The Fermion-Scalar system and the Bethe-Salpeter equation in Minkowski space

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In Collaboration with

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and some results....

- J. H. Nogueira et al, PRD 100, 016021 (2019) *Solving the Bethe-Salpeter Equation in Minkowski Space for a Fermion-Scalar system*
- W. de Paula et al EPJ C 77, 764 (2017) : *Light-cone singularities and structure* PRD 94, 071901(R) (2016): *Two-fermion bound systems*
- C. Gutierrez et al PLB 759, 131 (2016): *Spectra of excited states and LF momentum distributions*
- FVS EPJC 75, 398 (2015): *Scattering lengths for two scalars*
- FVS PRD 89, 016010 (2014): *Bound states and LF momentum distributions for two scalars*
- FVS PRD 85, 036009 (2012): *General formalism for bound and scattering states*

Outline

What we would like to learn and how

A fully covariant and non perturbative description of a bound system, with spin dof, useful for playing the phenomenology of the spin-$k_\perp$ correlations (Hadron tomography) .

Minkowski space and the QFT framework are necessary

Initial strategy:

i) to train and educate our physical intuition through *simple* applications of the Bethe-Salpeter equation (BSE) in Minkowski momentum-space, e.g.: fermion-scalar (new) and fermion-fermion (old) systems in ladder approximation ($\Rightarrow$ analytic behavior of the BS amplitude, interaction kernels, etc.)
ii) to extend, both phenomenologically and formally, the framework through the inclusion of gap-equations for the needed self-energy contributions ($\Rightarrow$ consistently cutting the tower of DSE's, etc.).

How:
i) Pivotal role of the Nakanishi Integral Representation (NIR) of the BS amplitudes (3- and 4-legs Transition Amplitudes) and self-energies (2-legs TA)
ii) Light-front (LF) variables, $x^\pm = x^0 \pm x^3$ and $\mathbf{x}_\perp \equiv \{x^1, x^2\}$, very suitable for managing analytic integration and spin dof in a very effective way, in Minkowski space, when needed.

★ Adding effects in a controlled way, as in NIR+LF, has a great methodological appealing for non perturbative phenomenological studies in Minkowski space.

Outputs: Valence probabilities and LF distributions.

Nakanishi Integral Representation - I

Nakanishi proposal for a compact and elegant expression of the full N-leg amplitude, written by means of the Feynman parametrization ($\to \vec{\alpha}$), $f_N(s) = \sum_{\mathcal{G}} \; f_{\mathcal{G}}(s)$ ($\mathcal{G} \equiv$ infinite graphs contributing to f_N):

Introducing the identity

$$1 \doteq \prod_h \int_0^1 dz_h \delta\left(z_h - \frac{\eta_h}{\beta}\right) \int_0^\infty d\gamma \; \delta\left(\gamma - \sum_l \frac{\alpha_l m_l^2}{\beta}\right)$$

with $\beta = \sum \eta_i(\vec{\alpha})$ and integrating by parts $n - 2k - 1$ times, the contribution from a graph is

$$f_{\mathcal{G}}(\tilde{s}) \propto \; \prod_h \int_0^1 dz_h \int_0^\infty d\gamma \frac{\delta(1 - \sum_h z_h) \; \tilde{\phi}_{\mathcal{G}}(\vec{z}, \gamma)}{(\gamma - \sum_h z_h s_h)}$$

$\tilde{\phi}_{\mathcal{G}}(\vec{z}, \gamma) \equiv$ a proper weight function, with $\vec{z} \equiv \{z_1, z_2, \; \dots \;, z_N\}$, compact real variables $\tilde{s} \equiv \{s_1, s_2, \; \dots \;, s_N\} \Rightarrow$ all the N *independent* scalar products, one can obtain from the external momenta

The dependence upon the details of the diagram, $\{n, k\}$, moves from the denominator $\to$ the numerator!!

