Baryon Structure, Distribution Amplitudes and diquark correlations

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In collaboration with:
J. Segovia, L. Chang, M. Ding and C.D. Roberts
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Form factors and diquarks

- Yesterday, arguments in favor of diquark correlations:
  - Spectrum
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  - Possible 0 crossing in the Neutron FF ratio
  - Flavour decomposition of FF (Q^2 behaviour depending of number of gluon involved)
- Impact on the x-dependent structure of the Nucleon (and excited states) of the presence of a diquarks:
  - Distribution amplitudes
  - pQCD prediction for high-Q^2 FF

Hadrons seen as Fock States

- Lightfront quantization allows to expand hadrons on a Fock basis:

$$|P,\pi\rangle \propto \sum_\beta \Psi_\beta^{q\bar{q}}|q\bar{q}\rangle + \sum_\beta \Psi_\beta^{q\bar{q},q\bar{q}}|q\bar{q},q\bar{q}\rangle + \ldots$$

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- Non-perturbative physics is contained in the N-particles Lightfront-Wave Functions (LFWF) Ψ^N
- Schematically a distribution amplitude φ is related to the LFWF through:

$$\varphi(x) \propto \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2}\Psi(x,k_\perp)$$

S. Brodsky and G. Lepage, PRD 22, (1980)

Nucleon Distribution Amplitudes

- 3 bodies matrix element:

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|P\rangle$$

- 3 bodies matrix element expanded at leading twist:

$$\langle 0|\epsilon^{ijk}u^i_\alpha(z_1)u^j_\beta(z_2)d^k_\gamma(z_3)|P\rangle = \frac{1}{4}\Big[\left(\not{p}C\right)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma V(z_i^-) + \left(\not{p}\gamma_5 C\right)_{\alpha\beta}\left(N^+\right)_\gamma A(z_i^-) - \left(ip^\mu\sigma_{\mu\nu}C\right)_{\alpha\beta}\left(\gamma^\nu\gamma_5 N^+\right)_\gamma T(z_i^-)\Big]$$

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- Isospin symmetry:

$$2T(x_1,x_2,x_3) = \varphi(x_1,x_3,x_2) + \varphi(x_2,x_3,x_1)$$

Evolution and Asymptotic results

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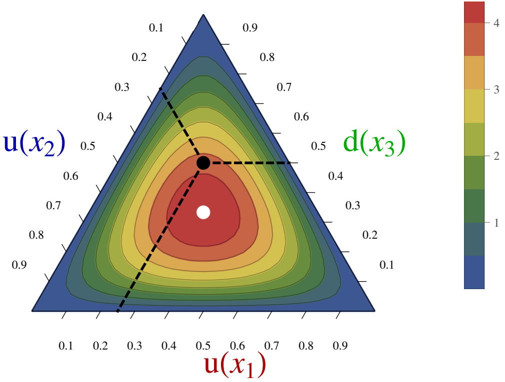
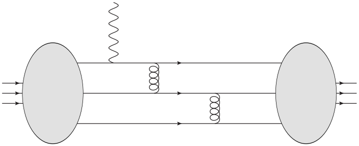


Form Factors: Nucleon case



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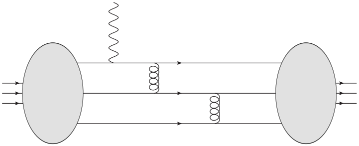

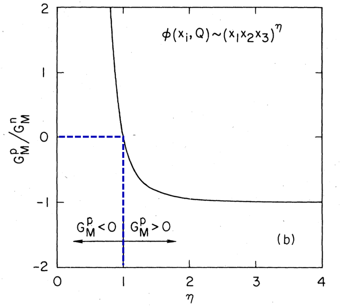

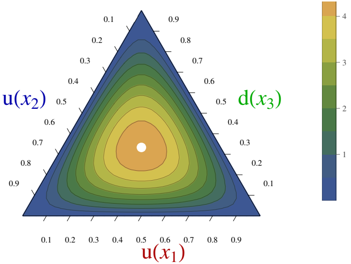

$\eta = 1$

S. Brodsky and G. Lepage, PRD 22, (1980)

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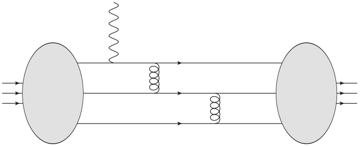
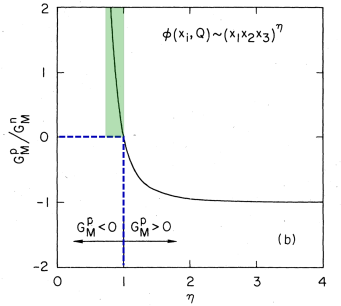
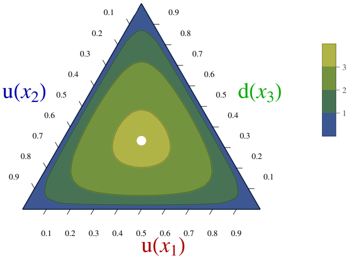
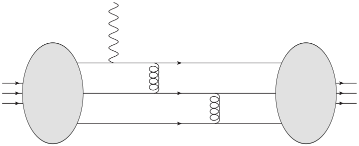


