Hadron Spectroscopy in the Diquark Model

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Diquark Correlations in Hadron Physics: Origin, Impact and Evidence
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Summary

Quark Model (QM) formalism

Exotic hadrons and their interpretations

Fully-heavy and heavy-light tetraquarks in the diquark model

The problem of baryon missing resonances

Strange and nonstrange baryons in the diquark model

Constituent Quark Models

- Complicated quark-gluon dynamics of QCD

  1. Effective degree of freedom of Constituent Quark is introduced:
     same quantum numbers as valence quarks
     mass ≈ 1/3 mass of the proton
  3. Baryons → bound states of 3 constituent quarks
  4. Mesons → bound states of a constituent quark-antiquark pair
  5. Constituent quark dynamics → phenomenological (QCD-inspired) interaction

- Phenomenological models

  $$H = \sum_i \sqrt{p_i^2 + m_i^2} + \sum_{i<j} -\frac{\alpha_{\mathrm{s}}}{r_{ij}} + \beta r_{ij} + V(\mathbf{S}_i, \mathbf{S}_j, \mathbf{L}_{ij}, \mathbf{r}_{ij})$$

  Coulomb-like + linear confining potentials + spin forces

- Several versions: Relativized Quark Model for baryons and mesons (Capstick and Isgur, Godfrey and Isgur); U(7) Model (Bijker, Iachello and Leviatan); Graz Model (Glozman and Riska); Hypercentral QM (Giannini and Santopinto), ...

- Reproduce reasonably well many hadron observables: baryon magnetic moments, lower part of baryon and meson spectrum, hadron strong decays, nucleon e.m. form factors ...

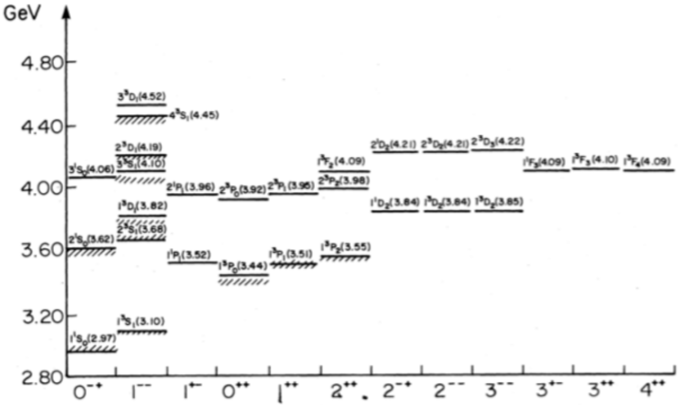
Relativized Quark Model for Baryons/Mesons

S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985)
S. Capstick and N. Isgur, Phys. Rev. D **34**, 2809 (1986)

- Relativized QM Hamiltonian
  $$H = \sum_i \sqrt{p_i^2 + m_i^2} + \sum_{i<j} V_{\rm conf}(r_{ij}) + V_{\rm hyp}(r_{ij}) + V_{\rm so}(r_{ij})$$
- Confining potential
  $$V_{\rm conf}(r_{ij}) = -\left(\tfrac{3}{4}c + \tfrac{3}{4}\beta r_{ij} - \frac{\alpha_{\rm s}(r_{ij})}{r_{ij}}\right) \mathbf{F}_i \cdot \mathbf{F}_j$$
- Hyperfine interaction
  $$V_{\rm hyp}(r_{ij}) = -\frac{\alpha_{\rm s}(r_{ij})}{m_i m_j}\left[\frac{8\pi}{3}\mathbf{S}_i \cdot \mathbf{S}_j \ \delta^3(\mathbf{r}_{ij}) + \frac{1}{r_{ij}^3}\left(\frac{3 \ \mathbf{S}_i \cdot \mathbf{r}_{ij} \ \mathbf{S}_j \cdot \mathbf{r}_{ij}}{r^2} - \mathbf{S}_i \cdot \mathbf{S}_j\right)\right] \ \mathbf{F}_i \cdot \mathbf{F}_j$$
- Spin-orbit interaction
  $$V_{\rm so}(r_{ij}) = -\frac{\alpha_{\rm s}(r_{ij})}{r_{ij}^3}\left(\frac{1}{m_i} + \frac{1}{m_j}\right)\left(\frac{\mathbf{S}_i}{m_i} + \frac{\mathbf{S}_j}{m_j}\right) \cdot \mathbf{L} \ \mathbf{F}_i \cdot \mathbf{F}_j$$
- Relativistic effects
  Inserted in the Hamiltonian by means of phenomenological parameterization

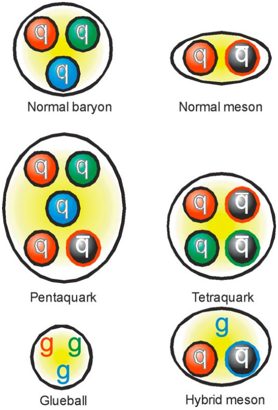
Charmonium spectrum in the relativized QM

S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985)



Exotic Hadrons: Possible Interpretations

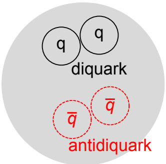
See e.g. Y. R. Liu *et al.*, Prog. Part. Nucl. Phys. **107**, 237 (2019)