The SAME formal expression for the denominator of ANY diagram $\mathcal{G}$ appears

NIR - II

The full N-leg transition amplitude is the sum of infinite diagrams $\mathcal{G}(n,k)$ and it can be formally written as

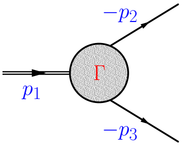
$$f_N(\tilde{s}) = \sum_{\mathcal{G}} f_{\mathcal{G}}(\tilde{s}) \propto \prod_h \int_0^1 dz_h \int_0^\infty d\gamma \frac{\delta(1-\sum_h z_h)\ \phi_N(\vec{z},\gamma)}{(\gamma - \sum_h z_h s_h)}$$

where

$$\phi_N(\vec{z},\gamma) = \sum_{\mathcal{G}} \tilde{\phi}_{\mathcal{G}}(\vec{z},\gamma)$$

is called a Nakanishi weight function and it is REAL (γ is non compact, while $\vec{z}$ is compact).

Application: 3-leg transition amplitude → vertex function for a scalar theory (N.B. for fermions → spinor indexes)



$$f_3(\tilde{s}) = \int_0^1 dz \int_0^\infty d\gamma \frac{\phi_3(z,\gamma)}{\gamma - \frac{p^2}{4} - k^2 - zk\cdot p - i\epsilon}$$

with $p = p_1 + p_2$ and $k = (p_1 - p_2)/2$

The expression holds at any order in perturbation-theory !

Natural choice as a general trial function for obtaining actual solution of BSE

A vertex function $f_3(\tilde{s})$ (N.B. $\tilde{s} \equiv$ all the independent scalar products involving the external momenta) with one leg on mass-shell is related to the two-body BS amplitude Φ_{BS}. Schematically (G_1 and G_2: constituent propagators)

$$\Phi_{BS} = G_1 \otimes G_2 \otimes f_3(\tilde{s})$$

Milestones

★ The BSE for the celebrated Wick-Cutkosky model (1954), i.e. two massive scalars interacting through a massless scalar can be **exactly solved** by using an integral representation, like the one introduced by Nakanishi.

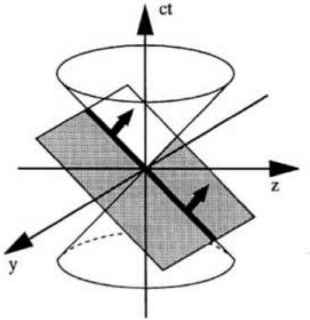
★ ★ The generalization to massive exchange was validated numerically over the years...

- Kusaka et al, exploited the *uniqueness* of the NIR weight-function for a two scalar system (they used standard variables); [PRD **56** (1997)]
- Carbonell and Karmanov properly integrated both sides of the BSE of two-scalars, exploiting Light-Front variables, without reverting to *NIR uniqueness*; [EPJA **27**, 1 (2006)].
- ⇑ Dorkin, Beyer, Kaptarin and Semikh investigated the two-fermion system within NIR + Wick rotation (⇒ a critical behavior in absence of vertex form factors was pointed out) [FBS **42**, 1 (2008)]
- ⇑ C-K evaluated a fermionic system, by using LF variables (!) (critical behavior confirmed) [EPJA **46** 387 (2010)]

- Frederico, Viviani and GS, extended the NIR+LF formalism to the scattering-state BSE [FSV PRD **85**, 036009 (2012)]and successfully cross-checked the two-scalar results, *with and without uniqueness*, using integration LF variables in a different context [FSV PRD **89**, 016010 (2014)]. The scattering BSE in the zero-energy limit was also calculated [FSV EPJC 75, 398 (2015)].
- $\Uparrow$ de Paula & FVS extended the study of the two-fermion system by clarifying and fixing a difficulty with spin dof was [dFSV PRD 94, 071901(R) (2016); EPJ C 77, 764 (2017)].
- $\Uparrow$ Nogueira et al , have investigated for the first time the fermion-scalar bound system [PRD 100, 016021 (2019)].

Projecting BSE onto the LF hyper-plane $x^+ = 0$

- NIR contains the *needed freedom* for exploring non perturbative problems, once the Nakanishi weight functions are taken as unknown REAL quantities.
- Even adopting NIR, BSE still remains a highly singular integral equation in the 4D Minkowski momentum space. BUT exploiting an expression á la Nakanishi for the BS amplitude, then its analytic structure is displayed in full
- Noteworthy, in the LF framework one recovers a probabilistic interpretation by expanding the BS amplitude on a Fock basis, and then singling out the *valence component*. Hence the probability of finding two constituents in the fully interacting two-body state can be evaluated.



