Diquark correlations in light-cone distribution amplitudes of the baryon octet

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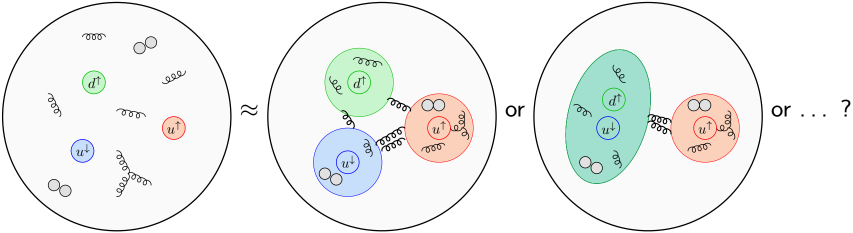
Baryon wave functions

Schematically:

$$|B\rangle = |qqq\rangle + |qqqg\rangle + |qqq\bar{q}q\rangle + \dots$$

- very complicated
- one needs taylor-made approximations / effective descriptions for different situations
- e.g.,

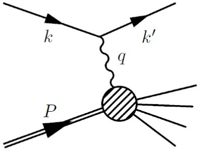


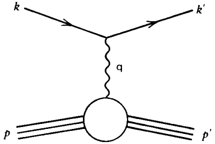
Inclusive vs. exclusive processes

inclusive **vs.** **exlcusive** (at high Q^2)

(DIS) $e + p \to e + X$ — $e + p \to e + p$ (elastic e-p scattering)

only e^- measured — all particles measured





relevant non-perturbative information

does not discriminate between Fock states

Parton Distribution Function (PDF)

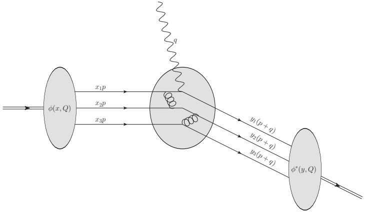
probability amplitude to find parton with a given momentum fraction

only Fock states with few partons relevant at high Q^2

Light-cone Distribution Amplitude (LCDA)

describes distribution of the momentum within a Fock state

What are light-cone distribution amplitudes



- **LCDAs**: distribution of the lightcone-momentum within a specific Fock state
- in **hard exclusive processes**: Fock states are increasingly power-suppressed with a rising number of partons ⇒ 3q contribution most important!

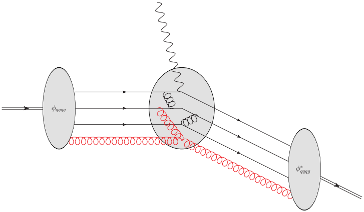
> ⇒ at high momentum transfer the 3-quark contribution plays the most important role

actually, its a bit more complicated...

$\underline{Q^2 \gtrsim 50\,\mathrm{GeV}^2?:}$ Form factor $=$ DA $\circ\, T_H \circ$ DA* (Factorization)

$\underline{Q^2 \gtrsim 1\,\mathrm{GeV}^2:}$ Form factor $\xleftrightarrow{\text{LCSR}}$ DA (Light cone sum rules)

What are light-cone distribution amplitudes



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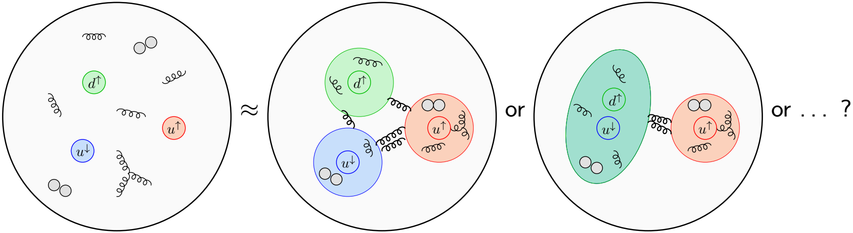
Baryon wave functions

Schematically:

$$|B\rangle = |qqq\rangle + |qqqg\rangle + |qqq\bar{q}q\rangle + \dots$$

- considering three-quark LCDAs we are only sensitive to the leading Fock state
- instead of



Baryon wave functions

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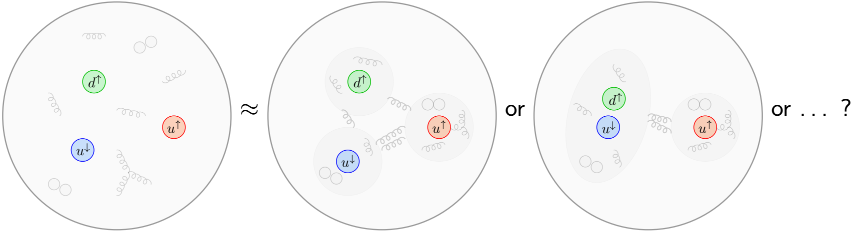
- we will see



$\Rightarrow$ we may find diquark correlations, but certainly no diquark (even if there was one)

What diquark correlations would one naively expect in our case?

Diquark correlations are known to be large if ...

- one has large angular momentum
- heavy quarks are involved

see, e.g., Anselmino M. et al., Rev. Mod. Phys. **65** (1993) 1199

We consider:

- ground-state baryons (from the $J^P = \frac{1}{2}^+$ octet)
- quark content: up, down, strange

$\Rightarrow$ only mild diquark correlations to be expected

LCDAs: connection to three-quark baryonic wave function

- full three-quark baryonic wave functions still very complex
  $\Rightarrow$ reduce complexity by introducing DAs

Wave function Ψ

$$\Psi(x, k_\perp) = \langle 0 | \epsilon^{ijk} f^i(x_1, k_{1\perp}) g^j(x_2, k_{2\perp}) h^k(x_3, k_{3\perp}) | B \rangle$$

$$\Phi(x, \mu) = Z(\mu) \int_{|k_\perp| \leq \mu} [d^2 k_\perp] \; \Psi(x, k_\perp)$$

Three-quark DAs Φ:

- transverse quark momenta are integrated out
- only sensitive to light-cone momentum fractions x_1, x_2, and x_3
- encode the momentum distribution of valence quarks at small transverse separations

