Diquark Correlations in Generalized Parton Distributions

Workshop on Diquark Correlations in Hadronic Physics
ECT* Trento, September 23-27, 2019

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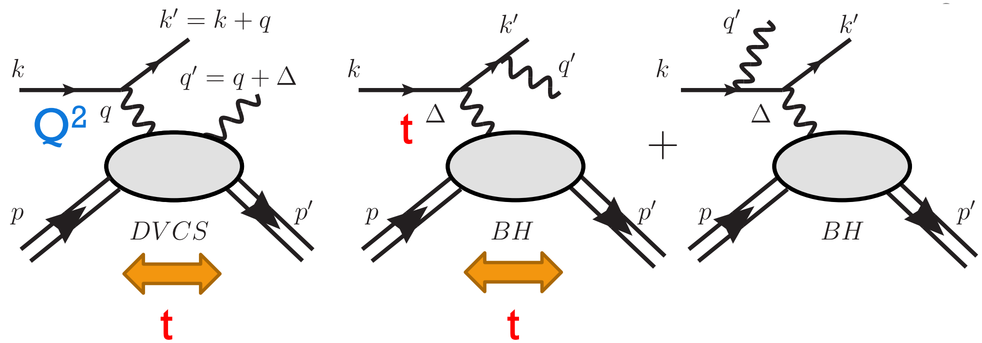
Outline

Diquark correlations in GPDs: exploring momentum and spatial configurations

Extracting GPDs from data is becoming a reality: the Center for Nuclear Femtography effort

Diquark structures in GPDs

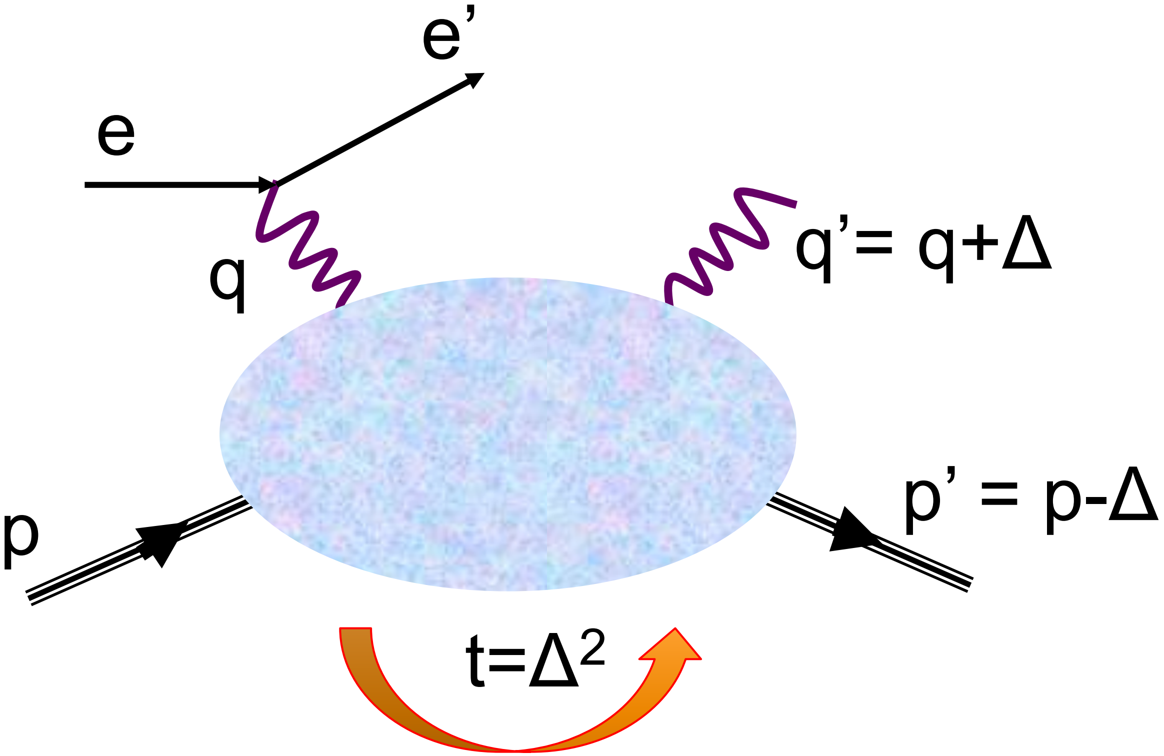


$$\frac{d^5\sigma}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}\left[|T_{\rm BH}|^2 + |T_{\rm DVCS}|^2 + \mathcal{I}\right]$$

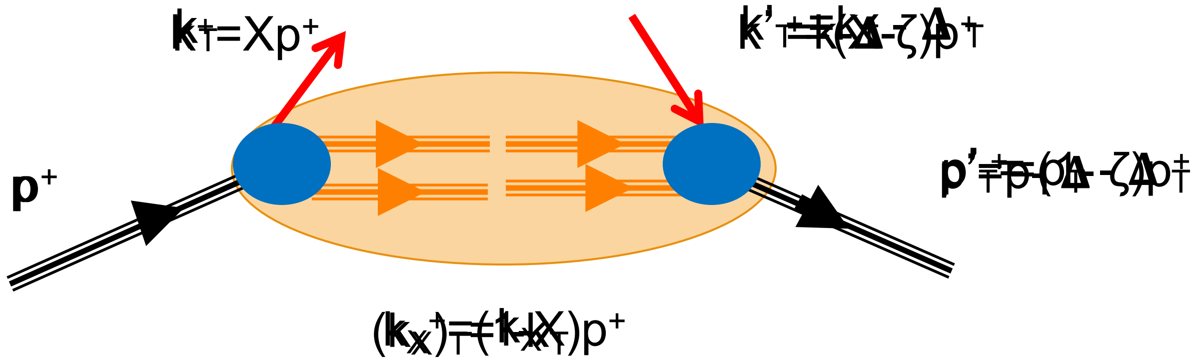
- The Bethe-Heitler cross section is exactly calculable using the electromagnetic form factors at low four-momentum transfer squared, t

- The pure Q^2 dependent DVCS and the BH-DVCS Interference term need to be separated from one another

Diquark structures in GPDs



Diquark structures in GPDs



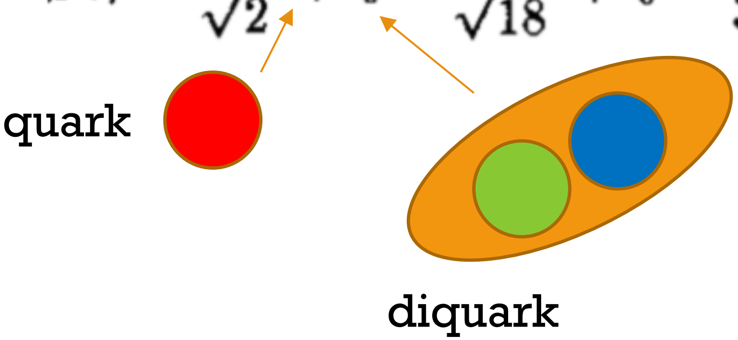
- <u>Intermediate X region</u>

The dominant soft factor is dominated by proton splitting into a valence quark and a diquark spectator

- <u>Low X</u>
  1. Gluons and sea quarks become important
  2. For gluons the spectator is a color octet baryon
  3. For sea quarks the spectator is a tetraquark/excited diquark pair

Spin-isospin structure in SU(4)

$$|p\uparrow\rangle = \frac{1}{\sqrt{2}} u\uparrow S_0^0 + \frac{1}{\sqrt{18}} u\uparrow T_0^0 - \frac{1}{3} u\downarrow T_0^1 - \frac{1}{3} d\uparrow T_1^0 - \sqrt{\frac{2}{9}} d\downarrow T_1^1.$$



$\mathrm{S} \rightarrow spin\ 0, isospin\ 0$

$\mathrm{T} \rightarrow spin\ 1, isospin\ 1$

Matrix element for unpolarized case
(assuming recoiling particles are in S-wave)

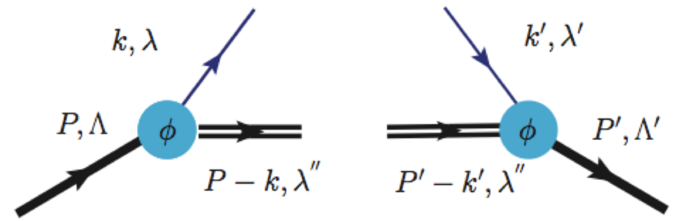
$$
\begin{aligned}
F^u &= \frac{3}{2}F^{S=0} - \frac{1}{6}F^{S=1} \\
F^d &= -\frac{1}{3}F^{S=1},
\end{aligned}
$$

Knowing the diquark spin allows to separate out the u and d quark contributions

Dirac Structure

Scalar diquark   Axial Vector diquark

$S=0$	$S=1$
$\phi^*_{\Lambda'\lambda'}\phi_{\Lambda\lambda}$	$\phi^{\mu}_{\Lambda'\lambda'}\left(\sum_{\lambda''}\epsilon^{*\,\lambda''}_{\mu}\epsilon^{\lambda''}_{\nu}\right)\phi^{\nu}_{\Lambda\lambda}$

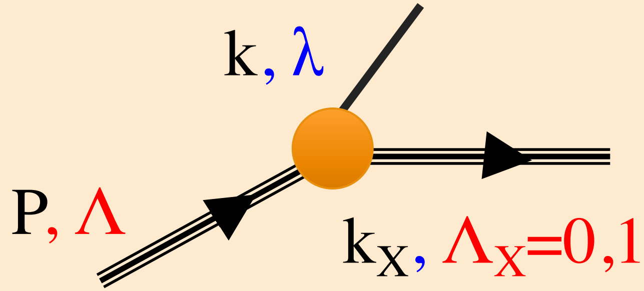


The diquark has structure

$$\phi_{\Lambda,\lambda}(k,P)=\Gamma(k)\frac{\bar{u}(k,\lambda)U(P,\Lambda)}{k^2-m^2}$$

$$\phi^*_{\Lambda'\lambda'}(k',P')=\Gamma(k')\frac{\overline{U}(P',\Lambda')u(k',\lambda')}{k'^2-m^2},$$

