Neutron Stars: Celestial Laboratories for Dense Matter

Shriya Soma
Supervisor: Debades Bandyopadhyay
Saha Institute for Nuclear Physics (2018-2019)

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Frankfurt Institute for Advanced Studies (FIAS)

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Introduction

Theorized in 1932:
Landau - Proposed astrophysical objects where atomic nuclei come in close contact, forming one gigantic nucleus!
Chadwick - Discovery of a Neutron

Discovered in 1967:
Jocelyn Bell & Anthony Hewish

In the Golden Jubilee Year of the Discovery, a Binary Neutron Star Merger event was detected!

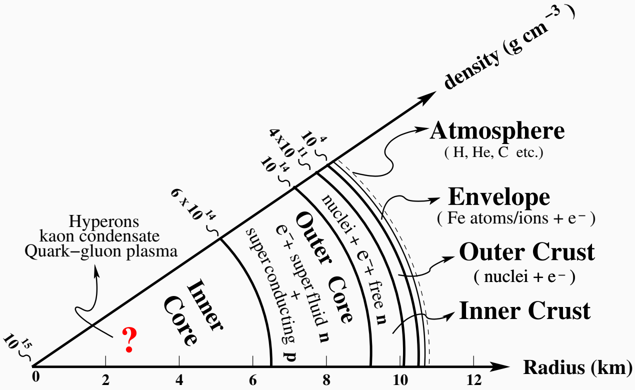


Figure 1: Structure of a Neutron Star

Neutron Star Observables

- **Mass** $\lesssim 2.14\ M_{\odot}$
- **Radius** $\sim 12 - 15km$
- **Moment of Inertia** $\sim 10^{45} g\ cm^2$

Binary Neutron Star Merger Events

- Tidal Properties

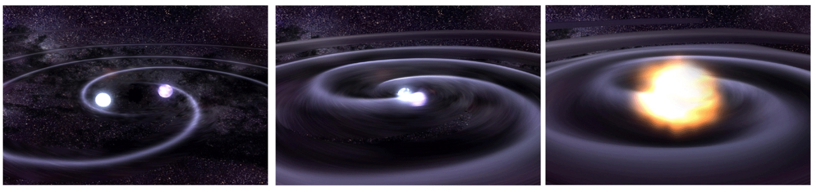


Figure 2: [Credit: NASA]

Theoretical Model for EoS of Dense Matter in the Core

- Nuclear matter in ground state [Neutron Stars are cold objects]
- Strong nucleon-nucleon interactions, via exchange of mesons (σ, ω, ρ).
- Chemical equilibrium

$$n \rightarrow p + e^{-} + \bar{\nu}_e \quad ; \quad p + e^{-} \rightarrow n + \nu_e$$

- The Lagrangian Density:

$$\mathcal{L}_B = \bar{\psi}_B(i\gamma_\mu\partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu - \frac{1}{2}g_{\rho B}\gamma_\mu\bar{\tau}_B \cdot \bar{\rho}^\mu)\psi_B$$

$$+\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu \cdot \rho^\mu - U(\sigma)$$

where,

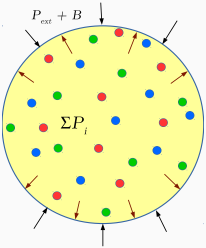
$$U(\sigma) = \frac{1}{3}bm_B(g_{\sigma B}\sigma)^3 + \frac{1}{4}c(g_{\sigma B}\sigma)^4 \quad ; \quad \omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$$

$$\mathcal{L}_\ell = \sum_\ell \bar{\psi}_\ell(i\gamma_\mu\partial^\mu - m_\ell)\psi_\ell$$

$$\mathcal{L}_{total} = \mathcal{L}_B + \mathcal{L}_\ell$$

Bag Model - For Quark Stars (QSs)

Strange Quark Matter with equal proportions of u, d, and s quarks could be the ground state of matter.
[E. Witten, Phys. Rev. D **30**, 272 (1984)]