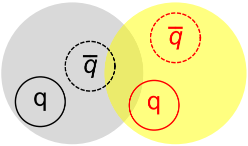
- $X(3872)$ [now $\chi_{c1}(3872)$]. Discovered by Belle in 2003 in $B^{\pm} \to K^{\pm} X$ ($X \to J/\psi \pi^+ \pi^-$).
  First observed exotic state. Its properties are not completely compatible with those of a $c\bar{c}$ state
- Exotics: Meson and baryon states whose properties and/or quantum numbers cannot be described by considering $q\bar{q}$ or qqq degrees of freedom only
- Multiquark states:
  1. baryons made up of more than 3 valence quarks
  $qqqq\bar{q}$ → pentaquarks
  $qqqqqq$ → exa-quarks or dibaryons
  2. mesons made up of more than a $q\bar{q}$ pair
  $qq\bar{q}\bar{q}$ → tetraquarks
- Hybrid mesons/baryons: hadrons made up of qqq or $q\bar{q}$ valence quarks plus gluonic degrees of freedom
- Glueballs: particles consisting of gluonic degrees of freedom only, without valence quarks

Tetraquarks

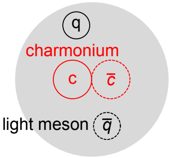
- Tetraquarks: bound states of four valence quarks/antiquarks ($qq\bar{q}\bar{q}$)
  R.L. Jaffe, Phys. Rev. D **15**, 267 & 281 (1977)
- Combine 4 quarks in terms of $qq/\bar{q}\bar{q}$ or $q\bar{q}$ substructures:
  1. Compact Tetraquark model: $\left[[qq]_{\bar{\mathbf{3}}_c}[\bar{q}\bar{q}]_{\mathbf{3}_c}\right]_{\mathbf{1}_c}$
  2. Meson-meson molecular mode: $[[q\bar{q}]_{\mathbf{1}_c}[q\bar{q}]_{\mathbf{1}_c}]_{\mathbf{1}_c}$
  3. Hadro-charmonium model: $\left[[q\bar{q}]_{\mathbf{1}_c}[Q\bar{Q}]_{\mathbf{1}_c}\right]_{\mathbf{1}_c}$



compact tetraquark



molecule



hadro–charmonium

- Possible mixing between $q\bar{q}$ and $q\bar{q}q\bar{q}$ components
  Unquenched Quark Model (UQM)

Exotic Meson Candidates. Hidden-charm sector

State	J^{PC}	$M_{\rm exp}$ (MeV)	Γ (MeV)	Observing Process	Experiment
$X(3872)$	1^{++}	3871.69 ± 0.17	< 1.7	$B^\pm \to K^\pm \pi^+ \pi^- J/\psi$	Belle
$Z_c(3900)$	1^{+-}	3886.6 ± 2.4	28.1 ± 2.6	$e^+ e^- \to \pi^+ \pi^- J/\psi$	BESIII
$Y(4008)$	1^{--}	4008 ± 40	226 ± 44	$e^+ e^- \to \gamma_{\rm ISR} \pi^+ \pi^- J/\psi$	Belle
$Z_c(4020)^\pm$	1^{+-}	4024.1 ± 1.9	13 ± 5	$e^+ e^- \to \pi^+ \pi^- h_c$	BESIII
$X(4140)$	1^{++}	4146.8 ± 2.5	19^{+8}_{-7}	$\gamma\gamma \to \phi J/\psi$	CDF
$Z_c(4240)^\pm$	0^-	$4239 \pm 18^{+45}_{-10}$	$220 \pm 47^{+108}_{-74}$	$B^0 \to K^+ \pi^- \psi(2S)$	LHCb
$Y(4260)$	1^{--}	4230 ± 8	55 ± 19	$e^+ e^- \to \gamma_{\rm ISR} \pi^+ \pi^- J/\psi$	BaBar
$X(4274)$	1^{++}	4273^{+19}_{-9}	56^{+14}_{-16}	$B^+ \to J/\psi \phi K^+$	CDF, LHCb
$Y(4360)$	1^{--}	4341 ± 8	102 ± 9	$e^+ e^- \to \gamma_{\rm ISR} \pi^+ \pi^- \psi(2S)$	Belle
$Z_c(4430)^\pm$	1^+	4478^{+15}_{-18}	181 ± 31	$B \to K \pi^\pm \psi(2S)$	Belle
$X(4500)$	0^{++}	4506^{+16}_{-19}	92 ± 29	$B^+ \to J/\psi \phi K^+$	LHCb
$Y(4630)$	1^{--}	4634^{+8}_{-7}	92^{+40}_{-24}	$e^+ e^- \to \Lambda_c^+ \Lambda_c^-$	Belle
$Y(4660)$	1^{--}	4643 ± 9	72 ± 11	$e^+ e^- \to \gamma_{\rm ISR} \pi^+ \pi^- \psi(2S)$	Belle
$X(4700)$	0^{++}	4704^{+17}_{-26}	120 ± 50	$B^+ \to J/\psi \phi K^+$	LHCb

Exotic Mesons. The well-known case of the $X(3872)$

- $X(3872)$: $J^{\rm PC} = 1^{++}$ quantum numbers, narrow width (< 1.2 MeV)
- Mass Problem:
  Experimental mass: 3871.69 ± 0.17 MeV [PDG]
  Incompatible with QM predictions. For example, Relativized QM → 3.95 GeV
  X(3872) very close to $D^0 \bar D^{*0}$ threshold ⇒ Importance of threshold effects?
- Peculiar decay modes:
  Main decay modes are into $D^0 \bar D^0 \pi^0$ (Br > 0.4) and $D^0 \bar D^{*0}$ (Br > 0.3)
  $J/\psi\rho$ (isospin violating, Br > 0.032) and $J/\psi\omega$ (Br > 0.023) decay modes
  Decay properties not compatible with those of a $c\bar c$ state
- Several interpretations:
  $D\bar D^*$ meson-meson molecule
  Compact tetraquark (diquark-antidiquark) state
  $c\bar c$ core + meson-meson pair loops (Unquenched Quark Model interpretation)
  Hadro-charmonium

Meson-meson molecular model

See e.g. N. A. Törnqvist, Z. Phys. C **61**, 525 (1994)

