Elementary particle masses from a non-perturbative anomaly

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Talk based on

- R. Frezzotti and G.C. Rossi, Phys. Rev. D92 (2015) 054505, arXiv:1811.10326 (PoS - Lattice 2018) and work in preparation
- R. Frezzotti, M. Garofalo and G.C. Rossi, Phys. Rev. D93 (2016) 105030
- S. Capitani *et al.*, Phys.Rev.Lett. 123 (2019) 061802; S. Capitani *et al.* EPJ Web Conf. 175 (2018) 08008 and 08009

a challenging lattice project carried out in 2017-'19 in collaboration with

⋆ P. Dimopoulos and G.C. Rossi (Univ. of Roma Tor Vergata and Centro Fermi)

⋆ M. Garofalo (INFN – Roma Tor Vergata; Univ. of Edinburgh)

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Outline

- Introduction
- Mechanism in simplest Toy Model:

  basic ideas & lattice demonstration
- Mechanism in an extended Toy Model with $g_W > 0$
- Towards a realistic BSM model: new interactions & matter
- Higgs as a bound state & sub-TeV effective Lagrangian
- Outlook & Conclusions

Assuming you'll be skeptical, hope you can get curious ...

Standard Model: success, limits and paradox

SM: very successful, but an extension is necessary to account for observed

- neutrino masses and mixings
- dark matter
- baryogenesis (larger CP violation needed)
- dark energy & quantum aspects of gravity

SM "paradox": phenomenologically incomplete but renormalizable

In spite of its theoretical self-consistency, a completion of the SM at some (what?) high energy scale is required by experimental facts.

SM renormalizability makes hard to guess this scale $\Rightarrow$ different scenarios:

- SM valid up to energies $E \leq 10^{16} \div 10^{19}$ GeV (big desert)
- SUSY or New Dynamics (non-perturbative?) at $E >$ few TeV

The low energy ($E < 1$ TeV) Lagrangian cannot differ much from the SM one.

The hierarchy problems for the EW scale and fermion masses are entangled,

see e.g. Bardeen, Hill, Lindner, Phys. Rev. D41 (1990) ... Composite Higgs Models.

Elementary particle masses: hierarchy – NP origin?

SM describes the masses of elementary fermions and weak bosons ($W^{\pm}$, Z^0)

- well established symmetry pattern: $SU(2)_L \times U(1)_Y \xrightarrow{SSB} U(1)_{em}$

  but no hope of explaining flavour mixings or mass orders of magnitude

- $m_{W,Z} \sim m_h \sim v \sim 10^{-13}\Lambda_{GUT} \sim 10^{-16}\Lambda_{Planck}$

  no extra symmetry when $m_h \sim v$ gets small $\Rightarrow$ un–naturalness

- $m_t \sim 10^5 m_u \,, \quad m_\tau \sim 10^4 m_e \,, \quad m_e > 10^7 m_{\nu_i}$ $\Rightarrow$ huge hierarchy

A possible alternative: a novel intrinsically non-perturbative (NP) mechanism

at work in non-Abelian gauge models with fermions and auxiliary scalars where

1) chiral symmetries ($\chi_{L,R}$) acting on fermions and scalars are exact

2) fermionic chiral transformations ($\tilde{\chi}_{L,R}$) are explicitly broken at the UV scale ...

... to minimal extent (critical model) at low energy: NP "anomaly" $\Leftrightarrow$ fermion mass

Renormalizable models with NP $\tilde{\chi}$ "anomaly"

Anomaly: in the quantum Effective Lagrangian 1PI vertices of NP origin violate $\tilde{\chi}_{L,R}$,

yielding mass $\propto \Lambda_T$ for elementary fermions and (if $g_W \neq 0$) weak bosons

Physical interest: if a new non-Abelian gauge interaction (coupling g_T) exists and

- ⋆ Λ_T is the theory's RGI scale – phenomenologically $\Lambda_T \gtrsim 5$ TeV
- ⋆ mass coefficients are controlled by the "particle's largest gauge coupling"
- ⋆ new fermions feeling the new interactions and with masses $\sim \Lambda_T$ exist
- ⋆ a composite Higgs bound state (h) gets formed in the WW+ZZ channel

At scale $E < 1$ TeV one expects: $\Gamma^{NG}_{\rm LE} \supset (c\Lambda_T^2 + c'\Lambda_T h + \ldots)\frac{1}{2}{\rm Tr}(D_\mu^{W,B} U^\dagger D_\mu^{W,B} U) +$

$+ \Lambda_T[c_1^t \bar{q}_L \tilde{u} q_R^t + c_1^b \bar{q}_L u q_R^b] + \Lambda_T[0 \bar{\ell}_L \tilde{u} \ell_R^\nu + c_1^\tau \bar{\ell}_L u \ell_R^\tau] + \text{other generations terms} + \mathrm{O}(\Lambda_T^{-1})$

with $U = [\tilde{u}|u] = \exp\left(i\zeta^j \tau^j / \sqrt{c}\Lambda_T\right)$, $\zeta^{1,2,3}$ GB fields, $u \overset{SU(2)_L \times U(1)}{\sim} \phi$

NP mechanism for elementary particle masses

A common NP & "natural" mechanism for both fermion and weak boson masses

⇒ basis for a number of models, which must satisfy experimental constraints, e.g.

⋆ SM-like 1:2 relation between W-mass and WWh coupling up to $\mathrm{O}(p^2/\Lambda_T^2)$

⋆ effective mass $m_f \propto f\bar{f}h$ effective coupling for all SM fermions (f) up to $\mathrm{O}(p^2/\Lambda_T^2)$

⋆ no tree level FCNC: due to SM-like form of all fermion effective mass terms

⋆ EW precision tests: S-parameter bounds "ok" owing to $m_{Tf}^{eff} \sim \mathrm{O}(1)\Lambda_T$

⇒ insights on the new physics scale Λ_T and on fermion hierarchies, e.g.

⋆ $m_t^{eff} = \mathrm{O}(\alpha_s^2|_{\Lambda_T})\Lambda_T\,,\;\; m_\tau^{eff} = \mathrm{O}(\alpha_Y^2|_{\Lambda_T})\Lambda_T\,,\;\; m_W^{eff} = \mathrm{O}\Big(g_W|_{\Lambda_T}((4\pi)^3 N_{Tf})^{-1/2}\Big)\Lambda_T\,,$

⋆ $m_{\rm neutrino}^{eff} \simeq 0$, possibly $m_b^{eff} = \mathrm{O}(\alpha_{S,W,Y})m_t^{eff}$, ... 2nd and 1st generation ...

⋆ possible gauge coupling unification: $\Lambda_{GUT} \simeq 10^{18}$ GeV, $m_{\rm neutrino}^{eff} \sim \frac{\Lambda_T^2}{\Lambda_{GUT}}$
[Frezzotti-Garofalo-Rossi, PRD 93 (2016) 105030]

Outline

Mechanism in simplest Toy Model:

basic ideas & lattice demonstration

The simplest $d = 4$ toy model

- ⋆ one strong interaction: SU(3) gauge field A_μ , dynamical RGI scale Λ_S
- ⋆ one Dirac fermion doublet: $Q = (f_U, f_D)^T$, a triplet of SU(3)$_{gauge}$
- ⋆ one scalar doublet: $\varphi = (\varphi_2 - i\varphi_1, \varphi_0 - i\varphi_3)^T$, SU(3)$_{gauge}$–neutral

Lagrangian – using Φ-matrix notation: $\Phi = [\tilde{\varphi}|\varphi]$, $\tilde{\varphi} = -i\tau^2\varphi^*$

$$\mathcal{L}_{\rm toy}(Q, A, \Phi) = \mathcal{L}_{kin}(Q, A, \Phi) + \mathcal{V}(\Phi) + \mathcal{L}_{Wil}(Q, A, \Phi) + \mathcal{L}_{Yuk}(Q, \Phi)$$

- $\mathcal{L}_{kin}(Q, A, \Phi) = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{Q}_L \gamma_\mu \mathcal{D}_\mu Q_L + \bar{Q}_R \gamma_\mu \mathcal{D}_\mu Q_R + \frac{1}{2} \mathrm{Tr}\left[\partial_\mu \Phi^\dagger \partial_\mu \Phi\right]$
- $\mathcal{L}_{Wil}(Q, A, \Phi) = \frac{b^2}{2} \rho \left(\bar{Q}_L \overleftarrow{\mathcal{D}}_\mu \Phi \mathcal{D}_\mu Q_R + \bar{Q}_R \overleftarrow{\mathcal{D}}_\mu \Phi^\dagger \mathcal{D}_\mu Q_L \right)$
- $\mathcal{L}_{Yuk}(Q, \Phi) = \eta \left(\bar{Q}_L \Phi Q_R + \bar{Q}_R \Phi^\dagger Q_L \right)$
- $\mathcal{V}(\Phi) = \frac{\mu_0^2}{2} \mathrm{Tr}\left[\Phi^\dagger \Phi\right] + \frac{\lambda_0}{4} \left(\mathrm{Tr}\left[\Phi^\dagger \Phi\right]\right)^2 , \qquad \Phi \equiv [-i\tau^2 \varphi^* | \varphi]$

dimensionful UV cutoff: $\Lambda_{UV} \sim b^{-1} \to \infty$, $\hat{\mu}^2_\Phi = Z^{-1}_{\Phi^\dagger\Phi} [\mu_0^2 - \tau_0 b^{-2}]$

Toy model: symmetries & renormalizability

• $\mathcal{L}_{kin}(Q,A,\Phi) = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{Q}_L\gamma_\mu\mathcal{D}_\mu Q_L + \bar{Q}_R\gamma_\mu\mathcal{D}_\mu\, Q_R + \frac{1}{2}\mathrm{Tr}\big[\partial_\mu\Phi^\dagger\partial_\mu\Phi\big]$ • $\mathcal{V}(\Phi) = \ldots$

• $\mathcal{L}_{Wil}(Q,A,\Phi) = \rho\frac{b^2}{2}\big(\bar{Q}_L\overleftarrow{\mathcal{D}}_\mu\Phi\mathcal{D}_\mu Q_R + \bar{Q}_R\overleftarrow{\mathcal{D}}_\mu\Phi^\dagger\mathcal{D}_\mu Q_L\big)$ • $\mathcal{L}_{Yuk}(Q,\Phi) = \eta\,\big(\bar{Q}_L\Phi Q_R + \bar{Q}_R\Phi^\dagger Q_L\big)$

1) chiral symmetries $\chi_L \times \chi_R$ acting on fermions Q and scalars Φ are exact

$\chi_L\ :\ \tilde{\chi}_L \otimes \chi_L^\Phi$ and $\chi_R\ :\ \tilde{\chi}_R \otimes \chi_R^\Phi$ with

$$\tilde{\chi}_L : Q_L \to \Omega_L\, Q_L\,,\ \bar{Q}_L \to \bar{Q}_L\,\Omega_L^\dagger\,, \quad \chi_L^\Phi : \Phi \to \Omega_L\Phi\,, \quad \Omega_L \in \mathrm{SU}(2)_L\,,$$
$$\tilde{\chi}_R : Q_R \to \Omega_R Q_R\,,\ \bar{Q}_R \to \bar{Q}_R\Omega_R^\dagger\,, \quad \chi_R^\Phi : \Phi \to \Phi\Omega_R^\dagger\,, \quad \Omega_R \in \mathrm{SU}(2)_R\,,$$

Symmetries (Poincaré, T, P, C, SU(3) gauge, $\chi_L \times \chi_R$) $\Rightarrow$ renormalizability,

no UV divergent $\Lambda_{UV}\bar{Q}Q$ terms, only $\mathrm{O}(\Lambda_{UV}^{-2} \sim b^2)$ cutoff effects on the lattice

2) fermionic chiral transformations $\tilde{\chi}_L \times \tilde{\chi}_R$ are explicitly broken by $\mathcal{L}_{Wil}$ and $\mathcal{L}_{Yuk}$

⋆ if $\rho \neq 0$ or $\eta \neq 0$ ⋆ effective $\tilde{\chi}$–breaking is minimal at $(\rho, \eta_{cr}(\rho,\ldots),\ldots)$

NP terms in the NG phase quantum effective action

What $d \leq 4$ operators may appear in Γ, the quantum Effective Lagrangian (EL)?