3q DAs: Definition

$$\langle 0|q^{a}_{\alpha}(a_1 n)q^{b}_{\beta}(a_2 n)q^{c}_{\gamma}(a_3 n)|B(p,s)\rangle$$
$$= \int [dx] e^{-i\, n\cdot p \sum_i a_i x_i} \Big[V_1^B(x_1,x_2,x_3)(\not{p}C)_{\alpha\beta}(\gamma_5 u_+^B(p,s))_\gamma + \dots \Big]$$

- color antisymmetrization and Wilson-lines are not written out explicitly
- a, b and c are flavor indices; α, β and γ are Dirac indices; n is a light-like vector
- the x_i are momentum fractions; $[dx] = dx_1 dx_2 dx_3\, \delta(1-x_1-x_2-x_3)$
- on the r.h.s. one has 24 different structures and the same number of different DAs:

	twist-3	twist-4	twist-5	twist-6
scalar		S_1^B	S_2^B	
pseudoscalar		P_1^B	P_2^B	
vector	V_1^B	V_2^B, V_3^B	V_4^B, V_5^B	V_6^B
axialvector	A_1^B	A_2^B, A_3^B	A_4^B, A_5^B	A_6^B
tensor	T_1^B	T_2^B, T_3^B, T_7^B	T_4^B, T_5^B, T_8^B	T_6^B

ME decomposition by Braun et al., Nucl. Phys. **B589** (2000) 381

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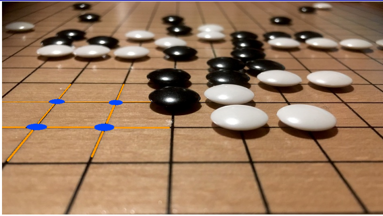
- on the l.h.s. one has to **choose** the **correct flavor content**
- the order of flavors is relevant for the symmetry properties of the DAs
- a convenient choice is:[1]

$$N \equiv p \mathrel{\hat{=}} uud\,, \qquad n \mathrel{\hat{=}} ddu\,, \qquad \Sigma^+ \mathrel{\hat{=}} uus\,, \qquad \Sigma^0 \mathrel{\hat{=}} uds\,,$$
$$\Sigma \equiv \Sigma^- \mathrel{\hat{=}} dds\,, \qquad \Xi \equiv \Xi^0 \mathrel{\hat{=}} ssu\,, \qquad \Xi^- \mathrel{\hat{=}} ssd\,, \qquad \Lambda \mathrel{\hat{=}} uds\,.$$

- we consider one representative from each isospin multiplet

[1] see, e.g., Franklin J., Phys. Rev. **172** (1968) 1807

Lattice QCD in a nutshell

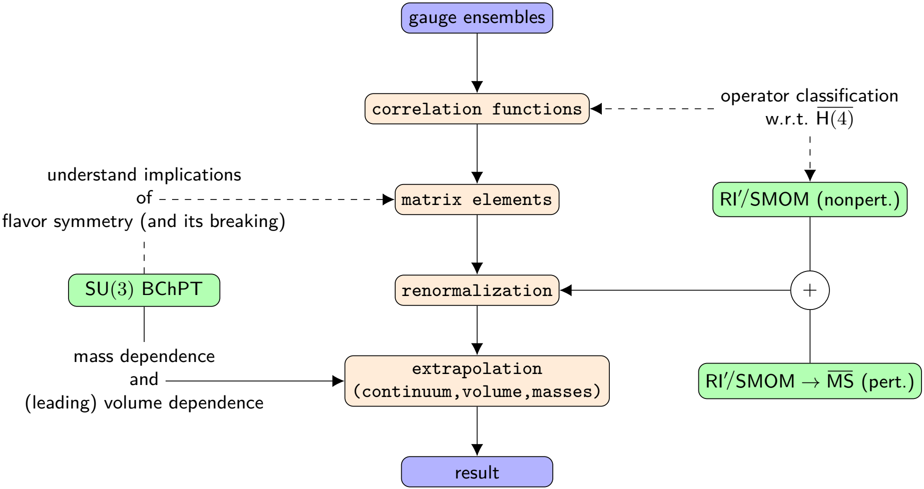


- evaluate pathintegral numerically on a 4D lattice
- the quark fields q live on lattice sites
- the gauge field U is represented by 3×3 matrices on the links between the sites
- after integrating out fermionic degrees of freedom, e.g.,

$$\langle q(x)\bar{q}(y)\rangle = \frac{1}{Z}\int \mathcal{D}U \det(M[U])e^{-S_E[U]}\left(M[U]\right)^{-1}_{xy}$$

  $M \equiv$ Dirac matrix
- one considers Euclidean space-time (i.e., imaginary times)
  $\Rightarrow \det(M[U])e^{-S_E[U]}$ can be used as weight in a Monte-Carlo integration
- small problem: we cannot evaluate quark fields at light-like separations

Overview: Lattice analysis



Moments $\longleftrightarrow$ matrix elements of local operators

Moments of DAs

$$V^B_{lmn} = \int [dx]\; x_1^l x_2^m x_3^n V^B(x_1, x_2, x_3)$$

- unlike the full DAs the moments can be directly evaluated on the lattice
- for that purpose we define local operators such as

$$\mathcal{V}^{B,000}_{\rho} = \epsilon^{ijk}\left(f^{Ti}(0) C \gamma_\rho g^j(0)\right)\gamma_5 h^k(0)$$
$$\mathcal{V}^{B,001}_{\rho\nu} = \epsilon^{ijk}\left(f^{Ti}(0) C \gamma_\rho g^j(0)\right)\gamma_5 \left[iD_\nu h(0)\right]^k$$

- consider specific linear combinations to avoid operator mixing, e.g.,

$$\mathcal{O}^{B,000}_{\mathcal{V}} = -\gamma_3 \mathcal{V}^{B,000}_3 + \gamma_4 \mathcal{V}^{B,000}_4 \,, \;\ldots\;, \text{ (see below)}$$