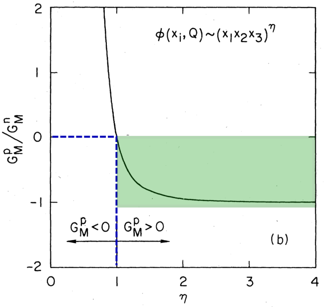


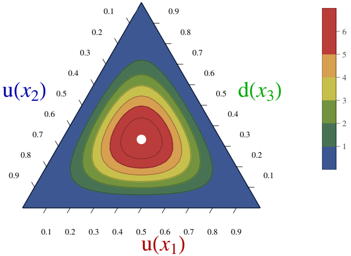


$\eta = 0.5$

S. Brodsky and G. Lepage, PRD 22, (1980)







$\eta = 2$

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Asymptotic DA and vanishing FF

When $Q^2 \to \infty$, $\varphi \to \varphi_{\rm as}$ and become fully symmetric under permutations. One obtains:

$$F_p^1 \propto \int \frac{[\mathrm{d}x_i][\mathrm{d}y_i]}{Q^4} \varphi_{\rm as}(x_i)\varphi_{\rm as}(y_i) \left[(5e_u + e_d)H_1(x_i, y_i) + (e_u + 2e_d)H_2(x_i, y_i)\right]$$

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Caveat: Leading Order analysis only

Some previous studies of DA

- QCD Sum Rules
  - V. Chernyak and I. Zhitnitsky, Nucl. Phys. B 246 (1984)
- Relativistic quark model
  - Z. Dziembowski, PRD 37 (1988)
- Scalar diquark clustering
  - Z. Dziembowski and J. Franklin, PRD 42 (1990)
- Phenomenological fit
  - J. Bolz and P. Kroll, Z. Phys. A 356 (1996)
- Lightcone quark model
  - B. Pasquini *et al.*, PRD 80 (2009)
- Lightcone sum rules
  - I. Anikin *et al.*, PRD 88 (2013)
- Lattice Mellin moment computation
  - G. Bali *et al.*, EPJ. A55 (2019)

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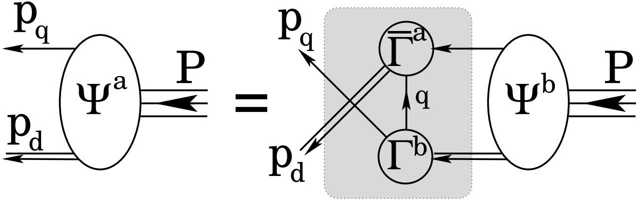

Baryon and Diquarks

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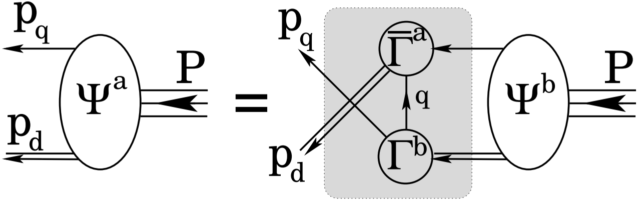

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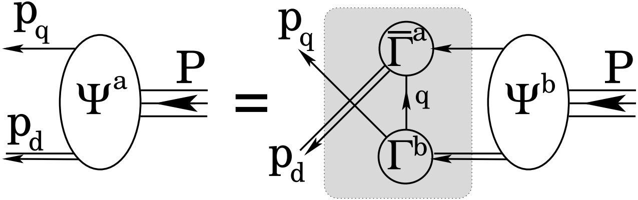

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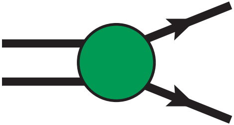
- Can we understand the nucleon structure in terms of quark-diquarks correlations?

Faddeev WF Model

- Algebraic parametrisation inspired by the results obtained from DSEs and Faddeev equations.
- It is based on Nakanishi representation, which is proved to be a good parametrisation of Green functions at all order of perturbation theory.
- We also assume the dynamical diquark correlations, both scalar and AV, and compare in the end with Lattice QCD results.
- This is a work in progress, an update of the previous baryon PDA work toward more realistic results



At all order of perturbation theory, one can write (Euclidean space):

$$\Gamma(k, P) = \mathcal{N} \int_0^{\infty} \mathrm{d}\gamma \int_{-1}^{1} \mathrm{d}z \frac{\rho_n(\gamma, z)}{(\gamma + (k + \frac{z}{2}P)^2)^n}$$

We use a "simpler" version of the latter as follow:

$$\tilde{\Gamma}(q, P) = \mathcal{N} \int_{-1}^{1} \mathrm{d}z \frac{\rho_n(z)}{(\Lambda^2 + (q + \frac{z}{2}P)^2)^n}$$

Nucleon Distribution Amplitude

- Operator point of view for every DA (and at every twist):

$$\langle 0|\epsilon^{ijk}\left(u_{\uparrow}^{i}(z_1)C\not{n}u_{\downarrow}^{j}(z_2)\right)\not{n}d_{\uparrow}^{k}(z_3)|P,\lambda\rangle \to \varphi(x_1,x_2,x_3),$$

Braun *et al.*, Nucl.Phys. B589 (2000)