- Meson-meson molecules: bound states of two mesons
  Binding energy is relatively small
  Emergence of molecular states only close to meson-meson thresholds
- Binding: one-meson exchange forces and/or contact interactions
- One-pion-exchange (OPE) potential
  $V_\pi(q) = -\frac{g^2}{f_\pi^2}\left(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\right) \frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q})}{q^2+m_\pi^2}$
  $\boldsymbol{\sigma}_i$ and $\boldsymbol{\tau}_i$ $(i=1,2)$ → spin and isospin matrices acting on light quarks i
  $f_\pi \approx 132$ MeV → pion decay constant
  m_π → pion mass
  g → dimensionless coupling constant
- Molecule wave function
  Obtained by combining the wave functions of the two meson
  Consider also relative orbital angular momentum between two mesons

Hadro-charmonium (hadro-quarkonium) model

S. Dubynskiy and M. B. Voloshin, Phys. Lett. B **666**, 344 (2008)
J. Ferretti, Phys. Lett. B **782**, 702 (2018)

- Heavy $Q\bar{Q}$ state embodied in a light $q\bar{q}$ one
  ($Q = c$ or b; $q = u, d$ or s)
- Binding provided by multiple gluon-exchange forces
  QCD analog of van der Waals-type interactions
- Parametrization of the heavy quarkonium-light meson interaction:

$$V_{\rm hq}(r) = \begin{cases} -\frac{2\pi\alpha_{Q\bar{Q}} M_{\rm M}}{3R_{\rm M}^3} & \text{for} \quad r < R_{\rm M} \\ 0 & \text{for} \quad r > R_{\rm M} \end{cases}$$

$R_{\rm M}$ = light meson radius, $M_{\rm M}$ = light meson mass
$\alpha_{Q\bar{Q}}$ = quarkonium diagonal chromo-electric polarizability

- Hamiltonian:

$$H_{\rm hq} = M_{Q\bar{Q}} + M_{\rm M} + V_{\rm hq}(r) + T_{\rm hq}$$

$T_{\rm hq} = \frac{k^2}{2\mu}$, k = relative momentum, μ = reduced mass of the system

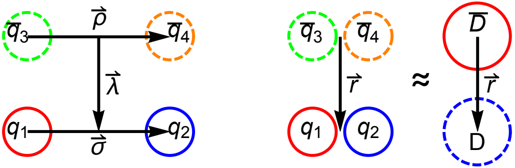
Unquenched quark model (UQM)

K. Heikkila, S. Ono and N. A. Törnqvist, Phys. Rev. D **29**, 110 (1984)
J. Ferretti, G. Galatà and E. Santopinto, Phys. Rev. C **88**, 015207 (2013)

- Bare meson $|\mathrm{A}\rangle$ develops a meson-meson continuum component $|\mathrm{BC}\rangle$
$$|\psi_\mathrm{A}\rangle = \mathcal{N}_\mathrm{A}\left[|\mathrm{A}\rangle + \sum_{\mathrm{BC}}\int d\mathbf{k}\,|\mathrm{BC}\mathbf{k}\rangle \frac{\langle \mathrm{BC}\mathbf{k}|\,H_{q\bar q}\,|\mathrm{A}\rangle}{M_\mathrm{A} - E_\mathrm{B} - E_\mathrm{C}}\right]$$
$|\mathrm{A}\rangle = \left|Q\bar Q\right\rangle$; $|\mathrm{BC}\rangle = \left|q\bar Q - Q\bar q\right\rangle$; $H_{q\bar q}$ = pair-creation operator
- UQM Hamiltonian
$H_\mathrm{UQM} = H_\mathrm{A} + H_\mathrm{BC} + H_{q\bar q}$
$H_\mathrm{A}(H_{\mathrm{B,C}}) \to Q\bar Q$ $(q\bar Q)$ bare meson Hamiltonian of A (B and C) → REL QM
$H_{q\bar q} \to$ couples A to BC meson-meson components by creating a light $q\bar q$ pair
- Physical Mass of a meson A:
$M_\mathrm{A} = E_\mathrm{A} + \Sigma(M_\mathrm{A})$ = bare mass + self-energy correction
- Self-energy corrections
$$\Sigma(M_\mathrm{A}) = \sum_{\mathrm{BC}}\int_0^\infty d\mathbf{k}\,\frac{\left|\langle \mathrm{BC}\mathbf{k}|\,H_{q\bar q}\,|\mathrm{A}\rangle\right|^2}{M_\mathrm{A} - E_\mathrm{B} - E_\mathrm{C}}$$
Sum over a complete set of intermediate states $|\mathrm{BC}\rangle$

Compact tetraquark (diquark-antidiquark) model (I)

- Compact tetraquarks: diquark-antidiquark bound states
  4 body problem → 2 body one



- Diquark: Effective bosonic degree of freedom
  2 strongly correlated quarks with no internal spatial excitation

Compact tetraquark (diquark-antidiquark) model (II)

- Possible diquark color configurations:
  color antitriplet ($\bar{\mathbf{3}}_{\rm c}$, attractive interaction)
  color sextet ($\mathbf{6}_{\rm c}$, repulsive interaction) $\Rightarrow$ $\bar{\mathbf{3}}_{\rm c}$ is favored
- Diquark WF: $\Psi_{\rm D} = \Psi_{\rm space}\Psi_{\rm color}\Psi_{\rm spin-flavor}$ is antisymmetric (Pauli principle) $\Rightarrow$ $\Psi_{\rm spin-flavor}$ is symmetric
- Diquark spin-flavor wave function: decomposed in terms of
  $S = 0$, $\bar{\mathbf{3}}_{\rm f}$ (scalar diquark)
  $S = 1$, $\mathbf{6}_{\rm f}$ (axial-vector diquark)
  OGE calculations $\Rightarrow$ scalar configuration is energetically favored

Compact tetraquark (diquark-antidiquark) model (III)