The valence component is formally obtained by integrating on $k^- = k^0 - k^3$ the BS amplitude. This mathematical step is equivalent to restrict the *LF-time* x^+ in Φ_{BS} to the null plane, i.e. $x^+ = 0$

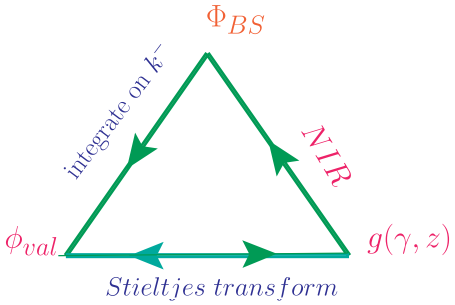
A regular integral equation equivalent to BSE

$$\text{Valence w.f.} = \psi_{n=2}(\xi, k_\perp) = \frac{p^+}{\sqrt{2}}\,\xi\,(1-\xi)\int \frac{dk^-}{2\pi}\,\overbrace{\Phi_b(k,p)}^{\text{BS Amplitude}} =$$

$$= \frac{1}{\sqrt{2}}\xi\,(1-\xi)\,\underbrace{\int_0^\infty d\gamma'\,\frac{g_b(\gamma', 1-2\xi; \kappa^2)}{[\gamma' + k_\perp^2 + \kappa^2 + (2\xi-1)^2\,\frac{M^2}{4} - i\epsilon]^2}}_{\text{NIR}}$$

with $\kappa^2 = 4m^2 - M^2$ and $M = 2m - B$.($B \equiv$ binding energy)

The step for recovering the probabilistic interpretation strongly suggests to apply the projection on both sides of BSE. This can be actually done by introducing NIR !!



N.B. The valence w.f. $\psi_{n=2}$ is a generalized Stieltjes transform (invertible) of the Nakanishi weight funct. g_b (Carbonell, Frederico, Karmanov PLB 769 (2017), 418). This observation enforces the idea that NIR can be a suitable and general trial function for solving BSE.

LF projection of the homogeneous BSE in ladder approx.

$$\Phi(k,p) = G_0(k,p)\ \int d^4k'\ \mathcal{K}_{BS}(k,k',p)\ \Phi(k',p)$$

$$\xrightarrow{NIR+LF}$$

$$\text{valence w.f.} \ \propto\ \int_0^\infty d\gamma' \frac{g_b(\gamma',z;\kappa^2)}{[\gamma'+\gamma+z^2m^2+(1-z^2)\kappa^2-i\epsilon]^2} =$$

$$= \alpha\ \int_0^\infty d\gamma' \int_{-1}^1 dz'\ V_b^{LF}(\alpha;\gamma,z;\gamma',z')\ \ g_b(\gamma',z';\kappa^2).$$

with $V_b^{LF}(\alpha;\gamma,z;\gamma',z')$ determined by the irreducible kernel $\mathcal{I}(k,k',p)$ **(!)** and α is the coupling constant ($\equiv g^2/16\pi$ for the scalar case).

In turn, by adopting an orthonormal basis (Laguerre $\times$ Gegenbauer) for expanding $g_b(\gamma,z;\kappa^2=4m^2-M^2)$, the integral equation becomes a generalized eigen-equation, with eigenvalue α , and the eigenvector composed by the coefficients of the expansion.

★ ★ If the eigen-equation admits a solution, for a given mass M of the system, then we know how to reconstruct the whole BS amplitude

Spin dof and BSE

Adding spin dof is a challenge, both on formal and numerical sides.
While projecting ladder BSE with spin dof onto the null plane, one faces with integrals that could become singular for some values of an external variable.