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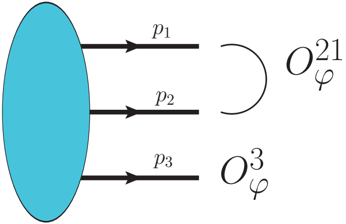

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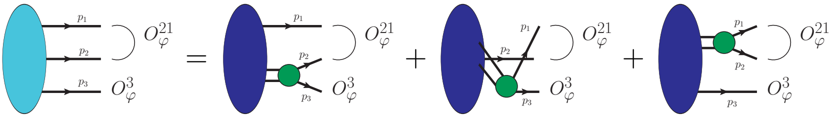

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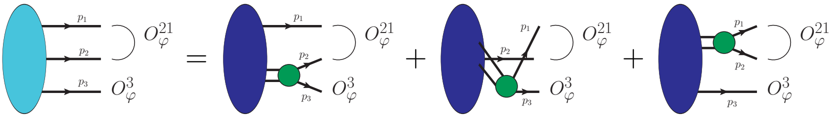

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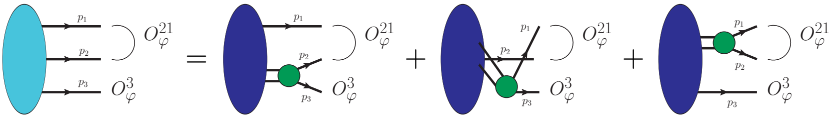

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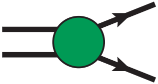
- Our ingredients are:
  - Perturbative-like quark and diquark propagator
  - Nakanishi based diquark Bethe-Salpeter-like amplitude (green disks)
  - Nakanishi based quark-diquark amplitude (dark blue ellipses)

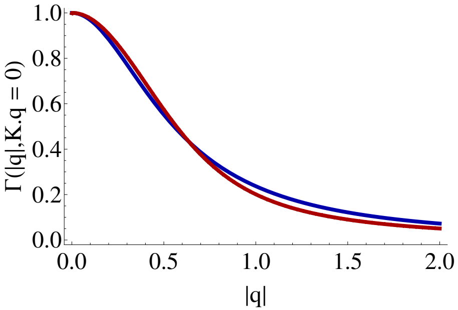
Scalar Diquark BSA

The model used:



$$= \mathcal{N} \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{(\Lambda^2 + (q + \frac{z}{2}K)^2)}$$

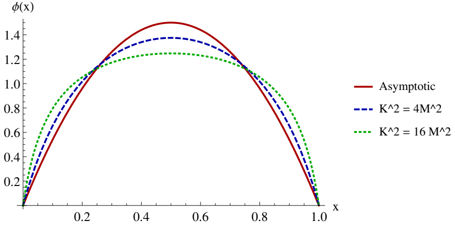
Comparable to scalar diquark amplitude previously used:



red curve from Segovia *et al.*,Few Body Syst. 55 (2014) 1185-1222

$$\phi(x) \propto 1 - \frac{M^2}{K^2} \frac{\ln\left[1 + \frac{K^2}{M^2} x(1-x)\right]}{x(1-x)}$$

Scalar diquark



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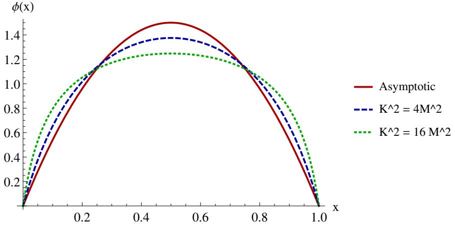
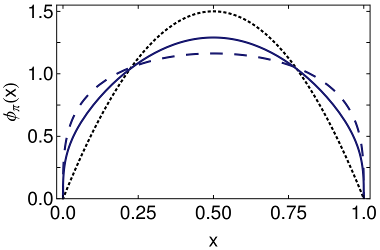


Pion



Pion figure from L. Chang *et al.*, PRL 110 (2013)

Diquark DA

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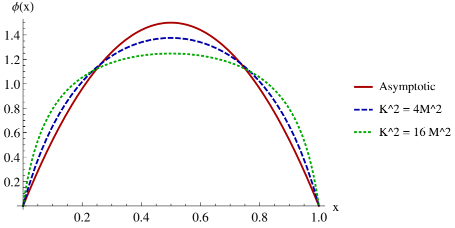

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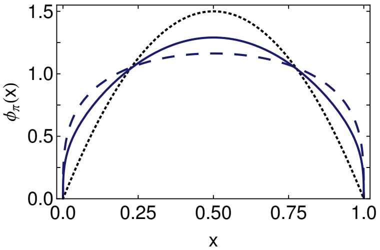
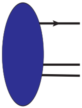


Pion figure from L. Chang *et al.*, PRL 110 (2013)

- This results provide a broad and concave meson DA parametrisation
- The endpoint behaviour remains linear

