



# Prospects for determining the top quark Yukawa coupling at future $e^+e^-$ colliders

Alexander Mitov

Cavendish Laboratory



Based on: [Boselli, Hunter, Mitov arXiv:1805.12027](#)

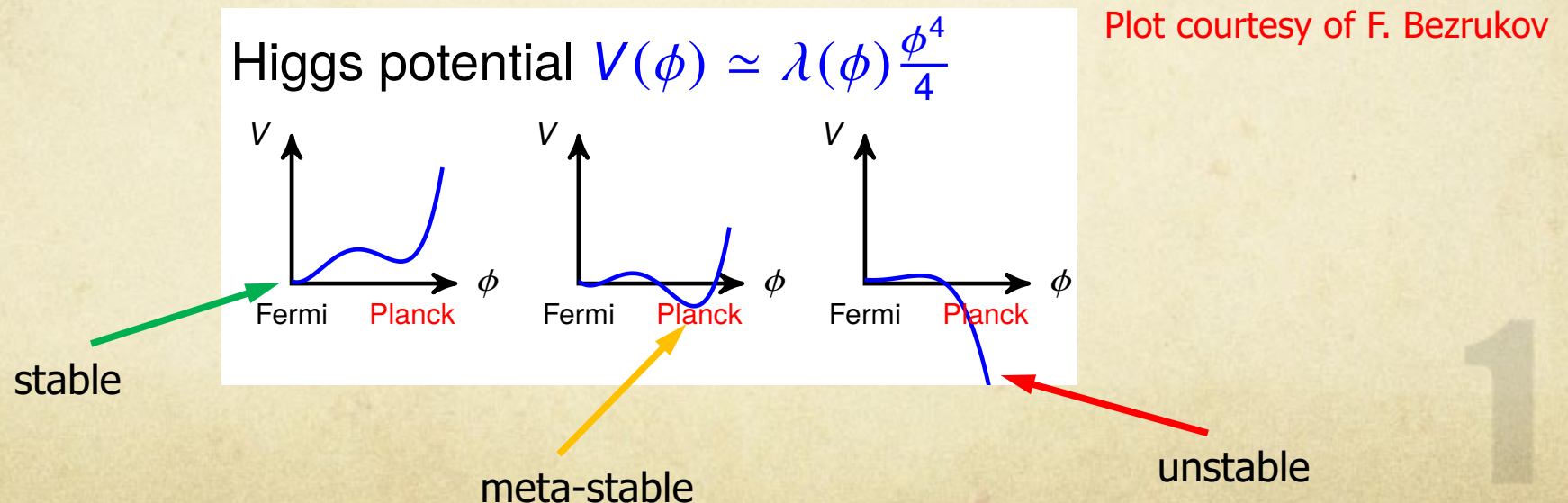
# Intro: why the top mass?

- ✓ It is a fundamental parameter of the SM
- ✓ Its precision affects many precision observables in the SM.
- ✓ Its precision affects the searches for new physics.
- ✓ However, the most relevant case is: extrapolation of the SM to very high energies.
  - ✓ Once the Higgs boson was found (and the mass measured quite precisely)  $m_{\text{top}}$  is the SM parameter that mostly parametrically affects SM predictions
  - ✓ Prime example: stability of EW vacuum (also Higgs inflation,...)

See talks by:

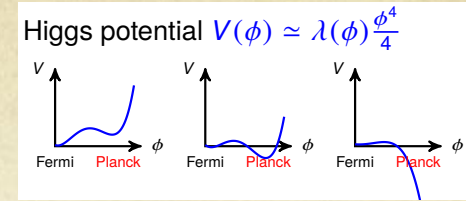
Davide Melini

Silvia Ferrario Ravasio



# Intro: why the top mass?

- ✓ Here is how  $m_{\text{top}}$  enters the game:
- ✓ Take the pole-masses  $m_{\text{top}}$  and  $m_h$  as input parameters. Then:



$$\lambda(\mu) = \frac{G_\mu}{\sqrt{2}} m_h^2 + \text{loop corrections}$$

$$y_t(\mu) = \frac{\sqrt{2}}{v} m_t + \text{loop corrections}$$

$\overline{MS}$ - running parameters

Defs:

$$\mathcal{L} = \frac{y_t}{\sqrt{2}} h \bar{t} t$$

$$G_\mu = \frac{1}{\sqrt{2}v^2} + \text{loop corrections}$$

Size of loop effects:

$\bar{\mu} = M_t$	$\lambda$	$y_t$
LO	0.12917	0.99561
NLO	0.12774	0.95113
NNLO	0.12604	0.94018

All numbers on this slide adapted from Buttazzo et al arXiv:1307.3536v4

- ✓ In other words in SM both  $\lambda$  and  $y_t$  are derived parameters. Their values are:

$$\lambda(\mu = m_t) \approx 0.126 - 0.00004 \left( \frac{\Delta m_t}{1\text{GeV}} \right) + 0.000412 \left( \frac{\Delta m_h}{0.2\text{GeV}} \right) \pm \dots$$

Where:  $\Delta x \equiv x - x^{\text{ref}}$

$$\lambda(\mu = m_{\text{PL}}) \approx -0.0143 - 0.0066 \left( \frac{\Delta m_t}{1\text{GeV}} \right) + 0.0026 \left( \frac{\Delta \alpha_s}{0.001} \right) + 0.0006 \left( \frac{\Delta m_h}{0.2\text{GeV}} \right) \pm \dots$$

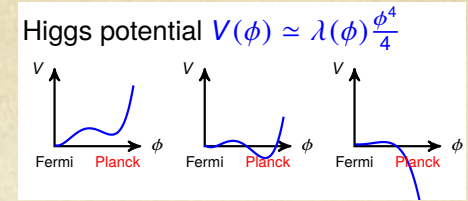
$$y_t(\mu = m_t) \approx 0.9369 + 0.0056 \left( \frac{\Delta m_t}{1\text{GeV}} \right) - 0.0006 \left( \frac{\Delta \alpha_s}{0.001} \right) \pm \dots$$

$$y_t(\mu = m_{\text{PL}}) \approx 0.3825 + 0.0051 \left( \frac{\Delta m_t}{1\text{GeV}} \right) - 0.003 \left( \frac{\Delta \alpha_s}{0.001} \right) \pm \dots$$

Driven by  $m_{\text{top}}$ , not  $m_h$ !

# Intro: why the top mass?

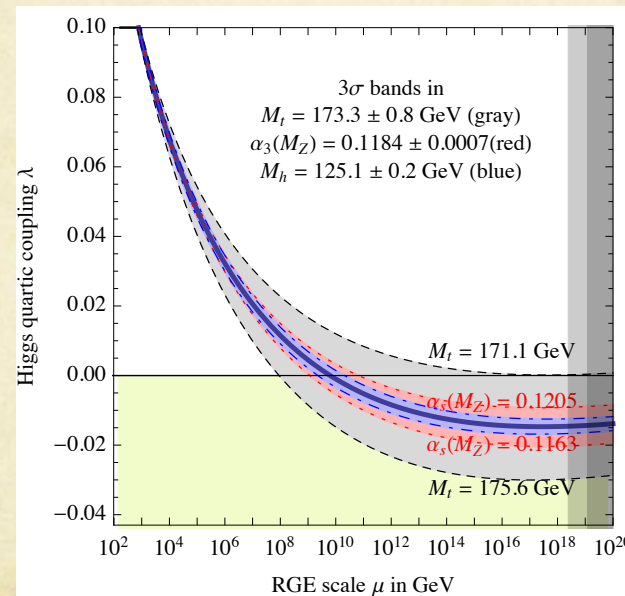
$$\lambda(\mu = m_{\text{PL}}) \approx -0.0143 - 0.0066 \left( \frac{\Delta m_t}{1 \text{ GeV}} \right) + 0.0026 \left( \frac{\Delta \alpha_s}{0.001} \right) + 0.0006 \left( \frac{\Delta m_h}{0.2 \text{ GeV}} \right) \pm \dots$$



- ✓ The effective potential can be non-negative all the way to  $m_{\text{PL}}$  if the top mass were **lower** than the current world average by about 2 GeV.
- ✓ Stated differently, stability requires:

Buttazzo et al arXiv:1307.3536v4

$$M_t < (171.53 \pm 0.15 \pm 0.23_{\alpha_3} \pm 0.15_{M_h}) \text{ GeV} = (171.53 \pm 0.42) \text{ GeV}$$



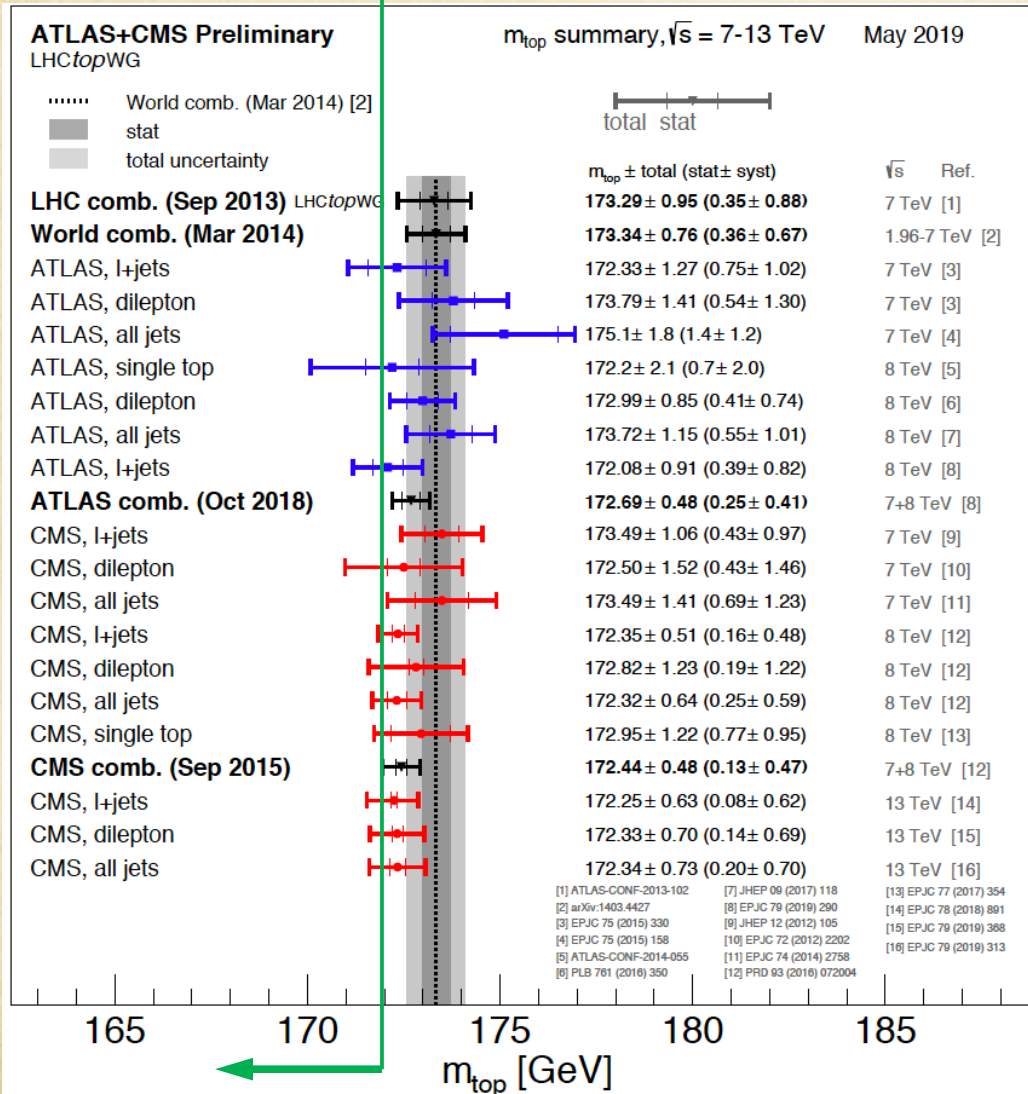
So, what is the value of  $m_{\text{top}}$  and how well do we know it?