Parameters



Parameters	H	E	$\widetilde{H}$	$\widetilde{E}$
m_u (GeV)	0.420	0.420	2.624	2.624
M_X^u (GeV)	0.604	0.604	0.474	0.474
M_Λ^u (GeV)	1.018	1.018	0.971	0.971
u quark				
$\mathcal{N}_u$	2.043	1.803	0.0504	1.074
χ^2	0.773	0.664	0.116	1.98
m_d (GeV)	0.275	0.275	2.603	2.603
M_X^d (GeV)	0.913	0.913	0.704	0.704
M_Λ^d (GeV)	0.860	0.860	0.878	0.878
d quark				
$\mathcal{N}_d$	1.570	-2.800	-0.0262	-0.966
χ^2	0.822	0.688	0.110	1.00

quark mass

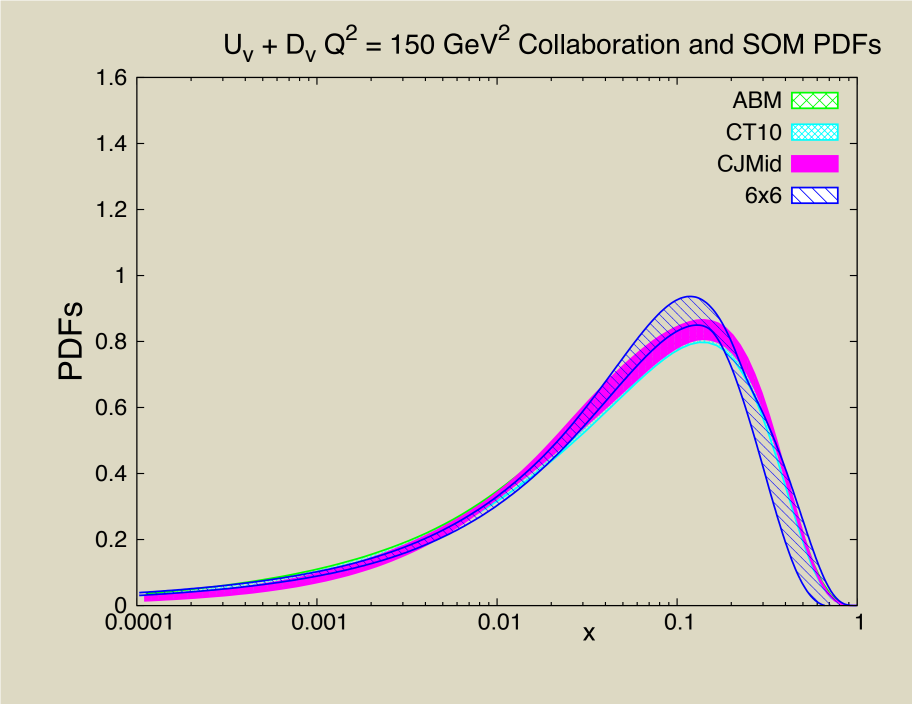
diquark mass

monopole mass

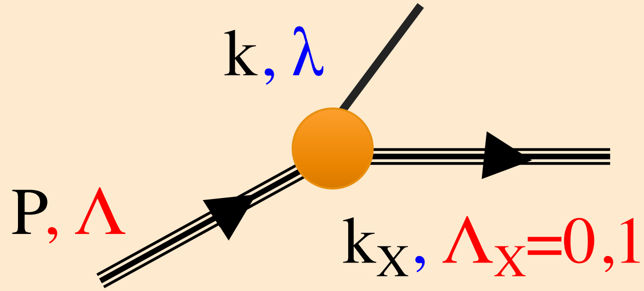
normalization

$$H_q(x,0,0;Q^2) = q(x,Q^2)$$

These parameters were fixed by taking the forward limit of GPDs



Parameters



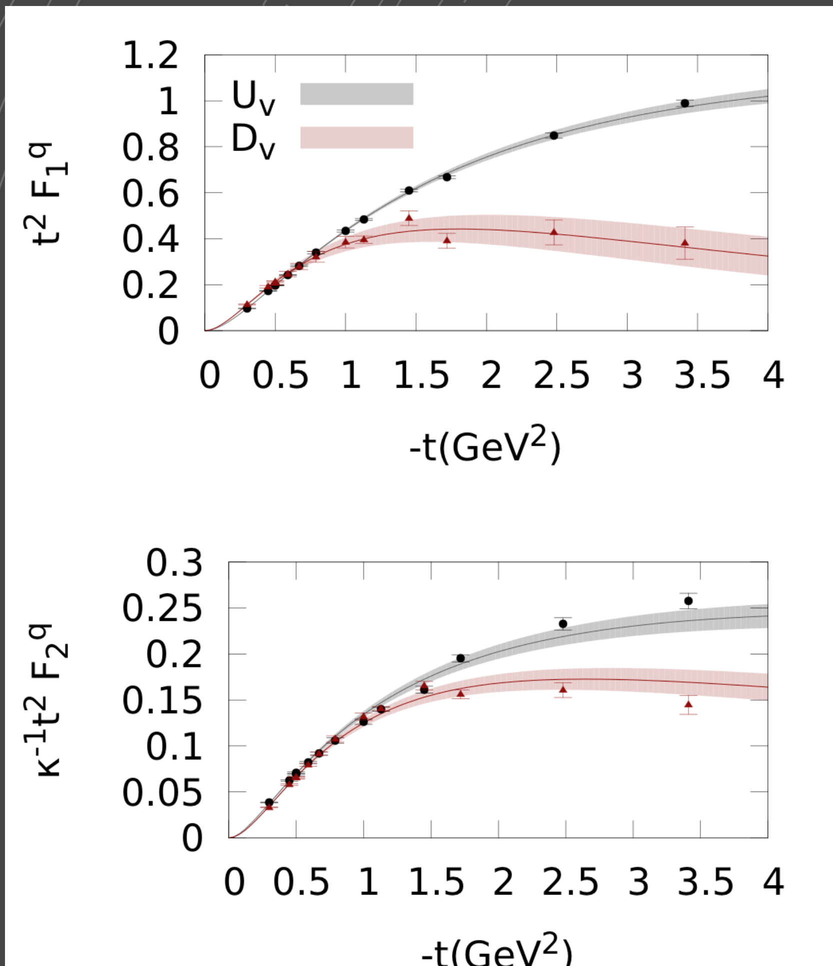
Parameters	H	E	$\widetilde{H}$	$\widetilde{E}$
m_u (GeV)	0.420	0.420	2.624	2.624
M_X^u (GeV)	0.604	0.604	0.474	0.474
M_Λ^u (GeV)	1.018	1.018	0.971	0.971
α_u	0.210	0.210	0.219	0.219
α'_u	2.448 ± 0.0885	2.811 ± 0.765	1.543 ± 0.296	5.130 ± 0.101
p_u	0.620 ± 0.0725	0.863 ± 0.482	0.346 ± 0.248	3.507 ± 0.054
$\mathcal{N}_u$	2.043	1.803	0.0504	1.074
χ^2	0.773	0.664	0.116	1.98
m_d (GeV)	0.275	0.275	2.603	2.603
M_X^d (GeV)	0.913	0.913	0.704	0.704
M_Λ^d (GeV)	0.860	0.860	0.878	0.878
α_d	0.0317	0.0317	0.0348	0.0348
α'_d	2.209 ± 0.156	1.362 ± 0.585	1.298 ± 0.245	3.385 ± 0.145
p_d	0.658 ± 0.257	1.115 ± 1.150	0.974 ± 0.358	2.326 ± 0.137
$\mathcal{N}_d$	1.570	-2.800	-0.0262	-0.966
χ^2	0.822	0.688	0.110	1.00