At $(\eta, \rho) \neq (0,0)$ we expect $\quad \Gamma = \Gamma_{d\leq 4,\ \hat{\mu}^2_\Phi} + \Delta\Gamma^{NG}_{d\leq 4,\ \hat{\mu}^2_\Phi} + \Gamma_{d>4,\ \hat{\mu}^2_\Phi}\ ,\quad$ with

$$\Gamma_{d\leq 4,\ \hat{\mu}^2_\Phi} = \tfrac{1}{4}(F\cdot F) + \bar{Q}\not{D}Q + \tfrac{1}{2}\,\mathrm{Tr}\left[\partial_\mu\Phi^\dagger\partial_\mu\Phi\right] + \tfrac{\hat{\mu}^2_\phi}{2}\,\mathrm{Tr}\left[\Phi^\dagger\Phi\right] + \tfrac{\hat{\lambda}}{4}\left(\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\right)^2 +$$

$$+(\eta - \bar{\eta}(\eta,\rho,\ldots))[\bar{Q}_L\Phi Q_R + \text{ h.c.}]\quad ,\qquad \text{where } \bar{\eta}(\eta,\rho,\ldots) \text{ is independent of } \hat{\mu}^2_\Phi,$$

and NP $d \leq 4$ terms if $\chi_L \times \chi_R$ is realized à la Nambu–Goldstone (NG), i.e. $\hat{\mu}^2_\Phi < 0$,

$$\Delta\Gamma^{NG}_{d\leq 4,\ \hat{\mu}^2_\Phi} = \theta(-\hat{\mu}^2_\Phi)[c_1\Lambda_S(\bar{Q}_L U Q_R + \text{ h.c.}) + (c_2\Lambda_S^2 + \tilde{c}\,\Lambda_S R)\tfrac{1}{2}\mathrm{Tr}(\partial_\mu U^\dagger\partial_\mu U)]\ ,$$

where $\quad \Phi = v_\Phi + \sigma + i\vec{\tau}\vec{\pi} = RU\ ,\qquad R = (v_\Phi + \zeta_0)\ ,\qquad U = \exp[i v_\Phi^{-1}\tau^k\zeta_k]\ .$

$\Rightarrow$ as $\eta \to \eta_{cr} = \bar{\eta}(\eta_{cr}, \rho, g_0^2, \lambda_0)$ the $\tilde{\chi}_L \times \tilde{\chi}_R$ symmetry is enhanced (critical model)

$\Rightarrow$ NP "anomaly" in $\tilde{\chi}$ restoration from the term $\propto c_1\Lambda_S$ and analogous $d > 4$ terms

$\Rightarrow$ at $\eta = \eta_{cr}$: NP fermion mass $\sim \mathrm{O}(\rho|\rho|g_s^4)\Lambda_S$, with $\Lambda_S =$theory's dynamical scale

Key ingredients for NP fermion mass generation

Crucial : $\tilde{\chi}$–breaking $d > 4$ Lagrangian terms, criticality, SSB of $\chi_L \times \chi_R$ invariance

- were $\rho = 0$, UV regulated model would be $\tilde{\chi}$–symmetric at $\eta_{cr} = 0$: no NP mass
- at $\rho \neq 0$, η_{cr} is the η-value for which no term $[\bar{Q}_L \Phi Q_R + \text{ h.c.}]$ occurs in the EL
- in the Wigner phase, i.e. $\hat{\mu}_\Phi^2 > 0$, $U = \Phi/\sqrt{\Phi^\dagger \Phi}$ is not well defined, no $\Delta\Gamma^{NG}_{d\leq 4,\ \hat{\mu}_\Phi^2}$
- in the NG phase SSB of $\chi_L \times \chi_R$ invariance, strong interactions and $\tilde{\chi}$–breaking

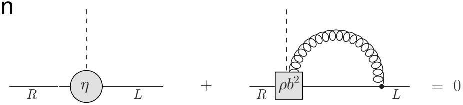
  due to $\mathcal{L}_{Wil+Yuk}{}^{\eta=\eta_{cr}(\rho,\dots)} = \frac{b^2}{2}\rho\big(\bar{Q}_L \overleftarrow{\mathcal{D}}_\mu \Phi \mathcal{D}_\mu Q_R + \text{h.c.}\big) + \eta_{cr}\big(\bar{Q}_L \Phi Q_R + \text{h.c.}\big)$

  induce $\tilde{\chi}$–SSB, with $\mathcal{L}_{Wil+Yuk}{}^{\eta=\eta_{cr}(\rho,\dots)}$ selecting one of the degenerate vacua

⋆ for $\hat{\mu}_\Phi^2 > 0$ one can easily evaluate η_{cr} by enforcing $\tilde{J}_\mu^{L,i}$ (or $\tilde{J}_\mu^{R,i}$) conservation

$$x \neq 0\,, \qquad \partial_\mu \langle Z_{\tilde{J}} \tilde{J}_\mu^{L,i}(x)\, O^i(0)\rangle = (\eta - \bar{\eta})\langle[\bar{Q}_L \tau^i \Phi Q_R - \text{h.c.}](x) O^i(0)\rangle + \mathrm{O}(b^2) = 0$$

i.e. the low energy cancellation



NP $\tilde{\chi}$ anomaly in simplest toy model: remarks

- NP anomaly in $\tilde{\chi}$ restoration was conjectured in PRD92 (2015) [RF & GCR] and is "demonstrated" via lattice simulations [Bonn-Rome group, PRL 123 (2019)]
- it shows up as RG-invariant terms in the renormalized $\tilde{\chi}$ Schwinger–Dyson Eqs.

  at $\rho \neq 0$, $\eta = \eta_{cr}(\rho)$ $\quad \partial_\mu \langle Z_{\tilde{J}} \tilde{J}^{L,i}_\mu(x)\, O^i(0)\rangle|^{\hat{\mu}^2_\Phi<0}_{x\neq 0} = \mathrm{O}(c_1 \Lambda_S b^0) + \mathrm{O}(b^2)$

  in the NG phase of the critical model – but $\partial_\mu Z_{\tilde{J}} \tilde{J}^{L,i}_\mu = 0 + \mathrm{O}(b^2)$ if $\hat{\mu}^2_\Phi > 0$
- at scales $p \ll \Lambda_{UV}$, any set of $d > 4$ $\tilde{\chi}$–breaking operators is equivalent to

  $\rho b^2 \frac{1}{2}(\bar{Q}_L \overleftarrow{\mathcal{D}}_\mu \Phi \mathcal{D}_\mu Q_R + h.c.)$ + $\eta_{cr}(\rho)\times$ Yukawa term with a suitable ρ:

  one has *universality classes* of massive critical models labelled by ρ where

  Γ contains $\tilde{\chi}$–violating NP terms: $\Delta\Gamma^{NG}_{d\leq 4,\ \hat{\mu}^2_\Phi} + \mathrm{O}(\Lambda^2_S/v^4_\Phi)[(\bar{Q}_L U Q_R)(\bar{Q}_R U^\dagger Q_L)] + \ldots$
- value of v_Φ matters for quantitative details of the toy model, but is irrelevant for $\Delta\Gamma^{NG}_{d\leq 4,\ \hat{\mu}^2_\Phi}$ / fermion mass existence and physics in more realistic models ($g_W \neq 0$)

Arg1: origin of NP vertex corrections at $O(b^2\alpha_s\Lambda_s)$

Consider small-b^2 expansion of formally $\tilde{\chi}_L \times \tilde{\chi}_R$ invariant correlators

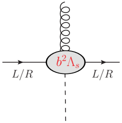
$$\langle O(x,x',\dots)\rangle\Big|_{cr}^{R} = \langle O(x,x',\dots)\rangle\Big|_{cr}^{F} - b^2\langle O(x,x',\dots)\int d^4z\,[L_6^{\tilde{\chi}br} + L_6^{\tilde{\chi}co}](z)\rangle\Big|_{cr}^{F} + O(b^4)$$

$$O(x,x',\dots) \Leftrightarrow A_\mu^b A_\nu^c\,\sigma\,,\ \ Q_{L/R}\bar{Q}_{L/R}\,\sigma\,,\ \ Q_{L/R}\bar{Q}_{L/R}A_\mu^b\,\sigma$$

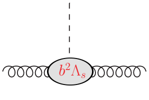
- $\langle\dots\rangle|^R$ = UV-Regulated $\qquad \langle\dots\rangle|^F$ = Formal correlator
- $\mathcal{L}_{Yuk} + \mathcal{L}_{Wil} \quad \Longrightarrow \quad L_6^{\tilde{\chi}br} \to \tilde{\chi}$-violating, d = 6 Symanzik LEL terms
- $\tilde{\chi}$-invariant $\quad O \to b^2\langle O \int L_6^{\tilde{\chi}br}\rangle|_{cr}^{F} \neq 0$, only due to dynamical $\tilde{\chi}$SB

  (otherwise it would vanish due to $\tilde{R}_5 \equiv [Q \to \gamma_5 Q\,,\, \bar{Q} \to -\bar{Q}\gamma_5] \in \tilde{\chi}_L \times \tilde{\chi}_R$ symmetry)
- dimensional arguments $\Rightarrow$ NP $\quad b^2\,O(\alpha_s|\rho|\Lambda_s)$ corrections occur in several $\tilde{\chi}$–conserving vertices ...

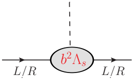
$\Delta\Gamma^{b\mu}_{Q\bar{Q}A\Phi} \quad \Longrightarrow$



$\Delta\Gamma^{bc\,\mu\nu}_{AA\Phi} \quad \Longrightarrow$



$\Delta\Gamma_{Q\bar{Q}\Phi} \quad \Longrightarrow$

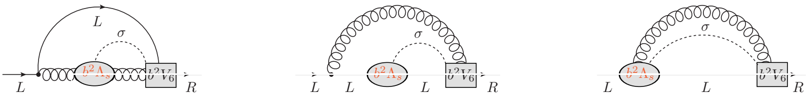


Arg2: from NP vertex corrections to fermion mass

$\Delta\Gamma_{AA\Phi,Q\bar{Q}\Phi,\ldots} = b^2\Lambda_s \mathrm{O}(|\rho|\alpha_s)\, w_{\mathrm{analytic}}(\mathrm{mom})\, F_{AA\Phi,Q\bar{Q}\Phi,\ldots}(\frac{\Lambda_s^2}{\mathrm{mom}^2})$ occur for

$p^2 \ll b^{-2}$; conjecture: they persist up to $p^2 \sim b^{-2} \to \infty \quad \Leftrightarrow \quad F_{\ldots}(0) = \mathrm{O}(1)$

- self-energy diagrams like



- give (e.g. central panel – surviving in quenched approximation)

$$\underline{\underline{m_Q^{eff}}} \propto g_s^2\rho|\rho|\alpha_s(\Lambda_s)\int^{1/b}\frac{d^4k}{k^2}\frac{\gamma_\mu k_\mu}{k^2}\int^{1/b}\frac{d^4\ell}{\ell^2+m_\sigma^2}\frac{\gamma_\nu(k+\ell)_\nu}{(k+\ell)^2}\cdot$$

$$\cdot b^2\gamma_\rho(k+\ell)_\rho\, b^2\Lambda_s\gamma_\lambda(2k+\ell)_\lambda \sim \underline{\underline{g_s^2\rho|\rho|\alpha_s(\Lambda_s)\Lambda_s}}$$

with the b^4 factor compensated by the two-loop quartic divergency

- in $\Gamma^{NG}_{4\,\hat{\mu}_\Phi^2}$ a NP mass term $\supset c_1\Lambda_S[\bar{Q}_L Q_R + \bar{Q}_R Q_L]\,, \quad c_1\Big|_{LO} = k_{LO}\rho|\rho|\, g_S^4$

Numerical Demonstration – 1: lattice formulation

First study with A_μ, Q, Φ field, needs lattice setup with exact $\chi_L \times \chi_R$ symmetry.

Simplified by quenched approximation: with no sea quark effects "NP anomaly"

is still expected, but can use naive *valence* lattice fermions + $\tilde{\chi}$-breaking terms

$$S_L = b^4 \sum_x \left\{ \mathcal{L}_k^g[U_\cdot] + \mathcal{L}^s(\Phi) + \bar{Q}(x) D_L[U_\cdot, \Phi] Q(x) \right\} + b^4 \mu \sum_x \left\{ \bar{Q}(x) i \mu \gamma_5 \tau^3 Q(x) \right\} ,$$

$$\mathcal{L}_k^g[U_\cdot] \, : \text{YM action} \, , \quad \mathcal{L}^s(\Phi) = \tfrac{1}{2} \operatorname{Tr}[\Phi^\dagger(-\partial_\mu^* \partial_\mu)\Phi] + \tfrac{m_\phi^2}{2} \operatorname{Tr}[\Phi^\dagger \Phi] + \tfrac{\lambda_0}{4} (\operatorname{Tr}[\Phi^\dagger \Phi])^2 \, ,$$

with $\Phi = \varphi_0 \mathbb{1} + i \varphi_j \tau^j$. In terms of $F \equiv [\varphi_0 \mathbb{1} + i \gamma_5 \tau^j \varphi_j]$ the Dirac operator D_L reads

$$D_L[U_\cdot, \Phi] Q(x) = \gamma_\nu \widetilde{\nabla}_\nu Q(x) + \eta F(x) Q(x) - b^2 \rho \tfrac{1}{2} F(x) \widetilde{\nabla}_\nu \widetilde{\nabla}_\nu Q(x) +$$

$$-b^2 \rho \tfrac{1}{4} \left[(\partial_\nu F)(x) U_\nu(x) \widetilde{\nabla}_\nu Q(x+\hat{\nu}) + (\partial_\nu^* F)(x) U_\nu^\dagger(x-\hat{\nu}) \widetilde{\nabla}_\nu Q(x-\hat{\nu}) \right]$$

Fermion mass term $\propto \mu$: technical device needed as IR cutoff in quenched approx.,

it breaks softly the $\chi_L \times \chi_R$ symmetry and is safely removed by $\mu \to 0$ extrapolation.