Quarks are in Chemical Equilibrium

$$d \to u + e^- + \nu_e \quad ; \quad s \to u + e^- + \nu_e$$

$$\mu_d = \mu_u + \mu_e \quad ; \quad \mu_s = \mu_d$$

$$\mathcal{L} = \sum_q \bar{\psi}_q (i\gamma_\mu \partial^\mu - m_q - g_{Vq}\gamma_\mu V^\mu)\psi_q - B + \sum_\ell \bar{\psi}_\ell (i\gamma_\mu \partial^\mu - m_\ell)\psi_\ell$$

Equations of State

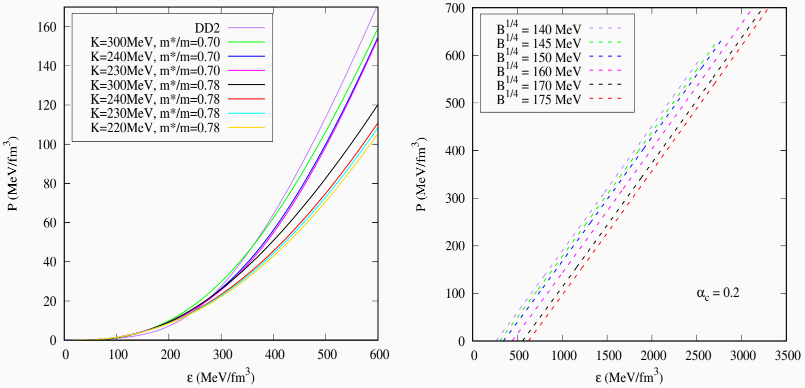


Figure 3: Equations of State for Neutron stars (left panel) and Quark stars (right panel)

- Higher compression modulus - stiffer EoS.
- Lower effective mass - stiffer EoS.
- Higher Bag constant - softer EoS.

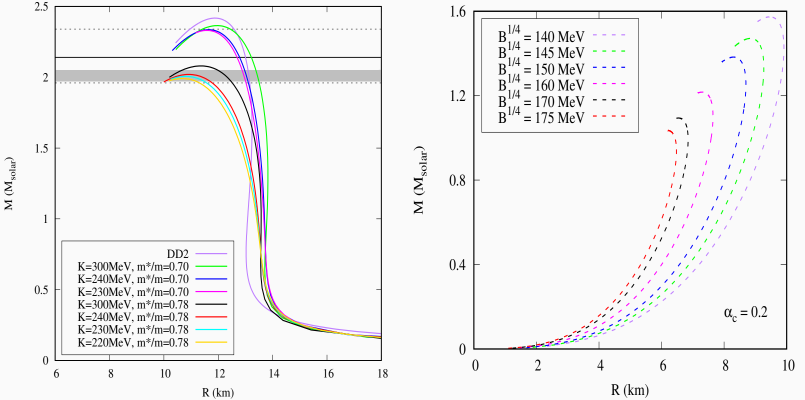


Figure 4: Mass-Radius Relations for NSs (left panel) and QSs (right panel)

Highest NS mass : $2.14^{+0.20}_{-0.18}\ M_{\odot}$ (95.4% credibility interval)

$2.14^{+0.10}_{-0.09}\ M_{\odot}$ (68.3% credibility interval)

[H. T. Cromartie *et al.*, Nature Astronomy, (2019), doi:10.1038/s41550-019-0880-2]

Metric for a massive rotating star

$$ds^2 = e^{2\nu(r,\theta)}dt^2 - e^{2\lambda(r,\theta)}dr^2 - e^{2\mu(r,\theta)}\left[r^2 d\theta^2 + r^2 \sin^2\theta\left(d\phi - \omega(r,\theta)dt\right)^2\right]$$