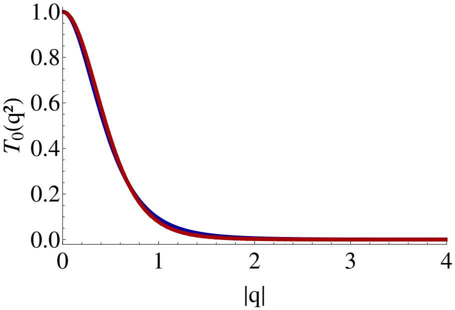
Nucleon Quark-Diquark Amplitude

Scalar diquark case



$$= \mathcal{N} \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)\tilde{\rho}(z)}{(\Lambda^2 + (\ell - \frac{1+3z}{6}P)^2)^3}, \quad \tilde{\rho}(z) = \prod_j (z - a_j)(z - \bar{a}_j)$$

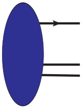
Fits of the parameters through comparison to Chebychev moments:



red curve from Segovia *et al.*,

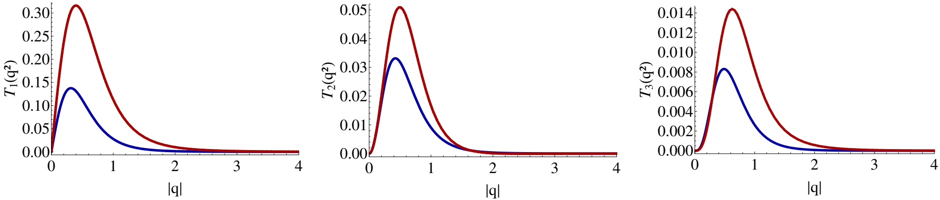
Nucleon Quark-Diquark Amplitude

Scalar diquark case



$$= \mathcal{N}\int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)\tilde{\rho}(z)}{(\Lambda^2 + (\ell - \frac{1+3z}{6}P)^2)^3}, \quad \tilde{\rho}(z) = \prod_j (z-a_j)(z-\bar{a}_j)$$

Fits of the parameters through comparison to Chebychev moments:



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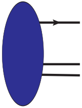

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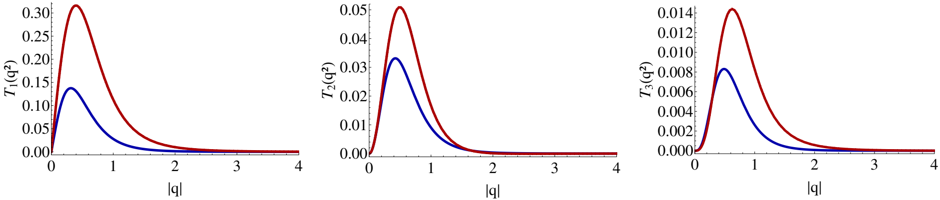

red curves from Segovia *et al.*,

Modification of the $\tilde{\rho}$ Ansatz ? $\tilde{\rho}(z) \to \tilde{\rho}(\gamma, z)$?

Axial-Vector Cases

- At the diquark level:
  - Richer tensorial structure $\rightarrow$ parallel with the vector meson: chiral even/ chiral odd structures $\rightarrow$ longitudinal/transverse polarisation
  - Logarithmic divergences appear $\rightarrow$ need for renormalisation Issue: renormalisation is not included $\rightarrow$ divergences "removed by hand" (additional model dependence)
- At the quark-diquark level:
  - Improve our $\tilde{\rho}$ function
  - Similar situation than in the scalar case

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Work in progress

Mellin Moments

- We do not compute the PDA directly but Mellin moments of it:

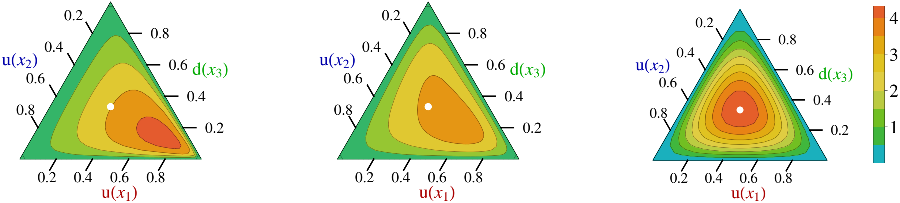
$$\langle x_1^m x_2^n \rangle = \int_0^1 \mathrm{d}x_1 \int_0^{1-x_1} \mathrm{d}x_2 \; x_1^m x_2^n \varphi(x_1, x_2, 1 - x_1 - x_2)$$

- For a general moment $\langle x_1^m x_2^n \rangle$, we change the variable in such a way to write down our moments as:

$$\langle x_1^m x_2^n \rangle = \int_0^1 \mathrm{d}\alpha \int_0^{1-\alpha} \mathrm{d}\beta \; \alpha^m \beta^n f(\alpha, \beta)$$

- f is a complicated function involving the integration on 6 parameters
- Uniqueness of the Mellin moments of continuous functions allows us to identify f and φ