Numer. Demonstr. – 2: lattice renormalization & η_{cr}

Lattice setup preserves symmetries of the formal model + doubling symmetry

$\Rightarrow$ all doubler species enter in the 1PI E L with common ρ and $(\eta - \bar{\eta})$

Quenching: gauge and Φ configurations are independently renormalized

⋆ Explore $\rho = 1.96, 2.94, 0$: renormalization conditions for m_Φ^2, λ_0, v_Φ, g_0^2

$$M_{\zeta_0}^2 r_0^2 = 1.284(6)\,, \quad \lambda_R \equiv \frac{M_{\zeta_0}^2}{2v_R} = 0.441(4)\,, \quad v_\Phi^2 r_0^2 = 1.458(2)\,, \quad r_0^2 F_{h\bar{h}}(r_0) = 1.65$$

g_0^2 decreased s.t. b^2/r_0^2 varies by ~ 2.2, criticality fixes $\eta = \eta_{cr}(\rho, g_0^2, \lambda_0)$

⋆ Large volume: $M_{PS}L \geq 4.7$ for three lattice spacings, several μ and η-values

⋆ Simple correlators give info on η_{cr}, M_{PS} and axial $\tilde{\chi}$–current matrix elements:

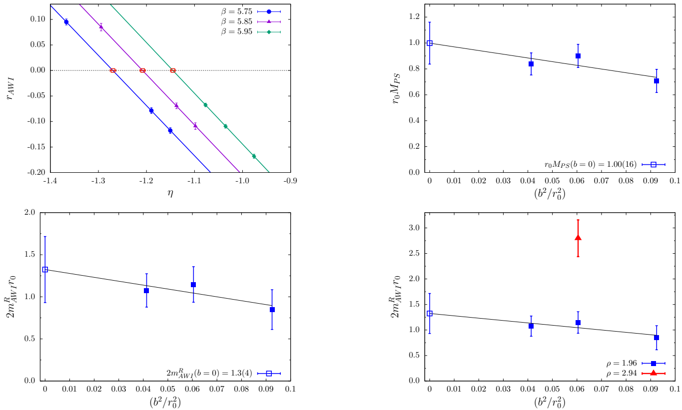
we set $P^i = \bar{Q}\gamma_5 \frac{\tau^i}{2} Q\,,\ D_P^i = \bar{Q}_L \left\{\Phi, \frac{\tau^i}{2}\right\} Q_R - \bar{Q}_R \left\{\frac{\tau^i}{2}, \Phi^\dagger\right\} Q_L$ and define

$$\eta_{cr} \quad \text{s.t.} \quad r_{AWI}\Big|_{\mu\to 0}^{\eta\to\eta_{cr}} = 0\,, \qquad r_{AWI} = \frac{\sum_{\mathbf{x},\mathbf{y}} \langle P^1(0)\partial_0^{FW}\tilde{A}_0^{1,BW}(x)\,\varphi_0(y)\rangle}{\sum_{\mathbf{x},\mathbf{y}} \langle P^1(0)\tilde{D}_P^1(x)\varphi_0(y)\rangle}$$

Numer. Demonstr. – 3: evidence for NP fermion mass

we measure M_{PS} from $\sum_{\mathbf{x}} \langle P^1(0) P^1(x) \rangle$ and the matrix element ratio

$$m^R_{AWI} \equiv \frac{Z_{\tilde{A}} \sum_{\mathbf{x}} \partial_0 \langle \tilde{A}^i_0(x) P^i(0) \rangle}{2 Z_P \sum_{\mathbf{x}} \langle P^i(x) P^i(0) \rangle} \Bigg|_{\eta_{cr}} \propto c_1 \Lambda_S \,, \quad \text{since} \quad \tilde{A}^i_\nu = \tilde{J}^{L\,i}_\nu - \tilde{J}^{R\,i}_\nu$$



$$\rho = 1.96: \quad r_0 M_{PS} = 0.93\ (0.09)_{\rm stat}[0.10]_{\rm syst}\,, \quad 2 r_0 m^R_{AWI} = 1.20\ (0.39)_{\rm stat}[0.19]_{\rm syst}$$

Outline

Mechanism in an extended Toy Model

with weak + strong interactions

Gauge: $\mathrm{SU}(3)_T \times SU(3)_S \times SU(2)_L$

RG-invariant scale: Λ_T

Toy model with weak interactions: overview

Dirac fermions: $Q \in (3_T, 3_S)$, $q \in (1_T, 3_S)$; L/R-handed $\leftrightarrow$ $SU(2)_L$ doublets/singlets

$$\mathcal{L}_{\rm toy}(Q, q, G, A, \Phi, W) = \mathcal{L}_{kin}(Q, q, \ldots) + \mathcal{V}(\Phi) + \mathcal{L}_{Wil}(Q, q, \ldots) + \mathcal{L}_{Yuk}(Q, q, \Phi)$$

- $$\mathcal{L}_{kin} = \frac{1}{4}F^G \cdot F^G + \frac{1}{4}F^A \cdot F^A + \frac{1}{4}F^W \cdot F^W + \bar{Q}_L\gamma_\mu\mathcal{D}^{G,A,W}_\mu Q_L + \bar{Q}_R\gamma_\mu\mathcal{D}^{G,A}_\mu Q_R$$
$$+\bar{q}_L\gamma_\mu\mathcal{D}^{A,W}_\mu q_L + \bar{q}_R\gamma_\mu\mathcal{D}^{A}_\mu q_R + \frac{1}{2}\mathrm{Tr}\big[\Phi^\dagger \overleftarrow{\mathcal{D}}^W_\mu \mathcal{D}^W_\mu \Phi\big]$$
- $$\mathcal{L}_{Wil} = \frac{b^2}{2}\rho\big(\bar{Q}_L \overleftarrow{\mathcal{D}}^{G,A,W}_\mu \Phi \mathcal{D}^{G,A}_\mu Q_R + h.c. + \bar{q}_L \overleftarrow{\mathcal{D}}^{A,W}_\mu \Phi \mathcal{D}^{A}_\mu q_R + h.c.\big)$$

$SU(2)_L$ gauge symmetry: $W^{1,2,3}_\mu$ bosons & covariant derivatives on $\psi = Q,\ q$, e.g.

$$\mathcal{D}^{A,W}_\mu q_L = (\partial_\mu - ig_s\lambda^a A^a_\mu - ig_w\tfrac{\tau^i}{2}W^i_\mu)q_L \qquad \bar{q}_L\overleftarrow{\mathcal{D}}^{A,W}_\mu = \bar{q}_L(\overleftarrow{\partial}_\mu + ig_s\lambda^a A^a_\mu + ig_w\tfrac{\tau^i}{2}W^i_\mu)$$

Global $SU(2)_L \times SU(2)_R$ invariance, if W's transform (as $\in su(2)_L$) under $\tilde{\chi}_L$:

$$\chi_L \equiv \tilde{\chi}_L \otimes \chi^\Phi_L \qquad \text{and} \qquad \chi_R \equiv \tilde{\chi}_R \otimes \chi^\Phi_R \qquad \text{with}$$

$$\tilde{\chi}_L:\ Q[q]_L \to \Omega_L\, Q[q]_L\,,\ \bar{Q}[q]_L \to \bar{Q}[q]_L\,\Omega_L^\dagger\,, \qquad \chi^\Phi_L:\ \Phi \to \Omega_L\Phi\,, \qquad \Omega_L \in SU(2)_L\,,$$
$$W_\mu \to \Omega_L W_\mu \Omega_L^\dagger\,,$$
$$\tilde{\chi}_R:\ Q[q]_R \to \Omega_R Q[q]_R\,,\ \bar{Q}[q]_R \to \bar{Q}[q]_R\Omega_R^\dagger\,, \qquad \chi^\Phi_R:\ \Phi \to \Phi\Omega_R^\dagger\,, \qquad \Omega_R \in SU(2)_R\,,$$

$g_W > 0$: minimal $\tilde\chi$–breaking in the quantum EL

$\tilde\chi_{L,R}$ and $\chi^{\Phi}_{L,R}$ transf.s are no symmetries ... term $\mathrm{Tr}\big[\Phi^\dagger \overleftarrow{\mathcal{D}}^W_\mu \mathcal{D}^W_\mu \Phi\big]$ breaks $\tilde\chi_L$, too

For the quantum EL we expect $\Gamma = \Gamma_{d\le 4,\ \hat\mu^2_\Phi} + \Delta\Gamma^{NG}_{d\le 4,\ \hat\mu^2_\Phi} + \Gamma_{d>4,\ \hat\mu^2_\Phi}$, with

$$\Gamma_{d\le 4,\ \hat\mu^2_\Phi} = \tfrac{1}{4}(F\cdot F)^{G,A,W} + \bar Q \not{\mathcal{D}} Q + \bar q \not{\mathcal{D}} q + \tfrac{\hat\mu^2_\phi}{2}\,\mathrm{Tr}\left[\Phi^\dagger\Phi\right] + \tfrac{\hat\lambda}{4}\left(\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\right)^2 +$$

$$+(\eta - \bar\eta_L(\eta,\rho,\ldots))\sum_{\psi=Q,q}[\bar\psi_L\Phi\psi_R + \text{ h.c.}] + \tfrac{1}{2}(1-\bar\gamma(\eta,\rho,\ldots))\,\mathrm{Tr}\left[\mathcal{D}^W_\mu\Phi^\dagger\mathcal{D}^W_\mu\Phi\right] \quad ,$$

and NP $d \le 4$ terms if $\chi_L \times \chi_R$ is realized à la Nambu–Goldstone (NG), i.e. $\hat\mu^2_\Phi < 0$,

$$\Delta\Gamma^{NG}_{d\le 4,\ \hat\mu^2_\Phi} = \theta(-\hat\mu^2_\Phi)\sum_{\psi=Q,q}[C_1\Lambda_T(\bar\psi_L U\psi_R + \text{h.c.}) + (C_2\Lambda_T^2 + \tilde C\Lambda_T R)\tfrac{1}{2}\mathrm{Tr}(\mathcal{D}^W_\mu U^\dagger \mathcal{D}^W_\mu U)]$$

where $\Phi = v_\Phi + \sigma + i\vec\tau\vec\pi = RU$, $R = (v_\Phi + \zeta_0)$, $U = \exp[i v_\Phi^{-1}\tau^k\zeta_k]$.

Criticality: $\eta_{cr} = \bar\eta_L(g_T, g_S, g_W; \mu_0, \lambda_0; \eta_{cr}, \rho_{cr})$, $1 = \bar\gamma(g_T, g_S, g_W; \mu_0, \lambda_0; \eta_{cr}, \rho_{cr})$

- ⋆ absence of Yukawa and ζ_0-kinetic terms from the EL $\Leftrightarrow$ minimal $\tilde\chi$–breaking
- ⋆ canonical $\zeta_0^c = (1-\bar\gamma)^{1/2}\zeta_0$ becomes decoupled, with $v^c_\Phi = (1-\bar\gamma)^{1/2} v_\Phi \to 0^+$

$g_W > 0$: minimal $\tilde\chi$–breaking in the $\tilde\chi$–SDE

the bare Schwinger Dyson equations (SDE) associated to $\tilde\chi_L$ transformations read

$$\partial_\mu \Big\langle \tilde J^{L,\,i}_\mu(x)\,\hat O(0)\Big\rangle = \Big\langle \tilde\Delta^i_L \hat O(0)\Big\rangle\delta(x) - \eta\Big\langle \textstyle\sum_{f=Q,q}\Big(\bar f_L \frac{\tau^i}{2}\Phi f_R - \bar f_R \Phi^\dagger \frac{\tau^i}{2} f_L\Big)(x)\,\hat O(0)\Big\rangle +$$

$$-\frac{b^2}{2}\rho\Big\langle \textstyle\sum_{f=Q,q}\Big(\bar f_L \overleftarrow{\mathcal D}^{A,W}_\mu \frac{\tau^i}{2}\Phi \mathcal D^A_\mu f_R - \bar f_R \overleftarrow{\mathcal D}^A_\mu \Phi^\dagger \frac{\tau^i}{2}\mathcal D^{A,W}_\mu f_L\Big)(x)\,\hat O(0)\Big\rangle +$$

$$+\frac{i}{2} g_W\Big\langle \mathrm{Tr}\Big(\Phi^\dagger[\tfrac{\tau^i}{2}, W_\mu]\mathcal D^W_\mu \Phi + \Phi^\dagger \overleftarrow{\mathcal D}^W_\mu [W_\mu, \tfrac{\tau^i}{2}]\Phi\Big)(x)\hat O(0)\Big\rangle$$

$$\tilde J^{L,\,i}_\mu = \textstyle\sum_{f=Q,q}\Big\{\bar f_L\gamma_\mu \frac{\tau^i}{2} f_L - \frac{b^2}{2}\rho\Big(\bar f_L \frac{\tau^i}{2}\Phi\mathcal D^A_\mu f_R - \bar f_R \overleftarrow{\mathcal D}^A_\mu \Phi^\dagger \frac{\tau^i}{2} f_L\Big)\Big\} + g_W\,\mathrm{Tr}\Big([W_\nu, F^W_{\mu\nu}]\frac{\tau^i}{2}\Big)$$

upon renormalization the effective $\tilde\chi_L$–SDE of the quantum theory are expected to be

$$Z_{\tilde J}\partial_\mu\Big\langle \tilde J^{L,\,i}_\mu(x)\hat O(0)\Big\rangle = \Big\langle\tilde\Delta^i_L\hat O(0)\Big\rangle\delta(x) - (\eta - \bar\eta_L)\Big\langle \textstyle\sum_{f=Q,q}(\bar f_L\frac{\tau^i}{2}\Phi f_R - h.c.)(x)\,\hat O(0)\Big\rangle +$$

$$+(1-\bar\gamma)\,\frac{i}{2} g_W\Big\langle \mathrm{Tr}\Big(\Phi^\dagger[\tfrac{\tau^i}{2}, W_\mu]\mathcal D^W_\mu\Phi + \Phi^\dagger\overleftarrow{\mathcal D}^W_\mu[W_\mu, \tfrac{\tau^i}{2}]\Phi\Big)(x)\hat O(0)\Big\rangle + \ldots + \mathrm{O}(b^2)$$

Criticality: $\eta_{cr} = \bar\eta_L(g_T, g_S, g_W; \mu_0, \lambda_0; \eta_{cr}, \rho_{cr})\,,\quad 1 = \bar\gamma(g_T, g_S, g_W; \mu_0, \lambda_0; \eta_{cr}, \rho_{cr})$

$\Rightarrow$ $\tilde\chi_L$ and also $\tilde\chi_R$ restored at low momenta, up to ... possible NP terms $\sim \Lambda_T$ ...

