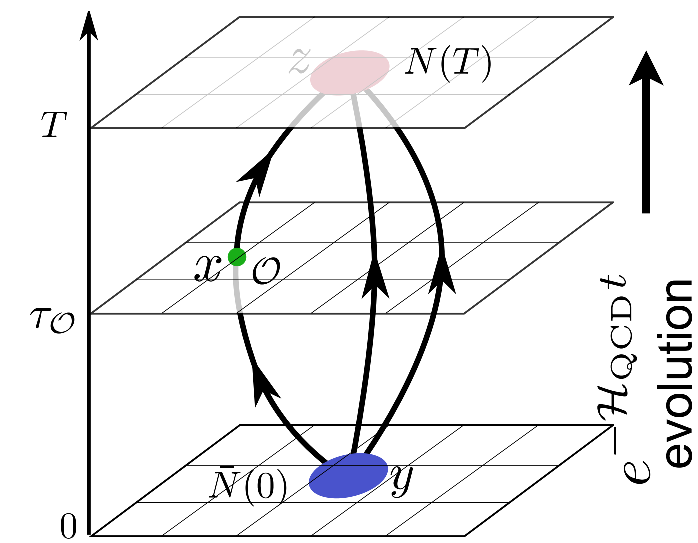


"connected"

\+



"disconnected"



Each quark line = $(\not{D}+m)^{-1}\cdot\psi$

$$\langle N(T)\mathcal{O}(\tau)N(0)\rangle = \sum_{n,m} Z_m e^{-E_n(T-\tau)} \langle n|\mathcal{O}|m\rangle e^{-E_m\tau} Z_n^*$$

$$\xrightarrow[T\to\infty]{} Z_{00}e^{-M_N T}\Big[\langle P'|\mathcal{O}|P\rangle + \mathcal{O}\big(\underbrace{e^{-\Delta E_{10}T}, e^{-\Delta E_{10}\tau}, e^{-\Delta E_{10}(T-\tau)}}_{\text{excited states}}\big)\Big]$$

Ground state form factors

Fit and throw away

Systematic effects
- excited states
- discretization errors
- finite volume
- unphysical (heavy) pion mass *

Nucleon Form Factors: Recipy on a Lattice

- Generage lattice ensemble

$$P(A_\mu) \propto e^{-S[A_\mu]} \prod_q \det(\not{D} + m_q)$$

- Compute nucleon correlation functions

2-point $$\langle N(\vec{p}', t_{\text{sep}})\,\bar{N}(0)\rangle = \sum_y e^{-i\vec{p}'\vec{x}} \langle N_{t_{\text{sep}},\vec{y}}\,\bar{N}(0,\vec{0})\rangle$$

3-point $$\langle N(\vec{p}', t_{\text{sep}})\,\mathcal{O}(\vec{q},\tau)\,\bar{N}(0)\rangle = \sum_{y,z} e^{-i\vec{p}'\vec{x}+i\vec{q}\vec{z}} \langle N_{t_{\text{sep}},\vec{y}}\,\mathcal{O}_{z,\tau}\,\bar{N}(0,\vec{0})\rangle$$

- Extract ground-state matrix elements (2-state fits typical)

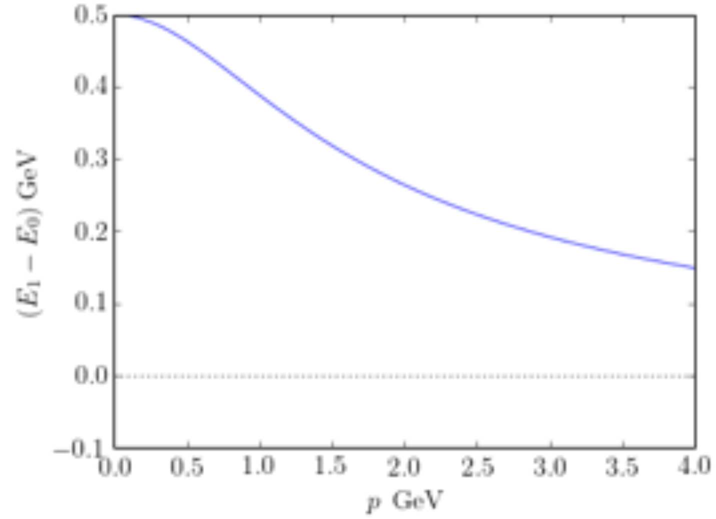
$$\begin{aligned}\langle N(p', t_{\text{sep}})\,\mathcal{O}(\tau)\,\bar{N}(p,0)\rangle \sim e^{-E_0'(t_{\text{tsep}}-\tau)-E_0\tau}\Big[&\langle 0(p')|\mathcal{O}|0(p)\rangle \\ &+ \langle 0(p')|\mathcal{O}|1(p)\rangle e^{-\Delta E_{01}\tau} + \langle 1(p')|\mathcal{O}|0(p)\rangle e^{-\Delta E_{01}'(t_{\text{sep}}-\tau)} \\ &+ \langle 1(p')|\mathcal{O}|1(p)\rangle e^{-\Delta E_{01}\tau-\Delta E_{01}'(t_{\text{sep}}-\tau)}\Big]\end{aligned}$$

- Reduce m.e. to form factors: fit over momentum combinations, polarization, etc

$$\langle p', \sigma'|J^\mu|p,\sigma\rangle = [\bar{u}'\gamma^\mu u]F_1 + [\bar{u}'\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}u]F_2$$

Challenges for Structure at Large Momentum

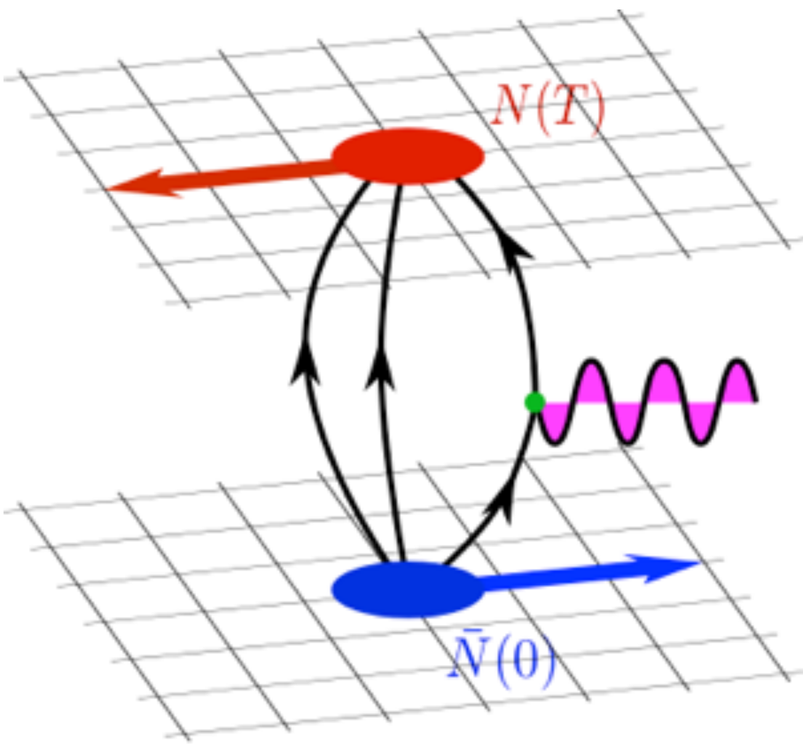
- Multiscale problem: $a \ll p_N^{-1}, Q^{-1}; \quad 4m_\pi^{-1} \lesssim L$
- Discretization effects:
  O(a) Correction to current operator
  $$(V_\mu)_I = [\bar{q}\gamma_\mu q] + c_V\, a\, \underbrace{\partial_\nu[\bar{q}i\sigma_{\mu\nu}q]}_{\propto Q}$$
- Stochastic noise grows faster with T [Lepage'89]:
  $$\begin{array}{lll} \text{Signal} & \langle N(T)\bar{N}(0)\rangle & \sim e^{-E_N T} \\ \text{Noise} & \langle |N(T)\bar{N}(0)|^2\rangle - |\langle N(T)\bar{N}(0)\rangle|^2 & \sim e^{-3m_\pi T} \\ \text{Signal/Noise} & & \sim e^{-(E_N - \frac{3}{2}m_\pi)T} \end{array}$$
- Excited states: boosting "shrinks" the energy gap
  $$E_1 - E_0 = \sqrt{M_1^2 + \vec{p}^2} - \sqrt{M_2^2 + \vec{p}^2} < M_1 - M_0$$
  - N(~1500): pN→1.5 GeV ⇒ ΔE = 500 → 300 MeV



- Large p_N: no EFT for extrapolation in m_π, L (lattice size)

Reduction of lattice correlator noise is <u>crucial</u>

Accessing Large Q^2 : Breit Frame on a Lattice



Minimize $E_{in,out}$ for target Q^2:

$$Q^2 = (\vec{p}_{in} - \vec{p}_{out})^2 - (E_{in} - E_{out})^2$$

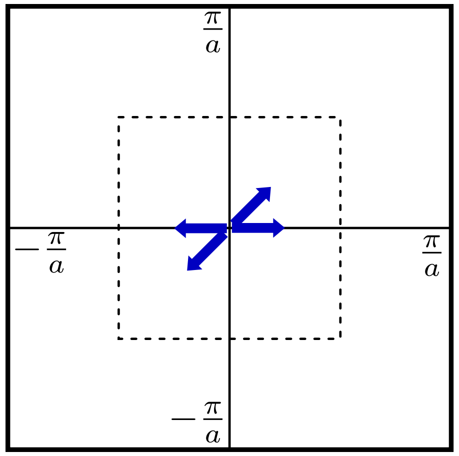
Back-to-back

$$Q^2 = 4\vec{p}^2$$

For $(Q^2)_{max}$ = *10 GeV²*

$$|\vec{p}| = \frac{1}{2}\sqrt{Q^2_{\rm max}} \approx 1.6\ \rm GeV \quad (E_N \approx 1.9\ GeV)$$

Nucleon momentum ~ Brillouin zone ⇒ unknown distortion of lattice nucleon Dirac operator



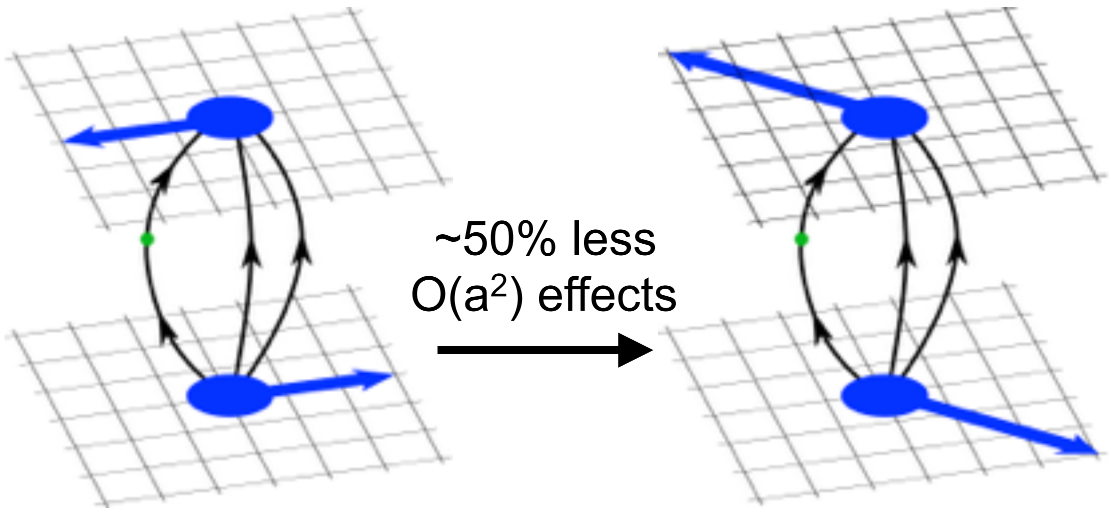
lattice kinematics for $Q^2 \approx 10\ \rm GeV^2$

$$\langle N\bar{N}\rangle^{-1}(p) \stackrel{?}{=} -i\not{p}^{\rm lat} + m_N$$

$$p^{\rm lat}_\mu = k_\mu + O(k^3)$$

⇒ expect O(a^2) corrections

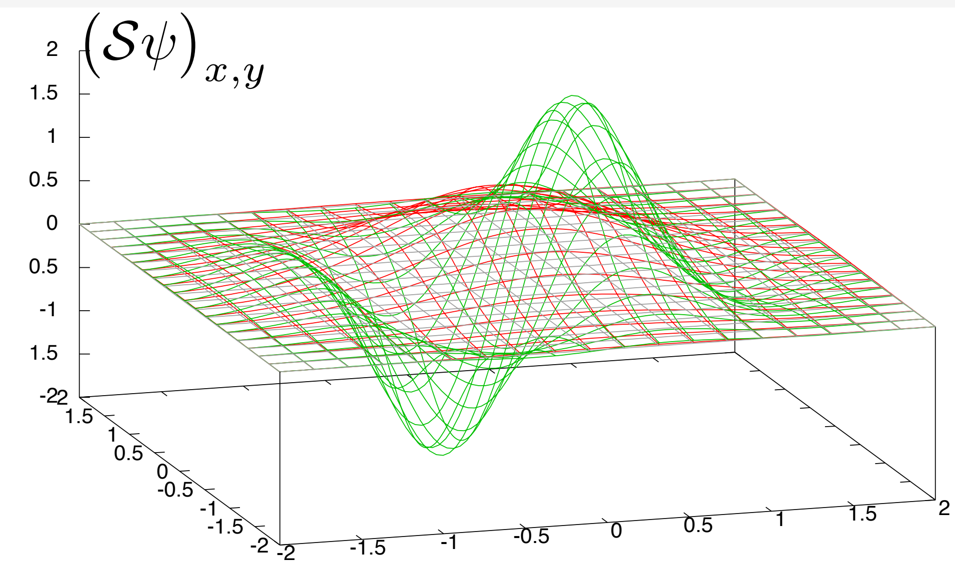
from lattice nucleon polarizaton



High-momentum Hadron States on a Lattice

Nucleon operator is built from ≈Gaussian smeared quarks

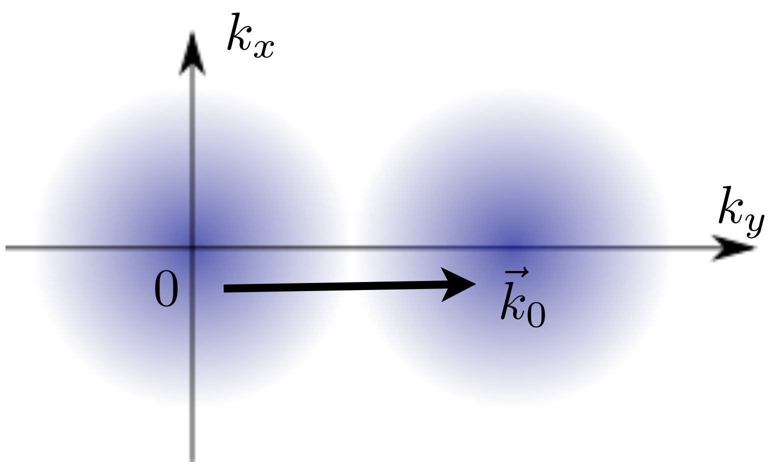
$$N_{\rm lat}(x) = (\mathcal{S}\,u)^a_x \left[(\mathcal{S}\,d)^b_x\, C\gamma_5\, (\mathcal{S}\,u)^c_x\right] \epsilon^{abc}$$



Gaussian shape in momentum space :
reduced overlap with quark WFs in a boosted nucleon

$$\mathcal{S}_{\text{at-rest}} = \exp[-\frac{w^2}{4}(i\vec{\nabla})^2] \sim exp(-\frac{w^2\vec{k}^2_{\rm lat}}{4})$$