Correlation functions: spectral decomposition

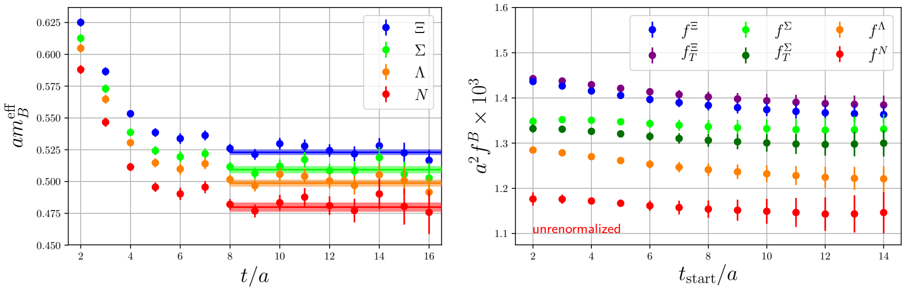
Two-point function

$$C_{\mathcal{O}} = (\gamma_+)_{\tau'\tau} \sum_{\mathbf{x}} e^{-i\mathbf{p}\mathbf{x}} \langle \mathcal{O}_\tau(t,\mathbf{x}) \bar{\mathcal{N}}^B_{\tau'}(0,\mathbf{0}) \rangle$$

$$= \frac{\sqrt{Z^B_{\mathbf{p}}}}{2E_B} (\gamma_+)_{\tau'\tau} \sum_\lambda \langle 0 | \mathcal{O}_\tau(0) | B(\mathbf{p},\lambda) \rangle \, \bar{u}^B_{\tau'}(\mathbf{p},\lambda) e^{-E_B t} + \dots$$

- insert full set of states
- at large t excited states are negligible
- parity projection: $\gamma_+ = (\mathbb{1} + k\gamma_4)/2$ with $k = m_{B^*}/E_{B^*}$ ($k=1$ also fine)
- source currents: $\mathcal{N}^N = \big(u^T C\gamma_5 d\big) u\,, \quad \mathcal{N}^\Sigma = \big(d^T C\gamma_5 s\big) d\,, \; \dots$
- quark fields in source currents smeared to enhance ground state overlap

$$\langle 0 | \mathcal{N}^B(0,\mathbf{0}) | B(\mathbf{p},\lambda) \rangle \equiv \sqrt{Z^B_{\mathbf{p}}}\, u(\mathbf{p},\lambda)$$



→ using $\sum_\lambda u^B(\mathbf{p},\lambda)\bar{u}^B(\mathbf{p},\lambda) = \not{p} + m$ one obtains, e.g.,

$$\mathcal{O} = \mathcal{N} \text{ (smeared current)}: \qquad C_\mathcal{N} = Z_B \frac{m_B + kE_B}{E_B} e^{-E_B t} + \dots$$

$$\mathcal{O} = \mathcal{O}_\mathcal{V}^{B,000} \text{ (local current)}: \qquad C_\mathcal{O} = V_{000}^B \sqrt{Z_B} \frac{E_B(m_B + kE_B) + kp_3^2}{E_B} e^{-E_B t} + \dots$$

→ simultaneous fit to smeared-smeared and smeared-point correlation functions (fit range: $t_{\rm start} < t < 20a$; in this case choose $t_{\rm start} = 10a$)

→ obtain normalization constants $f^B \equiv V_{000}^B$ and $f_T^B \equiv T_{000}^B$ and first moments

Renormalization procedure

$$\text{Lattice} \xrightarrow{\text{nonpert. ren.}} \text{RI}'/\text{SMOM} \xrightarrow{\text{continuum PT}} \overline{\text{MS}}$$

- bare lattice values have to be renormalized
- in the end we should be able to give our results in the popular continuum $\overline{\text{MS}}$ scheme
- this scheme cannot be implemented directly on the lattice
- we use a nonperturbative RI′/SMOM scheme for the lattice renormalization
  - renormalization condition: fix vertex function to Born term at the renormalization point
  - renormalization point: use non-exceptional (i.e., symmetric) momentum configuration

e.g., p_{SMOM}: $\quad p_1^2 = p_2^2 = p_3^2 = (p_1+p_2)^2 = (p_1+p_3)^2 = (p_1+p_2+p_3)^2 = \mu^2$

- we use continuum perturbation theory to convert from RI′/SMOM to $\overline{\text{MS}}$ (at 2 GeV)

Operator classification: $\overline{\mathrm{H}(4)}$

- on the lattice: continuous $\mathrm{O}(4)$ symmetry broken to hypercubic $\mathrm{H}(4)$ symmetry
- fermions on a lattice transform under the spinorial hypercubic group $\overline{\mathrm{H}(4)}$ (a discrete double cover group of $\mathrm{H}(4)$ with 768 elements)
- five irreducible spinorial representations: $\tau_1^{\underline{4}}$, $\tau_2^{\underline{4}}$, $\tau^{\underline{8}}$, $\tau_1^{\underline{12}}$, $\tau_2^{\underline{12}}$
- operators belonging to different representations cannot mix

	no derivatives dimension 9/2	1 derivative dimension 11/2	2 derivatives dimension 13/2
$\tau_1^{\underline{4}}$	$\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4, \mathcal{O}_5$	$\dots$	$\mathcal{O}_{DD1}, \mathcal{O}_{DD2}, \mathcal{O}_{DD3}, \dots$
$\tau_2^{\underline{4}}$			$\mathcal{O}_{DD4}, \mathcal{O}_{DD5}, \mathcal{O}_{DD6}, \dots$
$\tau^{\underline{8}}$	$\mathcal{O}_6$	$\mathcal{O}_{D1}, \dots$	$\mathcal{O}_{DD7}, \mathcal{O}_{DD8}, \mathcal{O}_{DD9}, \dots$
$\tau_1^{\underline{12}}$	$\mathcal{O}_7, \mathcal{O}_8, \mathcal{O}_9$	$\mathcal{O}_{D2}, \mathcal{O}_{D3}, \mathcal{O}_{D4}, \dots$	$\mathcal{O}_{DD10}, \mathcal{O}_{DD11}, \mathcal{O}_{DD12}, \mathcal{O}_{DD13}, \dots$
$\tau_2^{\underline{12}}$		$\mathcal{O}_{D5}, \mathcal{O}_{D6}, \mathcal{O}_{D7}, \mathcal{O}_{D8}$	$\mathcal{O}_{DD14}, \mathcal{O}_{DD15}, \mathcal{O}_{DD16}, \mathcal{O}_{DD17}, \mathcal{O}_{DD18}, \dots$