- Different approaches to study tetraquark properties:
  Potential models
  Algebraic models
  QCD calculations
  Mass inequality relations
  and so on
- Diquark models for tetraquarks: Several versions
  L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, PRD **71**, 014028 (2005)
  E. Santopinto and G. Galatà, PRC **75**, 045206 (2007)
  D. Ebert, R. N. Faustov, V. O. Galkin and W. Lucha, PRD **76**, 114015 (2007)
  M. N. Anwar, J. Ferretti and E. Santopinto, PRD **98**, 094015 (2018)
  and so on

Relativized tetraquark model. Hamiltonian

M. Naeem Anwar, J. Ferretti, F.-K. Guo, E. Santopinto and B.-S. Zou, EPJ C **78**, 647 (2018)
M. Naeem Anwar, J. Ferretti and E. Santopinto, Phys. Rev. D **98**, 094015 (2018)

- Hamiltonian: $H^{\rm REL} = T + V_{D\bar D}(r)$
- OGE + linear confining potentials:
  $$V_{D\bar D}(r) = \beta r + G(r) + \frac{2\mathbf{S}_D\cdot\mathbf{S}_{\bar D}}{3m_D m_{\bar D}}\nabla^2 G(r) - \frac{3\mathbf{S}_D\cdot\hat{\mathbf{r}}\;\mathbf{S}_{\bar D}\cdot\hat{\mathbf{r}} - \mathbf{S}_D\cdot\mathbf{S}_{\bar D}}{3m_D m_{\bar D}}\left(\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r}\right) G(r) + \Delta E$$
- Coulomb-like piece:
  $$G(r) = -\frac{4\alpha_{\rm s}(r)}{3r} = -\sum_k \frac{4\alpha_k}{3r}\,{\rm Erf}(\tau_{D\bar D k} r) \;; \quad \tau_{D\bar D k} = \frac{\gamma_k\sigma_{D\bar D}}{\sqrt{\sigma^2_{D\bar D}+\gamma_k^2}}$$
  $$\sigma_{D\bar D} = \sqrt{\frac{1}{2}\sigma_0^2\left[1+\left(\frac{4m_D m_{\bar D}}{(m_D+m_{\bar D})^2}\right)^4\right] + s^2\left(\frac{2m_D m_{\bar D}}{m_D+m_{\bar D}}\right)^2}$$
  S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985)
  W. Celmaster, H. Georgi and M. Machacek, Phys. Rev. D **17**, 879 (1978)
- Eigenvalue problem: Solved by means of a numerical variational method (harmonic oscillator basis)

Relativized tetraquark model. Model parameters and diquark masses

M. Naeem Anwar, J. Ferretti, F.-K. Guo, E. Santopinto and B.-S. Zou, EPJ C **78**, 647 (2018)
M. Naeem Anwar, J. Ferretti and E. Santopinto, Phys. Rev. D **98**, 094015 (2018)

- Model parameters: Extracted from previous studies or fitted to the experimental masses of hidden-charm XYZ ($c\bar{c}q\bar{q}$ or $c\bar{c}s\bar{s}$) states

Parameter	Value	Parameter	Value	Parameter	Value
α_1	0.25 †	γ_1	2.53 fm^{-1} †	$M_{cq}^{\rm s}$	1933 MeV †
α_2	0.15 †	γ_2	8.01 fm^{-1} †	$M_{cq}^{\rm av}$	2250 MeV
α_3	0.20 †	γ_3	80.1 fm^{-1} †	$M_{cs}^{\rm s}$	2229 MeV
σ_0	9.29 fm^{-1} †	β	3.90 fm^{-2}	$M_{cs}^{\rm av}$	2264 MeV
s	1.55 †	ΔE	-370 MeV		

The values denoted by the symbol † are extracted from previous studies.
$q = u$ or d quark. Superscripts: s = scalar diquark, av = axial-vector diquark

- Diquark masses: calculated by binding qq pairs via OGE + confining potential

Flavor content	Scalar diquark mass [MeV]	Axial-vector diquark mass [MeV]
qq	691	840
qs	886	992
ss	–	1136
qc	2099	2138
sc	2229	2264
cc	–	3329
qb	5451	5465
sb	5572	5585
cb	6599	6611
bb	–	9845

J. Ferretti, Few Body Syst. **60**, 17 (2019)

Hidden-charm $[qc][\bar{q}\bar{c}]$ and $[sc][\bar{s}\bar{c}]$ Tetraquarks (I)

M. Naeem Anwar, J. Ferretti and E. Santopinto, Phys. Rev. D **98**, 094015 (2018)

- Hidden-charm tetraquarks in the relativized $D\bar{D}$ model

State; J^{PC} ($q\bar{q}c\bar{c}$)	$E^{\rm th}$ [MeV]	$E^{\rm exp}$ [MeV]	State; J^{PC} ($s\bar{s}c\bar{c}$)	$E^{\rm th}$ [MeV]	$E^{\rm exp}$ [MeV]
$X(3872)$; 1^{++}	3872	3871.69 ± 0.17	$X(4500)$; 0^{++}	4509	$4506 \pm 11^{+12}_{-15}$
$Z_{\rm c}(3900)$; 1^{+-}	3872	3886.6 ± 2.4	$X(4700)$; 0^{++}	4653	4704^{+17}_{-26}
$Z_{\rm c}(4020)$; 1^{+-}	4047	4024.1 ± 1.9	$X(4140)$; 1^{++}	4159	4146.8 ± 2.5
$Z_{\rm c}(4430)$; 1^{+-}	4517	4478^{+15}_{-18}			
$Y(4008)$; 1^{--}	3960	4008 ± 40			
$Y(4260)$; 1^{--}	4253	4230 ± 8			
$Y(4360)$; 1^{--}	4353	4341 ± 8			
$Y(4630)$; 1^{--}	4642	4634^{+8}_{-7}			
$Y(4660)$; 1^{--}	4670	4643 ± 9			
$Z_{\rm c}(4240)$; 0^{--}	4253	$4239 \pm 18^{+45}_{-10}$			