Fortunately, the prototype of such singular integrals was studied by Yan in the context of the field theory in the Infinite Momentum frame,(PRD 7 (1973) 1780).
★One needs a simple generalization

$$\mathcal{I}_n(\beta, y) = \int_{-\infty}^{\infty} \frac{dx}{\left[\beta x - y \mp i\epsilon\right]^{2+n}} = \pm \frac{2\pi i \; \delta(\beta)}{(n+1)! \left[-y \mp i\epsilon\right]^{1+n}}$$

Differently, in the explicit covariant LF framework, the singular behavior of the relevant integrals was pragmatically healed by introducing a suitable smoothing function (Carbonell & Karmanov EPJA **46**, 387 (2010)).

BSE for Fermion-Scalar systems

The BSE, without both self-energy and vertex corrections, for a $J^\pi = [1/2]^+$ state reads

$$\Phi^N(k,p,J_z) = G_0(p/2-k)S(p/2+k) \int \frac{d^4k'}{(2\pi)^4}\ i\mathcal{K}^{Ld}(k,k',p)\ \Phi^N(k',p,J_z)\ ,$$

with

$$G_0(q) \ = i\ \frac{1}{(q^2 - m_S^2 + i\epsilon)}\ , \qquad S(q) \ = i\ \frac{\not{q} + m_F}{(q^2 - m_F^2 + i\epsilon)} \qquad (1)$$

The ladder approximation for scalar and vector exchanges

$$i\mathcal{K}_s^{Ld}(k,k',p) = -i\ \lambda_S^s \lambda_F^s\ \frac{1}{(k-k')^2 - \mu^2 + i\epsilon}\ ,$$

and

$$i\mathcal{K}_v^{Ld}(k,k',p) = -i\ \lambda_S^v \lambda_F^v\ \frac{(\not{p} - \not{k} - \not{k}')}{(k-k')^2 - \mu^2 + i\epsilon}$$

with μ the mass of the exchanged boson.

From general principles, the BS amplitude of the fermion-scalar system, $J^\pi = [1/2]^+$, contains two unknown scalar functions ϕ_i, without exchange symmetry,

$$\Phi_{BS}(k,p) = \Big[O_1(k)\ \phi_1(k,p) + O_2(k)\ \phi_2(k,p)\Big]\ U(p,s)$$

with

$$O_1(k) = \mathbb{I}\ , \qquad O_2(k) = \frac{\not k}{M}\ , \qquad (\not p - M)\ U(p,s) = 0\ .$$

One easily obtains the 2-channel system of integral equations for $\phi_i^{s(v)}$

$$\Rightarrow\quad \phi_i^{s(v)}(k,p) = \frac{i}{(p/2-k)^2 - m_S^2 + i\epsilon}\ \frac{i}{(p/2+k)^2 - m_F^2 + i\epsilon} \int \frac{d^4k'}{(2\pi)^4}$$
$$\times\ \frac{(-i\lambda_S^{s(v)}\lambda_F^{s(v)})}{(k-k')^2 - \mu^2 + i\epsilon}\ \sum_{j=1,2} \mathcal{C}_{ij}^{s(v)}(k,k',p)\ \phi_j^{s(v)}(k',p)$$

Then, by using the NIR of $\phi_i^{s(v)}(k,p)$

$$\phi_i(k,p) = \int_{-\infty}^{\infty} d\gamma' \int_{-1}^{1} dz' \frac{g_i(\gamma', z'; \kappa^2)}{[k^2 + z' p \cdot k - \kappa^2 - \gamma' + i\epsilon]^3} \quad ,$$

with $g_i(\gamma, z; \kappa^2) \equiv$ Nakanishi WFs, $\kappa^2 = \bar{m}^2 - \frac{M^2}{4}$, and $\bar{m} = (m_F + m_S)/2$
we expand the Nakanishi WFs on a basis

Laguerre Polys $\times$ *Gegenbauer Polys*

$\Rightarrow$ the familiar (in the NIR approach!) generalized eigenvalue problem is devised

$$A \; g = \alpha \; B \; g$$

.

N.B. differently from both the two-boson and two-fermion systems, the Gegenbauer polynomials involved in the fermion-scalar system we are investigating do not have definite parity.