# Intro: how well do we (think) we know the top mass?

See talk by Silvia Ferrario Ravasio

✓ And the latest LHCTopWG combination:

$$M_t < (171.53 \pm 0.15 \pm 0.23_{\alpha_3} \pm 0.15_{M_h}) \text{ GeV} = (171.53 \pm 0.42) \text{ GeV}$$



✓ At face value, the World Average is more than  $3\sigma$  away from stability.

✓ In practice, the most-precise LHC measurements are almost consistent with stability!

# Why the top-Yukawa coupling?

- ✓ We would like to measure  $y_t$  directly and verify its SM value **Recall**
- ✓ If BSM physics is present  $y_t$  can be modified:

$$y_t(\mu) = \frac{\sqrt{2}}{v} m_t + \text{loop corrections}$$

- ✓ How to measure  $\Delta y_t$ ?
- ✓ And here is the puzzle:
  - ✓ At the LHC we may be able to measure  $y_t$  with 5%-10% precision (at HL-LHC)
  - ✓ A 100 TeV hadron collider can measure  $y_t$  with 1% precision
- ✓ What about  $e^+e^-$  colliders?

Mangano, Plehn, Reimitz, Schell, Shao '15

- ✓ Usual wisdom: obtain  $y_t$  from  $t\bar{t}h$  final states.
  - ✓ This offers clean(er) interpretation of the measurement
  - ✓ However, we need a 500GeV c.m. energy to produce  $t\bar{t}h$ !
- ✓ Accessible only at CLIC and ILC (among all proposed colliders)
- ✓ Existing studies show that  $y_t$  can be measured with few % at CLIC and ILC
- ✓ Such a prospect is a bit underwhelming, isn't it?

# Why the top-Yukawa coupling?

- ✓ Why is the precision from  $t\bar{t}h$  so low?
- ✓ Answer: luminosity is low, despite the very good sensitivity of the  $\sigma$ -section w/r to  $y_t$ .
- ✓ In this work we ask the question: how can one do better (if possible at all)?
- ✓ Clearly, one has to look at different observables; ideally ones with high expected event yields.
- ✓ We consider events with a single Higgs in the final state but no top quarks.
  - ✓  $t\bar{t}$  final states also have some sensitivity to  $y_t$ . It is low  $O(10\%)$ ; has been studied in the context of  $m_{\text{top}}$  determination
- ✓ We consider all proposed colliders (CEPC, CLIC, FCC-ee, ILC)
- ✓ A great benefit from using single Higgs final states: they can be produced in (relative) abundance at all energies and at all colliders!
- ✓ Where does the  $y_t$  sensitivity come from in such processes?
- ✓ From coupling of the Higgs to top quarks in loops (working in the NWA for the Higgs)