[G.Bali et al, PRD93:094515; arXiv:1602.05525]:
improve the overlap with large-P_N nucleon by shifting the quark trial shape in momentum space (*"momentum smearing"*)



$$\mathcal{S}_{\vec{k}_0} = \exp[-\frac{w^2}{4}(-i\vec{\nabla} - \vec{k}_0)^2] \sim \exp(-\frac{w^2(\vec{k}_{\rm lat} - \vec{k}_0)^2}{4})$$

Modified smearing operator

$$\left[\mathcal{S}_{\vec{k}_0}(\psi)\right]_x = e^{+\vec{k}_0\vec{x}}\mathcal{S}(e^{-\vec{k}_0\vec{y}}\psi_y) \sim e^{+\vec{k}_0\vec{x}} \cdot \text{smooth fcn.}(x)$$

Modified covariant smearing operator in lattice*color space

$$\left[\mathcal{S}_{\vec{k}_0}\right]_{x,y} = e^{+i\vec{k}_0\vec{x}}\left[\mathcal{S}\right]_{x,y} e^{-i\vec{k}_0\vec{y}}$$

$\Longleftrightarrow$

Smearing with twisted gauge links

$$\Delta_{x,y} \longrightarrow e^{+i\vec{k}_0\vec{x}}\Delta_{x,y}e^{-i\vec{k}_0\vec{y}}$$

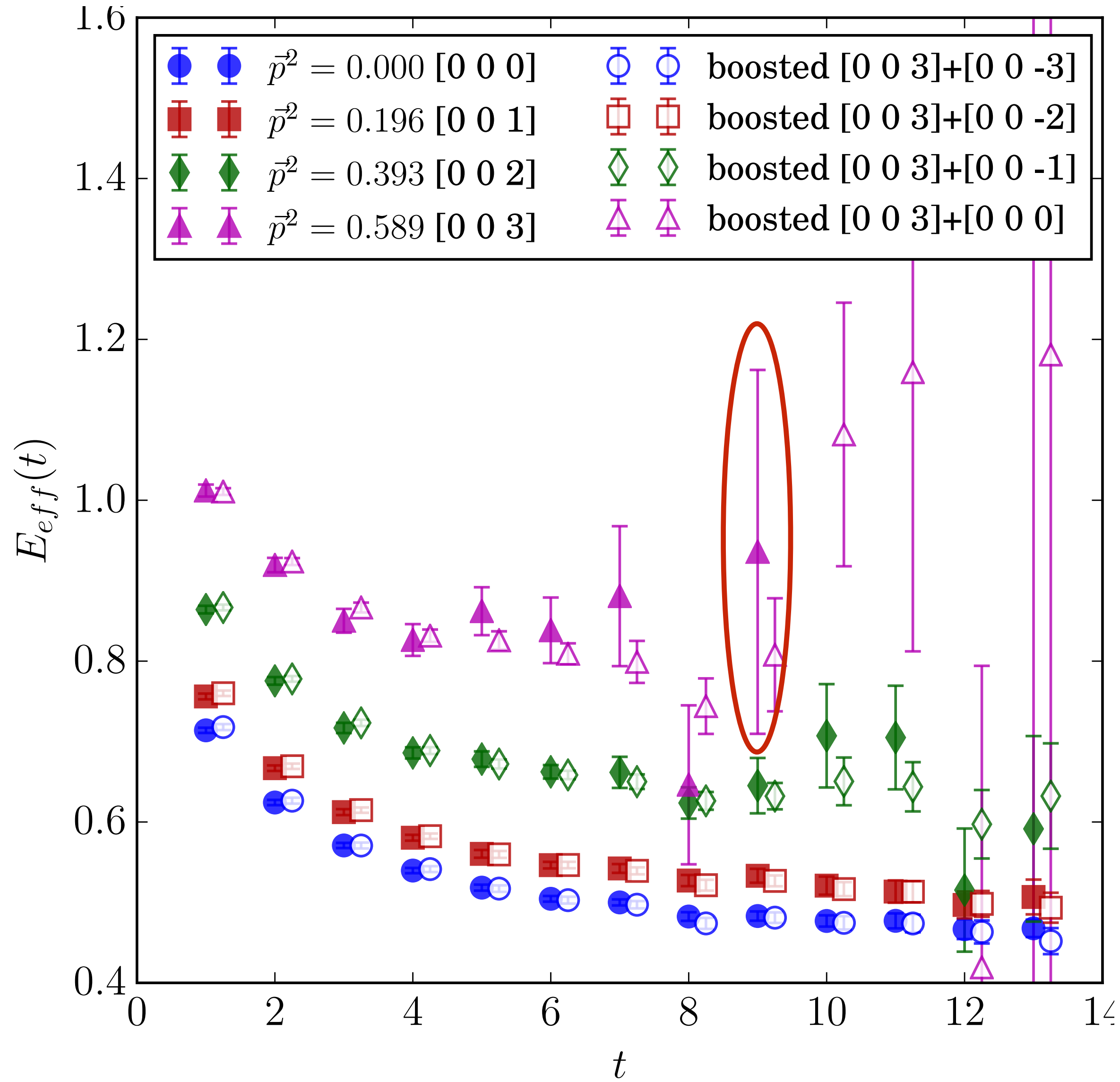
$$U_{x,\mu} \longrightarrow e^{-ik_\mu}U_{x,\mu}$$

Signal Gain in Effective Energy

Comparison of nucleon interpolating fields constructed from "boosted" and "regular" quarks

- Nucleon Effective Energy
  (D5: m_π = 280 MeV, a=0.094 fm, 32^3x64)

$$E_{eff} = \frac{1}{a} \log \frac{C_{N\bar{N}}(t)}{C_{N\bar{N}}(t+a)}$$



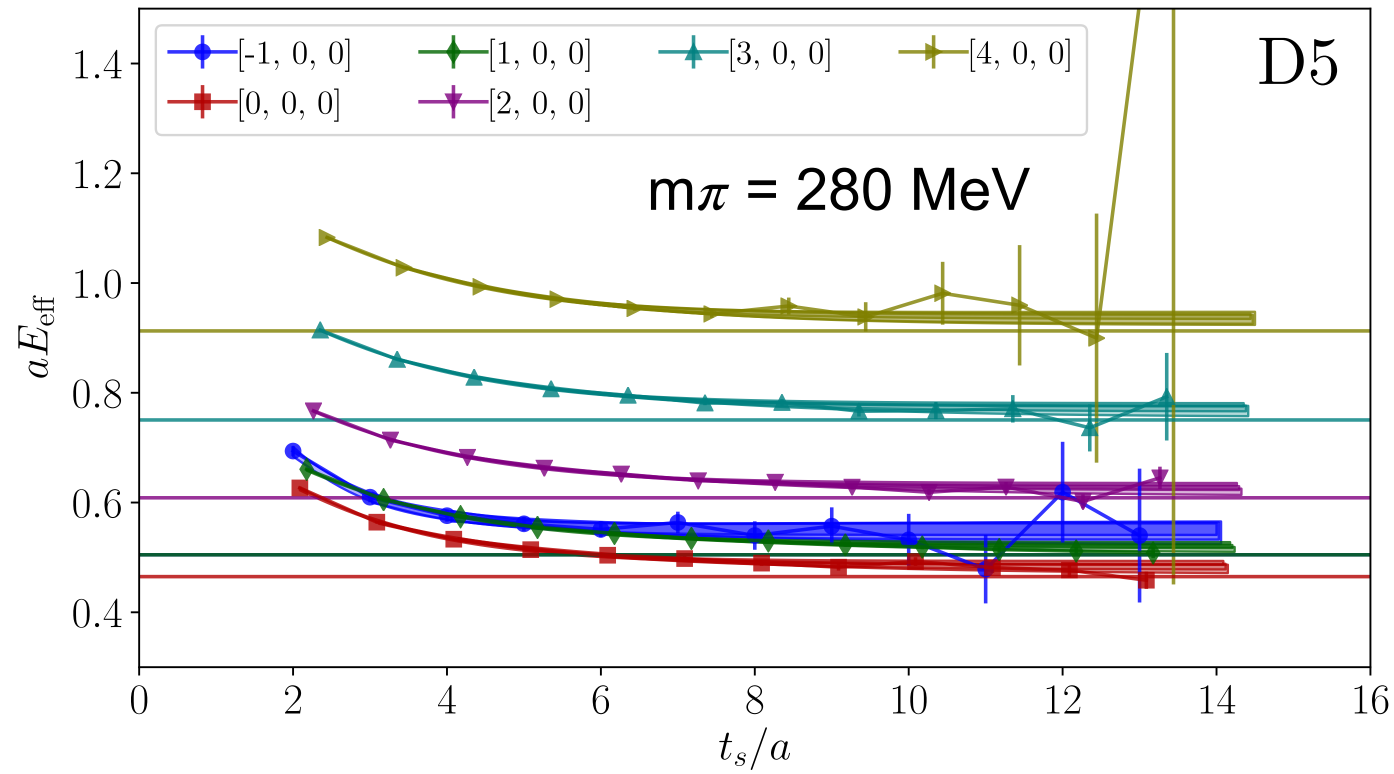
- Effect of quark "boost" increases with P_N
- Standard technique in lattice calculations of quasi-PDF, TMD, ...

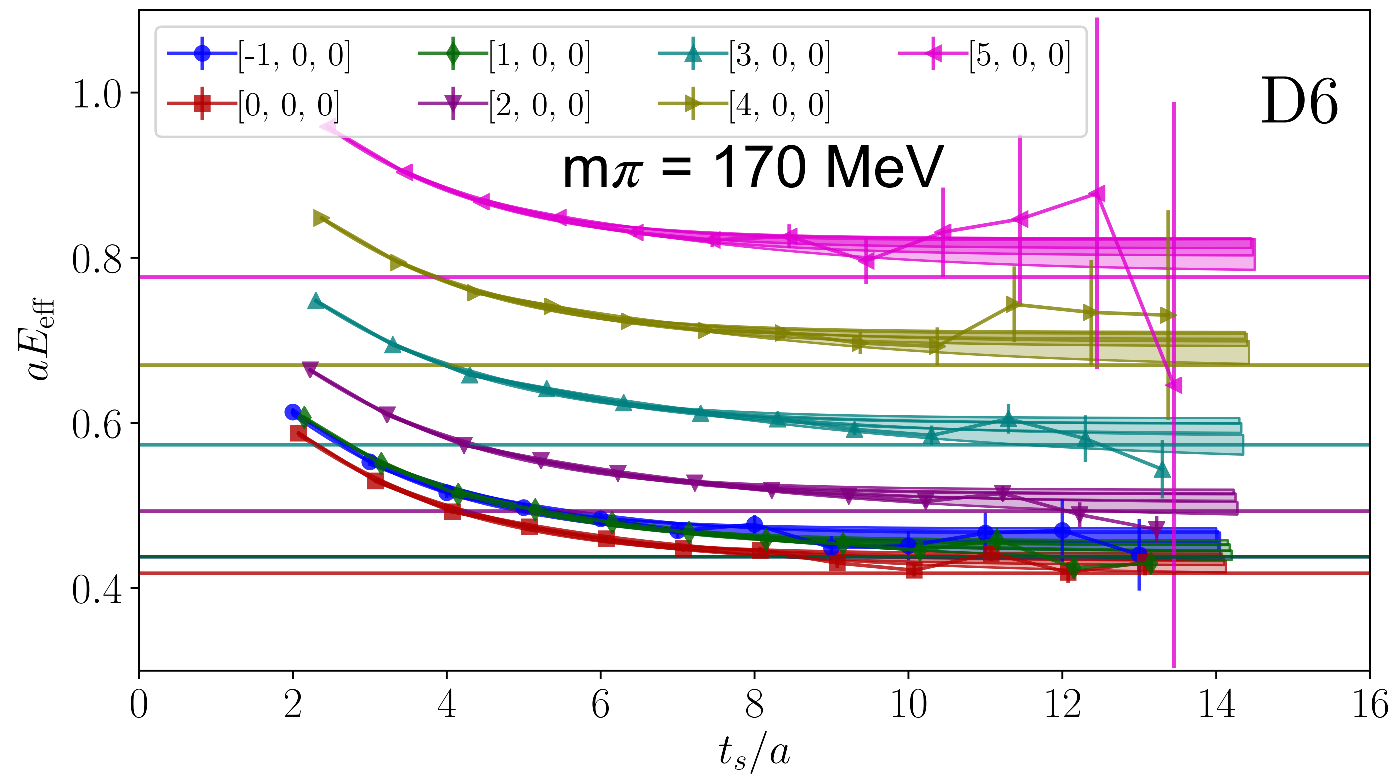
QCD Simulation Parameters

- two Nf=2+1 Wilson-clover ensembles, produced by JLab/W&M lattice group
- similar lattice spacing a ≈ 0.09 fm
- two different light quark masses (m_π = 280 and 170 MeV)
- large physical volume L≳ 3.8 $(m\pi)^{-1}$

D5-ensemble: $\beta = 6.3$, $a = 0.094$ fm, $a^{-1} = 2.10$ GeV		
$32^3 \times 64$, $L = 3.01$ fm	$a\mu_l$	-0.2390
	$a\mu_s$	-0.2050
	κ	0.132943
	$C_{\rm sw}$	1.205366
	m_π (MeV)	280
	$m_\pi L$	4.26
	Statistics	86144
D6-ensemble: $\beta = 6.3$, $a = 0.091$ fm, $a^{-1} = 2.17$ GeV		
$48^3 \times 96$, $L = 4.37$ fm	$a\mu_l$	-0.2416
	$a\mu_s$	-0.2050
	κ	0.133035
	$C_{\rm sw}$	1.205366
	m_π (MeV)	170
	$m_\pi L$	3.76
	Statistics	50176