Former results

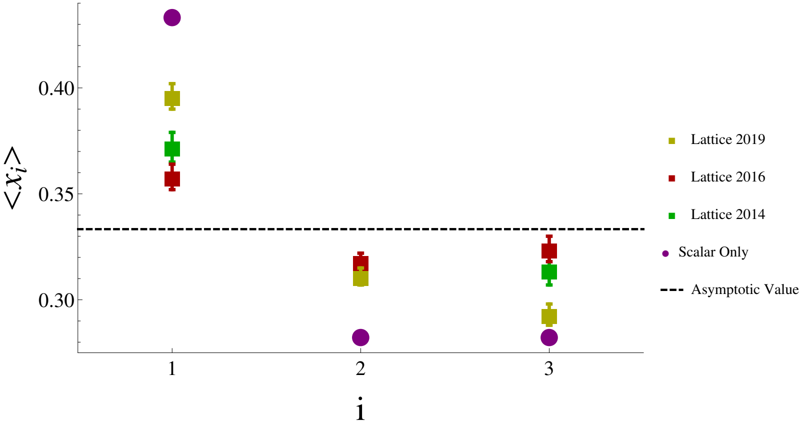


Scalar diquark Only | Nucleon DA | Asymptotic DA

- Typical symmetry in the pure scalar case
- Results evolved from 0.51 to 2 GeV with both scalar and AV diquark
- Nucleon DA is skewed compared to the asymptotic one
- It is also broader than the asymptotic results
- These properties are consequences of our quark-diquark picture

Comparison with lattice

$$< x_i >_\varphi = \int \mathcal{D}x \; x_i \varphi(x_1, x_2, x_3)$$



Lattice data from V.Braun *et al*, PRD 89 (2014)
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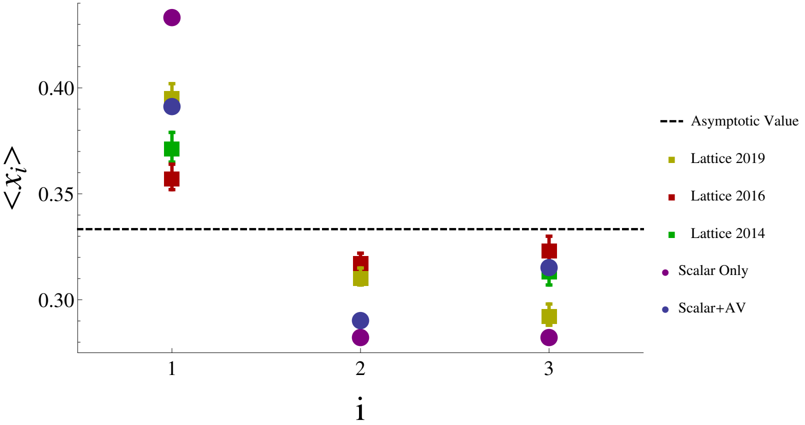

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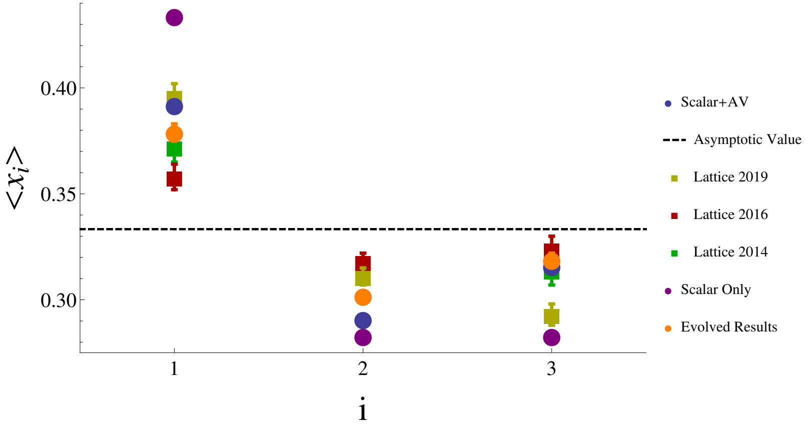

Lattice data from V.Braun *et al*, PRD 89 (2014)
G. Bali *et al.*, JHEP 2016 02
G. Bali *et al.*, EPJ. A55 (2019)

Summary so far

Achievements

- **DSE compatible** framework for Baryon PDAs.
- Based on the Nakanishi representation.
- First results from exploratory work (2017).

Work in progress/future work

- **Priority**: Stabilise the improved Nakanishi Ansätze.
- Better handling of logarithmic divergences.
- Improvement of propagators and additional tensorial structures
- Calculation of the Dirac form factor
- Higher-twist PDA (completely unknown)

Beyond PDA

Hadrons seen as Fock States

- Lightfront quantization allows to expand hadrons on a Fock basis:

$$|P,\pi\rangle \propto \sum_\beta \Psi_\beta^{q\bar{q}}|q\bar{q}\rangle + \sum_\beta \Psi_\beta^{q\bar{q},q\bar{q}}|q\bar{q},q\bar{q}\rangle + \dots$$

$$|P,N\rangle \propto \sum_\beta \Psi_\beta^{qqq}|qqq\rangle + \sum_\beta \Psi_\beta^{qqq,q\bar{q}}|qqq,q\bar{q}\rangle + \dots$$

- Non-perturbative physics is contained in the N-particles Lightfront-Wave Functions (LFWF) Ψ^N
- Schematically a distribution amplitude φ is related to the LFWF through:

$$\varphi(x) \propto \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2}\Psi(x,k_\perp)$$

S. Brodsky and G. Lepage, PRD 22, (1980)

Counting the Nucleon LFWFs

- At the three body level one can define:

$$|P,\uparrow\rangle^{3\,\text{body}} = |P,\uparrow\rangle^{3\,\text{body}}_{\downarrow\downarrow\downarrow} + |P,\uparrow\rangle^{3\,\text{body}}_{\{\downarrow\downarrow\uparrow\}} + |P,\uparrow\rangle^{3\,\text{body}}_{\{\uparrow\downarrow\uparrow\}} + |P,\uparrow\rangle^{3\,\text{body}}_{\uparrow\uparrow\uparrow}$$

- In total, one has 6 independent LFWFs carrying a given amount of OAM.