Regge intercept

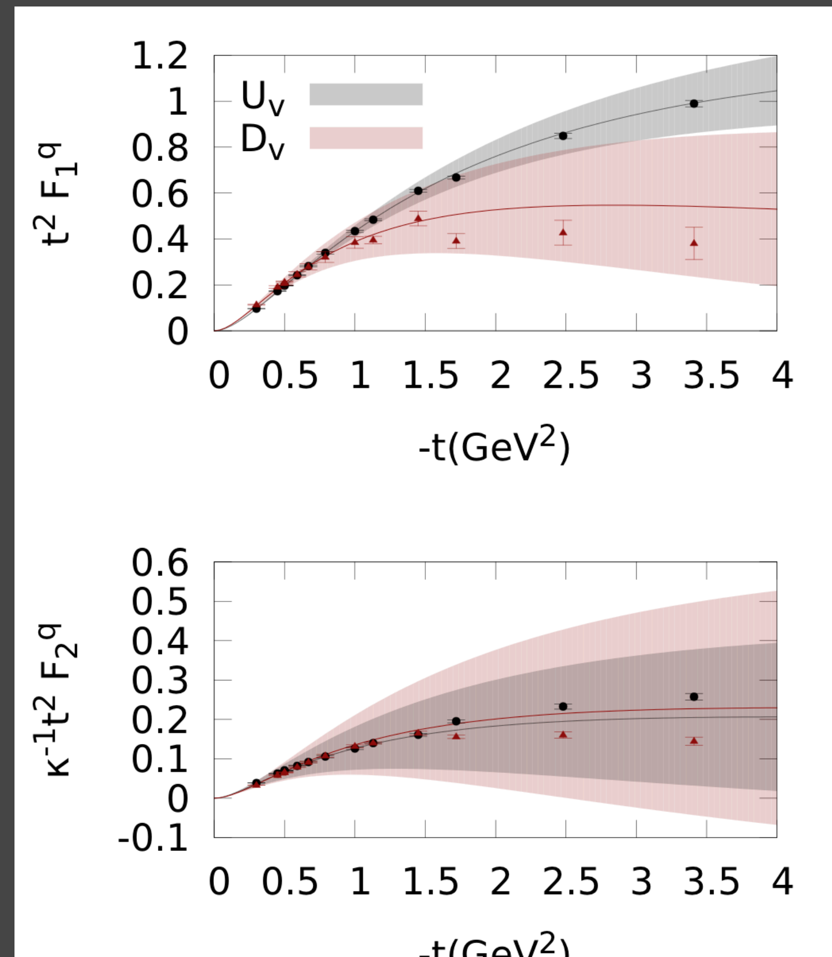
Regge slope

Regge interactions

Parameters fixed by integrating GPDs to form factors



with flavor separated form factors



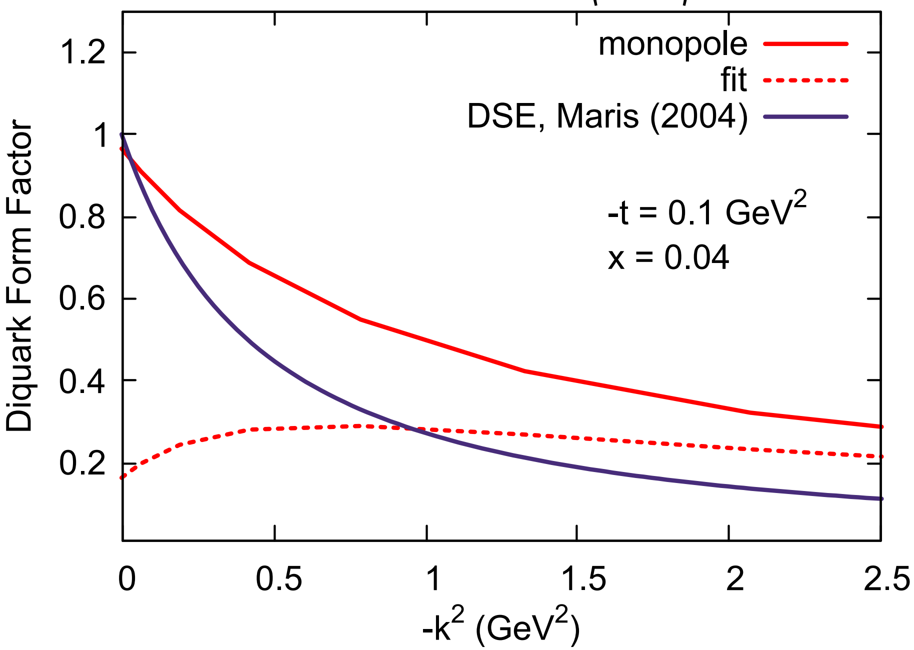
without flavor separated form factors

G.Cates et al, Phys. Rev. Lett. 106, 252003 (2011)

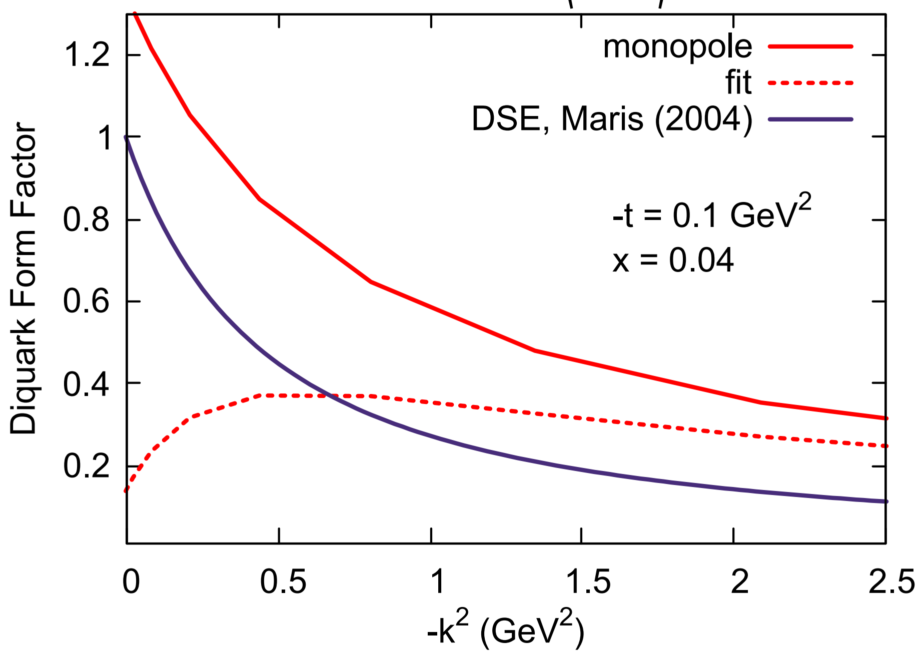
Diquark Form Factor

$$\Gamma(k) = g\frac{k^2 - m^2}{k^2 - M_\Lambda^2}\frac{1}{k^2 - M_\Lambda^2}$$

ud $\left\langle\sqrt{r^2}\right\rangle = 0.48\ fm$



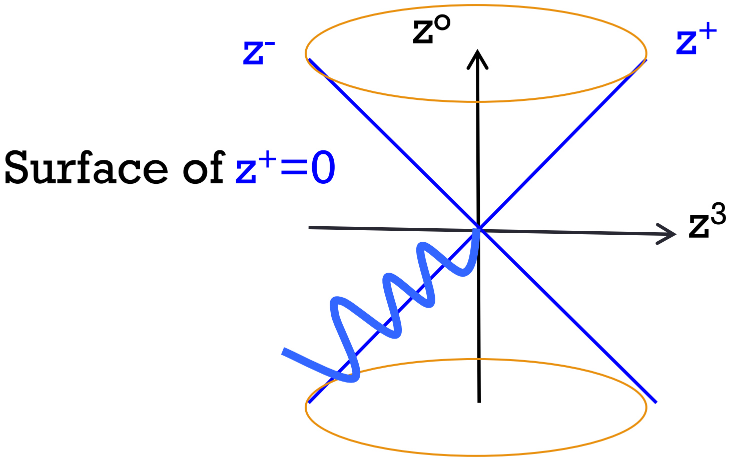
uu $\left\langle\sqrt{r^2}\right\rangle = 0.56\ fm$



DSE, Maris $\left\langle\sqrt{r^2}\right\rangle = 0.77\ fm$

Proton charge radius $\left\langle\sqrt{r^2}\right\rangle = 0.87\ fm$

Probing Spatial Correlations



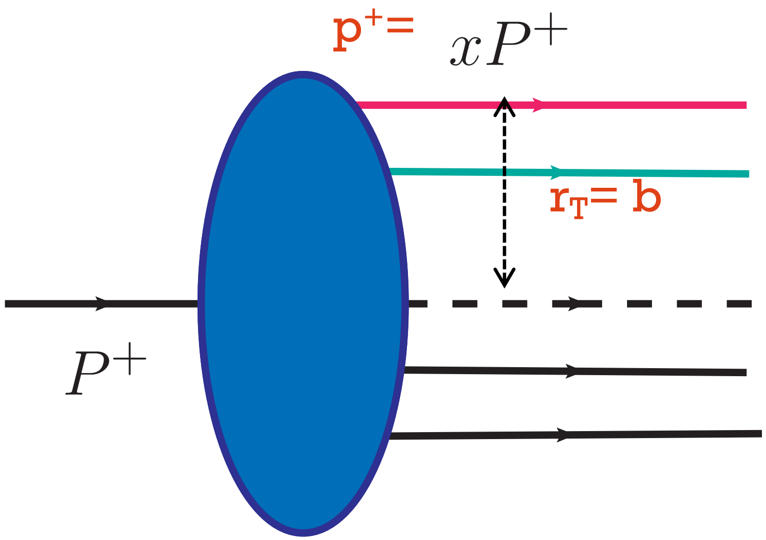
$$p^{\pm} = \frac{p^{o} \pm p^{3}}{\sqrt{2}}$$

Fourier transform

$$(pz) = \underline{p^{+} z^{-}} + \underline{p^{-} z^{+}} - p_T \cdot z_T$$

p^{+} is conjugate to z^{-} and p^{-} is conjugate to z^{+}

The Proton Relativistic Wave Function: Poincaré Invariance



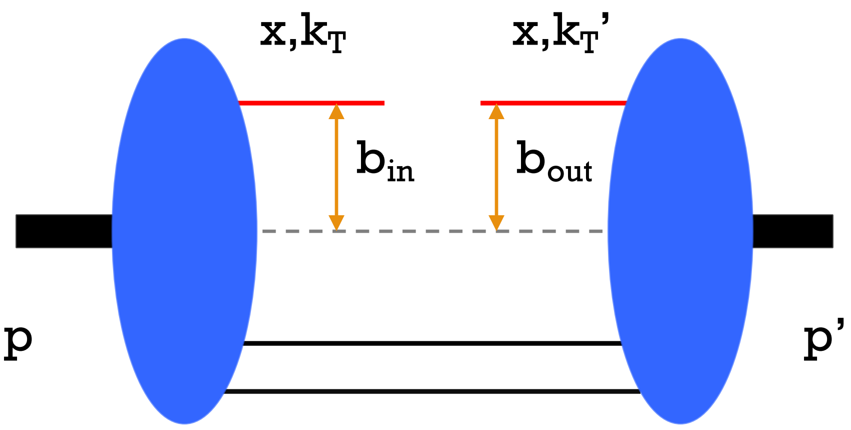
Center of P^+

$$\vec{R}_T = \frac{1}{P^+}\sum_i (x_i P^+)\vec{r}_T^{\,i}$$

- P^+ plays the role of mass
- "The subgroup of the Poincaré group that leaves the surface z^+=const invariant, is isomorphic to the Galilean group in 2D"
- We can disentangle the transverse components from the time components in boosts ➔ boosts in transverse plane are kinematical