Relativistic Nucleon Energies on a Lattice





- Effective energy

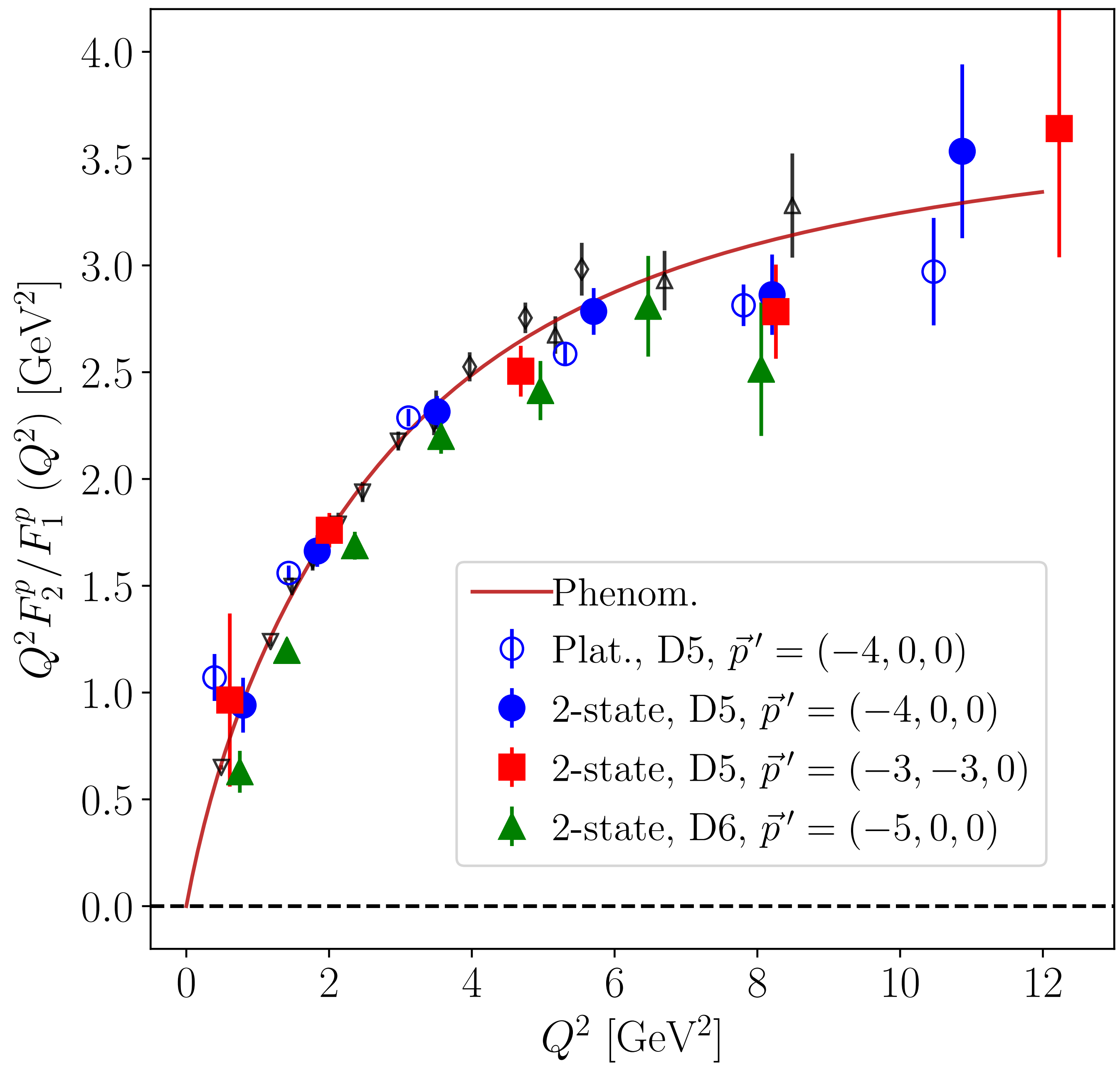
$$E_{eff} = \frac{1}{a} \log \frac{C_{N\bar{N}}(t)}{C_{N\bar{N}}(t+a)}$$

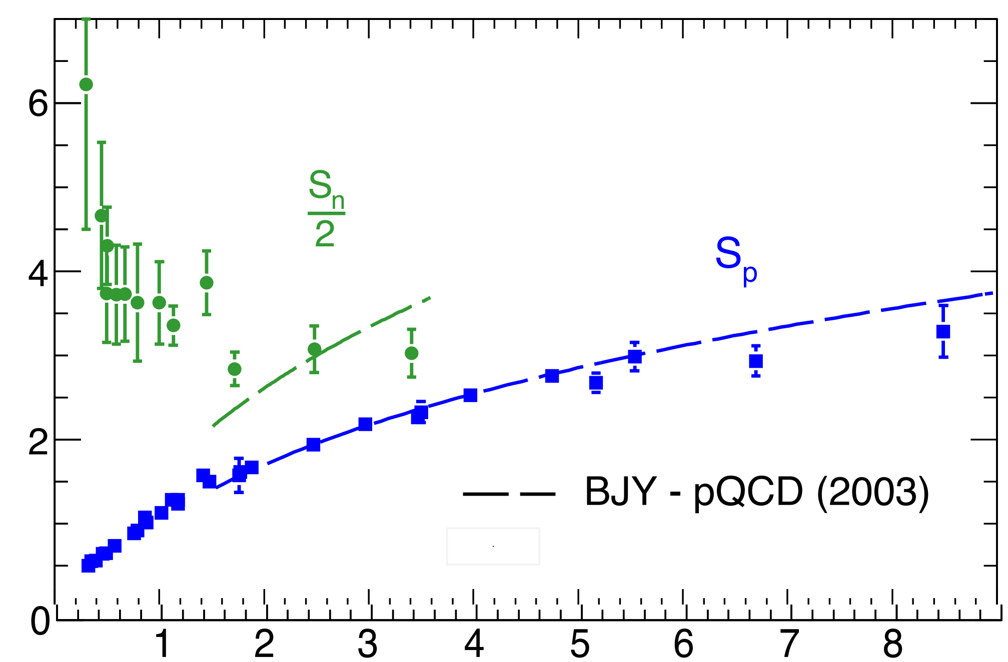
- Straight lines: continuum dispersion relation with m_N from [1602.07737]

F_{2p}/F_{1p} Form Factor Ratio, Proton

- *No disconnected diagrams*
- *No discretization corrections*

- Black points: experiments
- Phenomenology curves : [Alberico et al, PRC79:065204 (2008)]



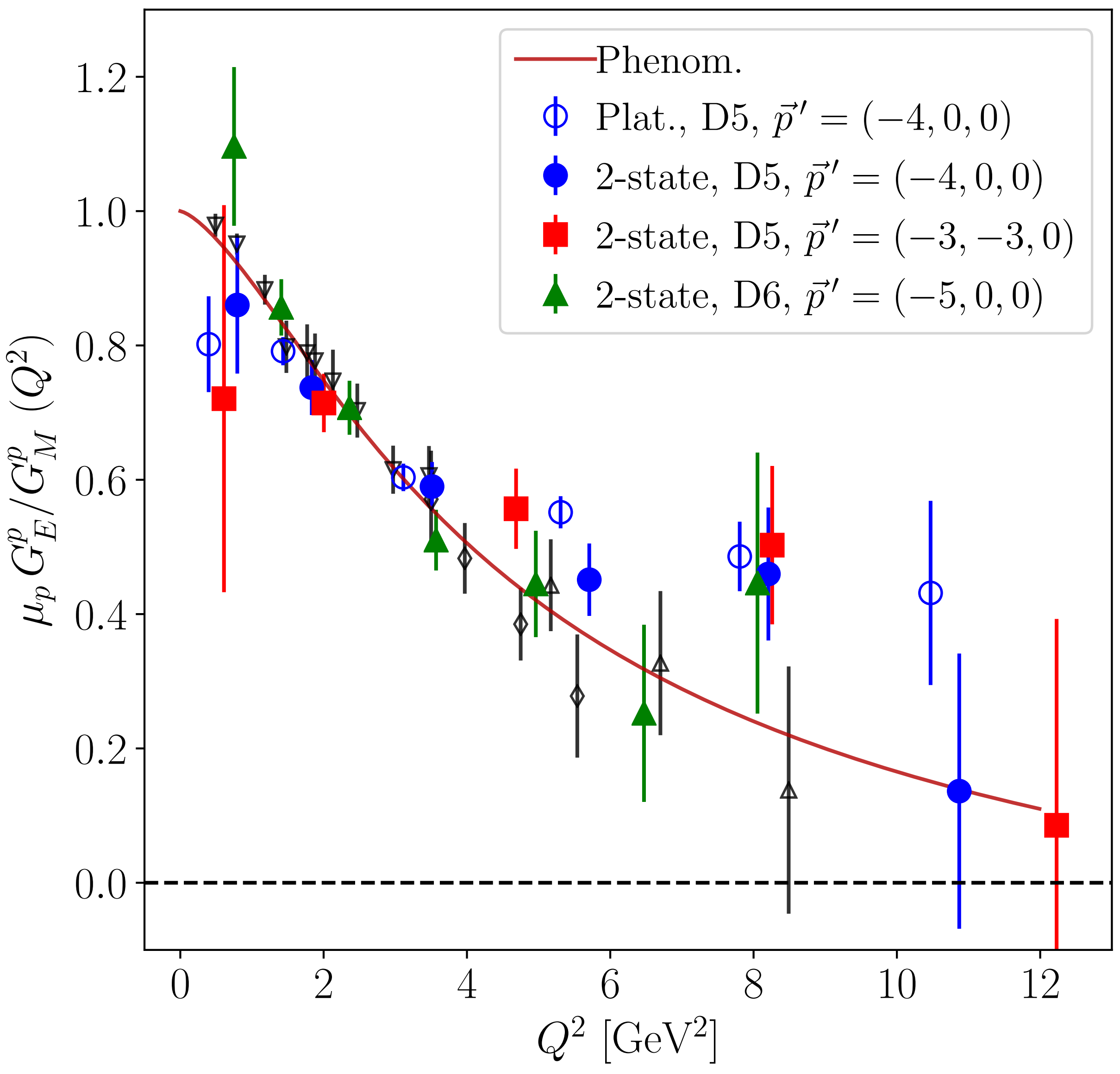


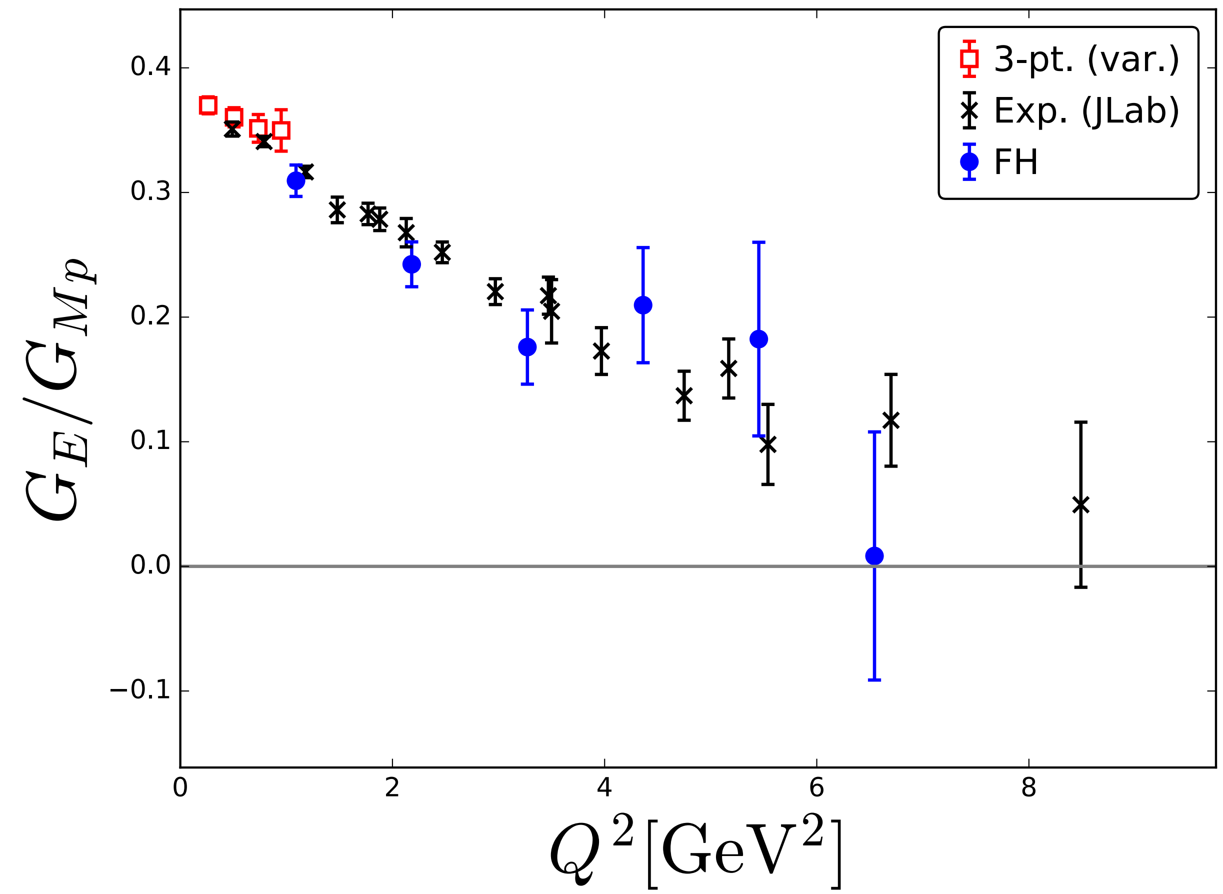
[G.D.Cates, et al, PRL106:252003 (2011)]

G_{Ep}/G_{Mp} Form Factor Ratio, Proton

- No disconnected diagrams
- No discretization corrections

- Black points: experiments
- Phenomenology curves : [Alberico et al, PRC79:065204 (2008)]
- Combined D5, D6 with 2-state fits analysis





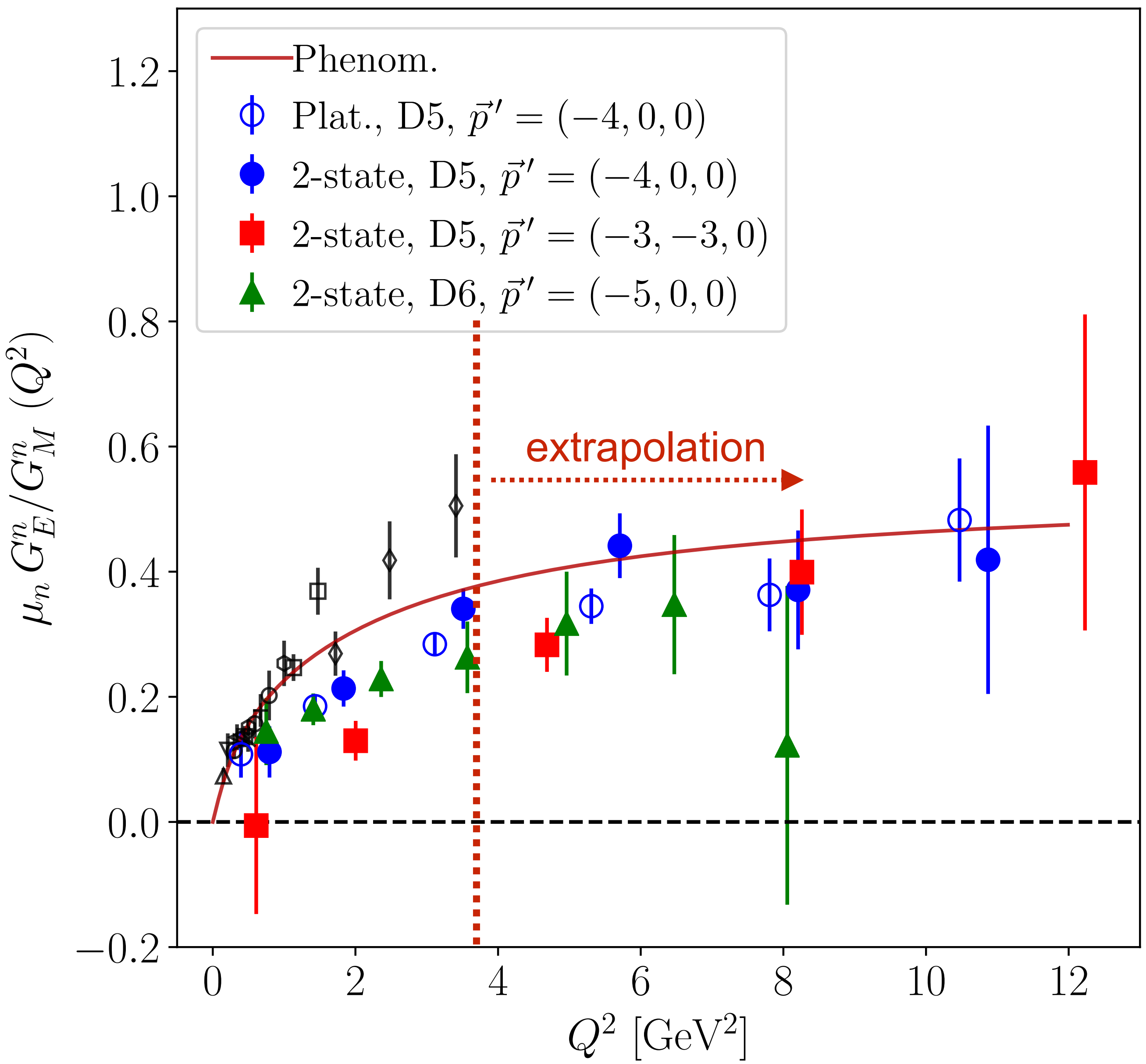
Earler calculation: Feynman-Hellman method (a=0.074 fm, mπ=470MeV)