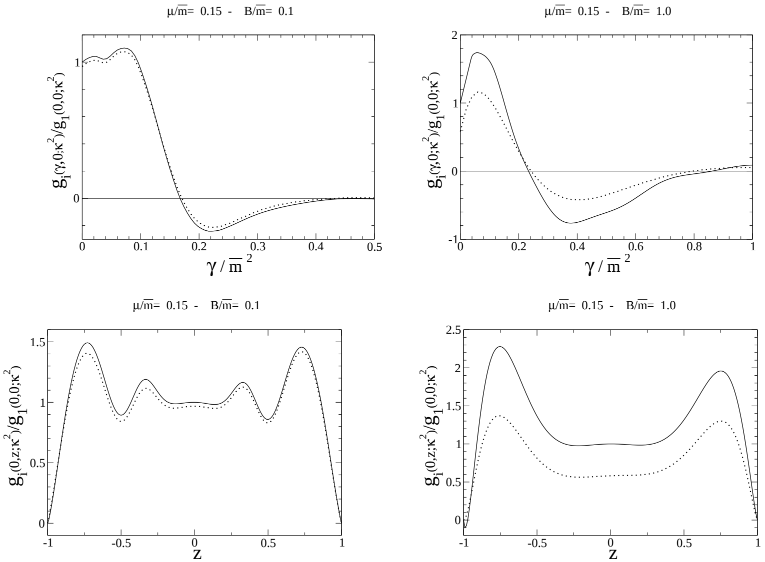
Scalar coupling $\alpha^S = \lambda_S \lambda_F/(8\pi m_S)$, for $m_F = m_S$ and $\mu/\bar{m} = 0.15,\ 0.50$ (with $\bar{m} = (m_S + m_F)/2$). First column: the binding energy in unit mass of $\bar{m}$, i.e. $B/\bar{m}$. Second and fourth columns: coupling constants α^S_M, obtained by solving the BSE in Minkowski space. Third and fifth columns: Wick-rotated results, α^S_{WR}

$B/\bar{m}$	$\alpha^S_M(0.15)$	$\alpha^S_{WR}(0.15)$	$\alpha^S_M(0.50)$	$\alpha^S_{WR}(0.50)$
0.10	1.506	1.506	2.656	2.656
0.20	2.297	2.297	3.624	3.624
0.30	3.047	3.047	4.535	4.535
0.40	3.796	3.796	5.451	5.451
0.50	4.568	4.568	6.404	6.404
0.80	7.239	7.239	9.879	9.879
1.00	9.778	9.778	13.738	13.738

For increasing values of $B/\bar{m}$, the size of the system shrinks, and repulsion starts to sizably oppose the binding. This can be heuristically understood: the fermion-scalar vertex, for on mass-shell fermions, contains the scalar density $\bar{u}\ u$, and the Dirac matrix γ^0 generates a minus sign in front of the contribution produced by the small components, more and more relevant when the system becomes more and more relativistic.

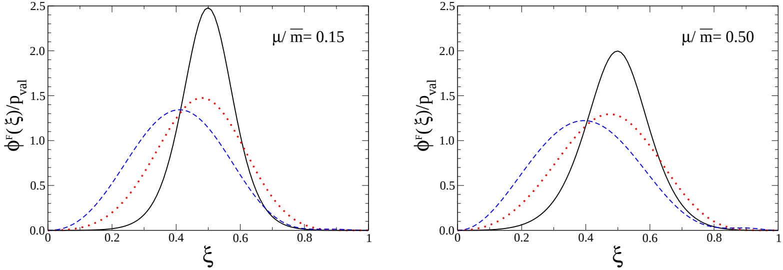
Valence probabilities, for the scalar exchange with $m_F = m_S$. $P_{val}^{NoA} \equiv \; \downarrow\Uparrow$ (constituent spin is anti-aligned to the system spin) - $P_{val}^{A} \equiv \; \uparrow\Uparrow$ First column: the binding energy in unit mass of $\bar{m}$.