- Spectrum: several *XYZ* exotic states are accommodated
- The number of predicted states is much larger than in the above table (the large majority of them is not shown here)
  $\Rightarrow$ Emergence of a very rich spectrum (several missing states)

Hidden-charm $[qc][\bar{q}\bar{c}]$ and $[sc][\bar{s}\bar{c}]$ Tetraquarks (II)

M. Naeem Anwar, J. Ferretti and E. Santopinto, Phys. Rev. D **98**, 094015 (2018)

- $[qc][\bar{q}\bar{c}]$ and $[sc][\bar{s}\bar{c}]$ tetraquarks
  1. Heavy-light $[qc][\bar{q}\bar{c}]$ and $[sc][\bar{s}\bar{c}]$ systems ($q = u$ or d)
  2. Provide an explanation of the nature of the $X(3872)$ and other X-type states
  3. Explain the nature of $Z_{\rm c}$ states, like $Z_{\rm c}(3900)$, which contain a $c\bar{c}$ pair and are charged

HOWEVER

- $\chi_{\rm c}(2P)$ charmonium and $[qc][\bar{q}\bar{c}]$ tetraquark multiplets
  1. The $\chi_{\rm c}(2P)$ multiplet contains $h_{\rm c}(2P)$, $\chi_{\rm c0}(2P)$, $\chi_{\rm c1}(2P)$ and $\chi_{\rm c2}(2P)$ states
  2. If $X(3872)$ is accommodated in a tetraquark multiplet $\Rightarrow$ $\chi_{\rm c1}(2P)$ is missing
  3. Experimental search for a $\chi_{\rm c1}(2P)$ state, with $M \approx 3.90 - 3.95$ GeV, to complete the $\chi_{\rm c}(2P)$ multiplet

- UQM interpretation is favored

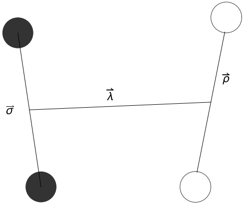
  J. Ferretti, G. Galatà and E. Santopinto, Phys. Rev. C **88**, 015207 (2013)

  J. Ferretti and E. Santopinto, Phys. Lett. B **789**, 550 (2019)

$bb\bar{b}\bar{b}$ Ground-State Tetraquark (I)

M. Naeem Anwar, J. Ferretti, F.-K. Guo, E. Santopinto and B.-S. Zou, EPJ C **78**, 647 (2018)

- Ground-state $bb\bar{b}\bar{b}$ configurations investigated by means of:
  1) Non-relativistic QM
  2) Relativized diquark-antidiquark model



1) Non-relativistic QM interaction

$$\mathcal{H}^{\rm NR} = \sum_{i=1}^{4} T_i + \sum_{i<j} \frac{\boldsymbol{\lambda}_i}{2} \cdot \frac{\boldsymbol{\lambda}_j}{2} \frac{\alpha_s}{|\mathbf{r}_i - \mathbf{r}_j|}$$

Parameters fitted to $b\bar{b}$ spectrum

2) Relativized $D\bar{D}$ model interaction

$$\mathcal{H}^{\rm rel} = T_{D\bar{D}} - \frac{4\alpha_{\rm s}}{3r} - \beta r - V_{\rm spin}(r)$$

Same values as in the previous slides

- Eigenvalue problems: Solved by means of a numerical variational method (harmonic oscillator basis)

$bb\bar{b}\bar{b}$ Ground-State Tetraquark (II)

M. Naeem Anwar, J. Ferretti, F.-K. Guo, E. Santopinto and B.-S. Zou, EPJ C **78**, 647 (2018)

- Calculated $bb\bar{b}\bar{b}$ Ground-State Mass

Reference	Mass [GeV]
This Work (NR)	18.72 ± 0.02
This Work (REL)	18.75 ± 0.05
Karliner *et. al.*	18.862 ± 0.025
Bai *et. al.*	18.69 ± 0.03
Berezhnoy *et. al.*	18.754

1. To be compared with twice the η_b ($b\bar{b}$) mass, 18.798 GeV
2. The results support the possible emergence of a stable $bb\bar{b}\bar{b}$ state

- Search for $\Upsilon(0^{++}/2^{++}) \rightarrow \Upsilon(1S)\mu^+\mu^-$ in *pp* collisions at LHC

CMS Collaboration, JHEP **1705**, 013 (2017); LHCb Collaboration, JHEP **1810**, 086 (2018)

1. Up to now, no effect has been measured
2. $\Upsilon(0^{++}/2^{++}) \rightarrow \Upsilon(1S)\mu^+\mu^-$ width too small to be observed at the LHC?
   A. Esposito and A. D. Polosa, Eur. Phys. J. C **78**, 782 (2018)

- Recently, RHIC claimed the observation of a fully-*b* tetraquark state

L. C. Bland *et al.* [A$_N$DY Collaboration], arXiv:1909.03124

Peak observed in double $\Upsilon(1S)$ production

$M = 18.12 \pm 0.15$ (stat) $\pm$ 0.6 (sys) GeV

The problem of baryon missing resonances

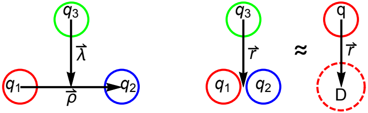
- Missing resonances: States predicted by quark models with no corresponding experimental counterparts
- QMs predict an eccessive number of states
- Possible explanations:

  1. Some baryon states may be very weakly coupled to single-pion channels. Then, one should look for two-pion, three-pion, eta decay channels ...