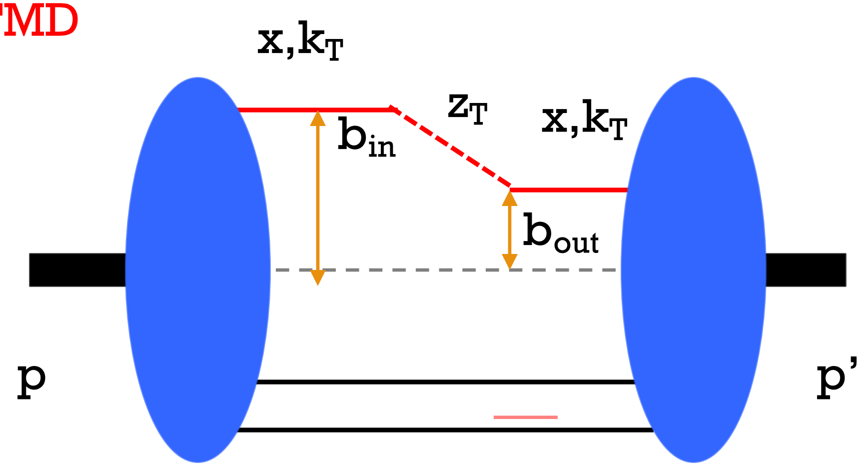
Correlation Functions

GPD



$$\Delta_T = k'_T - k_T \overset{F}{\leftrightarrow} b = \frac{b_{in} + b_{out}}{2}$$

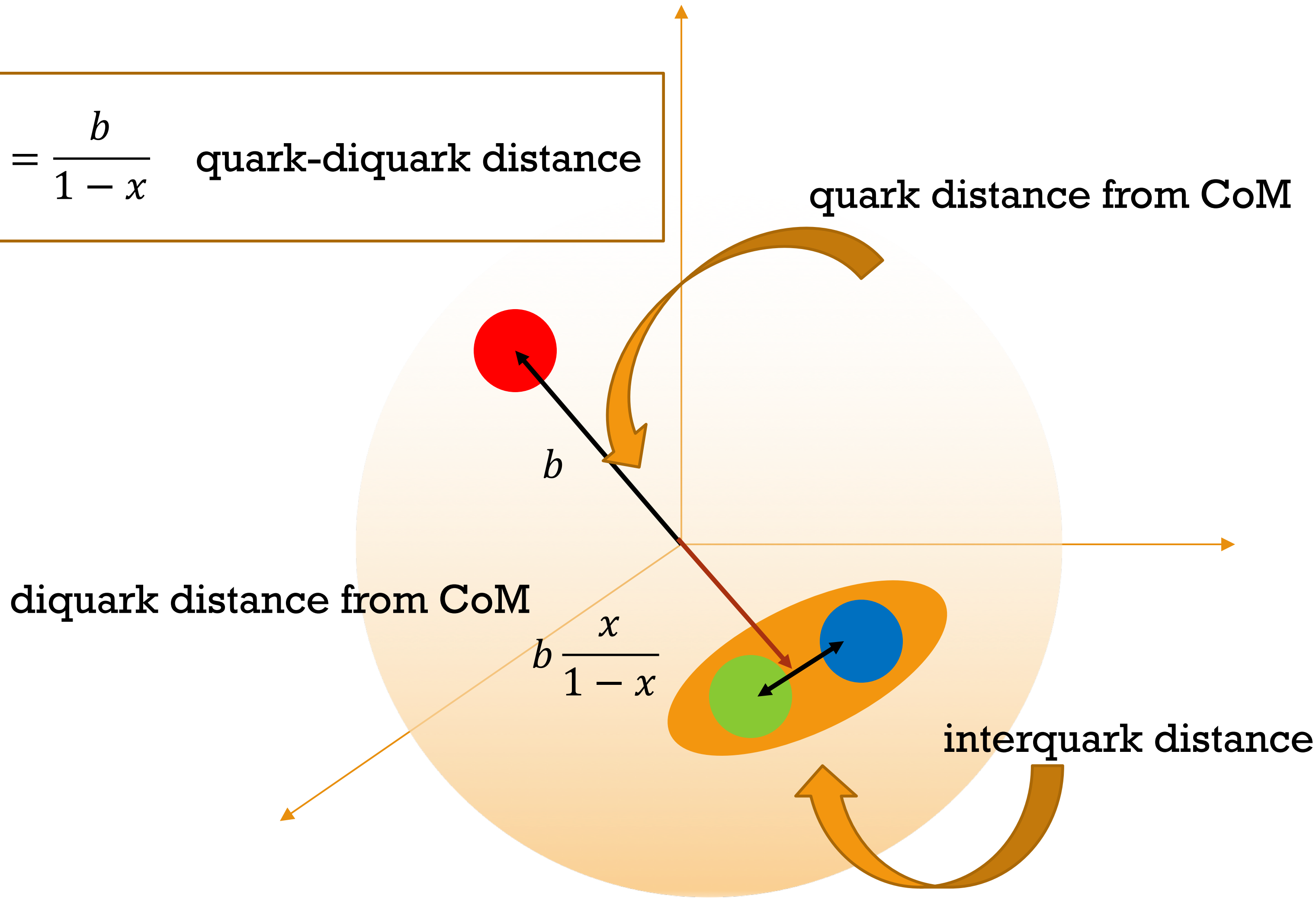
TMD



$$\bar{k}_T = \frac{k_T + k'_T}{2} \overset{F}{\leftrightarrow} z_T = b_{in} - b_{out}$$

Probing diquark transverse spatial correlations

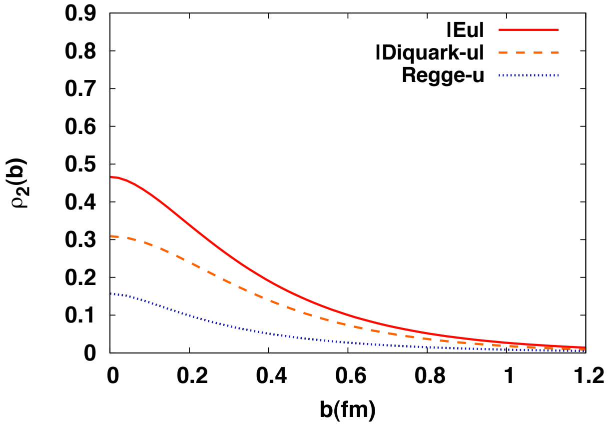
$$y = \frac{b}{1-x}$$ quark-diquark distance



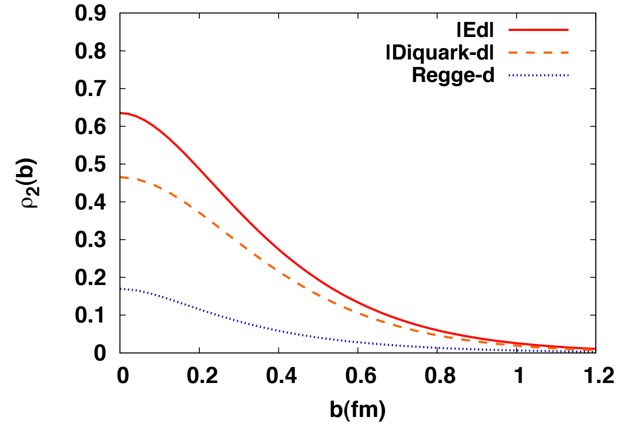
Once we know the quark position, we can establish the diquark position:

a form of color entanglement?

Pauli

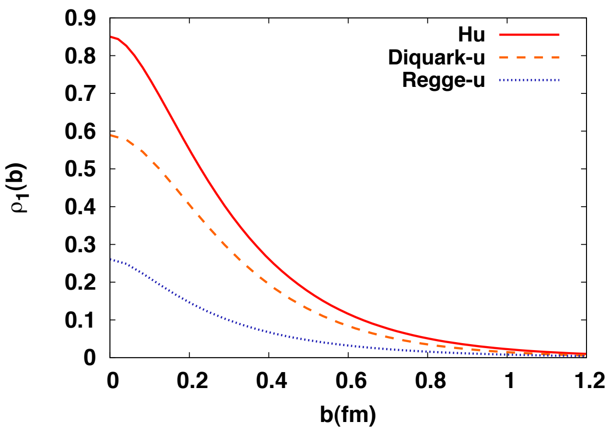


|Eu|
|Diquark-u|
Regge-u
$\rho_2(b)$
b(fm)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0.2
0.4
0.6
0.8
1
1.2

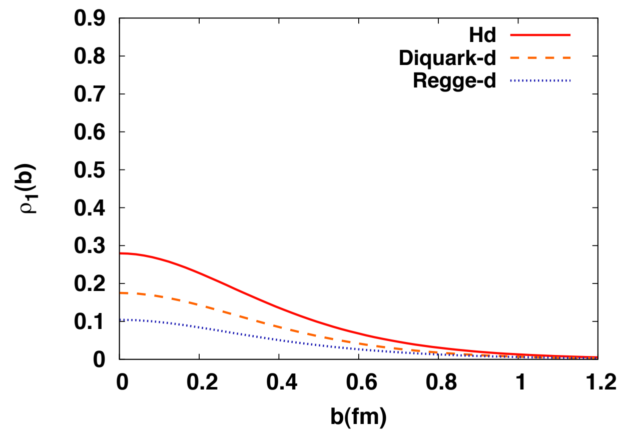


|Ed|
|Diquark-d|
Regge-d
$\rho_2(b)$
b(fm)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0.2
0.4
0.6
0.8
1
1.2

Dirac



Hu
Diquark-u
Regge-u
$\rho_1(b)$
b(fm)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0.2
0.4
0.6
0.8
1
1.2