	$\mu/\bar{m} = 0.15$			$\mu/\bar{m} = 0.50$		
$B/\bar{m}$	P_{val}	P_{val}^{noA}	P_{val}^{A}	P_{val}	P_{val}^{noA}	P_{val}^{A}
0.10	0.81	0.02	0.79	0.88	0.03	0.85
0.20	0.77	0.03	0.74	0.85	0.05	0.80
0.30	0.76	0.05	0.71	0.84	0.07	0.77
0.40	0.75	0.06	0.69	0.83	0.09	0.74
0.50	0.76	0.07	0.69	0.83	0.11	0.72
0.80	0.81	0.13	0.68	0.88	0.18	0.70
1.00	0.90	0.19	0.71	0.98	0.25	0.73



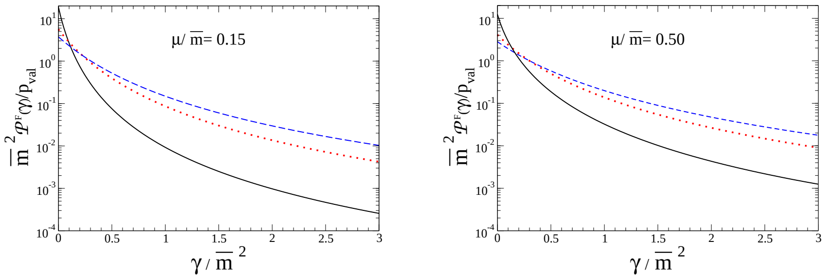
g_i for an equal-mass $(1/2)^+$ system, and for a scalar exchange. Solid line : g_1. Dotted line: g_2. Notice that the normalization factor is the same for both NWFs, namely $g_1(0, 0; \kappa^2)$. (Nogueira et al PRD 100, (2019))

Equal mass case $m_S = m_F$, scalar exchange



Longitudinal LF distributions for a fermion in the valence component for $\mu/\bar{m} = 0.15$ (left panel) and for $\mu/\bar{m} = 0.50$ (right panel), for a scalar exchange. Solid line: $B/\bar{m} = 0.1$. Dotted red line: $B/\bar{m} = 0.5$. Dashed blue line: $B/\bar{m} = 1.0$. (Nogueira et al PRD 100, (2019))

Equal mass case $m_S = m_F$, scalar exchange



Transverse LF distributions for a fermion in the valence component for $\mu/\bar{m} = 0.15$ (left panel) and for $\mu/\bar{m} = 0.50$ (right panel), for a scalar exchange. Solid line: $B/\bar{m} = 0.1$. Dotted red line: $B/\bar{m} = 0.5$. Dashed blue line: $B/\bar{m} = 1.0$.(Nogueira et al PRD 100, (2019)) Vector coupling $\alpha^V = \lambda_S \lambda_F/(8\pi)$, for $m_F = m_S$ and $\mu/\bar{m} = 0,\ 0.15,\ 0.50$. First column: the binding energy in unit mass of $\bar{m}$, i.e. $B/\bar{m}$. Second, fourth and sixth column: coupling constants α^V_M, obtained by solving the BSE in Minkowski space. Third, fifth and seventh column: Wick-rotated results, α^V_{WR}, with a numerical uncertainty for $B/\bar{m} = 0.5$ due to some instabilities in the Gaussian quadrature adopted.

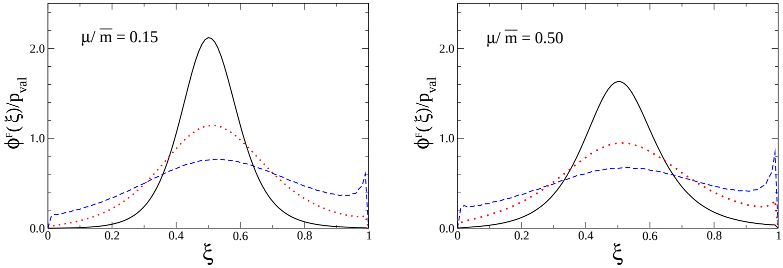
$B/\bar{m}$	$\alpha^V_M(0)$	$\alpha^V_{WR}(0)$	$\alpha^V_M(0.15)$	$\alpha^V_{WR}(0.15)$	$\alpha^V_M(0.50)$	$\alpha^V_{WR}(0.50)$
0.10	0.513	0.513	0.608	0.609	0.849	0.854
0.20	0.758	0.761	0.823	0.823	1.009	1.015
0.30	0.936	0.938	0.979	0.978	1.127	1.129
0.40	1.074	1.074	1.107	1.097	1.225	1.216
0.50	1.189	$1.18 \pm .03$	1.214	$1.19 \pm .03$	1.311	$1.28 \pm .04$