Operator classification: $\overline{\mathrm{H}(4)}$

- in the continuum operators with different number of derivatives do not mix
- on the lattice there is an additional dimensionful quantity: the lattice spacing a
- schematically: $Dqqq$ can now mix with $\frac{1}{a}qqq$ (problematic in the continuum limit)
- for leading twist normalization constants and first moments:
  $\rightarrow$ we can avoid the problem
- normalization constants/first moments mix among each other

	no derivatives dimension 9/2	1 derivative dimension 11/2	2 derivatives dimension 13/2
$\tau_1^{\underline{4}}$	$\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4, \mathcal{O}_5$	...	$\mathcal{O}_{DD1}, \mathcal{O}_{DD2}, \mathcal{O}_{DD3}, \ldots$
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SU(3) BChPT

- in QCD chiral symmetry is spontaneously broken:

$$\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R \xrightarrow{\text{SSB}} \mathrm{SU}(3) \text{ flavor} + 8 \text{ Goldstone bosons}$$

- explicit breaking due to non-zero quark masses
  - ⇒ the Goldstone bosons acquire a mass
  - ⇒ additional SU(3) flavor symmetry breaking if the quark masses are not equal
- using ChPT the explicit breaking can be treated perturbatively

ChPT

- yields correct description of SU(3) breaking
- on top of that: some higher order terms are included

⇒ **ChPT** is a most suitable tool to understand SU(3) symmetry and its breaking

Hadronic fields

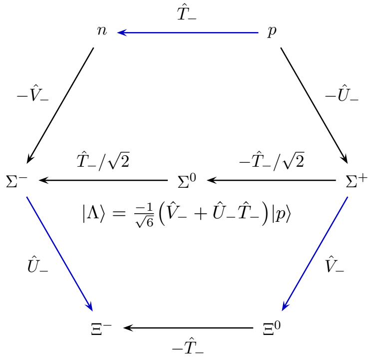
- meson fields:

$$\phi = \sqrt{2}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

$$u = \exp\left(\frac{i\phi}{2F_\pi}\right)$$

- octet baryons:

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix} \;\hat{=}$$



Three quark operators

- the quest: construct 3q operator in terms of hadron fields
- first step: use $q = q_L + q_R$ to make use of chiral symmetry

$$
\begin{aligned}
q_\alpha^a(a_1 n) q_\beta^b(a_2 n) q_\gamma^c(a_3 n) = {} & \mathcal{O}^{abc}_{RR,\alpha\beta\gamma(a_1,a_2,a_3)} + \mathcal{O}^{abc}_{LL,\alpha\beta\gamma(a_1,a_2,a_3)} \\
& + \mathcal{O}^{abc}_{RL,\alpha\beta\gamma(a_1,a_2,a_3)} + \mathcal{O}^{abc}_{LR,\alpha\beta\gamma(a_1,a_2,a_3)} \\
& + \mathcal{O}^{cab}_{RL,\gamma\alpha\beta(a_3,a_1,a_2)} + \mathcal{O}^{cab}_{LR,\gamma\alpha\beta(a_3,a_1,a_2)} \\
& + \mathcal{O}^{bca}_{RL,\beta\gamma\alpha(a_2,a_3,a_1)} + \mathcal{O}^{bca}_{LR,\beta\gamma\alpha(a_2,a_3,a_1)}
\end{aligned}
$$

- i.e., there are **only two** different **types of operators**: chiral-even ($\mathcal{O}_{RR}/\mathcal{O}_{LL}$) and chiral-odd ($\mathcal{O}_{RL}/\mathcal{O}_{LR}$)

$$
\mathcal{O}^{abc}_{XY,\alpha\beta\gamma(a_1 n,a_2 n,a_3 n)} = q^a_{X,\alpha}(a_1 n) q^b_{X,\beta}(a_2 n) q^c_{Y,\gamma}(a_3 n)
$$

Ansatz for the effective operator

$$\mathcal{O}^{abc}_{XY,\alpha\beta\gamma}(a_1 n, a_2 n, a_3 n) = \int [dx] \sum_{i,j} \sum_{k=1}^{k_j} \mathcal{F}^{i,j,k}_{XY}(x_1,x_2,x_3)\, \Gamma^{i,XXY}_{\alpha\beta\gamma\delta}\, B^{j,k,XXY}_{\delta,abc}(z)$$

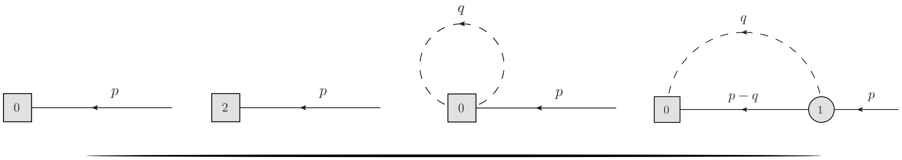
- the functions $\mathcal{F}$ are distribution amplitudes
  - $\rightarrow$ play the role of **LEC**s
- Γ: i labels the possible Dirac structures (6 for chiral-even and 6 for chiral-odd)
  - $\rightarrow$ Lorentz indices are contracted with n or derivatives acting on B
- B: contains hadron fields; e.g., $B^{1,1,RRL}_{\delta,abc} = (u\,B_\delta)_{aa'}(u)_{bb'}(u^\dagger)_{cc'}\varepsilon_{a'b'c'}$
  - $j = 2,3 \rightarrow$ structures with quark mass matrix (contained in χ^+)
  - $k = 2,3,\ldots \rightarrow$ different positions of baryon-octet field B_δ and χ^+