$\hat{\mu}^2_\Phi > 0$: restoration of $\tilde{\chi}$–symmetry @ (ρ_{cr}, η_{cr})

Wigner phase at (ρ_{cr}, η_{cr}) : $\mathcal{V}(\Phi)$ has a single minimum $(\hat{\mu}^2_\Phi > 0)$ $\Rightarrow$ renormalized

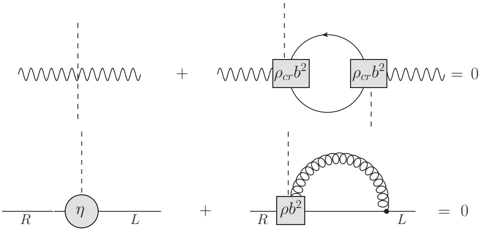
SDE's of $\tilde{\chi}_{L(R)}$: $Z_{\tilde{J}} \partial_\mu \Big\langle \tilde{J}^{L(R),\,i}_\mu(x) \hat{O}(0) \Big\rangle = \Big\langle \tilde{\Delta}^i_{L(R)} \hat{O}(0) \Big\rangle \delta(x) + \mathrm{O}(b^2)$

with no NP operators $\sim \Lambda_T$ in rhs as, due to $v_\Phi = 0$, effective U-field is undefined

Equivalently, $\Gamma^{Wig} = \Gamma_{d\le 4,\ \hat{\mu}^2_\Phi} + \Gamma^{Wig}_{d>4,\ \hat{\mu}^2_\Phi}$ has no Yukawa or Φ-kinetic terms

$$\Gamma_{d\le 4,\ \hat{\mu}^2_\Phi} = \tfrac{1}{4}[F^G \cdot F^G + F^A \cdot F^A + F^W \cdot F^W] + \sum_{\psi=Q,q} [\bar{\psi}_L \mathcal{D}^{\ldots,W} \psi_L + \bar{\psi}_R \mathcal{D}^{\ldots} \psi_R] + \mathcal{V}^{Wig}_{eff}[\Phi]$$

Φ decoupled at low energy $\leftrightarrow$ evaluation of (ρ_{cr}, η_{cr}) by imposing LE cancellations

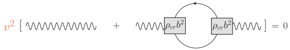


$\rho_{cr} \sim \mathrm{O}(1/\sqrt{N_F^{\rm tot}})$

$\hat{\mu}^2_\Phi < 0$: dynamical $\tilde{\chi}$SB & NP masses @ (ρ_{cr}, η_{cr})

NG phase at (ρ_{cr}, η_{cr}) : $\mathcal{V}(\Phi)$ has many degenerate minima $(\hat{\mu}^2_\Phi < 0)$ $\Rightarrow$

- ⋆ by def. of (ρ_{cr}, η_{cr}) <u>no</u> O(v) W-mass and <u>no</u> O(v) $Q(q)$-mass terms

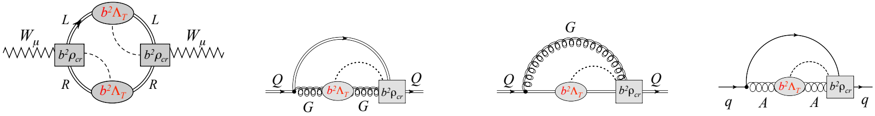


- ⋆ under dynamical $\tilde{\chi}$SB vacuum polarized by residual O($b^2 v$) $\tilde{\chi}$-breaking terms
- ⋆ interplay of O(b^2) $\tilde{\chi}$–breaking and $\tilde{\chi}$SB dynamics induces <u>RG-invariant</u> O($\Lambda_T b^0$)

  $\tilde{\chi}$–breaking operators in renormalized SDE's $\Leftrightarrow$ the 1PI E L reads

$$\Gamma_4 = \Gamma_{d\le 4,\, \hat{\mu}^2_\Phi} + \sum_{\psi=Q,q} \Lambda_T C_{1,\psi} [\bar{\psi}_L U \psi_R + h.c.] + C_2 \Lambda_T^2 \tfrac{1}{2} \,\mathrm{Tr}\, [U^\dagger \overleftarrow{\mathcal{D}}^W_\mu \mathcal{D}^W_\mu U] + \Gamma_{d>4,\, \hat{\mu}^2_\Phi}$$

$(M_W^{eff})^2 = g_W^2 C_2 \Lambda_T^2$ and $m^{eff}_{Q/q} = C_{1,Q/q} \Lambda_T$ from a common mechanism



NP masses in $\Gamma^{NG}_{\rm loc}$ @ (ρ_{cr}, η_{cr}) : remarks

⋆ Effective NP masses modulated by gauge couplings & loop suppression factors:

$m^{eff}_{Q/q} = C_{1,Q/q}\Lambda_T$: $C_{1,Q} = \mathrm{O}(\alpha_T^2|_{\Lambda_T}\rho_{cr}^2 N_Q)$, $C_{1,q} = \mathrm{O}(\alpha_S^2|_{\Lambda_T}\rho_{cr}^2 N_T N_Q) \sim (4\pi)^{-2}$

$(M^{eff}_W)^2 = g_W^2 C_2 \Lambda_T^2$: $C_2 = \mathrm{O}(\rho_{cr}^4 N_T N_Q (4\pi)^{-3}) \sim (N_T N_Q)^{-1}(4\pi)^{-3}$

here $N_S = N_T = 3$, $N_Q = N_q = 2$ and we use $\rho_{cr}^{-2} \overset{\rm crit}{\simeq} N_F^{\rm tot} = N_S(N_T N_Q + N_q) = 24$

⋆ Absence in $\Delta\Gamma^{NG}_{d\leq 4}$ of term $\tilde{C}\Lambda_s R\,\mathrm{Tr}\,[\mathcal{D}^W_\mu U^\dagger \mathcal{D}^W_\mu U]$ is specific to the critical model:

as $\rho_{cr}^2 - \rho^2 \to 0^+$ & $\eta \to \eta_{cr}$ we have $1-\bar{\gamma} \to 0^+$, $m^2_{\zeta^0} \sim |\hat{\mu}^2_\Phi|/(1-\bar{\gamma}) \to +\infty$

$\Rightarrow$ decoupling of ζ^0 . For Γ^{NG}_4 to describe the decoupling of ζ^0 , e.g. in

$WW \to WW$ amplitudes, it is necessary that $|\tilde{C}| \leq O((1-\bar{\gamma})^{1/2}) \xrightarrow{\rho\to\rho_{cr},\eta\to\eta_{cr}} 0$

⋆ Canonical normalization of NP kinetic term for GB's $\Rightarrow$ $U = \exp(i\vec{\zeta}\vec{\tau}/\sqrt{C_2}\Lambda_T)$

basic GB's provide longitudinal d.o.f.'s for massive W's (SU(2) unitarity gauge)

⋆ Further $\tilde{\chi}$-breaking terms in Γ^{NG} : $\frac{1}{\Lambda_T^2}[(\bar{Q}_L U Q_R)(\bar{Q}_R U^\dagger Q_L)]$, $[\mathrm{Tr}\,(D^W_\mu U^\dagger D^W_\mu U)]^2$, ...

Outline

Towards a realistic BSM model:

new interactions and matter

Dynamical NP mass: towards a realistic model

Might our mechanism be responsible for the effective mass of elementary particles ?

Consistency of such an hypothesis with experimentally observed masses requires

- new strong SU(N_T) interaction with RGI scale $\Lambda_T > M_W \gg \Lambda_{QCD}$
- new set of fermions subjected to the new force (besides to SM interactions)
  - $Q_L \in (N_T, 3, 2; Y_Q^L)$, $L_L \in (N_T, 1, 2; Y_L^L)$
  - $Q_R^u \in (N_T, 3, 1; Y_R^u)$, $L_R^u \in (N_T, 1, 1; Y_L^u)$
  - $Q_R^d \in (N_T, 3, 1; Y_R^d)$, $L_R^d \in (N_T, 1, 1; Y_L^d)$

  with (irrep. of SU(N_T), SU$(3)_S$, SU$(2)_L$; $Y = Q_{em} - T_3$) ; besides SM fermions, e.g.
  - $q_L \in (1, 3, 2; 1/6)$, $\ell_L \in (1, 3, 2; -1/2)$
  - $t_R \in (1, 3, 1; 2/3)$, $\nu_R \in (1, 1, 1; 0)$
  - $b_R \in (1, 3, 1; -1/3)$, $\tau_R \in (1, 1, 1; -1)$
- a bound state (composite higgs): binding through new fermions & T-strong force;

needed for PT unitarity in $WW \to WW$ scattering [Lee – Quigg – Thacker 1977]

UV complete Lagrangian: towards a realistic model

$$\mathcal{L}^{BSMM} = \frac{1}{4}\left(F^B F^B + F^W F^W + F^A F^A + F^G F^G\right) +$$

$$+ \sum_{g=1,2,3}\left[\bar{q}_L^g D^{BWA} q_L^g + \bar{q}_R^{g\,u} D^{BA} q_R^{g\,u} + \bar{q}_R^{g\,d} D^{BA} q_R^{g\,d} + \bar{\ell}_L^g D^{BW} \ell_L^g + \bar{\ell}_R^{g\,u} \partial\, \ell_R^{g\,u} + \bar{\ell}_R^{g\,d} D^B \ell_R^{g\,d}\right] +$$

$$+\left[\bar{L}_L D^{BWG} L_L + \bar{L}_R^u D^{BG} L_R^u + \bar{L}_R^d D^{BG} L_R^d\right] + \left[\bar{Q}_L D^{BWAG} Q_L + \bar{Q}_R^u D^{BAG} Q_R^u + \bar{Q}_R^d D^{BAG} Q_R^d\right] +$$

$$+ \sum_{\text{fermions } f}\left[\eta_{f,cr}^u (\bar{f}_L \tilde{\phi} f_R^u) + \frac{1}{2} b^2 \rho_f^u (\bar{f}_L W_f^u(\tilde{\phi}, \mathcal{D}, ...) f_R^u) + \text{h.c.}\right] +$$

$$+ \sum_{\text{fermions } f}\left[\eta_{f,cr}^d (\bar{f}_L \phi f_R^d) + \frac{1}{2} b^2 \rho_f^d (\bar{f}_L W_f^d(\phi, \mathcal{D}, ...) f_R^d) + \text{h.c.}\right]$$

$d = 6, 8, \ldots$ $\tilde{\chi}$-breaking terms: mass for t, c, u & τ, μ, e ; no ν mass

[appropriate $d = 8, 10, \ldots$ $\tilde{\chi}$ terms: $\alpha_{S/W/Y}$–suppressed mass for b, s, d]

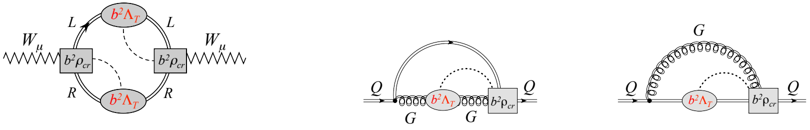
$\tilde{\chi}$-symmetry restoring $\Rightarrow$ $\sum_{f=1}^{N_{ferm}^{tot}} \rho_f^2 (1 + \mathrm{O}(\rho_f^2)) = O(1)$ and $\eta_f = \eta_{f,cr}(\{\rho\})$

Even if $\rho_f \simeq \rho_{cr}$ $\forall f$ describing isospin & generations requires further assumptions.

Computing $M_W^{eff}/M_{\mathrm{T-meson}} \sim M_W^{eff}/\Lambda_T$: expected $\ll 1$

$(M_W^{eff})^2 = g_W^2 C_2 \Lambda_T^2$: $\quad C_2 = \mathrm{O}(\rho_{cr}^4 N_T N_{Q+L}(4\pi)^{-3}) \sim \mathrm{O}((N_T N_{Q+L})^{-1}(4\pi)^{-3})$

$M_{\mathrm{T-meson}} \simeq 2M_{Q/L}^{eff}\mathrm{O}(1) = \mathrm{O}(2\alpha_T^2|_{\Lambda_T}\rho_{cr}^2 N_{Q+L})\Lambda_T \sim \mathrm{O}(2\Lambda_T)$ are expected from



$M_W^{eff}/M_{\mathrm{T-meson}} \sim g_W\sqrt{C_2}\,\mathrm{O}(1) \sim \mathrm{O}((N_T N_{Q+L})^{-1/2}(4\pi)^{-3/2}) \sim 10^{-2}$

numerically computable with controlled O(20%) errors $\Rightarrow$ little hierarchy [Pomarol]

i) $M_{\mathrm{T-meson}}$: $\sum_{\vec{x}}\langle \bar{Q}\gamma_5 Q'(x)\bar{Q}'\gamma_5 Q(0)\rangle \overset{|x_0|\,\mathrm{large}}{\propto} e^{-M_{\mathrm{T-meson}}|x_0|}$ (quenched approx.?)

ii) $(M_W^{eff})^2$: shifted above zero due to the double W-pole, with residue

computable from $\sum_y e^{i(p\cdot y)}\langle J^{\mathrm{weak}}_{\mu,Q}(y)J^{\mathrm{weak}}_{\lambda,Q}(0)\rangle$ at $g_W = g_Y = 0$

results depend / give hints on N_T (e.g. 3,2) and on N_{Q+L} (e.g. $N_c + 1 = 4$ doublets)

Hypercharge: M_Z/M_W, lepton masses

- $(M_Z^{eff})^2 = \frac{g_W^2+g_Y^2}{g_W^2}(M_W^{eff})^2$, $\quad M_\gamma^{eff} = 0$