[Chambers et al (CSSM), PRD96: 114509]

G_{Ep}/G_{Mp} Form Factor Ratio, Neutron

- No disconnected diagrams
- No discretization corrections

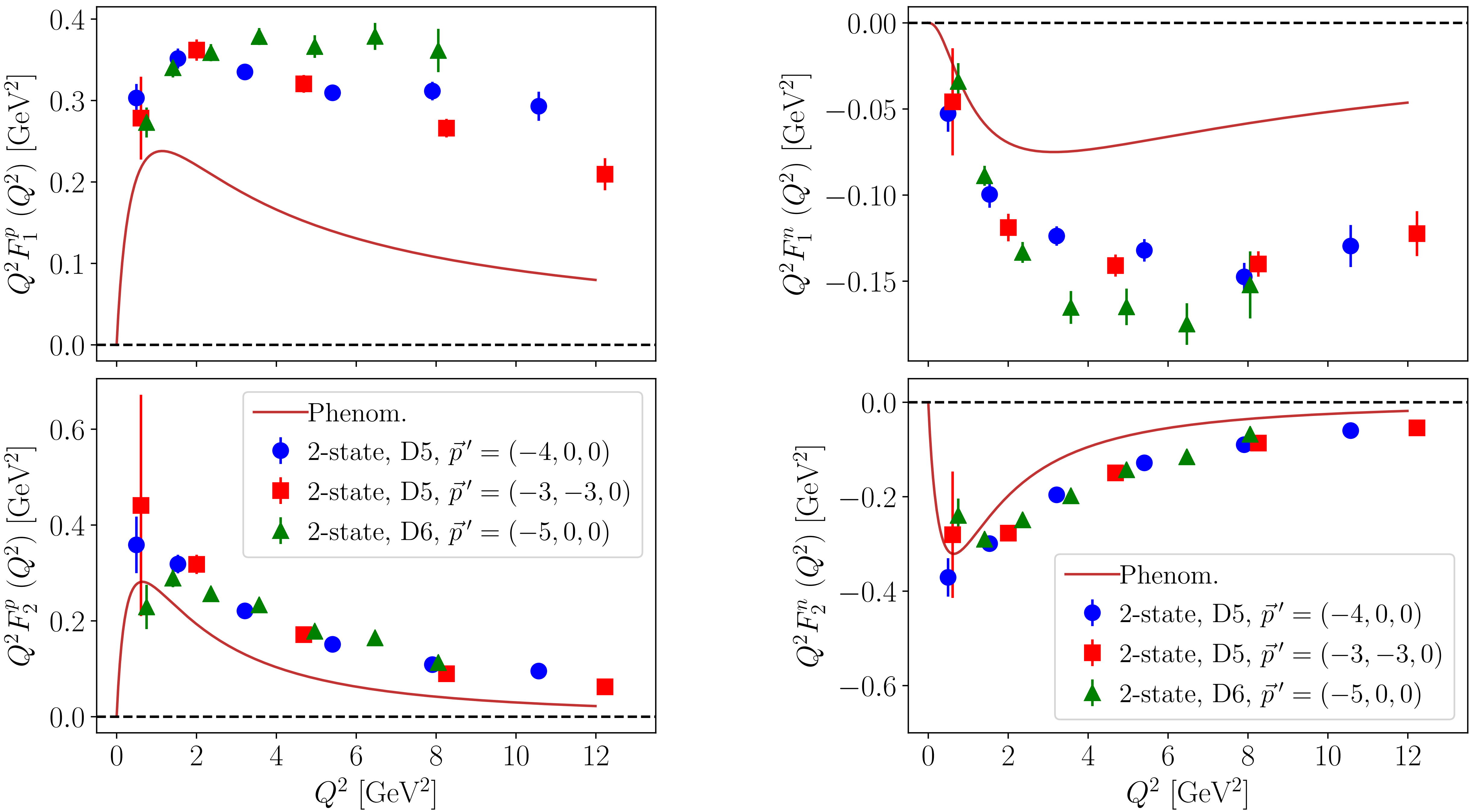
- Black points: experiments
- Phenomenology curves : [Alberico et al, PRC79:065204 (2008)]
- Combined D5, D6 with 2-state fits analysis



Nucleon Form Factors

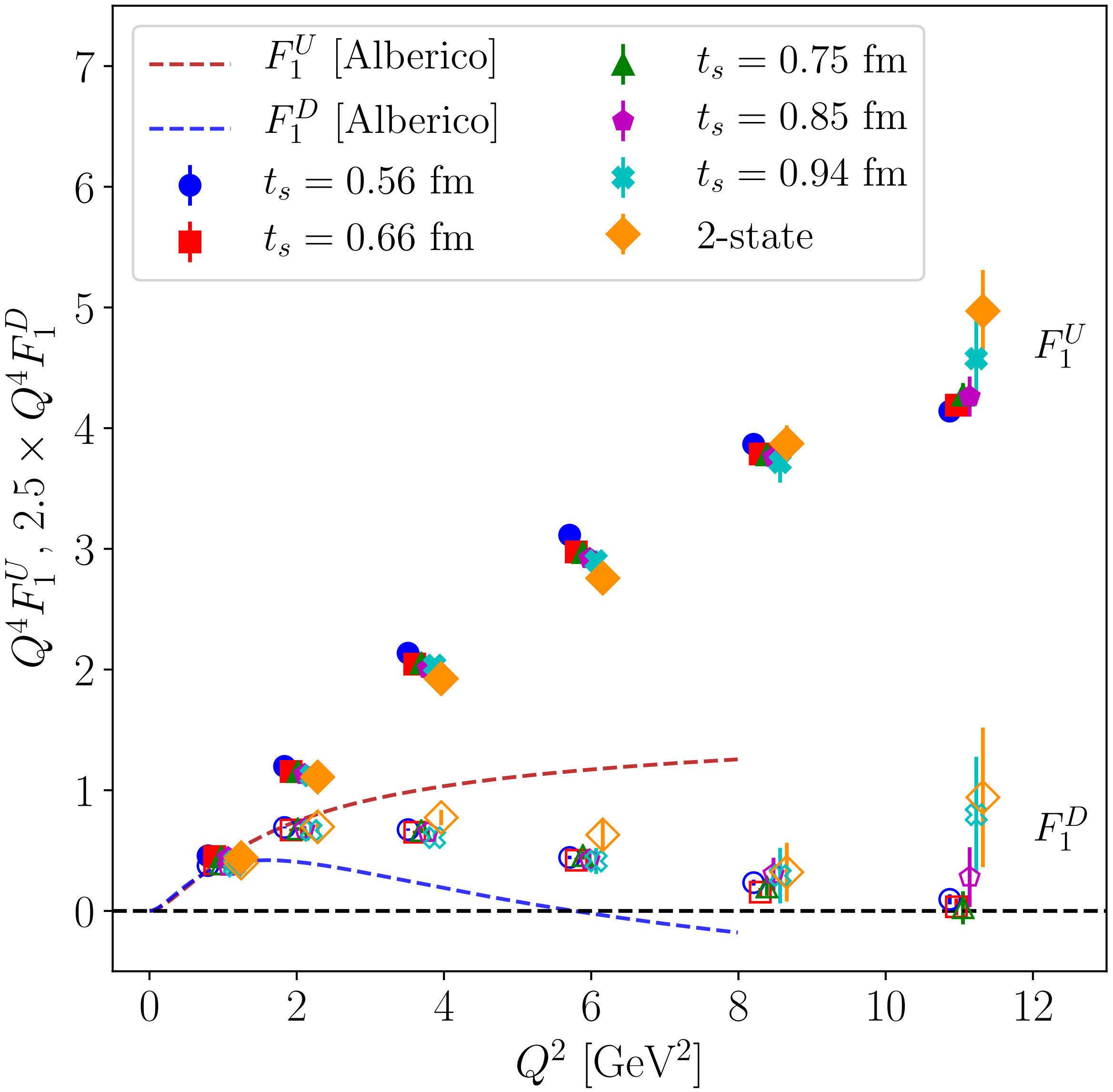
- No disconnected diagrams
- No discretization corrections

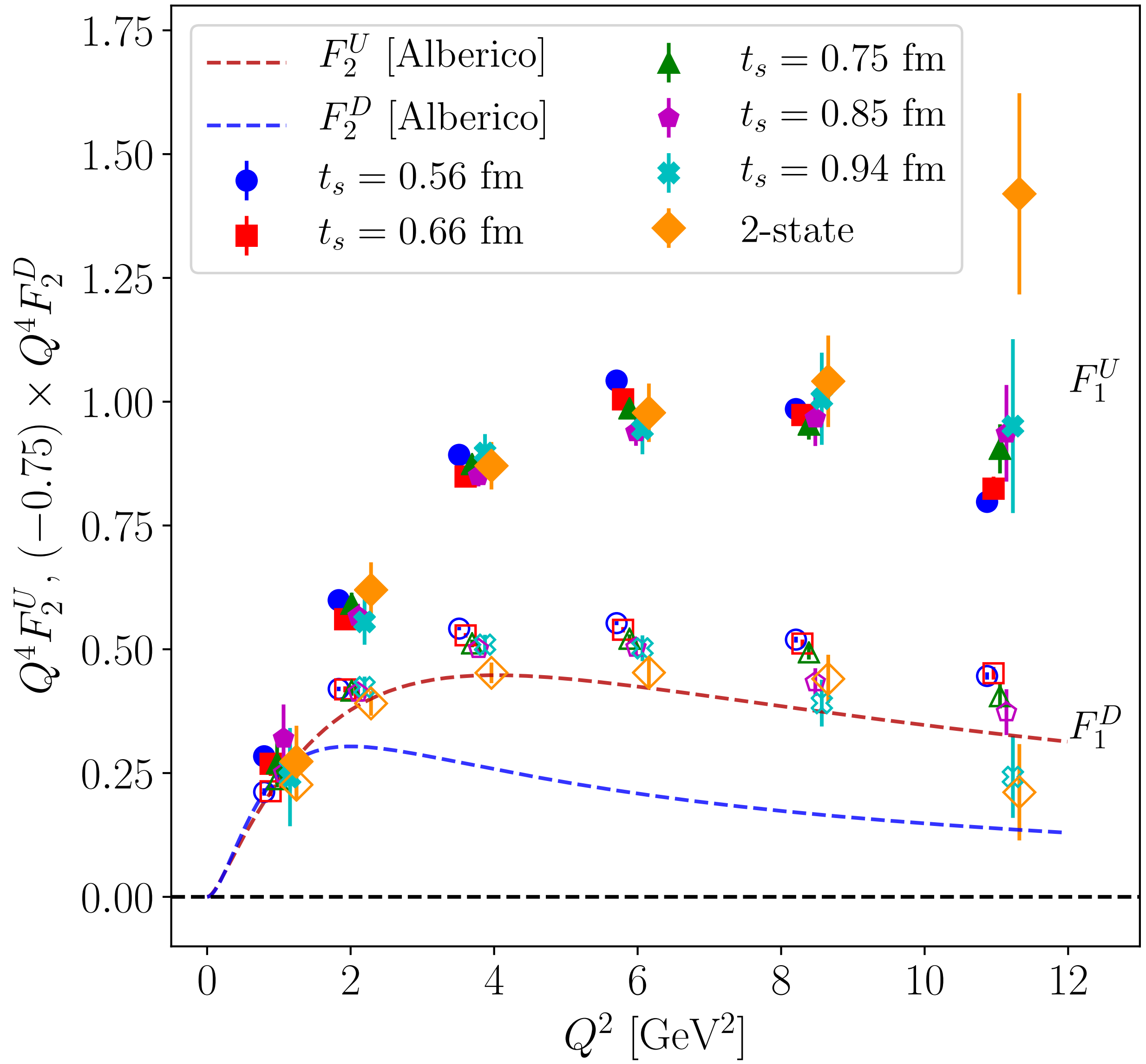
- Phenomenology curves : [Alberico et al, PRC79:065204 (2008)]