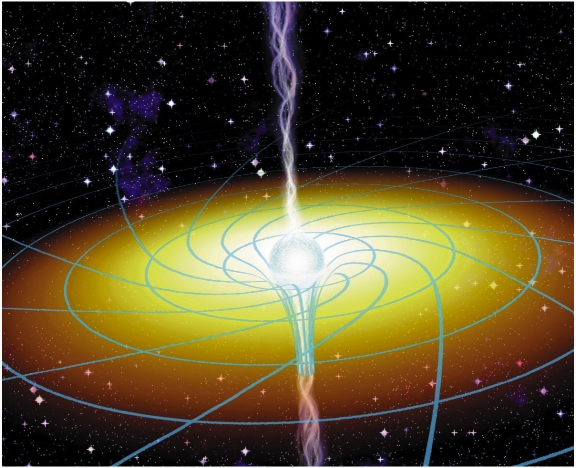


Figure 5: https://www.nap.edu/read/10079/chapter/8#117 Credit: Joe Bergeron

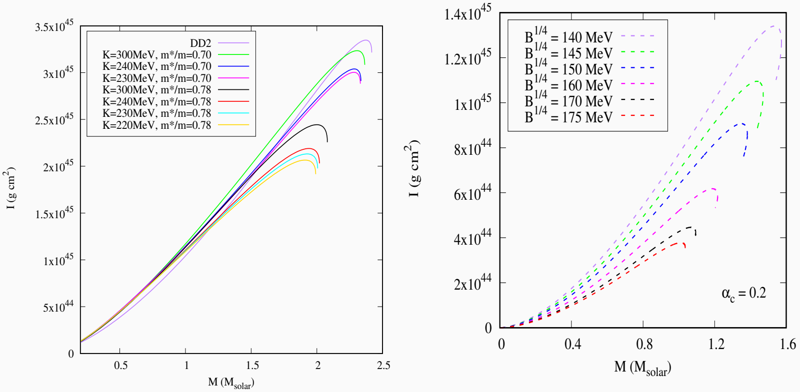


Figure 6: $I - M$ Relations for Neutron stars (left) and Quark stars (right) rotating at a frequency of 100Hz

[J. B. Hartle, Astrophys. J. **150** (1967) 1005.]

$Q - M$ Relations

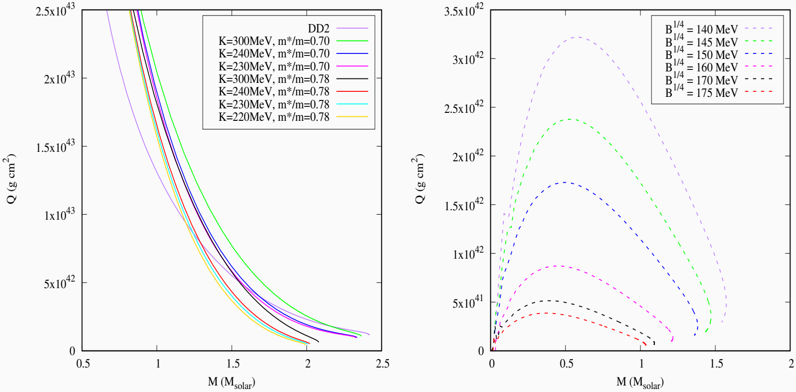
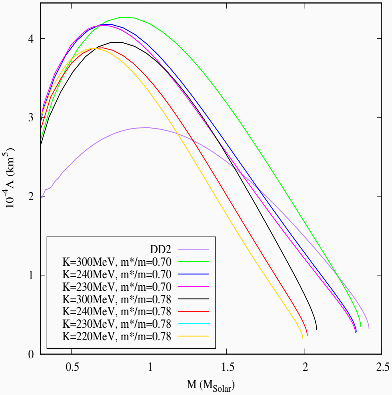
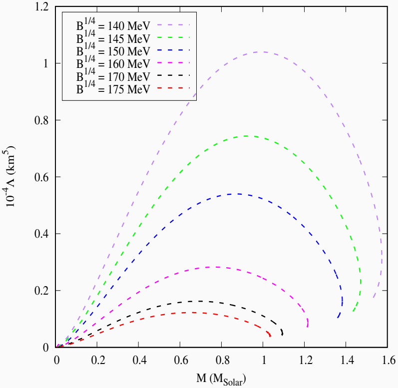


Figure 7: $Q - M$ Relations for Neutron stars (left panel) and Quark stars (right panel)

- Softer EoS $\Rightarrow$ Easily Compressed $\Rightarrow$ Stars are centrally condensed $\Rightarrow$ Smaller Deformation