$\Gamma_{LE}^{NG} \supset C_2\Lambda_T^2 \frac{1}{2}\mathrm{Tr}\,[D_\mu^{W,B} U^\dagger D_\mu^{W,B} U] \supset C_2\Lambda_T^2[g_W^2 \sum_{j=1}^3 (W^j \cdot W^j) + g_Y^2 B\cdot B + 2g_W g_Y W^3 \cdot B]$

$\Rightarrow$ diagonalization in W^3–B sector gives massless γ and $M_Z^2 = (g_W^2 + g_Y^2)C_2\Lambda_T^2$

owing to the custodial $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry of $\mathcal{L}^{BSMM}$ in the $g_Y \to 0$ limit

- prediction $m_\tau^{eff}/m_{\rm top}^{eff} \sim \alpha_Y^2|_{\Lambda_T}/\alpha_S^2|_{\Lambda_T} \simeq 0.01$ from

$$\mathcal{L}_{W+Y}^{\rm top} = \frac{1}{2}b^2\rho_{cr,\,t}[(\bar q_L \overleftarrow{\mathcal{D}}_\mu^{BWA}\tilde\phi \mathcal{D}_\mu^{BA} t_R) + \mathrm{h.c.}] + \eta_{cr,\,t}[\bar q_L \tilde\phi t_R]$$

$$\mathcal{L}_{W+Y}^{\tau} = \frac{1}{2}b^2\rho_{cr,\,\tau}[(\bar\ell_L \overleftarrow{\mathcal{D}}_\mu^{BW}\phi \mathcal{D}_\mu^{B} \tau_R) + \mathrm{h.c.}] + \eta_{cr,\,\tau}[\bar\ell_L \phi \tau_R]$$

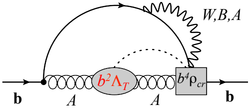
$\Rightarrow \quad m_{\rm top}^{eff} = \mathrm{O}(g_S^4|_{\Lambda_T}(\frac{1}{4\pi})^2\rho_{cr,\,t}^2 N_{Q+L}N_T)\Lambda_T \qquad m_\tau^{eff} = \mathrm{O}(g_Y^4|_{\Lambda_T}(\frac{1}{4\pi})^2\rho_{cr,\,\tau}^2 N_{Q+L}^Y N_T)\Lambda_T$

Isospin splitting: approximate symmetries? why?

Imposing further approximate symmetries one can get $m_{\rm b}^{eff} < m_{\rm top}^{eff}$ and $m_{\nu}^{eff} = 0$:

1) $\mathcal{L}^{BSMM}$-invariance under $f(x) \to f(x) + \text{const}$, $\bar{f}(x) \to \bar{f}(x) + \text{const}$ as $g_{S,W,Y} \to 0$
for all fermion species f [Goltermann & Petcher, 1990]

2) $\mathcal{L}^{BSMM}$-invariance under $b_R(x) \to -b_R(x)$, $\bar{b}_R(x) \to -\bar{b}_R(x)$ as $g_{S,W,Y} \to 0$



$$\mathcal{L}^{\rm b}_{W+Y} = \tfrac{1}{2} b^4 \rho_{cr,\,b} [(\bar{q}_L [\overleftarrow{\mathcal{D}}^{BWA}_{\mu}, \overleftarrow{\mathcal{D}}^{BWA}_{\nu}] \mathcal{D}^{BW}_{\mu} \phi \mathcal{D}^{BA}_{\nu} b_R) + \text{h.c.}] + \eta_{cr,\,b} [\bar{q}_L \phi b_R]$$

$$\mathcal{L}^{\nu}_{W+Y} = \tfrac{1}{2} b^2 \rho_{cr,\,\nu} [(\bar{\ell}_L \overleftarrow{\mathcal{D}}^{BW}_{\mu} \tilde{\phi}\, \partial_\mu \nu_R) + \text{h.c.}]\,, \qquad \eta_{cr,\,\nu} = 0$$

in fact leads to: $\quad m_{\rm b}^{eff} = \mathrm{O}(g_S^4|_{\Lambda_T} (\frac{1}{4\pi})^3 g^2_{S,W,Y} \rho^2_{cr,\,b} N_Q N_T) \Lambda_T \quad$ and $\quad m_{\nu}^{eff} = 0$

Outline

Higgs as a bound state and

sub-TeV effective Lagrangian

125 GeV Higgs boson (h): a $WW + ZZ$ bound state?

New T-strong force can lead to physical effects from Q-, L-fermions at scales $\ll \Lambda_T$

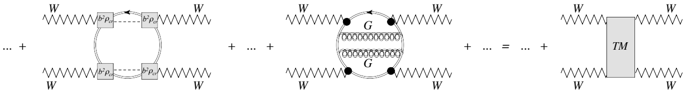
In $G_{V_3}(p) = \int dt d^3x d^3y d^3z \; e^{-ip_0 t - i\vec{p}\vec{x} + i\vec{p}\vec{z}} \; V_3^{-2} \Big\langle W(\vec{x},t) W(\vec{y},t) W^\dagger(\vec{z},0) W^\dagger(\vec{0},0) \Big\rangle$

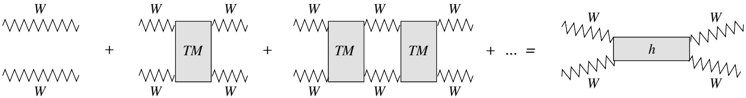
due to a large effective WW-WW coupling $\Delta_0^2 = \mathrm{O}(\Lambda_T^{2-2})\, g_W^4\, 4M_W^2$ one expects

$$G_{V_3}(p) \;\Rightarrow\; \frac{g_{analyt}(p^2)}{p^2 + 4M_W^2}\Big\{1 + \frac{\Delta_0^2(p^2)}{p^2 + 4M_W^2} + \dots\Big\} = \frac{g_{analyt}(p^2)}{p^2 + 4M_W^2 - \Delta_0^2(p^2)} \overset{p^2 \simeq -M_h^2}{\longrightarrow} \frac{g_W^2 M_W^2}{-s + M_h^2}$$

$V_3 \to \infty$: besides a cut for $-p^2 > 4M_W^2$, sum over all T-meson exchanges yields a pole

at $\quad p^2 = -M_h^2 = -4M_W^2 + \Delta_0^2(-M_h^2) \quad \leftrightarrow \quad M_h = 2M_W\Big(1 - \mathrm{O}\big(\frac{(m^{eff}_{Q/L})^2}{M^2_{\mathrm{T-meson}}}\big) g_W^4\Big)^{1/2}$





Arguments for one $WW + ZZ$ bound state

- ⋆ T-strong force $\Rightarrow$ mass $m_{Q,L} \sim \Lambda_T$, scalar T-meson exchange in t-channel $\Rightarrow$

WW-WW coupling $\overset{M_W^2 \ll \Lambda_T^2}{\sim} \frac{m^{eff}_{Q/L}}{\Lambda_T^2} \langle 0|\bar{Q}Q|\mathrm{T_{meson}}\rangle \frac{\mathrm{O}(1-10)}{M_T^2} \langle \mathrm{T_{meson}}|\bar{Q}Q|0\rangle \frac{m^{eff}_{Q/L}}{\Lambda_T} \sim$ large

two–W interaction attractive & strong over distances $\sim \Lambda_T^{-1} \lesssim M_W^{-1}$

- ⋆ In non-relativistic approximation ($\frac{\Delta_0^2}{4M_W^2} \ll 1$) $W - W$ scattering can be seen as

– a particle of mass $\mu_W = \frac{M_W}{2}$ in a potential well of height $|V_0| < 2M_W$, size $\lesssim M_W^{-1}$

– satisfying the condition that implies just one bound–state, viz. $\mu_W |V_0| \mathrm{size}^2 < 1$

- ⋆ QFT approach à la Bethe-Salpeter & Lüscher [Comm.Math.Phys.105(1986)]

→ Lüscher's effective Schröedinger Eq. with E-dependent potential ($E \in [0, 4M_W]$)

→ evaluate the effective $WWWW$ coupling through (quenched) lattice simulations

- ⋆ In custodial approximation: only $WW + ZZ$ energy is shifted; the corrections

are $\mathrm{O}(\alpha_Y(3N_Q + N_L))$: about 10%, depending on the Y-charges of Q, L

LE effective action & experimental constraints

A mass mechanism, not yet a unique model $+$ many experimental constraints, e.g.

⋆ at $E < 1$ TeV one expects: $\Gamma^{NG}_{\rm LE} \supset (c\Lambda_T^2 + c'\Lambda_T h + \ldots)\frac{1}{2}{\rm Tr}(D^{W,B}_\mu U^\dagger D^{W,B}_\mu U) +$

$+ \Lambda_T[c_1^t \bar{q}_L \tilde{u} q_R^t + c_1^b \bar{q}_L u q_R^b] + \Lambda_T[0\bar{\ell}_L \tilde{u} \ell_R^\nu + c_1^\tau \bar{\ell}_L u \ell_R^\tau] +$ other generations terms $+ \; {\rm O}(\Lambda_T^{-1})$

with $U = [\tilde{u}|u] = \exp\left(i\zeta^j\tau^j/\sqrt{c}\Lambda_T\right)$, $\zeta^{1,2,3}$ GB fields, $u \overset{SU(2)_L\times U(1)}{\sim} \phi$

⋆ up to ${\rm O}(p^2/\Lambda_T^2)$ SM-like relation between W-mass and WWh coupling

⋆ SM fermion mass $m_f^{eff} \simeq y_f^{eff} 2M_W g_W^{-1}(1 + {\rm O}(\alpha_W))$, y_f^{eff} effective $f\bar{f}h$–coupling

⋆ no tree level FCNC: due to SM-like form of all fermion effective mass terms

⋆ EW precision tests: S-parameter bounds “ok” owing to $m_{Q,L}^{eff} \sim {\rm O}(1)\Lambda_T$

⇒ key check on Λ_T: (W [top] mass) / (T-meson mass) $\sim \dfrac{M^{eff}_{W[top]}}{\Lambda_T}$ computable

⇒ prediction: (h mass) / (W mass) computable [by theory + lattice simulations]

Predictivity in models with $\tilde\chi$-breaking & NP mass

In a renormalizable model at the critical point of minimally broken $\tilde\chi$ symmetry:

$\partial_\mu \tilde J_\mu^{L,i} = 0$, (Wigner phase)

$\partial_\mu \tilde J_\mu^{L,i} = \sum_f c_{1,f} \Lambda_T \mathcal{D}_f^{L,i} + \frac{ig_W}{2} c_2 \Lambda_T^2 \mathrm{tr}\Big(U^\dagger [\frac{\tau^i}{2}, W_\mu] D_\mu^{WB} U - \mathrm{h.c.} \Big)$, (NG phase)

⋆ RGI of l.h.s. $\Rightarrow$ RGI (& UV-finite) NP $\tilde\chi$-breaking terms on the r.h.s.

with $\mathcal{D}_f^{L,i} = [\bar f_L \frac{\tau^i}{2} U f_R - \mathrm{h.c.}]$ and $c_{1,f} = \mathrm{O}(\rho_{f,cr}^2)\, \alpha_{\mathrm{coup}(f)}^{n(f)} [1 + \mathrm{O}(\alpha \ldots)]$

⋆ effective masses$|_{\mathrm{scale}\ b^{-1}}$: $c_{1,f}\Lambda_T \leftrightarrow m_f^{eff}$, $c_2 g_W^2 \Lambda_T^2 \leftrightarrow (m_W^{eff})^2$

UV cutoff $b^{-1} \to \infty$ at fixed $M_{\mathrm{Tglueball}}$, M_{proton}, G_F, $\sin^2\theta_W \leftrightarrow \hat\alpha_{T,S,W,Y}$

$\tilde\chi$-symmetry $\Rightarrow \sum_{f=1}^{N_{ferm}^{tot}} \rho_{f,cr}^2 (1 + \mathrm{O}(\rho_{f,cr}^2)) = O(1)$ entails bounds for the $\rho_{f,cr}$'s

$\rightarrow$ $\rho_{Q,cr}$, $\rho_{L,cr}$ control m_Q^{eff}, m_L^{eff}, as well as m_W^{eff}, m_Z^{eff}

$\rightarrow$ $\rho_{t,cr}$ controls m_t^{eff}, ... $\rho_{\tau,cr}$ controls m_τ^{eff}, ...

$\Rightarrow$ perhaps one can choose the $\tilde\chi$-breaking action terms for all flavours f such that all the $\rho_{f,cr}$ coefficients have similar (equal?) magnitude: ... theory of flavour?

Conclusions

Non-perturbative mass mechanism for fermions and EW gauge bosons:

- an unnoticed feature in gauge quantum field theory $\supset [A, \ Q, \ \Phi]$
- occurring if fermion chirality ($\tilde{\chi}$) is broken at the UV cutoff scale
- while the global chiral symmetry that once gauged describes the EW interactions is exact and forbids UV–power-divergent mass terms
- numerically demonstrated by (pioneering) lattice simulations

- maybe a solution to the naturalness problem: get EW & top mass scale
- . . . if there exists a new strong interaction with RGI scale of a few TeV,

  and new fermions subjected to it $\Rightarrow \dfrac{M_W}{M_{\mathrm{T-meson}}}, \dfrac{m_t}{M_{\mathrm{T-meson}}}$ computable;
- $\Rightarrow$ hinting at a “Higgs boson bound state” with computable M_h/M_W;
- $\Rightarrow$ giving insights on fermion hierarchies: m_τ/m_t, m_ν/m_τ , m_b/m_t

Outlook

Phenomenological prospects: consistency of mechanism with experiment requires embedding in a suitable UV complete gauge model with at least

- a new strong interaction with RGI scale $\Lambda_T > v_{SM}$ and $\sim$ a few TeV
- new fermions with mass $O(\Lambda_T)$ confined in detectable resonances
  [decaying via SM or new strong interactions, depending on model details]
- a composite Higgs boson: possibly given by a bound WW+ZZ state
  [as needed to have a perturbatively unitary LE description of WW-scattering]
- a low energy ($p < 1$ TeV) effective action very similar to the SM:
  deviations of LE couplings from SM under study, possibly $O(1/\Lambda_T^2)$,

Hints at $\Lambda_T \sim$ 5–10 TeV: to be checked in theory ... and by experiments

Backup slides

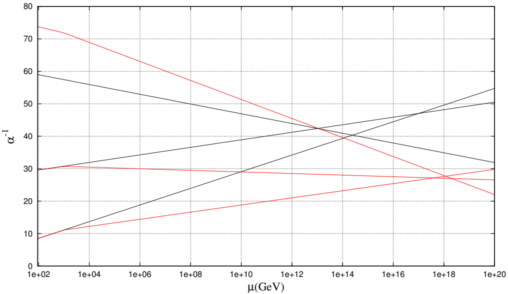
Backup slides follow ...