Light Flavor Contributions

- No disconnected diagrams
- No discretization corrections





Light Flavor Contributions

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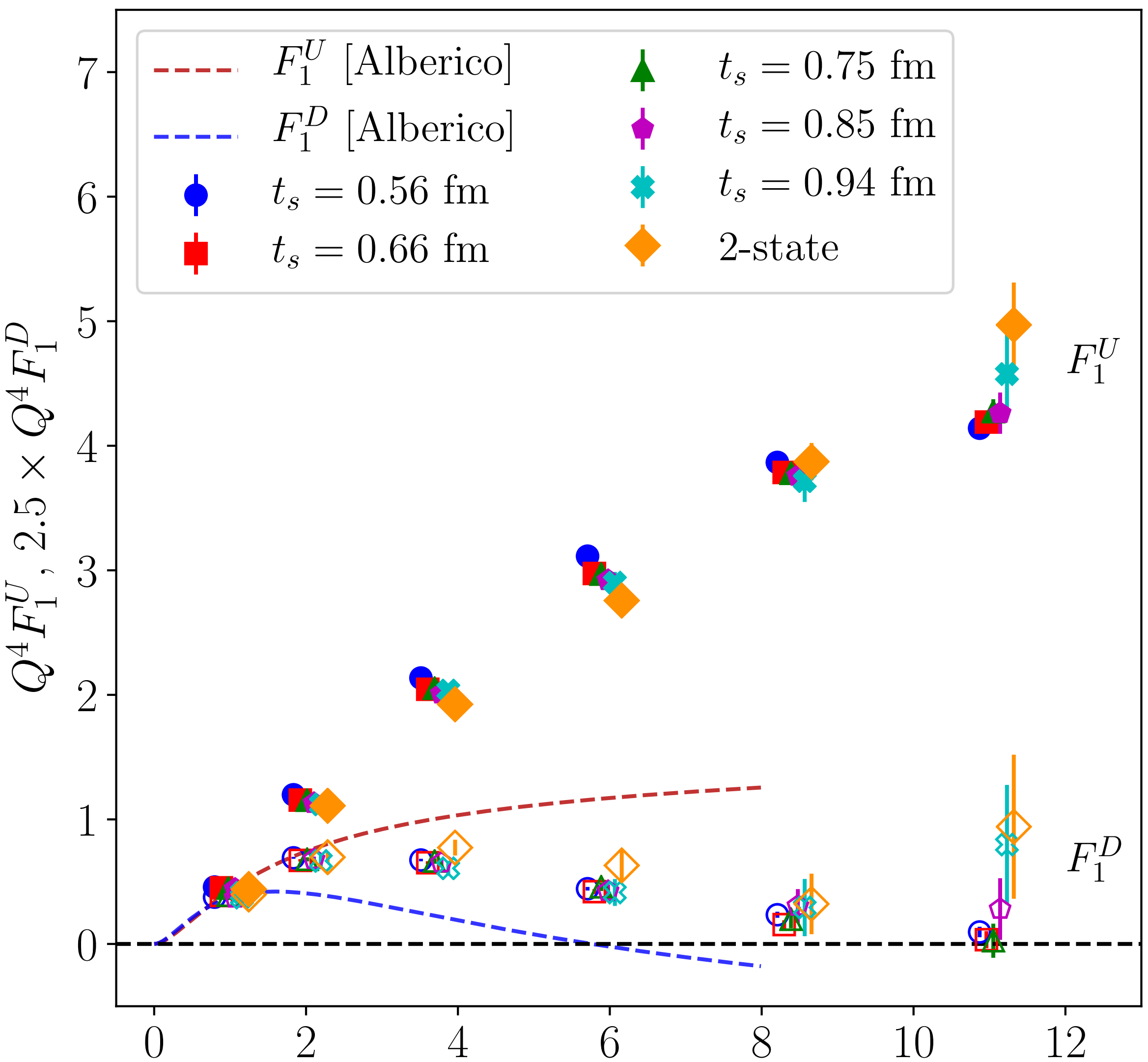
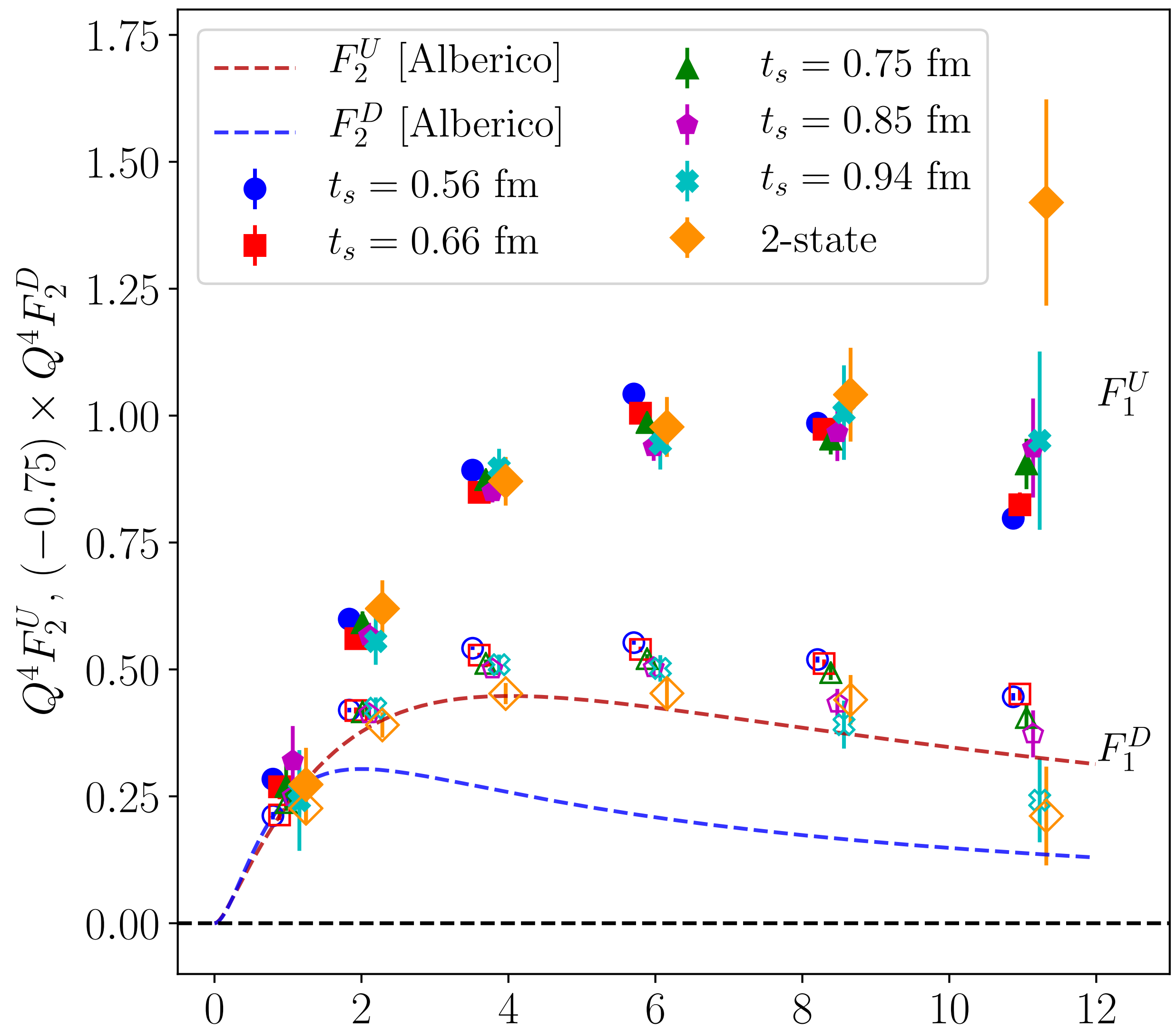
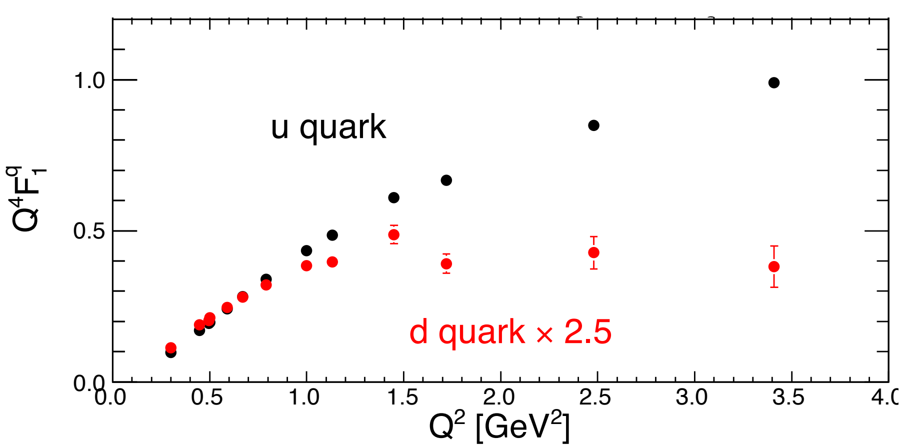
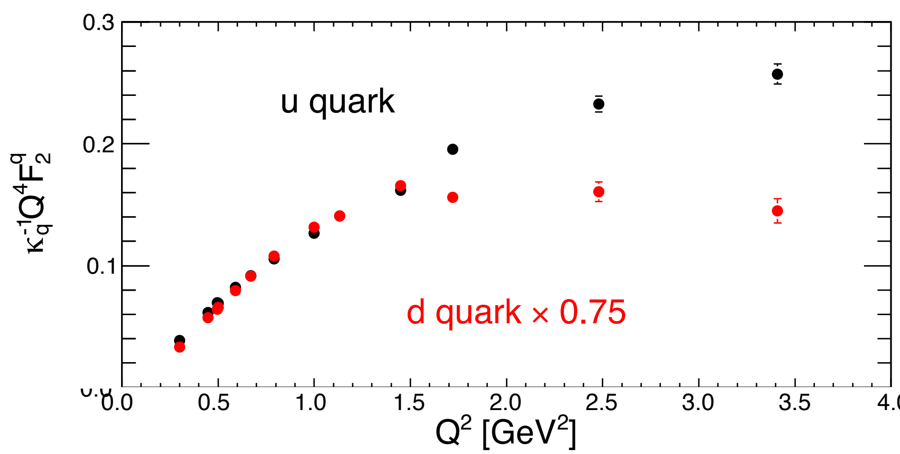








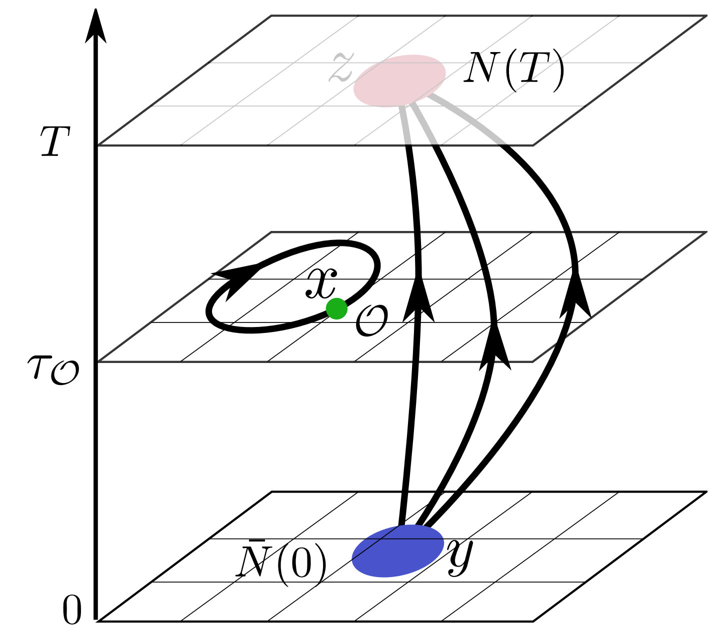
- Reproduce qual.features of flavor dependence [G.D.Cates, et al, PRL106:252003(2011)]
- Larger form factors: nucleon (+ exc.states?) on a lattice is more "compact"

Disconnected Quark Loops

- Stochastic evaluation: $\begin{cases} \xi(x) = \text{ random } Z_2\text{-vector} \\ E\left[\xi^\dagger(x)\xi(y)\right] = \delta_{x,y} \end{cases}$

$$\sum_x e^{iqx} \not{D}^{-1}(x,x) \approx \frac{1}{N_{MC}} \sum_i^{N_{MC}} \xi^\dagger_{(i)} \left(e^{iqx} \not{D}^{-1} \xi_{(i)}\right)$$

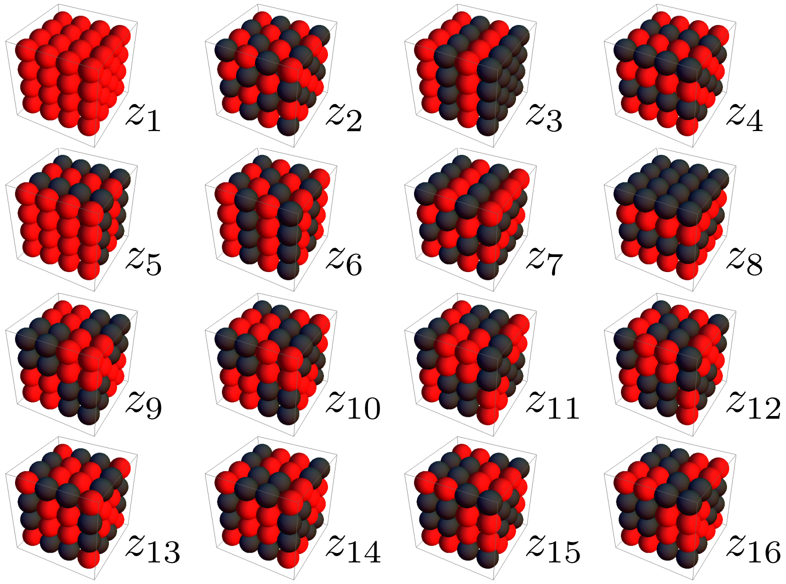
$$\mathrm{Var}\left(\sum_x \not{D}^{-1}(x,x)\right) \sim \frac{1}{N_{MC}}$$ (contributions from $\not{D}^{-1}(x \neq y)$)



- Exploit $\not{D}^{-1}(x,y)$ FALLOFF to reduce $\sum_{x\neq y} |\not{D}^{-1}(x,y)|^2$:

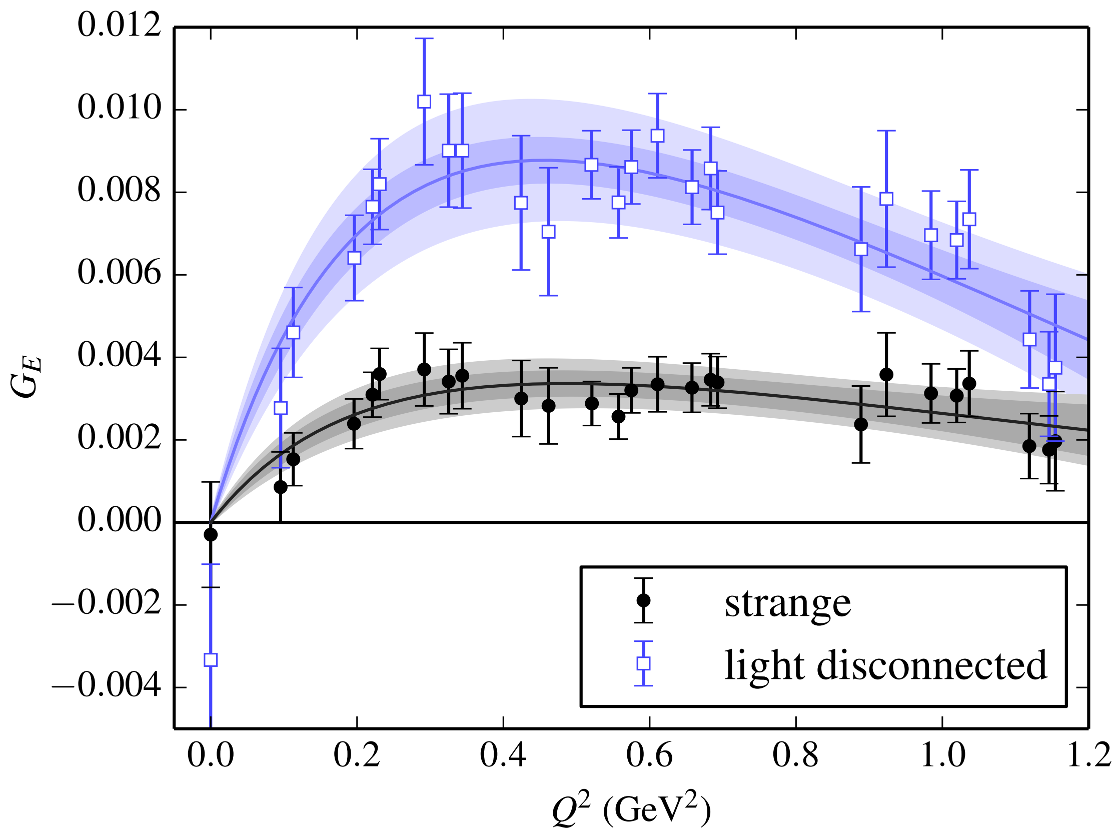
Hierarchical probing method [K.Orginos, A.Stathopoulos, '13] :
In sum over $N=2^{nd+1}$ 3D(4D) Hadamard vectors,
near-(x,y) terms cancel:

$$\frac{1}{N}\sum_i z_i(x) z_i(y)^\dagger = \begin{cases} 0, & 1 \leq |x-y| \leq 2^k, \\ 1, & x = y \text{ or } 2^k < |x-y| \end{cases}$$



- Further decrease variance by deflating low-lying, long-range modes [A.Gambhir's PhD thesis]