X. Ji, J.P. Ma and F. Yuan, Nucl.Phys. B652 (2003) 383-404

- Two sources of OAM : quark-diquark and inside the diquark itself.
- All 6 LFWFs are projections of the Faddeev Amplitude.

OAM and 3D structure

- Hadron 3D structure in coordinate space is encoded in GPDs
- GPDs can be computed as an overlap of LFWFs

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GPDs have to fulfill theoretical constraints (polynomiality, positivity...).
We have derived to consistent way to fulfill them a priori.

N. Chouika *et al.*, Eur.Phys.J. C77 (2017) no.12, 906
N. Chouika *et al.*, Phys.Lett. B780 (2018) 287-293

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Final Goals

- Impact of OAM on the 3D structure
- Impact of diquarks (and their own structure!) on the nucleon one

Beyond FF: TDA

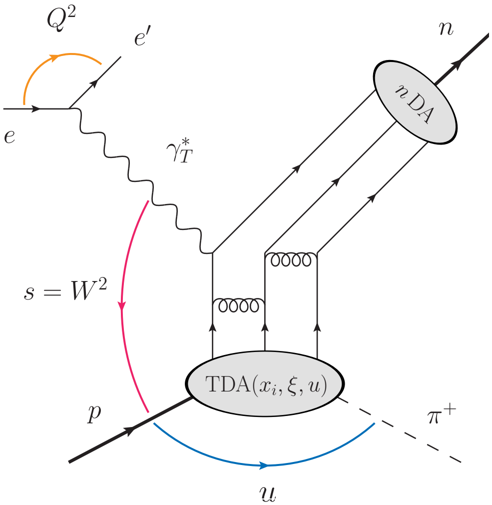


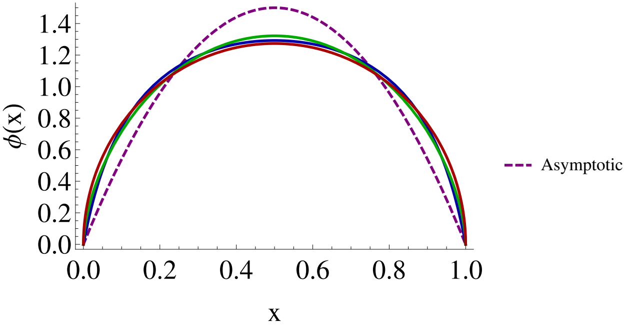
figure from K. Park *et al.*, Phys. Lett. B 780 340-345 (2018)

Addendum:
Form Factors in pQCD

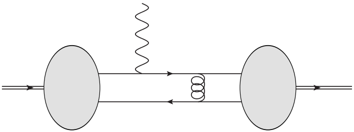
INFN Istituto Nazionale di Fisica Nucleare

$$\langle x^{-1}\rangle = \int_0^1 \mathrm{d}x \frac{\varphi(x)}{1-x} \qquad \phi_{\ln}(x) \propto 1 - \frac{\ln\left[1+\kappa x(1-x)\right]}{\kappa x(1-x)}$$

	$x(1-x)$	$\phi_{\ln}(x)$	$(x(1-x))^{\nu}$	$\sqrt{x(1-x)}$
$\langle x^{-1}\rangle$	3	3.41	3.66	4
$\frac{\langle x^{-1}\rangle}{\langle x^{-1}\rangle_{As}}$	1	1.14	1.22	1.33

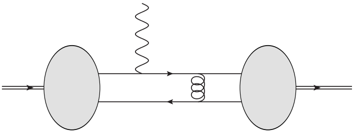


Form Factors



$$Q^2 F(Q^2) = \mathcal{N} \int [\mathrm{d}x_i][\mathrm{d}y_i] \varphi(x, \zeta_x^2) T(x, y, Q^2, \zeta_x^2, \zeta_y^2) \varphi(y, \zeta_y^2)$$

Form Factors



$$Q^2 F(Q^2) = \mathcal{N} \int [\mathrm{d}x_i][\mathrm{d}y_i] \varphi(x, \zeta_x^2) T(x, y, Q^2, \zeta_x^2, \zeta_y^2) \varphi(y, \zeta_y^2)$$

- LO Kernel and NLO kernels are known
- $T_0 \propto \frac{\alpha_S(\mu_R^2)}{(1-x)(1-y)}$
- $T_1 \propto \frac{\alpha_s^2(\mu_R^2)}{(1-x)(1-y)} (f_{UV}(\mu_R^2) + f_{IR}(\zeta^2) + f_{finite})$

R Field *et al.*, NPB 186 429 (1981)
F. Dittes and A. Radyushkin, YF 34 529 (1981)
B. Melic *et al.*, PRD 60 074004 (1999)