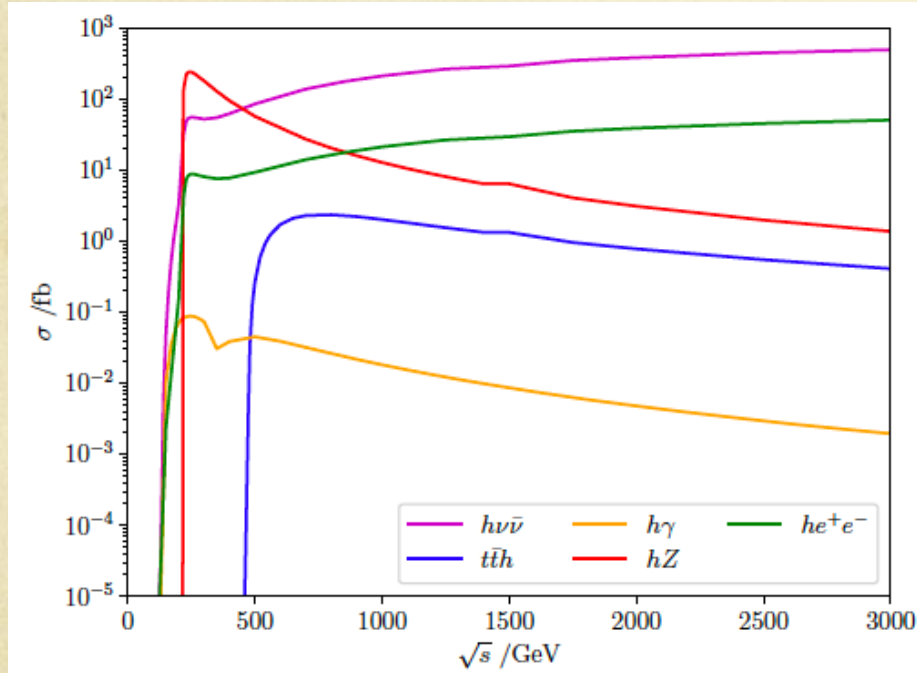
# Our approach

✓ We have identified 3 such loop-induced processes:

- ✓  $e^+e^- \rightarrow h\gamma$  (with  $h \rightarrow b\bar{b}$ )
- ✓  $h \rightarrow \gamma\gamma$  (from  $e^+e^- \rightarrow hZ/h\nu\nu/h\bar{e}e$ )
- ✓  $h \rightarrow g g$  (from  $e^+e^- \rightarrow hZ/h\nu\nu/h\bar{e}e$ )

✓ Here is a LO estimate of x-sections and event yields

	FCC-ee		CEPC
$\sqrt{s}$ (GeV)	240	350	240
$\mathcal{L}_{\text{int.}}$ ( $\text{fb}^{-1}$ )	$1.0 \cdot 10^4$	$2.6 \cdot 10^3$	$5.0 \cdot 10^3$
$\sigma_{hZ}$ (fb)	240	130	240
$\mathcal{N}_{hZ}$	$2.4 \cdot 10^6$	$3.38 \cdot 10^5$	$1.2 \cdot 10^6$
$\sigma_{\nu\bar{\nu}h}$ (fb)	54.4	54.7	54.4
$\mathcal{N}_{\nu\bar{\nu}h}$	$5.44 \cdot 10^5$	$1.42 \cdot 10^5$	$2.72 \cdot 10^5$
$\sigma_{eeh}$ (fb)	7.9	7.13	7.9
$\mathcal{N}_{eeh}$	$7.9 \cdot 10^4$	$1.85 \cdot 10^4$	$3.95 \cdot 10^4$
$\sigma_{h\gamma}$ (fb)	$8.96 \cdot 10^{-2}$	$3.18 \cdot 10^{-2}$	$8.96 \cdot 10^{-2}$
$\mathcal{N}_{h\gamma}$	896	82	448



	CLIC			ILC	
$\sqrt{s}$ (GeV)	350	1400	3000	250	500
$\mathcal{L}_{\text{int.}}$ ( $\text{fb}^{-1}$ )	$5.0 \cdot 10^2$	$1.5 \cdot 10^3$	$2.0 \cdot 10^3$	$2.0 \cdot 10^3$	$4.0 \cdot 10^3$
$\sigma_{hZ}$ (fb)	130	6.42	1.37	240	57.2
$\mathcal{N}_{hZ}$	$6.50 \cdot 10^4$	$9.6 \cdot 10^3$	$2.74 \cdot 10^3$	$4.80 \cdot 10^5$	$2.29 \cdot 10^5$
$\sigma_{\nu\bar{\nu}h}$ (fb)	54.4	293	498	55.0	85.2
$\mathcal{N}_{\nu\bar{\nu}h}$	$2.73 \cdot 10^4$	$4.39 \cdot 10^5$	$9.96 \cdot 10^5$	$1.10 \cdot 10^5$	$3.41 \cdot 10^5$
$\sigma_{eeh}$ (fb)	7.13	28.3	49.1	8.2	8.7
$\mathcal{N}_{eeh}$	$3.56 \cdot 10^3$	$4.24 \cdot 10^4$	$9.82 \cdot 10^4$	$1.64 \cdot 10^4$	$3.48 \cdot 10^4$
$\sigma_{t\bar{t}h}$ (fb)	-	1.33	0.41	-	0.27
$\mathcal{N}_{t\bar{t}h}$	-	1995	820	-	$1.08 \cdot 10^3$
$\sigma_{h\gamma}$ (fb)	$3.18 \cdot 10^{-2}$	$1.20 \cdot 10^{-2}$	$3.08 \cdot 10^{-3}$	$8.97 \cdot 10^{-2}$	$4.74 \cdot 10^{-2}$
$\mathcal{N}_{h\gamma}$	16	18	6	179	189