  S. Capstick and W. Roberts, Phys. Rev. D **49**, 4570 (1994)
  R. Bijker, J. Ferretti, G. Galatà, H. García-Tecocoatzi and E. Santopinto, Phys. Rev. D **94**, 074040 (2016)

  2. Consider models based on smaller number of effective degrees of freedom

  (e.g. quark-diquark model) ⇒ number of missing states decreases notably



Three-quark (left) vs quark-diquark model (right) pictures of baryons.

The problem of baryon missing resonances. Non-π decay channels

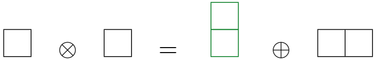
R. Bijker, J. Ferretti, G. Galatà, H. García-Tecocoatzi and E. Santopinto, PRD **94**, 074040 (2016)

Model	Resonance	Status	M [MeV]	$N\pi$	$N\eta$	ΣK	ΛK	$\Delta\pi$
	$N(1440)P_{11}$	****	1430-1470	$110 - 338$	$0 - 5$			$22 - 101$
U(7)	$^2 8_{1/2}[56, 0_2^+]$		1444	85	–	–	–	13
U(7)	$^2 8_{1/2}[56, 0_2^+]$		1444	108	–			22
hQM	$^2 8_{1/2}[56, 0_2^+]$		1550	105	–	–	–	12
	$N(1520)D_{13}$	****	1515-1530	102	0			342
U(7)	$^2 8_{3/2}[70, 1_1^-]$		1563	134	0	–	–	207
U(7)	$^2 8_{3/2}[70, 1_1^-]$		1563	102	0			342
hQM	$^2 8_{3/2}[70, 1_1^-]$		1525	111	0	–	–	206
	$N(1535)S_{11}$	****	1520-1555	$44 - 96$	$40 - 91$			< 2
U(7)	$^2 8_{1/2}[70, 1_1^-]$		1563	63	75	–	–	16
U(7)	$^2 8_{1/2}[70, 1_1^-]$		1563	106	86			14
hQM	$^2 8_{1/2}[70, 1_1^-]$		1525	84	50	–	–	6
	$N(1650)S_{11}$	****	1640-1680	$60 - 162$	$6 - 27$		$4 - 20$	$0 - 45$
U(7)	$^4 8_{1/2}[70, 1_1^-]$		1683	41	72	–	0	18
U(7)	$^4 8_{1/2}[70, 1_1^-]$		1683	71	83			15
hQM	$^2 8_{1/2}[70, 1_2^-]$		1574	51	29	–	0	4
	$N(1675)D_{15}$	****	1670-1685	$46 - 74$	$0 - 2$		< 2	$65 - 99$
U(7)	$^4 8_{5/2}[70, 1_1^-]$		1683	47	11	–	0	108
U(7)	$^4 8_{5/2}[70, 1_1^-]$		1683	29	7			79
hQM	$^4 8_{5/2}[70, 1_1^-]$		1579	41	9	–	–	85

Quark-Diquark Model

F. Wilczek, hep-ph/0409168; R. L. Jaffe, Phys. Rept. **409**, 1 (2005)
E. Santopinto, Phys. Rev. C **72**, 022201 (2005)

- Diquark wave function: $\Psi_{\rm D} = \Psi_{\rm space}\Psi_{\rm color}\Psi_{\rm spin-flavor}$
  $\Psi_{\rm D}$ is antisymmetric (Pauli principle)
- Baryon in color-singlet:
  $\Psi_{\rm color}$ is antisymmetric $\Rightarrow$ $\Psi_{\rm spin-flavor}$ is symmetric, $\Psi_{\rm space}$ being symmetric (diquark = S-wave particles)
- Diquark spin-flavor wave function → Young diagrams



$$\mathbf{6} \otimes \mathbf{6} = \mathbf{15}_{\rm A} \oplus \mathbf{21}_{\rm S}$$

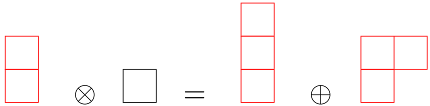
$\mathbf{15}_{\rm A}$ $SU(6)_{\rm sf}$ representation for diquarks is neglected

- $\mathbf{21}_{\rm S}$ decomposition in terms of $SU(2)_{\rm s} \otimes SU(3)_{\rm f}$:

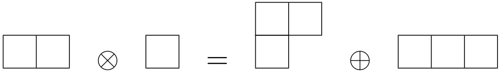
$[\bar{\mathbf{3}}_{\rm f}, S = 0]$ (scalar diquark; energetically favored)
$[\mathbf{6}_{\rm f}, S = 1]$ (axial-vector diquark)

Quark vs Quark-Diquark Model

- $\mathbf{20}_{\mathrm{A}}$ and $\mathbf{70}_{\mathrm{MA}}$ representations neglected in quark-diquark models



$$\mathbf{15}_{\mathrm{A}} \otimes \mathbf{6} = \mathbf{20}_{\mathrm{A}} \oplus \mathbf{70}_{\mathrm{MA}}$$



$$\mathbf{21}_{\mathrm{S}} \otimes \mathbf{6} = \mathbf{70}_{\mathrm{MS}} \oplus \mathbf{56}_{\mathrm{S}}$$

- State-space reduced with respect to three QMs
- No state belonging to the $\mathbf{20}_{\mathrm{A}}$ multiplet ever observed
- Possible explanation to the problem of missing resonances

Interacting Quark-Diquark Model (I)