- The UV scale dependent term behaves like:

$$f_{UV}(\mu_R^2) \propto \beta_0 \left(5/3 - \ln((1-x)(1-y)) + \ln\left(\frac{\mu_R^2}{Q^2}\right)\right)$$

- Here I take two examples:
  - the standard choice of $\zeta_x^2 = \zeta_y^2 = \mu^2 = Q^2/4$
  - the regularised BLM-PMC scale $\zeta_x^2 = \zeta_y^2 = \mu^2 = e^{-5/3}Q^2/4$

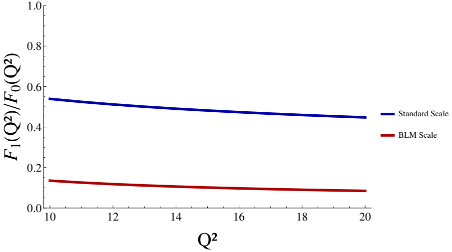
S. Brodsky *et al.*, PRD 28 228 (1983)
S. Brodsky and L. Di Giustino, PRD 86 085026 (2011)

- Take the PDA model coming from the scalar diquark:

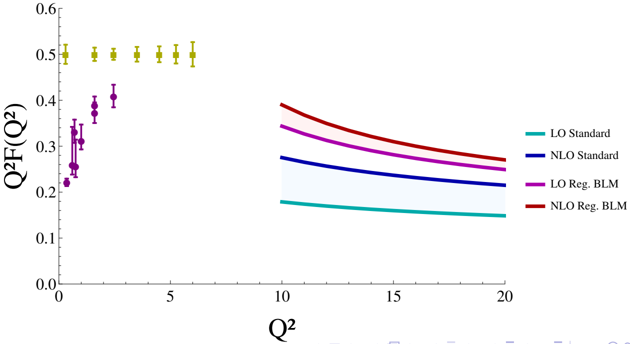
$$\phi(x) \propto 1 - \frac{\ln\left[1 + \kappa x(1-x)\right]}{\kappa x(1-x)}$$

κ is fitted to the lattice Mellin Moment

Pion FF

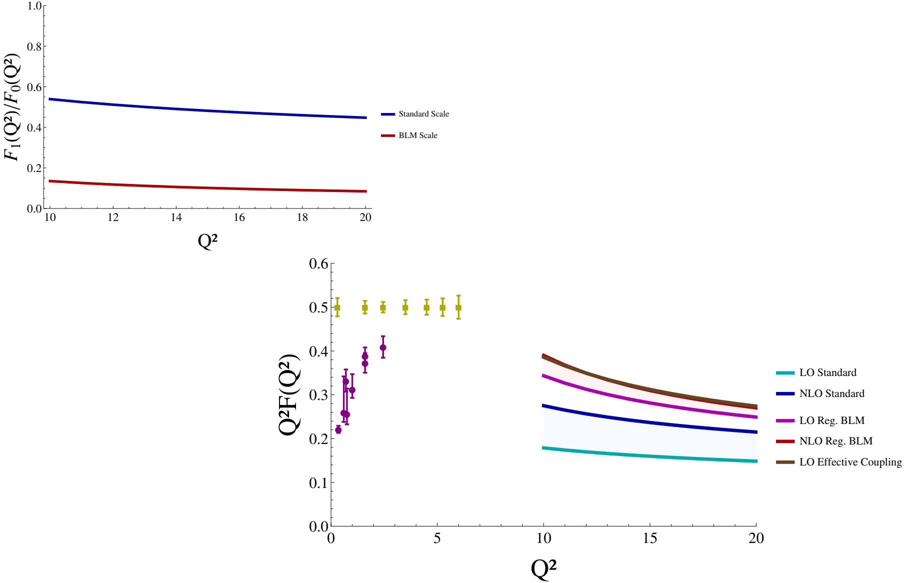


$F_1(Q^2)/F_0(Q^2)$
Q^2
Standard Scale
BLM Scale



$Q^2F(Q^2)$
Q^2
LO Standard
NLO Standard
LO Reg. BLM
NLO Reg. BLM

Pion FF



1.0
0.8
0.6
0.4
0.2
0.0
$F_1(Q^2)/F_0(Q^2)$
10
12
14
16
18
20
Q^2
Standard Scale
BLM Scale
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$Q^2F(Q^2)$
0
5
10
15
20
Q^2
LO Standard
NLO Standard
LO Reg. BLM
NLO Reg. BLM
LO Effective Coupling

Pion FF

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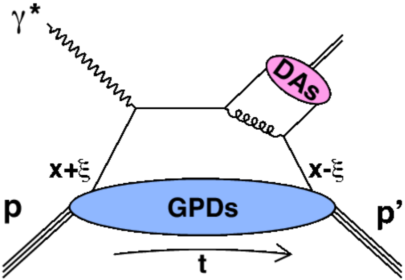
- BLM scale reduces significantly the impact of the NLO corrections and increase dramatically the LO one.