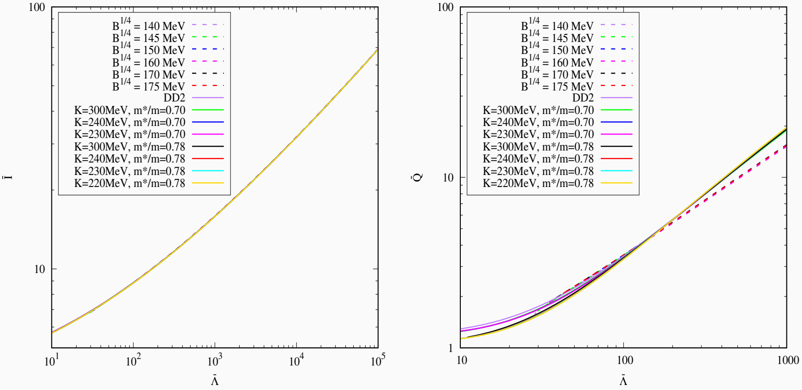
$\Lambda - M$ Relation





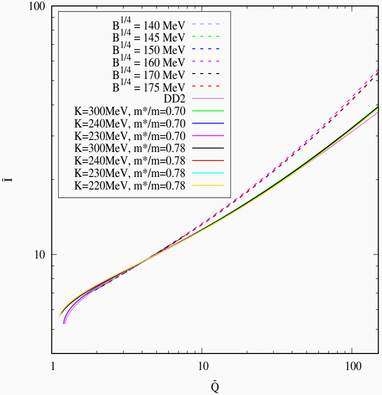
[T. Hinderer, Astrophys.J. 677 (2008) 1216-1220]

$I - Love - Q$ Relations

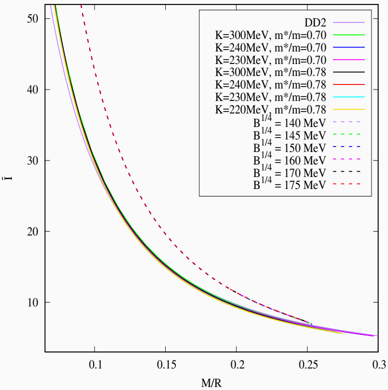


[K. Yagi and N. Yunes, Phys. Rev. D **88**, no. 2, 023009 (2013)]

$I - Q$

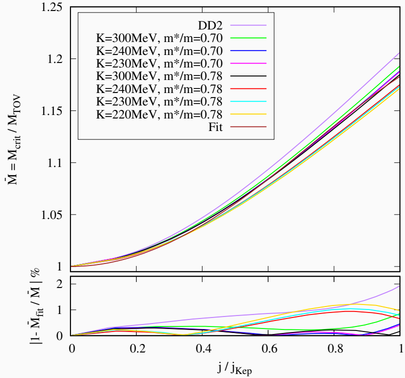


(e) Universal $I - Q$



(f) Universal $I - C(= M/R)$

Critical Mass - Angular Momentum



Quasi-Universal Relation

$$\frac{M_{crit}}{M_{TOV}} = 1 + a_2\left(\frac{j}{j_{Kep}}\right)^2 + a_4\left(\frac{j}{j_{Kep}}\right)^4.$$

$$M_{max} = M_{crit}(J = J_{Kep})$$

$$M_{max} = (1 + a_2 + a_4)\ M_{TOV}$$

$$\therefore M_{max} = (1.1829013 \pm 0.007747)\ M_{TOV}$$

- Critical mass is the maximum mass along a sequence of stars with constant angular momentum.
- At $j/j_{Kep} = 0$, the critical mass is M_{TOV}.
- Relevant range: $j/j_{Kep} \lesssim 0.5$.