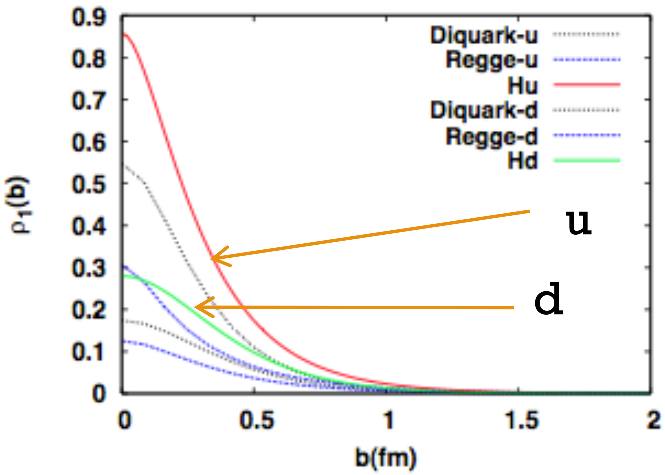
Hd
Diquark-d
Regge-d
$\rho_1(b)$
b(fm)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0.2
0.4
0.6
0.8
1
1.2

What do we obtain (or wish to obtain) from experiment

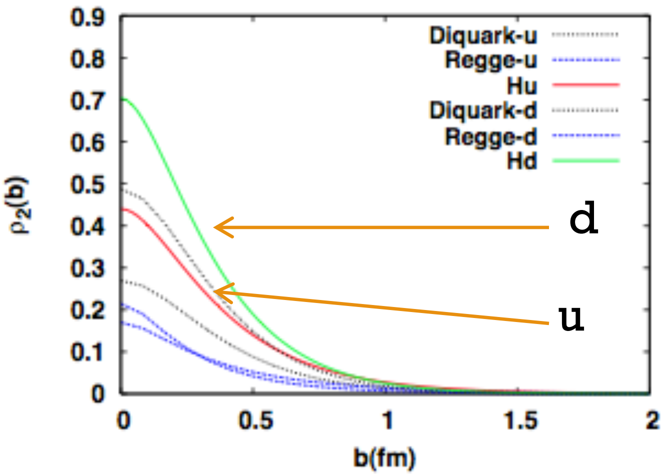
GPD

Transverse coordinate density distributions

Dirac



Pauli



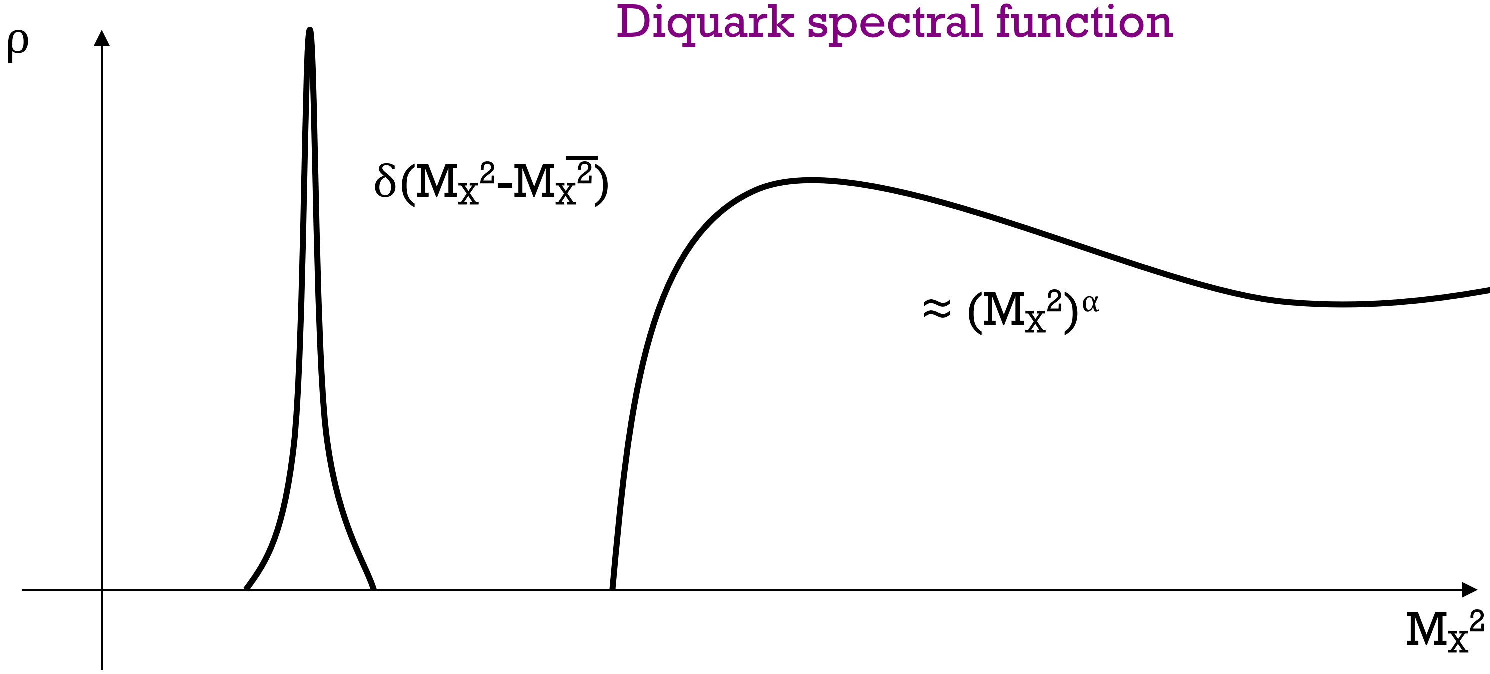
(using constraints from Jlab flavor separated form factor data, G. Cates et al, 2011)

Connection with Regge theory

Reggeization

$$\int_0^\infty dM_X^2 \rho_R(M_X^2) H(X,0,0) \sim X^{-\alpha(0)-1},$$

Diquark spectral function



Brodsky, Close, Gunion → DIS ('70s)
Gorshteyn & Szczepaniak (PRD, 2010)
Brodsky, Llanes, Szczepaniak arXiv:0812.0395

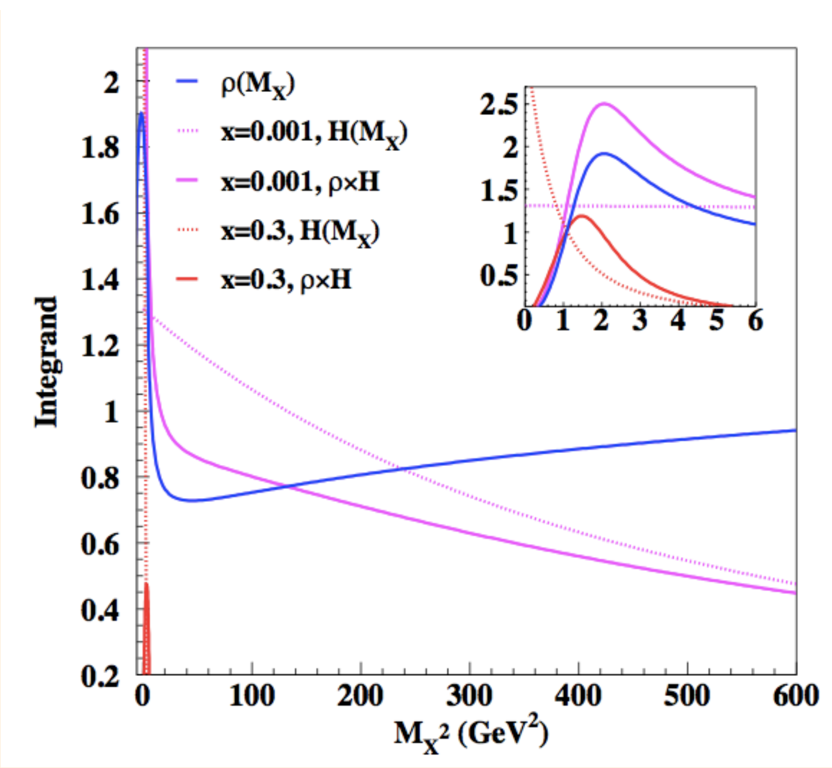
Reggeization: varying mass of the outgoing diquark system

Landshoff, Polkinghorn, Short '71
Brodsky, Close, Gunion '71
Applied to GPDs: Ahmad et al. '07, '09
Brodsky, Llanes Estrada '07

The mass dependence is regulated by a Spectral function

$$\rho(k_X^2, k^2) = (k_X^2)^{\alpha-1}\beta(k^2),$$

$$M_X^2 \rightarrow k_X^2$$



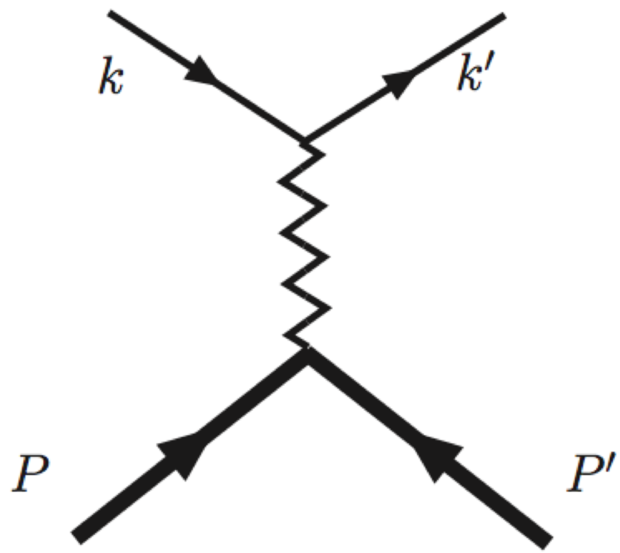
Convolution of GPD with mass spectral function