Ansatz for the effective operator

$$\mathcal{O}^{abc}_{XY,\alpha\beta\gamma}(a_1 n, a_2 n, a_3 n) = \int [dx] \sum_{i,j} \sum_{k=1}^{k_j} \mathcal{F}^{i,j,k}_{XY}(x_1,x_2,x_3)\, \Gamma^{i,XXY}_{\alpha\beta\gamma\delta}\, B^{j,k,XXY}_{\delta,abc}(z)$$

- $z = n \sum_i x_i a_i$ and $[dx] = dx_1 dx_2 dx_3\, \delta(1 - x_1 - x_2 - x_3)$
  $\rightarrow$ correct behaviour under translations in n direction
- **Parity:** relates left- and right-handed operators $\rightarrow \mathcal{F}_{RR} = -\mathcal{F}_{LL}$ and $\mathcal{F}_{LR} = -\mathcal{F}_{RL}$
- **CP** or **Time-reversal:** yield "reality condition" $\rightarrow \mathcal{F}^\dagger = \mathcal{F}$ up to an overall phase
- symmetry under **quark exchange**: relates different $\mathcal{F}$ to each other

Leading one-loop calculation



- loop integrals are (more or less) straightforward
- multiply with $\sqrt{Z}$ to take into account wave function renormalization
- we use IR regularization scheme[2]
- **main challenge:** handling the large number of different structures
- **last step:** obtain results for the 24 standard DAs $S^B_{1,2}$, $P^B_{1,2}$, V^B_{1-6}, A^B_{1-6}, T^B_{1-8} by matching to the general matrix element decomposition

[2] T. Becher and H. Leutwyler, Eur. Phys. J. **C9** (1999) 643

Results: Definition of octet DAs (leading twist)

$$\Phi^{B\neq\Lambda}_{\pm,3}(x_1,x_2,x_3) = \tfrac{1}{2}\Big([V_1-A_1]^B(x_1,x_2,x_3) \pm [V_1-A_1]^B(x_3,x_2,x_1)\Big)$$
$$\Pi^{B\neq\Lambda}_{3}(x_1,x_2,x_3) = T_1^B(x_1,x_3,x_2)$$
$$\Phi^{\Lambda}_{+,3}(x_1,x_2,x_3) = +\sqrt{\tfrac{1}{6}}\Big([V_1-A_1]^\Lambda(x_1,x_2,x_3) + [V_1-A_1]^\Lambda(x_3,x_2,x_1)\Big)$$
$$\Phi^{\Lambda}_{-,3}(x_1,x_2,x_3) = -\sqrt{\tfrac{3}{2}}\Big([V_1-A_1]^\Lambda(x_1,x_2,x_3) - [V_1-A_1]^\Lambda(x_3,x_2,x_1)\Big)$$
$$\Pi^{\Lambda}_{3}(x_1,x_2,x_3) = \sqrt{6}\,T_1^\Lambda(x_1,x_3,x_2)$$

- nice feature: **no mixing** under chiral extrapolation
- good behaviour in the SU(3) symmetric limit

$$\Phi^{\star}_{+,i} \equiv \Phi^{N\star}_{+,i} = \Phi^{\Sigma\star}_{+,i} = \Phi^{\Xi\star}_{+,i} = \Phi^{\Lambda\star}_{+,i} = \Pi^{N\star}_{i} = \Pi^{\Sigma\star}_{i} = \Pi^{\Xi\star}_{i}$$
$$\Phi^{\star}_{-,i} \equiv \Phi^{N\star}_{-,i} = \Phi^{\Sigma\star}_{-,i} = \Phi^{\Xi\star}_{-,i} = \Phi^{\Lambda\star}_{-,i} = \Pi^{\Lambda\star}_{i}$$

- the Λ baryon fits in nicely

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$$\Pi^{B\neq\Lambda}_{3}(x_1,x_2,x_3) = T^B_1(x_1,x_3,x_2)$$

$$\Phi^{\Lambda}_{+,3}(x_1,x_2,x_3) = +\sqrt{\tfrac{1}{6}}\Big([V_1-A_1]^\Lambda(x_1,x_2,x_3) + [V_1-A_1]^\Lambda(x_3,x_2,x_1)\Big)$$

$$\Phi^{\Lambda}_{-,3}(x_1,x_2,x_3) = -\sqrt{\tfrac{3}{2}}\Big([V_1-A_1]^\Lambda(x_1,x_2,x_3) - [V_1-A_1]^\Lambda(x_3,x_2,x_1)\Big)$$

$$\Pi^{\Lambda}_{3}(x_1,x_2,x_3) = \sqrt{6}\,T^\Lambda_1(x_1,x_3,x_2)$$

- similar definitions are possible for higher twist DAs
- the result automatically fulfills all known isospin constraints, e.g.,

$$\Phi^N_{+,3} = \Pi^N_3 \quad \hat{=} \quad 2T^N_1(x_1,x_3,x_2) = [V_1-A_1]^N(x_1,x_2,x_3) + [V_1-A_1]^N(x_3,x_2,x_1)$$

Results: Extrapolation formulas

Example: chiral-odd DAs

$$\bar{m}^2 \equiv (2m_K^2 + m_\pi^2)/3 \approx 2B_0(m_s + 2m_\ell)/3$$
$$\delta m^2 \equiv m_K^2 - m_\pi^2 \approx B_0(m_s - m_\ell)$$

$$\Phi^B_{\pm,i} = g^B_{\Phi\pm}(m_\pi, m_K, L)\left(\Phi^0_{\pm,i} + \bar{m}^2\bar{\Phi}_{\pm,i} + \delta m^2\ \Delta\Phi^B_{\pm,i}\right)$$

$$\Pi^B_i = g^B_\Pi(m_\pi, m_K, L) \times \begin{cases} \Phi^0_{+,i} + \bar{m}^2\bar{\Phi}_{+,i} + \delta m^2\ \Delta\Pi^B_i & \text{, if } B \neq \Lambda \\ \Phi^0_{-,i} + \bar{m}^2\bar{\Phi}_{-,i} + \delta m^2\ \Delta\Pi^B_i & \text{, if } B = \Lambda \end{cases}$$