[C. Breu and L. Rezzolla, Mon. Not. Roy. Astron. Soc. **459**, no. 1, 646 (2016)]

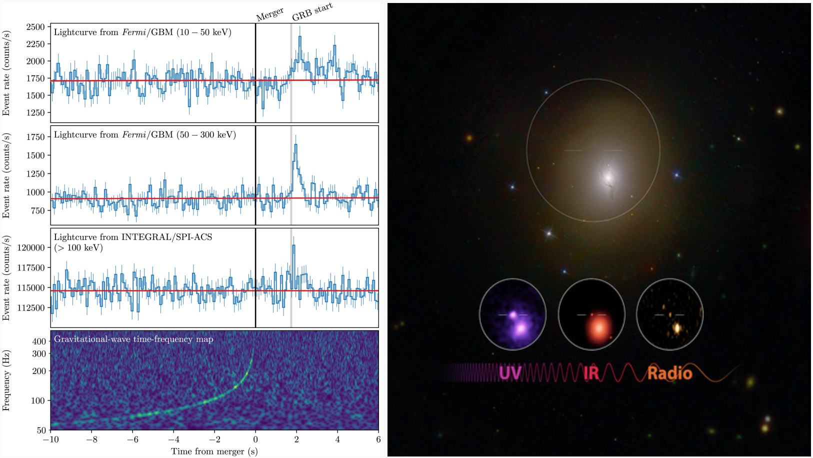
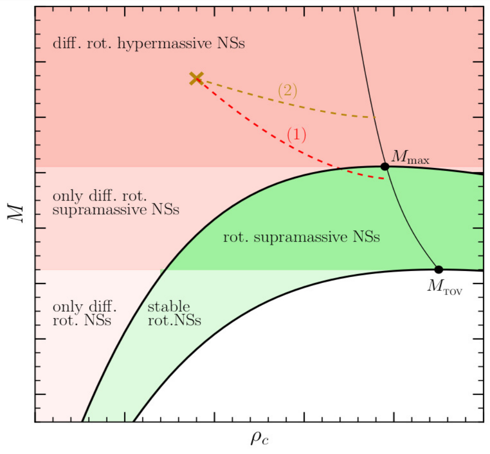


Figure 8: Multi-messenger detection of GW170817

[LIGO, Virgo, Fermi-GBM and INTEGRAL, Astrophys. J. **848** (2017) no.2, L13]

Fate of the merger remnant



[Rezzolla *et al*, Astrophys. J. Lett. **852**, 2018]

Upper Limit on maximum mass

Total mass of the merged product $\sim 2.73 M_{\odot}$
[LIGO, Virgo, Phys. Rev. X **9**, no.1, 011001, 2019]

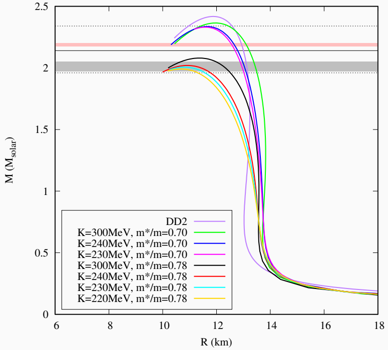
Mass loss $\sim 0.15 M_{\odot}$
[M. Shibata *et al.*, Phys. Rev. D **96**, 123012, 2017]

$\therefore$ Binary Merger Remnant $\sim 2.58 M_{\odot} \equiv M_{max}$

$$M_{max} = (1.1829013 \pm 0.007747)\ M_{TOV}$$

Maximum non-rotating mass:

$$\mathbf{M_{TOV} = \frac{2.58}{1.1829013} = 2.18 M_{\odot}}$$

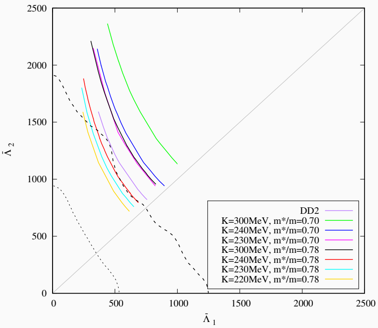


- Grey band : $2.01 M_{\odot}$ constraint with error [J. Antoniadis *et al*, Science **340**, 2013].
- Pink band : Upper bound on the maximum mass of a non-rotating neutron star.