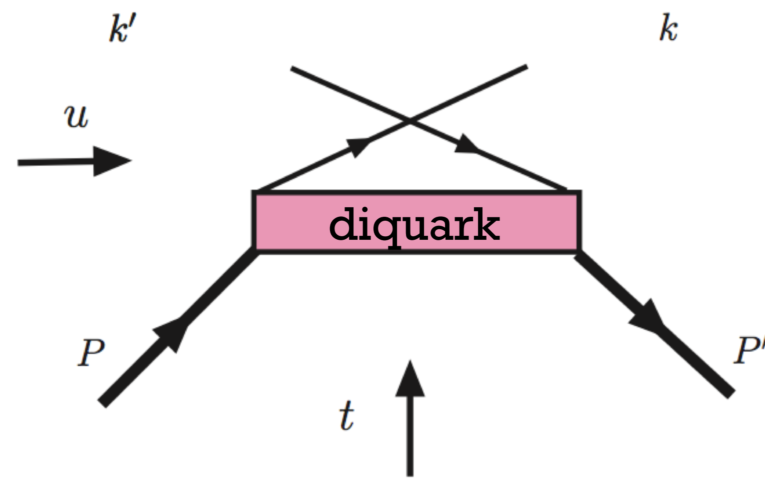
$$H(X,0,0) = \mathcal{N} \int_{M^2-m^2}^{\infty} dk_X^2 (k_X^2)^{\alpha-1} \int_{-\infty}^{\mathcal{M}^2(X,k_X^2)+M_\Lambda^2} dk^2 \beta(k^2) \overline{H}(X, M_\Lambda^2, k_X^2, k^2).$$

Regge behavior X≈0

$$H(X,0,0) \; = \; \mathcal{N} X^{-\alpha} \left[\int_0^{\infty} dz z^{\alpha-1} \int_{-\infty}^{-z} dk^2 \frac{m^2 - k^2 - z}{(k^2 - M_\Lambda^2)^4} \right]_{X \to 0},$$

Reggeon exchange $\approx$ u-channel diquark exchange





Full x and t dependence of Regge exchanges

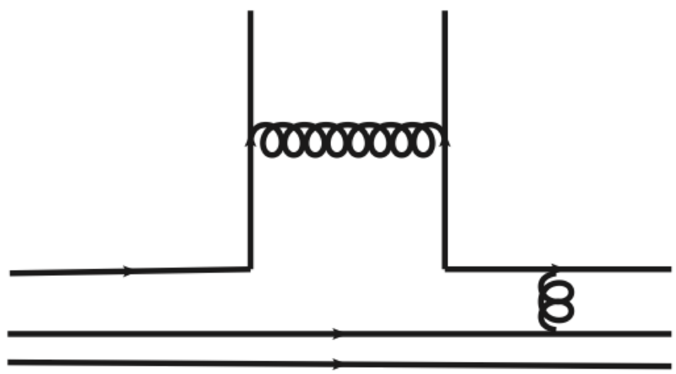
O. Gonzalez Hernandez et al.

arXiv 1206.1876

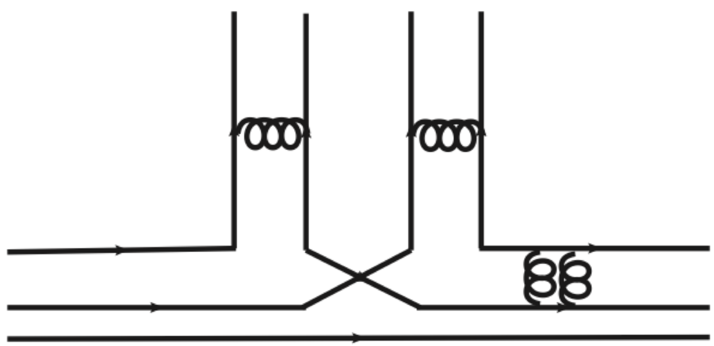
$$R_p^{\alpha,\alpha'} = X^{-[\alpha+\alpha'(X)t+\beta(\zeta)t]},$$