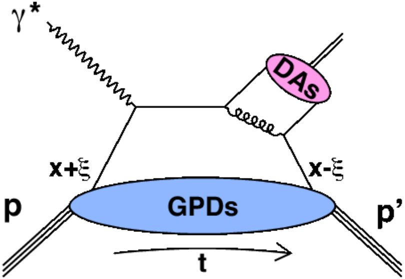
DVMP



- LO Transition Form Factor $\propto \langle x^{-1} \rangle$
- At NLO : $g_{UV} \propto \beta_0 \left(5/3 - \ln((1-u)(1-v)) + \ln\left(\frac{\mu_R^2}{Q^2}\right)\right)$
- Shape effects are also magnified

D. Müller *et al.*, Nucl.Phys. B884 (2014) 438-546

DVMP



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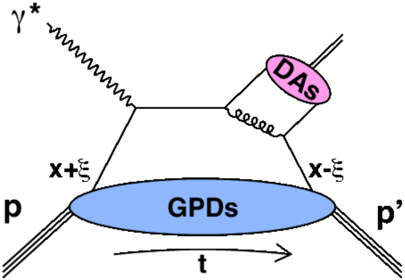
Bottom Line

A good knowledge of the PDA is a key point to perform reliable extraction of GPDs though DVMP

DVMP



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Optimism

Our understanding of PDA is much better today than 10 years ago

Summary

Summary and Conclusion

Diquarks and Distribution Amplitudes

- A formalism in which we have extended diquark correlations
- Computation of x dependent quantities: DA
- Impact of the nature and structure of the diquarks on the nucleon PDA
- Step 1: exploration completed
- Improvements are in progress

Beyond Distribution Amplitudes

- Lightfront wave functions calculations are achievable
- Contributions of the first Fock components to GPDs could be studied

Thank you for your attention

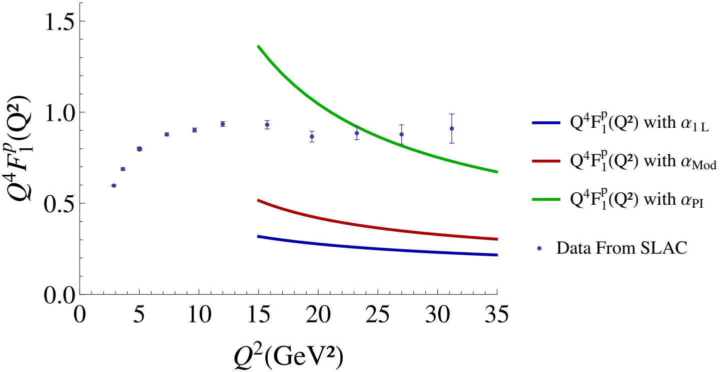
Back up slides

Proton case

- Unfortunately, only the LO treatment has been performed
  $\Rightarrow$ BLM scale is therefore unknown
- We use the Chernyak-Zhitnitsky formalism to compute the nucleon for factor with:
  - the CZ scale setting $\to \alpha_s(Q^2/9)\alpha_s(4Q^2/9)$
  - the pion BLM factor $\to \alpha_s(Q^2/9\, e^{-5/3})\alpha_s(4Q^2/9\, e^{-5/3})$

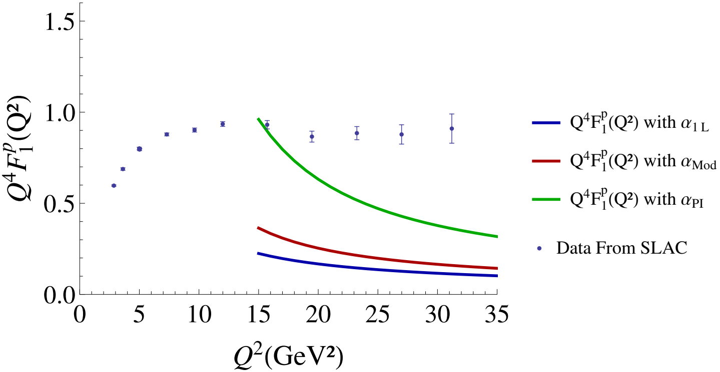
  and using both perturbative and effective couplings.

CZ scale setting with frozen PDA at $1\mathrm{GeV}^2$



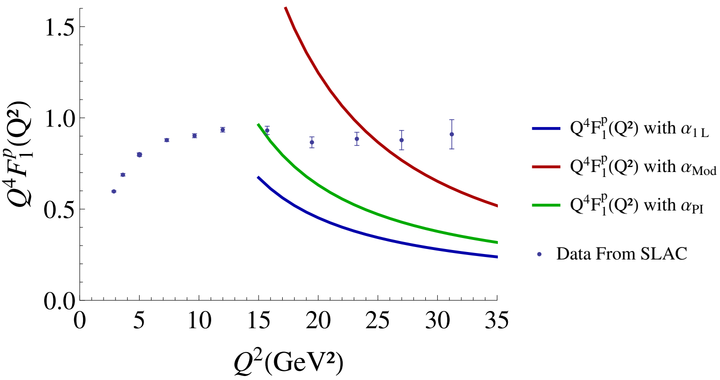
Data from Arnold et al. PRL 57

CZ scale setting + evolution



Data from Arnold et al. PRL 57

Pion BLM Factor + evolution



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  and using both perturbative and effective couplings.
- The data remain flat while the perturbative running show a logarithmic decreasing.
- More work are required to conclude on the validity of the perturbative approach:
  - Theory side : NLO + higher-twists?
  - Experimental side : more precise data to spot a logarithmic decreasing