- nice behaviour along the flavor symmetric line
- up to the fit parameters **extrapolation formulas** are **twist independent**
- complete non-analytic structure contained in prefactors; for $m_\pi = m_K$:

$$\lim_{m\to 0} g^B_{\mathsf{DA}}(m, m, \infty) = 1$$

$$g^N_{\Phi+} = g^\Sigma_{\Phi+} = g^\Xi_{\Phi+} = g^\Lambda_{\Phi+} = g^N_\Pi = g^\Sigma_\Pi = g^\Xi_\Pi$$
$$g^N_{\Phi-} = g^\Sigma_{\Phi-} = g^\Xi_{\Phi-} = g^\Lambda_{\Phi-} = g^\Lambda_\Pi$$

- constraints for $SU(3)_f$ symmetry breaking: e.g., $\Delta\Pi^\Sigma_3 = -\frac{1}{2}\Delta\Phi^\Sigma_{+,3} - \frac{3}{2}\Delta\Phi^\Lambda_{+,3}$

Parametrization of octet DAs

Example: leading twist DAs

$$\begin{aligned}
\Phi^B_{+,3} &= 120 x_1 x_2 x_3 \big(f^B \mathcal{P}_{00} + \varphi^B_{11} \mathcal{P}_{11} + \dots \big) \\
\Phi^B_{-,3} &= 120 x_1 x_2 x_3 \big(\varphi^B_{10} \mathcal{P}_{10} + \dots \big) \\
\Pi^{B \neq \Lambda}_3 &= 120 x_1 x_2 x_3 \big(f^B_T \mathcal{P}_{00} + \pi^B_{11} \mathcal{P}_{11} + \dots \big) \\
\Pi^\Lambda_3 &= 120 x_1 x_2 x_3 \big(\pi^\Lambda_{10} \mathcal{P}_{10} + \dots \big)
\end{aligned}$$

- distribution amplitudes can be expanded in a set of orthogonal polynomials

$$\mathcal{P}_{00} = 1 \,, \qquad \mathcal{P}_{11} = 7(x_1 - 2x_2 + x_3) \,, \qquad \mathcal{P}_{10} = 21(x_1 - x_3) \,, \qquad \dots$$

- the expansion coefficients have autonomous scale dependence[3]
- higher moments are suppressed in the asymptotic limit
- our definition separates symmetric from antisymmetric polynomials under $x_1 \leftrightarrow x_3$
- we only separate contributions which are orthogonal anyway!

[3] see, e.g., V. M. Braun et. al., Prog. Part. Nucl. Phys. **51** (2003) 311

Parametrization of octet DAs

Example: leading twist DAs

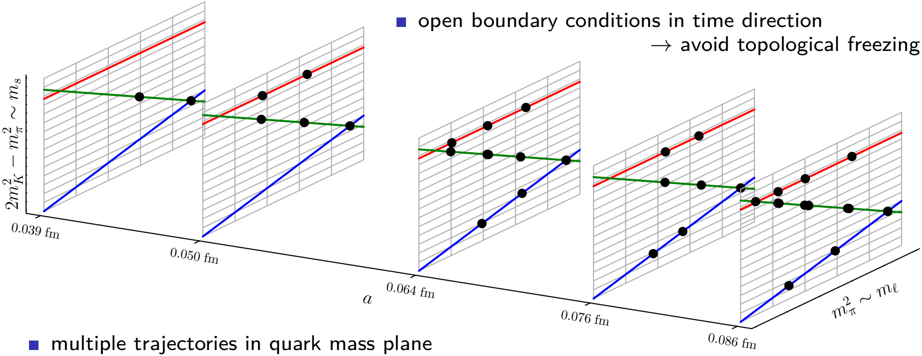
$$\Phi^B_{+,3} = 120 x_1 x_2 x_3 \big(f^B \mathcal{P}_{00} + \varphi^B_{11} \mathcal{P}_{11} + \dots \big)$$
$$\Phi^B_{-,3} = 120 x_1 x_2 x_3 \big(\varphi^B_{10} \mathcal{P}_{10} + \dots \big)$$
$$\Pi^{B\neq\Lambda}_3 = 120 x_1 x_2 x_3 \big(f^B_T \mathcal{P}_{00} + \pi^B_{11} \mathcal{P}_{11} + \dots \big)$$
$$\Pi^{\Lambda}_3 = 120 x_1 x_2 x_3 \big(\pi^\Lambda_{10} \mathcal{P}_{10} + \dots \big)$$

- additional leading twist normalization constants f^Σ_T and f^Ξ_T
- exact isospin symmetry $\Rightarrow f^N_T = f^N$ and $\pi^N_{11} = \varphi^N_{11}$
- quark mass dependence inherited from DAs; in particular:

$$f^\star \equiv f^{N\star} = f^{\Sigma\star} = f^{\Xi\star} = f^{\Lambda\star} = f^{N\star}_T = f^{\Sigma\star}_T = f^{\Xi\star}_T$$
$$\varphi^\star_{11} \equiv \varphi^{N\star}_{11} = \varphi^{\Sigma\star}_{11} = \varphi^{\Xi\star}_{11} = \varphi^{\Lambda\star}_{11} = \pi^{N\star}_{11} = \pi^{\Sigma\star}_{11} = \pi^{\Xi\star}_{11}$$
$$\varphi^\star_{10} \equiv \varphi^{N\star}_{10} = \varphi^{\Sigma\star}_{10} = \varphi^{\Xi\star}_{10} = \varphi^{\Lambda\star}_{10} = \pi^{\Lambda\star}_{10}$$

Coordinated Lattice Simulations gauge ensembles

- 40 ensembles
- $N_f = 2+1$ dynamical Wilson (clover) fermions
- open boundary conditions in time direction
  $\rightarrow$ avoid topological freezing



- multiple trajectories in quark mass plane
- wide range of lattice spacings $0.039\,\text{fm} \leq a \leq 0.086\,\text{fm}$
- large volumes (almost all ensembles have $m_\pi L > 4$)