A critical behavior of the coupling constant, due to the dimensionless character of the fermion coupling, manifests for $B/\bar{m} > 0.5$, as in the two-fermion case when a pointlike vertex is adopted (see Dorkin et al, FBS 42,1 (2008) and Carbonell-Karmanov JPA 46, 387 (2010)).

Valence probabilities for the vector exchange, with $m_F = m_S$. $P^{NoA}_{val} \equiv \downarrow\Uparrow$ - $P^{A}_{val} \equiv \uparrow\Uparrow$

	$\mu/\bar{m} = 0.0$			$\mu/\bar{m} = 0.15$			$\mu/\bar{m} = 0.50$		
$B/\bar{m}$	P_{val}	P^{noA}_{val}	P^{A}_{val}	P_{val}	P^{noA}_{val}	P^{A}_{val}	P_{val}	P^{noA}_{val}	P^{A}_{val}
0.10	0.69	0.01	0.68	0.73	0.02	0.71	0.75	0.04	0.71
0.20	0.62	0.02	0.60	0.64	0.03	0.61	0.66	0.05	0.61
0.30	0.57	0.03	0.54	0.58	0.04	0.54	0.60	0.06	0.54
0.40	0.53	0.04	0.49	0.54	0.05	0.49	0.55	0.07	0.48
0.50	0.50	0.05	0.45	0.50	0.05	0.45	0.52	0.07	0.45

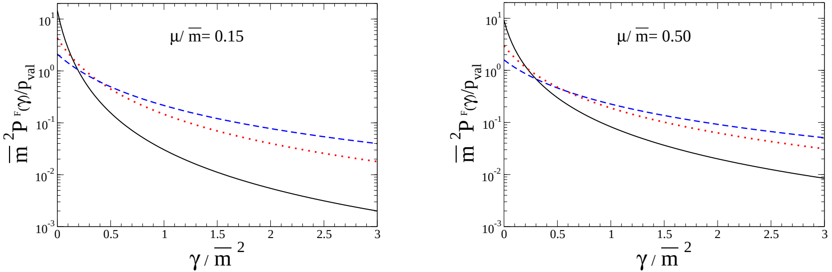
Equal mass case $m_S = m_F$, vector exchange



Longitudinal LF distributions for a fermion in the valence component for $\mu/\bar{m} = 0.15$ (left panel) and for $\mu/\bar{m} = 0.50$ (right panel), for a vector exchange. Solid line: $B/\bar{m} = 0.1$. Dotted red line: $B/\bar{m} = 0.3$. Dashed blue line: $B/\bar{m} = 0.5$.

For increasing $B/\bar{m}$, the size of the system shrinks and the helicity conservation becomes apparent, since the fermion mass starts to play a minor role.

Equal mass case $m_S = m_F$, vector exchange



Transverse LF distributions for a fermion in the valence component for $\mu/\bar{m} = 0.15$ (left panel) and for $\mu/\bar{m} = 0.50$ (right panel), for a vector exchange. Solid line: $B/\bar{m} = 0.1$. Dotted red line: $B/\bar{m} = 0.3$. Dashed blue line: $B/\bar{m} = 0.5$.

A mock nucleon

A state $(1/2)^+$ with a mass ratio $m_S/m_F = 2$ and a binding $B/\bar{m} = \ 0.1$.

