Hot neutron stars with microscopic equations of state

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Outline

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 - Beta Equilibrium and TOV Equations
- Results
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 - Neutron star structure
 - Temperature dependence of max mass
- Conclusion



Introduction

Gravitational waves and EOSs...



GW170817 First Cosmic Event Observed in Gravitational Waves and Light

- FT-EOS works as a critical input to BNS simulation;
- Analysis of BNS merger events provides constraints to EOS.
- * Right figure shows the snapshot of temperature profile at time = 5.94 ms after merger. [Hanauske+2019]

ZLIGO

LDM: LS [Lattimer&Swesty 1991] RMF: HShen [HShen+1998,2011] GShen-NL3 [GShen+2010] GShen-FSU2.1 [GShen+2010] SFHo [Steiner+2013] BHF+Chiral: BL [Bombaci&Logoteta 2018]



Ranges of baryon number density *n*, temperature *T*, net electron fraction $Y_e = n_e/n$, and entropy per baryon *S* encountered in the indicated astrophysical phenomena.

-	Core-collapse supernovae	Proto-neutron stars	Mergers of compact binary stars
$\frac{n/n_s}{T (MeV)}$	$10^{-8}-10$ 0-30 0.35-0.45	$10^{-8} - 10$ 0-50 0.01-0.3	$ \begin{array}{c} 10^{-8} - 10 \\ 0 - 100 \\ 0.01 - 0.6 \\ 0 - 100 \end{array} $
Y_e S (k _B)	0.35–0.45 0.5–10	0.01–0.3 0–10	0.01–0.6 0–100

*Table from Lattimer & Prakash 2016

Temp dependence of max mass

[Burgio&Schulze 2010] isentropic BHF | RMF

Table 2. Properties of (P)NS minimum and maximum mass configurations.

		Mmi	nimum n	nass	Maximum mass			
		M/M_{\odot}	<i>R</i> (km)	$ ho_{\rm c}/ ho_0$	M/M_{\odot}	<i>R</i> (km)	$ ho_{\rm c}/ ho_0$	
untrapped	LS				2.03	9.86	10.55	
$T = \overline{0}$	SKa				2.03	9.86	10.42	
	Shen				2.03	9.93	10.42	
trapped	LS	0.58	40	1.02	1.95	10.2	11.34	
S/A = 1	SKa	0.60	38	1.08	1.95	10.2	11.20	
	Shen	0.58	44	1.02	1.95	10.3	11.20	
trapped	LS	0.70	44	0.90	1.95	10.7	10.85	
S/A = 2	SKa	0.77	42	0.90	1.95	10.8	10.70	
	Shen	0.75	52	0.77	1.95	10.8	10.80	

Table 1. Characteristics of the maximum mass configurations for dif-ferent stellar compositions and temperatures.

Composition	T (MeV)	M/M_{\odot}	<i>R</i> (km)	$ ho_{ m c}/ ho_0$
	0	1.86	9.5	8.2
N, l	10	1.82	9.5	8.1
	30	1.73	9.7	7.7

[Nicotra+2006] isothermal



Star properties for matter in beta equilibrium at finite entropy in the BPAL potential model

EOS	S	$\frac{M_{\rm max}}{M_{\odot}}$	R (km)	$\frac{n_{\rm c}}{n_0}$ [P	rakash	+1997]
	0	1.933	10.420	7.343	590.2	0.0
BPAL 32	1	1.943	10.589	7.138	577.7	36.7
	2	1.974	11.136	6.506	482.8	71.5
	0	1.955	10.797	7.000	532.0	0.0
BPAL 33	1	1.966	11.020	6.719	507.0	33.3
	2	1.994	11.518	6.198	454.3	66.0

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Theory BHF, FCA, BETA-EQ, and TOV...



Brueckner-Hartree-Fock Theory

 $K(
ho,eta;\omega)=v_{NN}+v_{NN}\mathbf{Re}\sum_{k_1,k_2}rac{|k_1k_2
angle Q(1,2)\langle k_1k_2|}{\omega-\epsilon(k_1)-\epsilon(k_2)}K(
ho,eta;\omega)$ (1)

Nucleon-Nucleon Interaction
 $V_{NN} = v_2 + V_3^{eff}$ We consider here: BOB, V18, N93, and UIX





Fig. Two terms in the BHF level of BBG expansion. [Song+1998]

Single Particle Potential(Non-locality), cont. choice: for all k

$$U(k) = \sum_{k' < k_F} {f Re} \langle kk' | K(
ho,eta;\omega=e+e') | kk'
angle$$
 (З

Frozen Correlation Approximation

The finite temperature BHF should be done under Block & De Dominics formalism [Block & Dominicis 1958,1959] and the framework is constructed within BHF theory by our group [Baldo & Ferreira 1999].