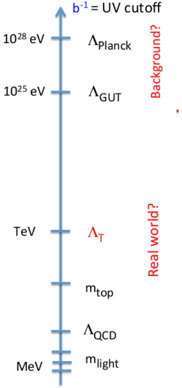
Towards a realistic field content / unification?

- A few “realistic” models seem phenomenologically viable
- Models with SM particles, T-gluons, T-quarks, T-leptons:
  - ⋆ can have cold DM candidates: e.m. neutral T–hadrons (with valence T–fermions and/or quarks)
  - ⋆ non-standard T-fermion hypercharges ⇒ gauge coupling unification?

$\Lambda_{GUT} \sim 10^{18}$ GeV: mass scale of the

$\tilde{\chi}$–unprotected particles? $\Lambda^2_{GUT} \leftrightarrow m^2_{\zeta_0^c}$?





Mechanism in Toy Model ($g_W = 0$): lattice study

Mechanism in Toy Model ($g_W = 0$): a numerical test on the lattice

Lattice study of $\mathcal{L}_{\rm toy}$ $(g_W = 0)$

First (as far as we know) lattice simulation of a $d = 4$ model with gluons, fermions, scalars:

quenching & naive lattice fermions $\Rightarrow$ χ-invariant formulation, doubling in valence

NP mass mechanism expected to be at work already in quenched approximation

$$S_{lat} = b^4 \sum_x \left\{ \mathcal{L}_{kin}^{YM,plaq}[U] + \mathcal{L}_{kin}^{sca}(\Phi) + \mathcal{V}(\Phi) + \overline{\Psi} D_{lat}[U,\Phi]\Psi \right\} , \quad \Phi = \varphi_0 1\!\!1 + i\varphi_j \tau^j ,$$

$$\mathcal{L}_{kin}^{sca}(\Phi) + \mathcal{V}(\Phi) = \frac{1}{2} \mathrm{Tr}\,[\Phi^\dagger(-\partial_\mu^* \partial_\mu)\Phi] + \frac{\mu_0^2}{2} \mathrm{Tr}\left[\Phi^\dagger\Phi\right] + \frac{\lambda_0}{4}\left(\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\right)^2 ,$$

$$(D_{lat}[U,\Phi]\Psi)(x) = \gamma_\mu \widetilde{\nabla}_\mu \Psi(x) + \eta F(x)\Psi(x) - b^2 \rho \frac{1}{2} F(x) \widetilde{\nabla}_\mu \widetilde{\nabla}_\mu \Psi(x)$$

$$- b^2 \rho \frac{1}{4} \Big[(\partial_\mu F)(x) U_\mu(x) \widetilde{\nabla}_\mu \Psi(x+\hat{\mu}) + (\partial_\mu^* F)(x) U_\mu^\dagger(x-\hat{\mu}) \widetilde{\nabla}_\mu \Psi(x-\hat{\mu}) \Big] ,$$

where $F \equiv \varphi_0 1\!\!1 + i\gamma_5 \tau^j \varphi_j$. Only derivatives $\widetilde{\nabla}_\mu = \frac{1}{2}(\nabla_\mu + \nabla_\mu^*)$ acting on fermions, with

$$\nabla_\mu f(x) \equiv \frac{1}{b}(U_\mu(x) f(x+\hat{\mu}) - f(x)) , \quad \nabla_\mu^* f(x) \equiv \frac{1}{b}(f(x) - U_\mu^\dagger(x-\hat{\mu})^f(x-\hat{\mu})) .$$

Term $\propto \rho$: Wilson-like, but with $d = 6$ $\Rightarrow$ fermion doublers do not decouple

Extension to 2 generations: $\bar{\Psi}_\ell D_{lat}[U,\Phi]\Psi_\ell + \bar{\Psi}_h D_{lat}[U,\Phi]\Psi_h$ in fermionic L_{lat}

Naive lattice fermions: from Ψ to flavour basis & $b \to 0$

Naive fermion action with $d = 6$ Wilson-like term *involving only* $\widetilde{\nabla}_\mu$ derivatives:

exact Spectrum Doubling [SD] symmetry $\Rightarrow$ all doublers equivalent @ $b > 0$

i) Wigner phase: |bare mass| $= 0$, ii) NG phase: |bare mass| $= |(\eta - \eta_{cr})v|$

Flavour content: 2 (isospin: I) $\times$ 4 (tastes: a) $\times$ 4 (replica: A) species per generation

$\Psi^I(x) \;\Leftrightarrow\; \chi^{A,I}(x) \;\Leftrightarrow\; q^{A,I}_{\alpha,a}(y)$, y_μ coarse, $x_\mu = 2y_\mu + \xi_\mu$ fine lattice coordinates

$\Psi(x) = \mathcal{A}_x \chi(x)\,,\quad \mathcal{A}_x = \gamma_1^{x_1}\gamma_2^{x_2}\gamma_3^{x_3}\gamma_4^{x_4}$ spin-diagonalizes S_{lat} in $\chi^A(x)$ basis: $A = 1, ..., 4$

$q^{A,I}_{\alpha,a}(y) = \frac{1}{8}\sum_\xi [\Gamma_\xi]_{\alpha,a}(1 - b\xi_\mu \widetilde{D}_{y,\mu})\chi^{A,I}(2y+\xi)\,,\quad \xi_\mu = 0, 1$ $q^{A,I}(y) \leftrightarrow$ flavour basis

Following Kluberg-Stern et al. ('83), ..., Sharpe et al. ('93), Luo ('96) and

adding scalars $\Rightarrow$ the classical b-expansion of S^{ferm}_{lat} (fermion sector) is

$$S^{ferm}_{lat} = \int d^4y \sum_A \{\bar{q}^{A,I}(y)(\gamma_\mu \otimes 1)D_\mu q^{A,I}(y) + (\eta - \bar{\eta})\,\bar{q}^{A,I}(y)\mathcal{F}^A(y)q^{A,I}(y) + \mathrm{O}(b^2)\}$$

$$\mathcal{F}^A(y) \equiv \varphi_0(2y)(1 \otimes 1) + s^A i\tau^j \varphi_j(\gamma_5 \otimes t_5)\,,\quad s_A = \pm 1\,,\ t_\mu = \gamma_\mu^*\ \text{taste matrices}$$

$\tilde{\chi}$–restoring, cutoff effects and key quark bilinears

- At a certain $\eta = \eta_{cr}(\rho, g_0, \lambda_0)$ the renormalized WTI's of $\tilde{\chi}_L \times \tilde{\chi}_R$ symmetry are restored up to cutoff effects simultaneously for all tastes and replica
- Exact $\chi_L \times \chi_R$ invariance $\Rightarrow$ only O(b^{2n}) cutoff effects [n integer]
- Pseudoscalar densities, vector and axial currents (isospin matrix $\tau \in \tau^{0,1,2,3}$):

  from Ψ (code) basis to the classical continuum expansion in flavour $q^{A,I}$ basis

$$\sum_\xi \bar{\Psi}^I(x)\gamma_5\tau_{IJ}\Psi^J(x)\Big|_{x=2y+\xi} = \sum_{A=1}^4 s_A \bar{q}^{A,I}(y)(\gamma_5 \otimes t_5)\tau_{IJ} q^{A,J}(y) + \mathrm{O}(b^2)$$

$$\sum_\xi \bar{\Psi}^I(x)\gamma_\mu\tau_{IJ}\Psi^J(x)\Big|_{x=2y+\xi}^{1pt\ split} = \sum_{A=1}^4 \bar{q}^{A,I}(y)(\gamma_\mu \otimes 1)\tau_{IJ} q^{A,J}(y) + \mathrm{O}(b^2)$$

$$\sum_\xi \bar{\Psi}^I(x)\gamma_\mu\gamma_5\tau_{IJ}\Psi^J(x)\Big|_{x=2y+\xi}^{1pt\ split} = \sum_{A=1}^4 s_A \bar{q}^{A,I}(y)(\gamma_\mu\gamma_5 \otimes t_5)\tau_{IJ} q^{A,J}(y) + \mathrm{O}(b^2)$$

Correlators with "family" non-singlet operators $\Rightarrow$ no disconnected diagrams:

e.g. $\langle \bar{\Psi}^I_\ell(x)\Gamma_W\tau_{IJ'}\Psi^{J'}_h(x)\ \bar{\Psi}^{I'}_h(\tilde{x})\Gamma_Z\tau_{I'J}\Psi^J_\ell(\tilde{x})\rangle$ & two degenerate quenched families

give info on η_{cr}, PS-meson mass, renormalized matrix elements of $\partial_\mu\bar{\Psi}^I\gamma_\mu\gamma_5\tau_{IJ'}\Psi^{J'}$

$L_{\mathrm{toy}}[Q, \bar{Q}, U, \Phi]$ with twisted mass and the SDE's of $\tilde{\chi}$

- Now $S_{lat}^{ferm} = \bar{Q} D_{lat}[U, \Phi_{smeared}] Q$ with locally smeared Φ for noise reduction

- Quenched (U, Φ)-configurations: known problem of exceptional conf.s with spurious zero modes of $D_{lat}(U, \Phi)$: at large $|\rho|$, $|\eta|$ enhanced by Φ-fluctuations

- Adding twisted mass term: $S_{lat}^{\mathrm{toy-tm}} = S_{lat} + b^4 \sum_x i\mu \bar{Q} \gamma_5 \tau^3 Q(x)$
robust IR cutoff $\Rightarrow$ soft breaking of $\chi_L \times \chi_R$ symmetry: take limit $\mu \propto \mu^{ren} \to 0$

- $\tilde{A}$ and $\tilde{V}$ renormalized SDE's (here generation non-singlet): $\eta_{sub} = \mathrm{O}(\eta - \eta_{cr})$

$$Z_{\widetilde{A}} \partial \widetilde{A}^1 - 2\eta_{sub} \widetilde{D}^{P1} \overset{\mathrm{O}(b^2)}{\simeq} \delta_{ph,NG} \, [\, \mathrm{NP\ term} \,]$$

at LE: NP term $\sim 2c_1 \Lambda_s (\bar{Q}_L \{U, \frac{\tau^1}{2}\} Q_R - \mathrm{hc})$

$$Z_{\widetilde{V}} \partial \widetilde{V}^2 - 2\eta_{sub} \widetilde{D}^{S2} - i2\mu P^1 \overset{\mathrm{O}(b^2)}{\simeq} \delta_{ph,NG} \, [\, \mathrm{NP\ term} \,]$$

NP term $\sim 2c_1 \Lambda_s (\bar{Q}_L [U, \frac{\tau^2}{2}] Q_R - \mathrm{hc})$

...

with $\widetilde{A}^1_\mu = \bar{Q} \gamma_\mu \gamma_5 \frac{\tau^1}{2} Q|^{1pt\ split}$, ... , $\widetilde{D}^{P1} = \bar{Q}_L \{\Phi, \frac{\tau^1}{2}\} Q_R - \mathrm{hc}$, $\widetilde{D}^{S2} = \bar{Q}_L [\Phi, \frac{\tau^2}{2}] Q_R - \mathrm{hc}$

Note: $Z_{\widetilde{A}} = Z_{\widetilde{V}}$ due to χ invariance; and with zero anomalous dimension if $\eta = \eta_{cr}$

Action parameters & renormalization conditions

Quenched fermion lattice setup: action reads [fermion isodoublet denoted now by Q]

$$S_{lat}^{\mathrm{toy-tm}} = b^4 \sum_x \left\{ \mathcal{L}_{kin}^{YM,plaq}[U] + \mathcal{L}_{kin}^{sca}(\Phi) + \mathcal{V}(\Phi) + \overline{Q} D_{lat}[U,\Phi] Q + i\mu \overline{Q}\gamma_5\tau^3 Q \right\},$$

- ⋆ bare couplings: $g_0^2 = 6/\beta$ (gauge), μ_0^2 & λ_0 (scalar mass & selfinteraction)
- ⋆ $\tilde{\chi}$ bare couplings: η (Yukawa), ρ (needs no UV-divergent CT); μ (soft mass)
- ⋆ quenching $\Rightarrow$ gauge and Φ sector couplings renormalize independently

- ⋆ for observables in NG phase a renorm. condition fixing v_Φ is also needed
- ⋆ soft mass $\mu^{ren} = Z_P^{-1}\mu$ and $\eta_{sub} = \mathrm{O}(\eta - \eta_{cr})$ will be extrapolated to zero ...