E. Santopinto, Phys. Rev. C **72**, 022201 (2005)

- Model Hamiltonian: $H = \Delta + \frac{q^2}{2\mu} + V_{\rm dir} + V_{\rm ex}$
  $\mu =$ reduced mass of the qD system
- Splitting between S and AV diquark configurations:
  $\Delta = B\delta_{S_{\rm D},1} + C\delta_0$ (δ_0 acts only on the ground-state)
- Direct potential $V_{\rm dir} = -\frac{\tau}{r} + \beta r$
- Exchange potential:
  $V_{\rm ex} = 2A(-1)^{L+1} e^{-\alpha r} \left[\mathbf{S}_{\rm D} \cdot \mathbf{S}_{\rm q} + \mathbf{T}_{\rm D} \cdot \mathbf{T}_{\rm q} + 2\left(\mathbf{S}_{\rm D} \cdot \mathbf{S}_{\rm q}\right)\left(\mathbf{T}_{\rm D} \cdot \mathbf{T}_{\rm q}\right)\right]$
  $\mathbf{S}_{\rm q,D}$ and $\mathbf{T}_{\rm q,D}$ $\rightarrow$ spin and isospin operators of the quark and the diquark
- Eigenvalues of pure Coulomb Hamiltonian: $E_{n,L} = -\frac{\tau^2\mu}{2n^2}$ $(n = 1, 2, ...)$
  Other interactions treated as perturbations
- Model parameters: fitted to experimental data

Relativized Interacting Quark-Diquark Model

J. Ferretti, A. Vassallo and E. Santopinto, Phys. Rev. C **83**, 065204 (2011)

- Model Hamiltonian:
  $H = E_0 + \sqrt{\mathbf{q}^2 + m_{\rm q}^2} + \sqrt{\mathbf{q}^2 + m_{\rm D}^2} + V_{\rm dir} + V_{\rm ex} + V_{\rm cont}$
  $E_0 =$ constant, $m_{\rm q,D} =$ quark and diquark masses
- Direct potential: $V_{\rm dir} = -\frac{\tau}{r}(1 - e^{-\mu r}) + \beta r$
  Coulomb-like potential regularized in the origin
- Exchange potential: $V_{\rm ex} = (-1)^{L+1} e^{-\sigma r} \left[A_{\rm S} \mathbf{S}_{\rm D} \cdot \mathbf{S}_{\rm q} + A_{\rm I} \mathbf{T}_{\rm D} \cdot \mathbf{T}_{\rm q}\right.$
  $\left.+ A_{\rm SI} \left(\mathbf{S}_{\rm D} \cdot \mathbf{S}_{\rm q}\right) \left(\mathbf{T}_{\rm D} \cdot \mathbf{T}_{\rm q}\right)\right]$
- Contact potential
  $V_{\rm cont} = \left(\frac{m_{\rm q} m_{\rm D}}{E_{\rm q} E_{\rm D}}\right)^{1/2+\epsilon} \frac{\eta^3 D}{\pi^{3/2}} e^{-\eta^2 r^2} \left(\frac{m_{\rm q} m_{\rm D}}{E_{\rm q} E_{\rm D}}\right)^{1/2+\epsilon} \delta_{L,0} \delta_{S_{\rm D},1}$
  δ-simulating function → Introduced to reproduce $\Delta - N$ mass splitting
- Eigenvalues of H: Numerical variational method (h.o. wave trial functions)
- Model parameters (14) fitted to experimental data [PDG]

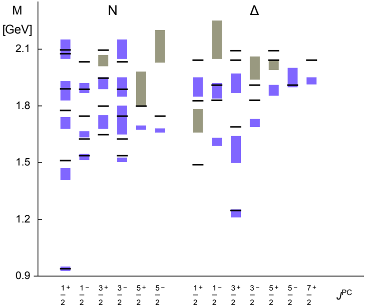
Extension to Strange Baryons (I)

E. Santopinto and J. Ferretti, Phys. Rev. C **92**, 025202 (2015)

- Model Hamiltonian:
  $H = E_0 + \sqrt{\mathbf{q}^2 + m_{\rm q}^2} + \sqrt{\mathbf{q}^2 + m_{\rm D}^2} + V_{\rm dir} + V_{\rm ex}$
- Direct potential: $V_{\rm dir} = -\frac{\tau}{r}(1 - e^{-\mu r}) + \beta r$
- Exchange potential → Gürsey-Radicati inspired interaction:
  $V_{\rm ex} = (-1)^{L+1} {\rm e}^{-\sigma r} \left[A_{\rm S} \mathbf{S}_{\rm D} \cdot \mathbf{S}_{\rm q} + A_{\rm I} \mathbf{T}_{\rm D} \cdot \mathbf{T}_{\rm q} + A_{\rm F} \boldsymbol{\lambda}_{\rm D} \cdot \boldsymbol{\lambda}_{\rm q}\right]$
  $\boldsymbol{\lambda}_{\rm q,D}$ = SU(3)$_{\rm f}$ Gell-Mann matrices
- Global fit to strange and nonstrange baryons
  Model parameters (18) fitted to the existing experimental data [PDG]

Extension to Strange Baryons (II). Nonstrange Spectrum

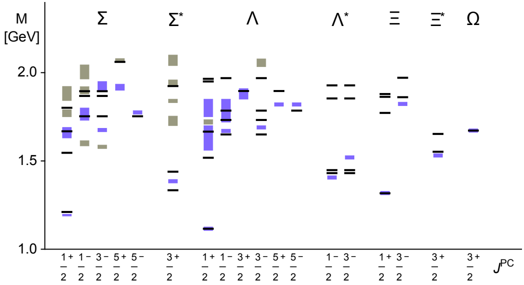
E. Santopinto and J. Ferretti, Phys. Rev. C **92**, 025202 (2015)



Comparison between the Relativized Interacting Quark-Diquark Model predictions (black lines) and the experimental data. Blue boxes stand for $***$ and $****$ PDG states, grey boxes for $*$ and $**$ PDG states.