Ignore the effects of finite temperature on the single particle potential, we introduce a simplified way called "FCA":

$$f = \sum_i [\sum_k n_i(k)(rac{k^2}{2m_i} + rac{1}{2}U_i(k)) - Ts_i]$$
 (1)

where

$$s_i = -\sum_k (n_i(k) {
m ln} n_i(k) + [1-n_i(k)] {
m ln} [1-n_i(k)])$$
 (2)

Beta Equilibrium

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$$B_1 o B_2 + l + \overline{
u_l} extsf{B}_2 + l o B_1 +
u_l$$

We impose following requirements with *n*, *p*, *e*, *mu*:

- Charge neutrality $x_p = x_e + e_\mu$
- Chemical equilibrium condition between leptons

$$\mu_e=\mu_\mu$$

- Chemical equilibrium condition between leptons and baryons

 $\mu_n - \mu_p = \mu_e$ We assume a cold crust (T=0) and then attach NV EOS in the medium-density range and BPS EOS for outer crust. Fixed transition point at $\rho = 0.08 {\rm fm}^{-3}$



Tolman-Oppenheimer-Volkoff Equations

$$rac{{\mathrm{d}} p(r)}{{\mathrm{d}} r}=-rac{Gm(r)\epsilon(r)}{r^2}rac{[1+rac{p(r)}{\epsilon(r)}][1+rac{4\pi r^3p(r)}{m(r)}]}{1-rac{2Gm(r)}{r}}$$

 $rac{\mathrm{d}m(r)}{\mathrm{d}r}=4\pi r^2\epsilon(r)$

$$\frac{dm_B}{dr} = 4\pi r^2 \frac{\rho m_N}{\sqrt{1 - 2Gm/r}}$$

Once the relation of internal energy density and pressure $p(\epsilon)$, i.e. the EOS, is given.

For a chosen central value of energy density, the numerical integration of the above equations provides the mass-radius relation.

3. Results

Free Energy, NS Structure, M-R Relation...



Free Energy of nuclear matter

We use the parametrization introduced in [Burgio & Schulze 2010]

 $rac{F}{A}(
ho,T)=(A_0+A_2t^2)
ho+B_0
ho^{B_1}+Ct^2\ln(
ho)+(D_0t^2+D_1t^{D_2})/
ho+E$

where $t \equiv T/(100 \text{ MeV})$ and is dimensionless.

	A ₀	A ₂	B ₀	B ₁	С	D ₀	D ₁	D ₂	E
BOB SNM	-65	-124	498	2.67	203	-105	122	2.20	-9
BOB PNM	57	-85	856	2.91	152	-32	43	2.47	4
V18 SNM	-60	-147	369	2.66	209	-66	85	2.32	-8
V18 PNM	37	-91	667	2.78	154	-52	62	2.28	6
UIX SNM	-174	-186	323	1.61	199	-136	153	2.16	-4
UIX PNM	24	-117	326	2.09	153	-85	94	2.16	6
N93 SNM	-42	-142	298	2.61	211	-64	87	2.35	-12
N93 PNM	67	-95	743	2.71	154	-35	46	2.44	4

Table 1. Parameterizations of free energy at finite temperature

Free Energy



Composition of stellar matter

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Mg-R/nb relation at T = 0

- BCPM [Sharma+2015]
- M_G/M_{\odot} Nicotra 2006 G [Nicotra+2006]
- Li 2008 N93/UIX G [Li+2008]
- Li 2012 N93 G [Li+2012]
- Constraint $2.17^{+0.11}_{-0.10}M_{\odot}$ from MSP J0740+6620 (68.3% credibility interval) [arXiv:1904.06759]



Neutron star structure

Thermal internal energy $\epsilon_{
m th} = rac{E}{V} = rac{E}{N} rac{N}{V} = rac{3}{2} T
ho$

significantly **overestimates** the thermal effects.