# Fit methodology

- ✓ Extract  $y_t$  from a  $\chi^2$  fit (assuming this is the only parameter to be fit; more later)

$$\chi^2(\Delta y_t) = \sum_{i=1}^{N_p} \sum_{j=1}^{N_d} \frac{[\mu_{ij}(\Delta y_t) - 1]^2}{\delta_{ij}^2}$$

Sums over these pairs of channels

Collider	$\sqrt{s}$ (GeV)	$\mathcal{L}$ ( $\text{fb}^{-1}$ )	$h \rightarrow gg$		$h \rightarrow \gamma\gamma$		$h \rightarrow b\bar{b}$	
			$hZ$	$\nu\bar{\nu}h$	$hZ$	$\nu\bar{\nu}h$	$h\gamma$	$t\bar{t}h$
FCC- $ee$	240	$1.0 \cdot 10^4$	1.4%	-	3.0%	-	4.4%	-
	350	$2.6 \cdot 10^3$	3.1%	4.7%	14%	21%	14%	-
CEPC	240	$5.0 \cdot 10^3$	1.2%	-	9.0%	-	6.2%	-
	350	$5.0 \cdot 10^2$	6.1%	10%	-	-	-	-
CLIC	1400	$1.5 \cdot 10^3$	-	5.0%	-	15%	-	8.0%
	3000	$2.0 \cdot 10^3$	-	4.3%	-	10%	-	12.5%
ILC	250	$2.0 \cdot 10^3$	2.5%	-	12%	-	10%	-
	500	$4.0 \cdot 10^3$	3.9%	1.4%	12%	6.7%	9.8%	9.9%

where:  $\mu_{ij} = \left( \frac{\sigma_i}{\sigma_i^{\text{SM}}} \right) \left( \frac{\Gamma_j}{\Gamma_j^{\text{SM}}} \right) \left( \frac{\Gamma_h}{\Gamma_h^{\text{SM}}} \right)^{-1}$

- ✓ SM above means, basically, that  $\Delta y_t = 0$
- ✓ One-sigma uncertainties  $\delta_{ij}$  are taken from the literature
  - ✓ An exception is  $e^+e^- \rightarrow h\gamma$  which is estimated by us based purely on the expected number of events (see previous slide). Likely to be optimistic

8

# Signal-strengths

- ✓ Here are the needed signal-strengths

$$\mu_{h\gamma} = \begin{pmatrix} \sqrt{s} = 240 \text{ GeV} \\ \sqrt{s} = 250 \text{ GeV} \\ \sqrt{s} = 350 \text{ GeV} \\ \sqrt{s} = 500 \text{ GeV} \end{pmatrix} = \frac{\sigma_{h\gamma}}{\sigma_{h\gamma}^{\text{SM}}} = 1 - \begin{pmatrix} 0.43 \\ 0.45 \\ 0.73 \\ 0.13 \end{pmatrix} \Delta y_t$$

$$\mu_{t\bar{t}h} = \begin{pmatrix} \sqrt{s} = 500 \text{ GeV} \\ \sqrt{s} = 1400 \text{ GeV} \\ \sqrt{s} = 3000 \text{ GeV} \end{pmatrix} = \frac{\sigma_{t\bar{t}h}}{\sigma_{t\bar{t}h}^{\text{SM}}} = 1 + \begin{pmatrix} 1.99 \\ 1.83 \\ 1.71 \end{pmatrix} \Delta y_t,$$