Disconnected & Strange Quark Contractions

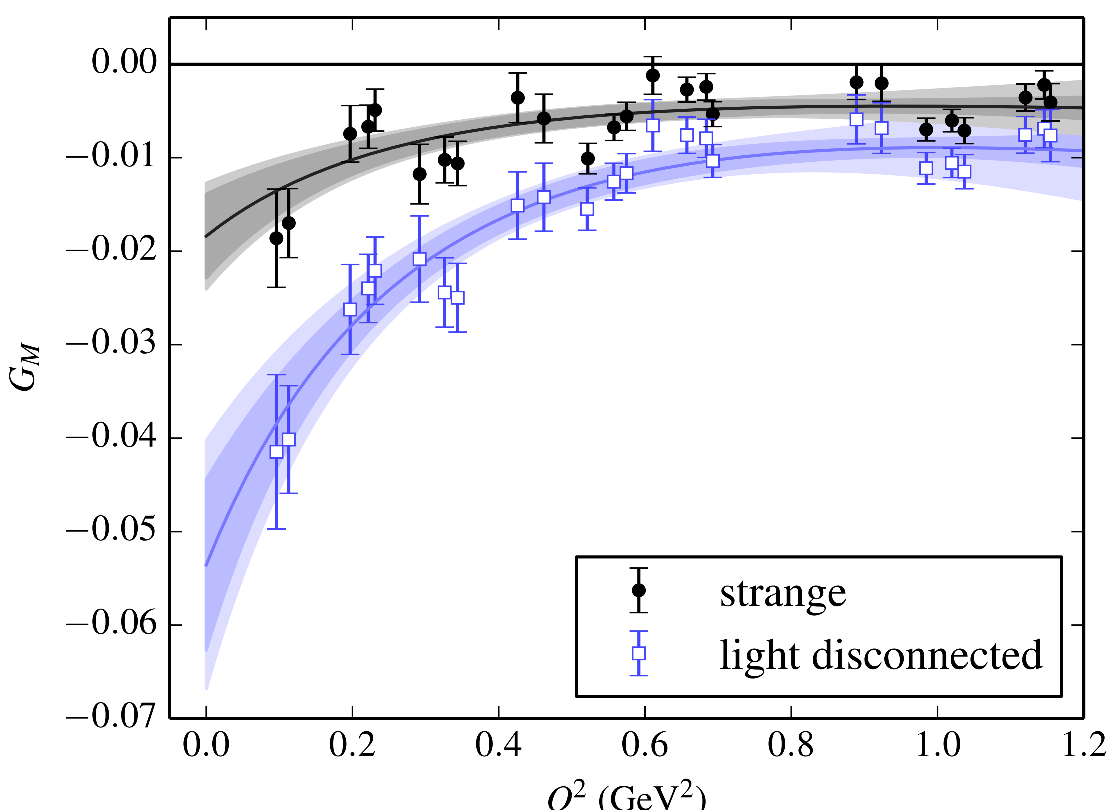


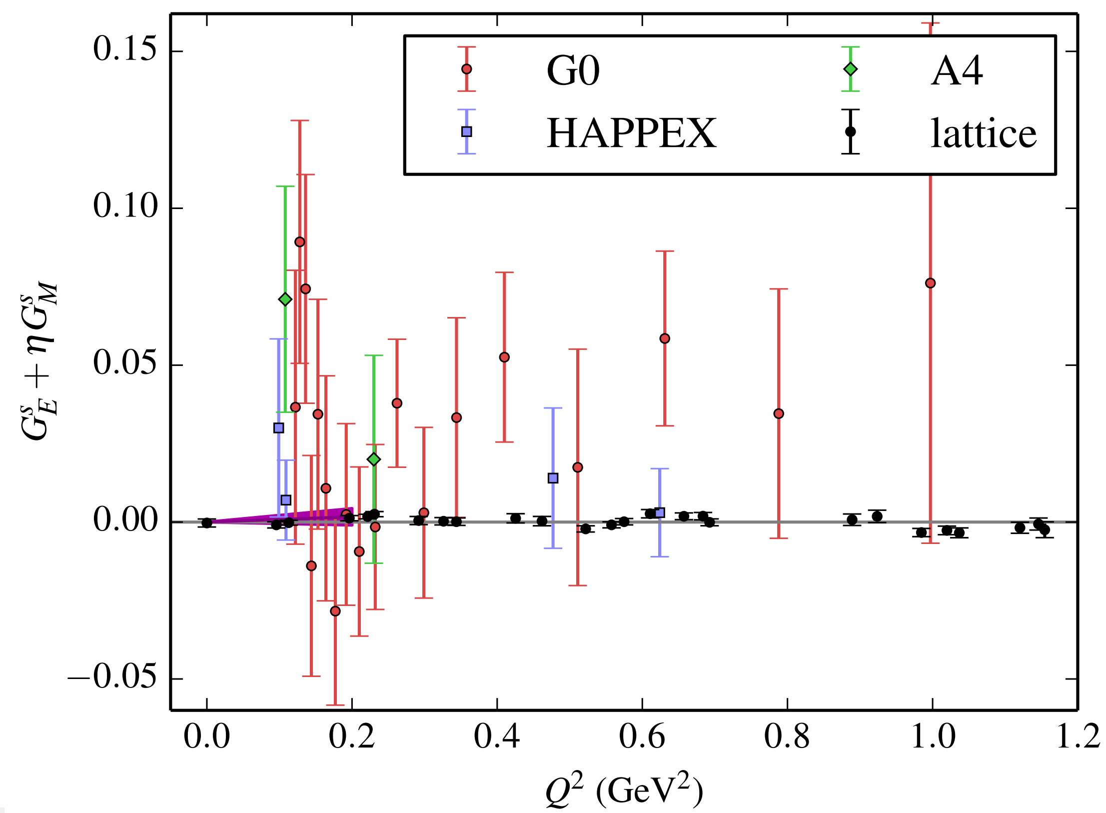
[J. Green, S. Meinel, et al; PRD92:031501 (2015)]

N_f=2+1 dynamical fermions, $m_\pi \approx$ 320 MeV
(the "coarse" JLab Clover ensemble)

$$|(G_E^{u/d})_{\rm disc}| \lesssim 0.010 \text{ of } |(G_E^{u-d})_{\rm conn}|$$
$$|(G_E^{s})_{\rm disc}| \lesssim 0.005 \text{ of } |(G_E^{u-d})_{\rm conn}|$$

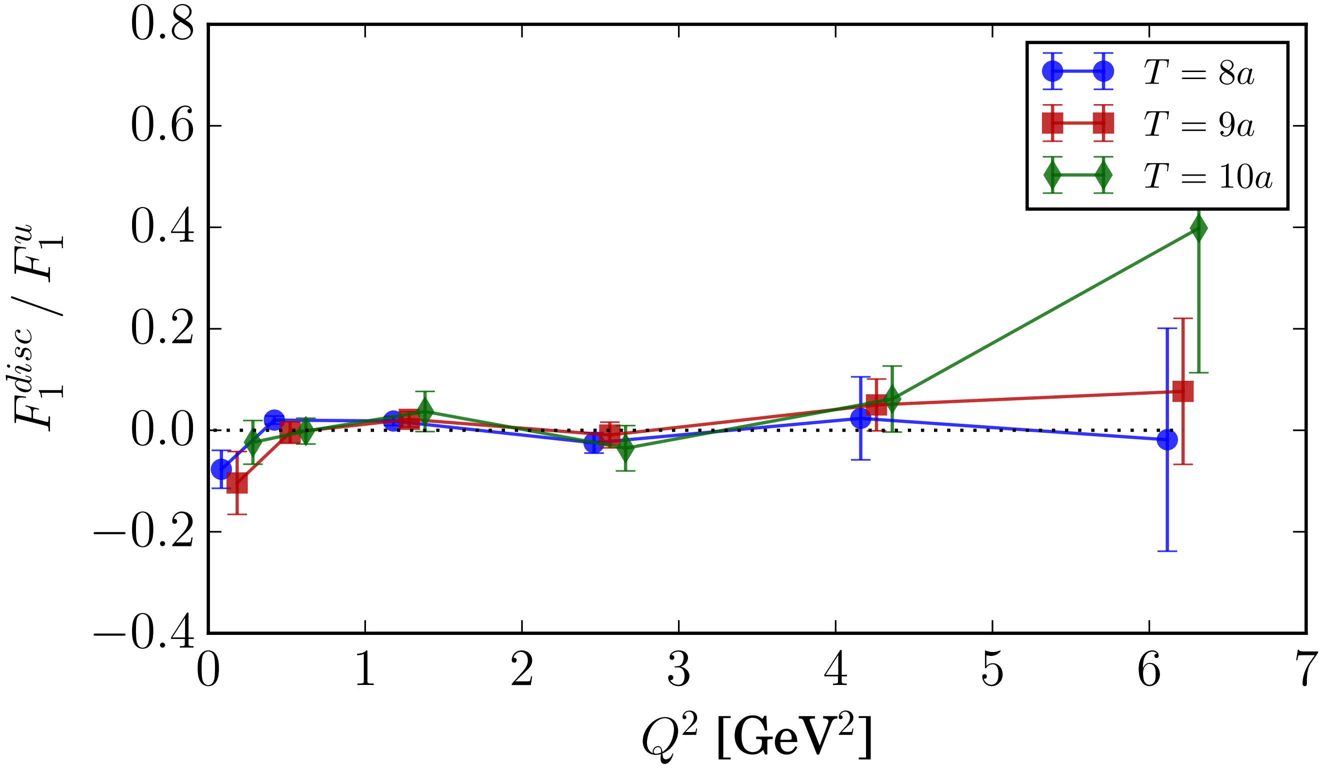
$$|(G_M^{u/d})_{\rm disc}| \lesssim 0.015 \text{ of } |(G_M^{u-d})_{\rm conn}|$$
$$|(G_M^{s})_{\rm disc}| \lesssim 0.005 \text{ of } |(G_M^{u-d})_{\rm conn}|$$

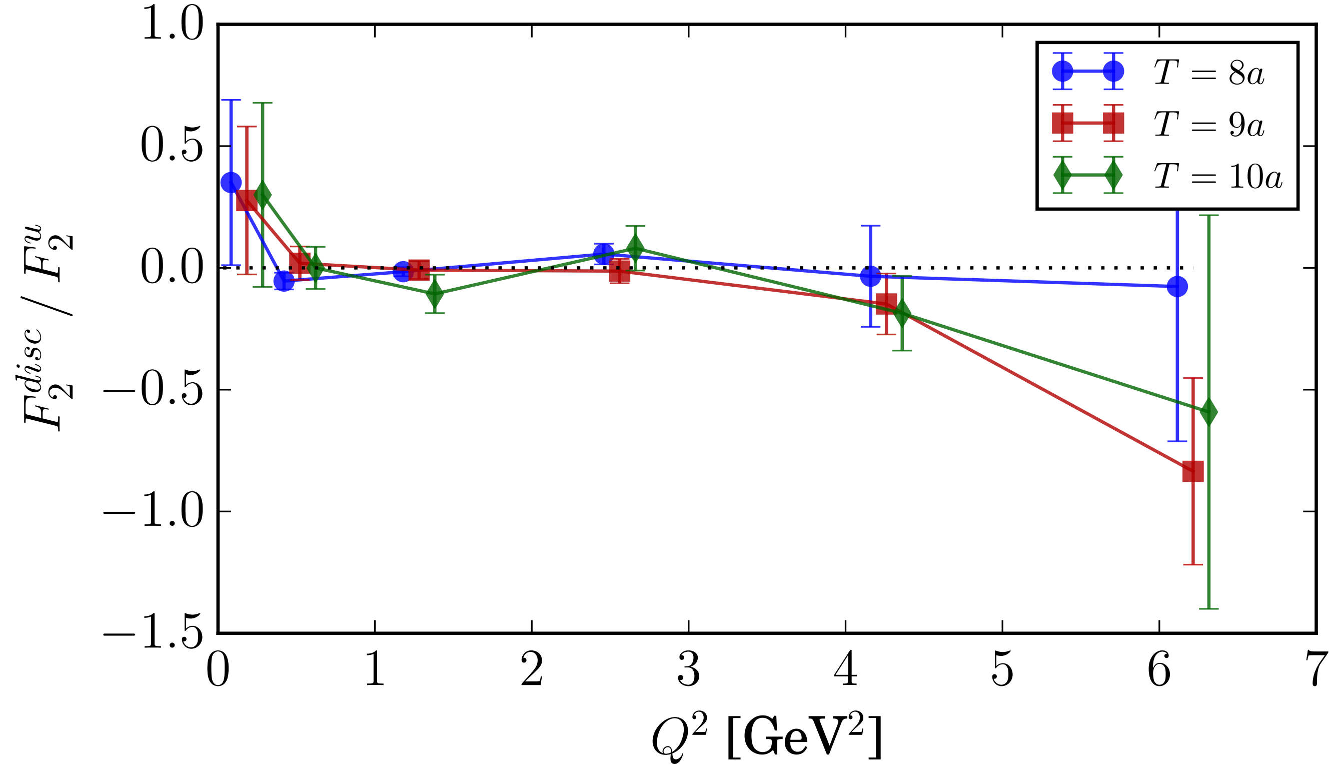




Disconnected vs. Connected : Large Q^2

Ratio of disconnected to connected(U) contributions





- D5 ensemble(mπ=280 MeV, a=0.094 fm)
- Indication for smallness of the strange form factor (up to $Q^2 \lesssim 4$ GeV^2)

$$|F^s_{1,2}| \overset{?}{\lesssim} |(F^{u/d}_{1,2})_{\rm disc}| \lesssim 0.1 |F^{u,d}_{1,2}| \quad (Q^2 \lesssim 4\ \mathrm{GeV}^2)$$

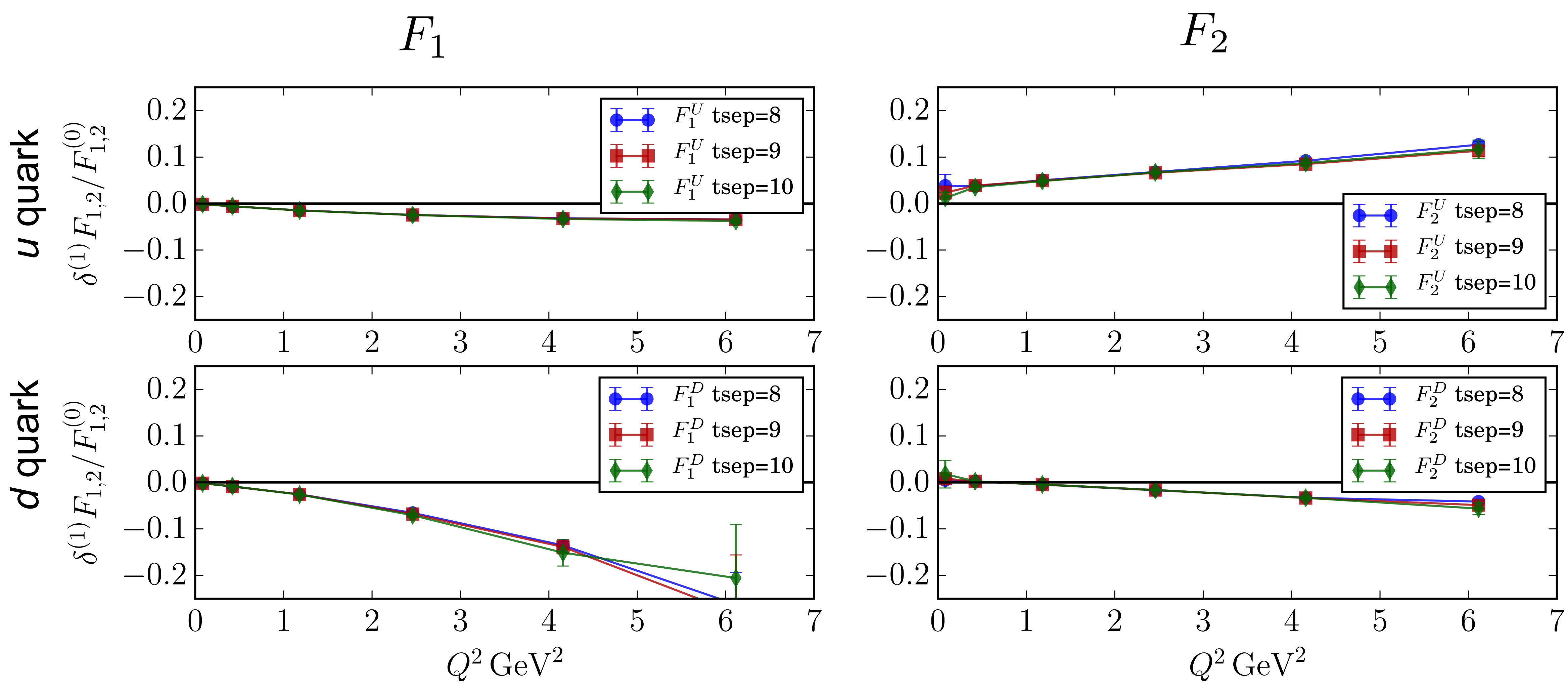
(study with increased statistics underway)