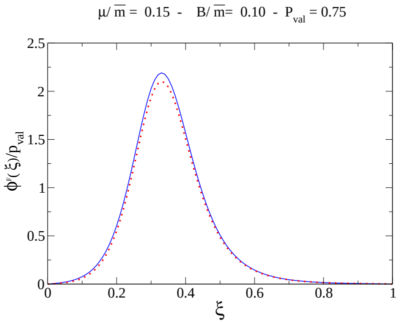
Two values of the exchanged-vector mass:
★ $\mu/\bar{m} = 0.15$, $\alpha^V = \lambda_S\lambda_F/(4\pi) \sim \ 1.3$. $M_N = 938\ MeV \quad \Rightarrow m_q = 330\ MeV$, $m_g = 33\ MeV$
★★ $\mu/\bar{m} = 0.50$, $\alpha^V = \lambda_S\lambda_F/(4\pi) \sim \ 1.8$. $M_N = 938\ MeV \quad \Rightarrow m_q = 330\ MeV$, $m_g = 110\ MeV$

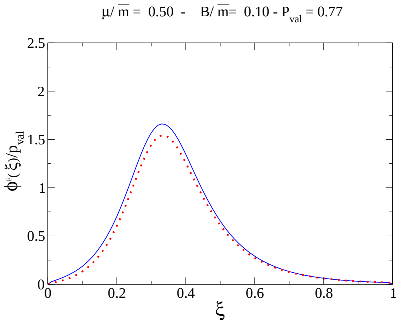
$$P_{val} = \uparrow\Uparrow \text{ (with OAM } L = 0) \quad + \quad \downarrow\Uparrow \text{ (with OAM } L = 1)$$

(N.B. small components have opposite parity $\rightarrow \gamma_0 \equiv diag\{1, -1\}$

$$< S_z^N >= 0.5\ P_\uparrow \quad - \quad 0.5\ P_\downarrow \ \sim \ 0.33$$

Valence contribution to the longitudinal-momentum distribution
$m_S = 2\ m_F$



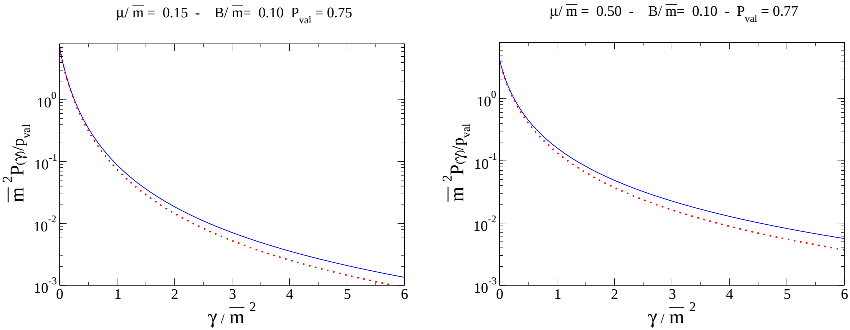


dotted line= $\uparrow\ \Uparrow$ with OAM $L = 0$ - $\xi \to$ Bjorken variable - No evolution applied

$m_q = 330\ MeV,\ m_g = 33\ MeV$ — $m_q = 330\ MeV,\ m_g = 110\ MeV$

$P_\uparrow = 0.71 \quad - \quad P_\downarrow = 0.4$ — $P_\uparrow = 0.70 \quad - \quad P_\downarrow = 0.7$

Valence contribution to the transverse-momentum distribution
$m_S = 2\ m_F$



dotted line= $\uparrow \Uparrow$ with OAM $L = 0$ - $\gamma = |\mathbf{k}_\perp|^2$ - No evolution applied

N.B. the ladder exchange of vector boson should govern the tail of the momentum distributions (also in more refined approaches)

Conclusions & Perspectives

- A systematization of the technique for solving BSE with (and without) spin dof has been reached, and the cross-check among results obtained by different groups, for different interacting systems (with kernels in ladder and cross-ladder contributions) has produced a clear numerical evidence of the validity of NIR for obtaining actual solutions.
- A general comment to be reminded: the LF framework has well-known advantages while analytical integrations are carried out, and its effectiveness is displayed in its full glory when the bound systems with spin dof are investigated.
- The numerical validation of NIR strongly encourages to face with needed improvements for approaching a continuous QFT in Minkowski space, e.g. including self-energies and vertex corrections evaluated within the same framework (work in progress on the gap equations)
- An interesting possibility for the tomography of the nucleon (i.e. the study of spin-$k_\perp$ correlations): Fragmentation functions?