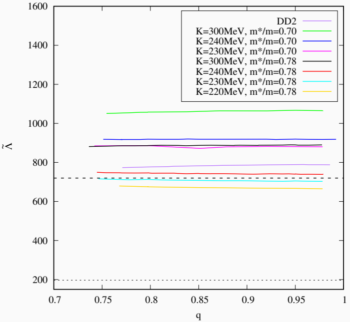
$M_{max} \lesssim 2.3$
[M.shibata *et al.*, Phys.Rev. D100 (2019) no.2, 023015]

Constraints on Tidal Deformability

$$\bar{\Lambda} = \frac{2}{3} k_2 \left(\frac{R}{M}\right)^5 \quad ; \quad \widetilde{\Lambda} = \frac{16}{13} \frac{[(M_1 + 12M_2)M_1^4 \bar{\Lambda}_1 + (M_2 + 12M_1)M_2^4 \bar{\Lambda}_2]}{(M_1 + M_2)^5}$$

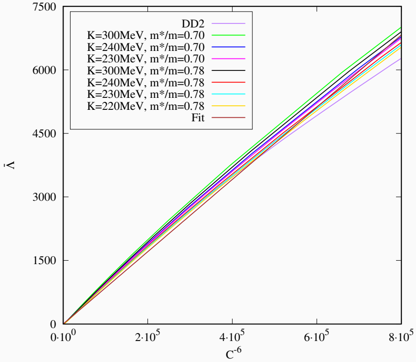


• Dashed and dotted curves : 90% and 50% confidence intervals
[LIGO, Virgo, Phys. Rev. X **9**, no.1, 011001, 2019].
• Grey Diagonal line :Λ_1=Λ_2 for $M_1 = M_2$.



• Lower bound: $\widetilde{\Lambda} = 197$
[Coughlin *et al.*, Mon.Not.Roy.Astron.Soc. 480 (2018) no.3]
• Upper bound: $\widetilde{\Lambda} = 720$
[LIGO, Virgo, Phys. Rev. X **9**, no.1, 011001, 2019]

Constraining Radius



$$\bar{\Lambda} = \frac{2}{3} k_2 \left(\frac{R}{M}\right)^5 = \frac{2}{3} k_2 C^{-5}$$

$$\bar{\Lambda} \simeq a C^{-6} \qquad \because k_2 \propto C^{-1}$$

[T. Zhao, J. M. Lattimer, Phys. Rev. D **98**, no. 6, 063020 (2018)]

$$\widetilde{\Lambda} = \frac{16}{13} \frac{[(M_1 + 12M_2)M_1^4 \bar{\Lambda}_1 + (M_2 + 12M_1)M_2^4 \bar{\Lambda}_2]}{(M_1 + M_2)^5}$$

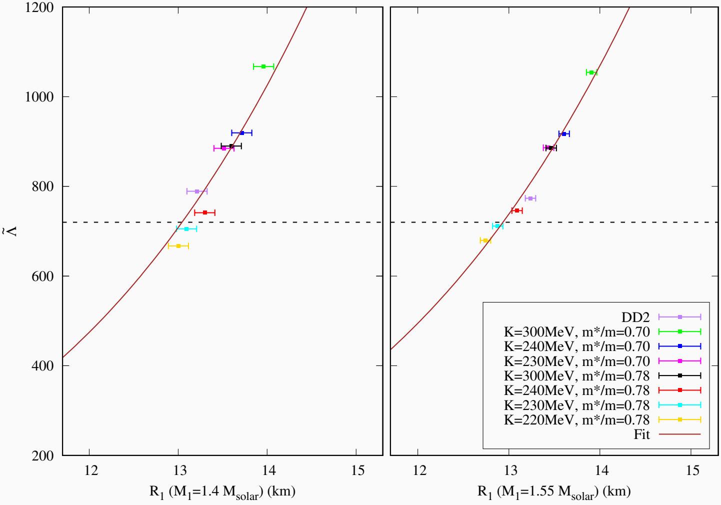
In the range $1.1M_\odot \lesssim M \lesssim 1.6M_\odot$, $R_1 \simeq R_2 \simeq R$.

$$\widetilde{\Lambda} = \frac{16a}{13} \times \left(\frac{R}{\mathcal{M}}\right)^6 \times \frac{q^{8/5}}{(1+q)^{26/5}} \left[12 - 11q + 12q^2\right]$$

$$\widetilde{\Lambda} = a' \left(\frac{R}{\mathcal{M}}\right)^6$$

- K = 220 (MeV), $m^*/m = 0.78$: 13.16 km
- K = 230 (MeV), $m^*/m = 0.78$: 13.28 km