O(a) Vector Current Correction

- *No disconnected diagrams*

Improved vector current $(V_\mu)_I = \bar{q}\gamma_\mu q + c_V\, a\partial_\nu \bar{q} i\sigma_{\mu\nu} q$

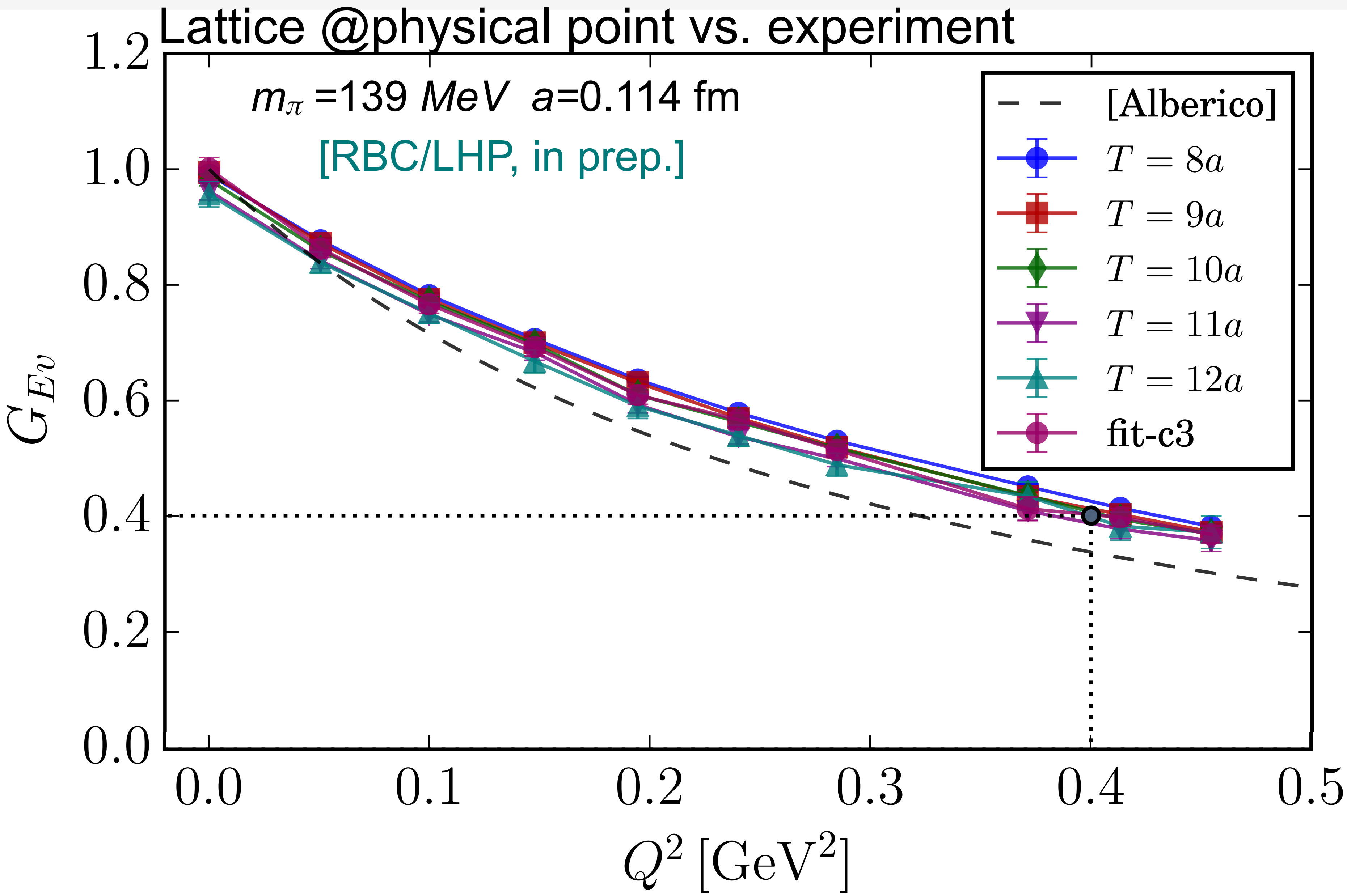
O(a^1) correction : form factors of $a\,\langle N|\partial_\nu(\bar{q} i\sigma^{\mu\nu} q)|N\rangle$



Relative magnitude of *O(a^1)* effects : {*O(a^1)* } / {*O(a^0)* } form factors
(assuming cV=0.05)

- improvement coefficient c_V : must be computed on lattice from WI
- perturbation theory: cV ≈ –0.01$C_F(g_0)^2$

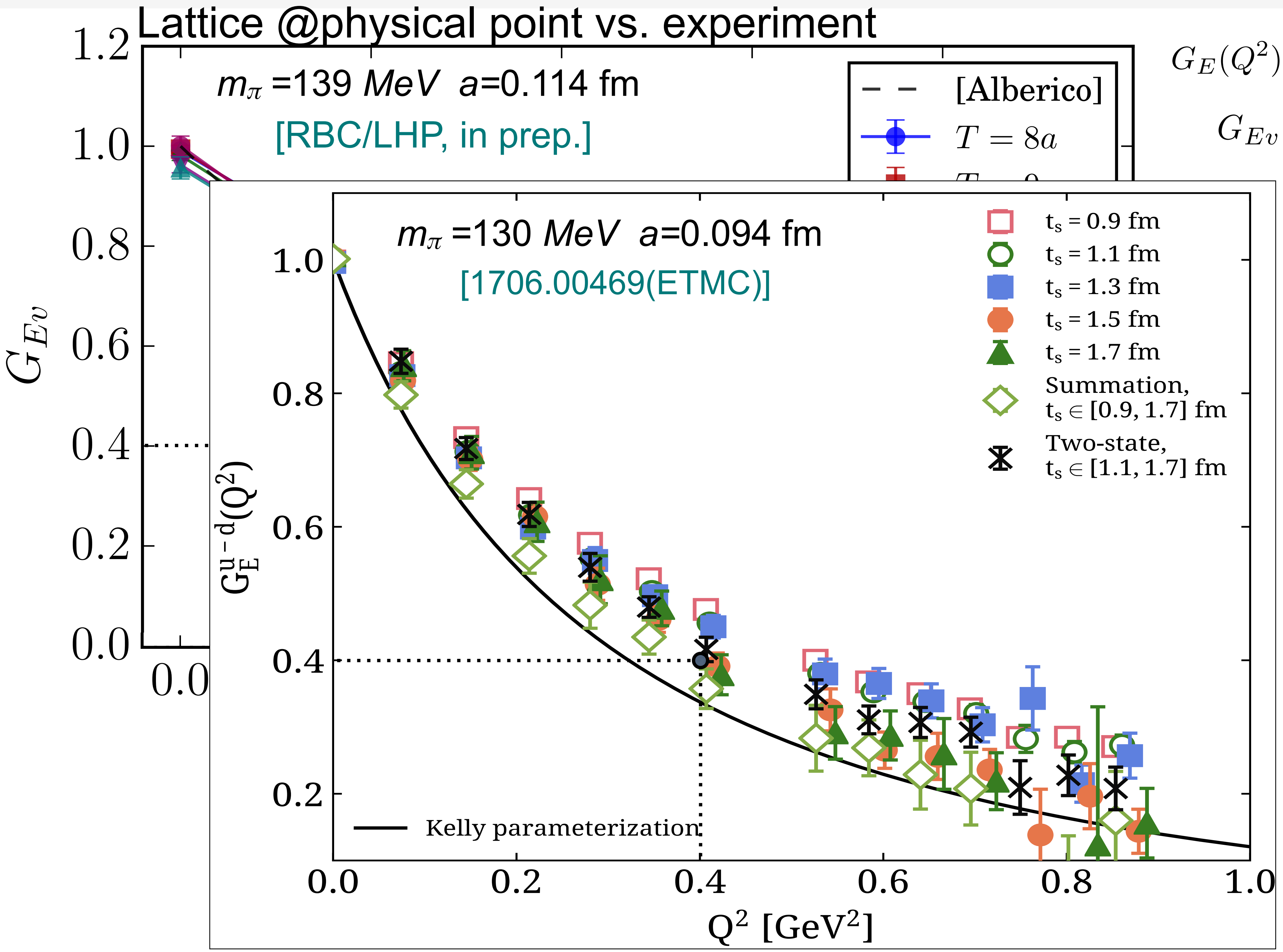
(u-d) Electric Form Factor for $Q^2 \lesssim 1$ GeV^2



$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M_N^2} F_2(Q^2)$$

$$G_{Ev} = G_{Ep} - G_{En}$$

(u-d) Electric Form Factor for $Q^2 \lesssim 1$ GeV^2



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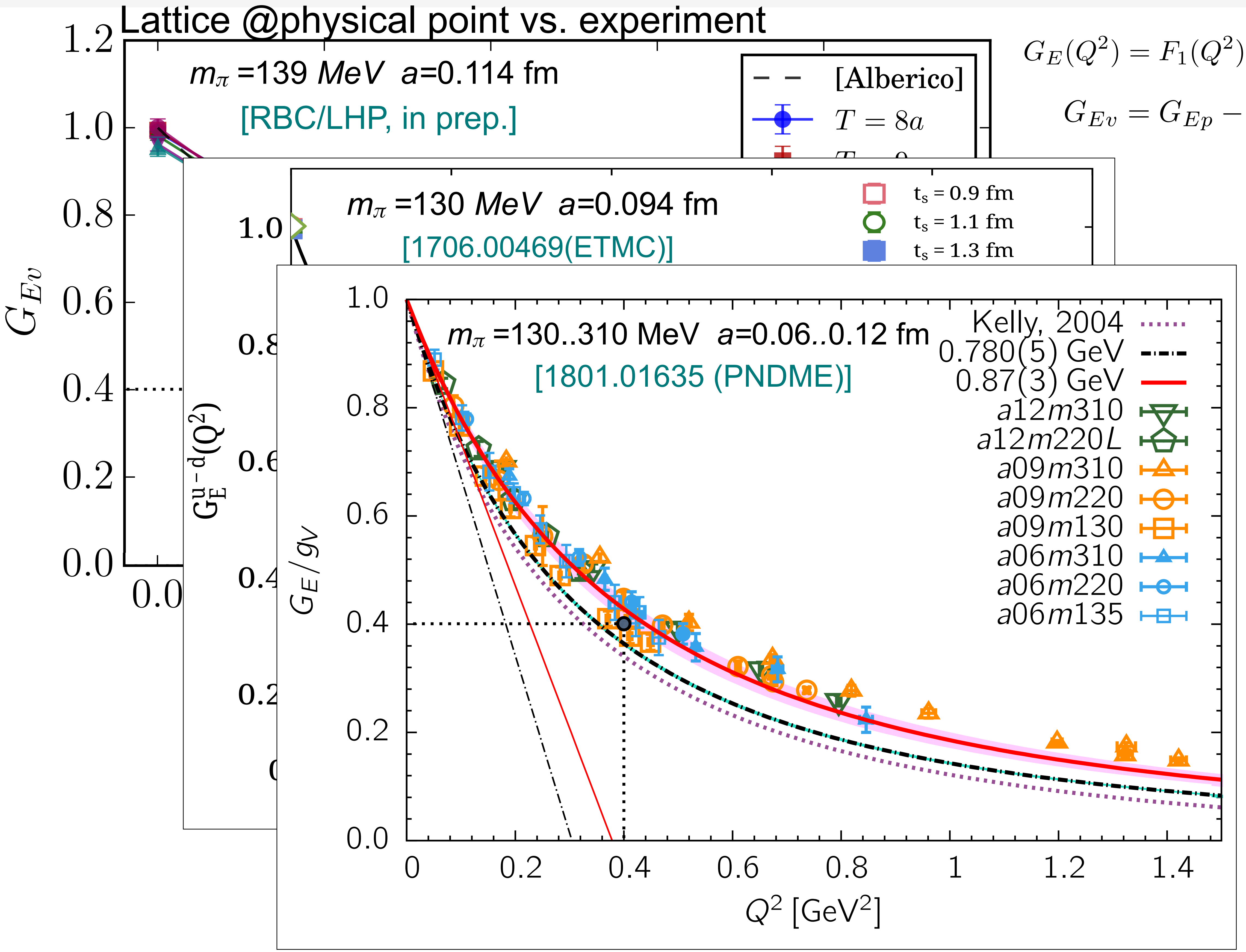
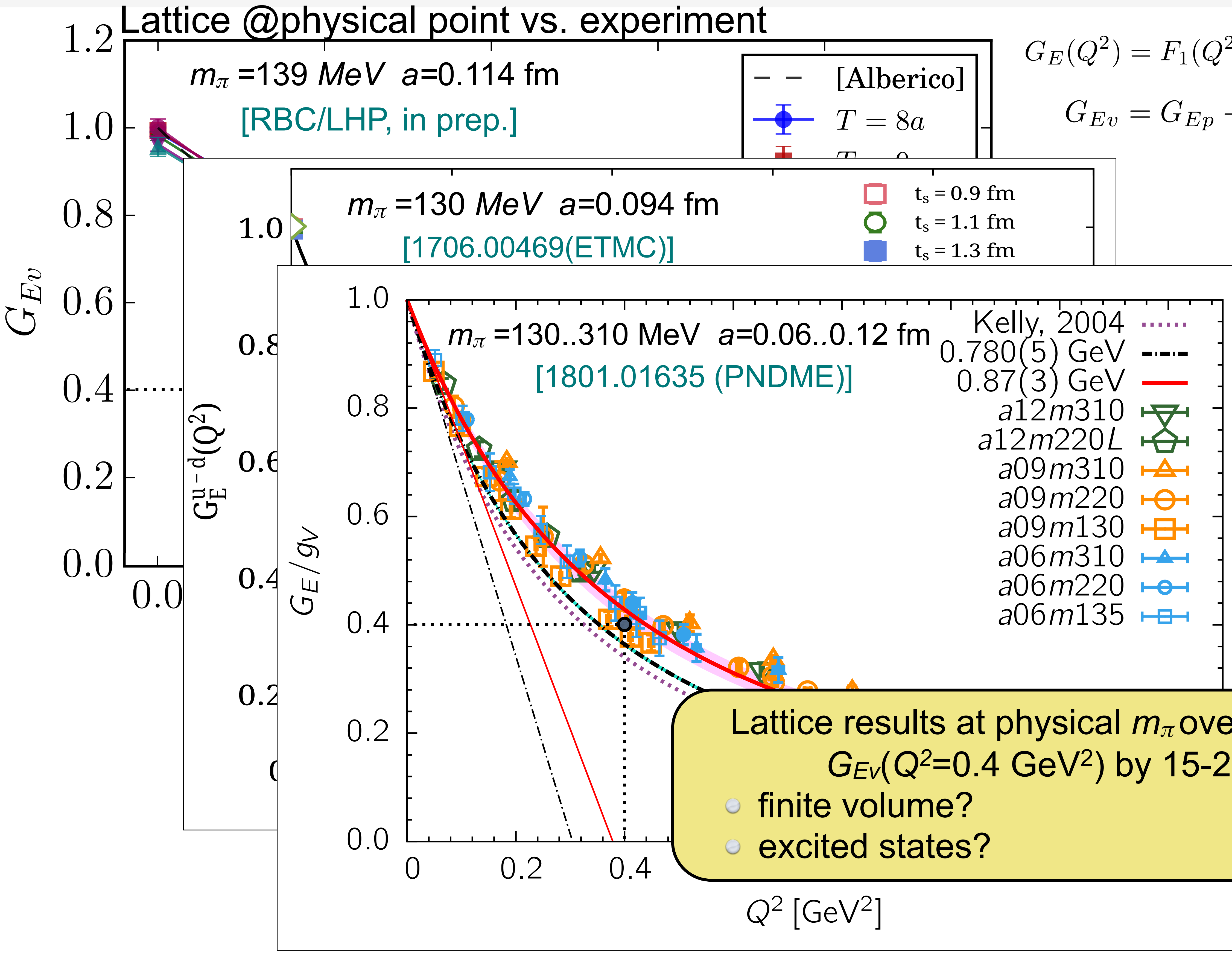