Extension to Strange Baryons (III). Strange Spectrum

E. Santopinto and J. Ferretti, Phys. Rev. C **92**, 025202 (2015)



Comparison between the Relativized Interacting Quark-Diquark Model predictions (black lines) and the experimental data. Blue boxes stand for $***$ and $****$ PDG states, grey boxes for $*$ and $**$ PDG states.

Extension to Strange Baryons (IV)

E. Santopinto and J. Ferretti, Phys. Rev. C **92**, 025202 (2015)

- Some missing states in the strange sector
  $\Rightarrow$ many resonances still to be discovered (strange spectrum is poorly known)
- Strange spectrum very important in the study of $\Lambda_{\rm b}$'s and pentaquarks
- $\Lambda_{\rm b}^0$ decay modes:
  1) $\Lambda_{\rm b}^0 \to J/\Psi \Lambda^*$ (conventional, involves hyperons)
  2) $\Lambda_{\rm b}^0 \to P_{\rm c}^+ K^-$ (pentaquarks)

  R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **115**, 072001 (2015); **117**, 082002 (2016); **117**, 082003 (2016)
- New pentaquarks recently discovered by LHCb:
  R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **122**, 222001 (2019)

Conclusion (I)

- Quark Model reproduces several properties of hadrons
  Spectrum (lower part), e.m. form factors of the nucleon, baryon magnetic moments, hadron strong decays, and so on
- Some issues at higher energies
  Emergence of exotic hadrons, (baryon) missing resonances, and so on
  Degree of freedom problems?
- $q\bar{q}$ (meson) and qqq (baryon) degrees of freedom
  may need to be replaced at higher energies

Conclusion (II)

- Emergence of exotic mesons at higher energies
  The properties of exotic mesons cannot be described in terms of a quark-antiquark valence pair
- Several interpretations for exotic mesons:
  Compact tetraquark states, meson-meson molecules, hadro-quarkonia, quarkonia + threshold corrections, ...
- Relativized diquark model: can accommodate several suspected *XYZ* exotics
  M. Naeem Anwar, J. Ferretti and E. Santopinto, Phys. Rev. D **98**, 094015 (2018)
- Possible emergence of fully-*b* tetraquarks: see the recent results from RHIC
  L. C. Bland *et al.* [A_NDY Collaboration], arXiv:1909.03124
  Peak observed in double-Υ production; $M = 18.12 \pm 0.15$ (stat) $\pm$ 0.6 (sys) GeV
  Results compatible with the relativized diquark model prediction for fully-*b* states: $M = 18.75 \pm 0.05$ GeV
  M. Naeem Anwar, J. Ferretti, F.-K. Guo, E. Santopinto and B.-S. Zou, EPJ C **78**, 647 (2018)

Conclusion (III)

- Three-quark models predict an excess of states
  (problem of missing resonances)
- Possible explanations:
  1. Some baryon states may be very weakly coupled to single-pion channels. Then, one should look for two-pion, three-pion, eta decay channels ...
  S. Capstick and W. Roberts, Phys. Rev. D **49**, 4570 (1994)
  R. Bijker, J. Ferretti, G. Galatà, H. García-Tecocoatzi and E. Santopinto, Phys. Rev. D **94**, 074040 (2016)
  2. Consider models based on smaller number of effective degrees of freedom
  (e.g. quark-diquark model) ⇒ number of missing states decreases notably
- Strange and nonstrange spectra calculated in the REL diquark model
  Global fit to strange and nonstrange baryons (single set of parameters)
  E. Santopinto and J. Ferretti, Phys. Rev. C **92**, 025202 (2015)
- Some missing states in the strange sector
  ⇒ many resonances still to be discovered (strange spectrum is poorly known)
- Strange spectrum very important in the study of Λ_b's and pentaquarks

Conclusion

Thank you for your attention!

Backup Slides

Relativized Interacting Quark-Diquark Model. Model parameters

J. Ferretti, A. Vassallo and E. Santopinto, Phys. Rev. C **83**, 065204 (2011)

Parameter	Value	Parameter	Value	Parameter	Value
m_q	200 MeV	$m_{[n,n]}$	600 MeV	$m_{\{n,n\}}$	950 MeV
τ	1.25	μ	75.0 fm^{-1}	β	2.15 fm^{-2}
$A_{\rm S}$	375 MeV	$A_{\rm I}$	260 MeV	$A_{\rm SI}$	375 MeV
σ	1.71 fm^{-1}	E_0	154 MeV	D	4.66 fm^2
η	10.0 fm^{-1}	ϵ	0.200		

Resulting values of the model parameters.

Extension to Strange Baryons (II)

E. Santopinto and J. Ferretti, Phys. Rev. C **92**, 025202 (2015)

Parameter	Value (Fit 1)	Value (Fit 2)	Parameter	Value (Fit 1)	Value (Fit 2)
m_n	200 MeV	159 MeV	m_s	550 MeV	213 Mev
$m_{[n,n]}$	600 MeV	607 MeV	$m_{[n,s]}$	900 MeV	856 MeV
$m_{\{n,n\}}$	950 MeV	963 MeV	$m_{\{n,s\}}$	1200 MeV	1216 MeV
$m_{\{s,s\}}$	1580 MeV	1352 MeV	τ	1.20	1.02
μ	75.0 fm^{-1}	28.4 fm^{-1}	β	2.15 fm^{-2}	2.36 fm^{-2}
A_S	350 MeV	-436 MeV	A_F	100 MeV	193 MeV
A_I	250 MeV	791 MeV	σ	2.30 fm^{-1}	2.25 fm^{-1}
E_0	141 MeV	150 MeV	ϵ	0.37	—
D	6.13 fm^2	—	η	11.0 fm^{-1}	—

Resulting values of the model parameters. The values denoted as "Fit 1" are obtained by fitting the mass formula to nonstrange and strange baryons, those denoted as "Fit 2" are fitted to the strange sector only.