$$\alpha'(X) \equiv \alpha'(1-X)^p \qquad \beta = 0.$$

Regge exchange



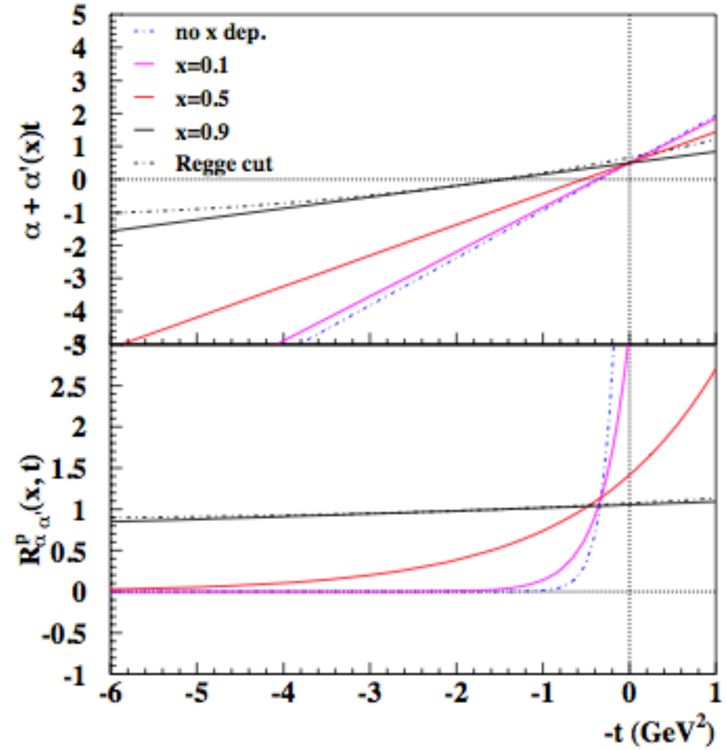
Regge exchange/diquark correlation



P.D.B.Collins and Kearney

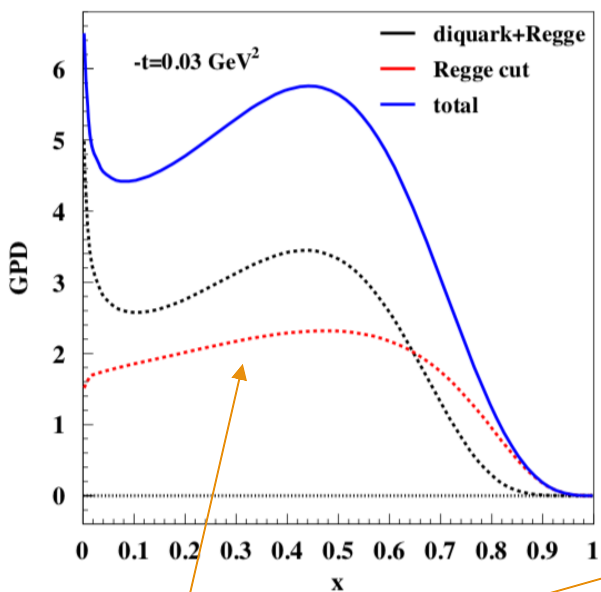
$$\alpha'(X) \equiv \alpha'(1-X)^p$$

$$R_p^{\alpha,\alpha'} = X^{-[\alpha+\alpha'(X)t+\beta(\zeta)t]},$$

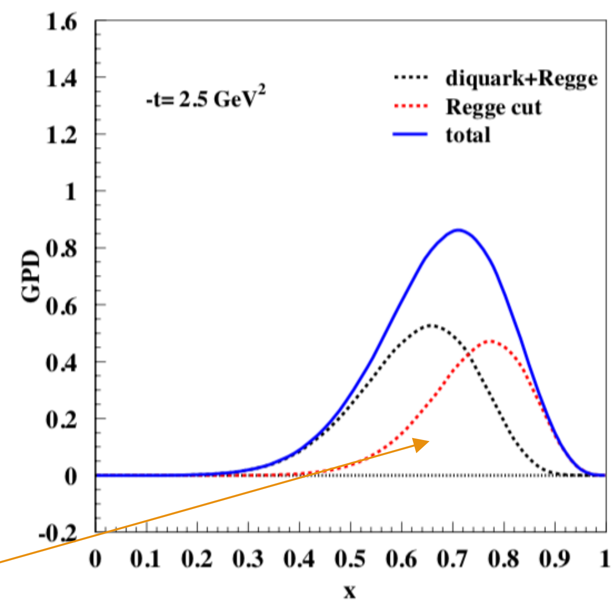


Diquark correlations in GPDs

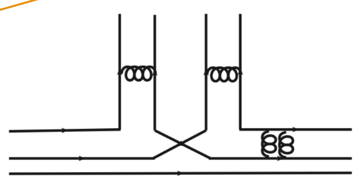
Small t



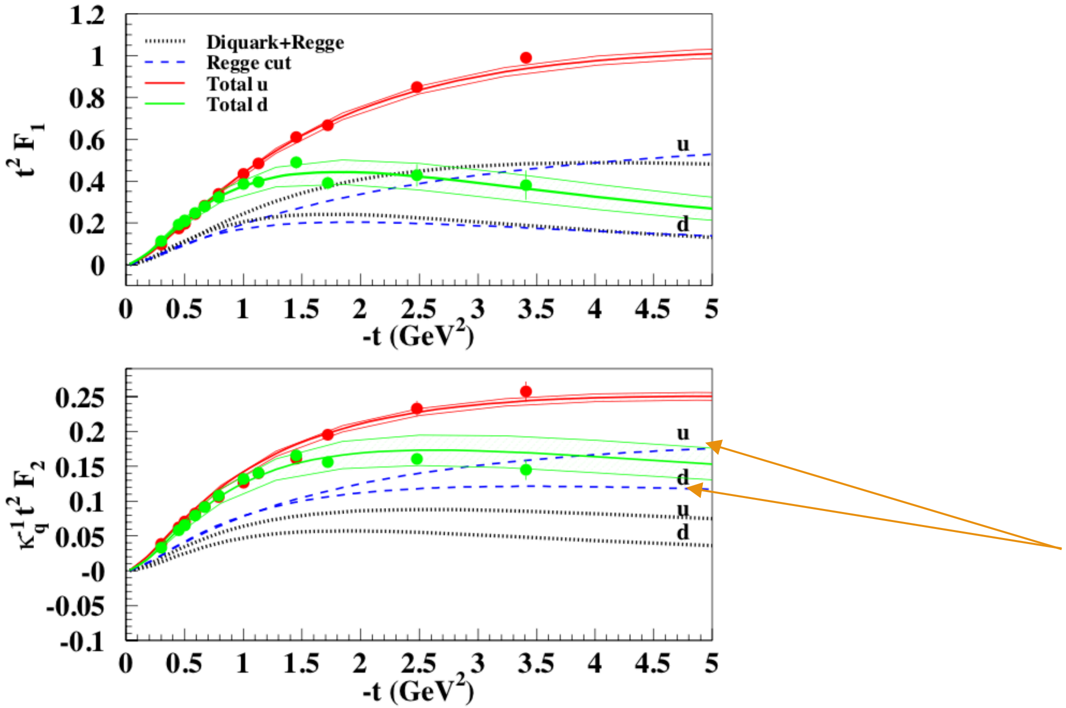
Large t

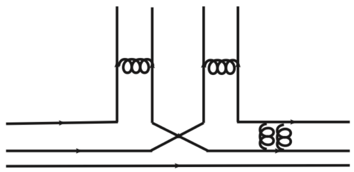


diquark correlation



Diquark correlations in Form Factors





diquark correlation

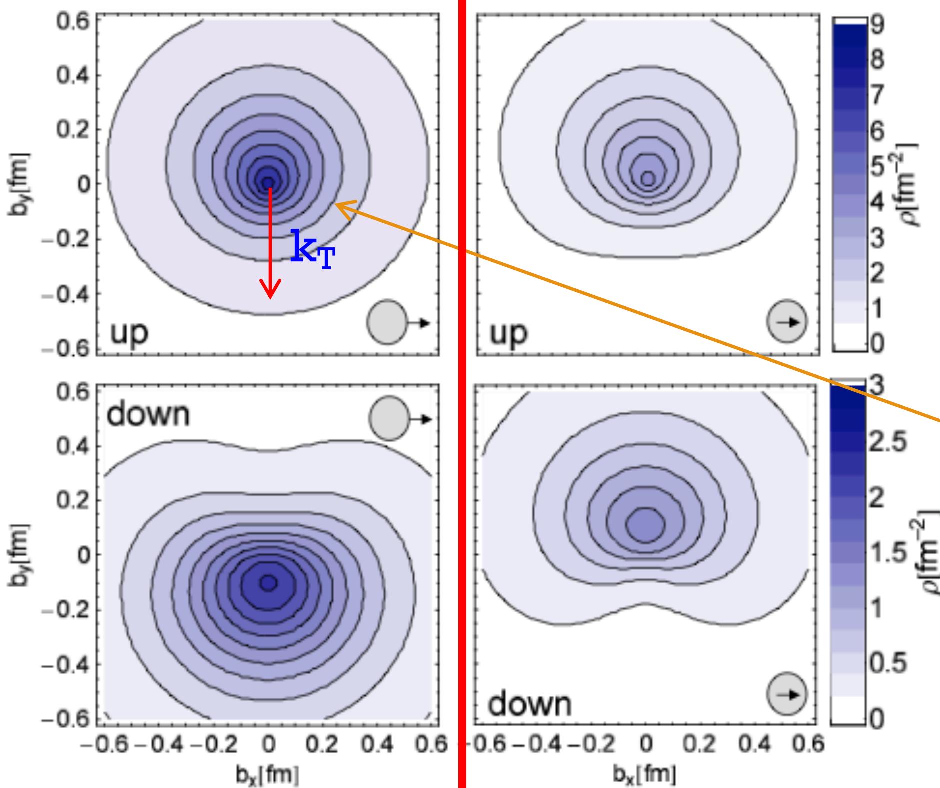
- The ratio $Q^2 F_2^q / F_1^q$ vs. Q^2 is linear, at variance with the proton and neutron ones.

- The ratio F_2^q / F_1^q vs. Q^2 shows a ``two components" behavior:

  Steeply decreasing at $Q^2 < 1.5$ GeV^2
  Flattens out at $Q^2 > 1.5$ GeV^2
  (this behavior is more pronounced for the u quark, the d-quark has an overall flatter behavior).

We are able to reproduce this behavior with diquark correlations, although we have no "cute" simple mechanism!

Note on Orbital Angular Momentum: for F_2 it works analogously to GPD E….

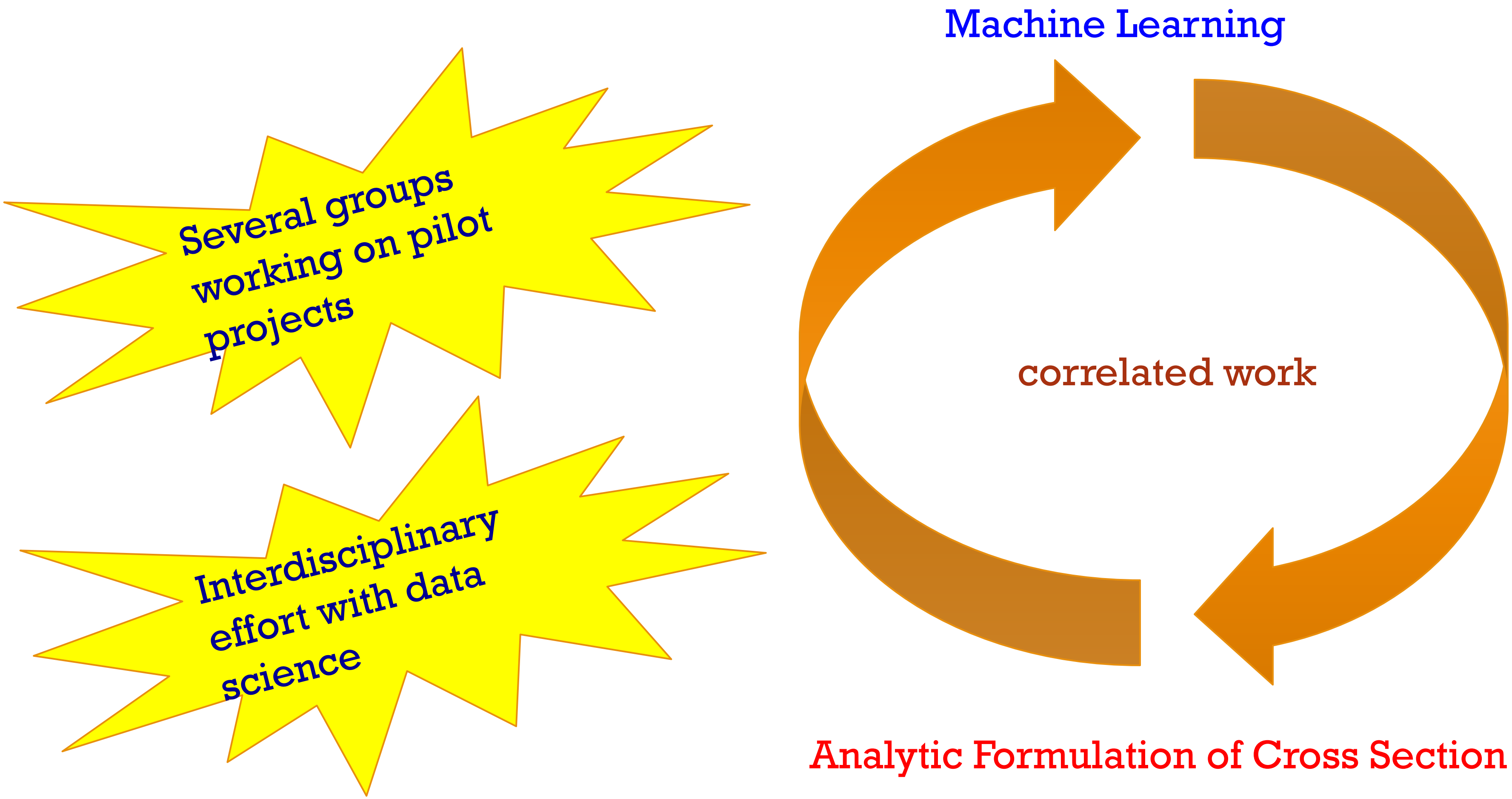


The net b corresponds to net k_T in the opposite direction (attractive color force due to FSI) (M. Burkardt)