(u-d) Electric Form Factor for $Q^2 \lesssim 1$ GeV^2



$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M_N^2} F_2(Q^2)$$

$$G_{Ev} = G_{Ep} - G_{En}$$

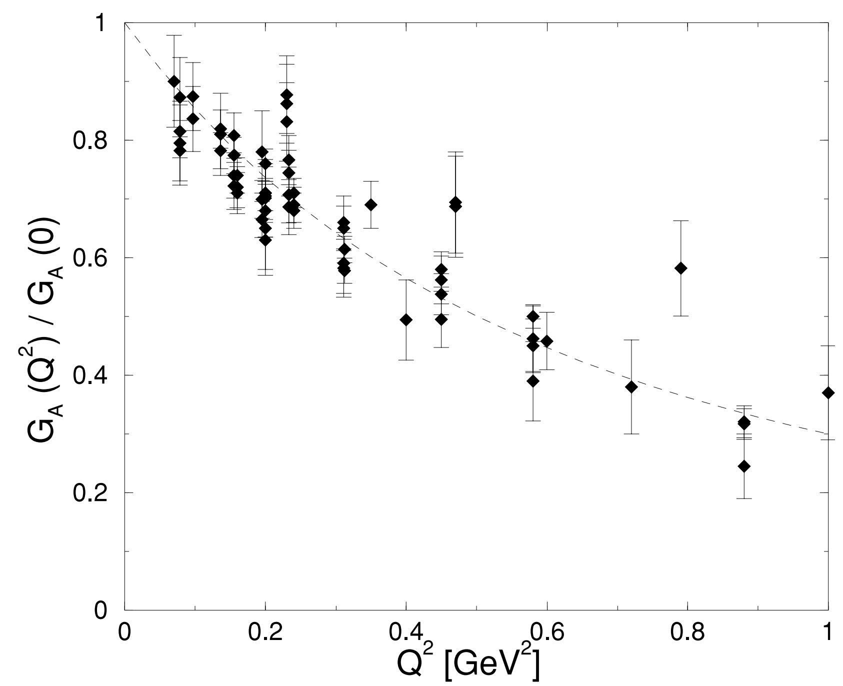
Lattice results at physical m_π overestimate $G_{Ev}(Q^2=0.4\text{ GeV}^2)$ by 15-20%

- finite volume?
- excited states?

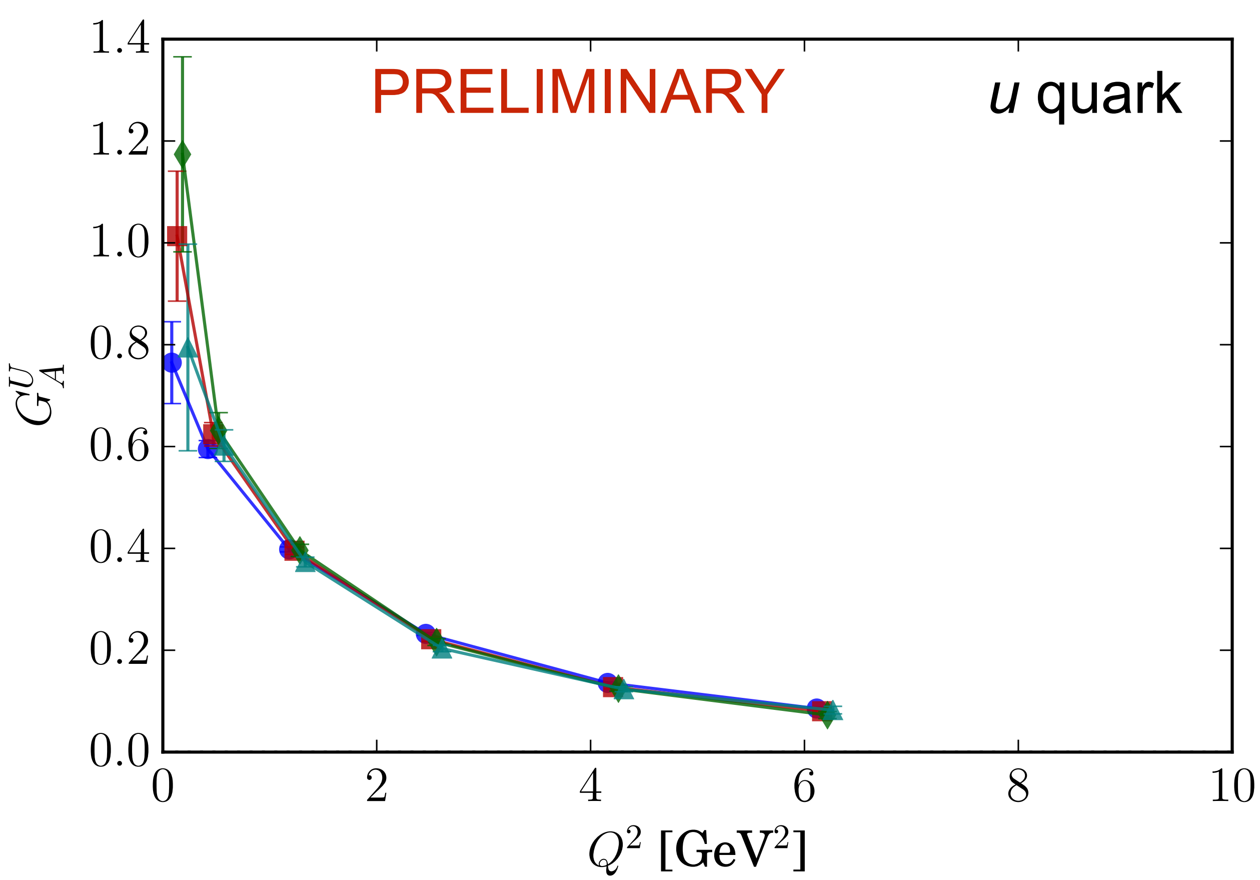
Axial Form Factors

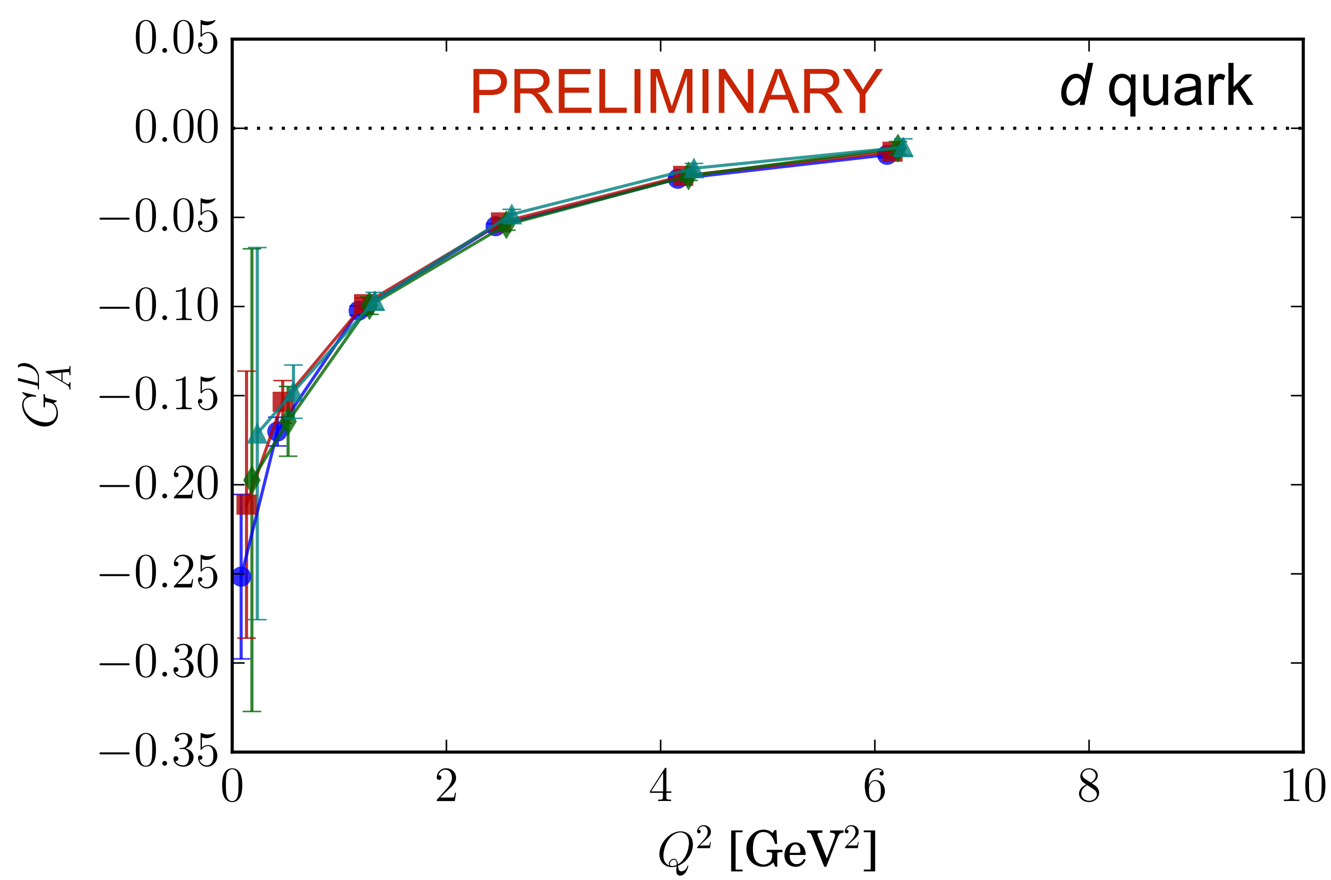
- *No disconnected diagrams*
- *No discretization corrections*

- $G_A(Q^2)$ are measured in ν-scattering, π-production;
  implications for neutrino flux norm. (e.g. in IceCube)
- Large uncertainty in $G_A(Q^2)$ from model dependence
  [B.Bhattacharya,R.Hill,G.Paz, PRD84:073006(2011)]
- Will be accessible in LBNF [see e.g. R.Petti's talk, POETIC'19]
  clean probe (only EW interaction)
  flavor separation in CC interactions



[V.Bernard et at, J.Phys.G28:R1(2002)]





Summary

- Initial results for high-momentum form factors up to $Q^2 \lesssim 10$ GeV^2
  (No quark-disconnected contributions)

- Form factor results overshoot experiment x(2 ... 2.5)
  Discretization? Non-physical quark masses? Excited states?
  ⇒ Will require ≳ $O(10^6)$ statistics

- G_{Ep}/G_{Mp} , F_2/F_1 , G_{En}/G_{Mn} are in qualitative agreement with experiment
  Universality between ground & excited nucleon states?

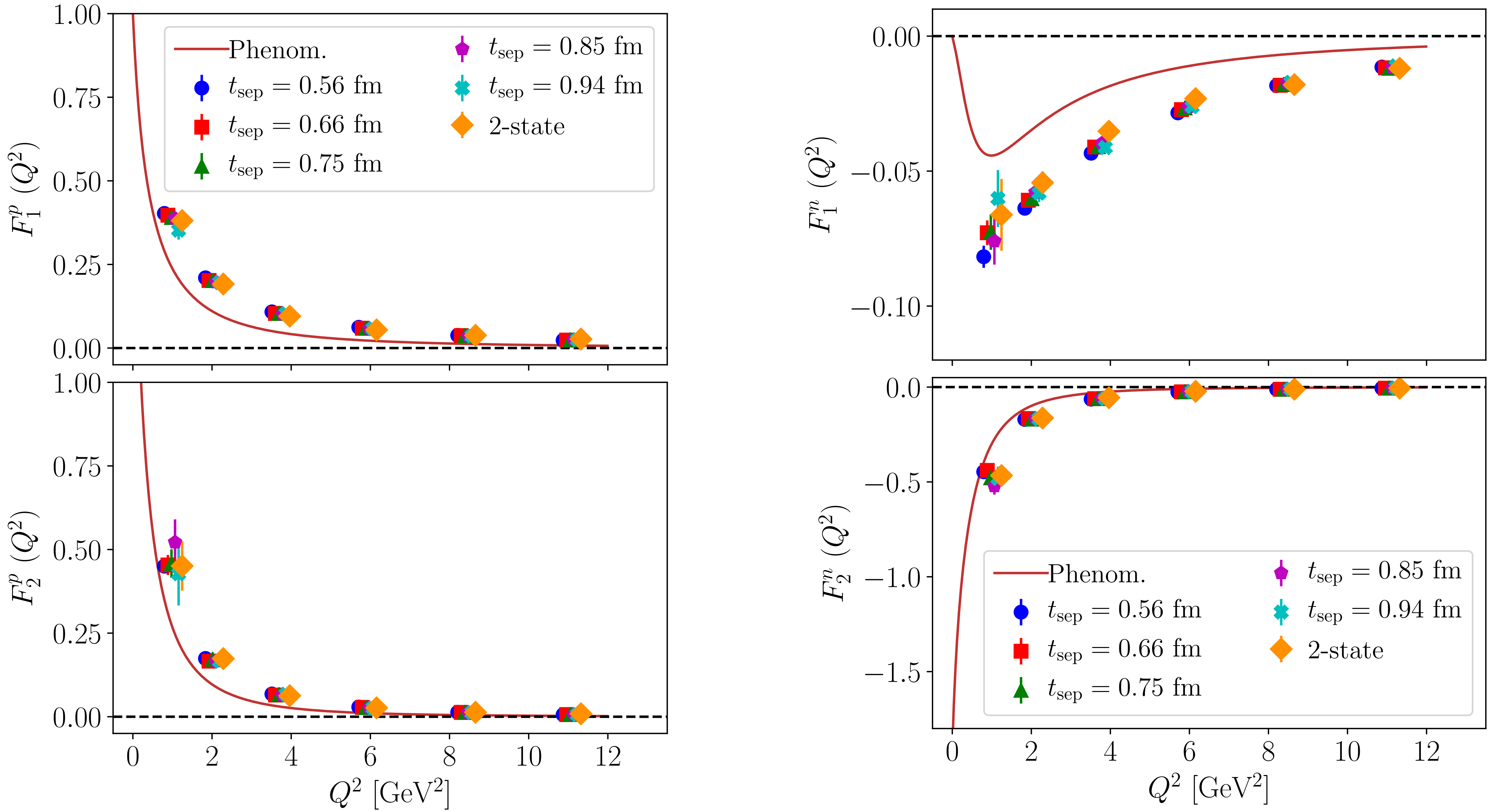
- O(a) discretization effects grow with Q^2
  Finer lattices + Non-perturbative vector current improvement needed

- Access to flavor-dependent contributions to nucleon <u>vector</u> and <u>axial-vector</u> form factors

- Comparison to experiment will (eventually) validate lattice methods for computing relativistic nucleon matrix elements
  Impact on lattice methodology for calculations of TMDs, PDFs, ...

BACKUP

Title



Phenom.
$t_{sep} = 0.56$ fm
$t_{sep} = 0.66$ fm
$t_{sep} = 0.75$ fm
$t_{sep} = 0.85$ fm
$t_{sep} = 0.94$ fm
2-state
$F_1^p(Q^2)$
$F_2^p(Q^2)$
$F_1^n(Q^2)$
$F_2^n(Q^2)$
Q^2 [GeV^2]