Continuum extrapolation

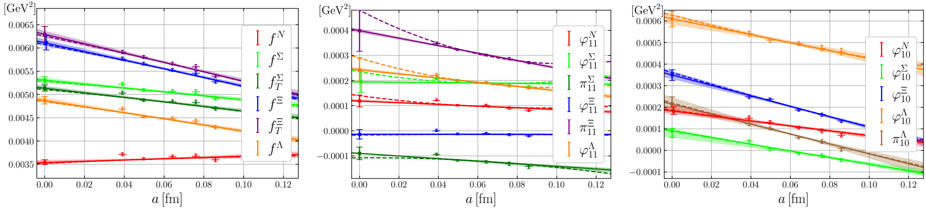
Ansatz for the lattice spacing dependence

$$\phi_{\text{lat}} = \left(1 + c_\phi^0 a + \bar{c}_\phi \bar{m}^2 a + \delta c_\phi^B \delta m^2 a\right) \phi_{\text{cont}} \,,$$

- ϕ is a wildcard for normalization constants and moments
- ϕ_{cont} corresponds to the volume and mass dependence in the continuum
- for $\delta m = 0$ flavor symmetry has to be exact also at $a \neq 0$
  $\rightarrow c_\phi^0$ and $\bar{c}_\phi$ are constrainted (in particular baryon-independent)
- however: the discretization effects can violate the SU(3) breaking constraints

$\Rightarrow$ perform simultaneous fit to all 40 ensembles

Continuum extrapolation

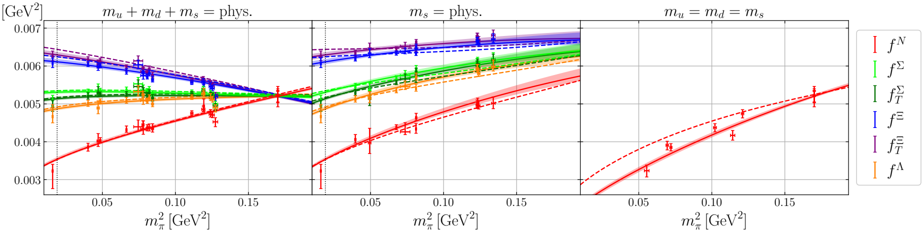


- for illustrative purposes:
  1. data shifted to physical masses and infinite volume
  2. then take average of all ensembles at the same lattice spacing (only 2 ens. at 0.039 fm)
- large discretization effects for normalization constants; even larger for moments
- for moments the effect can be a game changer (zero crossings)

  $\Rightarrow$ taking the continuum limit is pivotal

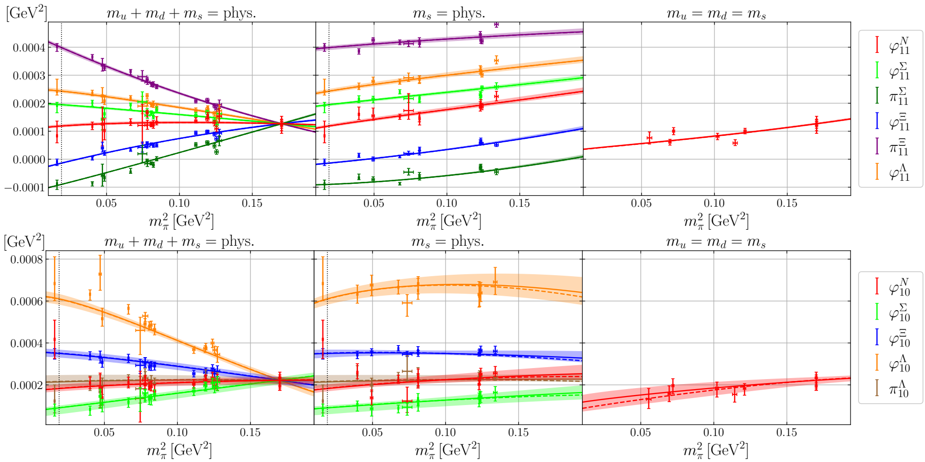
Anecdote: in an earlier study we only had data at $a = 0.086$ fm
$\rightarrow$ the accidental value $\pi_{10}^{\Lambda} \approx 0$ led to wrong conclusions

Chiral extrapolation: normalization constants



- for illustrative purposes: data shifted to $a = 0$
- at flavor symmetric case: baryons nicely fall ontop of each other
- we find strong SU(3) breaking effects: $(f_T^\Xi - f^N)/f^N \approx 78\%$
- far larger than estimated in QCD sum rules ($\lesssim 10\%$)

Chiral extrapolation: first moments



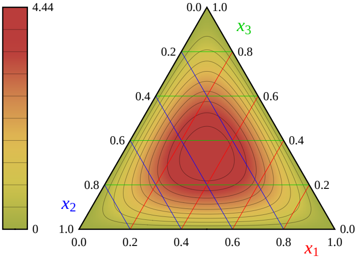
- for the moments SU(3) breaking effects are even larger (π_{11}^Σ changes sign)
- **BUT:** numerical values for moments quite small

Baryon 3q wave function (at leading twist & small transverse separation)

$$|B^{\uparrow}\rangle \sim \int \frac{[dx]}{\sqrt{x_1 x_2 x_3}} |fgh\rangle \otimes \Big\{ [V+A]^B(x_1,x_2,x_3)|{\downarrow\uparrow\uparrow}\rangle + [V-A]^B(x_1,x_2,x_3)|{\uparrow\downarrow\uparrow}\rangle \\ -2T^B(x_1,x_2,x_3)|{\uparrow\uparrow\downarrow}\rangle \Big\}$$

- shape in the asymptotic limit:

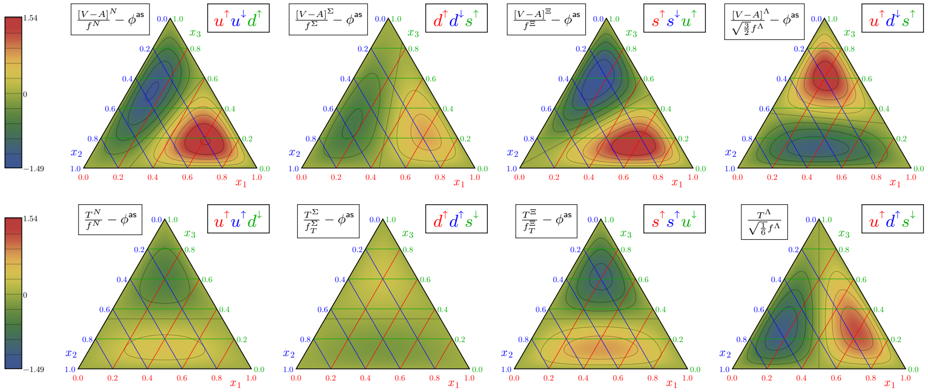
$$\phi_{\text{as}} = 120 x_1 x_2 x_3 = \frac{V^N}{f^N} = \frac{V^\Sigma}{f^\Sigma} = \frac{V^\Xi}{f^\Xi} \\ = \frac{T^N}{f^N} = \frac{T^\Sigma}{f_T^\Sigma} = \frac{T^\Xi}{f_T^\Xi} = \frac{-A^\Lambda}{f^\Lambda} \\ 0 = A^N = A^\Sigma = A^\Xi = V^\Lambda = T^\Lambda$$



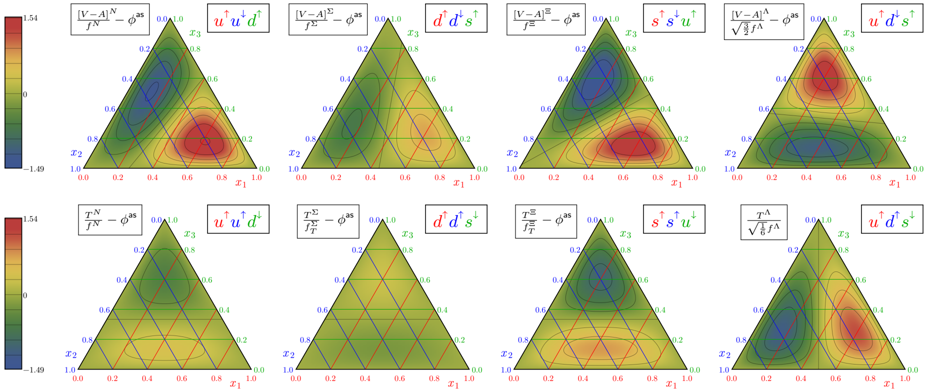
- use barycentric plots $(x_1 + x_2 + x_3 = 1)$

$$\phi_{\text{as}}^{\text{max}} = 120/27 = 4.44$$

see, e.g., Chernyak et al., Sov. J. Nucl. Phys. **48** (1988) 536



- deviations of $[V-A]^B$ (top) and T^B (bottom) from asymptotic shape
- from left to right the plots show the baryons N, Σ, Ξ, Λ
- $B \neq \Lambda$: shift towards strange quarks and towards the leading quark
- T^Λ: asymptotic limit vanishes by construction
- isospin symmetry: T^N can be obtained from $[V-A]^N$

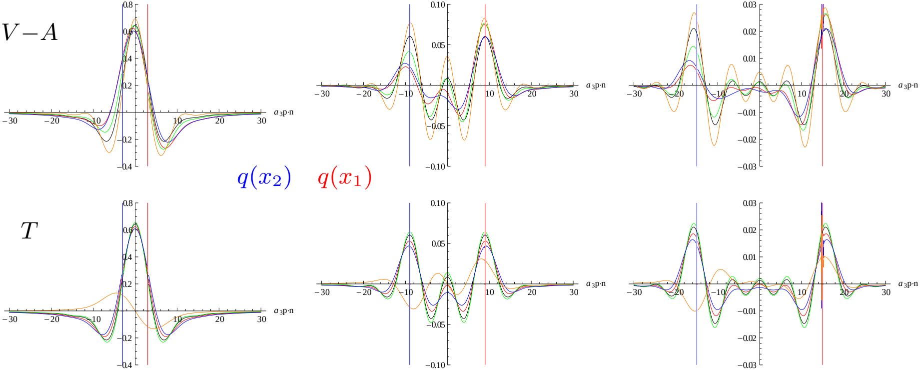


- due to isospin symmetry and symmetry under quark exchange:

$$|N^{\uparrow}\rangle \sim \int \frac{[dx]}{\sqrt{x_1 x_2 x_3}} [V-A]^N(x_1,x_2,x_3)\ \Big| u^{\uparrow}(x_1)\Big(u^{\downarrow}(x_2)d^{\uparrow}(x_3) - d^{\downarrow}(x_2)u^{\uparrow}(x_3)\Big)\Big\rangle$$

- momentum distribution shifted towards a leading u quark
- approximate symmetry under $x_2 \leftrightarrow x_3 \Rightarrow$ indicates scalar diquark correlation?

What does this mean in position space?



- consider $\mathrm{Re}\left\{\int [dx] e^{-in\cdot p \sum_i a_i x_i} DA(x_1, x_2, x_3)\right\}$ (DAs normalized as on last slide)
- $[V-A]^B$ (top) and T^B (bottom); asymptotic (black), N, Σ, Ξ, Λ
- **IF** two quarks are far apart → third quark close to one of them (**BUT** this is highly unlikely)
- in nucleon: opposite of my naive expectation for a scalar diquark happens (third quark prefers to be closer to the leading quark)

Summary

- determination of normalization constants and first moments of baryon octet DAs using lattice QCD
- not in this talk but in the article: results for higher twist normalization constants
- effect of higher moments ignored so far (second moments would be interesting)
- results can/should be used to cross-check diquark models and DSE calculations
  $\rightarrow$ see the talk by C. Mezrag yesterday

we find:

- limit of exact flavor symmetry: nicely fulfilled by the lattice data
- performing the continuum extrapolation is pivotal (in particular for the moments)
- normalizations: SU(3) breaking quite large
- deviations from asymptotic shapes: numerically small, but SU(3) breaking very large
- only very mild diquark correlations