- $r_0^2 F_{h\bar{h}}(r_0) = 1.65$ (g_s^2 from static quark-antiquark force) fixes $g_0^2 \Rightarrow$ scale $1/r_0$
- $(M_\sigma^{NG} r_0)^2 = 1.285$ & $(v_\Phi r_0)^2 = 1.458$ & $\lambda_{NP}^{NG} = (1/2)(M_\sigma^{NG}/v_\Phi)^2 = 0.4408$

  determine $m_{sub}^2 = \mu_0^2 - \mu_{cr}^2$ & $Z_\Phi = \hat{\Phi}_R/\hat{\Phi}$ & $\lambda_{NP}^{NG} = Z_\lambda \lambda_0$

- $\mu_{cr}^2 = \tau_{cr}(\lambda_0)/b^2$ evaluated from peak of $G_{\Phi\Phi^\dagger}(p=0,\mu_0^2,\lambda_0)$ in μ_0^2 @ large L

Simul. step 1: scalar sector renormalization

From several simulations of the $\lambda_0(\Phi^\dagger\Phi)^2$ theory with $12 < L/b < 24$ and $T = 2L$ $\Rightarrow$

scalar sector parameters matching renorm. conditions in NG phase & $\beta \leftrightarrow r_0/b$

[SU(3)-YM data for $\beta \leftrightarrow r_0/b$ from Necco and Sommer, Nucl.Phys. B622 (2002) 328-346]

β	r_0/b	$r_0^2 M_\sigma^2$	$r_0^2 v_R^2$	λ_{NP}	$b^2\mu_0^2$	λ_0	κ
5.75	3.29	1.278(4)	1.464(3)	0.437(2)	-0.5941	0.5807	0.132283
5.85	4.06	1.286(4)	1.459(3)	0.441(2)	-0.5805	0.5917	0.132000
5.95	4.91	1.290(5)	1.453(3)	0.444(2)	-0.5756	0.6022	0.131870

κ: code hopping parameter, s.t. $\kappa^{-1} - 2\kappa\lambda_0 - 8 = b^2 m_0^2$, $\eta_{code} = \eta\sqrt(2\kappa)$, $\rho_{code} = \rho\sqrt(2\kappa)$

Values of μ_{cr}^2 & μ_0^2, λ_0 parameters for simulations in Wigner phase at fixed $\mu_\Phi^2 r_0^2 > 0$

β	r_0/b	$(\mu_0^2 - \mu_{cr}^2)b^2$	$b^2\mu_{cr}^2$	$b^2\mu_0^2$	λ_0	κ
5.75	3.29	0.1119(12)	-0.5269(12)	-0.4150	0.5807	0.129280
5.85	4.06	0.0742(11)	-0.5357(11)	-0.4615	0.5917	0.130000
5.95	4.91	0.0504(10)	-0.5460(10)	-0.4956	0.6022	0.130521

Simul. step 2: η_{cr} determination in Wigner phase

Based on the renormalized form of the SDE's of $\tilde{A}^{1,2}$ transformations, e.g.

$$Z_{\widetilde{A}}\partial\widetilde{A}^1 = 2\,\eta_{sub}\,\widetilde{D}^{P1} + \mathrm{O}(b^2) \qquad \text{with} \qquad \eta_{sub} = (\eta-\eta_{cr})[1+\mathrm{O}(\eta-\eta_{cr})]$$

$$\widetilde{A}^1_\mu = \bar{Q}\gamma_\mu\gamma_5\tfrac{\tau^1}{2}Q \qquad \text{and} \qquad \widetilde{D}^{P1} = \bar{Q}_L\{\Phi,\tfrac{\tau^1}{2}\}Q_R - \mathrm{hc}\,,$$

$\eta_{cr}^{A,\cdots}(g_0^2,\lambda_0;\rho)$ is determined from $\lim_{\mu\to 0} r_{AWI}^{A,\cdots}[g_0^2,\lambda_0;\rho,\mu,\eta_{cr}^{A,\cdots}] = 0\,,$ where

$$r_{AWI}(x_0;y_0) \equiv \sum_{\mathbf{x}}\sum_{\mathbf{y}}\langle P^1(0)\partial^{FW}_{x,0}\tilde{A}_0^{1,BW}(x)\phi^0(y)\rangle \,/\, \sum_{\mathbf{x}}\sum_{\mathbf{y}}\langle P^1(0)D_P^1(x)\phi^0(y)\rangle$$

$y_0 = x_0+\tau$ with $\tau \simeq 0.6$ fm (to control noise from Φ propagator), $T\simeq 4.9$ fm ,

A: $x_0 \in (0.9, 1.8)$ fm [intermediate pion dominates before x_0, Φ state between x_0 and y_0]

B: $x_0 \in (2.7, 3.3)$ fm [vacuum dominates before x_0, "pion + Φ" state between x_0 and y_0]

- ⋆ $\mu\to 0$ limit from linear or parabolic fit in μ of $r_{AWI}^{A,B}|_{\dots,\mu\dots}$ (mean + spread added to err.)
- ⋆ $r_{AWI}^B - r_{AWI}^A = \mathrm{O}(b^2)$ entails $\eta_{cr}^B - \eta_{cr}^A = \mathrm{O}(b^2)$, η_{cr}^A with smaller lattice artifacts

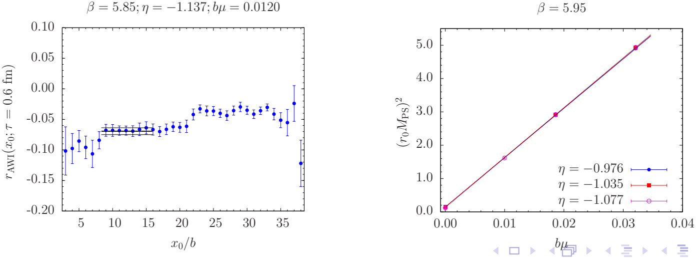
Step 2: simulation parameters, r_{AWI}–signal, M_π^2 vs. μ

β	$a^{-4}(L^3 \times T)$	η	$a\mu$	stat. $(N_U \times N_\Phi)$
5.75	$16^3 \times 32$	-1.1505	0.0180 0.0280 0.0480	60×8
($a \sim 0.152$ fm)		-1.1898	0.0180 0.0280 0.0480	
		-1.3668	0.0180 0.0280 0.0480	
5.85	$16^3 \times 40$	-1.0983	0.0224 0.0316 0.0387	60×8
($a \sim 0.123$ fm)		-1.1375	0.0120 0.0172 0.0224 0.0387 0.0600	
		-1.2944	0.0224 0.0387	
5.95	$20^3 \times 48$	-0.9761	0.0186, 0.0321	60×8
($a \sim 0.102$ fm)		-1.0354	0.0186, 0.0321	
		-1.0771	0.0100 0.0186, 0.0321	

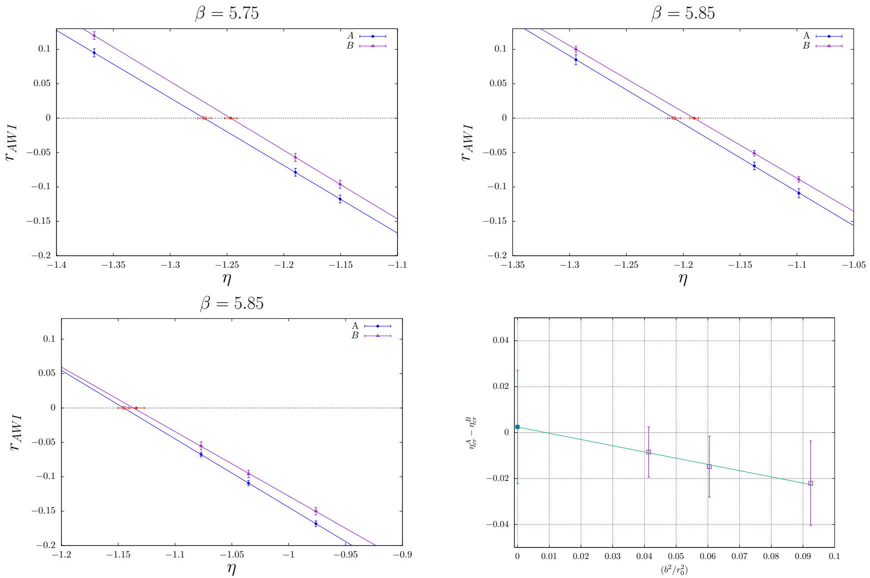
QCD-inspired lattice scale: assuming $r_0 = 0.5$ fm $\Rightarrow$ $L \in (2.0 - 2.4)$ fm and $T \simeq 4.9$ fm

r_0/b has $\sim 0.5\%$ errors; zero modes of fermionic $D_{lat}[U, \Phi_{smeared}]$ regulated by $\mu > 0$.

$\mathcal{V}(\Phi)$ with one minimum & SSB of $\tilde{\chi}$ due to strong interactions: vacuum polarized by $\mu > 0$



Step 2: $r_{AWI}^{A,B}$ vs. η and check of $\eta_{cr}^{B} - \eta_{cr}^{A} = \mathrm{O}(b^2)$



Results for $\eta_{cr}^{A,B}$ and $1/Z_P^{had}$, $Z_{\widetilde{V}}$, Z_η^{had}

β ρ	5.75 1.96	5.85 1.96	5.95 1.96	5.85 2.94
η_{cr}^A	-1.271(10)	-1.207(8)	-1.145(6)	-1.820(15)
η_{cr}^B	-1.249(8)	-1.192(6)	-1.136(6)	
$1/Z_P^{had}$	20.6(7)	20.1(7)	20.7(9)	22.7(1.1)
Z_V	0.95(2)	0.94(1)	0.97(2)	0.83(4)
Z_η^{had}	9.1(4)	13.9(3)	15.6(4)	

Need renormalization constants of a few composite (non–isosinglet) operators

to evaluate renormalized WTI fermion masses m_{AWI}^{ren}, μ^{ren}, ... in the NG phase

For computational convenience we define:

⋆ $1/Z_P^{had} = \langle 0|\bar{Q}\gamma_5 \frac{\tau^1}{2} Q|P^1_{meson}\rangle|_{\eta_{cr},\mu\to 0+}\, r_0^2 \equiv G_{PS}^{Wig}\, r_0^2$ eval. in Wigner phase

⋆ $Z_{\widetilde{V}}$ s.t. $Z_{\widetilde{V}}\langle 0|\partial_0 \widetilde{V}_0^2|P^1_{\rm meson}\rangle|_{\eta_{cr},\mu\to 0+} = 2\mu\langle 0|\bar{Q}\gamma_5 \frac{\tau^1}{2} Q|P^1_{\rm meson}\rangle|_{\eta_{cr},\mu\to 0+}$

evaluated in NG phase

⋆ $Z_\eta^{had} = |\partial(r_0 M_{P^1_{meson}})/\partial\eta|_{\eta_{cr},\mu\to 0+}$ evaluated in NG phase

Simul. step 3: NG phase around η_{cr} – Effective action

β	$a^{-4}(L^3 \times T)$	η	$a\mu$	stat. $(N_U \times N_\Phi)$
5.75 ($a \sim 0.152$ fm)	$16^3 \times 40$	-1.2714	0.0050 0.0087, 0.0131, 0.0183, 0.0277	60×1
		-1.2656	0.0131	
		-1.2539	0.0183	
		-1.2404	0.0131	
		1.2277	0.0131, 0.0183	
5.85 ($a \sim 0.123$ fm)	$20^3 \times 40$	-1.2105	0.0040, 0.0070, 0.0100, 0.0120	30×2
		-1.2068	0.0040, 0.0070, 0.0100 ,0.0224	
		-1.2028	0.0040, 0.0070, 0.0100, 0.0120, 0.0224	
		-1.1949	0.0100	
		-1.1776	0.0070, 0.0100, 0.0140, 0.0224 , 0.0316	
5.95 ($a \sim 0.102$ fm)	$24^3 \times 48$	-1.1474	0.0066, 0.0077, 0.0116, 0.0145, 0.0185	30×1
		-1.1449	0.0060, 0.0077, 0.0116, 0.0145	
		-1.1215	0.0077	
		-1.1134	0.0077, 0.0108	

Lattice correlators and derived observables with external momenta p in the range

$1/b \gg p \gg \Lambda_s$ can be described by an effective Lagrangian (including possible NP mass terms) written in terms of formal fermion, gauge and $\Phi \equiv (v + \zeta_0)\, U$ fields:

$\Gamma^{NG} = \Gamma_4^{NG} + b^2\Gamma_6^{NG} + b^4\Gamma_8^{NG} + \ldots$ where ...