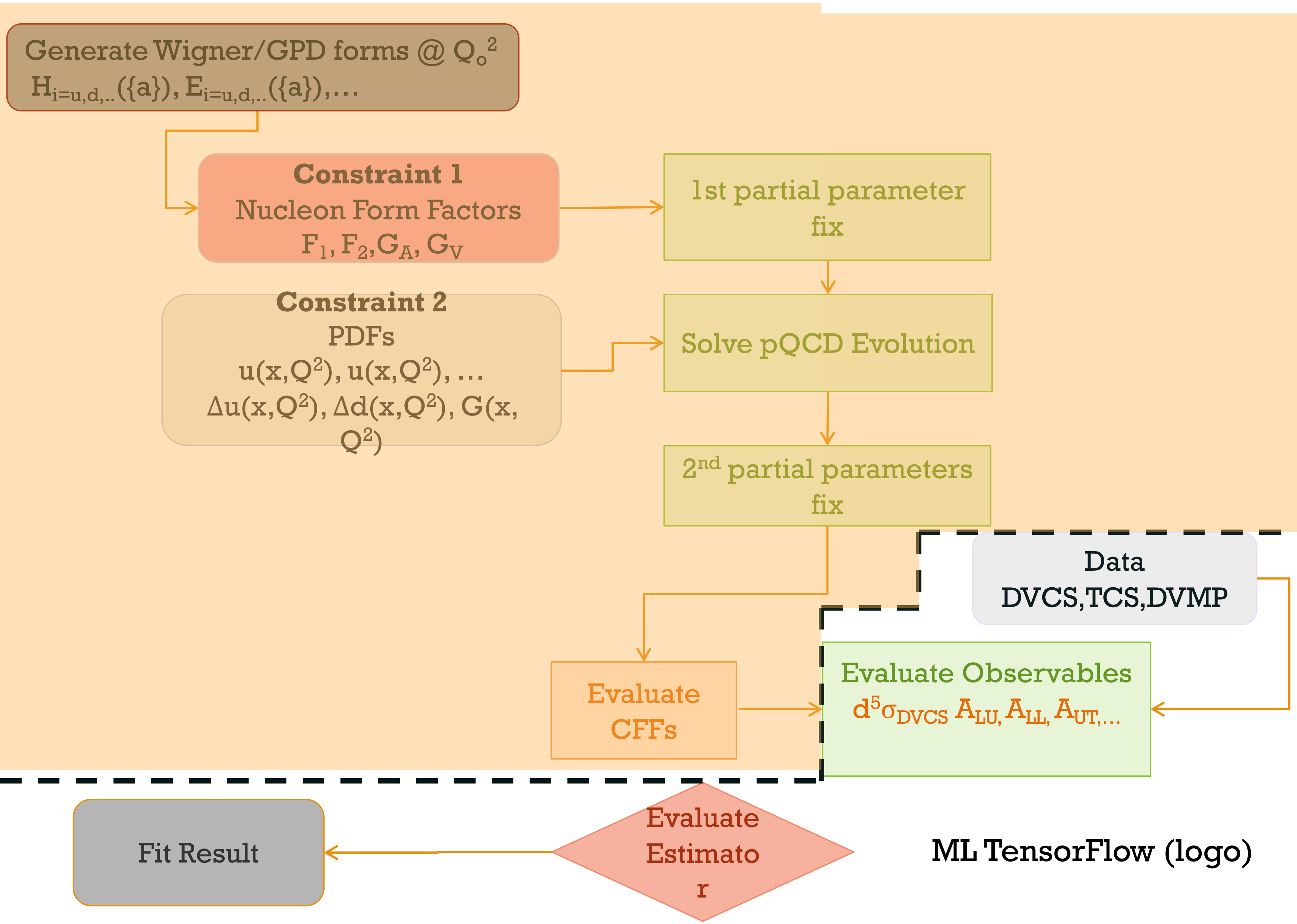
$$\left(A^X_{++,++}+A^X_{+-,+-}+A^X_{-+,-+}+A^X_{--,--}\right)+\left(A^X_{++,++}+A^X_{+-,+-}-A^X_{-+,-+}-A^X_{--,--}\right)$$

$$\approx H-i\Delta_2 E$$

Center for Nuclear Femtography will play an essential role for extracting 3D structure of the proton from data



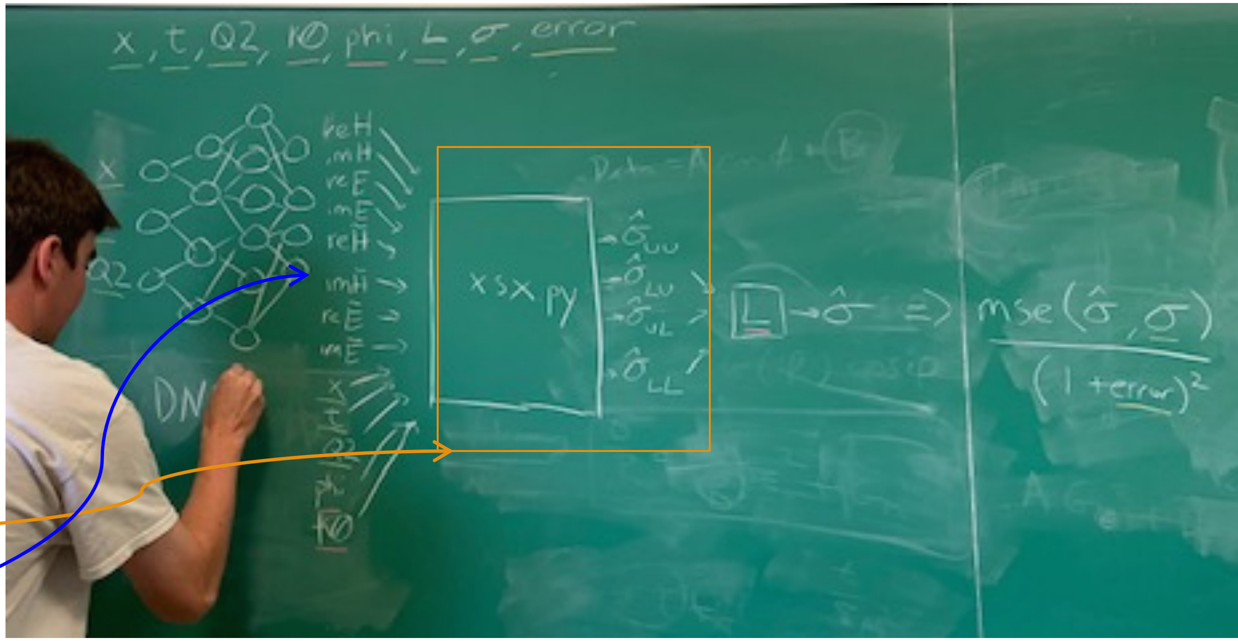
Flowchart/roadmap from data/observables to GPDs



Femtography Imaging with Neural Networks (FINN)

Strategy:

1. A fully connected neural network maps input kinematic data to a vector of eight form factors (see diagram).
2. Use a code developed by our Data Analysis Team to evaluate the cross sections and in terms of the CFFs.

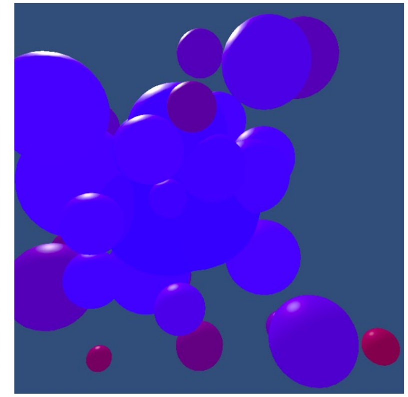


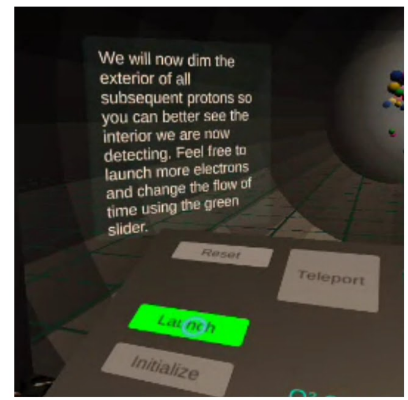
We translate the x-sec. code into TensorFlow

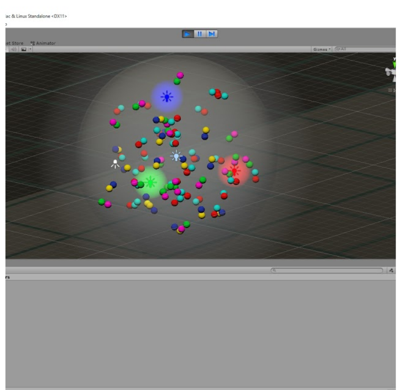
➔Automatically differentiable

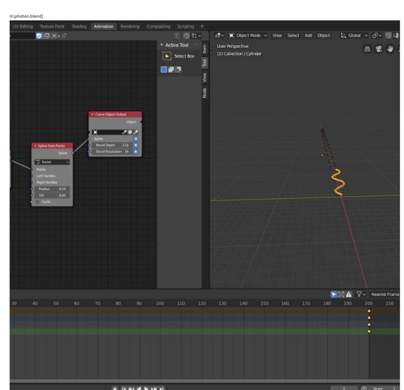
➔At variance with other efforts we can train CFF extraction network with backpropagation and variants of stochastic gradient descent.

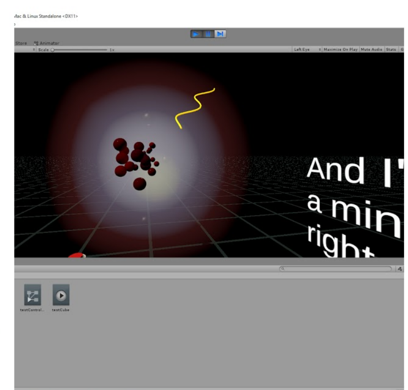
Outreach tool












SIWIF

NEW IDEAS/OBSERVATIONS

DIQUARK FORM FACTOR INSERTED IN PDF MODELS SERVES AS A REGULATOR

PHYSICAL IDENTIFICATION OF A REGULATOR

TOMOGRAPHY OF THE DIQUARK FORM FACTOR

Back up

GPDs in terms of quark-parton helicity amplitudes

$$
\begin{aligned}
A_{++,++} &= \int d^2k_\perp \phi^*_{++}(k',P')\phi_{++}(k,P) \\
A_{+-,+-} &= \int d^2k_\perp \phi^*_{+-}(k',P')\phi_{+-}(k,P) \\
A_{-+,++} &= \int d^2k_\perp \phi^*_{-+}(k',P')\phi_{++}(k,P) \\
A_{++,-+} &= \int d^2k_\perp \phi^*_{++}(k',P')\phi_{-+}(k,P).
\end{aligned}
$$

$$H = A_{++,++} + A_{+-,+-}$$

$$\frac{\Delta}{M} E = A^{T_y}_{++,++} + A^{T_y}_{+-,+-}$$