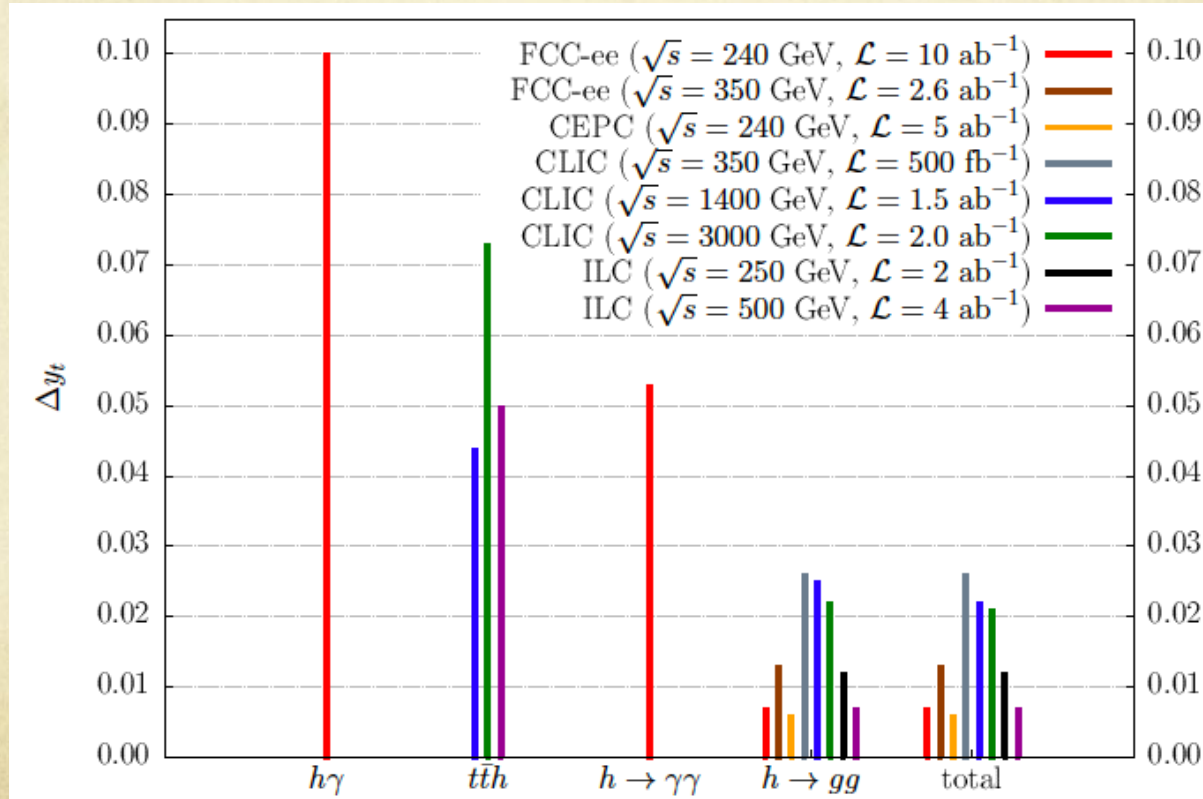
$$\mu_{h \rightarrow gg} = \frac{\Gamma_{h \rightarrow gg}}{\Gamma_{h \rightarrow gg}^{\text{SM}}} = 1 + 2\Delta y_t,$$

$$\mu_{h \rightarrow \gamma\gamma} = \frac{\Gamma_{h \rightarrow \gamma\gamma}}{\Gamma_{h \rightarrow \gamma\gamma}^{\text{SM}}} = 1 - 0.56\Delta y_t$$

- ✓ Derived by us at LO (full one loop):
  - ✓ Compute x-sections and decay widths for a number of values of  $\Delta y_t$ ,
  - ✓ Fit this with a quadratic polynomial,
  - ✓ Take the linear approximation for small  $\Delta y_t$ .
  - ✓ Bottom contribution to  $h \rightarrow g g$  neglected.
- ✓ Higher-order corrections in some cases have been included in the literature (CLIC 1.4 TeV).  
Abramowicz et al., arXiv:1608.07538
  - ✓ Slightly increases the expected precision