Low energy effective action in NG phase for $S_{lat}^{\rm toy-tm}$

$$\Gamma^{NG} = \Gamma_4^{NG} + b^2\Gamma_6^{NG} + b^4\Gamma_8^{NG} + \ldots \qquad \text{where}$$

$$\Gamma^{NG}_{4\,\rm loc} = \frac{1}{4}(G\cdot G) + \bar{Q}\not{\mathcal{D}}Q + \frac{1}{2}\,\mathrm{Tr}\left[\partial_\mu\Phi^\dagger\partial_\mu\Phi\right] + \frac{m_\Phi^2}{2}\,\mathrm{Tr}\left[\Phi^\dagger\Phi\right] + \frac{\lambda_\Phi}{4}\left(\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\right)^2 +$$

$$+\eta_{eff}\bar{Q}FQ + \mu_{eff}\bar{Q}i\gamma_5\tau^3 Q + c_1\Lambda_s\bar{Q}U_F Q + (\tilde{c}\Lambda_s R + c_2\Lambda_s^2)\frac{1}{2}\,\mathrm{Tr}\left[\partial_\mu U^\dagger\partial_\mu U\right]$$

$$\Gamma^{NG}_{6\,\rm loc} \supset O_6^{\tilde{\chi}-\rm inv}\,;\;[D_\lambda\bar{Q}_L\Phi D_\lambda Q_R + h.c.]\,,\;\ldots\,;\;[\textstyle\sum_{\Gamma_A\Gamma_B} c_{AB}(\bar{Q}\Gamma_A Q)(\bar{Q}\Gamma_B Q)]^{\tilde{\chi}-\rm inv}$$

here $\quad \eta_{eff} \propto \eta - \eta_{cr}\,, \quad \mu_{eff} \propto \mu$; vacuum choice (by "Φ-projection"): $\Phi \propto U \propto I$;

for $\tilde{\chi}$–breaking terms: since $\Phi = U(v+\zeta_0)$ separating out terms with v we get

$$\eta_{eff}\bar{Q}FQ + \mu_{eff}\bar{Q}i\gamma_5\tau^3 Q + c_1\Lambda_s\bar{Q}U_F Q = m_{eff}\bar{Q}U_F Q + \mu_{eff}\bar{Q}i\gamma_5\tau^3 Q + \eta_{eff}\bar{Q}\sigma_F Q =$$

$$= M_{eff}\,\bar{\Psi}\Psi + \eta_{eff}\bar{\Psi}e^{-i\frac{\omega}{2}\gamma_5\tau^3}\sigma_F e^{-i\frac{\omega}{2}\gamma_5\tau^3}\Psi + m_{eff}\bar{\Psi}e^{-i\frac{\omega}{2}\gamma_5\tau^3}(U_F-1)e^{-i\frac{\omega}{2}\gamma_5\tau^3}\Psi$$

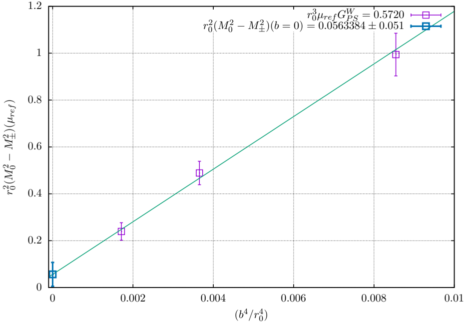
with $\quad m_{eff} \equiv \eta_{eff}\,v + c_1\,\Lambda_s\,, \qquad F \equiv (v+\zeta_0)U_F\,,\ \sigma_F \equiv \zeta_0 U_F\,,\ U_F = P_R U + P_L U^\dagger$

$M_{eff} \equiv \sqrt{m_{eff}^2 + \mu_{eff}^2}\,,\ \tan\omega = \mu_{eff}/m_{eff}\,, \quad \Psi = \exp(i\frac{\omega}{2}\gamma_5\tau^3)Q\,,\ \bar{\Psi} = \bar{Q}\exp(i\frac{\omega}{2}\gamma_5\tau^3)$

Scaling test in NG phase: $r_0^2(M_{P,0}^2 - M_{P,\pm}^2)$ at $\mu^{ren} > 0$

At fixed $\mu^{ren} > 0$ (here in *s*-quark range if $r_0 = 0.5$ fm) charged and neutral pion masses differ due to lattice artifacts – related to the misalignment in chiral space between $i\mu\bar{Q}\gamma_5\tau^3 Q$ and $b^2 v\bar{Q}D^2 Q$: however leading O(b^2) effects cancel in the pion mass difference and one expects (can actually prove starting from the lattice Γ^{NG} above)

$$r_0^2(M_{P,0}^2 - M_{P,\pm}^2) \propto b^4 \qquad \text{for generic } m_{eff} \text{ values}$$



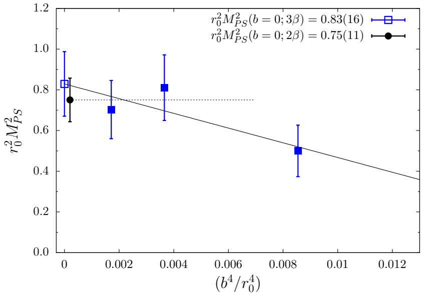
here data for $\eta = \eta_{cr}$ $\leftrightarrow$ O(b^4) scaling expected independently of $c_1\Lambda_s$ value

The "no mechanism hypothesis" and $M_{P,\pm}r_0(\mu \to 0+)$

Let us test the hypothesis of no NP mass generation: $c_1\Lambda_s = 0$

In this case, for $\eta = \eta_{cr}$, one expects $M_{P,\pm\,\mu\to 0} = M^{cont}_{P,\pm\,\mu\to 0} + \mathrm{O}(b^2) = \mathrm{O}(b^2)$

$\lim_{\mu\to 0+}[r_0^2 M_{P,\pm}^2](\mu)$ should have zero continuum limit with only $\mathrm{O}(b^4)$ artifacts

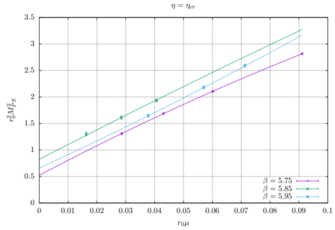


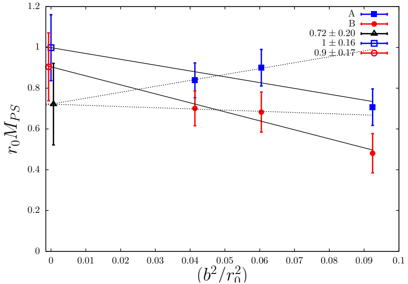
Hypothesis of $c_1\Lambda_s = 0$ (no mechanism) is not supported by numerical NP data

Assume $c_1 \Lambda_s \neq 0 \;\Rightarrow\; m_{eff}\big|^{\eta=\eta_{cr}} = c_1 \Lambda_s$: from the NG phase lattice effective action

- ⋆ at $\eta = \eta_{cr}$ expect $\quad M^2_{P,\pm}(\mu, \eta_{cr}) = 2B\sqrt{m^2_{eff} + \mu^2_{eff}} + \mathrm{O}(m^2_{eff} + \mu^2_{eff}) + \mathrm{O}(b^2)$
- ⋆ otherwise $\quad M^2_{P,\pm}(\mu, \eta) = M^2_{P,\pm}(\mu, \eta_{cr}) + \Delta_\eta(\eta - \eta_{cr}) + \Delta_{\eta\eta}(\eta - \eta_{cr})^2 + \mathrm{O}(b^2)$

$M_{P,\pm} r_0 \stackrel{\mu, b^2 \to 0+}{\longrightarrow}$ 0.87[17] [↔ (16)(07)]





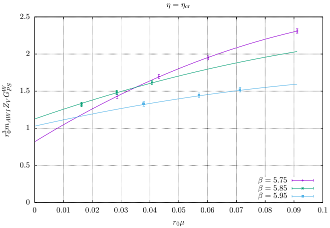
Limits $\mu \to 0+$ and $\eta \to \eta_{cr}$: use polynomial –up to 2nd order– fits in μ and $(\eta - \eta_{cr})$

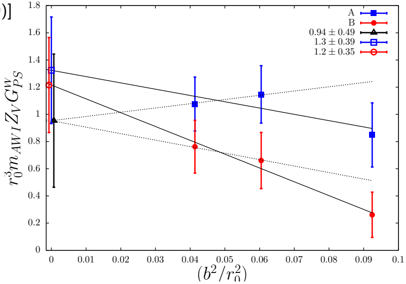
$2\,|m_{AWI}^{ren}|\, r_0\,(\mu \to 0+)$ at η_{cr} in the continuum limit

Assume $c_1\Lambda_s \neq 0 \;\Rightarrow\; m_{eff}|^{\eta=\eta_{cr}} = c_1\Lambda_s$: from the NG phase lattice effective action

- ⋆ $2m_{AWI}^{bare}(\mu,\eta) \equiv [\partial_0^{FW} \sum_{\vec{x}} \langle \tilde{A}^{1BW}_{0(\ell,h)}(\vec{x},x_0) P^1_{(\ell,h)}(0)\rangle] \,/\, [\sum_{\vec{x}} \langle P^1_{(\ell,h)}(\vec{x},x_0) P^1_{(\ell,h)}(0)\rangle](\mu,\eta)$
- ⋆ $Z_{\widetilde{A}}\partial \widetilde{A}^1 = 2\,\eta_{sub}\widetilde{D}^{P1} + 2\,c_1\Lambda_s\mathcal{P}^1 \quad \Rightarrow \quad$ expect $\quad m_{AWI} = \mathrm{O}((\eta-\eta_{cr}))v + \mathrm{O}(c_1\Lambda_s)$
- ⋆ renormalization: $\quad 2m_{AWI}^{ren} = (Z_{\widetilde{V}}/Z_P^{had})2m_{AWI}^{bare}$; $\quad \mu$–dependence is $\mathrm{O}(b^2)$

$2\,|m_{AWI}^{ren}|\, r_0 \;\xrightarrow{\mu,b^2\to 0+}\; 1.15[38]$ [$\leftrightarrow$ (37)(09)]





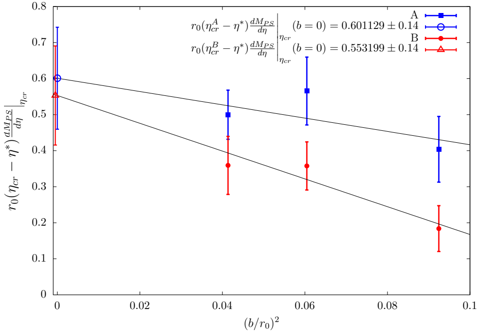
Limits $\mu \to 0+$ and $\eta \to \eta_{cr}$: use polynomial –up to 2nd order– fits in μ and $(\eta-\eta_{cr})$

$m_{AWI}(\eta_*, \mu = 0) \equiv 0$ and $[\eta_* - \eta_{cr}]^{ren}$ (continuum limit)

WTI fermion mass $\quad m_{AWI} = \mathrm{O}((\eta - \eta_{cr}))v + \mathrm{O}(c_1\Lambda_s) \quad \leftrightarrow \quad m_{eff} \equiv \eta_{eff}\, v + c_1\,\Lambda_s$

$\lim_{\mu\to 0+} m_{AWI}(\mu,\eta) = 0$ eventually at some $\eta = \eta_*$, with $\quad \eta_* = \eta_{cr} - v^{-1}\mathrm{O}(c_1\Lambda_s)$

$\star\ [\eta_* - \eta_{cr}]^{ren} = Z_\eta^{had}(\eta_* - \eta_{cr}) \quad$ with $\quad Z_\eta^{had} = [\partial(r_0 M_{P^1_{meson}})/\partial\eta]|_{\eta_{cr},\mu\to 0+}$



Determination of η_* from combined fit of $m_{AWI}(\mu,\eta)$: its uncertainty is $\ll \eta_{cr}$-error

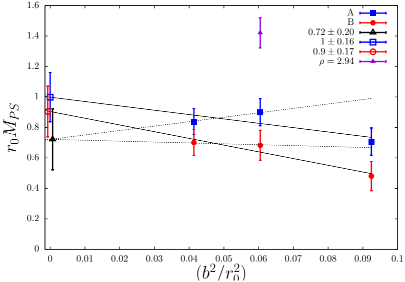
Upon increasing ρ: $M_{P,\pm}$ and $|m^{ren}_{AWI}|$ at η_{cr}

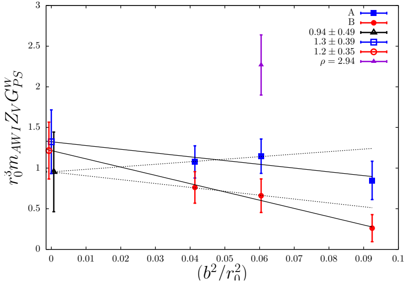
Study now lattice toy model with $\rho = 2.94$ – rather than $\rho = 1.96$ as previously:

Wigner phase: find that $\eta_{cr}(\rho)$ increases $\propto$ $\rho(1 + \mathrm{O}(\rho^2))$ – as expected

In $S_{lat}^{\mathrm{toy-tm}}$ the $\tilde{\chi}$ breaking terms (crucial for NP mass) are controlled by ρ :

mechanism details imply $M^2_{P,\pm} \sim |m^{ren}_{AWI}| \sim \mathrm{O}(\rho^2)$ [PRD92 (2015) 054505]



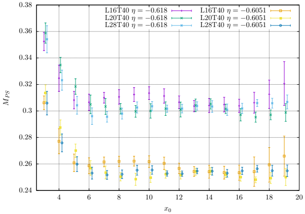


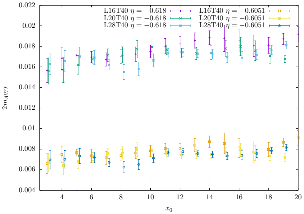
response to increase of ρ by a factor 1.5 is clear & consistent with expectation ...

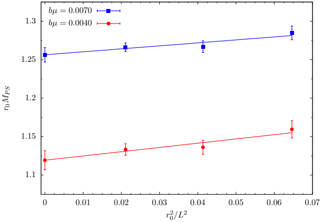
the observed NP fermion mass appears related to $\tilde{\chi}$ breaking at the UV cutoff scale

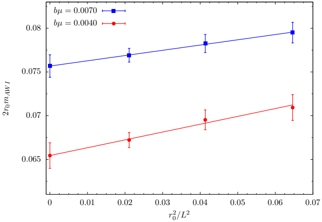
Check of finite size effects: $M_{P,\pm}$ and $|m_{AWI}^{ren}|$ ($\beta = 5.85$)

appropriate as NG phase spectrum contains massless (if $L \to \infty$) elementary GB's









the NP fermion mass observed at η_{cr} does not seem to vanish as $1/L^2 \to 0$ ...