Radius Constraints



- **Left** : $M_1 = 1.4M_\odot$ and $M_2 = 1.33M_\odot$. Upper limit on $R_1 = 13.04$ km.
- **Right** : $M_1 = 1.55M_\odot$ and $M_2 = 1.2M_\odot$. Upper limit on $R_1 = 12.94$ km.
- Fit : $\widetilde{\Lambda} = aR_1^5$ [C. Raithel, F. Özel, et al. Astrophys.J. 857 (2018) no.2, L23]

Summary

- Relativistic Mean-Field Model and Bag Model
- TOV and Hartle Thorne approximation
- Universal Relations : $I\!-\!Love\!-\!Q$ and quasi-universal relations of critical mass and angular momentum.
- GW170817 and its implications on Neutron Star EoS
  - Upper limit on maximum mass
  - EoS constraints from Tidal Properties
  - Radius Constraints

Outlook

- Hybrid Stars
- Finite temperature EoS

Thank You!

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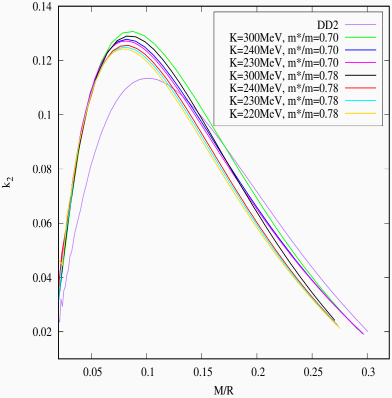
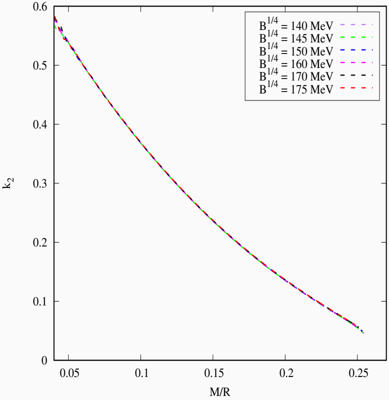
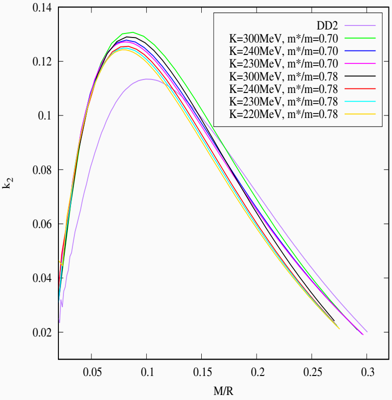
$k_2 - C$