# Results: top-Yukawa precision prospects

Collider	$\sqrt{s}$ (GeV)	$\mathcal{L}$ ( $\text{fb}^{-1}$ )	$h \rightarrow gg$	$h \rightarrow \gamma\gamma$	$h\gamma$	$t\bar{t}h$	total
FCC- <i>ee</i>	240	$1.0 \cdot 10^4$	0.7%	5.3%	10%	-	0.7%
	350	$2.6 \cdot 10^3$	1.3%	21%	19%	-	1.3%
CEPC	240	$5.0 \cdot 10^3$	0.6%	16%	14%	-	0.6%
CLIC	350	$5.0 \cdot 10^2$	2.6%	-	-	-	2.6%
	1400	$1.5 \cdot 10^3$	2.5%	27%	-	4.4%	2.2%
	3000	$2.0 \cdot 10^3$	2.2%	18%	-	7.3%	2.1%
ILC	250	$2.0 \cdot 10^3$	1.2%	21%	23%	-	1.2%
	500	$4.0 \cdot 10^3$	0.7%	10%	75%	5.0%	0.7%



## Top-Yukawa precision prospects: few comments

- ✓  $h \rightarrow g g$  leads, by far, among all loop-induced processes
- ✓ This process offers potential  $y_t$  precision of about 0.6-0.7% at
  - ✓ 240 GeV CEPC and FCC-ee
  - ✓ 500 GeV ILC
- ✓  $h \rightarrow g g$  is better than  $t\bar{t}h$  for all energies and colliders by a factor of at least 2 (CLIC) and 2 to up to 7 (ILC)
- ✓  $e^+e^- \rightarrow h\gamma$  allows 10% determination. It is not great, but is comparable to HL-LHC
- ✓  $h \rightarrow \gamma\gamma$  allows about 5-6% precision at FCC-ee 240 GeV
- ✓ CLIC can measure  $y_t$  with precision of 2-2.5% (combining loop-induced and  $t\bar{t}h$ )
- ✓ ILC can measure  $y_t$  with precision of 1% or even better (combining loop-induced and  $t\bar{t}h$ )

11

## Limitations, assumptions and possible improvements

- ✓  $e^+e^- \rightarrow h\gamma$ : no detector simulation, efficiencies or background estimates. All done at LO.
- ✓  $m_{\text{top}}$ : we assume perfect knowledge of the top mass. This is OK since already after HL-LHC this error will be negligible
- ✓ Lack of proper EFT treatment:
  - ✓ We assume  $\Delta y_t$  is the only source of deviation from SM and so is the only parameter to fit
  - ✓ However, assuming BSM, no reason to have just one source of deviation from SM
  - ✓ Multiple Wilson coefficients will enter. This will dilute the expected precision on  $y_t$ .
  - ✓ However, after HL-LHC there will be many constraints on those coefficients.
- ✓ Assumed perfect knowledge of SM predictions.
  - ✓ In reality all is at LO (although fully one loop effects included)
  - ✓ NLO effects can be computed with some effort (2-loop amplitudes)
  - ✓ Realistic cuts imposed, etc.
- ✓ All of the above need to be done but we do not expect to change the picture qualitatively!

## Conclusions

- This work tries to address the question: is it really not possible to measure  $y_t$  at a future  $e^+e^-$  collider with precision better than 4-5%?
- Such a prospect would be disheartening given we expect 5%-10% from HL-LHC and 1% from a 100 TeV hadron collider
- This is an exploratory work. Its precision level is basic; still, we believe it is adequate in order to get a global picture about what is the ultimate possibility for measuring  $y_t$  at any one of the future  $e^+e^-$  colliders. Much more refined studies have already been done

Durieux, Gu, Vryonidou, Zhang '18

Robson, Roloff '18

Durieux, Irles, Miralles, Peñuelas, Pöschl, Perelló, Vos '19  
de Blas, Durieux, Grojean, Gu, Paul '19

- We consider *indirect* determination from loop-induced single Higgs processes
- Our findings are very promising. We find  $y_t$  can be measured with precision as high as 0.6%
- This is almost an order of magnitude better than from purely  $t\bar{t}h$  final states and 10 times better than the extraction from  $t\bar{t}$  discussed in the Fcc-ee Conceptual Design Report (2018)
- Such precision measurements can be done at any future  $e^+e^-$  colliders, especially at 240 GeV runs with  $hZ$  final states.
- Our work is very preliminary and can be made more precise in a number of ways
- We hope it provides useful input to the current discussion about which collider to build!

13