Competitions

- 1. Thermal pressure Ĵ
- 2. 🛛 Baryonic pressure 🖶
- 3. Leptonic pressure Ĵ

Fig. Internal energy density and pressure of beta-stable matter at T=0 and the change of these thermal quantities at T=50 MeV.



Neutron star structure



17 matter at T = 50 MeV for the different EOSs.

Neutron star structure





- on different EOSs.
- The decrease of max mass is almost negligible, at most 1%.
- Density is still the dominant feature in neutron star mergers. [arXiv:1907.12760]





Tabulation EOSs in HShen Format Crust: HShen EOS 2011 version Finite temperature equation of states based on BHF theory

equation-of-state nuclear-physics many-body-theory Manage topics

4 commits	រ្វិ 1 branch	🛇 0 releases		🤽 1 contributor				
Branch: master New pull request			Create new file	Upload files	Find file	Clone or download 🗸		
😰 lujiajing1126 add affil					Latest co	ommit f 645c85 on Jun 8		
.gitattributes		add EOSs	version1	4 months ago				
README.md		add affil		4 months ago				
eos_LU2019_BOB_HShen_v20190326.tab.gz		add EOSs version1				4 months ago		
eos_LU2019_N93_HShen_v20190509.tab.gz		add EOSs version1				4 months ago		
eos_LU2019_UIX_HShen_v20190510.tab.gz		add EOSs version1			4 months ago			
eos_LU2019_V18_HShen_v20190508.tab.gz		add EOSs version1				4 months ago		

README.md

FT-EOS

this repository provides finite temperature equations of state (EOSs) built with BHF theory

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Available on



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Conclusion

- Present convenient parameterizations of FT-EOSs within Brueckner theory
- Discuss the density dependence of the adiabatic index
- Temperature dependence of the maximum NS mass
- A set of EOS tabulations is also provided.
- Antonio will talk about the numerical simulation results on Thursday



Thanks for your attention

Appendix

Numerical methods



Beta Equilibrium

Chemical Potentials

Bayron [Baldo+1998, 2000; Baldo & Ferreira 1999]

$$\mu_{p.n}(
ho_N,eta)=\mu_{p,n}(
ho_N,0)-(eta^2\pm 2eta-eta^2
ho_Nrac{\partial}{\partial
ho_N})E_{
m sym}(
ho_N)$$
 (1)

where $\mu_{p,n}(\rho_N, 0)$ is the chemical potential of a nucleon in symmetric matter (+ for p, – for n), and in particular

$$[\mu_n-\mu_p](
ho_N,eta)=4eta E_{
m sym}(
ho_N)$$

$$\mu_{p,n}(
ho_N,0)=f+p/
ho=f+
horac{\partial f}{\partial
ho}$$

At finite temperature, $E_{
m sym}$ should be replaced by $F_{
m sym}$.

Beta Equilibrium

- Chemical Potentials
 - Leptons [Shapiro & Teukolsky 2008]
 - in the natural units: $\, c = {oldsymbol \hbar} = k_B = 1 \,$

$$\mu_{e,\mu} = rac{2}{h^3} \int_0^{p_{ ext{cutoff}}} rac{E_{e,\mu}}{1 + e^{(E(k) - \mu)/T}} \mathrm{d}p^3 = rac{1}{\pi^2} \int_0^{p_{ ext{cutoff}}} p^2 rac{E_{e,\mu}}{1 + e^{(E(k) - \mu)/T}} \mathrm{d}p$$

For electron (ultrarel.):

 $E_e(k)=\hbar k=197.33({
m MeV/fm})k$ For muon (rel.):

$$E_{\mu}(k) = \sqrt{m_{\mu}^2 + (\hbar k)^2} = \sqrt{(m_{\mu}^2) + (197.33 ({
m MeV/fm})k)^2}$$



Temperature Dependence

Prakash+1997 got increasing max masses with temperature in the RMF framework. Also confirmed by Kaplan+2014.

Possible reasons of decreasing max mass:

- interaction part in RMF? [Nicotra+2006]

$$f = \sum_i [\sum_k n_i(k) rac{k^2}{2m_i} + rac{n_i^{T=0}(k)}{2} U_i(k) - T s_i]$$

where $n_i^{T=0}(k)$ is a step function.

- **Three-Nucleon Force** (TNF)
 - Free symmetry energy -> larger isospin asym.
 - Free energy of SNM