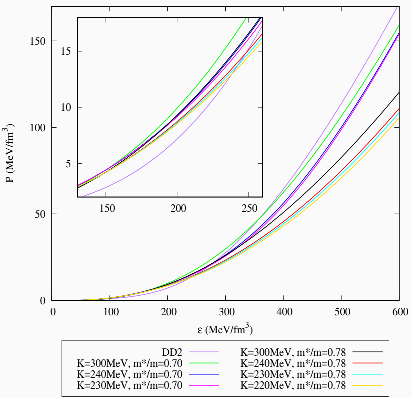
Compactness, $C = M/R$



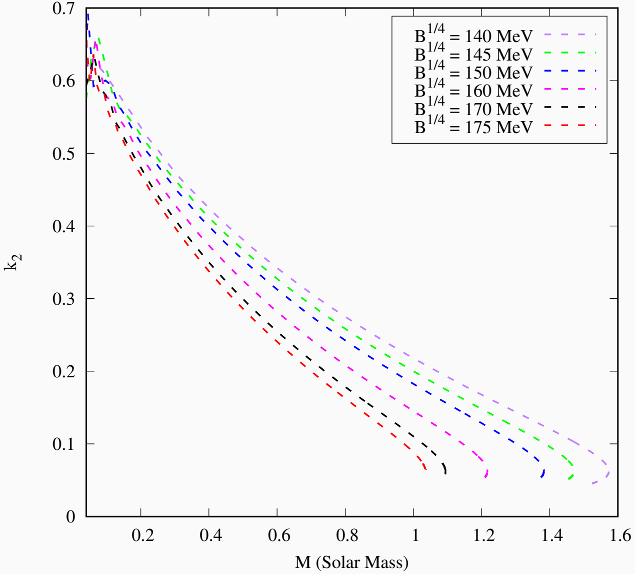


Dependence of Love Number on Compactness



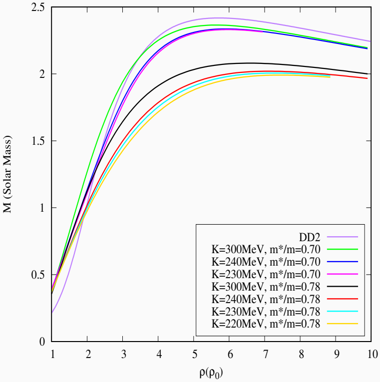


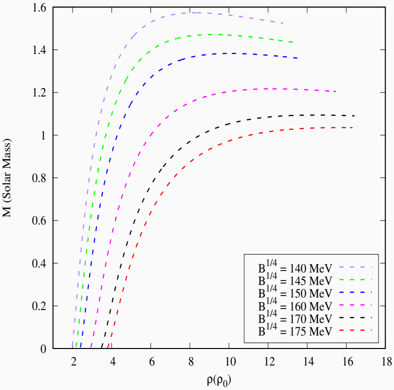
k_2 - Mass

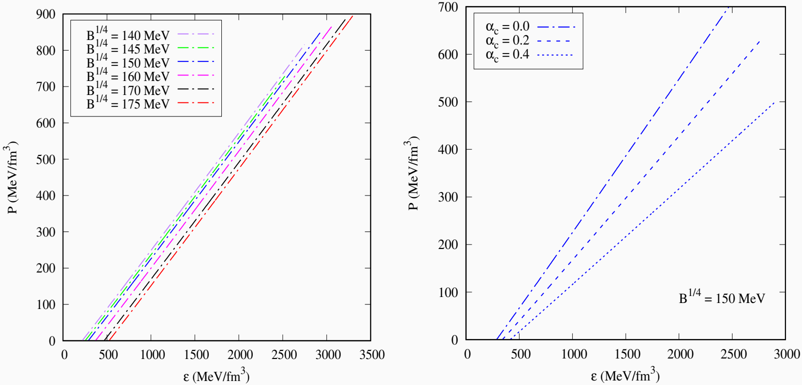


$B^{1/4}$ = 140 MeV
$B^{1/4}$ = 145 MeV
$B^{1/4}$ = 150 MeV
$B^{1/4}$ = 160 MeV
$B^{1/4}$ = 170 MeV
$B^{1/4}$ = 175 MeV
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
k_2
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
M (Solar Mass)

Mass - Central Density







B^1/4 = 140 MeV
B^1/4 = 145 MeV
B^1/4 = 150 MeV
B^1/4 = 160 MeV
B^1/4 = 170 MeV
B^1/4 = 175 MeV
P (MeV/fm^3)
ε (MeV/fm^3)
α_c = 0.0
α_c = 0.2
α_c = 0.4
B^1/4 = 150 MeV
P (MeV/fm^3)
ε (MeV/fm^3)

RMF Model with Density Dependent Couplings

The Lagrangian Density:

$$\mathcal{L}_B = \bar{\psi}_B(i\gamma_\mu\partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu - \frac{1}{2}g_{\rho B}\gamma_\mu\bar{\tau}_B\cdot\bar{\rho}^\mu)\psi_B$$

$$+\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu\cdot\rho^\mu$$

$$g_{\alpha B} = g_{\alpha B}(\rho_0)f_\alpha(\rho_B/\rho_0) \tag{1}$$

where,

$$f_\alpha(x) = a_\alpha\left[\frac{1+b_\alpha(x+d_\alpha)^2}{1+c_\alpha(x+d_\alpha)^2}\right] \tag{2}$$

$\alpha = \sigma, \omega$

For the ρ meson,

$$g_{\rho B} = g_{\rho B}(\rho_0)e^{-[a_\alpha(x-1)]} \tag{3}$$